Research in Mathematics Education
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Kakoma Luneta *Editor*

# Mathematics Teaching and Professional Learning in sub-Sahara Africa

Springer

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Editor

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# Foreword

When I was invited to attend the 2020 meeting of the South African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), I was fascinated by the opportunity to visit for the first time in my life a country in sub-Saharan Africa, regarded by geneticists as the birthplace of the human race. I did not anticipate the exciting experience I would have by meeting so many members of the thriving community of mathematics education researchers and teachers who attended SAARMSTE, some of whom have contributed to this volume on *Mathematics Teaching and Professional Learning in Sub-Sahara Africa*. I had done a bit of homework and read two reviews of papers published by South African researchers on mathematics education. The reviews identified multilingual classrooms as a focus in research and I noted that this was also present in the conference programme. I searched the programme for words such as *culture* and *indigenous* and knew that I would be finding out about new ideas in research on teaching and learning. I tried to learn to count in isiXhosa, one of the languages mentioned in the papers, and diagnosed myself as having learning difficulties in mathematics. I spent a bit more than a week in South Africa, and the level of energy and enthusiasm for research, teaching and learning I experienced meeting researchers and teachers made me wish I were 20 years younger and could work alongside them. Their creativity in engaging with different communities and their commitment to drawing on the indigenous cultural resources were palpable. To my great relief, nobody discovered my lack of number sense in isiXhosa.

Reading this volume brought back to me the excitement of being amongst researchers and teachers seeking to balance the *international* with the *indigenous*, a balancing act that was at the root of my own thinking when, with my colleagues Analucia Schliemann and David Carraher, I started to study street mathematics. But when I compare my thinking at that time with the thinking recorded in this volume, there is no doubt that the researchers who produced this volume are hundreds of miles ahead of where I was. They are completely aware of the apparent contradictions between mathematics as an abstract discipline that is international and its cultural nature, which must be reconciled with the local needs, resources and modes of thinking in order to become school mathematics in any particular community. The results of a quick search of terms in this volume tell the beginning of the story in this book. The stem *cultur** appears 230 times, the stem *communit** 147 times, the word *diversity* 76 times, the word *ethnomathematics* 65 times and *indigenous* 18 times. The clear focus on culture does not lead to an opposition between local and international or between concrete and abstract: the word *model* appears 105 times and technology 97 times.

The tensions that arise when one tries to connect mathematics and everyday knowledge are explicitly acknowledged, and they are not seen as leading to a dead end but demanding innovative approaches to teaching mathematics in sub-Saharan classrooms. The different chapters cover a diversity of ideas. For example, it is suggested that cultural artefacts can be explored in the mathematics classroom to transform teaching that is currently based on telling into teaching based on reflection and exploration. Manipulatives can be used not just as concrete representations, but also as a springboard to visualization, which entails encouraging students to use mental images that are necessarily abstract, although they represent specific characteristics of that which is represented. Problem-based learning is recommended as an approach that uses simulated but realistic problems that allow for collaborative learning of abstract mathematical tools, such as algebra.

Besides the focus on teaching and learning, this volume also offers contributions relevant to teacher education and policy. School systems are only as good as the teachers that constitute the system; this is explicitly recognized through the inclusion of chapters that consider the challenges of upgrading mathematics teachers' knowledge, of building communities of teachers who can continue to improve their own practice and of promoting teachers' professional growth through continued professional development. It is suggested that the power of teachers' reflections can be harnessed through engagement with their narratives about their own personal histories and about their classroom practices. These chapters show that the meaning of the word *community* in this volume is not its restricted sense, to refer to a group of people living in a particular local area, but its wider sense, to refer to a group of people who consider themselves a community because they share the same activity. The teachers' community of practice goes beyond the local context to embrace the professional education themes that are common to all of us who teach, such as the gap between research and the curriculum and the distance between the intended and the enacted curriculum, which are topics tackled in the volume.

Throughout the volume one can find numerous references to research reported in unpublished master's and doctoral theses. These references enrich the volume because the sources would not be easily found by the international readers and reveal the struggle to publish in developing countries. A review of publications included in this volume indicates that the African journals have offered a significant contribution to maintaining the African community of teachers and researchers in contact and are likely to continue to do so in the future. I personally believe that it is so very important for educational communities outside Europe and North America to have their own journals so that they can deal explicitly with the tension between the international and the indigenous that I have described. Each country and region faces its particular challenges; even though researchers and teachers may be inspired by theories and solutions developed elsewhere, the process of appropriation and assimilation to one's own country or region is a valuable work that needs a forum for discussion.

I strongly recommend this volume to everyone interested in mathematics education research and practice. My expressed admiration for this volume is not reduced by the suggestion with which I end this foreword. As I identify strongly with the aims and ambitions that are likely to be the motivation for this volume, I believe that it is only natural that I would seek in the lists of references used in the various chapters the commonalities between their sources and the sources of my own thinking. Of course, I found that many of the authors who have inspired me, such as D'Ambrosio, Freire, Lave and Duval, were prominent in the lists of references. But I was puzzled about the lack of reference to the original works of scholars who have been a source of inspiration to me, such as Brousseau, (Michael) Cole, Freudenthal (one reference), Hoyles and Noss (one reference), Piaget, Thompson and Vergnaud. Schemas of action, cultural psychology, quantitative reasoning, conceptual field, the pay-off of making invisible mathematics visible, didactical contract and realistic mathematics education are expressions that represent ideas that I find incredibly valuable. I end this foreword by emphatically recommending these authors and their ideas to researchers in mathematics education in the sub-Saharan region.

University of Oxford Terezinha Nunes
Oxford, UK
February 2021

# Contents

# About the Editor

**Kakoma Luneta** is an Associate Professor of Mathematics Education in the Faculty of Education at the University of Johannesburg, South Africa, where he has been a faculty member since January 2005. Dr Luneta holds a PhD in Mathematics Teacher Education from the University of Witwatersrand and a master's degree in mathematics teacher education from the University of Sussex. His research interest is in mathematics education; mathematics/numeric cognition and professional development and mentorship of mathematics teachers. He has supervised over 15 PhD students and over 20 master's students to completion and has written 4 books and published several articles in the field of mathematics education.

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**Jef Peeraer, PhD** has developed an expertise in educational innovation and change in general and teacher education and continuing professional development in particular, with more than a decade of experience in the design, implementation, monitoring and evaluation of education projects in various settings in Europe, Asia and Africa. Jef Peeraer is a Sociologist by training (MA) and holds a PhD in Education Sciences. He is an education practitioner with extended training experience in multi-cultural settings. Through policy dialogue and sector coordination, he has supported the education sector strategic planning and policy development. He managed the implementation of both small-scale and country-wide initiatives in Vietnam, Uganda and Rwanda. Currently, Jef Peeraer is the Programme Manager of VVOB—Education for Development in Rwanda, where he manages the implementation of three projects in Basic Education.

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# Part I
# Commentary

**Gibbs Y. Kanyongo**

The publication of this book *Mathematics Teaching and Professional Learning in sub-Sahara Africa* is timely considering the recognition by most African countries of the need to boost performance in mathematics. The importance of mathematics education is a well-known fact, and mathematics education goes beyond merely improving proficiency in test scores. Building a solid mathematics foundation helps students develop critical thinking skills, to help them solve real-life problems. With the emphasis on STEM by most countries, it is important that the curriculum of mathematics education is relevant to the twenty-first century, and teachers should be equipped with the requisite skills to allow them to effectively deliver mathematics instruction to meet the twenty-first century demands. Therefore, mathematics can no longer be taught using old methods which emphasize memorization and studying for the test. This part explores various innovative strategies to teach mathematics in sub-Sahara Africa.

In Part I, the authors focused on the various instructional strategies which are informed by indigenous mathematics teaching artefacts in sub-Sahara Africa. The first chapter by Marc Shafer presents an argument for the use of a *Geoboard* as both a visualization and a teaching tool. He presents a strong argument on how the use of visualization supports conceptual approach to teaching mathematics, and he expertly looked at the connections between manipulatives (both physical and virtual) and visualization processes and discusses how this connection between the two can be used for a visual approach to conceptual teaching of mathematics.

The author looks at a case study research project in Namibia, which used a *Geoboard* as a physical manipulative to teach the properties of quadrilaterals. Not only was the use of the *Geoboard* significant for the purpose of conceptual teaching, it was also significant considering the rural context of the school in which

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instructional resources are scarce. His findings showed that manipulative enabled teachers to quickly and effectively illustrate and facilitate explorations into the properties of quadrilaterals, thus adopting a visual approach to teaching mathematics.

The second chapter, *The Role of Number Sense on the Performance of Grade Learners 12 in Mathematics: A Case of Oshana Education Region, Namibia,* utilized both qualitative and quantitative methods to look at the factors that influence the development of number sense and its influence on mathematical performance among Grade 12 learners. The study provided valuable insight on the development of number sense and its influence in mathematics achievement, as well as some of the obstacles to teaching and learning mathematics in Namibian schools. Number sense is an important mathematical concept which involves the understanding of numbers and their magnitude, and this forms the foundation for understanding mathematical operations.

The authors look at the relationship between number sense and academic performance in mathematics, and they found a strong correlation between the two. However, I consider this to be a first step to understanding the nature of the relationship between number sense and mathematics achievement; correlation on its own does not imply causation, so the next step to understanding this relationship should involve conducting causal studies to establish a causal relationship between number sense and academic performance in mathematics.

In the third chapter *Using a Community of Practice Strategy to Strengthen Teaching and Learning of Mathematics in Rural Areas*, Benita Nel used Wenger's social theory of learning to look at the effects of professional isolation of mathematics teachers in rural schools in South Africa and the use of community of practice strategy to address the challenges associated with professional isolation. The study utilized qualitative data of teachers' views of how the rural school community PD programme assisted in (or hindered) breaking the cycle of their professional isolation. The establishment of community of practice in rural areas is an effective way to connect teachers with each other at a professional level and encourage collaboration, networking and even mentorship to support each other. The author acknowledges the small sample size of the study which limits the generalization of the findings. This, however, should not discount the findings as they can be used as a first step in further studies on community of practices in rural areas. Future research should build on these findings to look at the issue of community of practice and professional isolation using large-scale studies.

The fourth chapter *Exploring a Framework for Discourse-Based Mathematics Instruction in Secondary Schools in an Ethiopian Context* by Mekonnen Legesse and Tadele Ejigu provides a model for mathematical discourse-based teaching and learning of mathematics in secondary schools in Ethiopia. The chapter was guided by the methodological tool, The Teaching for Robust Understanding (TRU) in Mathematics framework as a framework for mathematical discourse-based instruction. The authors provided the Mathematics Discourse-Based Instruction, which consists of five instructional components: *Planning for instruction*, *Designing learning tasks, Doing mathematics, Discussing: Engaging in the classroom discourse, and Reflecting.*

The effectiveness of this model in teaching mathematics was not presented in this chapter. However, the authors reported that findings of the evaluation of this model are reported in Legesse et al. (2020). Analyzing the effects of mathematical discourse-based instruction on eleventh-grade students' procedural and conceptual understanding of probability and statistics. *Studies in Educational Evaluation 67 (2020) 100918*, and that the model has been found to be effective in improving students' mathematics performance.

The chapters *Ethnomathematics as a Fundamental Teaching Approach* by France Machaba and Joseph Dhlamini and *Use of Cultural Artefacts in the Teaching of Mathematics in Africa: The Case of Uganda* by Janet Kaahwa are somewhat related as they both look at how indigenous culture, cultural artefacts and surrounding practices relate to mathematics. These are both important articles because the concept of ethnomathematics is quite relevant in sub-Saharan African, in which culture in an integral part of people's way of life, just like any other society. However, in most sub-Saharan African countries, there is a strong connection between the people and surrounding practices and indigenous culture. Therefore, pedagogical practices in mathematics should take advantage of these strong connections since students can easily relate to the surrounding practices and indigenous culture. Further, limited instructional resources in most schools in sub-Sahara Africa require teachers to improvise with reading available resources from their surroundings.

The chapter *Mathematics Teacher Educators' Experiences of Using Technology-Based Instruction in South Africa* by Jayaluxmi Naidoo looks at the use of technology-based mathematics instruction in inspiring social change and encouraging teacher success to accommodate diversity within higher education milieus. This is an important and timely chapter due to the rapid increase in diversity among educational institutions; it means lecturers should devise instructional strategies to accommodate the diversity among their learners in terms of races, gender, socio-economic status and linguistic backgrounds. On issues of diversity, the author argues that lecturers should act as agents of change when it comes to ensuring equity and addressing injustices in society. The author notes that lecturers need to possess technology-based instructional strategies and technological pedagogical content knowledge to be effective as agents of social change. The author further presents an argument on how lecturers can create an inclusive and enriching context that would encourage teacher success and support mathematics instruction.

The chapter *Enhancing Learners' Retention of Algebraic Knowledge Through Problem-Solving-Based Learning* by Tuhafeni I M Kaufilua and Helena Miranda, utilizes quantitative research approach, with a non-equivalent comparison-group quasi-experiment research design to look at the effects of problem-based learning approach on senior secondary school learners' retention of algebraic knowledge and skills in Namibia. The authors did not find significant difference in mean retention scores of learners taught algebra using Problem-Based Learning (PBL) and those taught using a traditional instruction approach when students were assessed immediately after instruction took place. After 3 weeks have passed, results showed that there was a significant difference in retention of algebraic knowledge between learners taught algebra using Problem-Based Learning (PBL) and those taught using a traditional instruction approach, in which those in the PBL group scored significantly higher. These findings have important potential policy implications, as suggested by the authors that professional development workshops on instructional approaches in PBL be held in schools to train teachers on these pedagogical approaches.

The final chapter in this part, *Jacks' Story: How Storytelling Enhances Mathematics Instruction in Lesotho* by Ajayagosh Narayanan, looks at how storytelling enhances mathematics instruction in Lesotho using an ethnographic approach through classroom observations, one-to-one interviews and focus group interviews. This is an important chapter as it provides information on how a teacher's professional identity helps them effective at delivering mathematics instruction in the classrooms. Allowing teachers to explore their identity and have them connect this identity and experience to their classroom curriculum can be a powerful tool in instructional delivery.

# Chapter 1
# Manipulatives as Mediums for Visualisation Processes in the Teaching of Mathematics

**Marc Schäfer**

## Introduction

The purpose of this chapter is twofold. First, it argues for a visualisation approach to teaching, using the well-known *Geoboard* as both a teaching and a visualisation tool. Second, it narrates a professional teacher development programme that formed the heart of a research project that interrogated how a visual approach to teaching can be achieved.

This chapter begins by discussing notions of visualisation in mathematics and how the use of visualisation can support a conceptual approach to teaching. I will draw from literature and discuss some of the implications that one can infer from these readings. This discussion will specifically look at the interplay between manipulatives and visualisation processes and how this can be harnessed for a visual approach to conceptual teaching. The chapter will then specifically look at how the use of manipulatives can frame a visual approach to teaching. I will draw from one particular Namibian case study research project, which used a physical manipulative (the *Geoboard* in this case) as a means to teach the properties of quadrilaterals. Research in visualisation processes in the teaching of mathematics is once again gaining traction as we continue to search for examples of best classroom practice.

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## Backdrop

My observations of the many mathematics classrooms that I have visited in my long career as a mathematics teacher educator reveal that mathematics classrooms sadly often do not exude the desired inspiration and positive energy expected from an exciting learning environment. They are often drab, naked and devoid of any mathematical curiosities or interesting illustrations that relate to the beauty and intrigue of mathematics. Of course it is more difficult to adorn the walls of a mathematics classroom than a geography classroom, with, for example, interesting maps or pictures of beautiful landscapes or fascinating and obscure geomorphological features. This chapter argues that the inherent visual nature of mathematics is all too often neglected and should be revisited. Textbooks all over the world are often the only source of visual information that is at the disposal of the teacher. As Nghifimule (2016) from Namibia suggests, in many developing countries, where there is often a shortage of reading materials and a lack of access to these materials, the textbook becomes the only resource text that teachers can draw from. This implies then that the visual materials that teachers have access to are sourced mainly from textbooks. There are, however, a myriad of other opportunities to access and manufacture visualisation materials to facilitate a learning process that is genuinely conceptual and interesting. By their very nature, textbooks seem to avoid the use of visuals for concepts that are abstract in nature and not immediately visually accessible. In her analysis of the nature of visualisation objects in the algebra and geometry chapters of three Namibian mathematics textbooks, Nghifimule (2016) found that on average, 75% of these objects used pertained to geometry, a domain in mathematics that, it can be argued, is inherently more visual than algebra in any case. It is, however, this imbalance of illustrating and visually representing mathematical concepts that reinforces only a very selective use of visualisation objects in the teaching of mathematics.

## Visualisation Processes in the Teaching of Mathematics

Many definitions and perceptions of visualisation abound in the literature. Often when writing and theorising about visualisation, many researchers typically define visualisation in mathematics only in the context of producing and using imagery, both physical (external) and mental (internal). For example, visualisation can be seen in the context of producing and using diagrams, graphs and figures in a mathematics environment, on the one hand, and taking a broader perspective and view of visualisation as an intricate construct that involves both product and process, on the other. For example, Arcavi (2003) quite eloquently proposes that visualisation is "the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (p. 217). Despite the emphasis on process, this definition, however, is still imagery bound. This chapter argues that visualisation processes can also apply to processes that are embodied with our actions and our ways of communicating, such as gestures and language. Either way, using visualisation processes is cognitively desirable. It stimulates learners' minds to identify patterns and trends, make connections to real-life situations and make abstract ideas more accessible and imaginable. With regard to the latter, Rudziewicz et al. (2017) aptly suggest that visualisation processes enable learners to "see the unseen". This is particularly important when engaging in mathematical concepts and ideas, such as irrational numbers, that are obscure and not as tangible as those that are imminently visible and touchable, such as geometric shapes. It is asserted that being able to "see the unseen" is a sign of deeper understanding of a particular concept (Natsheh & Karsenty, 2014). With regard to using visualisation processes, Mudaly (2010, p. 65) observes that "viewed pictures often create clearer images in our minds because of the symbols attached to what we see, accompanied by other sensory perceptions".

Duval (2014) argues that making use of visualisation processes is a cognitive process that is underscored by reasoning, whereby learners think through what they see and hear (including words) in order to make sense of the mathematical idea under consideration. This chapter thus argues that it is a process integral to any form of mathematising. It therefore goes without saying that visualisation facilitates a process in learners to develop a deeper and richer mathematical understanding than they otherwise might have developed from other learning processes. As Figueiras and Arcavi (2014) so appropriately assert that in the context of teaching, if applied meaningfully, visualisation can "visually evoke real-life experiences" that form the basis of contextualising mathematics in the learner's own environment. Visualisation can thus be a process of harmonising what learners see in their environment to what they think or imagine in their minds (Nemirovsky & Noble, 1997).

Notions of the interplay between visualisation and mathematics are of course nothing new in the context of theorising and researching strategies of teaching and learning. Cobb et al. (1992) referred to the connection between mathematics in learners' minds (i.e. their imaginations) and the mathematics in their environment as a form of dualism. Zazkis et al. (1996) also expounded on the concept of visualisation as a link between what learners think and what they see by stating that:

> [a]n act of visualization [that is, the process of visualization] may consist of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard, or computer screen, of objects or events which the individual identifies with object(s) or process(es) in her or his mind. (p. 441)

From a teachers perspective, the pedagogy of visualisation has interesting and significant implications. The plethora of different curriculum statements all over the world refer to the use of visualisation processes in different ways. The National Curriculum for Basic Education of Namibia, for example, aligns visual skills with communication and graphicacy:

> A high level of communication skills, more than just functional literacy, numeracy and graphicacy, is essential in a knowledge-based society..... Visual communication plays an increasingly important role in a knowledge-based society, and learners need to develop good visual communication skills in understanding, investigating, interpreting, critically analysing, evaluating, and using a wide range of visual media and other sources of aural and visual messages. (Namibia. MoE, 2010).

Similarly, the National Curriculum Statement Grades R – 12 in South Africa aims to produce learners that are able to:

...communicate effectively using visual, symbolic and/or language skills in various modes. (RSA. DoBE, 2011).

Incorporating a visual pedagogy thus seems appropriate and indeed desirable if the classroom is to be a space where learners are indeed offered an opportunity to learn and hone the skills that the curriculum statements above are calling for. It is thus incumbent on the mathematics teacher to create learning and teaching environments where such skills can be developed.

One way of embedding a visual approach to teaching is through the use of manipulatives.

## Manipulatives

Manipulatives are objects, physical or virtual, that the user can consciously or unconsciously handle and transform to meet certain objectives. It is in the manipulation of these objects that learning and teaching outcomes can be achieved. For manipulatives to foster meaningful learning or teaching, they should be objects that have a visual appeal that inspire curiosity and an interest to explore and experiment. Manipulatives do not only describe and illustrate a mathematical idea, but they can also develop mathematical concepts and can be used as mathematising devices.

Manipulatives have been used as learning and teaching aids in many classroom and home environments over the centuries. Instinctively, young babies soon after birth clutch and manipulate objects as they maneuvre them close to their mouths in order to suckle them – the first tentative learning steps in perceiving their environment, on the one hand, and satisfying their sucking instinct, on the other. Teaching manipulatives or teaching aids are featured in all corners of the world as devices that enable the teacher to conceptually illustrate or model mathematical ideas and concepts. We should all be familiar with the Cuisenaire rods for configuring numerical proportions in the early-grade mathematics classroom, for example.

But the mystery remains as to why then these manipulatives or teaching aids remain a relatively rare sight in my classroom observations.

It goes without saying that manipulatives are inherently visual, whether these manipulatives are tactile, as in physical manipulatives, or digital as in virtual manipulatives. In both cases the visuality of the manipulative is key for the conceptual development of a mathematical idea.

In his research Dzambara (2012) engaged with 75 mathematics teachers from 25 secondary schools in Windhoek, the capital city of Namibia. The purpose of his research study was to audit the availability and use of mathematics teaching aids in secondary schools in that city. He analysed the distribution and use of 12 different types of teaching aids ranging from different chalkboard instruments, charts and posters to physical objects such as geometric models. In broad brushstrokes Dzambara (2012) found that whilst some teaching aids such as charts and posters, chalkboard 30 and 60 degree set squares, chalkboard rulers, protractors and compasses, mathematical sets for learners and improvised teaching aids were used regularly (i.e. on a daily basis), other teaching aids were only used moderately (i.e. used as frequently as possible) to never used. These included physical manipulatives such as geometric models and computers. Devices such as graph boards, interactive white boards and, surprisingly, the *Geoboard* were used by only a handful of teachers. On average, across all the teaching aids, Dzambara (2012) found that 11% of teaching aids were used on a daily basis, 40% were used as frequently as possible, and 49% were never used. More accurately, the chalkboard 30 and 60 degree set squares were used daily by 25 teachers, as frequently as possible by 39 teachers and never by 11 teachers. Charts and posters were used daily by 12 teachers, as frequently as possible by 51 teachers and never used by 12 teachers. Physical objects other than geometric models were used daily by 5 teachers, as frequently as possible by 27 teachers and never used by 37 teachers. Geometric models/shapes were used on a daily basis by 2 teachers, as frequently as possible by 29 teachers and never used by 44 teachers. It is interesting to note that 47% of the teaching aids were school purchases, 35% accounted for personal purchases and only 11% of the teaching aids were bought directly by the Ministry of Education. The rest were donations or gifts. From Dzambara's (2012) work, it is apparent that quite overwhelmingly, teaching aids are not used in in Namibia's most urban area. This despite the fact that 96% of his participating teachers agree that the use of teaching aids in mathematics classes promotes learners' participation and interest in mathematics. This is corroborated by the work of Suydam and Higgins (1977) on the use of manipulatives, suggesting that "lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do non-manipulative lessons" (p.83). This however presumably assumes that the use of the relevant manipulative was appropriate, well planned and aligned with the learning and teaching objectives. An unmediated manipulative operating in a pedagogical vacuum has only limited value. It would be interesting to extend Dzambara's work to the rural areas of Namibia where access to resources and infrastructure is compromised and limited and where the poverty gap is most pronounced. Kraft (2014) quotes the Namibia Statistic Agency that "people in rural areas are twice as likely to be poor compared to those in urban areas with about 37.4 percent of people living in rural areas being poor compared to 14.6 percent in urban areas". This would indicate that the purchase and acquisition of teaching aids may not be seen as a priority and thus be neglected.

## The Interplay between Manipulatives and Visualisation

Due to the tactile nature of manipulatives, the link between using them and visualisation seems self-evident. Manipulatives can provide the entry point to visualising a particular mathematical concept, particularly if the concept is at first obscure and a little nebulous. Take the well-known problem of calculating the shortest distance for a spider to crawl from the top corner of a room to the diagonally opposite corner at the bottom of the room. The use of a physical model of a rectangular prism may enable the problem solver to visualise the problem quite easily as opposed to sketching this three-dimensional scenario on a plane piece of paper first. The strength of manipulatives lies in their potential to model real-life situations and thus provide for a visual representation of a particular problem-solving situation. Essentially there are two different types of manipulatives. These are physical and virtual manipulatives.

## Physical Manipulatives

Physical manipulatives, also sometimes referred to as concrete objects or physical models, are objects or devices that are made of concrete materials such as wood, plastics, paper, clay and polystyrene. Drawings on paper would also fall under this category. Heddens (1986) makes the point that physical manipulatives from the learners' real world are strategically and typically used to visualise and represent mathematical ideas in a way that can clarify these ideas more simply and effectively than without them. It is thus unclear why more teachers do not make use of physical manipulatives in their teaching. Sets of physical manipulatives in many colours, shapes and sizes are available on the market. These however come at a financial cost, which often deters schools and teachers from purchasing them. As will be discussed below, substantial costs can be saved when physical manipulatives are self-made using materials that are readily available in the immediate surroundings of the school and the learners.

In their narrative about proficient teaching, Kilpatrick et al. (2001) posit that the use of physical manipulatives should be a key feature of teaching as they provide a means to link informal knowledge and intuition to mathematical abstraction; they can be used as mathematical representations to clarify ideas and support reasoning and build understanding; and they enhance and enrich conceptual understanding and inspire mathematical talk in the classroom.

The physical manipulative that is featured in this chapter is the *Geoboard*.

## Virtual Manipulatives

These manipulatives, also referred to as digital manipulatives, are virtual and non-tactile in nature as they occur on digital devices such as tablets, computers and smartphones. The extent of their manipulation potential and capacity depends on the complexity and comprehensiveness of the software that drives the manipulative. In computer-aided design (CAD) programmes, for example, it is possible to move a three-dimensional representation of, say, a cuboid, and view it from any perspective or angle one wishes. This capacity can of course be very powerful in enabling the user or consumer to visualise obscured sides and corners of shapes and thus enable him/her to see the unseen. The plethora of manipulation games available for children's use on their tablets can, however, at times be quite overwhelming. The quality and efficacy of these games vary greatly from game to game and require a critical eye to determine this quality. +The same applies to mathematical software. A useful indicator for quality is the extent of the versatility of the software and the degree to which the user is able to take control of the manipulation capacity to suit their needs.

## Case Study: The Geoboard

### *Introduction*

This interpretivist case study took place in the North Eastern Region of Kunene in Namibia. It was framed by an intervention programme that involved three selected Grade 7 teachers from three different schools.

The aim of the study was to investigate and analyse the use of *Geoboards* as visualisation tools to teach the properties of quadrilaterals to Grade 7 learners. The research questions that framed the study were as follows:

- What are the affordances of the utilisation of *Geoboards* as visualisation tools in the teaching of the properties of quadrilaterals in Grade 7 classes?
- What are selected teachers' experiences of using *Geoboards* as visualisation tools in teaching the properties of quadrilaterals, as a result of participating in an intervention programme?
- How do the participating teachers make use of the Van Hiele phases in their teaching of quadrilaterals using the *Geoboard*? (Matengu, 2018).

Due to limited space, the third research question will not be reported on in this chapter.

The case study was constructed within the context of an intervention programme that involved working with three purposefully selected teachers over a period of one term. The intervention programme involved regular meetings where *Geoboards* were designed and manufactured. Ideas about how these *Geobaords* could be put to good use were discussed and workshopped. Each teacher then planned a short teaching and learning programme which consisted of each teacher teaching three lessons which were video-recorded. These video-recordings were then analysed in conjunction with stimulus-recall and focus group interviews in order to answer the research questions above.

## The Geoboard

The *Geoboard* is a well-known manipulative that unfortunately has not found much traction in Namibia. Matengu (2018) cited Miranda and Adler (2010) who observed that in general, "Namibia is one of the many countries in which the use of manipulatives in mathematics classrooms is not common practice".

The *Geoboard* usually consists of a flat square piece of wood into which small nails are driven in a pattern of repeated squares. The nails then serve as little posts around which elastic bands can be placed to form various plane shapes. This manipulative is easy to manufacture and serves the exploration of shapes very well as it is quite versatile and transportable. It is particularly useful to explore area and perimeter of shapes such as quadrilaterals.

## The Intervention Programme

The intervention programme consisted of the researcher and the three teachers meeting on a regular basis. In the first instance, they all made a *Geoboard* themselves. A local carpenter was solicited to assist with the sourcing and cutting of the square base boards to size. A square lattice of points was then constructed on a piece of paper and placed on the square board. The points served as markers where the nails were then hammered in. See Fig. 1.1.

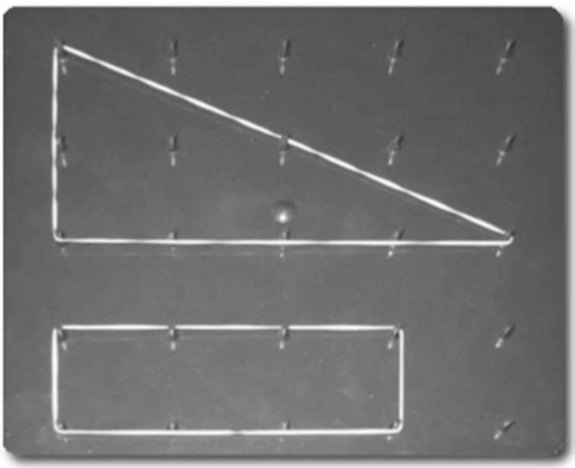

**Fig. 1.1** A typical *Geoboard* consisting of a 4 × 4 lattice of squares

The nails now serve as posts around which elastic bands can be placed to form plane geometry shapes, such as a right-angled triangle and rectangle as can be seen in Fig. 1.1.

The group then discussed and planned a total of nine lessons, i.e. three lessons per teacher, where the first lesson constituted a pilot lesson. Each lesson focused on the properties of different quadrilaterals.

All the nine lessons were video-recorded at times that suited the researcher and the teachers. Stimulus-recall interviews were then conducted to collaboratively analyse the lessons in order to critically reflect upon them. After this process of interviews, a focus group interview enabled the participating teachers to collectively deliberate about and reflect on their experiences.

## *Analysis of the Lessons*

In order to enable the researcher and the teachers to make sense of the lessons in terms of how the *Geoboard* was utilised as a visualisation tool, the analytical tool in Fig. 1.2 was used as a lens through which to observe each lesson.

The work of Presmeg (1986) and Duval (1995) were the main sources that inspired and informed this framework.

*Concrete pictorial imagery* (PI) refers to the formation of concrete images or representations. Teahen (2015) argues that concrete visual representations are important objects for learners to visualise mathematical operations. The *Geoboard* is an ideal device to form concrete visual representations of geometric shapes. The characteristics of these shapes can then be deliberated on.

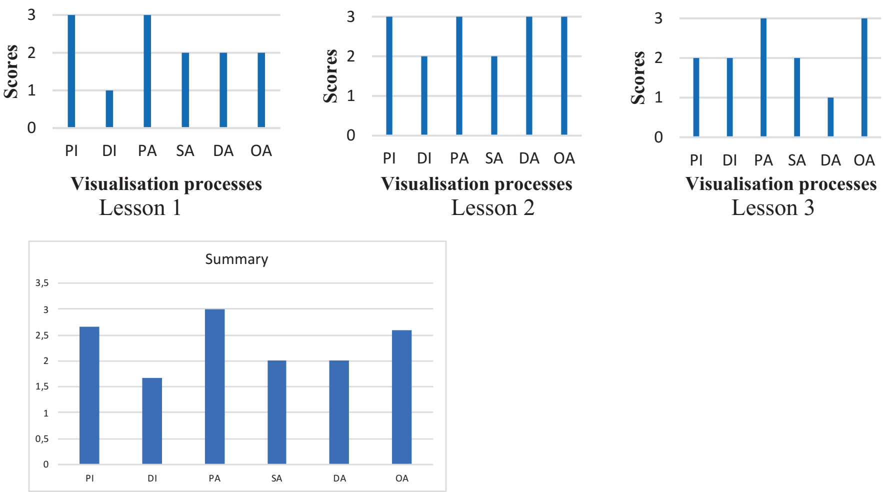


**Fig. 1.2** Bar graphs showing evidence of the visual imagery and apprehensions used in Mr. Jones' three lessons

*Dynamic imagery* (DI) refers to the dynamic nature of a visual representation. Typically in classroom settings, static images such as those in textbooks or posters on the wall are used. Dynamic images however are those that change and move and thus can capture the imagination in exciting and novel ways. Once again the *Geoboard* is an ideal instrument where the user can change the look of a shape in an instant by repositioning and moving the elastic bands to different positions, thus creating a new shape or altering an existing shape, thereby manipulating its original characteristics.

*Perceptual apprehension* (PA) is what an image evokes. Matengu (2018) argues that it is often the very first perception that an individual experiences when confronted with an image, such as a geometric shape on the *Geoboard.* For any image to have an impact on a learner, it must evoke perceptual apprehension (Duval, 1995). As Duval (1995) continues to suggest, "nothing is more convincing than what is seen" (p.12). It is however important that a learner needs to move beyond only perceptual apprehension. Samson and Schäfer (2011) suggest that one way of moving beyond perceptual apprehension is by seeing (or showing) an image in multiple ways. The *Geoboard* is well suited to this as a learner can construct a triangle, for example, in multiple ways, using different configurations of the pins and stretching the elastic bands in different ways.

*Sequential apprehension* (SA) is understanding the mathematical implications of the constructed shape or figure. Once again, the *Geoboard* lends itself well to exploring the mathematical implications of a constructed shape. In Fig. 1.1, taking a $1 \times 1$ square as one-unit square, for example, enables a learner to explore the area of the triangle by counting the number of squares and then conjecturing the relationship of the sides of the triangle with respect to this area.

*Discursive apprehension* (DA) is about articulating the mathematical ideas inherent in an image or representation. Duval (1995) makes the interesting observation that it is through speech and engagement, i.e. through a discursive process, that the mathematical properties of a shape start to make sense to a learner. Although this discursive process can be silent and personal, it goes beyond just a personal apprehension.

*Operative apprehension* (OA) involves the actual operation on a figure. According to Duval (1995) this can involve mental or physical manipulation depending on the nature of the figure or representation. Operative apprehension can also involve transforming the figure.

In the analysis of the three lessons per teacher, the researcher together with the teachers used the above framework to reflect on their respective lessons. This proved to be a very fruitful exercise to not only answer the research questions but also as a means to critically reflect on their practice. Each interview took the form of a stimulus-recall interview where the conversation between the researcher and the teacher was guided by the video recording of the specific lesson they were both watching. The interview questions were loosely framed by and aligned with the criteria in the analytical framework (see Table 1.1 below). The video recordings could be paused at any time to probe either further or deeper.

**Table 1.1** Analytical framework

| Type of visual imagery and apprehension criteria | Visualisation process indicators | Coding | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| Concrete pictorial imagery PI | There is evidence of the use of *geoboards* that encourages learners to form mental pictures of the properties of quadrilaterals. | | | | |
| Dynamic imagery DI | There is evidence that the teachers encouraged the manipulation of static *geoboard* figures to dynamic processes by changing the position(s) of rubber bands and number of pegs to transform shapes. | | | | |
| Perceptual apprehension PA | There is evidence that the teacher used the *geoboards* to assist learners to simply recognise basic shapes. These are not necessarily relevant to the constructions of quadrilaterals. | | | | |
| Sequential apprehension SA | There is evidence that the teachers facilitated the independent construction of shapes using the *geoboards*. Learners are encouraged to construct and describe shapes on their own. | | | | |
| Discursive apprehension DA | The teachers encouraged learners to verbally describe the properties of the constructed shapes. | | | | |
| Operative apprehension OA | The teacher sets problems for the learners to solve using the *geoboard.* | | | | |

Adopted from Presmeg (1986) and Duval (1995).

**Table 1.2** Coding descriptors

| Coding | Categories | Descriptions (visualisation process) |
|---|---|---|
| 0 | No evidence | There is no evidence of visualisation processes in the use of *geoboards* |
| 1 | Weak evidence | There is little evidence of visualisation processes in the use of *geoboards* (1–2 incidences) |
| 2 | Medium evidence | There is sufficient evidence of visualisation processes on the use of *geoboards* (2–3 incidences) |
| 3 | Strong evidence | There is abundant evidence of visualisation processes, more than three incidences on the use of *geoboards* |

Although the interviews and their analysis were mostly qualitative in nature, the researcher and the respective teachers also quantitatively captured the extent to which the visual imagery and apprehension criteria were evident. This was done by using the coding framework with coding descriptors in Table 1.2 above.

## *Findings and Discussion*

### Mr. Jones' Lessons

In all of Mr. Jones' three lessons, there was strong evidence that his use of the *Geoboard* evoked perceptual apprehension – see Fig. 1.2. He made extensive use of the *Geoboard* to illustrate properties of various quadrilaterals. He often demonstrated the construction of a quadrilateral in a step-by-step fashion using sequential apprehension to extend his original construction. On one occasion he constructed two perpendicular lines and asked the learners to complete the shape. Mr. Jones used the *Geobaord* dynamically relatively seldomly. On one occasion he asked the learners to transform a square with five pins on each side to one with seven pins on each side. On another, he asked the learners to change the size of a square without dismantling the original shape. Mr. Jones often encouraged learners to discuss the properties of the quadrilateral he constructed on the *Geoboard.* He mostly encouraged discursive apprehensions by asking strategic questions. Learners were given tasks that required them to construct lines of symmetry, diagonals and missing line segments of shapes, as can be seen in Fig. 1.3. There was thus medium to strong evidence of using operative apprehensions in Mr. Jones's lessons.

### Ms. Ruth's Lessons

Ms. Ruth's lessons showed strong evidence of using the *Geoboard* in the context of both perceptual and operative apprehension. She however did not make use of the *Geoboard* dynamically – see Fig. 1.4. On one occasion she turned the *Geoboard* on its side, however, to demonstrate a different orientation of a particular shape. In her second lesson, she used the shapes on the *Geoboard* to engage the learners in articulating their observations about the similarities and differences of the shapes, thus

**Fig. 1.3** Learners engaging in operative apprehension when doing activities on the *Geoboard*

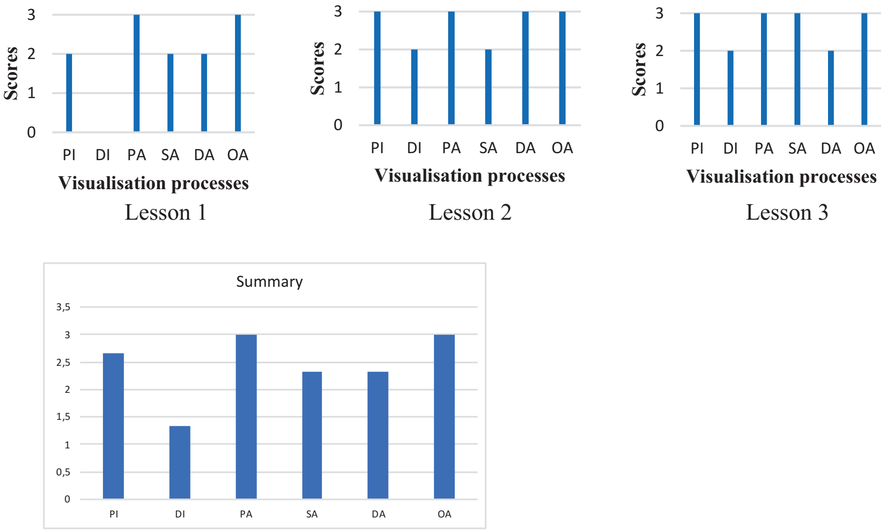


**Fig. 1.4** Bar graphs showing evidence of the visual imagery and apprehensions used in Ms. Ruth's three lessons

**Fig. 1.5** The use of dot papers to supplement the *Geoboard*

eliciting from them the essential properties of these shapes. It was interesting to note that Ms. Ruth also made use of dot papers (dots arranged in a similar square lattice as the nails on the *Geoboard*) to supplement the *Geoboard* activities she gave to the learners – see Fig. 1.5.

## Ms. Smith's Lessons

Ms. Smith was very effective in exploiting the pictorial imaging capacity of the *Geoboard* – see Fig. 1.6. The images that she produced were very striking and evoked strong perceptual apprehensions in the learners – see Fig. 1.7.

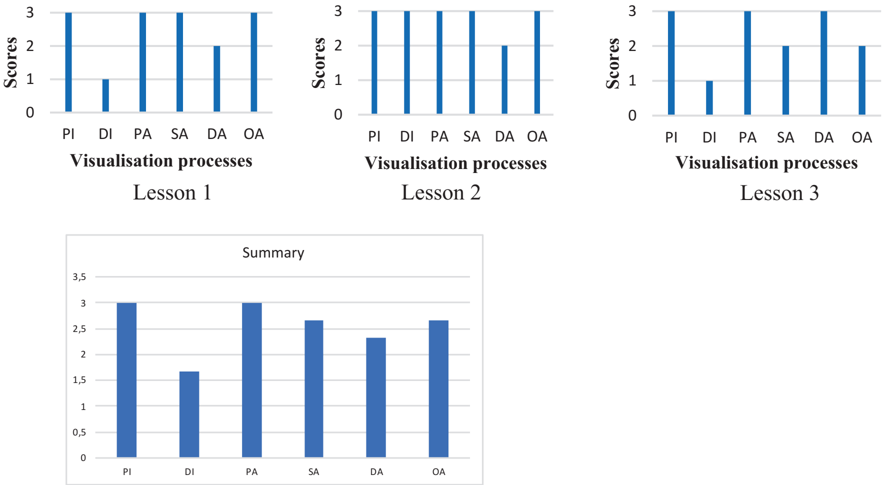


**Fig. 1.6** Bar graphs showing evidence of the visual imagery and apprehensions used in Ms. Smith's three lessons

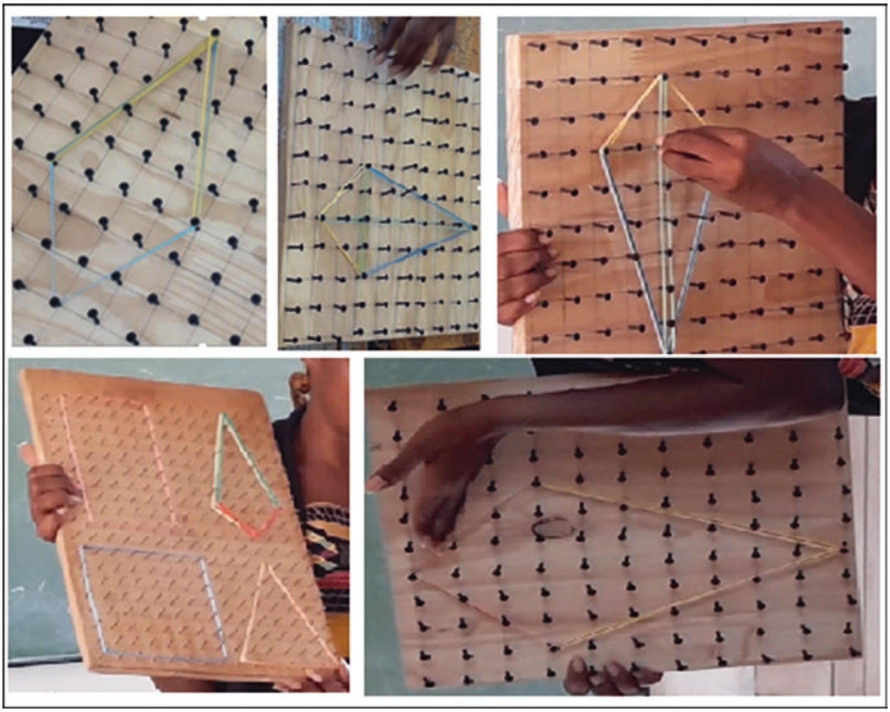

**Fig. 1.7** Shapes with strong visual appeal on Ms. Smith's *Geoboard*

### Interviews

In the focus group interview, the teachers were generally very positive about their experiences of teaching with a *Geoboard.* They found it a powerful manipulative to visually represent quadrilaterals and other geometric shapes. Matengu (2018) wrote that they alluded to the dynamism that a *Geoboard* affords, particularly in "forming and transforming shapes" (p.110). Ms. Smith particularly commented on the flexibility of the *Geoboard*. She often turned and re-orientated her board.

The teachers also alluded to the practicality of the *Geobaord.* Mr. Jones, for example, emphasised that a *Geoboard* does not need cleaning – it is just a matter of replacing and changing the elastic bands to create a new shape. The teachers also acknowledged that the construction and manipulation of quadrilaterals on the *Geoboard* was much easier than on a chalkboard.

All the teachers observed that the *Geoboard* brought "excitement and increased participation during teaching" (Matengu, 2018, p. 110). Interestingly they also suggested that the *Geoboard* helped cater for diverse learners. They said that the manipulative was user-friendly and thus encouraged interaction amongst learners.

There were also limitations to using the *Geoboard* that were identified by the teachers. Ms. Ruth, for example, found that her *Geoboard* should have been bigger in size for all her learners to be able to observe her constructions. In relation to this, it is important that all the learners have access to a *Geoboard.* As much as the teachers used the *Geoboard* as a demonstration tool, it should be used as an exploration tool by the learners. The teachers also noted that not all geometric shapes are constructible on a *Geoboard.* It is thus important that teachers plan carefully and strategically. It was noted by Mr. Jones, for example, that when exploring lines of symmetry, the sides of a selected quadrilateral should consist of an odd number of pins in order to easily find a mid-point.

## Conclusion

The above case study illustrated how the use of a simple, easy-to-make manipulative enriched a series of lessons that focused on the properties of quadrilaterals. The manipulative enabled the participating teachers quickly and effectively to illustrate and facilitate visual explorations into the properties of quadrilaterals, thus adopting a visual approach to teaching mathematics. The above case is of particular significance in the context of a rural school environment where reliable and quality access to digital technologies, such as the Internet and computer software, is still very limited. It is thus incumbent on mathematics teachers in these environments to look for alternative means to either source or make manipulatives that can be used as tools for a visual pedagogy. This case study showed that the *Geoboard* is a highly appropriate and effective manipulative that enables teachers and learners to make mathematics just a little more visible.

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# Chapter 2
# The Role of Number Sense on the Performance of Grade Learners 12 in Mathematics: A Case of Oshana Education Region, Namibia

**Shiwana T. Naukushu, Choshi Darius Kasanda, and Hileni M. Kapenda**

## Introduction

After independence, the education system in Namibia was reformed, and four main goals that entailed accessibility, quality, equitability and democracy in education were set (Ministry of Education and Culture (MEC), 1993). Namibia also developed a Vision 2030 document that anticipates that the country will be developed and industrialized by the year 2030 (National Planning Commission (NPC), 2003). NPC (2003) further states that the development of science and mathematics is crucial and needs to be strengthened in order to speed up the realization of Vision 2030. The development and understanding of mathematics and sciences require considerable numerical sense. In the Namibian context, the level of number sense and its influence on academic performance in mathematics is still underresearched (Clegg, 2008). Clegg holds the view that since the adoption of the International General Certificate of Secondary Education (IGCSE) curriculum and later the Namibia Senior Secondary Certificate (NSSC) curriculum, students do not seem to display greater proficiency with number sense. Further, he notes that no research has been done to find out whether number sense influences the academic performance of Grade 12 learners in mathematics in Namibia.

The Directorate of National Examinations and Assessment (DNEA) (2019) indicates that over the past 5 years, mathematics has been among the subjects where learners showed poor performance at Grade 12 level. Therefore, a problem of persistent poor performance in mathematics at Grade 11 and 12 levels in Oshana region exists (DNEA, 2019; SACMEQ, 2018). This poor academic performance in

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mathematics may be attributed to the lack of number sense as noted by DNEA (2019). Other researchers such as Menon (2004a, 2004b), Wessels (2006), Menlos and Dole (2000), Sowder and Kelin (1993), Steen (1999), Tansheng (2007), and Thomas (2001) noted that there is a strong positive correlation between the individual number sense and understanding of Mathematics. To date, there are few studies carried out in Namibia to address the factors affecting the development of number sense among Grade 11 and 12 learners and its influence on academic performance. It is against this background that this study investigated the factors that influence the development of number sense and its influence on the mathematical performance of Grade 12 learners in the Oshana education region.

Consequently, this study sought to answer the following questions:

1. What is the state of number sense among Grade 12 mathematics learners in the Oshana education region?
2. What is the influence of number sense on the academic performance in mathematics of Grade 12 learners in the Oshana education region?

The results of this study might contribute to knowledge creation by availing new information on the development of number sense and its influence on the academic performance in mathematics of Grade 12 students in Namibia. This study might also provide valuable information on the status of number sense development and highlight the hindrances to learning and teaching of mathematics in Namibia.

## Theoretical Framework and Literature Review

This study was informed by the constructivism paradigm, and although constructivism is not a new educational theory, it has gained traction in the global arena in the twenty-first century (Tan, 2017). According to Hersant and Perrin-Glorian (2005), the constructivism theory refers to the idea that learners construct their own knowledge, i.e., each learner individually and socially constructs his/her own knowledge from interacting with their own environment. Tan (2017, p.239) further states that "constructivism ass a theory of knowing presupposes that there is no fixed body of truths from the real world that are discovered by scholars, contained in textbooks, mastered by the teachers and subsequently transmitted to the learners, hence learners need to construct the knowledge for themselves (either individually or collectively)." This study therefore aimed at finding out whether the experiences of the Grade 12 learners in the Oshana education region had equipped them with the mathematical experiences and meaning that will enable them to build enough number sense.

Number sense has been defined in different ways. Menon (2004a, 2004b) defines number sense as an intuitive understanding of numbers, their magnitude, and how they are affected by operations. Hilbert (2001), on the other hand, defines number sense as the sense of what numbers mean and the ability to perform mental mathematics by thinking about numbers and making comparisons among them. Burn

(2004) defines number sense as well-organized number information that enables an individual to understand numbers and number relationships to solve mathematical problems. From Burn's definition, it follows that for a person to have a strong sense of numbers, one needs more information on how numbers behave and interrelate. Zanzali (2005) views number sense as the sound understanding of numbers and relationships between magnitudes of numbers. Number sense should stress the importance of the magnitudes of numbers, that is a learner with enough number sense should picture in his or her mind how much a given number represents. Johansen (2004), on the other hand, defines number sense as the ability to use numbers to compute accurately, to self-correct by detecting errors, and to recognize the result as reasonable. It is important to recognize the reasonableness of the results obtained, and learners who possess number sense should be able to produce answers that are reasonable.

From literature definitions, number sense can be summarized by these five key components: understanding the meaning and size of numbers; understanding equivalence with numbers; understanding meaning and effects of operations; understanding counting and computation strategies; and estimation without calculating.

Number sense has been studied widely in other countries. In the United Kingdom (UK), the concept of number sense is usually developed from the beginning of primary school starting at Key Stage 1 (Grades 1–3) (Clegg, 2008). Clegg further indicates that in Key Stage 2 (Grades 4–7), learners are expected to have a solid foundation of number sense and are expected to do most of the calculations mentally. There is thus a lot that Namibia can learn from the UK and other countries in terms of building strong number sense among the Namibian learners. It was noted by Wessels (2006) that Singapore came third in the Third International Mathematics and Science Study (TIMSS) of 1998, because the mathematics teachers work hard with their learners as far as the development of number sense is concerned. The Netherlands have also worked hard on the development of mathematical understanding and the development of number sense and hence gained the first position in the 1998 TIMSS (Schoenfeld, 2006). Teachers in the Netherlands develop the number sense among their learners based on Realistic Mathematics Education (RME), which stresses that the teaching of mathematics should be connected to the learners' reality. Inclusion of number sense in the mathematics curriculum appears to bear fruit as far as improving students' performance in mathematics is concerned. It is therefore important for Namibia to learn from other countries to see if adopting number sense at lower grades will help achieve reasonable mathematical academic performance at higher grades.

## Methodology

This study used a quantitative survey design. Quantitative variables take on numerical values and are usually obtained by measuring or counting (De Vos et al., 2002). The numerical information is an empirical representation of the abstract ideas

(Neuman, 2003, p. 171). Hence, the academic performance of learners in mathematics and in the number sense test was assessed in terms of the numerical scores. Therefore, this study possessed characteristics of a quantitative study. Particularly, this was a correlational study as it tried to establish the relationship between the number sense and the academic performance of the learners. Correlational studies look at the relationship between certain variables or how one variable is affected by another variable (Davis, 2014). The teachers' views about the factors that influence the development of number sense among the learners were also studied. In order to determine the current level of number sense among the learners in the Oshana education region, this study adopted and adapted the number sense test from McIntosh et al. (1997). Marks of learners for their regional mid-year mock examination were used in this study as well as a questionnaire with open-ended and closed-ended questions.

## Findings of the Study

This section presents the data about the state of number sense among the learners in the Oshana education region as well as the relationship between their number sense and academic performance in mathematics.

## Level of Number Sense of Learners

The evaluation of the teachers' awareness on number sense was the first point to consider since they are the ones that directly deal with the students (Wessels, 2006). Based on their assessment during the teaching number sense test, the teachers were asked to comment on the levels of number sense of their learners. The responses of the teachers are given in Table 2.1.

The marks for all the learners in each school were categorized into four levels (levels 1–4). These levels are standardized as adapted from McIntosh et al. (1997). Level 1 is the weakest, and level 4 is the strongest number sense ability. Figure 2.2 shows the overall percentage of learners who fell in the various levels depending on their numerical abilities.

**Table 2.1** Teachers' rating of the levels of their learners' number sense ($n$ = 18)

| Statement | Frequency | Percentage |
|---|---|---|
| Very strong | 4 | 22.2 |
| Strong | 2 | 11.1 |
| Weak | 4 | 22.2 |
| Very weak | 8 | 44.4 |
| Total | 18 | 100 |

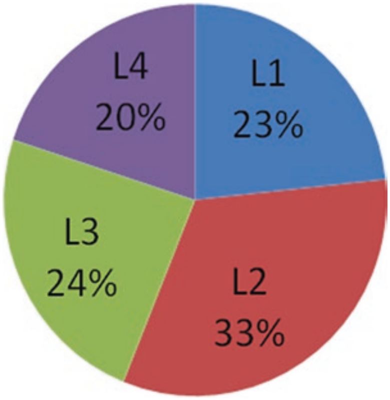


**Fig. 2.1** Number of learners in the four levels on the number sense test

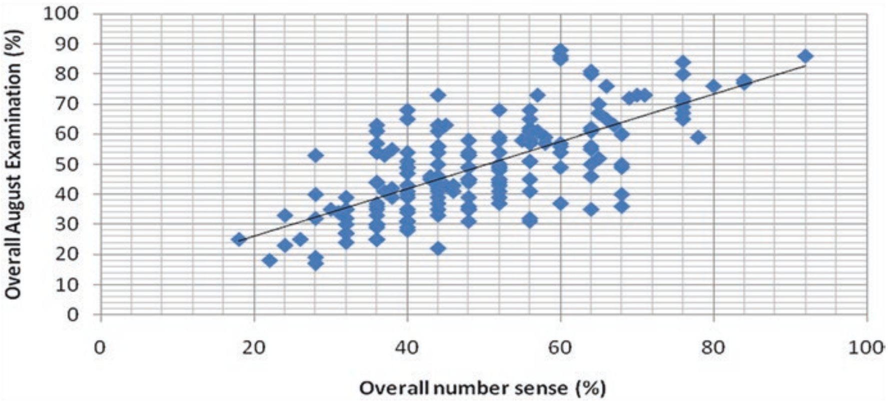


**Fig. 2.2** Scatter plot of number sense test % versus the mathematics August marks ($N = 181$)

The overall performance of learners according to their attainment levels in the number sense test as shown in Fig. 2.1 indicates that more than 50% (i.e., 56%; $N = 181$) of the sampled mathematics learners had weak number sense (levels 1 and 2). On the other hand, less than 50% of the learners ((44%) $N = 181$) were found to exhibit strong number sense (levels 3 and 4). Hence, the total number of learners that had weak number sense is more than that of the ones that have a strong number sense. The findings thus seem to indicate that there was a numerical deficiency among the learners, which might compromise their comprehension of the mathematical content and therefore might inhibit their academic performance in mathematics.

## The Influence of Number Sense on Academic Performance in Mathematics

This section presents the comparison between learners' performance in mathematics and in the number sense test. The academic performance in this section was measured by using the mathematics August examination marks, since this examination was used as a standard for every school in the Oshana education region. The teachers were asked to comment on the effect of number sense based on their teaching experiences. Table 2.2 shows the responses of the teachers.

Table 2.2 shows that 88.9% ($N = 18$) of the teachers perceived a positive influence of number sense on academic performance. None of the teachers believed that number sense affected the mathematical academic performance negatively. Two (11.1%) ($N = 181$) of the teachers were of the opinion that there was no relationship between number sense and academic performance. The number sense scores of learners were also compared with their academic performance in mathematics. A scatter plot comparing the scores of the learners in mathematics and number sense test was drawn for all the learners that participated in this study.

Figure 2.2 shows a correlation coefficient of $r = 0.702$, which indicates that there was relatively a strong positive relationship between the learners' number sense and their academic performance in mathematics examinations. This finding is in line with the responses of teachers (87.5%) from Table 2.2 that shows that learners' understanding of number sense has a positive influence on their academic performance in mathematics. These results are also in line with similar researches (Menon, 2004a, 2004b; Wessels, 2006; Menlos & Dole, 2000; Sowder & Kelin, 1993; Steen, 1999; Tansheng, 2007; and Thomas, 2001), which indicated that the learners' number sense has an influence on their academic performance in mathematics. Wright (1996) indicated that learners are required to have some numerical competencies and understanding in order to be able to comprehend the high school mathematics curriculum. Accordingly it is imperative that Namibian learners have this numerical competency to understand school mathematics.

This study also tried to compare the performance of schools in their mathematics mid-year (August) examination of 2010 with the number sense test performance and the regional rankings of schools in mathematics. Spearman's rank correlation coefficient of $(\rho) = 0.85$ was found between the ranking of the school in the August examination and the ranking on the number sense test. This again agrees with the work of Menon (2004a, 2004b), Wessels (2006), Menlos and Dole (2000), Tansheng

**Table 2.2** Teachers' opinions of the influence of number sense on academic performance ($n = 18$)

| Influence of number sense on academic performance in mathematics | Frequency | Percentages |
|---|---|---|
| Influences positively | 16 | 88.9 |
| No relationship | 2 | 11.1 |
| Influences negatively | 0 | 0 |
| Total | 18 | 100 |

(2007), and Thomas (2001), who indicated that there is a positive correlation coefficient between the performance of learners on a number sense test and their academic performance. This implies that the learners who have a weak number sense are likely to perform lower in mathematics, and the ones that have strong number sense stand a better chance of performing well in mathematics.

## Conclusions and Recommendations

It can be concluded that more than 56% ($N = 181$) of the Grade 12 learners in the Oshana region have a numerical deficiency, which might affect their understanding and performance in mathematics. In addition, the moderately high positive correlation coefficient found in this study seems to indicate that the development of number sense in mathematics has an influence on the academic performance of Grade 12 learners in Namibia. There is therefore a need for mathematics teachers to ensure that their learners are adequately exposed to number sense if better performance in mathematics is to be realized. Deliberate exposure of learners to aspects related to the development and maintenance of number sense proficiency is essential for Namibian learners in the Oshana education region.

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# Chapter 3
# Using a Community of Practice Strategy to Strengthen Teaching and Learning of Mathematics in Rural Areas

**Benita P. Nel**

## Introduction

Teaching, unlike many other professions, does not necessarily promote a 'shared culture'. Hence, teachers often work in isolation. Some professional development (PD) initiatives even assume that teachers work alone, which may further encourage a sense of professional isolation. Not surprisingly, Jita and Ndlalane (2009) noted that a significant number of "teachers work in isolation in their schools and classrooms" (p. 63). Botha (2012) underscores that at provincial, district and school leadership levels, teachers are not always encouraged to work together in shared spaces. This is predominant, especially among teachers in rural areas because of spatial isolation, where schools are significant distances apart. A considerable number of South African schools are in rural settings. Here, teachers generally work in silos compared with their urban school colleagues. More so, teachers in these schools work under extremely difficult conditions, and district officials' visits are few and far apart. Given the resulting limited professional support, these teachers are often less equipped than their urban counterparts (South African Council of Educators [SACE], 2011). Subsequently, when their learners' performance is unsatisfactory – despite continuous interventions – these teachers can become despondent.

Despite the advantageous lower pupil-teacher ratio in rural schools, mathematics teachers, at times, experience unique struggles. They often teach all the learners in the different grades due to limited subject specialists. This results in their heavy workloads (Nel, 2015). Besides the lack of professional support, these teachers also lack significant material support (Gardiner, 2008). In addition to a scarcity of special teaching and learning facilities, information technology (IT) and Internet

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coverage are also limited (Maringe et al., 2015). Consequently, there is a high teacher turnover in rural schools (Monk, 2007) as many prefer to teach in urban schools. Moreover, these mathematics teachers often lack the required qualifications and skills (Maringe et al., 2015) as rural schools struggle to attract qualified teachers (South African National Tutor Services (SANTS), 2015; See also Monk (2007). This leaves a small group of teachers equipped to teach these subjects—spread over a wide geographical area.

One reason for teachers' spatial, but then also professional, isolation could be related to the lack of adequate training of many departmental school officials in the key aspects of good leadership and curriculum delivery skills (Dufour et al., 2006). Consequently, many of these district officials struggle to support staff in establishing co-operative structures and supportive professional environments. This, the researcher concedes, is a broad challenge. However, teachers in rural schools experience specific challenges that hinder their ability to interact constructively with peers. Hence, there is a dire need for teachers in rural areas to work together. De Clercq and Phiri (2013) concurred that this would enable teachers to develop a sense of belonging and create a shared platform to exchange ideas and, in the process, break their isolation.

This contribution argues for a community of practice (CoP) strategy to address these challenges. A CoP can be broadly defined as 'a group of people who share an interest in a domain of human endeavor and engage in a process of collective learning that creates bonds between them' (Wenger, 1998, p. 1). It is a space where people can learn through their collective involvement (Wenger, 1998).

This article aims to answer the following question: How can the effects of professional isolation of mathematics teachers in a specific rural setting be addressed? This article opened with a review of the literature on the relationship between professional isolation and related concepts and PD. Subsequently, the article will describe the theoretical framework of this research. This will be followed by an empirical section where the specific mathematics PD programme under investigation will be described—which was implemented in a rural setting in South Africa. Thereafter, the research methodology will be described, followed by results and discussions, conclusions and implications.

## Literature Review

In the South African context, the traditional PD model is expert-driven as well as once-off workshops. Contrary to this are models in which teachers themselves are active learners in a CoP (Mak & Pun, 2015). A CoP model of PD endeavours needs to acknowledge and use participants' expertise as a valuable resource (Buchanan & Khamis, 1999). Subsequently, new knowledge and skills can be developed, shared and transferred to improve their own teaching practices. To ensure a productive interaction, CoP members should know each other and identify those who can be of assistance to them (Steyn, 2008). Islam (2012) observes that 'the group dynamics within the CoP plays a key role in the success, failure and effectiveness of the CoP' (p. 26). The PD programme in this study did not intend to create a CoP. The participants met regularly in the context of a joint enterprise. Different ways and intensities can influence the outcome of collaborations. Some collaborations may not even add value to teaching and learning (Little, 2000). This is partly due to the tension between balancing teacher autonomy and teacher initiative on issues of practice and principles (Little, 2000) and how these issues are managed. This study took place in a particular context with a set of key concepts, already introduced, which I will explain now and specifically relate it to PD.

## *Rurality*

This term is used for everything that is not metropolitan or urban (Monk, 2007). As indicated in the opening remarks, in the South African context, a significant portion of schools falls under this descriptor (Mpahla & Okeke, 2015). These rural schools are characterised by highly impoverished backgrounds of learners as most parents are low-skilled workers, and unemployment is widespread (Monk, 2007). Furthermore, rural learners face English language challenges as they have 'little opportunities to live, think, and work in a language other than their mother tongue' (Gardiner, 2008, p. 20). These conditions hinder the delivery of quality education in rural areas (Giordano, 2008) and contribute to rural learners' poor performance in mathematics and to PD failure.

## *Isolation*

Teaching can be a lonely profession at times because some teachers do not embrace the notion of a 'shared culture' amongst peers (Borg, 2012, p. 301). Giordano (2008) identifies the lack of, or inadequate, professional support from colleagues as one of the difficulties of teaching. Professional support is further hindered as 'most schools are organized in an eggcrate manner, making professional collaboration difficult' (Cookson, 2005, p. 14). This increases teachers' isolation. The rural schools chosen for this study had one or two Further Education and Training (FET[1]) Mathematics specialist teachers. Because the rural mathematics teachers might have no colleagues teaching in that phase, they have limited opportunities to discuss problems in mathematics teaching and student learning. This means that interaction between colleagues is limited to cordial, everyday talk (Ostovar-Nameghi & Sheikhahmadi, 2016). The causes of teachers' isolation can include limited time to engage in dialogue with colleagues about teaching practices, rural schools being

[1] FET in South Africa refers to the last 3 years in high school with learners' ages around 16–18 years.

significant distances apart (spatial isolation), schools' tight schedules, and their heavy workload, which is due to their being scarce-skill specialists, to name but a few. However, specific research on the isolatedness of teachers in the South African context has a gap in the literature. Arnold et al. (2005) claimed that despite ample research on rural education, rural schools and rural communities, the quality of this research is poor. Barter (2009) argued that something is missing in the literature around rural education—there is not enough literature on the 'right kind of research' (p. 240). This contribution thus attempts to address this gap.

### *Limited Opportunities*

There are several key elements that limit opportunities for teaching and learning in rural areas. These aspects include poorly equipped teaching and learning facilities, poor learner performance measured in terms of the 40% performance mark, ineffective PD workshops done by the Department of Education – they were too short, and the facilitators were incompetent—and the redeployment of teachers into rural mathematics and science posts without the required qualifications (Maringe et al., 2015). The fact that a significant number of teachers do not stay where they work (Ebersohn & Ferreira, 2012) is another limitation in rural schools as they have little time to strengthen relationships with the learners and community after school as they need to leave with a lift club. This means also that extra PD interventions after school are not really possible. While some rural schools might have specialist facilities like laboratories and libraries, they are white elephants and often used for storage. South Africa's public schools have district officials assigned to monitor and support teachers—in the quest for quality teaching. However, district officials' visits to some rural area schools are few and at long intervals apart; thus, teachers receive limited support (Giordano, 2008). Hence, where there is only one FET mathematics teacher at a school, with no peer support or regular visits by the district official, this teacher lacks the support needed from a subject specialist. Consequently, this teacher ultimately works on his/her own. Botha (2012) claimed that at various levels in South Africa, leaders are not encouraging teachers to work together in shared spaces, causing these educators to work in isolation.

A proposed PD intervention to address teacher isolation in rural areas in the literature is to utilise practices that increase collaboration efforts among teachers. This can be achieved by creating platforms where professional conversations and networks, whether face-to-face or online, can be established (Davidson & Dwyer, 2014). In this way, reciprocal learning can occur among teachers. This can potentially result in teachers improving their skills, knowledge, practices and, ultimately, their learners' performance (Ostovar-Nameghi & Sheikhahmadi, 2016). Indeed, the improvement also applies to teachers' confidence, their resource collection (Davidson & Dwyer, 2014) and their motivation (Little, 2000). The creation of such platforms in a rural context is difficult and requires careful planning, with financial implications pertaining to travelling and securing conducive venues. However, to create the above-described spaces is possible and might yield significant benefits. These benefits include reducing teacher isolation, increasing teacher confidence, bringing theory and practice together, as well as exchanging resources, skills and knowledge. These interventions are underpinned by a specific theoretical framework.

## Theoretical Framework

The theoretical framework of this study is Wenger's social theory of learning. From this stance, learning can occur in a CoP or his notion of socially situated practices (Wenger, 1998). This theoretical framework suits this study as it investigates the co-participation of a group of mathematics teachers, from a rural setting, who met over 3 years to engage in communal activities. Their objective was to make sense of aspects of their teaching. The overarching principle in this conceptualisation is that knowledge is situated in experiences, as well as in the interaction and critical reflection among CoP members (Buysse et al., 2003). Thus, increased engagement in a CoP can influence individual members' knowledge and beliefs (Borg, 2012). A CoP fosters collaboration among peers, as participants work towards a common purpose (Wenger, 1998). A CoP's focus—as a form of social learning theory—is active learning and social participation (Steyn, 2008). A CoP can also be a space where new knowledge is created—through facilitation—and where collaborative workplace learning can occur (Mayne et al., 2015). However, Andrew and Ferguson (2008) grapple with the following question: How can PD, especially in a rural setting, incorporate CoP as learning vehicles?

## Data Sources and Methodology

The article draws on the author's PhD study where a mathematics PD programme in a rural setting was implemented due to the poor performance of Grade 12 learners in a South African circuit. This research was part of a broader study involving mathematics and science teachers' professional development. The aim of this broader PD programme was to improve the competence and performance of the participating teachers. The programme used content training workshops, school support and monitoring visits, on-site mentoring, PD through professional learning clusters and a CoP as interventions. Residential workshops at off-site venues were scheduled over three consecutive days. Roads were not maintained, and access to the area was therefore difficult.

This study evaluated only the mathematics component of the broader study. The research question emanates from the evaluation study. Five schools (of which one later withdrew) were selected, where the focus was on FET phase mathematics teachers. The participants consisted of four males and one female with varied qualifications and teaching experiences. All these participants taught at least one other subject besides mathematics over different grades, leaving them with significant workloads. The participants' experience of their involvement in and the value of their CoP will be discussed later.

This study focused on human (social) interaction among a group of teachers in the context-specific setting of rural schools. The epistemological and ontological stance taken by the researcher is that knowledge is constructed through human beings' purposive interaction with each other and with the world.

This study employed a qualitative approach to explore rural mathematics teachers' view of how the investigated rural school community PD programme assisted in (or hindered) breaking the cycle of their professional isolatedness. Furthermore, for this study, a single case design which comprised of the schools involved in the programme was selected.

All five FET band participants and one mentor—who was also the facilitator of the investigated PD programme—were included in this study. They consisted of one teacher per school, except for one school where two teachers participated. This number of participants was sufficient for an in-depth study on strengthening the teaching and learning of mathematics in a specific rural context.

The main techniques used to collect the data were document analysis of mentor reports, semi-structured interviews with participants and observation of participants in their natural setting (the classroom). At least one face-to-face semi-structured interview of approximately 30 mins was conducted with each of the participants as well as with the mentor. These interviews were audio-recorded and later transcribed. The researcher also undertook at least two classroom observations per participant, using the same observation schedule elaborated for the programme, to ensure consistency. The different data sources, primary and secondary, enabled data triangulation, where the views of the different teachers and the mentor/facilitator were accommodated in the quest to enhance the study's trustworthiness (Lietz et al., 2006).

The interview questions were focused on the research questions of the PhD study, and isolatedness was a theme that emerged out of the data (see Appendix A). These teachers were being interviewed in English, which was not their home language, and they sometimes struggled to express themselves in English.

The data was analysed using a thematic approach. Gibson (2006) defines the latter as an approach 'dealing with data that involves the creation and application of "codes" to data' where 'codes' allude to the 'creation of categories in relation to data' (p.1). The following themes were derived from analysing the data using an inductive approach, keeping the research question in mind: benefits of peer interaction in the CoP, fostered reciprocal learning through the creation of a conducive environment and improvement of skills through collaborative workplace learning.

Ethical clearance from both the provincial Department of Education and the institution of study—with protocol number 2013/ISTE/30, was received. Consent was also received from the schools and teachers involved in the study. To protect their identity, pseudonyms were assigned to them: MK, NK, SB, RB and SR for the teachers and AK for the mentor/workshop coordinator. Similarly, the names of the schools were not revealed.

## Results and Discussions

This section presents and discusses the results pertaining to the professional isolation and strengthening of their teaching and learning in a CoP of rural area teachers involved in the investigated PD programme in South Africa. The themes will now be discussed.

### *Theme 1: Benefits of Peer Interaction in the CoP*

It became apparent, when studying the programme reports and school visit reports, that some of the participants needed development in their content knowledge, lesson preparation and lesson delivery. In different school visits and mentor reports, the absence of written lesson plans and the lagging behind with the syllabus (due to ineffective use of class time) were reported. It can be viewed that the lagging behind goes hand in hand with the absence of lesson plans. Interviews revealed that the participants were comfortable working in groups in improving their content knowledge, as expressed by RB:

> "The workshops were fascinating because in terms of the new syllabus for NCS, we were struggling with finance (as a topic) …."

It must be noted that financial mathematics was not included in the previous syllabus. Hence, some of the teachers were not confident when teaching on that topic. Since some of the teachers were the only single FET mathematics teachers at their respective schools, the newly formed CoP fostered reciprocal learning through the creation of a conducive environment for improving the participants' skills, knowledge and practice. Equally interesting was the use of the word 'fascinating' in the above excerpt. This alluded to the excitement and satisfaction emanating from working with peers—having worked alone in the past.

The data from the interviews showed that the mathematics PD workshops created an environment where teachers could share their skills and knowledge while grappling with different aspects of their profession. In an interview, SB remarked:

> … they (the workshops) give us an opportunity that we met as colleagues. In the cluster (workshops) we discussed about work, you see; something we lacked before.

This excerpt indicated that rural schools' specialist teachers needed to interact with each other, as they felt isolated at their respective schools. This participant's use of 'something we lacked before' unequivocally highlighted the need for collaboration. This phrase also revealed the teacher's sense of professional isolation, especially when using the word 'before'.

In the relatively small groups of five, the participants received individual support from their peers, knowing who could assist in different situations. This can be confirmed by SB who revealed how they were:

> … helping each other because we are not on the same level as far as the teaching Mathematics are concerned; the confidence we have with the subject (differed). So, it helped in the sense that we share it with some colleagues.

They were no longer grappling with different challenges on their own in their respective schools, nor did they continue to feel swallowed up by the masses in the bigger districts' circuit meetings. The participating specialist teachers felt that they were actively involved and listened to in the CoP. Their working together was experienced as beneficial to their PD—in different ways. Also from my observation, the CoP provided an avenue where reciprocal learning was possible through the creation of a conducive environment where teachers' skills, knowledge and practices were improved.

### *Theme 2: Fostered Reciprocal Learning through the Creation of a Conducive Environment*

When teachers meet in forums, it does not automatically translate into a conducive environment where they learn from each other. What then in this programme enhanced a conducive environment that fostered reciprocal learning? MK expressed his enjoyment of working in pairs to complete the given exercises:

> "… (One) feels free and … comfortable there because I can just ask you – you are my peer … You see another (method of attempting the problem) … That is (how) we're working in Mathematics together"

This extract indicates how being part of a CoP helped MK value his peers' input and advice and exposed him to new problem-solving methods. Collective learning, which created bonds between members (Wenger, 1998), was fostered. This enriched MK's knowledge and understanding and increased his range of possible methods to use to solve mathematical problems. All this occurred because he was comfortable sharing with and enquiring from his peers. This highlights the professional support that the participants received from their peers, as they were well enough acquainted with each other to interact freely. Hence, they knew where to get support, if needed.

The workshop-related discussions also focused on 'work', putting these teachers' PD at the centre. Their deliberations in the workshops led to open discussions that placed learning, within the context of their daily teaching, at the centre. In this regard, RB commented on their 'regrouping as teachers ….' On further probing, what RB meant by 'regrouping' was that they worked in pairs or in groups of three, assisting each other while working on exercises. SB clarified RB's remark by saying that, they 'never' met like this; they grappled with challenges on their own, at their respective schools:

> Ja, we're working together … it helped during that time … Because they provided that platform; we never met before; … I mean we never met being 5 at one time talking about our experiences at work you know or helping each other, but it did give us that opportunity that we can interact with each other.

This extract highlighted the fact that these teachers were not previously part of a mathematics CoP ('we never met before'). They might have met as a cluster or a circuit, but not in a way that enabled them to collaborate regarding their challenges and frustrations ('talk about our experiences at work')—in a smaller group where they could receive individual support from peers ('helping each other'). Clearly, the participating teachers are no longer engulfed by the masses (the entire circuit), their voices are heard, and they feel part of a group consisting of less than seven teachers ('interact with each other'). Before, these teachers were passive observers or listeners in their gatherings, but now, they are actively involved and being listened to ('we can interact'). For these teachers, working together has been helpful and developmental ('it helped during that time'). The CoP provided an avenue where teachers could interact and be transformed while co-constructing knowledge.

The participants also realised the value of a CoP where all members are not on the same level of mathematical skills, knowledge and teaching experience. This is evident in an excerpt from the interview with SB:

> Though, like we taught with other colleagues, you might find that 2 of us are moving at a faster rate, some are lacking behind you know. But those who were faster, they …asked to help others … just to try to explain to other colleagues. So I think they (the interactions) were very good.

Clearly, the various levels of development among the members of the CoP added value to their learning experience. This knowledge and skills diversity created a conducive environment where those with more knowledge and skills helped the others to develop professionally. The above excerpt also alludes to the different abilities of the members of the CoP who can thus identify those who can act as leaders in different scenarios. Teachers' expertise was thus used in the PD as valuable resource, something that was impossible when they worked in isolation. In the CoP, they took responsibility for each other's development through their interaction as a group, making the CoP a conducive environment for their common growth as teachers. Clearly, a CoP fosters an environment characterised by social construction of knowledge and active dialogue among its members.

The improvement of lesson preparation and presentation was also addressed in workshops where participants presented to colleagues the lessons that they had prepared. The lesson presentations were then critiqued by the rest of the group, and feedback was given to the presenters—to improve their pedagogical skills. This is evident in the remark made by RB:

> "Ja, you know what I like about it (the workshop): We are open. We criticize so that someone can see whether his or her lesson was good or not. We talk. You go there and present your lesson. We bombard it with questions. Sir, you could have started this lesson in this way. Why are you starting that way?"

Because the environment was trustworthy, the CoP members felt free to make mistakes and be corrected. Hence, the critique was received in a way that promoted learning and development, and the group dynamics contributed to the effectiveness of the CoP. The fact that the teachers were mutually engaged directly affected their effectiveness in the classroom. This highlights a possible transfer of knowledge and skills to the participants' own practice in their classroom. Before, these teachers were working in isolation and did not have anyone they could engage with when they had challenges, or they could not find other ways of doing or teaching. So, their working together enabled them to get feedback and advice from each other.

With regard to lesson presentation, SR noted the following:

> Ja I learned a lot from them (peers) because I still remember that when we were presenting … I've learnt a lot from those teachers how they present their lessons. A lot indeed … How they approach ….

Teachers' excitement about working together was evident during the interviews. The lesson analysis and lesson study conducted in the CoP created opportunities to improve participants' pedagogical and lesson preparation skills when receiving valuable input and advice—in a supportive environment. This also generated opportunities to gain new knowledge, which the teachers should be able to transfer into their classrooms. The CoP fostered reciprocal learning through the creation of a conducive environment where members could experience other methods and ways of addressing problems.

## *Theme 3: Improving Skills through Collaborative Workplace Learning*

The participating teachers grappled with ways of improving their skills and, through interactions, became aware of areas where they could improve their lesson presentation and the setting of assessments. Indeed, by observing how others present or set assessments, one can reflect on one's own practice to find ways of improving it and grow professionally. Their experiences of togetherness encouraged learning. Concerning this, RB and SR noted the following:

> You're supposed to explain why (*you think a person should change the lesson*), but not in a negative way; in a way that will help you see it ... because some (participants) are experienced, some are new; … If the mistake will be in this way, avoid doing this and that ... Ja, that was helpful (RB).

> … I learnt a lot from them (lessons) … How they (should be) approach(ed) … the relationship between the teacher and the learners… now they have just given us the direction that we just do this, concentrate on this and (so on) …(SR)

The phrases 'not in a negative way', 'in a way that will help you see it…' and 'that was helpful' indicate that the participants benefited from their interactions. Those phrases also highlight the supportive environment in which they critiqued each other's work. SR also alluded to their improved skill in setting assessments:

> We formulate questions there at the workshop … So we just combine them to make a sort of a class test… questions and the memorandum ... Then we take that exercises, make sort of a book. So now time to time we just photo copy and just give the learners particular exercises… We have formulated those particular exercises.

Participants worked on assessments together and sharpened their skills – even ended up with a booklet that was used after the programme. An atmosphere of trust was developed in the CoP environment, so that colleagues were accepting of the given critique and used it for their development. Their increased engagement in the CoP thus enhanced their knowledge and skills as teachers.

My observation of the established CoP confirmed that it created a supportive environment where teachers felt free to request assistance, where they were comfortable to present lessons (even if these were not perfect) and receive critique. In the process, these teachers realised that there were other colleagues with the same challenges and that, together, they could devise strategies to address them. This togetherness alleviated their sense of isolation. The aspects that teachers found valuable in their development included increased content knowledge and improved pedagogical skills as well as classroom management and assessment skills.

## Conclusions

While a limitation of this study could be that it had a too small sample size to generalise, given the specific context, the following conclusions and implications did emerge. Social learning among the participants alleviated isolation. It assisted in addressing their challenges and was a means of replacing passive participation with a more active one. The participants also benefited where they supported one another and built confidence regarding the new content topics in the syllabus as well as lesson presentation and assessments. An environment that fostered reciprocal learning among the participants was created, and alternative methods for solving mathematical problems were shared. The knowledge and skills diversity within the group created an environment where participants developed each other. All this led to the improvement in teachers' skills through a form of collaboration.

Thus, the conscious establishment of a CoP in rural settings can yield significant advantages regarding the alleviation of teachers' sense of isolation. These advantages include teachers feeling less isolated, sharing of skills and experiences, receiving of constructive feedback from peers and building on it and hands-on learning concerning lesson presentation and assessment. A CoP enables individuals to incorporate what is learnt into their own practice. Social learning through a CoP is a tool to combat the professional isolation of mathematic teachers in rural settings while communal knowledge and skills can be utilised in a sharing culture and platform.

## Practical Implications

This study recommends that districts consciously establish CoPs in both rural and urban settings as the space for in-service PD. Doing so could potentially yield significant advantages regarding the alleviation of teachers' sense of professional isolatedness. A CoP's size tends to impact the ability of which participants can get to know each other on a personal and professional level. The members could possibly work together—on a continuous basis—over a long period. The study further recommends the creation of environments where teachers can even access each other's class and learn from one another—through observations.

## Further Research

Further comparative research is recommended in rural and urban contexts on professional isolation. This can shed more light into whether sample size or location affects the outcome of the study. Studies where a mixed method approach is used as well as research in areas other than where this study was conducted in are recommended. The use of these other variables might give more insight into how widespread the professional isolatedness of teachers is as well as how to alleviate it through the strategy of CoPs.

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## Teachers' Interview Schedule

1. Briefly share your experiences in the PD programme.
2. What in the lesson you prepared would you say was done differently from how you did it before your involvement in the programme?
3. Did the presentation in the workshops assist you in your development as teacher? Explain.
4. Would you say that the mentoring by the facilitator contributed to your development as teacher? Explain your answer.
5. Are there other developments that took place in you as teacher due to your involvement in the PD programme?
6. Can you recommend ways in which the programme can be improved in order to assist you better as a teacher?

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# Chapter 4
# Using Discourse-Based Mathematics Instruction in Secondary School Classrooms in Ethiopia

**Mekonnen Yimam Legesse, Kakoma Luneta, and Tadele Ejigu Wondimuneh**

## Introduction

Over the past few decades, reform efforts in mathematics education have highlighted the potential of mathematical discourse as an instructional approach for successful mathematics teaching and learning (Forman, 1996; Lampert & Blunk, 1998; Lampert, 1990; NCTM, 1991, 2000). As a result, several studies encourage teachers to orchestrate classroom discourse that fosters mathematical communication and social interactions, in which students engage in discourse practices of problem-solving, explaining, justifying, questioning, and agreeing and disagreeing with each other's ideas (NCTM, 1991, 2014; Pourdavood & Wachira, 2015; Stein, 2007).

Studies show that engaging students in mathematical discourse promotes mathematical understanding and thinking (NCTM, 2014; Smith & Stein, 2011; Stein, 2007; Stiles, 2016). Pourdavood and Wachira (2015) conducted a qualitative study that investigated discursive teaching strategies used by a secondary school teacher to engage students in problem-solving, reasoning, and proof. The study indicates the potential of discursive teaching strategies to promote mathematical understanding (Pourdavood & Wachira, 2015). However, creating a classroom learning environment that promotes productive mathematical discourse remains a challenging teaching practice for many teachers (Hufferd-Ackles, Fuson, & Sherin, 2004; Pirie & Schwarzenberger, 1988; Trocki, Taylor, Starling, Sztajn, & Heck, 2015). For instance, Adler and Ronda (2015) developed a framework for studying mathematical discourse in instruction in the South African educational context and illustrated how it can be used to investigate mathematics content made accessible to students

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as well as to interpret differences in teaching. The framework is characterized by four components: *exemplification*, *explanatory talk*, *learner participation*, and *the object of learning* (Adler & Ronda, 2015).

Moving from the global to local context, the teaching of mathematics for the development of students' mathematical understanding, problem-solving and reasoning skills, and mathematical confidence has been promised in the Ethiopian secondary school mathematics curriculum (Ministry of Education [MOE], 2009). Policy and curriculum documents encourage teachers to use instructional methods that increase students' learning outcomes. However, local studies indicate that poor mathematical performance and negative attitudes toward mathematics remain as critical problems for the majority of students (Engida & Zeytu, 2017; Dhoj & Verspoor, 2013; Ethiopian National Examination Agency [ENEA], 2010). Studies that focus on exploring alternative methods of teaching that support the development of students' mathematical understanding, problem-solving skills, and confidence are limited. Therefore, noticing the potential benefits of mathematics teaching and learning through mathematical discourse (e.g., Ballard, n.d.; Forman, 1996; Lampert & Blunk, 1998; NCTM, 1991, 2000, 2014; Smith & Stein, 2011), the present study aimed to develop a framework for the design and implementation of discourse-based mathematics instruction in secondary school classrooms, in particular, for use in an Ethiopian educational context. The learning goals articulated in a given curriculum serve as a starting point that informs the kinds of classroom instruction to be practiced (Gordon, 2016). The goals of mathematics teaching in the Ethiopian schools were considered when developing the framework for discourse-based instruction.

This study conceptualizes learning mathematics as participation in mathematical communication and social interactions, in which students engage in thinking, agreeing or disagreeing, talking about mathematical concepts and relationships (Cobb, Yackel, & Wood, 1992; Lampert, 1990; Sfard, 2008). Moreover, discourse-based mathematics instruction is characterized by active participation and engagement of students in the interactive classroom mathematical discourse practices orchestrated around carefully designed tasks. In the discourse-centered classrooms, teachers allow and facilitate students to talk about mathematical concepts, explain, justify, listen to each other and share ideas, and compare solution procedures (Lampert & Blunk, 1998; Pourdavood & Wachira, 2015; Stein, 2007).

## Theoretical Perspectives and Literature Review

Multiple theoretical perspectives are consistent with discourse-based mathematics instruction. From a constructivist perspective, "knowing and learning is constructed by individuals as they participate in and contribute to the classroom activities" (Pourdavood & Wachira, 2015, p. 18). Mathematics is about reasoning, discussing and sharing ideas, developing understanding, explaining how a problem can be solved and extended, and justifying and communicating how the solution is obtained (Maguire & Neill, 2006). From another perspective, "mathematics learning as a social endeavor that is achieved through discourse in the classroom between teacher and students, and among students" (Griffin et al., 2013, p. 9). Mathematics learning as constructive, social, and cognitive activity (Schoenfeld, 1994) takes place through participation in social interaction and communication (Smith & Stein, 2011). This paper supports the idea that mathematics should be taught in a way that reflects the nature of mathematics itself (Lampert, 1990; Schoenfeld, 1994). Thus, this calls for the need to create a classroom learning atmosphere that allows students to engage in sharing of each other's ideas, agreeing or disagreeing, questioning, thinking, and explaining of one's ideas to others (NCTM, 1991; Forman, 1996). Characteristics of classroom instruction that promote the development of students' understanding of mathematics include "(a) the nature of classroom tasks, (b) the role of the teacher, (c) the social culture of the classroom, (d) the kinds of mathematical tools that are available, and (e) the accessibility of mathematics for every student" (Hiebert et al., 1997, p. 2). Also, the TRU Math framework captures the attributes of powerful mathematical classrooms that foster mathematical understanding, thinking, and reasoning (Schoenfeld et al., 2014). According to Schoenfeld et al. (2014), the framework is "comprehensive that encapsulates what is known to be high-quality instruction; it *problematizes* instruction and focuses on what counts in instruction" (p. 16).

### *Describing Mathematical Discourse*

Pirie and Schwarzenberger (1988) defined mathematical discourse as a focused talk on a given mathematical task in which a small group of students work together to discuss and exchange their ideas with one another. Mathematical discourse practices include the use of language, symbols, and representations to express, think, and share mathematical ideas, comparing solution procedures, defining mathematical terms, explaining or justifying, asking questions for clarifications, solving word problems, and presenting solutions (Moschkovich, 2007). According to Pirie and Schwarzenberger (1988), classroom discourse plays two major functions: "the cognitive function of talk and communication function of talk" (p. 462). The cognitive function refers to one's readiness to participate and share ideas and thinking; it allows one to reflect on mathematical ideas while. Conversely, the communication function refers to the communication of one's idea and thinking to peers in the group or challenging of each other's reasoning and ideas (Pirie & Schwarzenberger, 1988). Crucially important is how well the classroom discourse is organized and structured to engage students in exchanging ideas and constructing meaning about mathematical contents (NCTM, 1991).

### *Mathematical Discourse from Participationist Perspective*

The participationist perspective views learning mathematics as "a process of enculturation into mathematical practices, including discursive practices (e.g., ways of explaining, proving, or defining mathematical concepts)" (Barwell, 2014, p. 332). Thus, mathematical learning is conceptualized as actively participating in mathematical discourse practices (Forman, 1996; Sfard, 2008).

### *Mathematical Discourse from a Sociocultural Perspective*

From a sociocultural perspective, which views learning mathematics as a form of active participation in classroom discourse and social activity (Forman, 1996; Goos, 1991), social interactions play a role as a means of teaching and learning mathematics in collaborative learning environments (Goos, 1991). Sociocultural practices include peer scaffolding and social norms (Fischler & Firschein, 1987; Goos, 1991; Maguire & Neill, 2006). Thus, the teaching and learning of mathematics should reflect the nature of mathematics as a social process and collaborative activity (Goos, 1991).

### *Mathematical Discourse from a Cognitive Perspective*

Cognitivists hold the view that when students are engaged in challenging mathematical tasks in explaining and justifying their solution procedures and methods to each other, they develop deep mathematical understanding and reasoning (Hodara, 2011). When students engage in collaborative small group work on mathematical tasks, they develop reflective, reorganized, and collective thinking (Goos, 1991; Hodara, 2011). The cognitive perspective regards social interaction as a factor that affects learning (Kumpulainen & Wray, 2002), where learning is viewed as "a variable that can be partly explained by the characteristics of interaction and social context" (Kumpulainen & Wray, 2002, p. 22).

### *Mathematical Discourse from Socio-Constructivism*

According to the constructivist theory, learning occurs through interaction with the learning environment (Maguire & Neill, 2006). For the construction of knowledge by students, this theory underlies the importance of teaching context, students' prior knowledge, and active interaction between students and the content in the context (Major, 2012).

From the socio-constructivist perspective, learning mathematics takes place through interactive discourse among students and between teachers and students (Sfard, 2008). Social-constructivists hold the view that learning is a social process in which students collaboratively work in groups on given tasks to construct new ideas and concepts based on existing prior knowledge and skills (Hein, 1991). Classroom activities that are collaborative, social, and meaningful promote students' co-construction of their knowledge through social interactions (Harasim, 2017).

## Contextualizing the Five Dimensions of the TRU Math Framework

A detailed explanation of the five dimensions of the TRU Math framework is presented in the work by Schoenfeld et al. (2014). The first two dimensions, the *Mathematics* and *Cognitive Demand*, focus on the "richness" of the mathematical content in terms of "productive struggle" (Schoenfeld et al., 2014, p. 7). The *Mathematics* dimension captures whether the mathematics in the classroom instruction is "coherent, focused and connected to other concepts" (Schoenfeld et al., 2014, p. 2). The mathematics dimension focuses more on the interactions between teacher and content than the interactions between student and content (Schoenfeld et al., 2014).

The cognitive demand dimension focuses on whether classroom interaction provides appropriate opportunities and support for students to actively engage in the learning task. It refers to the interaction between student and content and teacher and content but places a stronger emphasis on the student-content interaction (Schoenfeld et al., 2014). The other three dimensions of TRU Math focus on learning environments that ensure students have equitable access to learning content, develop positive mathematical disposition, and promote the use of assessment for learning.

The TRU Math framework guides directions of what to focus on as instructional components for the design and implementation of discourse-based instruction. The goals of mathematics teaching (MOE, 2009) were used as a hallmark for rationalizing the dimensions of TRU Math in an Ethiopian context (Table 4.1).

The contextualization of TRU Math dimensions may inform aspects of classroom mathematics instruction that include the following:

- *The Mathematics*; What should teachers do to help students understand the mathematical ideas of each lesson and develop reasoning and problem-solving skills and apply what students have learned to solve problems?
- *Cognitive demand*; What challenging problem situations should be offered to help students develop solid, applicable, and extendable mathematical knowledge and skills?

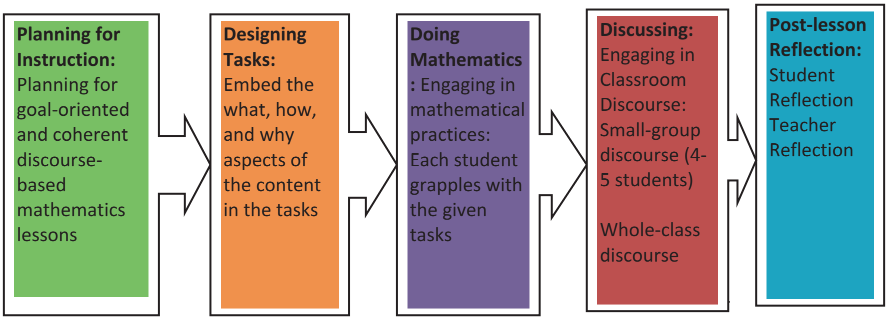
- *Access to mathematical content:* What types of mathematical tasks would engage students in the classroom discourse?
- *Agency, authority, and identities:* What learning environments should be created that encourage students to participate actively in classroom discourse practices?
- *Uses of Assessment:* What do teachers do to assess and improve student learning?

**Table 4.1** Contextualization of the TRU Math dimensions in an Ethiopian classroom context (MOE, 2009; Schoenfeld et al., 2014)

| Contextualization of TRU dimensions in Ethiopian math syllabus | An aspect of the dimensions of TRU Math during the classroom instruction |
|---|---|
| 1. *The mathematics*<br>Acquisition and development of solid, applicable, and extendable mathematical knowledge and skills | Goal-oriented, connected and logically sequenced mathematics lessons that focus on mastery and understanding of important algorithms, procedures, concepts, vocabularies, definitions, theorems, and proofs; promote knowledge integration by connecting students' prior and existing knowledge; connect mathematical concepts to real-life problems; and maintain the accurate use of mathematical language when writing proofs, deriving formulas, explaining theorems, and solving equations |
| 2. *Cognitive demand*<br>Challenging problem situations: Planning learning contexts and activities that engage students in kinds of mathematical thinking and reasoning | Select or design tasks that demand from students some degree of mathematical challenge and engage students in formulating, explaining, justifying, representing, and interpreting solution methods and strategies and solving problems |
| 3. *Access to mathematical content*<br>Active participation in collaborative learning settings | Sequence and present the tasks in a logical manner. Encourage students to participate in collaborative group work on given tasks (e.g., problem-solving tasks, proof-related tasks, open-ended tasks, and tasks that require students to provide explanations and justifications.<br>Use task clarification strategies to help and guide students when the given tasks give them some challenge and difficulty |
| 4. *Agency, authority, and identity*<br>Attitude, self-efficacy, and sensible: Students who are confident in their mathematics ability and who appreciate the elegance, power, and usefulness of mathematics | Include incorrectly solved tasks as part of the lesson; providing examples of real-life problem situations that can be formulated and represented using mathematical equations, diagrams, or graphs |
| 5. *Uses of assessment*<br>Review and feedback | The teacher interrogates students' understanding and misunderstanding by reviewing the main points of the lesson through questioning; providing constructive feedback to classwork and homework tasks |

## *Framing the Design of the Framework for DBMI*

Rationalizing the implications of the goals of mathematics teaching through the analytic lens of TRU Math framework, supported by a wide range of literature review on mathematics teaching and learning (e.g., Kilpatrick, Swafford, & Findell, 2001; NCTM, 2014; Schoenfeld et al., 2014; Smith & Stein, 2011), five instructional components, namely, *planning*, *designing*, *doing*, *discussing*, and *reflecting* were framed as elements of discourse-based mathematics instruction. Each component provides a set of guidelines that inform what teachers need to do to design and implement lessons that emphasize mathematical task-oriented classroom discourse.




A Framework for Discourse-Based Mathematics Instruction (DBMI)

The framework for DMBI centrally focuses on creating mathematical task-oriented classroom discourse in which students have opportunities to engage in challenging each other's idea, explaining, exemplifying, elaborating problem situations, and comparing and contrasting solution strategies (Ballard, n.d.; Gresham & Shannon, 2017; Hoyles, 1985; Lampert & Blunk, 1998). Creating a classroom learning atmosphere that involves students in explanation, justification, and exemplification is central to mathematics teaching and learning (Lampert & Blunk, 1998).

## *Discussion of the Components of the Framework for DBMI*

### Planning for Instruction

The planning for instruction is identified as a central element of mathematics teaching (Lampert, 2001, cited in Chazan, 2002; Superfine, 2008). It provides a set of guidelines that inform what teachers need to do to design classroom instruction that accommodates and integrates the "what," "why," and "how" aspects of mathematical content to be taught (Fujii, 2013; Kilpatrick et al., 2001). A well-connected and focused classroom instruction helps students develop the view that mathematics is an integrated whole, not a collection of isolated pieces of ideas and concepts (NCTM, 2000).

Planning for a lesson cannot be done without considering the interactions between the teacher and students and among students over the content of the lesson (Lampert, 2001, cited in Chazan, 2002, p. 188). Knowledge needed for teaching mathematics incorporates knowledge of the subject matter they teach, knowledge of students and their social and cultural contexts, and knowledge of how students learn mathematics (Goos, Stillman, & Valle, 2007). Lampert (2001) pointed out that the work of effective teaching begins with planning, in which the objectives stated in curriculum serve as starting points (cited in Chazan, 2002). Planning for instruction requires thinking and decision-making about how to organize, represent, and connect the mathematical goals of each lesson around big ideas (Daro, Mosher, & Corcoran, 2011).

Additionally, Lampert (2001) explained that:

> The work of teaching encompasses teaching while preparing a lesson, teaching while students work independently, teaching while leading a whole-class discussion, and teaching the whole class. The elements of teaching include teaching to establish a kind of classroom culture, teaching to deliberately connect contents across lessons, teaching to cover curriculum, and teaching students to be people who study mathematics in school (cited in Chazan, 2002, p. 189).

Lampert (2001) further explained how students are represented in planning lessons: while preparing a lesson, teachers think about students' ways of learning and prior knowledge as resources to anticipate what will happen in the classroom (Chazan, 2002).

## Designing Tasks for Classroom Discourse

The designing task component focuses on task choice that embeds core concepts, procedures, and applications. A lesson that incorporates cognitively demanding tasks instigates students to strive for mathematical understanding (Schoenfeld et al., 2014). Cognitively demanding tasks create more opportunities for students to foster their mathematical thinking and communication skills (Smith & Stein, 2011).

The types of questions and types of discourse elicited by the tasks utilized in the classroom instruction influence students' mathematical learning and attitudes (Anthony & Walshaw, 2007; Hiebert & Wearne, 1993; Stein & Smith, 1998). Tasks are the means through which the teacher and students communicate about mathematical contents and processes (Anthony & Walshaw, 2007) and allow students to talk, discuss, reason, and think about the content embedded within the tasks (Hoyles, 1985). The types of tasks may include computational tasks, proof-related tasks, graphing tasks, non-routine problems, and tasks that involve comparing and contrasting solution strategies and representations (Riccomini, Smith, Hughes, & Fries, 2015). Crafting higher-order questions that elicit students' mathematical thinking and reasoning should be part of the task design stage (Sullivan, 2011).

## Doing Mathematics

The “doing” mathematics component refers to each student’s independent engagement in the given tasks before engaging in small-group discussions. Students who get opportunities to individually engage in thinking about the mathematical ideas embedded within the given tasks generate one’s ideas and solution strategies to be shared during group discussions (Romano, 2014). When students individually think about the mathematical ideas embedded within the tasks, they communicate with their thinking, which is viewed as engaging in discourse practice (Lampert & Blunk, 1998; Sfard, 2008).

When doing mathematics becomes an integral component of mathematics instruction, students learn to apply their previous knowledge to solve, investigate, and explore problems (Flores, 2010) and develop a mathematical identity as “knower” and “doers” of mathematics (Van de Walle, Karp, & Bay-Williams, 2015, p. 39).

## Discussing: Engaging in the Classroom Discourse

The discussion component serves as a forum for interactive discourse to exchange and share students’ thinking and reasoning about mathematical ideas and concepts embodied within given tasks (Moschkovich, 2007). The component focuses on creating classroom discourse orchestrated around cognitively demanding tasks where the teacher and all students respectfully and actively engage and participate.

The kinds of talk and questions the teacher used to guide and monitor student interaction affect the quality of the classroom discourse (Hiebert & Wearne, 1993). The use of questioning strategies that demand students to “explain the reasons for their responses or define their positions will engage in deeper reflective, integrative thought than if they are asked to recall fact or rules” (Hiebert & Wearne, 1993, p. 397).

## Reflecting on Classroom Instruction

The *Reflecting* component constitutes the final stage of the DBMI lessons in which the teacher and students conduct a post-lesson assessment on the overall classroom instruction. It can help teachers assess how well students are moving on towards the mathematical goals of the lesson and to decide whether students are making appropriate connections between ideas within and across lessons. Reflection as an element of classroom lessons should illustrate what students can do with what they have learned to make connections within and across a series of mathematics lessons (NCTM, 2014).

> Students who reflect on what they do and communicate with others about it are in the best position to build useful relationships and connections in mathematics. If it is true that reflection and communication foster the development of connections, then classrooms that facilitate understanding will be those in which students reflect on and communicate about mathematics (Hiebert et al., 1997, p. 7).

Using questioning strategies, the teacher facilitates students to reflect on the ongoing instruction. Reflection through questioning strategies on classroom instruction can serve as a formative assessment strategy to assess students' learning of the main points of the lesson (Angelo & Cross, 1993). Allowing students to communicate through questioning-answering on what they have learned fosters their mathematical understanding and metacognitive skills (Van de Walle et al., 2015).

### *Discourse-Based Mathematics Lesson Design and Implementation*

Mathematical discourse as a method of teaching mathematics emphasizes active student participation in classroom discourse practices orchestrated around carefully designed tasks in which the core content topics to be taught are embedded. Students learn mathematics through explaining, justifying, questioning, and comparing solution strategies when they work together on given tasks in small groups leading to a whole-class discussion.

The design and implementation of discourse-based lessons can be carried out as described along with each component. Observing the planning for instruction through the five practices model (Smith & Stein, 2011), it would be reasonable to structure the DBMI lessons into individual work, small-group discourse, whole-class discourse, and reflection. A teacher begins the lesson by activating prior knowledge to enable students to engage in the mathematical task(s) and articulate the goals of each lesson. The classroom lesson goes through the stages of task presentation (3 min); individual work, each student grapples with the given task (3 min); small-group discussion (18 min); whole-class discussion (12 min); and reflection (6 min). Then the teacher facilitates and guides students' contributions while working on the tasks by applying the five practice models (Smith & Stein, 2011) (Table 4.2).

## Conclusion

Studies encourage teachers to engage students in collaborative, student-centered classroom discourse practices (e.g., NCTM, 2000, 2014; Pourdavood & Wachira, 2015). To help teachers orchestrate productive classroom discourse, this study attempts to contribute a framework for the design and implementation of discourse-based mathematics instruction. The framework not only guides teachers to design and implement discourse-based lessons but also helps teachers to make decision about the goal of each lesson. Furthermore, a teacher professional development program should be designed to help teachers gain knowledge and skills for the effective implementation of discourse-based instructional approach.

**Table 4.2** Description of mathematical discourse-based lesson design and implementation steps

| |
|---|
| Planning for instruction |
| • The planning begins by explicitly articulating the "what," "why," and "how" aspects of the mathematical topic and attempts to make a connection to previously taught topics (Fujii, 2013; Reeves, 2011)<br>• Set out the learning outcomes to be demonstrated after each lesson (Trocki et al., 2015)<br>• Plan questions that activate students' prior knowledge and elicit their thinking during the current lesson (Accardo & Kuder, 2017; Cirillo, 2013) |
| Designing tasks |
| • Select or construct tasks that embody core mathematical content and align with the goals of the lesson<br>• Ensure that the task designed provides access to all students<br>• Include incorrectly solved tasks to elicit student thinking (Accardo & Kuder, 2017)<br>• Craft task-specific questions to activate students' prior knowledge for the current lesson<br>• Plan strategies for selecting and sequencing the anticipated students' solutions strategies for each task |
| Doing Maths: Each student grapples with the given task |
| *Role of the teacher*<br>• Articulates the goal of the lesson and initiate the discourse by presenting the task and motivating students to start working on it<br>• Motivates students' thinking by asking questions that connect the previous lesson and current lesson<br>• Encourages students to think about the task and generate solution strategies (Goos, 1991; Hofmann & Mercer, 2015) through clarifying questions: "what do you think that…?" "What happens if you do…?" "What does the task ask you to do?"<br>• Appreciates the contribution of all students and use students' ideas to guide the group discussion (Anthony & Walshaw, 2007)<br>*Role of the students*<br>• Grasp the basic ideas of the given task and make contribution to group discussion |
| Discussing: Engaging in the classroom discourse |
| *Role of the teacher*<br>• Informs students to appreciate and respect each other's ideas and mistakes (Hofmann & Mercer, 2015; Store, 2014).<br>• Listens to what students talk about the given task<br>• Elicits student's thinking through questioning (Hiebert et al., 1997)<br>• Guides and scaffolds students' communication and discussion by posing questions that challenge each other's thinking (Goos et al., 2007; Pourdavood & Wachira, 2015)<br>• Encourages students to explain, justify, or share their solution strategies.<br>• Focuses on students' use of appropriate mathematical language.<br>• Motivates students to use representations to demonstrate their understanding.<br>• Identifies the source of students' disagreement if observed (Moschkovich, 2007)<br>• Guides students to reflect on what they have done on the task and what they have still stuck on for immediate feedback.<br>• Applies the five practice model (Smith & Stein, 2011) to guide and facilitate the classroom discourse<br>• Announces when the majority of students finish the discussion (Smith & Stein, 2011)<br>*Role of the students*<br>• Ask questions of each other for justification and explanation of solution strategies and listen to each other's responses to the given tasks and their teacher (Hiebert et al., 1997; Maguire & Neill, 2006)<br>• Contribute, share, and connect their ideas (Store, 2014).<br>• Appreciate the benefit of working together in small groups (Hofmann & Mercer, 2015)<br>• Talk about their ideas and thinking to each other (Moschkovich, 2007)<br>• Compare and evaluate their solution strategies and methods (Store, 2014). |

(continued)

**Table 4.2** (continued)

| Reflecting |
|---|
| *Teacher reflection on classroom teaching*<br>• Uses reflection as a means to assess students' progress (Angelo & Cross, 1993)<br>• Questions used for self-reflection on classroom teaching: What concepts were represented within the task? What procedures did the tasks require students to use?<br>• Did the lesson focus on making connections between concepts and procedures? If so, how so? If not, what should be done to make the connection in the next lessons?<br>• What questions were used to elicit student thinking and reasoning during the lesson? How did students respond to the questions?<br>• How well did the lesson engage all students in the task? How well did students work on a given task? Was it difficult? Does it invite all students?<br>• To what extent of the goal of the lesson has been achieved? What is the evidence for it?<br>• Reflects on strategies students used and the challenges they have faced.<br>• How did students working in small groups interact with each other?<br>• Elicits and guides students' reflection on the lesson (Goos, 1991)<br>*Student reflection on the lesson*<br>• What procedures have you learned in today's lesson? What concepts have you learned in today's lesson? How is today's lesson related to the previous lessons?<br>• What have you understood well in today's lesson? What have you not understood yet? How will you make sense of the whole lesson?<br>What problem-solving strategies have you learned? What challenges did you face in today's lesson? |

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# Chapter 5
# Ethnomathematics as a Fundamental Teaching Approach

**France Machaba and Joseph Dhlamini**

## Introduction

South African classrooms, like in other parts of Africa, consist of students drawn from a rich diversity of linguistic and cultural traditions who embrace mathematical contexts and practices from these geographical dispositions. A classroom provides a space to experience and share different cultures and social practices to advance the central dialogue of learning, and this is also manifested in mathematics classrooms. In a true cultural integration, a relationship develops between an abstract and everyday mathematics, as well as the language usage. Symbolic representation, which is at the heart of learning mathematics, becomes a critical component of teaching and learning in socially guided classrooms. A symbolic representation may be conceived as a visible item or object, which is a stand-in or a representative of a non-visible item or concept. Many mathematics classrooms operate on the assumption that all students should receive the same mathematical content, at the same time, and in the same way. Some mathematicians believe that there is a single approach to solve a mathematical problem (Orey & Rosa, 2008). This is an instructional assumption implying that culturally all students are similar and that the learning of mathematics is a linearly classroom event. However, teaching mathematics through ethnomathematics possibly debunks this myth and may open a myriad of instructional avenues to richly unpack educational content to students.

"Ethnomathematics" is regarded in mathematics education as the study of how mathematics is related to culture. It is the study that illuminates the parallel connection between mathematics and culture, thus emphasizing the critical role of one's culture in learning mathematics. The Greek word "ethno" refers to different cultural

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groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring (D'Ambrosio, 1990). According to D'Ambrosio (2001a), the term *ethno* covers "all of the ingredients that make up the cultural identity of a group: language, codes, values, jargon, beliefs, food and dress, habits, and physical traits" (p. 1). In addition, the Greek word "máthēma" means to explain, understand, and manage reality by counting, measuring, ordering, and so on (D'Ambrosio, 2001a, 2001b). Hence ethnomathematics focuses on how a culture relates to mathematics and how the recognition of the culture could enhance one's learning of mathematics. In so doing, ethnomathematics may be considered as the way in which various cultural groups are mathematizing, because it examines how both mathematical ideas and practices are processed and used in daily activities (p. 207). Some authors have viewed ethnomathematics as the study of anthropology and societal history (c.f. Cimen, 2014; D'Ambrosio, 2016; Kay, 1971). According to D'Ambrosio (2001b), "ethnomathematics is embedded in ethics, focused on the recovery of the cultural dignity of the human being" (p. 1).

All nations consist of a variety of cultures, and these variations may bring about several influences in response to mathematical knowledge. In its approach, ethnomathematics attempts to make a meaningful shift from formal and conventional academic mathematics to mathematics that is built from the surrounding practices and indigenous culture. Some ethnomathematicians study formal mathematics as an artifact of a particular culture or cultures. Ethnomathematicians refer to formal academic mathematics as *Eurocentric mathematics* or the *Western-European* view of mathematics (Powell & Frankenstein, 1997; Rosa & Orey, 2012). Western-European mathematicians posit that mathematics springs full-blown from the textbook or from the teacher's head while students are not accorded the opportunity to realize the role of human beings of various cultures in developing mathematical ideas (Zaslavsky, 1991). In this view the role of human culture is downplayed. The Western-European academic views have often displayed an outright refusal to acknowledge a cultural identity and its potential influence on mathematical learning (D'Ambrosio, 1990).

In this chapter, with more teaching and learning examples, we demonstrate the applicability of ethnomathematics as a fundamental teaching approach in mathematics classrooms. We argue that ethnomathematics can be a teaching approach, which focuses on students' background, their immediate environments integrated with the euro-centric mathematics in a practical way as demanded by the curriculum. To this end, we first define what ethnomathematics is, highlight the importance of ethnomathematics instruction[3] in African mathematics classroom, reflect on the demand of the use of ethnomathematics in the curriculum, and finally show how it can be used as a teaching approach in mathematics classroom.

## What Is "Ethnomathematics"?

Over the years several definitions of "ethnomathematics" have evolved. The term "ethnomathematics" was first used in the late 1960s by a Brazilian educator, mathematics philosopher, and mathematician, Ubiratan D'Ambrosio, to describe the mathematical practices of identifiable cultural groups (c.f. Cimen, 2014; D'Ambrosio, 1985). D'Ambrosio (1985) defines ethnomathematics as "the mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labor groups, children of certain age brackets and professional classes." (p. 45). Gerdes (1988) simply defines ethnomathematics as the mathematics implicit in each practice (see also Cimen, 2014). Ascher's (1991) definition of ethnomathematics seems to pay special attention to non-literate cultures, resonating the notion of "the study of mathematical ideas of a non-literate culture" (Cimen, 2014, p. 524). What seems to be similar though in the definitions is the commonality of practices by certain groups of people representing a particular culture.

Despite the authors' varying versions of ethnomathematics, almost all definitions share four common assumptions (Cimen, 2014). According to Cimen (2014), the first assumption has to do with the epistemology of ethnomathematics. In short, this assumption suggests that mathematics is the product of human creation (Bishop, 1988; Cimen, 2014). This view does not only present mathematics as a space within which a theory could be generated but also as a cultural tool relevant to society. The second assumption of ethnomathematicians is their attributions toward the anthropological and historical findings of mathematics (Cimen, 2014, p. 526). This dimension explains the fact that societies across the world experience similar problems. However, the ways of responding to these problems vary from one culture to another. According to Cimen (2014), the fact that the strategies used to solve problems vary from one culture to another "does not mean that these problems or realities themselves were different" (p. 526). This means that different cultures develop different problem-solving strategies to similar problems. The third assumption is that almost all ethnomathematicians emphasize that Western mathematics has been imposed on other cultural groups by colonization and share the emergent need of searching the derivations of the mathematics of third-world countries (Cimen, 2014). The fourth assumption is the fact that all ethnomathematicians remain located in mathematics and are also pursuing research in mathematics education (Cimen, 2014). Hence ethnomathematics share the applicability of their research findings with mathematical education research.

Given its natural link with culture, ethnomathematics may be different from its well-known and mainstream mathematics. According to Cimen (2014), D'Ambrosio uses the term cultural groups "in an expanded form that also covers different social groups within a society (such as carpenters, street sellers, etc.)" (p. 524). These groups would have their own language and specific ways of obtaining mathematical estimates and measures, and ethnomathematicians would then be keen to study their practices and techniques (Cimen, 2014; Gilmer, 1995).

## Exploring Mathematics Instruction in an African Context

Mathematics has manifested itself in various ways in African cultures. Gheverghese Joseph (1997) has argued that "Egypt was the cradle of mathematics" (p. 65). One could argue that since Egypt is in the African continent, mathematical knowledge was first developed in Africa as opposed to Greece (Arismendi-Pardi, 1999). Mathematical historian Rouse-Ball (1960) admitted that the "Greeks … were largely indebted to the previous investigations made by the Egyptians" (p. 1, cited in Arismendi-Pardi, 1999, p. 4). In addition, Rouse-Ball (1960) argued that "Greek geometry was derived from Egypt" (see also Arismendi-Pardi, 1999, p. 4). In addition, other authors have argued that Egyptians used the notion of a zero long before it was supposedly invented and accepted (Lumpkin, 1996). According to Arismendi-Pardi (1999, p. 5), mathematics is a cultural product, and evidence suggests that its genesis was from the African continent. Given this background, Africa must acknowledge the significant role it has played in the foundations of mathematics. This realization may deeply influence our ways of dealing with mathematical concepts in African classrooms. Gerdes (2001) has noted:

> When students, pupils, teachers or future mathematics teachers believe that 'mathematics' does not have any roots in 'their' culture, a 'cultural-psychological' blockage exists that hinders the teaching and learning of mathematical thinking, a blockage that turns the realisation of the full development of the mathematical potential of the learners impossible. (p. 1)

Africa, being rich with a diversity of cultural dimensions, could contribute meaningfully to the development of ethnomathematics and its instructional positioning in schools. The Western-European influence might have generated a thick 'cultural-psychological" blockage that inhibits any efforts to promote mathematical knowledge from a cultural perspective. It is, however, noteworthy that several efforts are being made to incorporate African indigenous knowledge in the teaching and learning of mathematics. African researchers are now beginning to share their mathematics education research findings with the rest of the world.

Many ethnomathematicians have considered the launch of ethnomathematics to be the plenary address of Ubiratan D'Ambrosio at the fifth International Congress of Mathematical Education (ICME) in Australia in 1984 (Shirley & Palhares, 2013). Since then ICME has continually included groups devoted to ethnomathematics. Apart from South Africa, African countries generally have been weakly represented at the ICME (Gerdes, n.d.). Most of the papers presented at the ICME have underscored the importance of cultural aspects in mathematics education in African countries. The most important is a variety of papers written by African authors, South African included, to highlight the significance of ethnomathematics in African classrooms (e.g., see Gerdes, n.d.; Ilyyana & Rochmad, 2018; Imswatama & Lukman, 2018; Laridon et al., 2005; Narayanan, 2011; Nkopodi & Mosimege, 2009; Nursyahidah et al., 2018). The findings of some of these studies are discussed in the next sections.

## Infusing Ethnomathematics into the Curriculum

In South Africa the Curriculum and Assessment Policy Statement (CAPS) for mathematics provides useful guidelines and principles to facilitate the classroom teaching and learning of mathematics. One of the principles highlights the "Valuing Indigenous Knowledge Systems (IKS): Acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the constitution" (Department of Basic Education (DBE), 2011, p. 5). This principle calls for the teaching and learning of mathematics to prioritize the indigenous knowledge. Mosimege (2016) argued that for the mathematics teachers to realize the principle of valuing IKS as pronounced in the CAPS documents for mathematics, there has to be a consideration and integration of ethnomathematical studies into various content areas. Mosimege (2016) further mentioned that without this effort, the principle of the integration of IKS in the teaching and learning of mathematics would not be fully realized. Admittedly, looking at CAPS, the phrase "Indigenous Knowledge" was only mentioned once in the introduction of CAPS document. No mention of indigenous knowledge was made in the specification and clarification of the content sections of the South African curriculum. This suggests that teachers should be familiar with the kind of indigenous knowledge that they should consider in their classroom instructions to facilitate effective learning of mathematical concepts. This may further suggest that the integration of ethnomathematics into the South African curriculum was an afterthought idea. The fact that it was just mentioned once in the entire curriculum package may suggest that it was not originally intended for classroom consideration.

However, in spite of these curriculum gaps and the assumptions made, which may not be realized by teachers in the classrooms, it is important to initiate and preserve a dialogue that seeks to highlight the role of ethnomathematics in African classrooms. In this section we draw from South African literature (Cherinda, 2002; Herawaty et al., 2018; Ismael, 2002; Maure et al., 2018; Mogari, 2002, 2014; Mosimege, 2012; Purkey, 1998; Widada et al., 2018) that emphasize the mathematics instruction that is grounded on cultural perspectives, and we also consider some possible tensions of integrating indigenous knowledge into the teaching and learning of mathematics in the South African context. This section focuses on cultural practices such as games – Morabaraba, Malepa, Moruba—and cultural artifacts such as round huts (rondavels), wire cars, beadwork, woven basketry, and the South African flag, South African architecture, and the Ndebele mural art, which are not tied strongly to any one ethnic or cultural group in South Africa (see Fig. 5.1). We then use these examples to unpack mathematical concepts embedded in them that teachers could infuse effectively into their mathematics instruction.

The South African flag is a symbol of national pride in which mathematical concepts are embedded. These concepts include the measurement of angles, lines, proportion, tessellation, symmetry, reflections, and geometrical shapes. Purkey (1998) used six groups of activities based on cultural acuities, such as the South African flag, architecture, and the Ndebele mural art, to unpack mathematical concepts


**Fig. 5.1** South African cultural artifacts

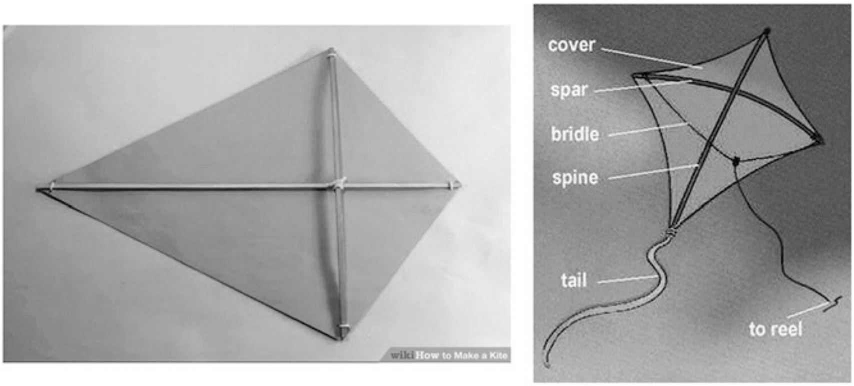


**Fig. 5.2** Examples of the construction of a kite

embedded in these cultural artifacts. With respect to South African architecture, the activities dealt with different types of huts designed by various cultural groups, such as the Basotho and Ndebele. One of the activities involved the measurements, the contractions of geometrical shapes, and the determination of area. Regarding the Basotho rectangular huts, the mathematical concepts that are embedded in them may be the right angles using Pythagorean triples, the theorem of Pythagoras, and the distance formula. For the colorful Ndebele mural art concepts such as translations, reflections and symmetry were explored (Purkey, 1998). One of the consequences of overdependence on foreign approaches to teaching mathematics is the lack of basic mathematical principles, which result to rote-learning and low achievement in mathematics, as could be seen in South Africa and other African countries today. Attempts to address this problem have emphasized that teachers should review their instructional strategies, such as integrating learners' cultures with mathematics. The latter could enhance students' active participation in mathematics learning.

The Mogari (2001) study involved the construction of kites in which a thick cotton, a newspaper page of tabloid size, and bamboo in the kite activity were used (see Fig. 5.2). Mathematical concepts embedded in the activities are angles, congruency, parallelism, area, shapes, and the properties of triangles, quadrilaterals, and hexagons.

Mogari (2002) noted that "the learning of mathematics can and will remain enjoyable if mathematics is taught in the context familiar to learners" (p. 53). The author further argued that students do not enter the classroom with blank minds, but bring with them ideas, conceptions, and experiences about mathematical concepts. This is also consistent with Piaget (1964, 2003) and Vygotsky (1978) theories of learning, suggesting that the construction of knowledge is dependent on what the student already knows (existing knowledge) and what the student must know or learn (new knowledge).

Furthermore, Mogari (2002) argued that mathematics learning that is facilitated through contextualized teaching approaches has potential to enhance logical and critical thinking and to improve the problem-solving skills of students by building new mathematical knowledge on what students already know. Mogari (2002) discovered that students could construct geometric concepts like straightness, angles, measurement, and congruence when children playfully construct miniature wired cars (see Fig. 5.3). This shows that the incorporation of the two discourses, namely, mathematics and real world context, could help students learn mathematics more effectively. In addition, Mogari (2002) argued that the latter teaching approach could facilitate the bridging of the "contextual gap" between school mathematics and mathematics outside the school in the area of geometrical or space-related mathematics. Mogari emphasized that bridging the gap could encourage and motivate students to learn mathematics in a more meaningful way. The author has maintained that mathematical concepts and ideas embedded in informal cultural practices could be identified to create awareness of the significance of ethnomathematical[4] activities to improve mathematics instruction (p. 53). In short, Mogari suggests that connecting mathematics to students' everyday experiences has potential to enhance the meaningful learning of mathematics.

In a slightly different context, Laridon and Presmeg (1998) conducted a study in South Africa that aimed to discover and apply mathematical concepts that are related to mountain bikes (see Fig. 5.4). Mathematical concepts discovered from

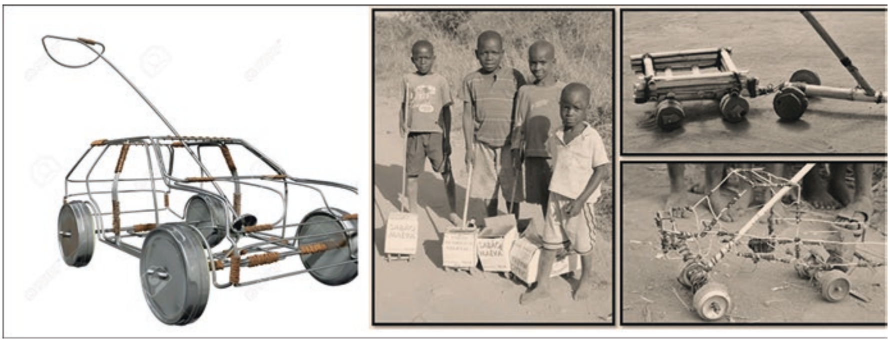

**Fig. 5.3** Pictures of children's wire cars

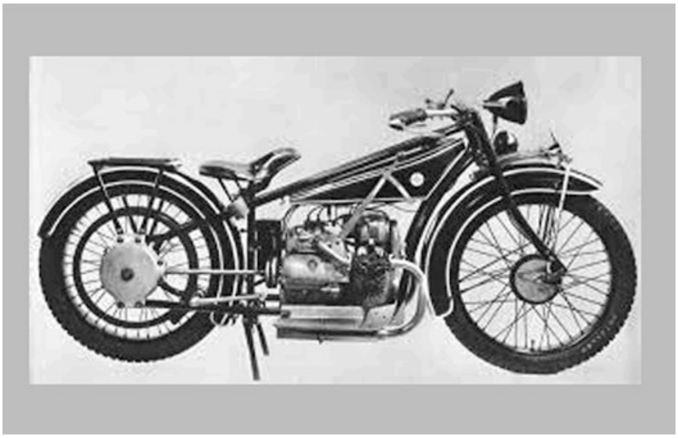

**Fig. 5.4** A picture of a motorbike

**Fig. 5.5** Pictures of the Malepa game

this study included the congruent circles, radius, triangles, and ratio concepts. These South African studies highlight instructional opportunities to facilitate the linking of mathematics to cultural practices to elevate the teaching and learning of mathematics in schools. The studies have highlighted the importance of merging culture and mathematics to enhance meaningful learning of classroom mathematics.

## *South African Localized Children's Games*

Some of the culturally specific games that are played in South Africa have a strong connection with the teaching and learning of mathematics, and these are Malepa (string figure gates), Morabaraba (played on a game board), Moruba (a Mancala-type game), Diketo, Ntimo, and Ma-dice (see Fig. 5.5).

Regarding the Malepa game, Mosimege (2016) stated that one person may play the game to determine the number of gates that can be made and learn the rules or the moves for making one gate and to make the next number of gates. When players are engaged in this game with other players, they talk about how the gates are made using relevant terminologies to show how the various manipulations are carried out. The following mathematical concepts are found in the analysis of the Malepa game or string figure gates (Mosimege, 2016, p. 27):

- Identification of a variety of geometric figures after making the different string figure gates: triangles and quadrilaterals (depending on how the string was stretched, quadrilaterals are also specified into squares and rectangles).
- Specification of relationships between various figures and generalization drawn from these relationships, for example, the number of triangles (y) is related to the number of quadrilaterals (x) by the formula $y = 2x + 2$.
- Symmetry in terms of performance of some steps in making the gates—an activity performed on one side being similar to the activity performed on the other side; exploration of the different kinds of symmetries and the related operations in the different gates—bilateral (reflection), rotational, and radical symmetries—deconstructing the gate along a specific line of symmetry which ensures that the string does not get entangled.

Nkopodi and Mosimege (2009), p. 385) listed some of the mathematical concepts that are found in the analysis of Morabaraba as follows:

- Identification of various quadrilaterals (squares) and the similarities and differences between them.
- Ratio and proportion between the lines and the squares making the complete Morabaraba board.

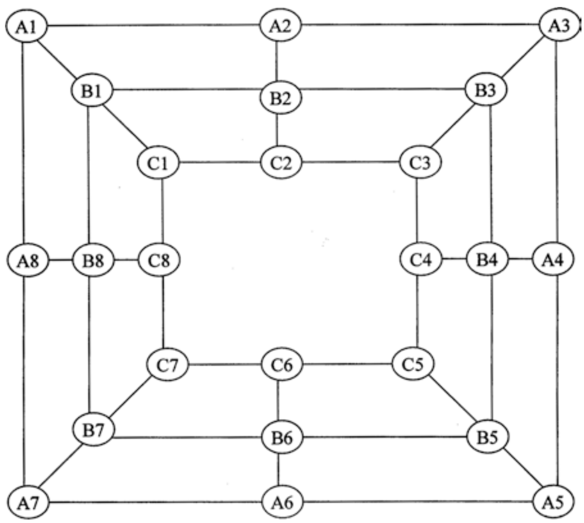


**Fig. 5.6** Picture of the Morabaraba board

- Symmetry is observed in at least three different instances, namely, (1) the various sides of the board, (2) within each side of the board, and (3) the placement of tokens and repetitive movements of the tokens on the boards.
- Logical deductions in the execution of the various steps of the game.
- The counting of tokens.
- Addition and subtraction of the tokens until a game is won based on the remaining number of tokens.

Nkopodi and Mosimege (2009) stated that as students engage in a variety of games on the Morabaraba board, they use various terms while engaging in the game. Therefore, teachers can use these terms to correct, introduce, and highlight some of the mathematical concepts as indicated above. Moreover, students may be encouraged to use the language of mathematics while playing the game so that understanding of the concepts can be encouraged. It must however be noted that some of the researchers have practiced caution in highlighting the importance of integrating cultural games and artifacts when teaching and learning of mathematics, and this aspect is addressed in the next section of our chapter.

## Possible Tensions in Connecting Mathematics and Everyday Knowledge

Other researchers have highlighted the possibility of unintended tensions when incorporating students' everyday life experiences into classroom mathematics instruction (Cooper & Dunne, 2000; Koirala, 1999; Lubieski, 2000; Nyabanyaba, 1999; Sethole, 2004). These researchers have argued that the inclusion of mathematics and students' "real-life" experiences may bring into play the following tensions: (1) mathematics may get lost or be watered down (backgrounding) when prioritizing contextualized instruction, (2) may expose differences between the attainment of the recognition and realization rules in relation to students with different socioeconomic backgrounds, and (3) may promote gender bias and impose unintended language demands. These issues will be discussed in some detail as follows.

### *The Backgrounding of Mathematics*

Koirala (1999) has argued that the teaching of mathematics using students' everyday context does not necessarily enhance their understanding of academic mathematics. The author gave a contextualized mathematical problem-solving task, involving aspects of everyday shopping, to a group of pre-service teachers. The students were enrolled in a course entitled "number system" and were taught by the author at the Eastern Connecticut State University. The author observed that pre-service teachers opted to solve the contextualized problem-solving task using concepts of simple division instead of using conventional and routine mathematical concepts and procedures. Koirala (1999) and Machaba (2017) have noted that although the learning of mathematics through everyday context could be fascinating for most of the students, many of them could not make a leap from their everyday experiences to academic mathematics. Koirala (1999) suggested that teachers need to make a concerted effort to help students connect every day and academic mathematics (p. 3).

Similarly, Nyabanyaba (1999) asserted that mathematics gets lost or may be backgrounded when aspects of contextualization are considered in mathematics lessons. Study participants were given a contextualized mathematics task based on the Premier Soccer League (PSL) football log. Students were able to respond to the PSL task without having to concentrate entirely on the mathematics embedded on the task (Nyabanyaba, 1999). Another example involved students' participation in the Independent Examination Board (IBM) where a student was asked to identify a soccer team that would eventually win the league and to predict the score with which the team would win. The score prediction was based on the comparison of two South African soccer league tables involving popular local teams like Orlando Pirates, Kaizer Chiefs, and Moroka Swallows. The student responded, "Moroka Swallows won 3-1, I know, because I was there when they played." This is one of the tensions that emerge when learners are dealing with contextualized tasks. Moreover, the study revealed that it is likely that differences in terms of gender between students could play out when responding to contextualized tasks, since not all students are able to understand the "key," such as the one used in football log tables. Therefore, Nyabanyaba (1999) maintained that teachers must be responsible when selecting tasks so as not to perpetuate inequalities. The author argued that teachers should intervene when students are dealing with such contextualized tasks.

In his investigation of the practical experiences and challenges of two teachers when attempting to incorporate everyday experiences into mathematics, Sethole (2004) found that "the foregrounding of social aspects of the everyday, as was the case in one of the teachers' classes, seemed to render the mathematics inaccessible" (p. 24). In other words, the meaningfulness of the context in relation to the lives and experiences of the teacher and the learners may overly emphasize the context such that mathematics may not be accessed (Machaba, 2017). Sethole (2004) used Dowling's concept of myth of reference and concept of participation to analyze the study data. The author likened the backgrounding of mathematics to a myth of participation, which entails a belief that mathematics is a tool needed to engage in the everyday experiences. For an individual to participate fully in a practice or a culture, mathematics is necessary. School activities which foreground non-mathematics and limit explicit reference to mathematics characterize this myth. In other words, mathematics is regarded as a "ticket to participation" (Dowling, 1998, p. 16).

In Bernstein's terms, this may suggest that the teacher and students in the Sethole (2004) study seem to pose the recognition rule (see Bernstein, 1958). However, they could not move to the epistemic level of mathematical production or recognize the legitimate text. Furthermore, Sethole found that the foregrounding of mathematical goals, as was the case in one of the teacher's classes, may motivate teachers to recruit what may be considered a dead-mock reality. Adler (2001) used the concept of transparency (in Lave & Wenger, 1991) to explain the leap from the context to the school mathematics. Adler (2001) and Lave and Wenger (1991) have argued that the context could be used as a resource or a tool to access mathematics. The context should be seen to be both visible and invisible. It needs to be visible so that it can be noticed and used and simultaneously invisible so that attention is focused on mathematics and not the context. Adler (2001) seemed to suggest that the context needs to be backgrounded so that mathematics could be in the forefront.

### *The Influence of Students' Socioeconomic Differences on their Learning*

Some researchers have argued that the inclusion of everyday life situations in mathematics creates disparities in performance between learners of diverse socioeconomic backgrounds. Cooper and Dunne (1998), Cooper (1992, 1998), Boaler (1997), and Nyabanyaba (2002) found that students coming from a poor family background did not leap from the context or their everyday knowledge to mathematical knowledge production. Students get stuck on their everyday knowledge possibly constraining the learning of mathematics. Cooper and Dunne's (2000) study in the UK illustrated differential access to "realistic" school mathematics assessment items across social classes. They provided an epistemic explanation for the poor performance of working-class children in the UK that crosses the boundaries between the "everyday" and the "school" concepts. The study showed that the working-class children remain tied to their "everyday" knowledge when faced with a realistic question and consequently fail to negotiate the boundaries between the everyday and the school context.

Cooper and Dunne (1998) illustrated with reference to Bernstein's (1996)'s "recognition rules" and "realization rules" the difficulties that students experience with the realistic mathematics items. Cooper (1998) used Bernstein's framework to argue that, even if students are quite familiar with the recognition rules, those who are from the working class may not know the realization rules required to perform effectively in "realistic" mathematics examination items. Cooper (1998) argued that middle-class children may successfully navigate from context to mathematics. The author uses various examples to confirm this claim. For instance, Cooper (1998) used the example of the student named Mike who is from a working-class environment. Mike's response to a realistic task remained tied to the material base of the question. The question required students to sort the rubbish from the sports field into one of the circles drawn. The rubbish to be sorted included a newspaper, a cool drink can, a bottle of mustard, a pen, and a carton of milk. The marking scheme indicated that it was acceptable for students to sort by "shape of container, by being edible or drinkable, and so on" (Cooper & Dunne, 2000, p. 50).

Mike's sorting categorized the materials as either paper or metal as Mike wrote "meatle." Another student, Diane, from the middle-class environment responded by demonstrating the recognition of both material and metacognitive sorting. Diane's task sorting prioritized two-dimensional over three-dimensional diagrams, although she also demonstrated awareness that sorting can be done at the material level. The investigation therefore reveals that Mike knew the recognition rule, but not the realization rule that Diane demonstrated. This literature influenced the author to look at how students' habitus and everyday context influence their learning of mathematics, not by focusing on a comparison of social class, but by remarking on some of the similarities and differences, if any, across the two schools with contrasting backgrounds.

In Lesotho, Nyabanyaba's (2002) study investigated the impact of "realistic" mathematics items on both students' access to the school-leaving examination. The study found that widespread poor performance characterized the Basotho students in the school-leaving mathematics examination. There was evidence that some of the students were aware of their choices, while others were frustrated by "confusing" the nature of the realistic context. Put differently, most of the students demonstrated "absences" in their responses. They did not even attempt to answer the realistic items that they found confusing and intimidating. According to Bernstein, the Basotho students did not know the recognition rule. However, some of them, who were regarded as coming from a rich family background, could demonstrate mathematical competencies after the researcher had assisted them to recognize the context. Nyabanyaba (2002, p. 1) argued that "the explanation for the difference in performance is more about a socio-economic or even socio-political than an epistemic boundary." In conclusion, most of Basotho students seemed to lack both the recognition and the realization rules, contrary to the working-class learners in Cooper and Dunne's (1998) study, who only know the recognition rules. It will be interesting to see whether the absences demonstrated in Nyabanyaba's (1998) study can be produced in a South African context, where social and economic inequalities are largely prevalent.

## *The Open-Contextualized Problem*

Boaler (1997, 1999) suggested that the infusion of mathematics into everyday life situations (open-contextualized problem) has an advantage in terms of making mathematics meaningful to learners. Boaler argued that when students are dealing with open, contextualized mathematical problems, they develop knowledge that could be applied outside the classroom situation. Contrary, to an "inert" knowledge (knowledge that is procedural, routine; knowledge gained through note learning that cannot be applied outside the classroom situation), Boaler argues for a "flexibale" knowledge (Knowledge that can be applied in different situation). Boaler therefore favors an open-ended, project-based approach to the contextualized mathematics problem. For example, Boaler (1997) argued that translocation of everyday life experiences to mathematics is more likely to take place if learners learn mathematics in a more integrated way through a contextualized, open-ended, problem-solving approach. She compares the mathematical experiences and achievements of learners in two working-class schools in the UK, schools she named Amber Hill and Phoenix Park.

The two schools adopted different approaches to the teaching and learning of mathematics. Phoenix Park was more open-ended, with learners spending most of their time working on problem-solving tasks. In contrast, mathematics lessons at Amber Hill consisted of rule-based, procedural activities with a lot of drill and practice. Learners from these two schools developed different forms of mathematical knowledge due to their different approaches. Learners at Phoenix Park were able to apply the knowledge they acquired inside the classroom to outside the classroom, that is, a flexible form of knowledge. For them, mathematics did not consist of rules, procedures, and algorithms. In contrast, learners at Amber Hill were unable to apply their knowledge to new problems and situations because their knowledge was rigid, consisting of remembering rules and procedures. This study suggests that the translocation from context to mathematics could only happen when mathematics is learned in a situation like the one of Phoenix Park where the classification between school mathematics and everyday experiences is weak (Bernstein, 1996).

Laridon et al. (2005) stated that teachers respond differently to ethnomathematical pedagogy. Some teachers are apprehensive about its use in the teaching and learning of mathematics in the classroom, while others do not know how to incorporate the two discourses. This could be due to the subject knowledge of teachers on how to infuse cultural games and artifacts into mathematics. Consequently, a teacher plays a vital role in mediating a transition from cultural context to mathematical concepts in the teaching and learning of mathematics.

## Conclusion

This chapter has highlighted the importance of ethnomathematics in enhancing the optimization of mathematics learning outcomes, not only in African but also in South African classrooms. The performance of South African learners in mathematics continues to be an area of major concern. This chapter has argued that poor mathematical performance could be linked to a seeming disconnection and disregard of students' everyday experiences in mathematics classrooms. These experiences are naturally tied to culture, which plays a substantial role in shaping students' perspectives and understanding when learning mathematics. South Africa has a notable cultural diversity to make mathematics instruction an exciting and enriching educational event that is prone to elevating learning outcomes. In this chapter, we have strongly argued that ethnomathematics is a cultural tool that should be utilized to generate productive instruction in mathematics classrooms. We have emphasized that it is an instructional approach that recognizes students' previous knowledge, background, the role that students' environment has played in informing their knowledge, and informal modes of problem solving.

Hence ethnomathematics uses this contextualized students' background as a springboard to generate a formally productive learning trajectory and experience. In the process mathematics teachers should have a sense of students' exposure and immersion to their respective cultures. The chapter has emphasized that while ethnomathematics may help students connect culture and mathematics to enrich their subject matter content, it may be difficult to bridge the often-perceived dichotomy between academic mathematics and daily life. Failure to achieve this integration may result in unintended educational tensions. Conventional teaching approaches have tended to disregard the meaningfulness of students' cultural background and its role in enhancing educationally productive mathematics classrooms. The new generation of students requires teachers who are innovative and, most importantly, who are sensitive to their lived experience and cultural encounters. To mitigate unintended educational tensions, we propose that mathematics teachers should be trained to create culturally rich mathematics tasks that are aligned to students' real-life experiences. With recent advances in technology and digitalization of instruction, we assert that further research should be done to infuse innovative approaches into ethnomathematics instruction to help mathematics teachers become effective in South African classrooms.

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# Chapter 6
# Use of Cultural Artefacts in the Teaching of Mathematics in Africa: The Case of Uganda

**Janet Kaahwa**

## Introduction

Mathematics should be studied by all because it is everywhere. Research on the ways mathematics is taught (e.g. Kasigwa,1990; Kikomeko– Mwanamoiza, 1991; Ssajjabbi, 1992) and the type of examinations set for learners (Magino,1988; UNCUST, 1999) concludes that mathematics learning for most learners in Uganda is by rote. Today in Ugandan schools, our experience as former primary students, teachers, pre-service teachers and consultants in pedagogical matters indicates that mathematics is taught mainly by telling. The teacher stands in front of the class and defines the topic and then proceeds to explain the concepts involved. He/she does an example of a calculation on the chalkboard and then gives an exercise to the learners (Ssajjabbi, 1992; Kikomeko–Mwanamoiza, 1991). This is not good enough to make children learn. They must participate in the learning process. Teachers should use teaching aids, develop activities in which they involve learners (Kaahwa, 2005) and act as guides. Teachers mostly claim not to have teaching aids (and that is mostly about commercially made teaching aids); however in the Rutindo project, the researchers engaged mathematics teachers in the use of cultural artefacts as teaching aids.

This chapter describes the research in which teachers were trained on methods that involve learners in the learning process. It also presents the views of community members concerning mathematics. Moreover, it describes the changed views of teachers and learners about mathematics after the project. Views about the project of the various stakeholders, such as the District Education Officer, the Curriculum Specialist for Mathematics in the National Curriculum Development Centre, the Teacher Educators and Family and Children's Worker from Family Life Network, are all presented.

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## Statement of the Problem

Several rural communities think that mathematics is mainly for boys (World Bank, 2016). Over the years, the resultant poor numeracy skills have become a greater barrier to economic and social well-being. In 2006 it was reported that "Sub-Saharan Africa accounts for the world's highest increase in total primary school enrolment", with more than 23 million children entering the classroom for the first time. Unfortunately, despite such enrolment, the teaching of mathematics has always been a source of concern. The PISA—Global studies, a survey by OECD across 34 countries in 2012, showed that 59% of 15–16 years old have phobia of mathematics (OECD, 2013). In Sub-Saharan Africa, most of the learners perform far below the international average: 31% of the primary six learners are classified as innumerate. In Uganda, for example, 38.8% of primary six learners are classified as innumerate, and the results of the 2018 teachers' examinations also show that teachers find mathematics to be the most difficult subject (Mukhaye, 2018). The World Bank also reports that teachers lack basic subject matter knowledge and pedagogical skills. For example, only 21% of primary four mathematics teachers could compare fractions, and 25% could assess learners' abilities (World Bank, 2019, May).

According to the last National Assessment of Progress in Education (NAPE) conducted in 2014, less than half of the primary six pupils acquired most of the competencies in Numeracy and English Literacy specified in the primary six curriculum (UNEB, 2015). In rural and disadvantaged areas, teaching mathematics is further compromised by poor physical conditions in schools and inadequate teaching and learning materials, as well as the shortage of well-qualified and trained teachers (World Bank, 2016). There has also been a recent influx of 1.2 million refugees, many of whom are children of school-going age.

The World Bank observes that children born today can be only 38% as productive when they grow up because of poor quality of education, less school years and the reality that some of the curricula do not respond to the needs of those who do not have pre-primary education. In Uganda, the US Ambassador, speaking at the Ministry of Education Sector review (Ahimbisiibwe, 2019,Sept.4), said that "currently, 1/3 of the children will survive to primary 7 and 15% who make it to primary 7 are unable to do simple arithmetic. A proportion of primary 2 pupils rated proficient in Mathematics dropped from 69.4% in 2008 to 41.5% in 2013".

This project therefore was conducted in a rural area in North Western Uganda and involved five primary schools in the Masindi District. It involved a needs assessment of the teachers, training the teachers on the new methods of teaching and observing them put into practice what they had been taught. It also involved the interviewing of the community concerning mathematics. The community was sensitised about mathematics and the learning of the children.

## Theoretical Framework

Mathematical knowledge is a set of prepositions together with their proofs (Ernest, 1991). The proofs are based on a reason with an assumed set of mathematical axioms as basis from which to infer mathematical knowledge. Knowing mathematics demands understanding its structures and the symbols. It involves understanding the interrelations among concepts and operations and the rules that can be used to manipulate and organise them to discover new patterns and properties (Resnick & Ford, 1981). To learn mathematics then is to form mental pictures of individual mathematics concepts and figure out the processes involved (Skemp, 1971). It also involves an acquisition of the ability to apply the knowledge acquired to real-life situations (Dienes, 1971). Teaching mathematics then entails helping learners to form conceptual structures, develop strategies and make connections between the mathematical symbolism and the surrounding reality (Onslow, 1991). Active learning is a form of social interaction that, according to Vygotsky, plays a fundamental role in the development of cognition. According to Vygotsky (1978), every function in the child's cultural development appears first between people (interpsychological) and then inside the child (intrapsychological). The potential for the cognitive development is in a certain time span, which Vygotsky terms "zone of proximal development" (ZPD). He said that full development during the ZPD depends on full social interaction. One of the ways to achieve social interaction is to use teaching aids from which one can develop activities capable of promoting active learning. Since teachers in Uganda rarely use teaching aids, this project suggested that they use cultural artefacts. D'Ambrosio (1991) coined a term "ethnomathematics" to mean techniques of understanding, explaining, managing and coping with reality in cultural environments in order to survive (which is one of the main endeavours of society). Gerdes (1985) said that mathematics is frozen in artefacts. Students unfreeze it out of them, and through cultural relevant activities, they can find what they learn meaningful (Davidson & Miller, 1998). Merttens (1995) demonstrated that one-way culture can be used in the teaching of mathematics. In one of the activities she gave the children, she asked them to find heights in terms of spoons, which are cultural artefacts.

## Treatment of Teachers as Learners

In the training of teachers, the project treated teachers as learners (which they were). Knowing that learning is a slow process and takes time, their training on artefacts use was done on a continuous basis. Although there were several workshops held and because it was a new concept, the teachers still needed more trainings on artefacts use in mathematics teaching and learning.

The project trained teachers to plan lessons and organise learner activities. In using these activities, the teacher acts as a guide, and the learners discover as they follow along. The project also taught teachers to add on the column of RELEVANCY in their lesson planning.

## Culture and Cultural Artefacts

Culture is a complex whole that includes beliefs, art, morals, law, custom and any other capabilities and habits acquired by man as a member of society (Tylor, 1903, p. 1). It is more than a collection of mere isolated bits of behaviour. It is the integrated total of learnt behaviour and traits that are manifest and shared by members of society. Culture is wholly the result of social invention; it can be seen as a social heritage since it is transmitted by precepts to each new generation (Shapiro, 1956).

Culture is ingrained in nature and in the human mind (Kaahwa, 2011). Human beings have, out of need, made things to use in life. These things are unique in every culture. Price-Williams et al. (1969) used pottery-making, a cultural practice, to show the role of skills in the growth of cognition. In Uganda, there are many tribes, and each tribe has a culture. Depending on the culture, there are varying artefacts (man-made objects). Cultural artefacts are always changing in line with cultural changes. For example, the present-day chairs are different from those of the past. Today jerrycans are used for water, unlike in the past when pots made of clay were used. Communities have evolved and adapted to modern trends. Industrial objects have substituted the traditional ones.

In the culture where the research was done, there are objects such as baskets, mats, motors, pestles and many others. The parents carry out daily activities with their school-going children. Mathematics being a human activity and a product of culture, these children experience a lot of it in their daily lives.

## Relationship of Mathematics to Culture and Cultural Artefacts

The project continuously trained teachers on using cultural artefacts in mathematics teaching. Below are some examples of cultural artefacts and their relations to mathematics (Fig. 6.1).

Baskets can be used to teach volume and cylinder, as presented in the artefact figures above. African baskets are woven through a variety of techniques, which form distinctive patterns to achieve the functionality the artefact serves, for example, shopping and storage. These varieties of patterns can significantly aid mathematics teaching and learning. A teacher can teach patterns, volume and circles.

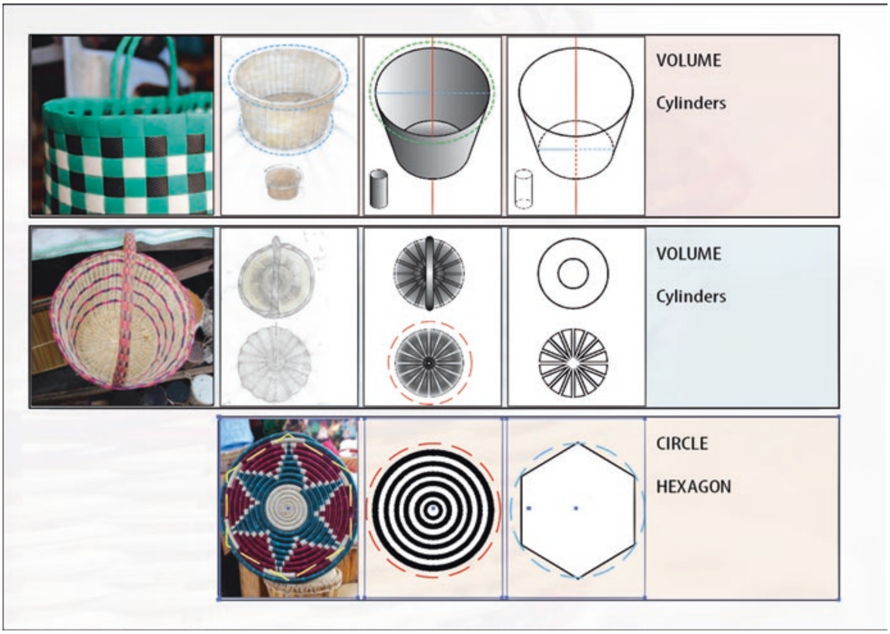


**Fig. 6.1** Baskets

## *Huts*

The African hut is a symbolic cultural object that reflects the traditions of its people. It is constructed from readily available local materials, such as wood, stone, mud (bricks), grass, palm leaves, branches, reeds or hides, using techniques passed down through generations. Huts serve as architectural structures for shelter and storage (granary). They are three-dimensional in form, normally with a pyramidal roof and circular or square walls, which can aid outdoor activities for mathematics instruction, especially in the rural areas. The teacher can take learners to a village homestead and teach triangles cubes, prisms, rectangles and prisms (Fig. 6.2).

## *Classroom*

In the classroom there are various objects to learn mathematics from. These can be brick walls, doors and windows. Walls are common attributes of constructions, such as houses, churches and hospitals. A teacher can take learners to a construction site and teach mathematics (Fig. 6.3).

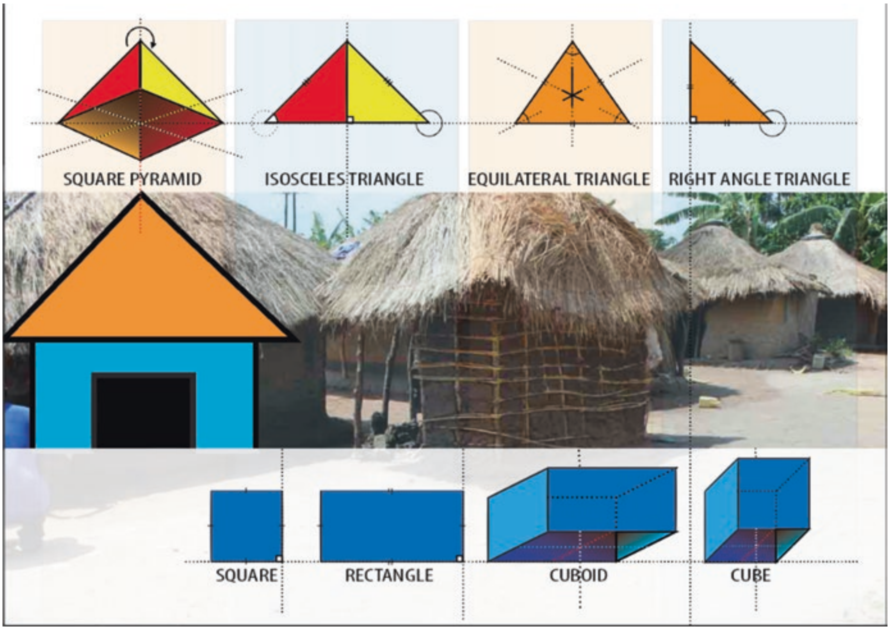


**Fig. 6.2** Huts

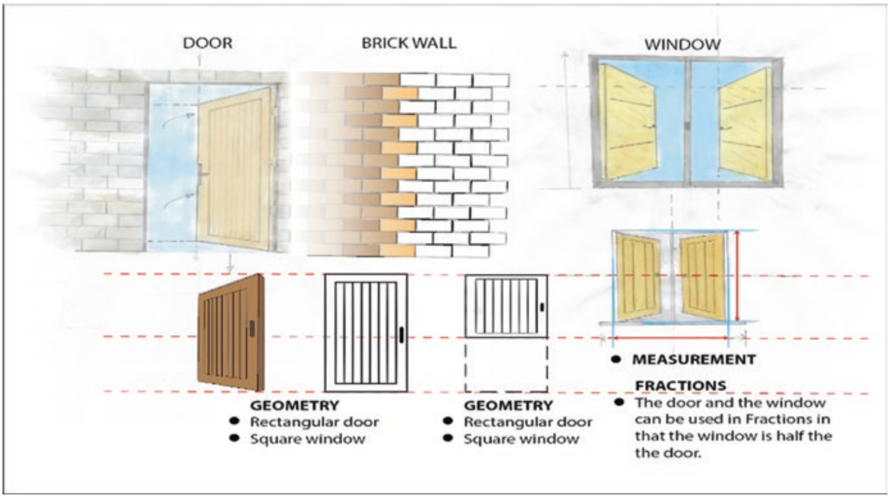


**Fig. 6.3** Classroom

## The Research Hypothesis

Teachers' use of cultural artefacts in the teaching of mathematics can make them change their attitudes towards mathematics and its teaching. It can also make the learners like the subject and enhance their mathematics' learning.

## The Research Project (Method)

Many learners have negative attitudes and fear towards mathematics because of poor teaching methods and discouragement. The project Rutindo Maths + Culture aimed at simplifying mathematics to children by training teachers to develop activities and design lessons to teach based on the things that the children are familiar with from their culture and environment.

First, the researchers created a survey that would allow the teachers to express their views and opinions openly, honestly and without restriction, confusion or judgment using a structured questionnaire. Therefore, a survey that was based on close-ended questions where the interviewee was able to provide concise answers as they deemed appropriate was designed. The wording of questions was kept as simple as possible. The survey was conducted in the intervention schools and the control schools. A total of 30 mathematics teachers from 5 selected (intervention) primary schools in the Masindi District responded to this survey. Every mathematics teacher was asked to respond to this questionnaire; thus no sampling was done.

Thus the project started with a baseline study, in which a needs assessment of teachers was conducted. In the five schools of the study, the attitudes of the teachers towards mathematics and its teaching were sorted for. Teachers' practices, such as scheming of work and lesson planning, as well as how often and when this was done were also established. They then undertook mindset training workshops after which they were trained on new methods of scheming the work and lesson planning. They were then trained on developing activities and using them in the classroom teaching.

Over 1 year the project trained 30 teachers in five primary schools in Masindi, a district in North Western Uganda, on how to use teaching/learning aids. About a thousand children were reached in this program. The research question that drove this study was: Can the use of cultural artefacts as teaching aids improve mathematics learning and attitudes of teachers and learners towards the subject?

Meanwhile there was classroom monitoring to see if and how the training was being put into practice. The project also surveyed the community's views concerning mathematics and carried out activities to change the mindset of parents, such as the negative influence of family and community on their children learning mathematics. Afterwards the views of the teachers and the learners about the new methods of teaching and mathematics were sorted for using interviews on randomly sampled individuals.

## Analysis

Data was analysed using both qualitative and quantitative methods. The qualitative data was obtained from key informant interviews and focus group discussions with teachers and community beneficiaries. Qualitative data also involved regular observation and recording of activities taking place in the project. The data was cleaned, summarised and categorised into thematic areas in accordance with the project objectives and used to complement the quantitative data. The quantitative data was obtained from structured questionnaires administered to selected learners (primary 5–primary 7), whose literacy competencies were judged to be appropriate. It was then analysed using Excel pivot tables for descriptive statistics, that is, frequencies, percentages, means and standard deviations, and presented in the form of tables and graphs.

### *Randomised Control Trials*

A randomised control trial survey was conducted at the beginning of the project in three schools selected to be control schools. A baseline was conducted on both the intervention schools and the control schools. Data was collected using a structured baseline tool. The data was analysed using Excel pivot tables advanced analysis package and reported in the form of tables and graphs. The baseline report on the control schools was later compared with the routine monitoring reports on the intervention schools to evaluate the effectiveness of the project intervention.

Detailed here under are the findings of the research.

## Baseline Study Findings

The baseline study revealed that the teacher-centred methods used lacked learners' activities. There was limited use of teaching/learning materials, especially concrete objects; no demonstration of relevance and connection of concepts to daily life; and limited understanding of the mathematics curriculum by teachers, especially its use and interpretation. The learning environment was limited to the four walls of the classroom setting, hence limiting outdoor activities. According to the teachers, there was insufficient availability of teaching/learning aids for mathematics (referring to commercially produced aids). Teachers had negative attitude towards mathematics and its teaching, as well as learners, parents and the community at large. Topics that learners failed to understand were the same that troubled teachers (algebra, fractions, integers, division and measurements, among others). Some topics were challenging because they were difficult to explain in the local language. Learners' challenges in mathematics learning were largely dependent on the teachers' failure to make and use learning aids while teaching. They failed to revisit and involve the learners in the learning activities; consequently leaners failed to ask questions and take lead of the lesson. Furthermore, the assessment was not thorough.

## Views and Attitudes of the Community Towards Mathematics

Five (5) people from the local community who were randomly sampled and interviewed about their views and beliefs towards mathematics mostly agreed that mathematics was important and useful in their everyday lives. Most of community members spoken to also acknowledged that their experience of learning mathematics was not good as it was filled with anxiety and fear; thus they abandoned the subject at first opportunity. Here are two examples from the conversation with the community members:

Isaak, Bodaboda, a motorcycle rider said:

> Mathematics is good. With my work, it helps me to calculate the fuel to use before I set off. If am to buy full tank, I can easily tell the amount of money to earn from it. I can tell how to give back the balance to my customers other than giving excess money. …So mathematics helps me to budget and balance everything well for my daily business ... Shortest route, distance, it's all about mathematics.

Rugongeza, a chef, said:

> Half of my classes were dodged because of mathematics. I hated it, I didn't enjoy it. Surprisingly, every day of my life today I'm working in mathematics. As a chef … with food, I deal with purchasing, portioning and cooking. When we are charging for a plate of food or a cup of coffee, we cost it. …So if you don't have mathematics in you, this is where many businesses are failing.

## The Research Intervention

After the baseline study findings, the project proceeded to put intervention measures in place. It carried out Mindset Change Workshops to address negative attitudes and ideas, such as fear of the subject and being ashamed in front of learners; thinking of mathematics as only a theoretical subject; believing that there is limited or no specific teaching/learning aids for mathematics instruction; thinking that teaching/learning aids are time-consuming; thinking that there are limited/no resources to purchase teaching/learning aids; thinking that there are complex topics and concepts; thinking that the national guides are not clear on how best to use them; having no interest in teaching mathematics but rather had to by need; and thinking that some learners are dull and just do not like the subject.

The research project also solicited for partner support from respective education specialists concerning education policy, guides and related matters and appropriate use of curriculum and other instructional materials and guides.

It also mapped out challenging concepts for teachers to teach and for learners to learn and cooperatively found solutions for them. It conducted teacher support trainings to address the identified challenges of inadequate preparation, which included scheming of work and lesson planning in cooperation with each other as a team.

Because teachers need support from the community, the project carried out activities to change the mindsets of the community (the community included the Parent–Teachers Association (PTA) and the School Management Committees (SMC)). These included mindset change workshops addressing the following:

The negative family and community influence of mathematics on learners. These included fear of embarrassment in front of their children (the learners); application of mathematics in daily life and in solving problems; and encouragement of positive parent support to learners.

## Findings of the Study after Intervention

At the end of the project, a survey was conducted. Below are the results.

### *Feelings Towards Mathematics as a Subject Now*

The teachers reported that they now enjoy teaching mathematics. A total of 13% indicated that there is limited time to do activities. Compared with the baseline findings demonstrating that 7% teach mathematics for lack of an option, the attitude seemed to have drastically changed. The situation now is how the teachers begin to integrate activities in the lessons.

### *Relationship between Teacher and Learner*

The teacher–learner relationships have improved 100% since the mindset change workshops. This training opened up the teachers' minds, and they began looking at mathematics teaching from a wider picture.

### *Most Difficult Subject to Teach*

When asked what was the most difficult subject to teach, 48% answered that it was SST (social studies), 35% answered English, and 17% answered science. Mathematics did not appear anywhere, reflecting a change of opinion when compared with the baseline study. More still it can be seen that the subjects (SST and English) that were indicated as the most difficult to teach do not involve much elements of mathematics. This is an indication that the teachers are now getting more accustomed to teaching mathematics and also enjoying it as compared to the past.

### Teaching Methodology Changed Teachers' Attitudes Towards Teaching Mathematics

On probing the teachers on the reasons why they now have a positive attitude towards mathematics, 96% of them said that the change in the teaching methodology from teacher-centred to leaner-centred had changed their attitude towards teaching mathematics.

### Artefacts Use Improving Learner Performance

When the teachers were asked why they thought the learners' performance had improved, 96% of them said that the use of artefacts had improved learner performance. They also noted that artefacts make the learners active and more involved in the learning as compared with the situation before when they did not use these artefacts at all. This has drastically improved their grades.

### Change in Teaching Methodology Improving Performance

When asked about the change in the teaching methodology and the pupils' performance, 87% of the teachers revealed that change in teaching methodology had greatly improved learner performance. Only 13% disagreed, showing that the majority believed that learning is taking place due to the change in teaching methods. They said that the class is more participative now compared with the situation before the project; the current change in teaching methods had increased formative learning, where learners were more exposed to why they were learning a given topic and how they could apply that knowledge in daily life.

## Preparing Schemes of Work As Often as Required

When asked whether they are preparing schemes of work now as often as required, 91% of the teachers said that they prepare schemes of work now as required. Only 9% do not. This reflects an improvement from the baseline study that indicated that only 68% of the teachers comply.

## New Lesson Plan Format Working for Teachers

When asked whether the new lesson plan format worked for them, 87% of the teachers agreed that the new lesson plan format was working well for them, whereas 13% indicated that they were not using the new format. Further discussions with the teachers who did not use the new format revealed that not all teachers were trained and guided in the formulation of the new format.

A total of 83% of the teachers revealed that the new lesson plan format is addressing the challenges that were faced before. Some of the challenges being addressed include the addition of lesson relevancy and evaluation criteria columns to their lesson plan. In the preliminaries section, consideration of the learning environment was also added.

As shown above, majority of the teachers (70%) have already started preparing their lesson plans using the new format (an intervention by the Maths + Culture project). Only 30% are still using the old format. However, there is a call for designed templates to make their work even easier.

## Views of the Teachers about the New Methods from the Interviews after Intervention

At the end of the project, the views of the teachers concerning new methods taught to them were sorted using interviews. They responded as follows:

The new methods make learners active, participate and learn faster. They make teachers deliver the content very well.

Basemera Maggorie, a teacher at Kitaguire Primary School, for example, responded as follows:

> …teaching methods… when the learners are also involved as the teacher guides them makes learners to pick the content faster than when the teacher demonstrates…. we were exposed to the use of cultural artifacts for example the use of sticks, stones and others when teaching. Their use helps the learners to be active, their participation very high and their attention is captured which helps the teacher to deliver the content very well.

The teachers' attitudes and those of the pupils towards mathematics changed. The performance of the pupils improved, and even slow learners began to enjoy the subject. The teachers learnt new methods.

Wasimire Saul, a teacher at Kitaguire Primary School, said the following:

> … with the intervention of Rutindo project, we were taken through a series of workshops which helped us to change our attitude towards the subject and consequently we also changed the learners' attitude towards the subject. They started loving it as a subject which they need to use in their daily life. We learnt involvement of the learners in the lesson which was minimal before. We learnt how to prepare teaching/ learning aids using cultural artifacts which the learners enjoyed so much.
>
> And we were also taken through procedures of assessment which has greatly improved the performance of the learners. ... Personally, I have loved the subject more because I have learnt that it's a subject to live with, a subject needed by everyone in daily life. …we have also learnt new methods of imparting knowledge to the learners, particularly those which involve the learners which was not the case before. The learning aids make the subject more real and practical and even what we would call the slow learners have come to enjoy the subject. By use of the various skills, touching, measuring they come to realize that it's not only memory tasking but they can also use their skills to do mathematics.

They learnt of the use of cultural artefacts as teaching aids. Earlier on they thought that upper primary pupils did not need teaching aids. Now they found out that the teaching aids worked for them too. They were also taught how to make lesson plans and add relevancy on the topic column.

Mutunzi Alex, a teacher at Kisiita School, said:

> But when rutindo project came in, more ideas, skills and methods of teaching pupils were involved and the teaching /learning aids …before Rutindo project… for upper primary. I thought that we use charts only but when it came and talked about cultural artifacts, we saw a lot of things our pupils in primary six and seven were missing ... When the project came, more things were developed, we can now use those artifacts to demonstrate, explain, guide our pupils. They can now participate in the subject and score highly. The Rutindo project trained us on how to make schemes of work, lesson plans. Even there is a step that was missing in the lesson plan- that of relevancy. After teaching the lesson, what is the relevancy? When the project came in that step was discovered.

> …the project introduced the learner centered methods whereby we involve learners in the teaching/ learning process and nowadays they are the ones I use. I was also not using teaching/ learning aids but when rutindo project came in, it introduced learning aids like the artifacts and the environment which improved on the learning process. I now use artifacts like bottle tops, sticks, straws, stones, baskets, threads, papyrus and mats to teach which has made the teaching/ learning process real and interesting in class.

The pupils can now understand mathematics better. All learners are involved in learning. Learners are able to make their own teaching aids from home. They learn mathematics at home as they make them. They are also doing problem solving as they make them. They involve their parents in the collection and making of artefacts. The teachers used to cane the children; now they do not anymore.

Kabahukya Sitenda, a teacher at Nyakakeeto Primary School, had this to say:

> …Now when rutindo project came we were taught on mindset whereby we were taught to begin by motivating learners. We were taught the new methods of teaching mathematics.

> Here we were taught to use artifacts like the counters, bottle tops, straws, the stones or leaves and others. So the children now can understand mathematics better. The project taught us use of the demonstration and group work methods where by all learners are involved in the teaching/ learning of mathematics. To add on we were taught on how to interact with these learners in the mathematics lesson so now we are able to interact with them. And we were taught how to make the artifacts in mathematics. Now after learning all these, children are able to learn mathematics and perform better in my classes. They are getting high marks than the previous years.
>
> And more so the learners are able to make their own artifacts. They make from homes after going back. And there is incidental learning in these children after the making the artifacts even if the teacher is not there. Again, the parents have been involved in collection of these artifacts. The learners are able to solve word problem in mathematics …These artifacts have made these learners interested and motivated…In the past years, I could come in class with my cane in the hand. And the children after seeing me entering the class, they started going outside, asking for permission "please teacher may I go out". You find in class very few learners when others are outside because I used to give them canes every day. But now when they see me coming to class with these artifacts, they begin clapping their hands, they start saying "yes yes teacher" now you find when the class is happy and ready to participate.

Pupils participated more in the lessons. The relationship between teachers and pupils has improved greatly.

Anand Drazoa, a teacher at Kyagwe Primary School, said:

> …When rutindo project came, it started by training teachers and involving children in doing some activities. Teachers were taught how to use local materials in conducting mathematics … sometimes children used to fear mathematics. I myself would say 'Mr. Math has come'. So because of the new systems and use of local materials, children started picking up. Even teachers themselves had interest in teaching mathematics. It was the frequent training which gave morale to the teachers. Hence it impacted teaching of mathematics by the use of local materials in the classes which encouraged children to participate even love mathematics more. More especially I have a child called Owenyiga Albert who was the slowest child in class. But now as I talk the boy is very active.
>
> Seeing me coming with those things in the class they begin singing the song: 'time for math,' everybody tunes to math. And most of the children feel responsible in giving out the learning aids. So if not planned properly you will find that the class will be disrupted because every child want to be responsible to give out the learning aids which are made out of local materials. Most of them were made by the children under the guidance of the teacher. Because of doing, I myself also felt that it has changed my life. I love math more. I feel the relationship between me and the children is increasing. I no longer put children at a gun point. I don't use sticks. So, I just guide each learner when it's time for group work. I am able to identify the learner who needs more help so at the end I have my successful lesson.

The teachers started to cater for individual learners and help them do problem solving. Teachers started to do continuous assessment and making a follow-up on the learners. Learners have started to ask where they do not understand.

Kakuru Solomon, a teacher at Kyagwe Primary School, said the following:

> …The project helped me to realize my mistakes and begin teaching math well. How catering for learners individually, helping them to solve problems, using real objects could improve on their learning. Their attitude towards math started improving. Then also making continuous assessment and making a follow up of these learners and their performance was mentored to me well by the rutindo project.

> Actually, the greater advantage which I have got in the project is that I came to realize that using visual learning aids, teaching /learning process is always amazing. And when I started using them always, my learners started understanding well something which was not like that before. Also, it motivated them to keep asking what they don't know, to always consult and ask where they see that they have failed to understand and I have always been helping them. Personally I have gained by improving on my teaching methods. One time there is a learner whom I tried to make a follow up on. She was in p.6 last year and she didn't like math at all. By the time I promoted her to p7 her performance was not good in math. But when I started helping her these days using learning aids, helping her individually, the girl started improving which was of a greater advantage on my side.

## Discussion and Conclusion

The project set out to train primary school teachers on the use of cultural artefacts as teaching aids in the teaching of mathematics.

It revealed that teachers had a negative attitude towards mathematics and its teaching before the project. This concurs with the Acting Academic Registrar at Kyambogo University who said, while releasing the results of primary school teachers that sat the 2017 grade three teachers' certificate examinations, that most teachers failed mathematics (Mukhaye, 2018). He also said that the overall performance of candidates had declined compared with that of the previous years and that primary school teachers find mathematics the most difficult among the subjects (Mukhaye, 2018). The World Bank (2019, May) also reported that teachers do not have basic subject matter knowledge and pedagogical skills. It also concurs with Kaahwa (2005), who said that teachers do not use teaching aids. One cannot simplify and put in form of activities what he/she does not understand. To teach mathematics is to help learners to form conceptual structures, develop strategies and make connections between the mathematical symbolism and the surrounding reality (Onslow, 1991). To do this one can use teaching aids from which one can develop activities capable of promoting active learning. Teaching aids from culture helps the learner to use "cultural environment" ("ethno" according to D'Ambrosio (1991)) to understand, explain, manage and cope with reality in order to survive. According to Gerdes (1985) people make cultural artefacts, and these artefacts exhibit mathematics. The mathematics frozen in these cultural artefacts is unfrozen by students when these are used as teaching aids.

Before the project, learners did not understand mathematics. It is also no wonder that community members reported that while in school, they had disliked it. Some of them dodged it only to find it in their jobs. Now learners are enjoying it and participating in it both in the classroom and at home. They even involve their parents, all because of teaching aids. Vygotsky (1978) said that social interaction plays a basic role in the development of cognition. So when teachers used teaching aids, learners started to understand mathematics, which had been hard to understand. This concurs with Skemp (1971) who said that to learn mathematics is to form mental pictures of individual mathematics concepts and figuring out the processes involved. It also involves an acquisition of the ability to apply the knowledge acquired to real-life situations (Dienes, 1971).

The relationship between teacher and learner improved. Teachers can now help individual learners. All this happened because of the use of teaching aids. There is no more dodging of classes, as one of the community members said in an interview. The children now enjoy the lessons.

In conclusion, the use of cultural artefacts as teaching aids makes teachers and learners like the subject. It also makes teachers like and enjoy teaching the subject; it makes the pupils enjoy learning mathematics and perform better in it.

## Way Forward

The teachers want this project to continue and spread to other teachers.

Wasimire Saul, a teacher at Kitaguire Primary School, said:

> …I would suggest that this project continues and other teachers either in Masindi or other parts of the country also get the opportunity of acquiring the same skills so that we can make the subject better.

Guma Alen, a teacher at Nyakakeeto Primary School, said:

> …I want to thank the rutindo project, the directors and I would like rutindo project to expand in the whole district. Other teachers also need to gain the knowledge which we have also gained.

Kabahukya Sitenda, a teacher at Nyakakeeto Primary School, said:

> …I encourage this project to continue with this program and involve in other teachers for better performance in schools.

The specialists also want the project to continue and spread to other teachers, for example:

Huuko John, a curriculum specialist (National Curriculum Development Center), said:

> …Whatever I saw and was able to contribute was impactful. To me it was a good program in trying to help the teachers in curriculum implementation because our teachers are not well versed with implementation. More schools or districts should be involved when funds permit. This can help improve our education especially in regard to mathematic which is a challenge.

Leya Drako, a human resource consultant (Family Life Network), said:

> …The project was unique. It reached teachers, parents and children to overcome mathematics anxiety. It was good to get children especially the girl child to overcome mathematics intimidation. The onus is to the educators since teachers think mathematics is a superior subject, difficult and complex to deliver, so this makes the learners intimidated at the subject. We need to help teachers consistently to recognize, identify and also encourage the learners by teaching mathematics as a hands-on subject related to every day's activities. Besides parents should be able to use the everyday objects and things we do at home to help

children learn at home because that way they will know mathematics is a familiar but not a difficult, strange subject.

Bitamazire Bazeleo, a senior education officer (Masindi District), said:

…The project truly was good and fit because it promoted practical work more than theory, more so it promoted resourcefulness and creativity among teachers which to me was an important impact observed. Since we have seen some tangible benefits from the project in our supervisions, we need more schools to be brought on board depending on funding so that more schools can be able to improve in the district and even beyond.

Nyabitaka Jennifer, an academic coach (Creative Learning Africa), said:

…The Math + Culture is promising because it uses culture to aid mathematics teaching and learning. The advantage is Mathematics and culture are so similar and this can easily work in other regions. The only question is, how does technology come in? What is Math and Culture suggesting to this regard as far as the future is concerned.

Batengu Basabasa, a project manager, said:

Teacher training programs should be initiated. The project has developed training manuals for future Activity Based Trainings with focus on culture.

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# Chapter 7
# Mathematics Teacher Educators' Experiences of Using Technology-Based Instruction in South Africa

**Jayaluxmi Naidoo**

## Introduction

Education plays an integral part in enlightening society by transforming the world in which we live. To transform education and the world we live in necessitates a transformation in how we teach and learn. This requires a change from the traditional 'chalk and talk' pedagogy, thereby creating a transformed education milieu. Also, as global society enters the Fourth Industrial Revolution (4IR), we require a transformation in instruction to cater to the needs of contemporary students. The 4IR involves progressive methods in which the use of technology is expected within society (Schwab, 2016).

Technology is ubiquitous in contemporary society, and within mathematics teacher education (Pyper, 2017), digital, audio and visual tools are gradually replacing traditional 'chalk and talk' instruction. Research suggests that the use of web-based technology, the Internet and the use of Information and Communication Technology (ICT) could bring about social change within the education sector (Du Plessis, 2013). Since all nations have distinctive and multifaceted contexts that define their educational policies (Aikenhead, 2017), the use of ICT to inspire social change within any society would be embraced. Social change refers to the changes that take place during interactions within society and leads to changes in thinking which affect the behaviour of society (Sharma & Monteiro, 2016).

Social change within South Africa is not novel; South Africa underwent a significant social change with the demise of the apartheid era resulting in a diverse democratic society. Hence, South African society may be depicted as multicultural and

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multilingual. Diversity in South Africa is accepted since South Africa consists of various linguistic, religious and cultural communities which include the 11 official languages,[1] as well as other linguistic communities from the rest of the world (Rodrigues, 2006). Language and cultural diversity enhance diversity within education milieus while simultaneously testing the effectiveness of a teacher's instruction (Yuan, 2018). Diversity within the ambits of this study encompasses issues of race, gender, ethnicity, socio-economic status, learning styles and English language ability.

Using ICT within education milieus has become the catalyst for change (Du Plessis, 2013). Society has accepted technology as part of contemporary life, and the use of technology-based mathematics instructional strategies is widespread (Ertmer & Ottenbreit-Leftwich, 2010). Consequently, within mathematics lecture rooms, the use of software programmes is becoming progressively regular; for example, the Geometer's Sketchpad programme[2] is used as a tool for mathematics instruction. Moreover, technology-based instructional (TBI) strategies are essential when teaching students who are diverse with respect to class, race and English language ability levels. By using TBI strategies, students are provided with education milieus that are relevant to their lives, thereby enriching learning, inspiring social change and encouraging teacher success (Qing, 2003). The use of TBI strategies transforms traditional educational milieu and promotes teacher and student success by improving and enhancing teaching and learning. Along similar lines, based on numerous studies in the field, the use of digital devices when teaching and learning mathematics has revealed positive influences on students' understanding of mathematics concepts (Clark-Wilson, 2017).

Moreover, within a transformed education milieu, all participants are empowered to achieve more; thus, notions of empowerment advance both teacher achievement and student dedication (Lawson, 2011). Within an empowered, transformed educational milieu, the teacher becomes a partner with the student, and this collaboration allows the student to take ownership of their individual learning experience (Rindner, 2004). Teachers who encourage notions of empowerment within their education milieus are prepared to hand over (some) authority over the learning process to students; this helps learning to become a collaborative rather than compelled activity (Lawson, 2011). To further examine the relationship between transformed education milieus, empowerment, social change and mathematics instruction, the purpose of this chapter is to report on a study that illustrated how social change within education empowered mathematics lecturers.[3] This empowerment led to the participants transforming their instruction, thereby transforming their educational

[1] The 11 official languages recognised in South Africa are Afrikaans, English, Ndebele, Northern Sotho, Sotho, Swazi, Tsonga, Tswana, Venda, Xhosa and Zulu.

[2] The Geometer's Sketchpad programme is a software programme for teaching mathematics.

[3] The words 'teacher', 'teacher educator' and 'lecturer' are used synonymously in this chapter.

milieus. This chapter focusses on a study that explored: *Mathematics teacher educators' experiences of using technology-based instruction in South Africa.*

## Using Technology-Based Mathematics Instructional Strategies

Teaching to accommodate diversity within education milieus is a mammoth task which requires that the teacher reflects while teaching. This reflection in and on instruction creates a conducive and engaging education milieu (Schön, 1987). Moreover, an education milieu is viewed as a community of practice (COP) (Anthony & Walshaw, 2009) whose members value and acknowledge the diversity of fellow members. The fundamental component of this type of community ought to encompass bringing together a variety of students and empowering them to embrace diversity. Similarly, the role of the teacher has changed in that they are considered as individuals who encourage the use of instruction to support students to participate in ethical and socially responsible ways within educational milieus (Le Cornu, 2010). Thus, teachers who encourage diversity within their education milieus accommodate and acknowledge students from different races, gender, socio-economic status and linguistic backgrounds to redress inequalities and confront issues that prevent access and success for any social group. These teachers are concerned with bringing about a positive change within existing social structures. Thus, empowered teachers who inspire social change and encourage teacher and student success are those who respect and promote student engagement (Anthony & Walshaw, 2009).

The use of TBI strategies provides students and teachers with numerous strategies that accommodate different learning styles within an education milieu. The accommodation of different learning styles is viewed as a means of transforming the way society views instruction. One must concede that while there are many challenges related to the use of TBI strategies, technology-mediated research is an emerging area of exploration within education (Dunne, 2009). Since technology is used as a tool for instruction (Qing, 2003), teacher knowledge, discussion and engagement are essential to ensure that these TBI tools are used appropriately. Similarly, Mishra and Koehler (2006) proposed that new technologies present both different opportunities and challenges for teachers. Thus, teachers ought to have the necessary skills to use TBI strategies effectively so that they can gauge what knowledge they would need to develop, support and empower their students to promote teacher and student success. In addition to content knowledge skills, when using TBI strategies, a proficient teacher ought to be efficient at using TBI tools successfully within the education milieu. This is called Technological Pedagogical Content Knowledge (TPACK). The notions of TPACK focuses on a teacher's expertise in combining TBI strategies to promote mathematics instruction within an education milieu (Koh & Sing, 2011).

## Conceptual Framing: Transforming Mathematics Education Milieus

Teacher success, university throughput and the need for adequate student support remain dominant concerns in national and international higher education milieus (de Klerk et al., 2017). Similarly, mathematics teacher educators globally are on a constant search to transform their education milieus to improve understanding and encourage teacher and student success through their mathematics instruction. The key concepts that informed the transformation of mathematics education milieus within this study were teacher success, social change, social diversity, empowerment and TBI strategies. These concepts are illustrated in Fig. 7.1 and are defined and discussed within the context of this study.

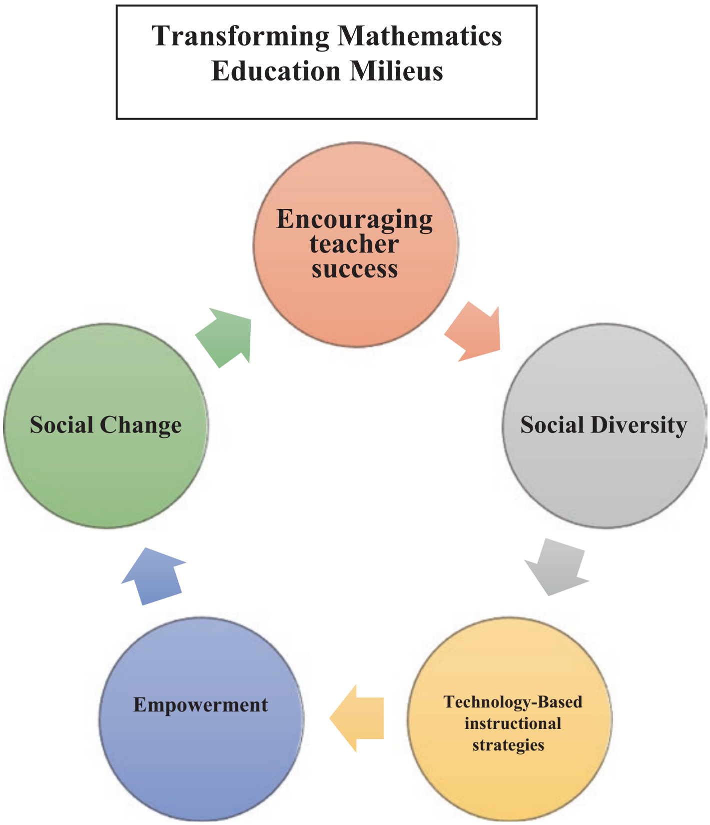


**Fig. 7.1** Key concepts that transformed mathematics education milieus within this study (Author's Construction)

### Encouraging Teacher Success

To create a transformed mathematics educational milieu and promote teacher success, the abstract nature of mathematical concepts ought to be eliminated which may be achieved through transforming mathematics instruction. Thus, to encourage teacher success, the education milieu ought to reflect empowering situations that develop the student as well as acknowledge and respect the culture, diversity and lived experiences of the student. Consequently, mathematics instruction ought to recognise the community, the milieu and the beliefs of both teachers and their students. Mathematics instruction ought to be student centred, contextualised and socially relevant.

### Social Change

Social change in this study implies the changes occurring in human communication and behaviour within the lecture rooms. Social change incorporates perceptions of how mathematics instruction has changed due to changes in society. Lecturing within this study is viewed as social change since it is student centred and driven by TBI tools as opposed to traditional lecture rooms where the lecturer/teacher educator dominated the lecture by using 'chalk and talk' instructional strategies.

### Social Diversity

Social diversity focuses on the differences between individuals. These differences could include aspects of race, gender, experiences, language, cultural contexts and worldviews. Reflecting on these differences will allow both teachers and students to construe this inclusive space. One way to construct mathematics instruction that promotes an inclusive area is to use self-reflection. However, reflecting on social diversity alone is insufficient to transform an education milieu. Social diversity ought to be accompanied by empowerment.

### Empowerment

Empowerment represents the ability of students to perform at their best and to participate confidently within their education milieu actively. Being an empowered lecturer/teacher educator implies that the lecturer/teacher educator is required to provide every student with the education that they deserve. This promotes teacher and student success.

### *Technology-based instructional strategies*

Within the context of this study, instructional strategies that include the use of computers, web-based technology, the Internet and the use of Information and Communication Technology (ICT) are referred to as TBI strategies. The use of TBI strategies within an education milieu signifies a transformation from the traditional 'chalk and talk' instructional practices. This implies a social change of what was once considered a norm at higher education Institutions.

## Research Methodology

### *Participants*

This study focused on mathematics teacher educators' instruction at one university in KwaZulu-Natal, South Africa. The 12 lecturers/teacher educators who were invited to participate in this study had a minimum of 6 years of teaching experience within higher education and a minimum qualification of a Master's degree either in mathematics education or pure mathematics. The necessary ethical clearance was obtained and granted by the relevant research ethics committee. Additionally, each teacher educator was provided with information about the study and completed an informed consent form. Nine teacher educators agreed to participate in the study.

### *Research Instruments*

#### Lecturer/Teacher Educator Questionnaire

The questionnaire comprised of both open- and close-ended questions and was developed to obtain information concerning each participant's professional development, instructional strategies, notions of student success and student demographics. Hence, the questionnaire included questions focusing on student success, instructional strategies, empowerment and social diversity.

#### Observation Schedule

The observation schedule focused on teacher educator interaction and instruction that empowered and motivated students to succeed in a transformed education milieu. The observation schedule was also developed to identify those instructional strategies that exemplified the use of technology-based instructional tools.

**Interview Schedule**

The interview schedule was developed to probe participants' responses to determine how each participant acknowledged social change and diversity to advance a transformed education milieu to promote student success. A semi-structured interview schedule was developed, and responses were probed to ensure clarification of responses.

## *Data Generation*

All research instruments were piloted with three randomly selected participants. The research instruments were also discussed with four colleagues within similar fields of research. The piloting of the research instruments and discussions with colleagues assisted in ensuring the validity and reliability of each research instrument. Following the pilot study and discussions with colleagues, each research instrument was modified. After this modification, the main study commenced with a purposive sample of five lecturers/teacher educators. This selection was based on the participants' responses to the questionnaire. After an analysis of the questionnaires, the observations and interviews proceeded.

## *Data Analysis*

Each questionnaire was analysed, whereby responses were manually entered onto a spreadsheet and codes were identified. An in-depth qualitative analysis of the completed questionnaires revealed that the five purposively selected participants used various technology-based instructional strategies when teaching within their education milieus. The participants used a combination of online discussion forums and TBI strategies, for example, Moodle,[4] cellphone technology, interactive whiteboards, PowerPoint presentations, overhead projectors (OHP), video clips, various mathematics websites, GeoGebra, Geometer's Sketchpad and graphical calculators. The use of these TBI strategies and online discussion forums exhibited a substantial transformation from the traditional 'chalk and talk' instructional strategy that once dominated higher education lecture rooms.

Consequently, these five participants were selected for lecture observations. The lecture observations were summarised and were then carefully examined to determine if there were any connections or patterns. The summarised observations were coded by acknowledging the key concepts and the conceptual framework within

[4] Moodle is a license free Open Source Learning Management System (LMS) used at the participating university.

which the study was embedded. Hence associations to the notions of student success, empowerment, social diversity, TBI strategies and transformed education milieus were noted.

Subsequently, the five purposively selected teacher educators participated in individual interviews. The interviews focused on probing each participant's experiences concerning their transformation in instruction that was observed at critical moments within the lecture observations. All transcribed interview data was read and reread in order to become familiar with the data, and then coding of the data ensued. Data coding was beneficial in identifying themes, which were reviewed by acknowledging the key concepts and conceptual framework within which the study was embedded. Through this qualitative analysis, themes and subthemes were identified. These are discussed in detail in the results and discussion section that follows.

## Results and Discussion

### *Inspiring Social Change to Transform the Mathematics Education Milieu and Encourage Teacher Success*

The participants acknowledged issues surrounding student diversity, students' sense of belonging, student learning styles and differentiated tasks to empower students and inspire social change. The social change inspired a transformed mathematics education milieu. This transformation supported teacher success. These notions are substantiated by the discussions that follow.

#### Diversity Enhances the Mathematics Education Milieu and Inspires Social Change to Encourage Teacher Success

Based on the interviews, most of the participants believed that the use of TBI tools enhanced their education milieu. The participants were aware of the diverse nature of their students' circumstances, insufficient educational readiness and the lack of opportunities available. Participation and collaboration were viewed as essential components to inspire social change and encourage teacher success. Likewise, Qing (2003) proposed that using TBI tools within an education milieu enhances learning by providing interactive or visual learning experiences for students. Similarly, Foote and Lambert (2011) maintained that without active student participation, learning could not occur. Moreover, the use of real-world experiences, enhancing learning and exposure to 'real-world' learning, is emerging across various disciplines within higher education (Ryan et al., 2009, p. 156). These notions are exemplified by the selected lecture observation and interview transcript excerpts that follow (Fig. 7.2):

[Lecturer B (LB) has been teaching how to teach Functions and Relationships to his diverse group of students for the past week. LB knew his students came from different backgrounds and many struggled with the language of instruction. LB used differentiated tasks to ensure his students knew the content and how to teach the content].
The lecturer started recapping and reflecting on the lecture that was taught two days ago [reflection on action]. LB used a power point presentation and the interactive white board to review key concepts that were taught [11 minutes] …
In the last 20 minutes of the lecture, the students were given a choice of two tasks to complete as an assessment activity. The assessment was due the following week. [The first task comprised of flow diagrams and equations that ought to be completed, the second task comprised of word problems which needed to be converted into equations before solving. The students could choose either task, since both tasks eventually displayed the same equations, although the first task catered for the student who struggled with language issues].

**Fig. 7.2** Observation of lecturer/teacher educator B's lecture (LBO)

> LAI: …I know my students…they don't want to talk …English is not their home language …I use technology in the lecture room…sometimes they [students][5] are asked to access Moodle before they come for the lecture…they have an idea of what I am going to do and feel more confident…they know what to expect and can answer and solve problems based on this knowledge…they come up with new and interesting methods…this is different from how we used to teach before…

> LDI: …by using videos and power points, students come up with new ideas for solving …I wanted this outcome…they are encouraged to do their presentations in front of the class as well…since they are diverse in terms of their abilities and strengths…this is recognized and students…work in pairs or groups…this allows them to demonstrate what they are good in…this has transformed my lecture room…

> LEI: …I start with video clips, pictures or transparencies to get them [students] talking as you saw…they [students] try examples on their own…group discussions take place…this section [the topic based on 3D shapes: prisms and pyramids] lends itself to group work…language is a problem, but when using diagrams and videos, this helps…this has changed the way my students learn….

By encouraging relevant, meaningful interaction, students were exposed to different ideas, alternative methods to problem-solving and innovative skills, thereby expanding and enriching the teaching and learning process. This is very different from traditional mathematics instruction and *…transformed the lecture room….* Additionally, encouraging students to create their approaches for problem-solving may assist in developing deep conceptual understanding of mathematical concepts and may *…demonstrate what they are good in….* Thus, including diverse tasks

[5] Words in square brackets within the interview excerpts have been added by the researcher to support the reader's understanding.

based on students' strength transformed the education milieu and …*changed the way students learn*…. This transformation from the traditional 'chalk and talk' instruction transformed the social interactions within each education milieu. The participants used their knowledge of content and students and their understanding of content and teaching to assist in selecting the most appropriate instructional strategy to encourage student participation. Additionally, when diversity concerning language became an obstacle within the education milieu, the use of diagrams, diverse tasks, online resources and video clips created a way of ensuring and promoting understanding. This development of knowledge inspired social change and transformed the education milieu, which encouraged teacher success.

### Promoting a Sense of Belonging: Inspiring Social Change to Encourage Mathematics Teacher Success

Students' responses to mathematics are primarily reliant on their abilities, experiences, interests and how mathematics is taught to them. For students to actively participate within an education milieu, they need to feel as if they are participants and not recipients. Students and teachers need to actively belong to a COP (Lave & Wenger, 1991) whereby they share joint participation and mutually engage with members of this community. This engagement may include discussions revolving around activities, completing class and homework tasks as well as interacting with teacher educators to acquire knowledge.

When the participants included meaningful and relevant mathematics instruction experiences, they created a sense of belonging which supported the social change to transform the education milieu and encourage teacher success. Based on the lectures that were observed, it was evident that students wanted to participate because they felt as if they belonged to this COP. Many participants indicated that their students were more accepting of TBI strategies since they could identify with the TBI tools and also assist the teacher educators at times if there were technical challenges. This is viewed as a reversal of roles from a traditional education milieu whereby the teacher educators were seen as the more knowledgeable other. This notion exhibits social change and a transformed education milieu.

The participants created an education milieu where the students felt that they belonged and could discuss their ideas without fear of being ridiculed. For example, when students required clarification, the teacher educator explained the problem and posed another similar mathematical question. Also, when it was not the abstract mathematical concept that needed clarification, but rather the language of the problem required explanation, then the use of interactive diagrams created a scaffold for mathematics instruction. These sentiments are echoed in the lesson observation and excerpts from the interview transcripts that follow (Fig. 7.3):

> LCI: …I tap into what interests them [the students]…I try to make the maths relevant…they [students] are not afraid to talk to me, especially when it is something they know more

[Lecturer A (LA) introduced the section Data Handling. The focus of LA's lecture was on how to teach Grade 9 learners collecting, organising and summarising data.] LA welcomed the students and introduced the topic of data handling [12 minutes]. LA encouraged the students to think about real world issues in which Grade 9 learners would be interested. LA used a video clip of a recent soccer match [6 minutes]. The students and lecturer discussed the different players and their averages with respect to scoring goals [10 minutes]. A list of player names, teams and goals scored over a period was displayed. LA asked students to draw a tally table to represent the data and to discuss the frequency table that would assist in summarising the data [12 minutes]. [LA walked around the lecture room offering support to the students as they worked in small groups]. Students were then asked to summarise the data using the measures of central tendency [16 minutes].
[LA used a slide to display the definitions of the measures of central tendency and the formulas used to calculate them. The students worked enthusiastically on the tasks since they could relate to the data. LA inspired social change in the lecture room because the lecture revolved around a real world context that appealed to the students, who worked well in the lecture because they could relate to what was being discussed, their voices were heard and this inspired a sense of belonging. This was evident in the atmosphere that permeated the lecture room].

**Fig. 7.3** Observation of lecturer/teacher educator A's lecture (LAO)

> about…I recently used the concept of Idols[6] when I was teaching probability…I used a video clip of an earlier episode…they knew more about the contestants than I did…it also created a change in the classroom…students felt important and felt that they could connect to what was being taught…

> LDI: …sometimes they [students] help me set up when I use the cell phone…my phone was not connected to the correct wireless network a…they quickly corrected me, and the lecture continued with their help…they felt…like an important part…without their help, the lecture would not have been successful…this is very different from how I taught five years ago…my teaching has changed…

> LEI: …they [students] know that I will listen to them …if they need help, they often go onto Moodle and send me a message…we can have online discussions…class participate voluntarily in the online discussions…there is no need to make an appointment with me…we just talk online…this is very different…we have changed how we teach now…we focus more on the student…need to feel…respected and valued….

As is evident, by making mathematics instruction relevant and meaningful to students, teacher success was promoted since students felt they were an *…important part…without their help…the lecture would not be have been successful…teaching*

[6] Idols is a television show in South Africa, which is a contest aimed to find the best young singer in South Africa.

*has changed….* By encouraging student voices to be heard within the education milieu promotes a sense of belonging, and students feel the need to make connections with concepts being taught, …[the students] *participate voluntarily in the online discussions…they need to feel respected and valued….* This type of mathematics instruction transformed the education milieu and encouraged teacher success.

### Acknowledging Learning Styles: Inspiring Social Change to Encourage Mathematics Teacher Success

The participants used their knowledge of content, students, teaching and curriculum (Ball et al., 2008) to ensure that the students' diversity concerning learning abilities and learning styles was acknowledged and catered for within the diverse education milieus. Furthermore, the acknowledgement of learning styles inspired social change and transformed the education milieu, which encouraged teacher success. Mathematics instruction within these various education milieus transformed to accommodate the needs of the diverse students. The participants' use of TBI strategies empowered them to successfully assist their different students and transform their education milieu, as clarified by excerpts from the interview transcripts that follow:

> LAI: …to help my diverse students I use technology…it also helps me to be more efficient… I … can rely on the videos and presentations to reach my students…this is very different from traditional ways of teaching…

> LBI: …I try to enhance my students' engagement and interaction with mathematics ideas…they all learn differently…I keep this in mind when teaching…I often use videos or the interactive whiteboard to enrich my lectures…

> LCI: …the graphical calculator is useful when working with graphs of functions…they [the students] can see how a graph shifts when values of variables are changed…they can work at the only pace…they follow the instructions on the OHP…saves time no need to redraw graphs…the calculator does this for them…they can practice this on their own and master this…

> LDI: …I try to model for them what good teaching is about…they will be teaching their classes soon…this makes a difference…

> LEI: …it is important to include everyone when teaching…all of them [students] are important…all of them come from different backgrounds…some learn through seeing, feeling or hearing…so I try to cater for this in my lectures…you would have observed this in my second lecture on surface area and volume…we projected pictures on the data projector and used 3D models….

By acknowledging the different learning styles within the education milieu, teacher success was promoted, and mathematics instruction was transformed. Participants used this type of mathematics instruction *…to model…what good teaching is about…*, a change in mathematics instruction …*helps* [the lecturer] *to be more efficient…,* thereby transforming the education milieu. The transformed education milieu encouraged student engagement and interaction with mathematics ideas.

### Accommodating Diversity: Differentiating Mathematics Tasks to Inspire Social Change and Encourage Teacher Success

The participants valued the fact that their students were diverse with respect to race, socio-economic status, ability level and linguistic background. They made efforts to accommodate and cater to their students' needs by teaching to accommodate diversity. The participants constructed different tasks to ensure that they provided for their diverse student population within their education milieus. These differentiated tasks empowered the participants to be efficient in accommodating for student diversity which inspired social change and encouraged teacher and student success within their transformed education milieu. These sentiments are supported by the interview transcript excerpts that follow:

> LBI: …to ensure active engagement in my class…I have different types of tasks that I display using the data projector…and I think about how it relates to what the curriculum requirements are with what knowledge they [students] have…I often use open-ended tasks or criterion-referenced tasks…this empowers them and brings about a positive change in the class…

> LCI: …I usually use assessments that provide students with real opportunities to show understanding…this means they need to be different…so I provide examples using Moodle…it could be a presentation of a topic…researching certain aspects of a problem…designing a task...I know that students are different and have different strengths and abilities…I try to cater for all of them…this makes a big difference with how they perform…this is very different from how I taught before…

> LEI: …I tend to have several different tasks that students can choose from…I work hard…to accommodate for different ability levels…even when allocating members to groups…rely on…knowledge of…students to ensure that the group is comprised of students of different ability levels…in this way, everyone benefits…these groups are displayed on Moodle and the notice boards….

Through the use of differentiated tasks, mathematics instruction was transformed, and students were empowered. Empowered students encourage teacher success since transforming mathematics instruction *...makes a big difference with how they* [the students] *perform….* Collaborative work that incorporates the voices of students with differing mathematics ability levels is beneficial in transforming the education milieu and encouraging teacher success.

### Inspiring Social Change to Transform Mathematics Instruction and Encourage Teacher Success

The participants transformed their mathematics instruction while they taught to encourage teacher success and inspire social change. Through their experience of mathematics instruction, the participants possessed adequate TPACK (Mishra & Koehler, 2006). This knowledge supported the participants to transform their instructional strategies successfully to inspire social change. The use of TBI strategies also assisted and empowered the teacher educator to make this transformation.

[Lecturer E (LE) asked students to prepare for the current lecture the previous week. Students were grouped in groups of 6 and each group was asked to prepare to teach a section within the Topic: Geometry of 2D shapes. A cooperative instructional strategy called the Jig Saw Method was used. In each group, the section under study was divided into subsections and each student was assigned to become an 'expert' on one of the subsections. Each student was expected to read and research their allocated subsection with the aim of becoming an expert in that subsection].
LE welcomed the students and passed the register to the students. LE reflected and recapped the previous lecture by using a power point. Key points for constructions were reviewed [17 minutes, LE was occasionally stopped by a student with a query or clarification, LE also used the white board to review construction techniques and discuss common misconceptions].
LE asked different groups to teach various sections within Geometry of 2D shapes. Group members came to the front of the lecture room with their prepared instructional material. Each group member took turns to teach their fellow students the subsection in which they had become an 'expert'. [LE often clarified concepts or added a bit of information to support what the student was teaching. LE wrote notes on the board to clarify or assist with understanding. LE transformed instruction and introduced students to a collaborative method of instruction. LE displayed confidence in the students and effectively handed control of the lecture over to the students. It was in this way that social change was inspired. A change from the traditional lecture milieu where the lecturer is the focus was observed, now the lecture was student focused].

**Fig. 7.4** Observation of lecturer/teacher educator E's lecture (LEO)

This is substantiated by the subsequent lecture observation in which the focus of the participant's lecture was geometry instruction (Fig. 7.4).

### Reflecting in and on Mathematics Instruction: Inspiring Social Change to Encourage Teacher Success

Participants showed evidence of reflecting on previous experiences and reflecting while teaching within their diverse education milieus, and as a result of this reflection (Schön, 1987), their mathematics instruction transformed to cater for the needs of their students. The participants showed evidence of viewing mathematical concepts through their …*students' eyes*… to reflect in and on the action. Seeing mathematics through …*students' eyes*… is essential for understanding students' meaning-making of mathematical concepts. Thus, based on the lecture observations, this transformation in mathematics instruction led to a positive shift in the education milieu.

This change of instruction was observed during the lecture observations. Based on the interview transcripts, it was evident that the use of TBI tools empowered participants to make their transformation in instructional strategies efficient and manageable. The practical instructional strategies inspired social change in the lecture room and enabled the students to succeed. These sentiments are reinforced as follows:

> LAI: …it is easier to change my teaching style when I use the net…I can find anything I need to assist my students on the net…I do things very differently now…I need to teach to accommodate all my students…

> LBI: …when I change my strategy I do so to make the content more accessible… downloading from the Internet makes my teaching more effective…I feel better as a lecturer…feel…I can do anything to make successful learning possible…

> LCI: …reflecting on my experiences is important…it helps me become a better lecturer…I change my examples and strategies as I teach…it helps my students…I have a bank of ideas and questions on my laptop that I can use on the interactive board….

Based on the evidence provided, reflecting in and on action …*makes successful learning possible*…. This transformed mathematics instruction and inspired teacher success. Teacher educators were encouraged to become better at their jobs which …*helps*…*students*…. Through a change in mathematics instruction …*content* [is] *accessible*…*and teaching* [is] *more effective*…. This type of reflective and transformed education milieu encouraged teacher success.

### Collaborative Mathematics Instruction: Inspiring Social Change to Encourage Teacher Success

Teachers who awaken notions of empowerment within their education milieus are prepared to hand over some control of the learning process to their students, which ensures that learning becomes collaborative rather than obligatory (Lawson, 2011). This transformation in the learning process inspired social change since this was different from the traditional learning process. Participants saw themselves as collaborators with their students. Apart from reflecting in and on their teaching, the participants used collaborative methods of instruction to engage students within their contexts. This is validated as follows:

> LAI: …word squares[7] are completed in groups before I start a new section…it is placed on Moodle…students know what words mean before they come to class…if they are having a problem with certain words they can look on the Internet for clues…they ask me for help on the online discussion forum…they all come from different backgrounds…they all don't have the same foundation knowledge required for math…we need to cater for this…in this way I have transformed my learning community….

[7] Word squares are similar to crossword puzzles and are used in mathematics as an activity to help students learn mathematics vocabulary. The clues for the word squares are often definitions of the words used in mathematics.

> LBI: …they [students] sometimes create WhatsApp groups for discussing in and outside class mathematics sections that they have difficulty with…I do not mind… something they are used to doing…they even help me…not too clued up on…Apps…through using the cell phone we have changed the traditional teaching and learning process…

> LCI: …I use a lot of group work…it helps the students who do not understand English well…they come from different backgrounds and language groups…they work together in carefully allocated pairs to solve word problems…they access videos and articles on the net on their cell phones while they are in lectures to help with understanding concepts…it's more interesting for them…it is not only my teaching…they can also lookup difficult words on the net….

Collaborative relations of power within the education milieu echo a sense of control with notions of being empowered to achieve more. The use of *…groupwork…helps the students who do not understand English well….* Students were encouraged to *…work together in carefully allocated pairs to solve word problems….* Students within these empowering education milieus knew that their ideas were welcomed and respected. This knowledge ensured participation within their contexts, and it was in this way that the participants inspired the social change to transform their education milieu and encourage teacher success.

## Conclusion

This chapter sought to explore mathematics teacher educators' experiences of using technology-based instructional strategies in South Africa. While there are no immediate solutions to alleviate all challenges experienced during mathematics instruction, this chapter provided the opportunity to consider how one could empower teacher educators and create an inclusive and enriching context that would encourage teacher and student success and support mathematics instruction. This chapter aimed to bridge the gap that exists in the literature concerning how supporting and catering for the needs of a diverse learning community inspired social change and encouraged teacher success. This chapter demonstrated that by using TBI strategies for mathematics, the participants were empowered to alleviate some issues associated with teaching within diverse contexts. By using TBI strategies, the participants were empowered to acknowledge the diverse nature of their settings, thus inspiring social change by transforming their education milieus to encourage teacher and student success within these education milieus. This also enabled the participants to become efficient in making the learning of complex mathematical concepts accessible to their students.

Through collaboration, the participants were able to achieve active participation regardless of their diverse contexts. Furthermore, empowering students to voice their ideas and promoting a sense of belonging within various contexts was paramount for meaningful student interaction, thereby encouraging teacher and student success. This is very different from the traditional education milieus, where the 'chalk and talk' instructional strategy is the focus, as was evident, what is of importance when teaching using TBI strategies is TPACK. This knowledge ensured that the appropriate TBI strategies were used to achieve favourable results within education milieus. This knowledge empowered the participants within this study to become efficient and successful in their mathematics instruction within their diverse education milieus.

Thus, to encourage teacher and student success, mathematics instruction ought to be carefully planned so that the diversity of students is acknowledged. When students collaborate within an education milieu, students feel a sense of belonging since their voices are heard and acknowledged. Students feel empowered to participate actively within the education milieu and are motivated to interact and collaborate actively within their contexts. Lecturers/teacher educators are encouraged to reflect in and on their activities within these contexts. Within this type of setting, mathematics instruction is transformed, thereby inspiring social change and promoting teacher and student success.

There are some recommendations based on the findings of this study. It is without a doubt that teachers are an essential resource that any country has, and hence they ought to be subject to investment. Higher education institutions ought to incorporate professional development initiatives within teacher training courses. Methodology modules may be introduced to assist teachers with transforming education milieus and mathematics instruction to encourage teacher and student success. Case studies reflecting successful research studies showcasing mathematics instruction strategies used to transform education milieus and promote teacher success ought to be included in these methodology modules. One must be mindful that students are unique concerning their interest and ability levels, so too, all education milieus have their diverse context within which they are situated. Teachers need to know what instructional strategies work and what methods ought to be adjusted so that they are useful within diverse settings, thereby promoting teacher and student success. The findings discussed within this chapter may be adapted in contemporary mathematics education milieus globally, to support teachers with the practical instruction of mathematics that leads to transformed mathematics education milieus.

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# Chapter 8
# Enhancing Learners' Retention of Algebraic Knowledge Through Problem-Solving-Based Learning

**Tuhafeni I. M. Kaufilua and Helena Miranda**

## Introduction

Mathematics is essential in our everyday life; it enables us to think logically and abstractly (Ministry of Education, 2010). Numeracy skills are used to communicate and solve problems, such as in sciences, technology and commerce. Because of this, mathematics is made compulsory for senior secondary school learners in Namibia and a prerequisite in almost all quantitative disciplines at tertiary level. There is, however, low retention of mathematical knowledge due to lack of interest, low retention as well as poor performance, because most learners have developed a dislike for mathematics (Kurumeh et al., 2012).

The poor performance in mathematics as described above can be linked to the poor instructional approaches adopted by teachers and other factors (Namibia: Ministry of Education, 2010). As a way to guard against this, the Ministry discourages learning through rote memorisation that leads to forgetting and encourages methods that ensure retention and understanding by presenting knowledge in the manner which builds on prior knowledge and experiences and reflects reality.

From a constructivist point of view, teachers are expected to acknowledge that learners bring along some mathematics ideas or prior conceptions to class and are able to construct new concepts and skills independently. The instructional approaches used may hinder or promote understanding and knowledge retention. Moreover, learners would find interest in learning concepts that they can apply to everyday life experiences, and this is an assumption of problem-based learning (PBL). Problem-based learning (PBL) is an "engaging instructional strategy in which learners are given triggers or realistic, simulated real life problems that are ill-structured, vague

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or ambiguous before they experience any training in a specific content area" (Tarmizi et al., 2010, p. 4684). In this study, PBL is considered as a teaching and learning approach that allows for active yet collaborative learning opportunities that are centred on solving ill-structured real-life algebraic problems through research or discovery.

As educators of mathematics in the Namibian context, we have observed with great concern that many learners at any level of learning do not recall much of what has been learnt from the previous grades. This makes it difficult for learners to fully comprehend new concepts. One of the areas in which this problem is highly manifested is algebra. Learners do not relate algebraic concepts to everyday life situations and hence find it difficult to understand algebraic problems. Despite the importance of mathematics in quantitative disciplines, it appears that teaching and learning strategies being employed are not effective. This may result in lower mathematics knowledge retention as well as high failure rates in the subject. Therefore, the study towards a Master's Degree conducted by Kaufilua (2019) assessed the effects of PBL approach on senior secondary school learners' retention of algebraic knowledge and skills. This paper is a brief report on that study.

The findings of this study revealed the need to advise curriculum planners and senior secondary school teachers in Ohangwena region and Namibia at large, with regard to designing and using instructional methods that reduce the memory load as well as raise learners' interest and knowledge retention. Such methods include but are not limited to PBL, an approach that encourages the retention of subject knowledge and thus improve the learners' performance in the subject. Additionally, the results may assist mathematics teachers in understanding the PBL approach to learning. Hence, the study made proper recommendations for mathematics classroom practices. It also adds to the literature on the topic and thus serves as a useful resource for other researchers who might want to investigate the effects of PBL approach on the retention in other branches of mathematics, other subjects or in other regions in Namibia or Africa at large.

## THE Cognitive Load Theory (CLT)

This study is guided by John Sweller's theory, known as the Cognitive Load Theory (CLT). Sweller (1994) suggests that when considering cognition, schema acquisition and automation are the core tools of learning. That is, learning occurs when knowledge and skills (schemas) are acquired and then processed and stored on to the long-term memory. Nevertheless, learners may not learn when they experience the cognitive load extremes (Sweller, 1994; Paas et al., 2004). Cognitive load extremes refer to "either excessively low load (underload) or excessively high memory load (overload)" (Paas et al., 2004), and this means that learning cannot take place when a learner experiences an underload or an overload on the working memory (WM). Hence the cognitive memory load, in particular the working memory (WM), should be optimised if learning has to take place.

The CLT theorises on "managing the working memory (WM) loads in order to facilitate the changes in long term memory associated with schema construction and automation" (Paas et al., 2004, p. 2). De Jong (2009) indicates that unlike the long-term memory, the capacity of the working memory is little and learning may not occur if the item to be learnt requires a capacity larger than that of the WM, unless instructional approaches that optimize the load of the WM are used and the cognitive load extremes are avoided. Consequently, the Cognitive Load Theory focuses on instructional control of these extreme cognitive loads and thus enables effective learning by managing the cognitive load (Paas et al., 2004). Moreover, Tarmizi and Bayat (2012) stressed that problem-based learning (PBL) is an "instructional strategy that may effectively increase learners' motivation and knowledge retention" due to the fact that PBL actively focuses on the utilisation of critical thinking skills in problem-solving (p. 346). Hence, this study chose PBL as an instructional approach that may optimise the cognitive of the WM and enhances transfer to the long-term memory as well as retention thereafter.

## Problem-Based Learning (PBL) Approach

Studies conducted on mathematics performance and retention rate found a link between poor performance and inappropriate teaching strategies as well as between low retention rate and inappropriate teaching strategies (Kurumeh et al., 2012; Kurumeh et al., 2016). On the same note, Gorghiu et al. (2015) criticised the traditional teaching which tends to passively engage learners in problem-solving and focuses mostly on the provision of already constructed knowledge. In this approach, the learners memorise and reproduce the learned knowledge in assessment activities. This indicates that the traditional teaching results in learning for immediate recall only. However, Gorghiu et al. (2015) propose what they also called a "discovery apprenticeship", a problem-based learning approach (p. 1866). Hence problem-based learning (PBL) was employed in this study.

Mari (2012) indicates that manipulating and adopting appropriate teaching strategies may increase the retention of knowledge in mathematics and science. Bearing this in mind, new effective teaching and learning strategies must be designed and existing ones deserve recommendations. Additionally, curriculum planners and developers worldwide advocate for a paradigm shift, from the teacher-centred to the learning-centred and effective mathematics learning strategies. Likewise, Gorghiu et al. (2015), and Tarmizi and Bayat (2012) identify problem-based learning (PBL) as an effective teaching and learning strategy that can enhance retention of numeracy skills. Hence this study focused on testing whether there is a significant difference between the retention scores of learners learning through PBL and those learning through the traditional teaching methods.

Abdullah et al. (2010) discovered some disadvantages that engage learners with mathematics through the traditional, teacher-centred approaches, which is preoccupied by exercises, algorithms and formula that need to be mastered. They say such approaches are of little use in other and unfamiliar situations that demand one to solve real-life mathematics problems. Thus, finishing the mathematics syllabus and mastering the questions and their solutions which may result in passing the examinations are the core activities of the traditional mathematics classroom environment. Problem-based learning environment, which is learner-centred on the other hand, affords the learners the opportunity to develop skills to accommodate and adjust procedures to fit unfamiliar situations.

Problem-based learning (PBL) is an engaging teaching and learning approach in which learners are given "ill-structured, vague, or ambiguous" real-life problems to solve before they are provided with any sort of teaching (Tarmizi et al., 2010, p. 4684). Learners learning through PBL have to solve problems through research or discovery. Furthermore, PBL is a teaching and learning strategy in which learners learn through reflecting on their experiences to solve problems instead of imposing already made knowledge on learners, a traditional teaching perspective (Kazemi & Ghoraishi, 2012; Carriger, 2016).

The PBL learners first do individual self-directed and self-regulating explorations on the ill-structured problems before retreating to their small groups to discuss and refine the learnt knowledge (Wood, 2003). Problems of this nature are viewed to be ill-structured since learners have insufficient information from teachers on how to generate the solutions. They are therefore required to seek the information and knowledge they need to acquire and apply in order to solve the problems (Kazemi & Ghoraishi, 2012; Tarmizi & Bayat, 2012).

The ill-structured problems are favoured in discovery learning because they allow "free inquiry" (Carriger, 2016, p. 93) as solving them begins with a question followed by investigating solutions, through which new knowledge is created, gathered and understood. Discussing, discovering and reflecting on newly discovered knowledge and experiences are some of PBL learning aspects that motivate learners to do further inquiry on a given concept. Kazemi and Ghoraishi (2012) stress that instead of generating a single answer, learners do research, discover solutions and make well-informed conclusions. PBL focuses on problem-solving, whereby learners seek for the knowledge required to solve the problems through research (Ribeiro, 2011).

## Learners' Role in the Problem-Based Learning Classroom

The problem-based learning (PBL) approach requires learners to take responsibility of their own learning, by solving problems and reflecting on their experiences (Hmelo-Silver, 2004). They do so by actively engaging in relevant, real-life problems. Consequently, the role of the PBL learners is to actively engage in the learning process and construct own understanding under the guidance of the tutor (Kazemi & Ghoraishi, 2012). According to Abdullah et al. (2010), PBL learners are active problem-solvers, contributors and participants in group discussions. In their groups, they have to work together as a team and share the available information, mathematical resources and notes in order to solve problems. Furthermore, PBL centres at teamwork, in which learners first discover information they need in order to solve problems in small cooperative learning groups (Hmelo-Silver, 2004). Thus, prior learning also has a role to play in PBL and discovery is the means to acquire it.

## Tutor's Role in the Problem-Based Learning Classroom

Since the learners construct their own understanding, the PBL teacher/tutor does not impose knowledge on learners. Wood (2003), Hmelo-Silver (2004) and Carriger (2016) explain that the PBL tutor's duty is to facilitate learning rather than to provide knowledge. Hence, the teacher presents the ill-structured problems to learners who formulate and analyse it and construct the conjectures about the solutions to such problems. Furthermore, the teacher is responsible for making sure that the group accomplishes the learning objectives while encouraging active participation by all group members (Wood, 2003).

Based on Kazemi and Ghoraishi (2012), the facilitator in a PBL classroom is "an expert learner, able to model good strategies for learning and thinking, rather than providing expertise in specific content" (p. 3853). Similarly, Abdullah et al. (2010) as well as Kazemi and Ghoraishi (2012) in their studies described the PBL teachers as facilitators of learning, who provide guidance to learners and their involvement fade away as learners take responsibility of their own learning process. Their description concurs with that of Hmelo-Silver (2004) who suggests that the PBL facilitator scaffolds learners' learning by modelling and coaching. However, these scaffoldings decline as learners become more knowledgeable in the concepts. In the same vein, the monitoring process remains the facilitator's duty even though scaffolding fades away through experience.

## Pros and Cons of Problem-Based Learning

According to Wood (2003), problem-based learning (PBL) brings about knowledge acquisition and retention, good communication skills, teamwork, problem-solving, active and self-directed learning, information sharing and acknowledgement of individual differences and views. It benefits the learners both academically and socially by elevating the learners' life skills. At the same time, PBL enhances learners' motivation and interest and fosters deep learning and real-life applications of knowledge. It also offers learners an opportunity to activate prior knowledge to construct new schema (Wood, 2003).

Despite its advantages, implementing problem-based learning (PBL) comes with challenges, which need to be tackled. De Simone (2014) identified resource-intensiveness and implementation dip as major challenges to problem-based learning.

Resource-intensiveness: Wood (2003) associates problem-based learning with "implications for staffing and learning resources" and the demands of a different timetabling approach, workload and assessment (p. 2). Additionally, De Simone (2014) stresses that in the implementation of PBL, changes in the planning, curriculum and assessment are required. Consequently, there is need for extra resources especially when PBL is to be offered to big groups. Furthermore, the setup and maintenance costs, technical know-how, access to library and online PBL aids need to be considered (De Simone, 2014).

Implementation dip: This is "a drop in performance and confidence as users (teacher in this case) encounter an innovation that requires new skills and understanding" (De Simone, 2014, p. 22). It is the incompetence that can result from the teachers' inability to cope and adjust to the newly implemented method and its demands. Despite that curriculum planners and designers as well as education policymakers are advocating for a learner-centred approach to teaching and learning; most curriculum implementers (teachers) still opt to use traditional and teacher-centred approaches. This could be resulting from the implementation dip imposed on the teachers by learner-centred approaches. That is, most often implementers cannot cope with the pressure and demand of the new approach.

De Simone (2014) stressed that once a new approach is introduced, PBL for instance, implementers would experience the need for more direction (the technical capacity) on how to go about the new method, the difficulties understanding their roles and roles of other stakeholders, difficulties working with others as well as the difficulties in understanding expectations. Hence, many teachers may struggle to keep up with the methods different from their usual method because of their inability to cope and adapt to the changes and demands in the new approach.

Once a new method or innovation is introduced, it may take longer for it to gain recognition. Moreover, only after it showcases good results that it may be considered successful. However, De Simone (2014) suggests that for PBL facilitators to succeed in implementing the approach and deal with the implementation dip, they must adapt to many roles and learn some new roles as well. This may not happen in an instant of time and may require some form of continuous professional development.

## Effects of Problem-Based Learning (PBL) on the Learning of Mathematics

Research on problem-based learning (PBL) is not recent. It dates back from the 1960s (Hmelo-Silver, 2004; Kazemi & Ghoraishi, 2012) and has been applied to different fields, yielding positive effects on academic performances. For instance, in a medical school program at McMaster University where its first application took place back in the 1970s (Oğuz-Ünver & Arabacioğlu, 2011; Tarmizi & Bayat, 2012), in high school economics (Mergendoller et al., 2001), high school mathematics, statistics in particular (Abdullah et al., 2010; Tarmizi & Bayat, 2012), university mathematics (Kazemi and Ghoraishi 2012), sciences (Gorghiu et al., 2015) as well as education management (Carriger, 2016), the use of PBL has marked some memorable milestones in research. Specifically, its positive effect on both performance and interest towards learning has been evidenced well enough (see, e.g. Üzel & Özdemir, 2012; Kazemi & Ghoraishi, 2012; Abdullah et al., 2010 and Carriger, 2016).

Even though it was first applied in medical schools, PBL has been studied and researched on in other fields including education, mathematics education in particular. For instance, Abdullah et al. (2010) concluded that problem-based learning (PBL) is effective and learners taught using this method consistently display better Mathematics communication skills and worked together well compared to those taught using traditional teaching methods. In their study, Abdullah et al. (2010) reported that learners agreed that the PBL instructional strategy was a more effective approach in explaining difficult mathematical concepts and led them to understand the content better. However, they found no significant differences as both the PBL and traditional teaching groups displayed positive interest and perception towards group work and valued the importance of helping others and working in teams. Additionally, Abdullah et al. (2010) posited that stimulating and engaging problems bring about a deep understanding and skill development as compared to traditional instruction. Carriger (2016) also indicates that PBL is superior to traditional lecture. He, however, suggests that a hybrid of these two teaching methods may antagonistically yield the best learning outcomes.

There is consistence in the results of research conducted on the effects that PBL has on the learning of mathematics. For example, Üzel and Özdemir (2012) investigated the achievements and attitudes of prospective teachers towards mathematics in problem-based e-learning and found a significant difference between the results of learners learning through problem-based learning (PBL) and those who were offered the traditional instruction approach. In their study, they reported that the studied prospective teachers showed positive attitude towards mathematics when taught through PBL compared to those taught through traditional instruction approaches.

When they investigated the effects of a problem-based learning (PBL) approach on attitude, misconceptions and mathematics performance among university students, Kazemi and Ghoraishi (2012) found that the PBL method positively affects learners' performance and attitude towards mathematics. Kazemi and Ghoraishi (2012) also conclude that the PBL teaching approach, in comparison with traditional methods, helps in minimising learners' misunderstandings and misconceptions of mathematics. Concurrently, Tarmizi and Bayat (2012) indicate the positive correlation that PBL has on the performance and teamwork skills.

In their study on the effects of PBL on the learning outcomes of an educational statistics course, Tarmizi and Bayat (2012) found a significant difference in the performance of their participants. They stated that the PBL group performed better than the traditional (conventional) group. Furthermore, Tarmizi and Bayat (2012) explained that the PBL approach afforded learners an opportunity to work collaboratively, discovering knowledge and solving real-life problems. Their results are consistent with that of Tarmizi et al. (2010). Both the former and the latter found that the PBL group outperformed the traditional teaching group in mathematics tests written. Upon data analysis, the latter further found that the majority of PBL learners (62.3%) scored A-B grades against 45.9 percent of the traditional group. Hence, they concluded that the PBL approach is effective and its effects on learning are statistically significant.

Praising the PBL approach, Tarmizi et al. (2010) showed how most learners prefer learning mathematics in groups rather than individually. This is another valid reason as to why the implementation of PBL in the learning of mathematics needs to be promoted and not discouraged. Additionally, Tarmizi et al. (2010) indicated that learners perceive PBL as an approach that enables people to discuss and clarify misconceptions as well as strengthen the development of skills transfer. Accordingly, PBL may ensure retention. Moreover, learners studied by Tarmizi et al. (2010) also expressed that PBL is an approach that offers them opportunities to do self-study and knowledge exploration, supplemented by the ability to construct own understanding and apply the learnt mathematics concepts to new situations. It also allows them to accept and appreciate different perspectives within groups.

In their study, Oğuz-Ünver and Arabacioğlu (2011) could not trace any negative effects of PBL and indicated that as an approach it allows learners to develop effective problem-solving, self-directed and lifelong learning skills. Similarly, De Simone (2014) echoed Oğuz-Ünver and Arabacioğlu's (2011) belief that fostering self-directed learning (SDL) and lifelong learning skills among others are key goals of PBL. Furthermore, Hmelo-Silver (2004) and De Simone (2014) are in agreement with the literature above, indicating that PBL enables learners to construct knowledge and acquire effective problem-solving, self-directed learning and effective collaboration skills in addition to helping them become intrinsically motivated. As a result, this approach also increases learners' interest in learning itself and in the subject being learnt.

Hmelo-Silver et al. (2007) add to the literature of the effects of problem-based learning (PBL) by indicating that the problem-based learning (PBL) learners are able to transfer the learnt knowledge to new situations, an ability that their traditionally taught counterparts rarely demonstrate. PBL learners also develop a deep understanding of the knowledge they construct and can freely and passionately communicate about it. However, PBL learners are more vulnerable to making errors compared to learners learning through traditional approaches (Hmelo-Silver et al., 2007). Despite their likelihood of making errors, Hmelo-Silver (2004) suggests that errors are essential when applying new knowledge; but, PBL learners stand a better chance to correct their misconceptions when feedback is provided by the teacher or tutor. According to Hmelo-Silver (2004), PBL learners retain much knowledge and for longer times than traditional instruction learners. This is what the current study tried to establish, in the Namibian context.

In summary, the reviewed literature indicates that problem-based learning (PBL) is effective as it brings about better communication skills and teamwork among the learners, good performances as well as positive attitudes towards mathematics.

Additionally, PBL affords learners an opportunity to do self-study, knowledge exploration, construction of own understanding besides developing effective problem-solving skills as well as self-directed and lifelong learning skills. Furthermore, the reviewed literature also indicates that PBL helps in minimising learners' misconceptions in mathematics. However, none of the reviewed literature or study investigated the effects that PBL has on knowledge retention and transfer of mathematics skills to long-term memory. Hence, this study was aimed at determining the effects of PBL on the Grade 11 learners' retention of algebraic knowledge and skills.

## Research Methodology

The study employed a quantitative research approach, a non-equivalent comparison-group quasi-experiment research design. Guided by John Sweller's theory, called the Cognitive Load Theory, the study was carried out over a span of 6 weeks. Two out of 12 class groups at a selected school were sampled using simple random sampling. Each of these two sampled classes was further assigned as either the experimental or the control group, using probability sampling. Both groups were taught by the same teacher (Author 1, while Author 2 played the role of a supervisor and moderator to the teaching sessions observed in visual recordings). The teaching was done using the PBL and traditional instruction approach for the experimental group and the traditional instruction for the control group, respectively. Fifteen lesson plans on the topic of algebra were used for each group. These lessons covered the learning objectives under the topic of algebra for the Namibia Senior Secondary Certificate (NSSC) Ordinary Level Mathematics syllabus.

The PBL learners were first given group work with ill-structured problems before they were taught. They solved the problem through discoveries, based on the explanations and examples given in their textbook, at the same time writing their self-explanation at each step of problem-solving. The learners then presented their solutions as a group. This was followed by the presentation of the lesson as reinforcement of the concept learnt. The control group, on the other hand, was taught using the traditional teaching approach, in which the teacher provides learners with rules, principles as well as worked examples and learners applied such rules and principles to other related problems.

The population consisted all the 480 learners in the 12 Grade 11 class groups at the selected school. All the 78 learners from the two sampled classes took part in the study. Three mathematics tests on algebraic knowledge, namely, the pre-test, post-test and retention test, were administered during the experiment. The pre-test was a diagnostic test and assessed the algebraic knowledge and skills that learners learnt from Grade 10. Moreover, the post- and retention test contained the same content and assessed the algebraic knowledge learnt during the experiment. Despite the post- and retention test being the same, questions in the retention test were reshuffled to change the order of questions and minimise the chances of learners recalling items in the order they have seen them previously.

## Findings

The data collected from the pre, post and retention tests were analysed, and the two proposed hypotheses were tested using the t-test at $\alpha = 0.05$. Furthermore, t-test in Microsoft Excel was used to test the null and alternative hypotheses of the study. The hypotheses of the study were as follows:

**Null Hypothesis** There is no significant difference in the mean retention scores of learners taught algebra using problem-based learning (PBL) and those taught using traditional instruction approach.

**Alternative Hypothesis** There is significant difference in the mean retention scores of learners taught algebra using problem-based learning (PBL) and those taught using traditional instruction approach.

## Biographical Information of the Participants

### *Gender*

A total of 78 Grade 11 learners took part in the study, of which 42 learners were from the PBL group and the other 36 learners were from traditional instruction group. Eighteen out of the 42 PBL learners were male and 24 were female. Moreover, 14 out of the 36 traditional instruction were male and 22 were female.

### *Age*

The ages of the participants are summarised in the following Table 8.1.

Table 8.1 shows that the learners were aged between 17 and 20, of which the majority of learners were 17 years old, with 32 and 24 learners for the PBL and traditional instruction group, respectively.

## Retention from Previous Grades as Assessed BY Pre-Test

The learners wrote a pre-test in order to identify and bridge the gap in their previous grade algebraic knowledge and skills through interventions. A t-test could not be run on the pre-test since the objective of this study was to strictly assess the effects of problem-based learning (PBL) on the Grade 11 learners' retention of algebraic knowledge and skills. Nevertheless, the pre(diagnostic)-test assessed learners on

**Table 8.1** Ages of participants

| Ages/years | Frequency | | Total |
|---|---|---|---|
| | PBL | Traditional instruction | |
| 17 | 32 | 24 | 56 |
| 18 | 4 | 6 | 10 |
| 19 | 4 | 5 | 9 |
| 20 | 2 | 1 | 3 |
| Total | 42 | 36 | 78 |

**Table 8.2** Group mean summary of learners' performance from the diagnostic pre-test on algebra

| Teaching and learning method | Participants | Mean |
|---|---|---|
| PBL | 42 | 61.14 |
| Traditional instruction approach | 36 | 62.97 |

Grade 10 algebraic knowledge and in spite of learners coming from different Grade 10s, whereby the teaching methods used are not known; the scores' means of both groups on the pre-test were relatively close to one another. This can be seen in Table 8.2.

On the comparison of groups' means of the pre(diagnostic)-test administered, there was a slight difference in the means. However, the means of the control group (62.97%) and experimental group (61.14%) were relatively close to one another, with the control groups' mean being a little bit greater than that of the experimental group with 1.83 percent. The diagnostic test assessed what the participants retained from Grade 10 and consequently found out that on average the participants, in both groups, fairly answered the items administered in the pre-test. Despite the fact that the participants might have been enrolled in Grade 10 from different schools before meeting in Grade 11 at the selected school, they still possessed adequate prior knowledge, resulting in an average of within $a \pm 1$ of each other. This pre-test was administered as a surprise test to minimise the effect that studying may have on the participants' performances.

## The Effects of PBL on Grade 11 Learners' Retention of Algebraic Knowledge at One Secondary School in Ohangwena Region

After the administration of the pre-test, intervention took place in the form of teaching the experimental and control groups using PBL and traditional instruction, respectively. Upon completion of the intervention, both groups sat for the same

**Table 8.3** *t*-test results summary for the post-test

| *t*-test two-sample assuming unequal variances | | |
|---|---|---|
| | PBL | Traditional instruction |
| Mean | 42.83 | 34.50 |
| Variance | 498.53 | 268.49 |
| Observations | 42.00 | 36.00 |
| Hypothesized mean difference | 0.00 | |
| df | 76.00 | |
| $t$ stat | 1.90 | |
| $P(T \leq t)$ one-tail | 0.03 | |
| $t$ critical one-tail | 1.67 | |
| $P(T \leq t)$ two-tail | 0.06 | |
| $t$ critical two-tail | 1.99 | |

post-test. The post-test was administered as surprise test to deal with the effect of studying on learners' retention while assessing immediate retention due to instructional intervention. Table 8.3 shows the results' summary for the post-test as it was analysed using the t-test.

T-test for independent samples was used to analyse data in this study. Based on their book, Johnson and Christensen (2012) argued that a t-test for independent samples should be used when dealing with quantitative dependent variable (e.g. retention in this case) and a "dichotomous" independent variable (e.g. teaching method, which is problem-based learning (PBL) and tradition instruction in this study) (p. 503).

Table 8.3 indicates that on average, participants in the PBL (mean score = 42.83%) class performed and retained much of what was learnt, in comparison to the traditionally taught counterpart whose mean score is 34.50%. From the results on the post-test above, the researcher compared the groups' means to determine the teaching method (PBL or traditional instruction approach) which enhances retention immediately after intervention.

Johnson and Christensen (2012) in rule 1 of testing the hypothesis indicates that "if the probability (p) value is less than or equal to the significance level", the null hypothesis should be rejected and hence concludes that the finding is statistically significant (p. 502). For this study, the degree of freedom (df) was 76; the sample sizes were $n_1 = 42$ and $n_2 = 36$ for PBL and the traditional instruction, respectively; and the probability value (p) for two tails obtained was equal to 0.06. The probability value ($p = 0.06$) found is greater than the significance level ($\alpha = 0.05$). Hence, the study fails to reject the null hypothesis based on the post-test results.

Johnson and Christensen (2012) also state that one should reject the null hypothesis, when the value of the t (statistics) obtained is larger in one of the two tails of the t distribution, that is, when t (statistic) > 1.99. However, for this study, $t = 1.91$ which is less than 1.99 and this further fails to reject the null hypothesis. Therefore, the researchers conclude that there is in fact no significant difference in the mean scores of learners taught algebra using PBL and those taught using traditional instruction approach based on the post-test scores. This simply means that both methods have an immediate effect on learning. That is, if assessed immediately after instruction took place, there may not be much of a difference between the mean scores of learners taught using problem-based learning and those who are taught using the traditional instruction. Narli (2011) revealed similar finding that both the traditional and alternative instructions have similar immediate effects after interventions. In this study the PBL group has performed slightly better than the traditional learners on comparison; however the difference in the mean scores was not statistically significant.

A time of at least 3 weeks was allowed to lapse before learners could sit for a retention test. This retention test was the same as the post-test; however the questions were reshuffled. Likewise the post-test, the retention test was a surprise test too. It focussed on assessing the long-term retention of algebraic knowledge and skills. The following table shows the results' summary for the retention test as it was analysed using the t-test. The table gives the analysis of data collected from the retention test of the two groups, which is used to compare the retention of learning by the learners taught using PBL and traditional instruction approaches, respectively (Table 8.4).

The analysis of the post-test revealed that the difference in the mean scores of the two groups was not significant. However, analyses of data collected through the retention test results indicated that PBL has post effects on the retention of algebraic knowledge. Analyses of data collected by the retention test results, with the degree of freedom (df) = 76 and $\alpha = 0.05$, showed the probability value $p = 0.00$ and $t$ (statistics) of 3.93. Since p value (0.00) < $\alpha$ (0.05) and $t$ (3.93) > 1.99, then the study rejects the null hypothesis. Hence, the retention test results showed a significant difference in the mean retention scores of the learners treated using PBL and those taught using the traditional instruction approach. Therefore, it was concluded that

**Table 8.4** *t*-test results summary of the retention test

| *t*-test two-sample assuming unequal variances | | |
|---|---|---|
| | PBL | Traditional instruction |
| Mean | 55.33 | 39.64 |
| Variance | 369.45 | 257.32 |
| Observations | 42.00 | 36.00 |
| Hypothesized mean difference | 0.00 | |
| df | 76.00 | |
| *t* stat | 3.93 | |
| $P(T \leq t)$ one-tail | 0.00 | |
| *t* critical one-tail | 1.67 | |
| $P(T \leq t)$ two-tail | 0.00 | |
| *t* critical two-tail | 1.99 | |

there is significant difference in the mean retention scores of learners taught algebra using problem-based learning (PBL) and those taught using traditional instruction approach.

Based on the findings above, the study made a few recommendations. The use of instructional methods that reduce the memory load as well as raise learners' interest and knowledge retention such as problem-based learning (PBL) should be encouraged in mathematics classrooms. The Ministry of Education, Arts and Culture should organise professional development workshops on instructional approaches of this kind so that learners understand subject content and can retain their understanding for longer periods. School libraries need to be equipped with teaching and learning materials that can be used in contemporary teaching approaches such as PBL. Finally, researchers in mathematics education may want to further assess the impact of PBL on prospective teachers' practices in the field after graduation.

## Conclusion

In this study, both the experimental (PBL) and control (traditional instruction) group were relatively close in terms of prior knowledge, giving an average of 61.14% and 62.97% for the experimental and control group, respectively. Based on the data analysis of the retention test results, the probability value $p = 0.00$, the t (statistics) obtained $t = 3.93$ and the significance level used $\alpha = 0.05$. Since $p < \alpha$ and $t > 1.99$, the study rejected the null hypothesis and accepted the alternative hypothesis. As a result, the study concluded that there is a significant difference in the retention of algebraic knowledge and skills between learners learning through PBL and those learning through the traditional instructional approach. Thus, PBL learners understood and retained more algebraic knowledge and skills than their traditional fellows. This further enables a conclusion that PBL and other instructional approaches that focus on working on ill-structured problems reduce the cognitive load of the working memory, hence ensuring retention and understanding as argued in the literature (e.g. Paas et al., 2004; De Jong, 2009; Tarmizi & Bayat, 2012).

On the contrary, when the content is learnt in isolation, that is one at time, which is a base to the traditional teaching, there is little connection from one item to another, thus imposing a cognitive load on the working memory (De Jong, 2009). Furthermore, this study proved that problem-based learning is better than teacher-centred methods because it yields better results. It is therefore recommended that educational policymakers and teaching advisory service providers assist teachers through formal development training practices where innovative approaches to teaching mathematics such as PBL can be explored and implemented. Further research in this area is also encouraged, to assess whether the findings of this study are the same elsewhere.

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# Chapter 9
# Jacks' Story: How Storytelling Enhances Mathematics Instruction in Lesotho

**Ajayagosh Narayanan**

## Introduction

Much has been written in the last decade on themes based on professional identities of mathematics teachers with notions on mathematics identity, teacher identity, and personal identity on one hand (Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005; Gee & Green, 1998) and classroom teaching and learning approaches on the other (Wenger, 1998; Lave & Wenger, 1991).

This chapter examines narratives of one teacher, Jack, on his classroom practices to understand how he shapes his professional identity and enhances mathematics learning. Wenger's (1998) social theory of learning is extensively used to understand the components of making sense of Jack's identity, with a focus on professional identity. The major question that this chapter explores is how Jack identifies himself as a teacher enhancing mathematics learning and how he makes sense of his professional identity. Two layers are embedded in this question: (1) Jack's purpose of enhancing mathematics learning through his classroom approach and (2) how Jack makes sense of his professional identity based on this particular classroom experience. The first layer explores Jack's role as a mathematics teacher, and the second one suggests how Jack's professional identity is being shaped through the experience that he attained from the classroom teaching. In order for a reader to understand this, I need to portray Jack's identity as a person.

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## Who Is Jack?

Jack is a Beginner Mathematics Teacher (BMT) working and living in Maseru, Lesotho. He obtained a diploma in education from Lesotho College of Education. He teaches mathematics and science in a secondary school. He considered himself as *a kind of person who loves teaching mathematics* (Jack Int). At some point in future, Jack wants to be a lecturer at the National University of Lesotho, like his grandmother. His dream to be a lecturer and a Master of Mathematics education is central to his designated identity. He is confident that he will attain that designated identity one day.

## Issues and Challenges

From the day a BMT starts working in a school, he becomes part of the everyday activities that unfold in the school and its classrooms. He attaches a personal meaning to the career he has chosen and begins to craft his own professional identity day by day. Flores and Day (2005, p. 1) observe that the first few years of teaching may be seen as a "two-way struggle in which teachers try to create their own social reality by attempting to make their work match their personal vision of how it should be." At the same time, he is also subjected to the culture of the school. While he struggles to understand how his students think, he realizes that he is also changing himself (Goldsmith & Shifter, 1997). In this scenario, a BMT asks many questions without necessarily knowing what the future holds for him. While exploring and trying to familiarize himself with the norms of the school, he struggles to find his own space in the mathematics classroom (Narayanan, 2016).

## Lesotho Context

The backdrop of the major study is the observation that the performance in mathematics classrooms at the secondary level of education in Lesotho is generally poor (Ministry of Education & Training, 2011). The Ministry of Education and Training is also aware of the concerns mathematics teachers have in this regard (MoET, 2005). However, Lesotho teachers' effectiveness is evaluated and measured by their communities against the academic performance of their learners (GEMS, 2013; MoET, 2006). These observations recommend that mathematics teachers require more sustainable school-based support for professional development (GEMS, 2013). By participating in these programs, a BMT could smoothly become part of the educational community. Belonging to multiple communities of practices according to Narayanan (2016) is equally important for teachers to shape their professional identity. Jack as a beginner indeed benefitted from such communities of practice when participated in activities organized by the educational organizations.

### *Wenger's Socioculturalism*

Use of identity in educational research has taken a new turn by filling the gap between the personal voices and the voices of a community through collective discourses shaping personal worlds by combining individual voices with the voice of a community (Sfard & Prusak, 2005). Gee and Green (1998) in this regard point out that what is learned at one point becomes a sociocultural resource for future learning for the community and the individual. The individual is the BMT who gives meaning to his/her teaching approaches (Wenger, 1998). Such approaches assist BMT to shape his/her professional identity whose purpose is to enhance mathematics learning, classroom practices, as well as meaningful participation of learners. As such activities are socially constructed, he/she is thus recognized by the school community as a *certain kind of a teacher*. Wenger (1998) suggests that our identities are constituted by what we are, what we will be, and what we are not. Identity also develops through "negotiated experiences of self" that links participation for individuals with learning through engaging in and contributing to the communities of practices (Wenger, 1998).

### *Positioning Learning as Central*

Flores and Day (2005, p. 1) point out that "learning to become an effective teacher is a long and complex process." While exploring legitimate peripheral participation, Lave and Wenger (1991) explain learning as a process from being a newcomer to becoming an old-timer. This process is part of the evolution of practices and for growing professional identities, as Wenger (1998) argues. Learning thus gains a central position in the social theory of learning. Through practices and participation, BMTs give meaning to their activities (Wenger, 1998). In shaping professional identity, such learning processes are meaningful. Therefore, learning becomes an integral aspect of social practice (Wenger, 1998; Lave & Wenger, 1991). Learning thus shapes and reshapes the professional identity within the context of the teaching approach that a beginner initiates for effective mathematics learning. Learning therefore is crucial not only for a learner in the classroom but also for the teacher who is engaged in teaching.

## *Identity*

Identity has been used in a variety of ways in mathematics education and related fields (Felix, 2014; Gresalfi & Cobb, 2011). In mathematics education, identity as one of the products of learning ranges from change in beliefs, attitudes, and approaches to teaching mathematics (Felix, 2014), to their stories (Sfard & Prusak, 2005; Connelly & Clandinin, 1999), and to the change in practice (Wenger, 1998). It also implies that if change in practice does not occur, then learning limits the shaping of identity and situating participation peripherally (Lave & Wenger, 1991). These views suggest a central role for identity.

At this juncture, views of some authors on identity are explored. Wenger (1998, p. 5) finds identity as "a way of talking about how learning changes who we are." Identity refers to the personal histories of becoming in the context of our communities. Gee (2001, p. 99) sees identity as a *certain kind of person* in each context and as a person's own narrativization. Sfard and Prusak (2005) elaborate that foregrounding a "person's own narrativization" and telling who one is an important element of identity. The key to these observations is learning. Figure 9.1 demonstrates how learning shapes BMT's identity by making sense of his/her actions.

Figure 9.1 suggests that meaning of learning is configured through a process of becoming a full participant within a sociocultural setting and practices. Wenger (1998, p. 5) elaborates this view in detail:

> *Meaning is a way of talking about our ability to experience the world (learning as experience); practice as a mutual engagement in action (learning as doing); community as a way of talking about the social configuration (learning as belonging) and identity as a way of talking about how changes who we are (learning as becoming).*

This observation pins down the two crucial themes within the context of this chapter, namely, learning by practice and learning through participation that shape one's identity, which is shifted to the center as demonstrated in Fig. 9.2.

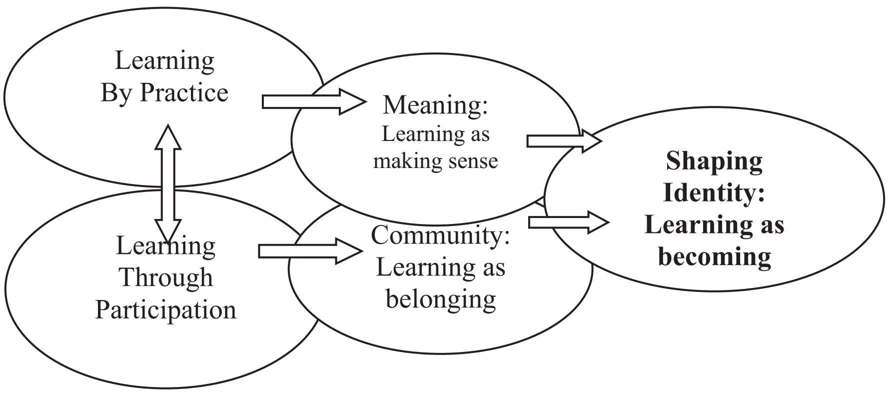


**Fig. 9.1** A demonstration of how identity is generated. (According to the author)

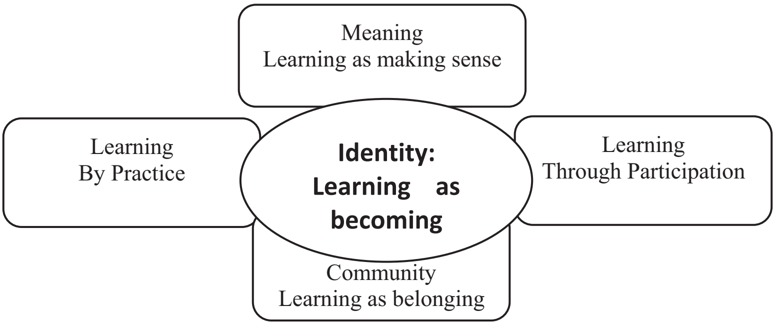


**Fig. 9.2** Adaptation of Wenger's social theory of learning that shapes identity

The dynamic nature of identity is an opportunity to continue learning (Wenger, 1998). Learning thus becomes an integral part of shaping one's professional identity. From a social context, the mathematics classroom is seen as a community that constitutes its own norms and practices for developing mathematical knowledge for learners (Goldsmith & Shifter, 1997). Identity, mathematics, and learning are thus linked to BMTs' classroom settings and the approaches he/she follows in the instructions (Sfard & Prusak, 2005). Jack's story presents how he used a particular classroom approach to teach a particular topic. Within this context, his story could narrate how his making sense of being a mathematics teacher shapes his teacher identity as well as how it enhances the mathematics instructions.

The identity discourse suggests that situated meanings shape the designated identity for beginners (Sfard & Prusak, 2005). Learning assists a beginner to understand "what is actually happening at the moment" that enables the BMT to shift his/her position from periphery to the center. This shifting is the key for a beginner to change his/her classroom practices accordingly, enhancing mathematics learning. These *actions* imply the active participation of a BMT in his/her classroom teachings. A BMT could thus produce meaning of his/her own actions that in turn shapes his/her identity as well as learners' mathematics learning (Sfard & Prusak, 2005; Wenger, 1998). Within this context, I find the classroom activities identify the BMT as "a certain kind of a teacher" and enhance his/her mathematics instruction.

## *Professional Identity*

This chapter considers professional identity as a single cable with various strands of identities. Therefore, the notion of different identities is complex and these do not exist individually. Professional identity consists of strands of personal identity, teacher identity, mathematics identity, as well as community of practice identity. Personal identity refers to where we come from and how we view ourselves within our communities of practice. Wenger (1998) in this regard observes that even our private thoughts make use of concepts, images, and perspectives that we understand through our participation in social communities. Flores and Day (2005) suggest that the personal response of a person and the emotional climate (care, joy, etc.) of a community influence one's identity. Teacher identity refers to the way "teachers feel about themselves professionally, emotionally and politically given the conditions of their work" (Jansen, 2001, p. 242). Mathematics identity, according to Grootenboer and Zevenbergen (2008), involves significant mathematical knowledge and skills, positive attitude towards the subject, and a sense of joy and satisfaction in undertaking mathematics practice for students and for teachers. Community of practice identity is a set of relations among persons, activity, and the world through meaningful participation (Lave & Wenger, 1991). Even though these identities are separately constructed, the chapter suggests that it is possible for these identities to overlap, intertwine, and enrich each other and shape the professional identity.

The various layers of identity that are discussed so far could be relevant only if these identities create an atmosphere for effective learning in the mathematics classrooms. Therefore, Jack's particular approach for introducing a topic should be discussed briefly in order to understand how effectual it is in the learning. The professional identity of a teacher is closely linked to students' mathematics identity and their learning which makes classroom instructions effective. Effective student learning incorporates elements of practice related to the classroom community, classroom discourse, the kinds of tasks that enhance students' thinking, and the role of teacher knowledge (Anthony & Walshaw, 2009). Within this context, Jack's classroom approach is justified to enhance mathematics learning.

## Methodology

An ethnographic approach was used to follow Jack's classroom activities that unfolds the meanings of situations and explores his stories which are actively constructed by the participants (Sfard & Prusak, 2005; Gee, 2001). The ethnographic approach provides various advantages to identity discourse that may unfold Jack's actions and their significance within the context of learning a mathematical concept that he explored in this classroom. Data collected through classroom observations, one-to-one interviews, and focus group interview were horizontally and vertically analyzed. The one-to-one interview provided a picture of "what was going on" in the classroom (Gee & Green, 1998) and helped to understand how he made sense of it. Focus group interview helped Jack to listen to other BMTs who participated in the study; thus he got the opportunity to reflect on his classroom activities in that light. In that way, he was able to build confidence in his own practice (Narayanan, 2016).

### *Analytic Tool*

The idea of **mathematizing** and **subjectifying** in the narratives of identity formation is borrowed from Heyd-Metzuyanim and Sfard (2012). Mathematics learning is seen as the interplay between these two activities that are used in this model as a tool to understand how Jack used storytelling as a method to introduce statistics. Mathematizing referred to any utterances indicating all mathematical terms or ideas that revolve around mathematical concepts, operational activities, and mathematical objects such as numbers or mathematical objectives such as sets, statistics, etc. Utterances are subjectifying when linked to mathematizing. When these two intertwine, the utterances make sense to the recipient and the actor. Within this context, the actor is Jack and the recipient is his class at large.

### *Sample of Vertical Analysis*

The following utterances from one of Jack's lessons in the main study are used to demonstrate how mathematizing and subjectifying are derived.

According to the definitions, the utterance, *multiply*, and its derivatives indicate mathematical actions and are therefore examples of mathematizing. Jack's utterances like *how do we* guide the learners to understand the term *multiply* in connection to the proposed mathematical activity. These mathematizing and subjectifying utterances are analyzed to portray how Jack makes sense of his approaches based on the situations. The situated meanings that are developed from Jack's utterances in the mathematics classroom are derived from subjectification.

The sample utterances are demonstrated to understand if these subjectifying utterances are reifying, endorsable, and significant. The reifying quality comes with the use of verbs such as have, is, can, etc. (Sfard & Prusak, 2005). The reifying utterances are then linked to other utterances (e.g., interviews) to identify if these are endorsable or significant. A story about a person counts as endorsable if the identity builder would say that it faithfully reflects the situation in the world.

## Discussion

A vertical picture of Jack's classroom activities under discussion is shown to illustrate how mathematizing utterances are reifying and endorsable:

> *I see that some of you have problems with **collecting data**. And then look at how the first student has collected the data. What if there are four buses at the same time? Do you think the student will have the time to write four times? So do you see the problem, he might make a mistake, right? Let us see the second student … She has decided, ok, let me try to draw a table … And then she told herself that the first, the first column is the column of vehicles, right? … You see!* (Jack Ob)

**Table 9.1** An example of categorizing the utterances in to subjectifying and mathematizing (from Narayanan, 2016)

| Utterance | Subjectifying | Mathematizing |
|---|---|---|
| Jack Ob: **We are going to look** at multiplication of decimals. How do we multiply decimals? | We are going to look at … | Multiplication of decimals. |
| | How do we … | Multiply decimals? |

**Table 9.2** Jack's story is reifying (from Narayanan, 2016)

| Indirect reification (Forming an opinion about actions of someone, self or something) | Direct reification (What does Jack observe about him or about others?) | Direct reification, Jack's conclusion in a certain way |
|---|---|---|
| *And then look at …* | *So, do you see the problem …* | *She has decided, ok, let me try to draw a table.* |
| | *And then she told herself …* | *You see!* |

These subjectifying utterances (Tables 9.1 and 9.2) demonstrate various levels of reifying actions within the context of mathematizing and mathematics learning. Though these utterances may not indicate any pattern in Jack's approach or any impact on him and the learners, these utterances reify because at one stage, he started identifying with the character (*She has decided*) and interpreted her thinking (*ok, let me try to draw a table*). Throughout the lesson, his monologue approach was consistent which flowed like a story. The story made sense for the class; therefore, these reifying utterances are endorsable.

## *How Jack Introduced the Topic, Statistics*

The topic that Jack planned during the observation day was *An Introduction to Statistics*. Jack started the lesson by describing how two students were assigned to count the number of vehicles that passed by on a particular day and time, like a story. These students then derived the mathematical part from the story by classifying, counting, and recording the type and number of vehicles that passed by. Jack then described the findings, justified their conclusion, and interpreted data to the class. This was an approach that he used to teach how to record data statistically. He then gave the students a task as quoted:

> *I got some examples of two students who were asked to collect data. They were asked to stand by the road-side, and then to record the type of vehicle that passes across the road. That is what I look at* (Jack Ob).

Considering the events that were observed, Jack's identity discourse took various levels. He used a storytelling approach to introduce the topic. His activities were mathematizing (*I see that some of you have problems with* ***collecting data***). In one way, his descriptive procedure of recording data was not free from the teacher-centered teaching approach. Students were preoccupied with the story about the student who collected the data but ignored the way that this child in the story performed it, which perhaps compromised students' mathematics learning. Jack's demonstration of the procedure was partially ignored or rejected by the class. Therefore, they failed to explore the assignment that Jack gave them afterward. Once Jack recognized this, he ended the lesson and suggested that it will be continued next day (*tomorrow* ***we*** *are going to do some work*).

What does this storytelling approach mean to Jack and his students? When Jack uttered *then I shall proceed tomorrow*, he was ensuring them that they would explore the topic further. The author asked him if he would change this particular approach of teaching in the future. He responded:

> *I don't know but actually because the same procedure that I used in A4, this one that I am teaching, I used in A3 and it worked faster in A3 because they are … more able than this one, so I still think I will do it but I will have to improvise somehow* (Jack Int).

Jack identified the importance of this approach that he employed in this class and indicated the future actions he might take to improve it further. Based on Jack's reflective thoughts, I elaborate the way he made sense of situated meaning of this particular classroom teaching.

## Making Sense of Jack's Experience Through Effective Mathematics Learning

### *Jack Reflects on His Teaching Experience*

How does Jack's classroom teaching differ from conventional classroom activities such as "chalk and talk style"? The most striking feature of his classroom experience was his meaningful thoughts on those teaching moments. He was concerned with students' attitude towards mathematics learning (Jack Int: *These kids are lazy*) and their *yes/no* responses to most of his questions.

He also stated that his philosophy in teaching was shaped by what he experienced in life (Jack Int: *It seems I suffered a lot. So, I don't want my students to be part of that suffering*). This inspired him to adopt a particular approach as a teacher towards his students (Jack Int: *If somebody failed, then let me be responsible*). This philosophy might have made his classroom practice different. Jack probably built his career through such perspectives to improve his teaching skills.

A key "concern" with Jack's classroom practice was the difficulty he had linking his instructional practices to students engaging with mathematical concepts (Jack Ob: *I am going to give you a puzzle … we are going to collect those data and record it*). On various occasions, he assumed that the students have the basic knowledge and therefore they should be able to learn mathematical concepts easily (e.g., how to record the data statistically). However, he realized that they did not perform the task as he expected (Jack Ob: *I see that some of you have problems with collecting data*). This is a serious concern because identity is conceptualized in terms of the ways teachers practice and participate in particular types of activities (e.g., classroom practices) that are shaped by their norms, values, and practices (Cobb et al., 2009). From my observation, I thus reached the following conclusions on Jack's classroom teaching.

## Jack's Postscript

Jack's narrative reveals how he made sense of his classroom activities that shaped his professional identity.

### *Jack's Teacher Identity*

Jack wishes to become a *facilitator*. His dream is to transform a mathematics learner to become a person, who is able to *think critically and straightforward* (FG Int). On many occasions his questions (Jack Ob: *Isn't it*?) received limited responses from them such as *yes/no* that probably showed that the learners were not fully engaged in the mathematics classroom.

Jack frequently uttered words like "us," "we," and "our," compared to the word, "I." He was a person who moved around the classroom to monitor students' work and guided them in their learning activities. He identified and fitted into a role of "being a good teacher" within this traditional framework, giving a different approach to mathematics instructions.

On many occasions, Jack initiated and controlled the interactions with his class. He accepted their *yes/no* responses and continued teaching and it was unusual for them to seek clarification in the classroom. When Jack reflected on this, he also revealed his worries, uncertainty, and confusion as a beginner teacher as well as the lack of enthusiasm that the learners had in the classroom (*I thought I was not going to make it*—Ref: Jack Int).

Jack had a vision to improve the students' future through their learning mathematics. He was becoming a mathematics teacher through his learning experience and negotiating new meanings while *doing the job* (Lasky, 2005). While exercising the "job," he expected his students to learn through the samples that he displayed and demonstrated in the classrooms.

### *Jack's Mathematics Identity*

Jack identified himself as a mathematics teacher, who saw the power of mathematics and mathematics teaching as **bringing change** in the lives of students (*it seems I suffered a lot. So, I don't want my students to be part of that suffering*). The following vignette narrates his philosophy as a mathematics teacher:

> *I want to have a positive feeling towards each and every mathematic topic … I want to create that sense … So I want* (the students) *to have a positive feeling towards each and every mathematic topic* (FG Int). *I* ***managed to do all the topics*** (despite being a beginner teacher - my emphasis and interpretation). *I managed to do them with quality* (FG Int).

Jack's classroom practices included demonstration of a few models and their imitation of these samples as assigned by him. Students' achievements thus linked to his mathematics and teachers' identity (Grootenboer & Zevenbergen, 2008). He considered mathematics as a tool to be successful in life. In his own words:

> *It is a tool … because we are living in the world of interaction, world of technology, world of science. If we don't learn mathematics, or if we haven't learned mathematics, such things become like off your sights* (Mathematics is an eye opener). *You see? So you will be part of the changing world. And … you should have mathematical skills that are going to help us with life* (FG Int).

These utterances identified Jack's views on learning mathematics. The way he emphasized, *you should have mathematical skills that are going to help us with life*, shows how he visualized the importance of learning mathematics. If mathematical identity is about having mathematical knowledge and skills, creating a positive attitude towards the subject, and making a sense of joy and satisfaction in undertaking mathematical practice, Jack's narratives demonstrated his mathematics identity that instilled students' mathematics identity too. This is the negotiated experiences that made sense for him because in his view, learning mathematics is enhanced in his mathematics classroom.

## *Jack's Communities of Practice Identity (Learning as Belonging)*

The way Jack engaged in teaching perhaps demonstrated a teacher-centered approach. At the same time, he encountered challenges in the localized curriculum because he was not given the opportunity to grasp it, though he identifies the change in curriculum as an opportunity towards positioning himself peripherally. In his words:

> *You may hope for the best, but expecting the worst. I am saying this because if you look at localizing the curriculum. So I am afraid that if this is* (implemented), *it is going to bring tension. And if it happens that I don't understand this, I have the tension. I am afraid, I might be schooling again, while I should be teaching* (FG Int).

These utterances indicated his willingness to learn further so that he improves his teaching skills on mathematics instruction.

In the classroom, Jack asked questions that students answered promptly. He probed their prior knowledge before starting lessons, but their responses at times discouraged him. He categorically asked the students if they understood him, and they dutifully answered "yes." In such situations, he used the terms, "we, *us*, and *our*" that indicated that they are "learning" together. They accepted the way he communicated with them and enjoyed his lessons. His approach could be linked to Wenger's (1998) arguments on identity: *I define who we are by the ways we experience ourselves through participation as well as by the ways we reify ourselves* (p. 149). That becomes identity as negotiated experience according to Wenger.

## *Jack's Personal Identity*

In identity discourse, the frequently asked questions such as who am I, where do I come from, where am I going, etc. are the indicators of one's personal identity (Wenger, 1998). Jack saw himself as a friendly person to students. He said, *I would like them to see me as a friend, a friend with a passion about the way they learn, and then how do they acquire knowledge, and how do they perform. That is how I see* (and) *how I want them to see me* (FG Int). These words portrayed a colorful designated personal identity for him. This aligns with Wenger (1998) suggesting *The experience of identity in practice is a way of being in the world … we often think about our identities as self-images because we talk about ourselves and each other – and even think about ourselves and each other – in words* (p. 151).

## *Jack's Professional Identity*

If professional identity for Jack was to project him as a "good" mathematics teacher, he considered that acquiring teaching skills was crucial for him. In this vignette, I provide Jack's dream of playing a role to build students' bright future:

> *As a mathematics teacher … I need to* ***create a way of perfecting students' way of learning****, you see? Because there are two types of learning involved. You learn or acquire skills with listening. Sometimes … I need to have a way of* ***getting … the acquired skills and then after acquiring skills, they should have reasoned****, because I think a student* (should) *have listening skills. That is when they can tackle the problems easily. You see? That is one of the things that I wanted to perfect in my teaching* (FG Int).

Jack's philosophy of being a teacher was based on the thought that he needed to create *perfecting students' way of learning* (FG Int). In this manner, students' learning is the central focus of his learning to become a teacher (Wenger, 1998). Through these utterances Jack is making sense of his classroom experience and exploring options to improve his teaching skills. This aligns with the argument that by recognizing and portraying his own reality, Jack would be gaining confidence to become a mathematics teacher sooner or later through similar learning experience (Gee, 2001; Wenger, 1998).

Jack negotiated meaning for himself and for the students by changing their attitude towards mathematics. The meanings that Jack extracted earlier assisted him to understand his professional identity. In conclusion, the following utterances are useful to understand his views on teaching and learning mathematics:

> *If the classroom is lively and very active and they* (teachers) *have good relationships with the students, then that attitude towards you also changes* (influences) *the attitude towards the subject … They grow up following certain ideas* (FG Int). ***I believe in work with interaction*** (Jack Int).

In my view, Jack could practice professional interaction so that at least he brings changes within himself. In his words, he is *dreaming of being a perfect teacher* (Jack Int). He may need to define and realize the meaning of *being a perfect teacher*, which should align with the learners' mathematics learning.

The main study could not consider students' mathematics identity or Jack's other classroom activities. This is a limitation that considered the analysis as superficial. However, such limitations can create opportunities for further studies focusing on learners' mathematics identity to understand how they link mathematics learning to their identity.

## Linking Jack's Experience to Mathematics Learning

Curriculum-based professional development programs for BMTs help "teachers deepen their understanding of mathematics content, students' mathematical thinking, and instructional strategies; and develop norms and practices for learning about teaching" (Borko, 2004, p. 10). Authors like Jansen (2001) and Flores and Day (2005) indicate that feelings could influence teacher identity. For instance, Jack had a personal identity crisis in the beginning of his teaching career. He thought, *I was not going to make it, but at the end of the day, I end up having C's and B's* (Jack Int 4.418). He was referring to the better performance that his students achieved. Wenger (1998) mainly focused on *learning* as a way to become a teacher, but Graven (2003) thought that Wenger is undermining the values of teachers and therefore raised the question, what does it mean for a person to be a teacher? Lasky (2005) argued that teachers' personal and cultural background has an impact on their career selection. Jack's way of introducing the lesson also is indicative of a "grandmother's approach." This deserves further study. My concern is how far Jack's storytelling approach is enhancing mathematics instruction. It is impossible to learn to play the violin by listening to stories about playing the violin (Nazieve, 2018). Within the context of Jack's classroom experience, he already observed that the students were interested in the story of the two children, but they lost the enthusiasm once he moved to the mathematical part (*I see that some of you have problems with **collecting data***).

The central point for Jack is to make sense of his classroom activities as *learning* to become a mathematics teacher by **participation**. I quote Jack: *It seems, I suffered a lot, so I don't want my students to be part of that suffering* (Jack, Int). These reflective utterances should be taken seriously, as they shape his action through a **negotiated experience**. Jack's experience therefore carries multiple layers of learning that indicate a possible negotiation for maximizing learning (Wenger, 1998). I re-emphasize: what is happening at the moment shapes designated identity as "presenting a different understanding of these happenings at a later stage" (Sfard & Prusak, 2005, p. 18). Learning therefore becomes central for Jack which changes him from who he is now. As Jack continues learning, identity becomes dynamic. Learning thus becomes an integral part of shaping his professional identity (Wenger, 1998). At the same time, does this understanding enhance mathematics learning in his classroom? He observed that his approach did not help much.

Jack's experience does not necessarily provide new light on identity but may suggest that the literature on identity needs to further explore the core issue, the process of learning, to become a teacher. That could open new doors for research to understand how learning through participation and practice make sense for a teacher that in turn could shape his teacher identity. For instance, there is a need for BMTs to understand how students see themselves in relation to their mathematics learning (Narayanan, 2016). I further observe that when a mathematics teacher enters a school community as a beginner, he/she comes with hope, dreams, and a revolutionary mind to change the mind of a learner. Thus, he/she starts to engage in participation as well as negotiating meanings. In other words, a BMT designs a mathematics identity for his/her students with his/her beliefs, attitudes, and approaches, which is his/her "personal identity." As time progresses, he/she realizes his/her teacher identity as linking with students' mathematics identity that could enhance mathematics learning. Understanding these multiple layers of identities are central to "making sense," in my view that opens the door linking teacher identity with students' mathematics identity.

As a teacher, Jack knew that *no topic is better than the other one* (Jack Int). He *chose the teaching strategy depending on the type of the topic* (Jack Int) and chose the instruction method. He engaged the students in various learning activities by giving them opportunities to demonstrate (Jack Ob) and was happy to observe the improvements in their mathematics learning and performance (Jack Int). This obviously indicated a change in mathematics learning in Jack's classrooms.

## Conclusion

The analysis of Jack's lesson ends here. This chapter tried to bring Jack's experience and his narrative closer to the readers who may be able to observe how he shapes his professional identity and how he enhances mathematics learning in classrooms.

This chapter also suggests a need for further research such as students' mathematics identity and how mathematics learning could be effective. I examined some of Jack's utterances in the light of professional identity. In his journey to become a mathematics teacher, he may continue learning because *learning is first and foremost the ability to negotiate new meanings: it involves our whole person in a dynamic interplay of participation and reification* that transforms one's identity (Wenger, 1998). Jack's role as a mathematics teacher is a change from being a student teacher to a beginner mathematics teacher, and with his dream to be a mathematics lecturer, he continues negotiating new experiences and thus shaping his teacher identity. By doing so, he could develop new strategies to enhance mathematics learning.

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# Part II
# Introductory Commentary

**Cyril Julie**

This segment gyrates around chapters on various professional development initiatives carried out in sub-Sahara Africa. It covers research-based programmes and summaries of such studies from Rwanda, Kenya, Malawi, Namibia, Zimbabwe and South Africa.

## Introduction

This part revolves around chapters on various continuing professional development (CPD) of mathematics teachers in some countries in sub-Saharan Africa. As comes through in the various chapters, as is the case globally, CPD of mathematics teachers is a complex activity. CPD falls under the domain of "Mathematics Teacher Education" as one of the ten objects phenomena or objects of study or *problématiques* being pursued in Mathematics Education as a discipline (Niss, 2007). Mathematics Teacher Education—together with Goals and Aims of Mathematics; Mastery of Mathematics by Students; Teaching of Mathematics and Policy Matters Linked to Mathematical Education—has been part of the research agenda in Mathematics Education since the early years of the emergence of Mathematics Education as discipline (see Klein, 1945; Begle, 1979; Freudenthal, 1980). The catalyst for the current burgeoning of CPD for mathematics teaching practitioners was the so-called reform movement. This movement, which Wittmann (2019) terms the "New Math Education", can be traced to the mid-1970s and early 1980s. There were, however, with the "New Maths" era, CPD offered to make teachers conversant with "new" content such as, for example, set theory, groups and fields. In South

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Africa, it was the school inspectors who offered these courses. An important arena for teachers to engage in professional development through their own volition was the congresses by mathematics teacher associations such as the National Council of Teachers of Mathematics (NCTM) in the United States of America (USA) and the Association of Teachers of Mathematics (ATM) in England. In addition there were CPD activities associated with the textbook development projects such as the School Mathematics Project (SMP), which started in 1961 in the United Kingdom and the University of Chicago School Mathematics Project (UCSMP) in operation in 1983 in the USA.

In order to get a sense of growth of research in CPD linked to mathematical education, two authoritative systematic reviews were consulted. The one review is a meta-analytic study (Timperley et al., 2007) and the other one is meta-synthetic (Cordingley et al., 2003). The table below shows that research reports on CPD in mathematical education came strongly to the fore from 2000 onwards.

| Review | 1989–1993 | 1994–1999 | 2000+ |
|---|---|---|---|
| Timperley et al. | 6 | 7 | 18 |
| Cordingley et al. | 0 | 2 | 4 |

The contributed chapters in this part should be read by keeping the above narrative in mind. The next section provides an overview of the chapters.

## Overview

The 9 chapters in the part cover 5 countries (number of chapters between brackets)—Zimbabwe (1), South Africa (4), Rwanda (2), Malawi (1) and Kenya (1). One South African study (Chap. 14) does not strictly fall within the realm of CPD and is best read with the chapters in Part I dealing with the initial professional education of teachers. The comments in this overview thus exclude this chapter.

Chapter 11 is different in that its empirical domain is not the normal sites and contexts of CPD. Rather it surveyed, for the period 2006–2015, CPD research primarily in South African journals to ascertain the dominant underlying paradigms of CPD in the country. It found that the majority of published studies had an interpretive underpinning.

For the 7 remaining studies the range for participants in the involved CPD initiatives was 12–89. With the exception of the Malawian case, all participants in these studies were teachers. The Malawian cohort were 89 teacher educators. Although 116 teachers attended the programme reported in Chap. 13, data were obtained from 35 teachers and the in-depth data were obtained from 10 participants.

Barring Chap. 15 with its focus on mathematics educators at colleges of education, two programmes, the Rwandan ones, had its focus on teachers in primary school with the rest focusing on teachers in high schools.

For two (Rwanda and Zimbabwe) of the programmes, participation was mandatory and participants were awarded certificates. In the case of Zimbabwe the certificate was a degree. One other programme (South Africa, Chap. 18) was an Honours degree-bearing one with the participation, due to the nature of such programmes, being voluntary. Participation in two other programmes (Chaps. 15 and 17) was also voluntary and the mandatory/voluntary nature of one (Chap. 16) is not reported.

Primarily qualitative data, obtained via the normal methods of recorded (audio and video) interviews and observations, were analysed. Where quantitative data were collected via questionnaires these were analysed to provide descriptive statistics of the data set.

A variety of perspectives underpinned the analysis of the data. These were the traditional grounded theory approach (Chap. 10), a framework for evaluating CPD practice comprising teachers, students and schools in an overlapping manner (Chap. 12), Engstrom's activity systems theory (Chap. 13), thematic analysis (Chap. 15), a conceptual framework of "professional noticing" (Chap. 16), Latour's (2005) notion of, "agent(s) of change" (Chap. 17) and narrative analysis linked to play in mathematics (Chap. 18).

The various foci of the CPD are teaching techniques consonant with mathematical process goals such as questioning and problem-solving (Chap. 10); changes in teaching practice as a result of coaching and mentoring (Chap. 12); challenges faced by mathematics teachers involved in a degree-awarding programme with reduced contact time compared to full-time prospective students following the same programme on a full-time basis (Chap. 13); implementation of lesson study and concept study (Chap. 15); lesson study on the teaching of a particular mathematical construct to a grade 8 class (Chap. 16); addressing the issue of "relevance" through observing the use of mathematics and science at an actual worksite so as to change teaching (Chap. 17) and play as teaching strategy to address affective domain issues (Chap. 18).

The major recommendation resulting from Chap. 10 is that mentor teachers need more support and that a need also exists for the development of useable strategies to deal with errors and misconceptions. In Chap. 12 a major challenge experienced was the lack of time to meaningfully implement a school-based cascade model of CPD. Reference is also made to resistances mentors face from their colleagues. The issue of how the internal dynamics of using a programme designed for preservice teacher education mitigates against the realization of the objective of enhancing relevant mathematical content knowledge of practising teachers is a major outcome alluded to in Chap. 13. The teacher educators in Chap. 15 were capacitated to identify mathematics topics that their students find difficult to understand and to teach. They could develop research questions to be investigated in their on-site lesson study sessions. Chapter 16 brought to the fore that shifts to a more reform literature-aligned practice are possible if lesson studies are implemented. The exposure to different strategies to deal with topics as was the case with the ordering of integers, where teachers found the use of a vertical number line fitted their context better than a horizontal one, was found beneficial by the teacher participants. The teachers reported on the changes of their teaching practices and expressed appreciation for the connection of mathematics to real-world situations in Chap. 17. The CPD initiative heightened their awareness about intra-class collaboration. Chapter 18 had as a major outcome that perceptions of mathematics were impacted and that mathematics can be role-played or demonstrated through everyday life situations.

## Concluding Comments

In deliberating about the focus of CPD for mathematics, Hiebert and Morris (2012) distinguish between those with primary focus on the improvement of the quality of teachers and those with primary focus on the improvement of the quality of teaching. Julie (2019) opines that this distinction should not be viewed as a dichotomy but rather as a continuum with the development of the teacher at one end of the continuum and the development of teaching at the other end. He also alludes to CPD research being more near the development of the teacher end. The contributions in this book on CPD also fall at this end of the continuum with two chapters (Chaps. 13 and 18) being more of a course evaluation by participants given its cohorts following prescribed, formal degree programmes. The location of studies at the teacher development end points to the need for research in the region nearer to the other end of the continuum.

Given this focus on the development of the teacher there is no reporting of the impact of the offered CPD programmes on the achievement by students. The enhancement of achievement by students is the one outcome expectation of governments, parents and schools. This is a complex matter since student assessment instruments can be of different kinds (see Julie, 2019). Which kind is used can have different consequences for CPD being offered. Again the need for the research and development of viable instruments to assess the impact of CPD on student achievement outcomes needs to be pursued.

In closing, the participants the programmes covered in this section are teachers and the CPD providers. Transforming education systems to enhance achievement outcomes of students requires collaborative work between all major role-players—officials in education ministries, school management, CPD providers, parents and teachers. It needs to be researched how the different strengths of these role-players can be harnessed to reach the goal of improved student achievement in mathematics, especially those students at the lower rungs of socio-economic ladder, which are the majority of school-goers in sub-Saharan Africa.

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# Chapter 10
# Mathematics Teachers Professional Development Perceived to Improve Instruction and Learning Outcomes in Rwanda

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## Introduction

In order to transform Rwandan society from agricultural to a knowledge-based economy that is one of the pillars of Rwanda's Vision 2020 consists in increasing the number of skilled people for the socio-economic development of the country. This pillar underlines the importance of education and training for equipping citizens with the knowledge, skills and attitudes they need to be entrepreneurial in their own learning, thinking and doing. In this perspective, improving the quality of education and training is the overarching goal of the Ministry of Education as reflected in Education Sector Strategic Plan 2013–2018 (Rwanda Ministry of Education (MINEDUC), 2015). However, the quality of education depends on many factors, but teachers and school leaders are the two most critical actors (Day & Leithwood, 2007). Evidence shows that teacher development improves teaching and learning and that effective school leadership is required for professional development of teachers (Glewwe & Muralidharan, 2016). The introduction of a competence-based curriculum (CBC) in Rwandan schools calls for comprehensive change and new thinking on instructional approaches in teaching and learning, focusing on a learner-centred approach. Therefore, teachers need to master a variety of methodologies to increase the quality of their teaching. Improving the quality of teaching is a career-long process; it is not finished when teachers graduate or after one training session (Killen, 2015). Various researchers (e.g. Beatty, 2000; Day et al., 2002) explored

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teacher collaboration and mutual support as agents in raising teacher performance. In addition, the provision of opportunities for teachers to reflect on their teaching and engage in dialogue about it with other teachers helps in building motivation and commitment (Day et al., 2002). Teachers learn from each other through building lessons, working together on real school improvement problems, drawing on best practice in developing solutions, feedback and taking part in coaching and mentoring, which are considered by many teachers as the most effective ways to improve their practice (Hampton et al., 2004).

In this regard, the Flemish Association for Development Cooperation and Technical Assistance (VVOB) in partnership with Rwanda Education Board (REB) and University of Rwanda-College of Education (UR-CE) has introduced and implemented a multiyear 'Leading, Teaching and Learning Together' Programme (2017–2021) in order to support the government of Rwanda to implement the competence-based curriculum which requires teachers' creativity and innovations for enabling students achieving learning outcomes, thus improving the quality of education. To date, many teachers still work largely in isolation from colleagues. This isolation can be especially difficult for new teachers who on arriving in a school are often left on their own to succeed or fail within the confines of their own classrooms—often likened to a 'sink or swim' experience (Ingersoll & Strong, 2011).

Based on a number of new ideas in education system in Rwanda, it is important to see how teachers cope with a number of educational changes very particularly in line with the introduced competence-based curriculum which seems to be much demanding to teachers whose academic background is hypothetically not match with competence-based curriculum. Within the $(LT)^2$ programme, mathematics school subject leaders (MSSLs) were selected at school level to receive a continuous professional development (CPD) programme specific to their career firstly as teachers and secondly as mentors of their fellows. The programme provides a wide overview of techniques and methods to equip mathematics subject leaders in primary education with knowledge and skills to support colleagues in the teaching of mathematics. Mathematical instructional aspects focused on include questioning, mathematics conversations, developing problem-solving skills, use of learners' errors and misconceptions, connecting multiple representations, games, motivating learners and assessment. However, from existing literature on teacher training (e.g., Vanderline & van Braak, 2010), there is still gap between what teachers received in the training and what they do in practices. Thus, the overall aim of this chapter consists of analysing how trained MSSLs demonstrate that the CPD programme has made a difference to their teaching capability and, ultimately, their performance in their mentoring role. This is achieved through analysing what mathematical aspects are applied in classrooms together with challenges faced by teachers.

In the next section, different mathematical aspects used in engaging learners in mathematics lessons are discussed. The section is followed by research methodological considerations, data presentation as well as key findings. Also, the common and specific challenges that impede mathematics teachers from applying the introduced mathematical aspects are explored and some proposals for addressing these both common and specific challenges are provided.

## Continuous Professional Development on Mathematics Instructional Techniques

Part of the competence-based curriculum framework in Rwanda is capacity building for teachers including continuous professional development for head teachers in school leadership, school management and school improvement planning and for teachers in coaching and mentoring (REB, 2015). Accordingly, teachers and new teachers should receive an orientation to the curriculum as part of their induction, in addition to mentoring and coaching at the school level. The Rwanda Ministry of Education has developed a school-based mentorship framework to guide the implementation of CPD activities. Our understanding of continuous professional development builds on the definition selected by the Rwanda Education Board that is 'learning continuously throughout one's career to improve performance. CPD is an umbrella term that covers all formal, non-formal and informal professional learning experiences over the duration of a teacher's career' (REB, 2015). We therefore define CPD programme in the present chapter as certified sustainable and formal professional development programme provided by an accredited learning institution building on existing practice and interventions in Rwanda, with formal learning and practicing activities and excluding one-off, 1-day or short residential courses. It is within this framework, we interrogate ourselves as trainers how do trained MSSLs demonstrate that the CPD programme has made a difference to their teaching capability and, ultimately, their performance as peers' mentors.

The following paragraphs describe key active teaching and learning techniques to foster competence-based teaching and learning in mathematics classrooms. Gray and Tall (1994) argue that mathematics has been notorious over the centuries for the fact that so many of the population fail to understand what small minority regard as being almost trivially simple. In Rwanda, many learners fail to achieve basic skills in numeracy (REB, 2017); and this will limit their capabilities to freely choose their professional career. Although some people do not need or use highly technical mathematics in their daily jobs, the complexity of daily life requires that we all can reason with numbers (National Research Council, 2002).

Ideally, students pursue learning not because they must but rather out of a desire to figure things out. This is called intrinsic motivation. To achieve this, techniques of motivating learners are at the heart of mathematical instructional aspects among others that were introduced to mathematics teachers in the CPD programme. These are questioning, mathematical conversation, problem-solving skills, mathematical representations, teaching and learning resources, games, group work, use of learners' errors and misconceptions and inclusive education.

Questioning is a key skill for teachers. During an average lesson, teachers ask tens of questions (Lemov, 2015). But what makes a question effective? And how can you use questioning to stimulate thinking, collaboration and motivation in your mathematics lessons? Unfortunately, many teachers don't use the power of questioning to stimulate thinking and learning fully. Upon hearing a correct answer, many teachers are happy to move on. Upon hearing a wrong answer, they correct it or ask another learner to give the correct answer. Some teachers consider a wrong answer as something that needs to be avoided as much as possible. Often, teachers move on without knowing why a learner gave an answer or if anybody else had other thoughts. However, answers of confident students are a bad guide to what the rest of the class is thinking (Van de Walle et al., 2015). Questions are not only about getting the right answer from learners but are about developing reasoning skills and the capacity to formulate one's thinking accurately. However, mathematics teachers are often concerned with concepts and numbers but lack the language and argumentation skills to support or challenge learners' answers. This is in particular observed in Rwandan education system which has recently adopted to use English as medium of instruction at all levels of education. Similarly, learners often get right answers to problems but cannot explain how they came up with those answers. Arguments for this paradox may vary, but the most recognised in the Rwandan education system like in most of the sub-Saharan African countries is the use of English as medium of instruction (Barrett & Bainton, 2016). Although young children may have a beginning understanding of mathematical concepts, they often lack the language to communicate their ideas. By modelling and stimulating discussions and paying attention to using correct mathematical terms, teachers can help students to express their ideas. It is also important to encourage talk among students as they explain, question and discuss each other's ideas and strategies while they are solving a given problem.

Being able to solve problems is a key objective of studying mathematics. Solving problems is at the core of what doing mathematics means (Burns, 2015). Learning mathematical rules and facts is important, but they are the tools with which we learn to do mathematics fluently. Problem-solving is about engaging with real problems, guessing, discovering and making sense of mathematics (Polya, 1945). For Polya, problem-solving is:

- Seeking solutions not just memorising procedures
- Exploring patterns not just memorising formulas
- Formulating conjectures (conclusion from observations without proof), not just doing exercises

Burns (2015) provides a structure in three phases useful for planning lessons that include problem-solving: introducing for launching the investigation, exploring for students to work independently or in groups and summarising for a classroom discussion to share results and talk about the mathematics involved. Mathematics is a language that uses many representations of ideas. Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas (National Research Council, 2002). There is no inherent meaning in symbols. Symbols always stand for something else. The meaning a symbol has for a child depends on what the child knows and understands about the concepts the symbol represents (Richardson et al., 2012). In order to support learning of symbols, teachers are most of the time advised to integrate learning material. There exist various teaching and learning resources such as human and material resources. Here, the focus is on material resources which are physical or manipulatives.

Physical materials or manipulatives are important tools for helping children make sense of mathematics. They can support learning and they are very effective for engaging students' interest and motivating them to explore ideas (Burns, 2015; Carbonneau et al., 2013). But to be effective, it is important teachers have a good insight in how manipulative materials can help children learn (Van de Walle et al., 2015); especially these materials need to be connected to the situations being modelled (National Research Council, 2001). Students also need the opportunity to reflect upon their actions with manipulatives and discuss the meaning they generate, so that the link between the manipulatives and the key mathematical ideas that they represent is clear (Clements & Sarama, 2014).

Practices have shown that games (Afari, 2012) and cooperative small groups (Brame & Biel, 2015) can be very useful to capture students' interest and provide alternative ways for engaging them in learning mathematics. Games can address various skills such as strengthening procedural knowledge, strategic thinking and creativity, while cooperative small group works have positive effects on both social skills and mathematics learning (Afari, 2012). However, this effect is dependent firstly on shared goals for the group and secondary individual accountability for the achievement of the group (Askew & Wiliam, 1995). When the purpose of the lesson is to develop fluency in a skill and there is little to discuss, then individual practice may be more suitable. Group work is useful when the purpose of the lesson is to develop conceptual understanding or problem-solving skills. In these cases, learners need to share alternative interpretations and approaches. There is a clear difference between working in a group and working as a group (Swan, 2005). It is common to see learners working independently, even when they are sitting together. Sometimes, one group member does the work and others copy the solution. In this case, they work in group not as the group.

Working individually or in cooperative small groups, students are likely to make errors or mistakes. As teacher's trainers, we observe that the reaction of many teachers in Rwandan classrooms when students make an error is to correct it as quickly as possible or to call upon another learner to provide correct answer. Within active learning framework, errors can provide teachers with valuable information about students' thinking and also can be used as starting points for powerful teaching. This is especially true for errors that are not just calculation errors but that reflect underlying misconceptions. Learners as well need to see errors as opportunities for learning, not as things to avoid. They must feel that it is valuable to offer an idea that might be incorrect and know that they will have the support of their teacher and fellow learners to resolve errors in their thinking. Successfully changing students' misconceptions consists of two steps. First, you need to discover and make explicit (elicit or expose) the misconceptions and, secondly, you need to help students to change them (Swan, 2001). Just telling students that a certain solution or strategy is wrong and giving them the correct answer is not enough. As the misconception looks intuitively correct to learners, they will easily revert to their old framework that includes the misconception. This is in contradiction of the Rwanda competence-based curriculum framework that dictates teachers to identify any students who are struggling mathematically and adjusting the learning environment to better enable them to learn. These practices are part of what is called inclusive education that is based on the idea that all learners have the capacity to succeed in mathematics and the recognition that diverse thinking is an essential and valued resource OECD (2010). In other words, inclusive education means tailoring teaching to meet the needs of each individual learner. But for the most difficult issue teachers face daily is how to meet the needs of so many students that vary greatly in terms of what they currently know, what they can do, their motivation, their personalities, etc. OECD (2010).

## Research Methodology

This study is qualitative under the interpretivism paradigm. Qualitative and quantitative data were collected through interview protocol, structured questionnaire and classroom observation whereby tools were developed on basis of investigating how mathematics school subject leaders (MSSLs) from six districts out of 30 country districts are applying instructional aspects described in the previous section at the same time mentoring and coaching their fellows. An MSSL is appointed by the school leader on the basis to have served at least 3 years in the same school, and all MSSLs of the six districts have to benefit of the CPD programme at different cohorts. The present book chapter is concerned with 39 MSSLs comprised of six females and 33 males that attended the first of the four cohorts. They are in range of 5–30 years of teaching experience. The six districts were selected based on the P6 learners' mathematics results in national examinations which revealed lower performance in mathematics especially for girls. Furthermore, these districts registered higher rate of dropouts at primary school level for the 3 years up to the baseline study in 2017. The formal training where participants met trainers from the University of Rwanda-College of Education (authors of the chapter) composed of four face-to-face sessions of 2 days. At the end of each session, participants were given a practical assignment to be submitted at the first day of next session. There is an interval of 1 month between two consecutive sessions to allow participants doing their assignment. After the third session, trainers visited each trainee in his/her school where they observed mathematics lessons, discussed with school administrators on observed changes in the trainee's behaviours and practices. For lesson observations, there were a pre- and post-lesson conversations. In the pre-lesson, the teacher/trainee (MSSL) shared with the trainer the lesson plan and key mathematical aspects he/she would like focus on during lesson delivery. During the lesson, the trainer would give more attention to these aspects. After the lesson, a joint reflection was conducted. This process was also part of training process where key steps of effective feedback encountered in the training were followed. Before the observant gave his/her observations on which aspects went well and any area for improvement, the teacher was given a space to share his/her feeling on the lesson taught. He/she was able to link lesson planning and actual teaching process. Furthermore, another mathematics teacher preferably a new one in school was interviewed to learn more about whether MSSLs are playing their role as mentors; but analysis of these interviews is presented in Maniraho, Mugabo, Habimana, Gafiligi and vande Walle Chap. 12. All 39 mathematics participant teachers were interviewed and observed while teaching. Furthermore, they all completed structured questionnaire at the same day. The collected data were manually analysed using the themes related to different mathematical aspects discussed in the literature review. The findings are presented both qualitatively and quantitatively but with descriptive interpretation. The overall chapter objective consists in sharing to what extent MSSLs are applying mathematical instructional aspects in Rwandan primary schools as result of their participation to the CPD programme. To this end, the following two sections present participants' views and observations from classroom practices.

## Implementation of Mathematics Aspects: MSSL Views

The elements of mathematical instructional aspects MSSLs reported to have applied in their teaching include questioning, mathematics conversation, problem-solving, using multiple representation of concept(s), group work, motivating learners, using learners' errors and addressing misconceptions, formative assessment and inclusive education. Furthermore, as their role in improving teaching and learning mathematics within their respective schools, MSSLs confirmed having coached and mentored their fellows. But this was at different levels as shown in the chart below (Fig. 10.1).

In addition, the MSSLs reported that they need more support in all aspects but at different levels as indicated in the chart below (Fig. 10.2).

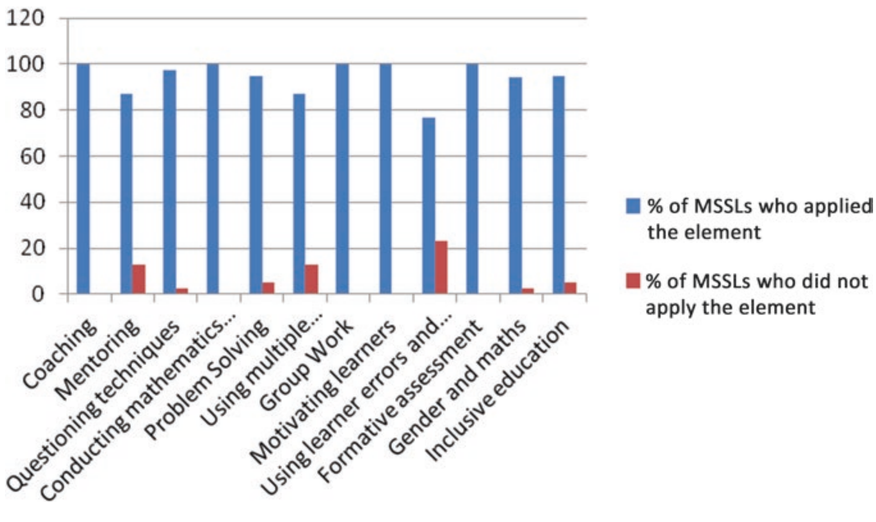


**Fig. 10.1** Elements applied by MSSLS

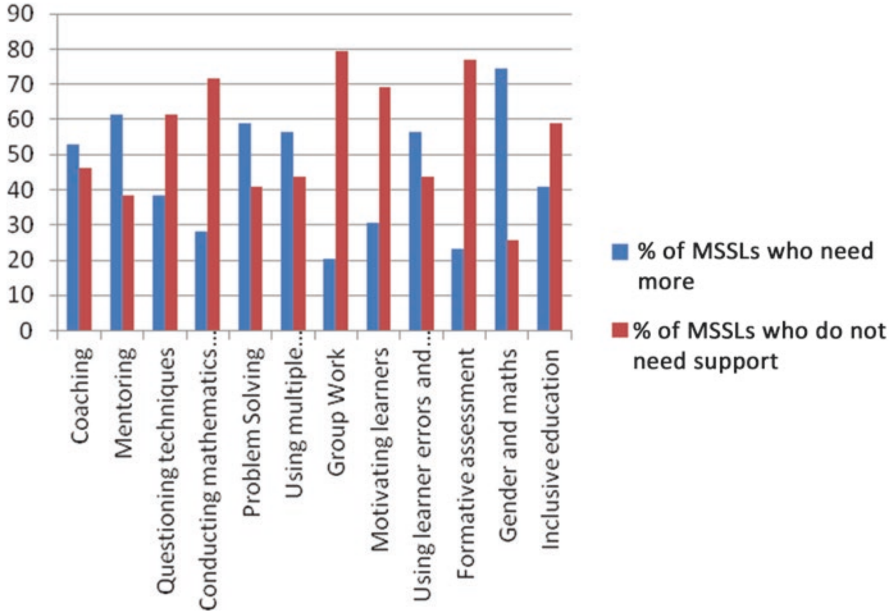


**Fig. 10.2** Elements for which MSSLs need more support

From the above chart, the aspects in which MSSLs need more support are, in order of priorities as follows: gender and mathematics, mentoring, problem-solving, using learners' errors and addressing misconceptions, using multiple representations of concept, questioning techniques, motivating learners, problem-solving, formative assessment and group work.

About coaching new teachers, MSSLs reported to have conducted coaching and mentoring activities with new teachers on an average of two new teachers per MSSL. Topics which were brought by coaches for discussions include, to list but few: preparing scheme of work, lesson planning, teaching table of multiplication, rounding off decimals, use of geometric tools, using mathematics conversation and errors and addressing misconception, how to teach proportions, how to make student reports, effective grouping and enhancing learners' problem-solving skills. However, the effectiveness of coaching or mentoring can be questionable as much as almost half participants share the desire of getting more support.

Though good practices were mentioned by participants, there exist also challenges that prevent them to apply mathematics instructional aspects or to play their mentorship role. The commonly reported ones are large class size, lack of teaching resources, heavy teaching load, lack of proficiency in English language of instruction, lack of ICT facilities and electricity, double shifting system, insufficient time to find teaching materials and using them effectively in a 40-min lesson even when they are available.

Other challenges are lack of support from some school leaders or fellow teachers who do not understand the role of MSSLs. Though the MSSLs use group work in the most cases, they are not satisfied on their effectiveness due to the period of 40 min given to a lesson. They continue to argue that this period does not give enough space for discussions. In this way, the quality of mathematical conversations and effectiveness of questioning are still area for investigation. Some MSSLs also highlight the absenteeism and dropout of some learners and incapacity to get teaching aids for every lesson to support learners with specific special education needs such as blindness. Change of mind of teachers was also mentioned as key challenge in coaching or mentoring process especially when it comes to fellow older than MSSL. The common and shared challenge is lack of time to conduct coaching conversations due to the heavy teaching load for every teacher.

## Implementation of Mathematics Aspects: Lesson Plans and Classroom Observations

The present sub-section discusses findings from MSSL lessons observed by trainers. The lesson observation tool and analysis of collected data were based on the mathematical aspects discussed in the literature review section. The table below summarises findings from lesson observations including lesson plan analysis. The lesson plan tool is nationally developed by Rwanda Education Board for all schools following national curriculum.

Before the lesson, the teacher shared with observant what mathematical aspect(s) he/she was going to emphasise on while delivering the lesson. After the lessons, reflective discussions were carried out. Both discussions and lesson observations (Table 10.1) demonstrated that teachers are trying and eager to use active teaching and learning techniques with focus on use of multiple representations, problem-solving, mathematical conversation, group work and questioning. Conversations between MSSLs and researchers revealed that positive changes in teaching of mathematics are being occurring as result of CPD programme. This change was also confirmed by respective school leaders through interviews with trainers as detailed in Maniraho et al. Chap. 12. Key area for improvement for teachers consists in using learners' errors and misconceptions to provide constructive feedback. It was observed that teachers are still collecting right answers without giving time and attention to wrong answers. In addition to what teachers point out, the language of instruction constitutes a key barrier for classroom interactions in general and in particular for mathematical conversations and questioning.

## Discussions

The actual teaching and learning of mathematics within competence-based curriculum framework is observed in the lesson planning and delivery. This is an opportunity for the teacher to practically integrate mathematics instructional aspects so that all learners can achieve learning outcomes. Findings of the present study show that given opportunities mathematics teachers can exploit different mathematical teaching techniques that engage actively learners though with some difficulties. Participant teachers' lesson plans indicate the use of a variety of mathematical instructional aspects discussed in the CPD course but during lesson delivery using questioning and small group works are the mostly commonly used.

While Burns (2015) suggests involving learners in investigation either independently or in groups and summarising for a classroom discussion to share results and talk about the mathematics involved, participant teachers believe that problem-solving skill is only acquired through world problems. Furthermore, observed lessons show that group works are well organised in terms of gender balance, and content to be discussed consists in replicating similar examples used by the teacher during the development of the lesson; questions that stimulate learners' solving and creativity skills are rarely observed. Questioning techniques are limited by language barrier from both teachers and learners as pointed by participants during reflective session. This leads to unavoidable 'question-right answer' that does not enhance learners' thinking and mathematical conversations; thus learners are likely to not demonstrate the understanding of the content. This leads to conclude that questioning is still teacher-learners directed with less involvement of the trio teacher-learner-learner. As consequence a social aspect of mathematics classroom (Koblitz & Wilson, 2014) is missing. It is argued that group works to some extent any active teaching technique including questioning enhance students learning mathematics and bring also social benefits. Students have opportunities to communicate with others and create a position in an argument by using objective facts to back themselves up instead of trying to persuade through emotions (Koblitz & Wilson, 2014).

In most cases, school leaders do not understand clearly the attributions of MSSL. Thus, there is observation of lack of facilitation to the MSSL to accomplish their attributions. In addition, fellow teachers still manifest resistance to try out new acquired teaching techniques by arguing to have limited time in exploiting techniques learnt within a lesson of 40 min. In other words, peers do not recognize and value the role of MSSL in their professional development. Looking at the percentage of those who need support in using learner's errors in enhancing learning (Fig. 10.2) and the observation on incorporating cross-cutting issues (Table 10.1), it can be deducted that teachers are not yet well prepared to implement the competence-based curriculum. Based on the above information in combination of lesson observation and interview, the following are areas where MSSLs still need more improvements: formulating instructional objectives including evaluation of competences that are addressed by the lesson, using assessed prior knowledge to introduce the new topic, enhancing learners' problem-solving skills (use more than one strategy to solve a particular problem), developing/using multiple mathematics representations, using a variety of resources (manipulative resources, games, ICT, etc.), eliciting and addressing learners' errors and misconceptions, motivating learners, addressing cross-cutting issues (gender, learners with special needs) and using language of medium of instruction.

Classroom observation (Table 10.1) shows that almost half (15 out of 39) of MSSLs use multiple representation especially when introducing lessons. The evaluation of learners' understanding/learning outcomes is mainly composed of exercises involving only calculations. Thus, students are not evaluated on multiple representation skills which lead to conclude that teachers are more conversant to the use of a symbol mathematics rather than providing learners access to mathematical concepts via multiple and varied representations. Because of the abstract nature of mathematics, people access mathematical ideas through the representations of those ideas in symbols (National Research Council, 2002). Thus, learners in the research setting are not supported to firstly develop a concrete level of understanding for a mathematics concept, while they are expected to use this foundation to link their conceptual understanding to abstract mathematics learning activities (ibid.).

## Conclusions and Implications

Findings reveal that questioning, group work, mathematical conversations and multiple representations of concepts are the most applied by participants in teaching and learning process. If effectively well implemented, all these instructional are considered to be likely contributing in producing high learning outcomes in mathematics at primary schools in Rwanda.

Of particularity mathematics and science teachers, irrespective of not being teachers of language, must ensure the proper use of the language of instruction by learners to help them communicate clearly and confidently and convey ideas effectively through speaking and writing (REB, 2015). To see this happen, the present chapter calls for educational stakeholders for equipping teachers with the skills required to support that all learners actively participate in learning process. The support should go through different steps involved in the teaching process with emphasis on lesson planning, delivery and assessment. Of particular interest, in order for all children to progress and enjoy all the benefits of education within an inclusive setting, they need the support of all actors involved in the provision of education. Training teachers cannot alone make inclusive education happen. The overarching goal of the Rwanda Ministry of Education to equipping all citizens with the knowledge, skills and attitudes will be achieved only if teachers are supported by school leaders, parents, local authorities, civil society and development partners in accessing appropriate resources and using different approaches and strategies so that all learners especially within large class size are actively and effectively engaged in lessons. It can be concluded that challenges for improving the quality of education as reflected in Education Sector Strategic Plan 2013–2018 for students achieving learning outcomes may not be alleviated by simply involving teachers in training on subject content or pedagogical content knowledge; rather simultaneous actions such as policy on teaching load, teaching and learning resources availability may be required.

**Table 10.1** Observations from lesson plans and classrooms

| | Observed aspects | Observations/comments of trainers |
|---|---|---|
| I | The lesson plan | |
| | Objectives | In general, all 39 MSSLs know how to formulate instructional objectives; but there is a need for some of them to include the three components of competence (knowledge/understanding, skills, attitudes/values) |
| | Learners and teacher's activities | All 39 lesson plans show learners and teachers' activities. However, some of these activities seem to demonstrate teacher-centred rather than learner-centred teaching approach. For instance, activity related to summarising lesson taught (providing key lesson points) was found among teacher's activity. There is a need to improve how activities are set and how they would be carried out |
| | Cross-cutting issues (gender, inclusiveness, etc.) | In general, MSSLs are considering only gender aspect which is addressed through group work organisation. Whenever another aspect of cross-cutting was mentioned in the lesson plan, classroom observations showed that this was not taken into consideration during the teaching and learning process |
| | Generic competences | 35 out 39 MSSLs include generic competences in their lesson plans with accordance to the lesson taught. But these competences are not assessed during the teaching and learning process |
| | Provision of teaching and learning resources. | Teaching aids and teaching materials are not yet clearly articulated in most of lesson plans |
| II | Lesson delivery | |
| | 1. Introduction: | |
| | Prior knowledge is assessed and is used to introduce the new topic | All 39 MSSLs assess the prior knowledge through two or three mathematical exercises on the previous lesson answered. But there is no smooth transition between the assessed prior knowledge with the new knowledge being taught. There is a need to adapt the learning content to level of learners or follow a logical sequence of different topics. For example, one new lesson was about the circumference of a circle. The MSSL started by asking learners to find the area of square and volume of cube. [This is what we mean by absence of smooth transition.] |
| | World problems/real-life situations are used to stimulate learners' interest | Depending on the topic, some MSSLs started lessons with a world problem which generally is solved in small groups |
| | 2. Lesson development: | |
| | The link between the introduction and the new topic | As mentioned above few MSSLs tried to link the introduction with the new lesson |
| | (a) Questioning techniques | Questioning technique is used by MSSLs, but it consists in question-'right' answer. Generally, when a learner X does not provide right answer, the question is directed to another student or teacher gives answer |

| | | |
|---|---|---|
| | (b) Mathematics conversations (was the mathematics language used appropriate?) | In general, all MSSLs know how to use mathematics language appropriately. However, there is a need to emphasise on the use of instruction language (English). It was observed a strong shift to the local language. |
| | (c) Problem-solving skills (more than one method was used to solve a particular problem) | All 39 MSSLs enhance learners' problem-solving skills through setting world problems |
| | (d) Use of multiple representations | 15 out of 39 MSSLs use multiple representation especially when introducing lessons. The evaluation of learners' understanding/learning outcomes is mainly composed of exercises involving only calculations. This means students are not evaluated on multiple representation skills |
| | (e) Using a variety of resources (manipulative resources, games, ICT, etc.) | In general, teaching aids are used in teaching, but generally manipulated by the teacher |
| | (f) Group work (are learners working in group or working as a group?) | In all observed lessons, there is a use of group work. However, in most cases, the task is similar to examples worked out in the lesson development |
| | (g) Motivation (are learners' given the opportunity to ask questions? Are learners challenged?) | All MSSLs are concentrated on asking questions to learners but not giving opportunities to learners to ask questions themselves. After discussion, they said it is because of time reserved to one lesson period, but they acknowledge the importance for learners to ask questions so that the mathematical conversation takes place and then the lesson is effective |
| | (h) Eliciting and addressing learners' errors and misconception | Errors and misconceptions are not used to engage learners in mathematical conversations. As mentioned above, question-right answer limits teachers to explore learner's understanding. This is a common observed teaching approach. However, during reflective period after the lesson, this aspect is well known by all MSSLs. MSSLs said that they are not emphasising on it as the lesson period is not enough for one lesson to be taught considering every aspect |
| | Addressing cross-cutting issues (gender, learners with special needs, etc.) | As said above, only one cross-cutting issue, gender, is considered. It is also addressed through mixing boys and girls in groups and calling upon alternatively boys and girls students for presenting group findings |
| | 3. Conclusion: | |
| | Summary | Five out of 39 observed MSSLs led learners to summarise key lesson points before the evaluation takes place |
| | The evaluation is procedural fluency focused rather than focusing on memorisation aspects | Evaluation is almost focusing on aspects of memorisation: simple duplication of examples used in the teaching process. There is absence of questions opening learners' mind about the application of what learnt |

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# Chapter 11
# Paradigms in Mathematics Teacher Professional Learning Research: A Review of South Africa's Literature for 2006–2015

**Mdutshekelwa Ndlovu**

## Introduction

In educational research and in the broader field of social scientific inquiry, paradigm wars are well known to have pitted the positivists against the interpretivists (constructivists) in a contestation for legitimacy and supremacy (e.g. Guba & Lincoln, 1994; Lincoln & Guba, 2005). This chapter seeks to explore the extent to which research paradigms played out in mathematics teacher professional learning/development (TPL/TPD) research in South Africa during the period 2006–2015. Although the literature seems to attribute paradigmatic uncertainty, mainly to young and early career researchers, this study's suspicion was that uncertainty affects even mid-career researchers as well, hence the need to investigate the paradigmatic landscape influencing contemporary research in mathematics teacher professional learning. Moreover, as Morgan (2007) observes, even experienced researchers working within a long-standing paradigm are often not explicitly conscious of the beliefs and practices that inform their research work.

Despite a large number of paradigms having proliferated educational research, Candy (1989) points out that they all distil down to three taxonomies or intellectual traditions: the positivist, interpretivist and critical. However, Morgan (2007) and Kivunja and Kuyini (2017) propose pragmatism as a fourth paradigm. Pragmatism is presented to be a compromise between positivism and interpretivism by acknowledging that we can learn from both worldviews about reality. Postpositivism also makes the same attempt of accepting the combination of positivist/and qualitative (interpretivist/constructivist) studies but remains largely positivist (Kivunja & Kuyini, 2017).

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## Theoretical Framework

Kivunja (2018) defines a theoretical framework as a structure, a theoretical coat hanger, consisting of concepts, their definitions and theories about the relationship between those concepts as expressed by experts in the extant scholarly literature. The main concept being investigated in this chapter is that of a paradigm together with its structure, different types thereof and their theoretical underpinnings. From a Kuhnian perspective, Willis (2007) views a paradigm to be a belief system, world-view or theoretical lens that guides both research and practice in a field. Mathematics teacher professional learning/development research is the field of choice in this study. Taylor and Medina (2013) define a paradigm more philosophically, and structurally, as comprising three main components: (a) *ontology*, a view of the nature of reality, whether it is internal (subjective/relative) or external (objective/independent) to the knower; (b) *epistemology*, a view of the nature and type of knowledge that can be produced and criteria for justifying it; and (c) *methodology*, the means by which knowledge is generated. These components are in line with Lincoln and Guba's (2005) structure. Kivunja and Kuyini (2017) separately add a fourth dimension: (d) *axiology*—ethical issues relating to values and ethics, the requirement that all research should aim at optimising positive outcomes or beneficence to research participants and prevent harm or maleficence to research subjects. This addition is in in keeping with Lincoln and Guba's (2005) criticism of the folding ontological and epistemological dimensions into one as well as the conflation of methodological and axiological dimensions into one. The main paradigmatic orientations and concepts of this study are examined in detail below using this theoretical coat hanger.

### *The Positivist Paradigm*

The traditional positivist paradigm, arising as a need for an alternative to theological and philosophical worldviews, posits that sensory experience (observation), experimentation and experience-based reasoning and, therefore, the scientific method of investigation or experimental (quantitative) evidence is the only legitimate gateway to authentic scientific knowledge and understanding of human behaviour. For Kivunja and Kuyini (2017), the paradigm is ideal for exploring cause-effect relationships relying on deductive logic, formulation and testing of hypotheses to provide explanations and make predictions based on quantitative measurements or outcomes. In terms of the four foundational dimensions of a paradigm, positivist ontology is realism. This realism is commonly referred to as *crude* or *naïve realism* (Morgan, 2007). That is, reality is assumed to exist, driven by immutable laws and mechanisms (Guba & Lincoln, 1994). From a realist's perspective knowledge is perceived to be out there, external and waiting to be discovered—by deterministic and reductionist means. Positivist epistemology is characterised to be *dualist and objectivist*. That is, the researcher and the participant (object or mathematics teacher) are assumed to be independent. The researcher is assumed to be capable of studying participants without influencing them or being influenced by them (Guba & Lincoln, 1994). The positivist methodology is *experimental and manipulative* in that questions and hypothesis are stated as propositions and subjected to hypothesis testing to verify or refute them after manipulating the independent/explanatory variable(s) being investigated and simultaneously controlling for confounding and extraneous variables very carefully. Positivist *axiology* (ethics) aims to optimize positive outcomes (beneficence) for humanity in general (Kivunja & Kuyini, 2017) and for research participants in particular (Mertens, 2015), by minimizing risk or harm and avoiding any wrongdoing.

## *The Interpretivist/Constructivist Paradigm*

The interpretivist or constructivist paradigm, which arose as an antithesis to positivism, aims to understand the world of human experience by getting into the heads of the participants to understand and interpret or reconstruct what they are thinking or meaning (Guba & Lincoln, 1994; Kivunja & Kuyini, 2017). From an anthropological perspective, Taylor and Medina (2013) refer to aim of this paradigm as a humanistic attempt to understand the culturally different other from their own perspective, by standing in their shoes or by looking through their eyes. A key ontological starting point for this paradigm is that reality is not out there, waiting to be discovered, but is socially constructed (Bogdan & Biklen, 1998). In other words, theory does not precede research in this paradigm. Rather it emerges from it by being grounded in data so generated. Furthermore, the positivist benchmarks of internal/external validity and reliability are substituted with four dimensions of trustworthiness and authenticity, namely, dependability, credibility, transferability and confirmability (Guba, 1981). Guba further argues that this (constructivist/interpretivist) paradigm takes into account that unlike matter, human behaviour, by its very nature, varies continuously, is contextual (situated) and amenable to multiple interpretations and, therefore, is more sensible to refer to dependability rather than reproducibility or reliability. Confirmability is used, in the place of objectivity, to refer to the degree to which an investigation can be confirmed by other researchers when biases and prejudices are eliminated or minimized. The ideas and experiences must be those of the participants rather than the researcher. In the interpretivist paradigm generalisability (external validity) is replaced by transferability, which refers to the provision of adequate contextual data so that other researchers can relate findings to their contexts of embodiment or situatedness (Guba, 1981; Lincoln & Guba, 1985).

In terms of the four foundational dimensions of a paradigm, interpretivist ontology is *relativist*. This relativism means that reality exists in the form of multiple, intangible mental constructions that are socially and experientially nuanced (Guba & Lincoln, 1994). In other words, knowledge is perceived to be internal, tentative and inductively constructed. Interpretivist epistemology is taken to be *subjectivist (and transactional)*. That is, the researcher and the participant (object or mathematics teacher) are assumed to be interdependent and the researcher is assumed to be incapable of studying participants without influencing them or being influenced by them (Guba & Lincoln, 1994). The interpretivist methodology is *naturalistic (hermeneutic and dialogic)* in that the variable and subjective nature of meaning construction implies that individual knowledge can be elicited through interaction between and among the researcher and the participants (through interviews, discourses, text messages and reflective sessions) to arrive at a more informed and more sophisticated consensus (Guba & Lincoln, 1994; Kivunja & Kuyini, 2017). The axiology is *balanced*, which suggests that the research findings ought to reflect the values of the researcher, trying to put forward a fair and unbiased account of the evidence.

## *The Critical Paradigm/Transformative Paradigm*

The critical paradigm locates its research in social justice and aims to confront the social, political and economic realities, which lead to social oppression, conflict, struggle and power relations in levels where these might happen (Kivunja & Kuyini, 2017). Due to its quest to influence politics to confront the grim realities of social and economic exclusion, systemic/structural inequalities and gross asymmetries of power and to strive to redress the situation, this paradigm is frequently referred to as the transformative paradigm. Taylor and Medina (2013) refer to aim of this paradigm as to empower participants to become critical and imaginative thinkers about whose interests should be (but are currently not being) served.

In terms of the four key dimensions of a paradigm, critical ontology aligns with *historical realism*, especially in relation to oppression; its epistemology is *transactional*, where the researcher interacts with the participants (as in the interpretivist paradigm); its methodology is *dialogic and dialectical* (as in the interpretivist paradigm) with the goal of transforming ignorance and misapprehensions into more informed consciousness; and, finally, its axiology abides by *cultural norms* (Guba & Lincoln, 1994; Kivunja & Kuyini, 2017; Taylor & Medina, 2013). When applied to education, Taylor and Medina (2013) admonish that critical enquiry must focus on raising consciousness among (in-service mathematics) teachers about (and away from) established values and beliefs underpinning seemingly natural traditional teacher-centred classroom discourses (NB: Evidently, there is no watertight boundary between the interpretivist paradigm and the critical paradigm, as both Guba & Lincoln, 2005 and Morgan, 2007 acknowledge).

## *The Pragmatic Paradigm*

This paradigm emerged from the premise that it is not possible to determine the truth about the real world only by reason of a single (deductive) method of enquiry as advocated by the positivists, nor is it possible to determine or access social reality

| Paradigm-> | Positivist | Interpretivist | Pragmatic |
|---|---|---|---|
| ❑ Ontology | ➢ *Realism* | • *Relativism* | ❖ *Non-singular* |
| ❑ Epistemology | ➢ *Objectivist* | • *Subjectivism* | ❖ *Relational* |
| ❑ Methodology | ➢ *Experimental* | • *Naturalistic* | ❖ *Mixed-methods* |
| ❑ Axiology | ➢ *Beneficence* | • *Balanced* | ❖ *Value-laden* |

**Fig. 11.1** Theoretical framework for paradigmatic orientations

only by virtue of the (inductive) methodology advocated by the interpretivists (Kivunja & Kuyini, 2017). In other words, a mono-paradigmatic approach to research is both limited and limiting in the ontological, epistemological, methodological and axiological claims it could make about social reality, including about mathematics teacher professional learning. Rather, some broker philosophers (e.g. Creswell, 2003, Alise & Teddle, 2010; Morgan, 2007) posit the need for a worldview that would promote research methods that could be more practical, inclusive and most appropriate to shed light and insights on the actual demeanour of research participants, which includes in-service mathematics teachers. In other words, pragmatism rejects the incommensurability or incompatibility of paradigms (Morgan, 2007).

Hence, in terms of the four key elements of a paradigm, the pragmatic ontology aligns to a *non-singular reality* (i.e. there is no single reality; all individuals have their own idiosyncratic interpretations of it); its epistemology is *relational* (i.e. research relationships are best defined by what the researcher considers to be appropriate to that particular inquiry); its methodology aligns with *mixed methods* (i.e. a combination of qualitative and quantitative methods); and its axiology is *value-laden* (i.e. the research must benefit people) (Kivunja & Kuyini, 2017).

Figure 11.1 summarises the theoretical framework.

## Context of the Research

Continuous teacher professional development or learning is a good feature of education systems in transition and/or in search of a new identity. More than two decades after the dawn of democracy, the South African education system is still in a perpetual state of construction and reconstruction, more so as it relates to the country's underperformance in international benchmark tests such as the Trends in International Mathematics and Science Study (TIMSS) and the Southern and East Africa Consortium for Monitoring Educational Quality (SACMEQ). Continuous teacher professional development is one lever through which to improve systemic performance—effectiveness, responsiveness and efficiency. Teacher competencies and adaptability to new curriculum demands are central to improving education systems as South Africa strives, not only to redress past imbalances in educational provision but also to cope with a rapidly changing twenty-first-century landscape induced by the 4th industrial revolution or 4IR.

Teachers are a key factor in determining the quality of education; hence, research in mathematics teacher professional learning can lead to a better understanding of what kinds of support are needed in order for them to perform their work more effectively. It is well known that the pre-service training of teachers can never be sufficient to produce experienced teachers, but only to licence them and ensure they acquire basic competences. A great deal of effort and financial support has gone into in-service teacher education or professional development (learning) in South Africa (e.g. Reddy, 2006), in an effort to modernise and democratise access to quality education for previously disadvantaged population groups. Mathematics as a school subject has been hardest hit by the critical shortage of appropriately qualified and experienced teachers in South Africa. Apart from a lack of experience, many teachers of mathematics in the first two decades of democracy have been either unqualified or inadequately qualified for the levels they teach. It has therefore been a twin problem of many of them needing support with content and pedagogy, even by their own admission (e.g. Ndlovu, 2014).

## The Research Questions

The overarching research question for this article can be stated thus: To what extent was SA mathematics teacher professional learning research, between 2006 and 2015, positivist, interpretivist or pragmatist? What has been the impact of the research as measured by the Google Scholar citation metrics up to June 2019?

## Research Methodology

This study adopted a literature review methodology that involved a systematic Google Scholar literature search (sampling) using the following key phrases: 'mathematics teacher professional learning research in South Africa', 'mathematics teacher professional development research in South Africa', 'mathematics teacher continuous professional development research in South Africa', 'mathematics in-service teacher education in South Africa', etc. The search was circumscribed to journal publications from 2006 to 2015. It excluded conference proceedings, books and book chapters as well as publications delisted by the Department of Higher Education and Training. The abstracts of the articles were subjected to content analysis to determine the paradigm that explicitly or implicitly guided the study. If the abstract was not informative enough, the methodology section of the article was scrutinised/perused to get clarity. Overall, 72 articles were deemed to satisfy the mathematics teacher professional criteria for the 10-year period under review. These articles were categorised using descriptive statistics.

## Results and Discussion

### *Journals Surveyed*

Figure 11.2 shows a cross section of journals surveyed in this study. The majority of the journals were local journals in which the majority of South African mathematics researchers publish. However, some of them are also recognised as international journals when indexed on international lists such as IBSS, ISI, Scopus, Norwegian and SciELO SA.

Pythagoras, an Elsevier indexed journal, had the highest number of articles (26) making up 36% of the articles reviewed (Bansilal & Mkhwananzi, 2014; Berger, 2013; Biccard & Wessels, 2015; Brijlall et al., 2012; Brodie, 2007; Brown & Schafer, 2006; Julie et al., 2011; Kazima & Adler, 2006; Lampen, 2015; Leendertz et al., 2015; Likwambe & Christiansen, 2008; Mbekwa, 2006; Mhlolo & Schäfer, 2012; Mhlolo et al., 2012; Molefe & Brodie, 2010; Mudaly & Moore-Russo, 2011; Mwakapenda & Dhlamini, 2009; Naidoo, 2012; Ndlovu, 2014; Posthuma, 2012; Shalem et al., 2013, 2014; Stols et al., 2008; Venkat & Adler, 2012; Wessels & Nieuwoudt, 2011, 2013). The second highest contributor was the African Journal of Research in Mathematics, Science and Technology Education (AJRMSTE), a Scopus listed journal, with 16% or 22% of articles reviewed (Adler & Pillay, 2007:

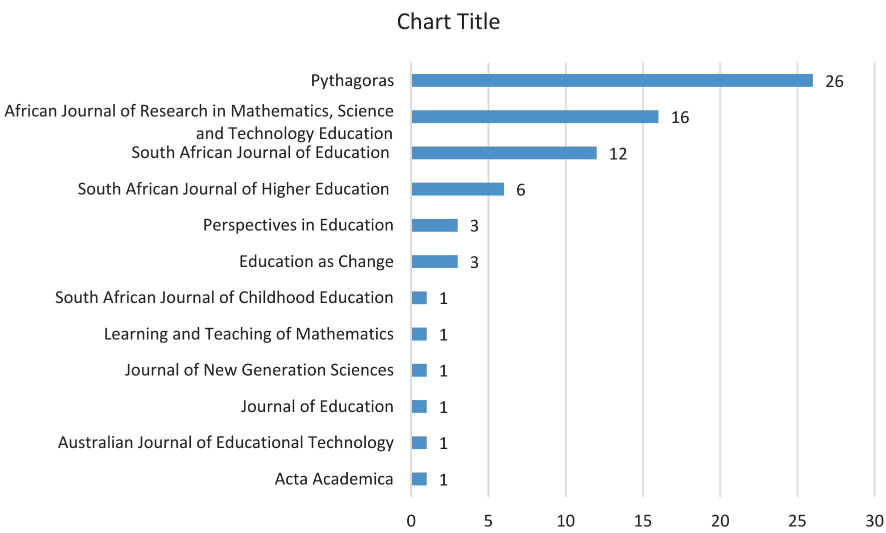


**Fig. 11.2** Cross section of journals surveyed

Adler & Ronda, 2015; Bansilal, 2011; Berger, 2011; Brodie & Sanni, 2014; Brodie & Shalem, 2011; Gierdien, 2014; Graven & Venkat, 2014; Julie, 2006; Naroth & Luneta, 2015; Plotz et al., 2012; Stols et al., 2015a, b; Tosavainen et al., 2012; Van der Merwe et al., 2010; Van Laren & Moore-Russo, 2012; Webb, 2015).

The South African Journal of Education (SAJE), indexed on both the ISI and IBSS lists, was the third highest contributor with 12% or 17% of the articles reviewed (i.e. Aldridge et al., 2009; Gierdien, 2008; Kazima et al., 2008; Maharaj, 2008; Ndlovu, 2011a, b, c, d; Nel, 2012; Ono & Ferreira, 2010; Pournara et al., 2015; Rakumako & Laugksch, 2010; Setati, 2008; Stols et al., 2015a, b; Van der Walt & Maree, 2007). Overall, these top three journals together accounted for 76% of the articles that were content analysed in this study.

The fourth largest contributing journal was the South African Journal of Higher Education with 8% of articles analysed (i.e. Bansilal, 2014; Fricke et al., 2008; Julie, 2009; Ndlovu, 2011b, c, d). The rest of the journals contributed the remainder (i.e. Bansilal et al., 2012, 2014; Brodie, 2013; De Clercq, 2014; Graven & Venkatakrishnan, 2013; Jita & Mokhele, 2012; Ndlovu & Mji, 2012; Pausigere & Graven, 2013, 2014; Stols & Kriek, 2011; Van der Merwe & Van der Merwe, 2008; Van Staden & Van der Westhuizen, 2013).

### *Distribution of Articles Among Paradigms*

Figure 11.3 shows how the articles distributed among the interpretivist, positivist and pragmatic paradigms corresponding to qualitative, quantitative and mixed-methods research approaches, respectively.

It is clear from the pie chart that the interpretivist paradigm had a lion's share of 64% influence on research on teacher professional learning. In the absence of explanations for this dominance, this study can only acknowledge that qualitative research was the most preferred by researchers in the field. It is also likely that it is more informative and accessible to a majority of readers. A major criticism, though, is that most qualitative studies tend to be small scale and therefore anecdotal rather and therefore not generalisable. However, a major strength is a deep understanding of thick descriptions of the cases studied. Good interpretive research opens a window for researchers to see the world from the perspective of the participants' lived experiences. As noted in Ndlovu (2018), such experiences related to teacher concerns about rapid curricular changes, a critical shortage of appropriately qualified, and appropriately deployed teachers. Massive attempts were made by researchers in the field to find solutions to the intractable question of mathematics knowledge for teaching, in relation to specific problematic topics. The positivist paradigm per se only influenced 5% of the research. However, this alone is misleading if we consider that the mixed-methods approaches accounted for 31% of the studies. Many researchers therefore combined quantitative and qualitative approaches in the pragmatic paradigm to draw from the strengths of both the positivist and interpretivist worldviews.

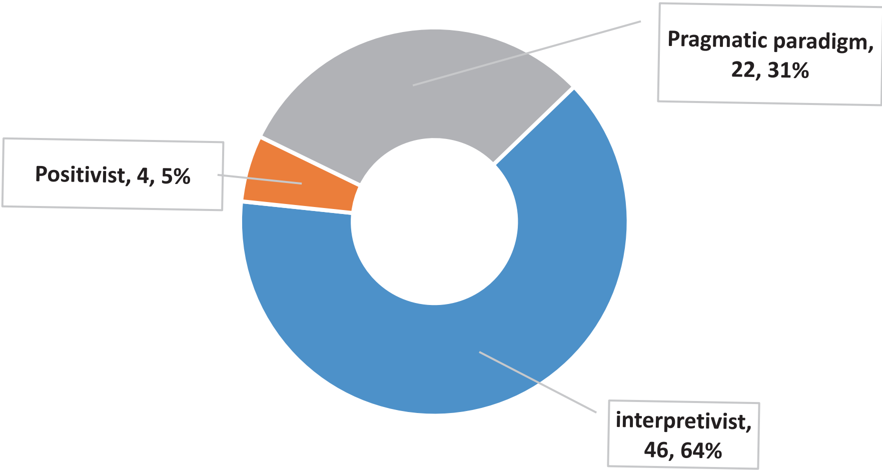


**Fig. 11.3** Distribution of articles among paradigms

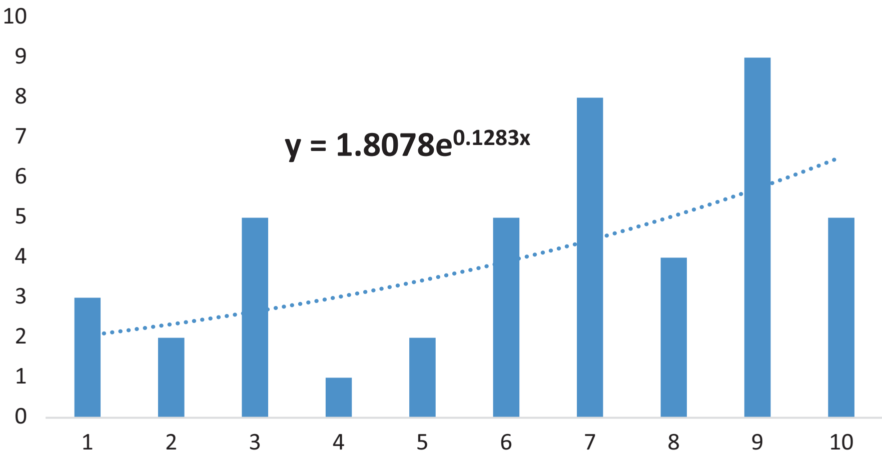


**Fig. 11.4** Rate of increase in interpretivist articles: 2006–2015

## *Comparative Analysis of Growth Rates of Article Outputs*

Figure 11.4 shows that during the 10-year period under review there was an exponential growth in qualitative studies, although not smoothly so. The exponential growth rate approximated the model shown on the figure with equation: $y = 1.8078e^{0.1283x}$.

Figure 11.5, on the other hand, shows that there was a curvilinear growth rate in solely quantitative studies best modelled by the quadratic equation: $y = 0.0379x^2$ –

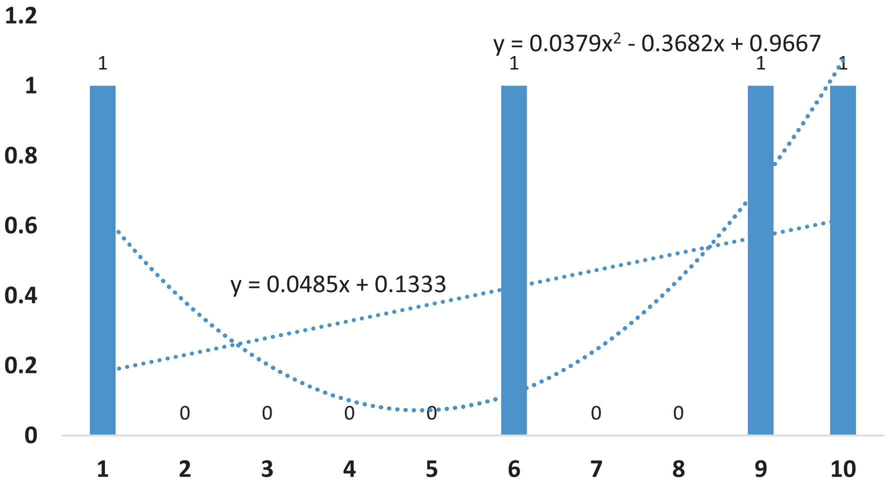


**Fig. 11.5** Publication rate for positivist articles

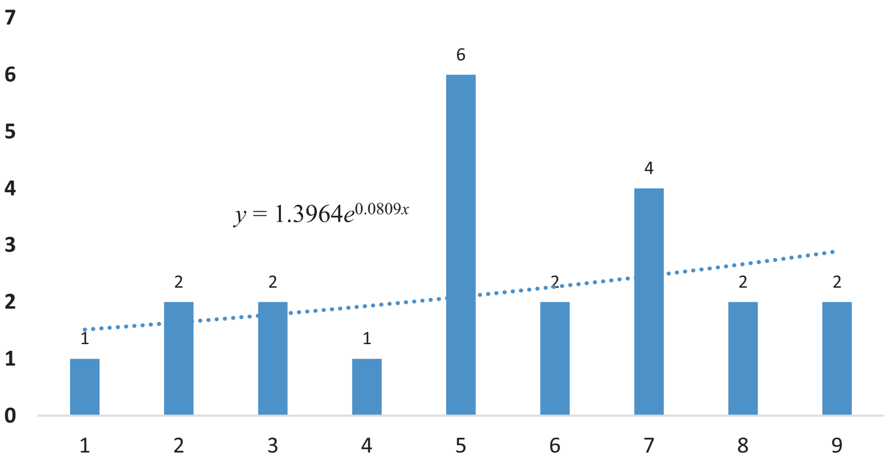


**Fig. 11.6** Publication rate of articles in the pragmatic paradigm

$0.3682x + 0.9667$. The average growth rate is best summarised by the trend line ($y = 0.0485x + 0.1333$) showing a much slower rate than for qualitative, interpretative studies during the period under review.

However, Fig. 11.6 shows that the growth rate in mixed-methods (pragmatic paradigm) studies was moderately exponential and could be approximately modelled by the equation: $y = 1.3964e^{0.0809x}$. This growth rate was more robust than that of quantitative studies alone but less robust than that of qualitative studies alone.

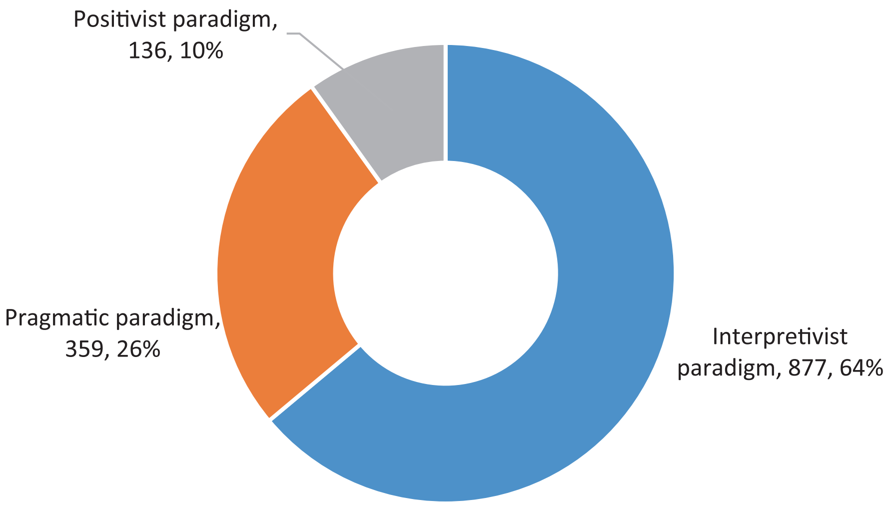


**Fig. 11.7** Google Scholar gross citation rates

## Comparative Analysis of Gross Google Scholar Citation Rates

Google Scholar citation rates for the articles were drawn in June 2019, four-and-a-half years after the youngest article was published. This allowed all younger/newer articles to gain some traction and older ones to loose steam or stabilise. Figure 11.7 shows that qualitative/interpretivist articles had the highest number of citations (877) making up a proportion of 64%. This proportion was identical to the proportion of number of qualitative articles. That meant that the number of citations was directly proportional to the number of articles.

With a cumulative total of 136, quantitative articles had the least citation rates. However, these citations were 10% of the total citations. This was double the proportion of 5% enjoyed by the gross number of quantitative articles. Mixed-methods articles had 359 citations making up a proportion of 26%. This was lower than the proportion of 31% enjoyed by the gross number of articles in this category.

## Comparative Analysis of Citation Rates per Capita (Average)

Figure 11.8 shows citation rates per capita to compare all the three paradigms.

It can be observed that the positivist paradigm had the highest per capita citation rate of 34:1. This was followed by the interpretivist with a per capita citation rate of approximately 20:1. The pragmatic paradigm had the lowest per capita citation rate

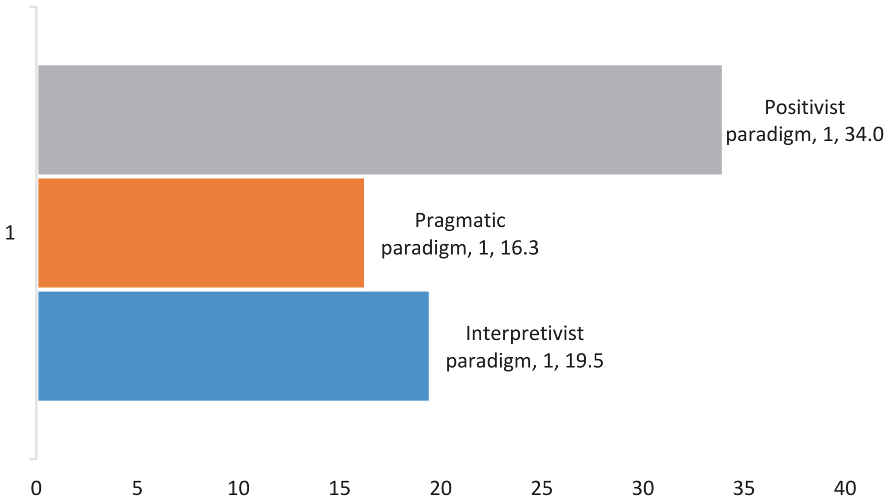


**Fig. 11.8** Google Scholar per capita citation rates

of approximately 16:1. This means that although the quantitative articles were much fewer in number, they had the highest citation rates per article on average.

## Comparative Analysis of Citation Rates per Article Cohort

Figure 11.9 shows the citation rates of qualitative (interpretivist paradigm) articles per cohort.

It can be observed from the graph that the third year cohort was the most cited, followed by a slump in the fourth year, and a resurgence from the 6th onwards. This will be investigated in the next section. For now suffice to say the overall trend can be modelled by the linear function $y = -0.3273x + 89.6$ or the exponential function $y = 43.369e^{0.0609x}$.

The linear equation has a negative gradient, which is expected in the sense that the older the articles are the more citations they are expected to garner.

Figure 11.10 shows the citation rates of quantitative (positivist paradigm) articles per cohort.

The sixth year article/cohort attracted the attention of the greatest number of researchers. Probably due to this outlier coming a little more than half way through the period of study, the overall trend in citations was upward. The linear function, $y = 0.8606x + 8.8667$, and quadratic function, $y = 0.3712x^2 - 3.2227x + 17.033$, are approximate models of the trend for this cohort of publications and show a positive slope thus affirming a marginal growth rate.

Figure 11.11 shows the citation rates of mixed-methods (paradigm paradigm) articles per cohort.

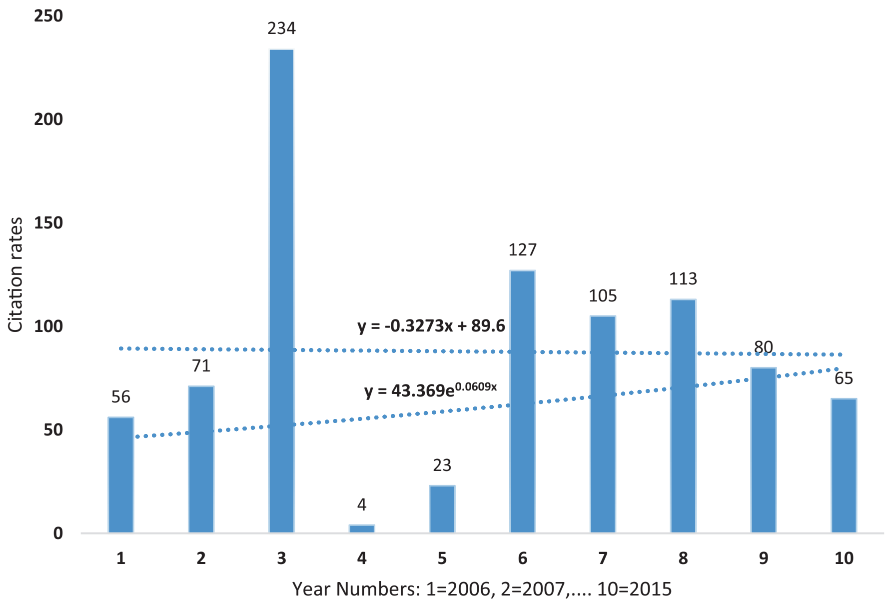


**Fig. 11.9** Citation rates of interpretivist articles per cohort

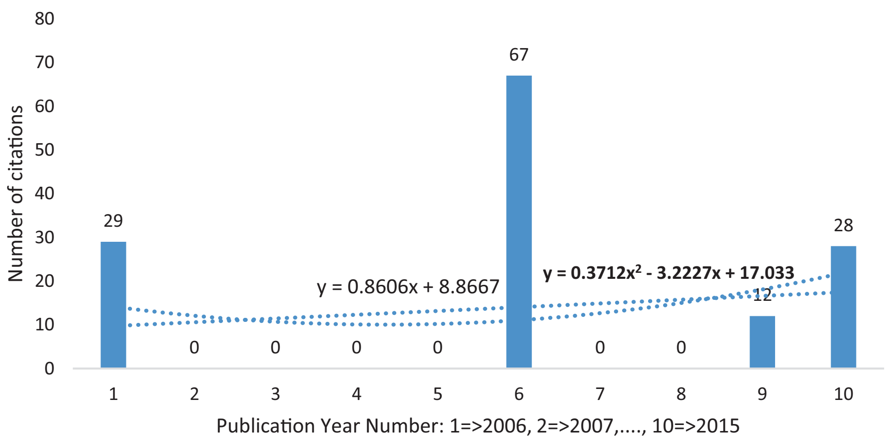


**Fig. 11.10** Citation rates of positivist articles per annual cohort

It can be observed that Year 3 (2008) cohort articles generated the highest interest from researchers over the period of investigation. However there was a lull immediately after that followed by resurgence in Year 5 (2010) which attracted the second highest attention of researchers. Overall the trend is best modelled by the linear equation $y = -5.45x + 67.139$ or the exponential function $y = 68.388e^{-0.155x}$, both with much more negative gradients when compared to the interpretivist paradigm.

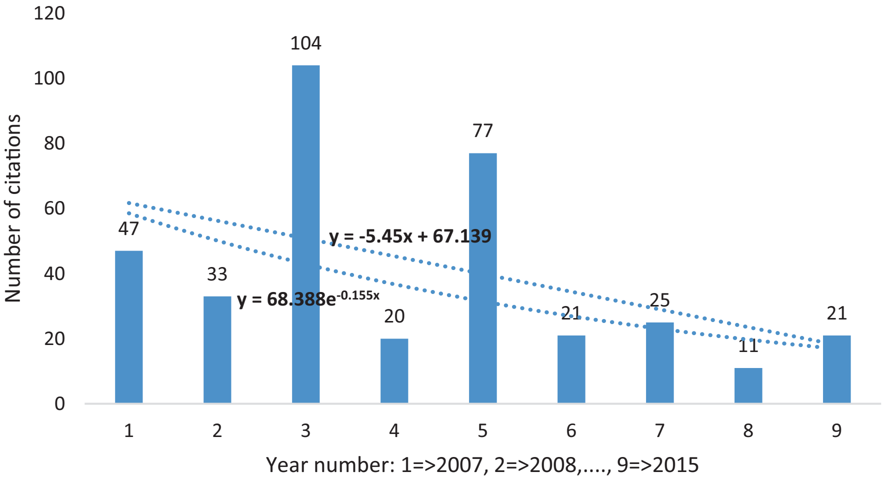


**Fig. 11.11** Citation rates of mixed-methods (pragmatic paradigm) articles per annual cohort

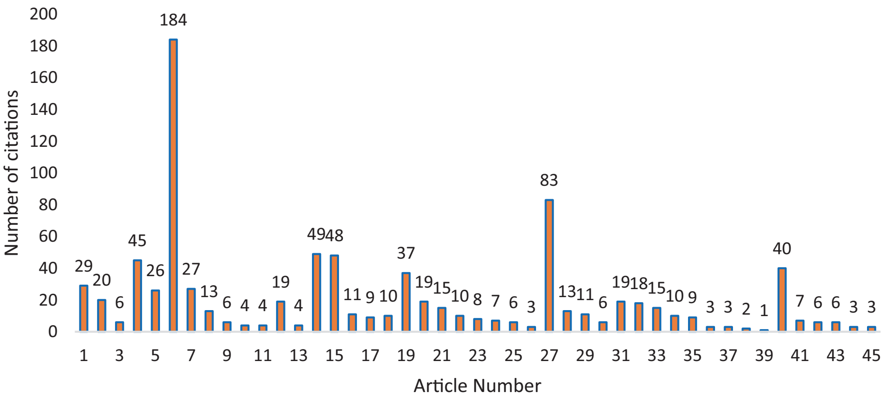


**Fig. 11.12** Google Scholar citation rates per qualitative article by chronological order

## Comparative Analysis of Citation Rates per Article

Figure 11.12 shows the citation rate per qualitative (interpretivist paradigm) article by chronological age.

Article 6 had the highest citation rate of 184. This was an article which raised the issue of language as a factor in mathematics teaching and learning, which teachers needed to be sensitised to as peculiar to the multilingual South African context (Setati, 2008). In particular, access to mathematics in a multilingual classroom was seen as obstructed by the language of (apartheid and colonial) domination. The second most frequently cited article (83 citations) problematised professional learning communities (PLCs) as a more sustainable (autonomous) vehicle of

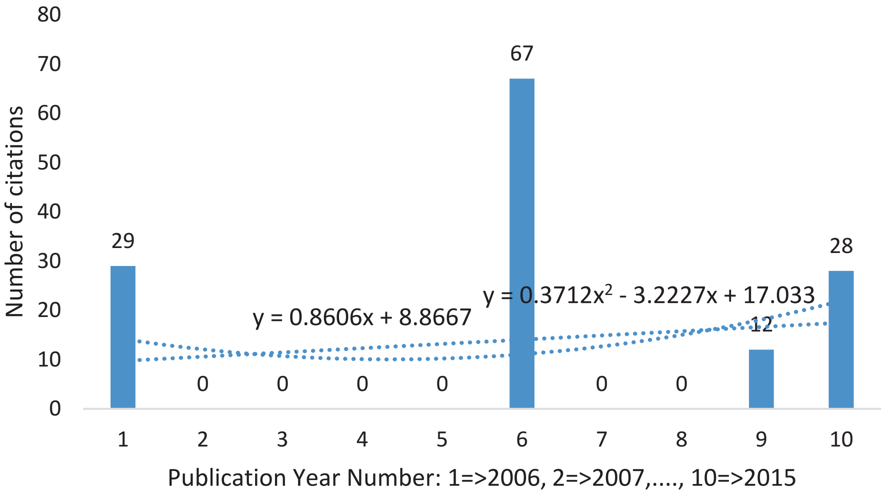


**Fig. 11.13** Citation rate of positivist articles by chronological age

mathematics teacher professional learning (Brodie, 2013). The article proposed a strong focus on error analysis (by means of video analysis of lessons) as a site for teachers to gain authentic mathematics knowledge for teaching. The third most cited interpretivist article, with 49 citations, grappled with the question of coherence and connections in mathematical discourses in instruction (Venkat & Adler, 2012).

Figure 11.13 shows the citation rate per quantitative (positivist paradigm) article by chronological age.

The most cited article (67 citations) in this paradigm addressed the attitude (or belief) question of why South African teachers were reluctant to teach their learners mathematics using dynamic geometry software packages such as free download GeoGebra and Sketchpad (Stols & Kriek, 2011) when these digital tools could in fact significantly enhance learner conceptual understanding of difficult mathematics concepts. Given that we are at the cusp of the fourth industrial revolution, the need to embrace and harness digital tools cannot be more apt than now. The second most cited positivist article, with 29 citations, addressed the issue of contexts preferred by teachers in the relatively new subject of mathematical literacy, introduced in 2006, with no precedent to learn from across the globe (Julie, 2006), at that time.

Figure 11.14 shows the citation rate per mixed-methods (pragmatic paradigm) article by chronological age.

Article 2, the most cited with 100 citations in this paradigm, was about utilising learning environment assessment to improve teaching practices among in-service teachers in a distance education programme (Aldridge et al., 2009). Faced with a massive increase in the number of learners taking mathematics in the South African school system coupled with a critical shortage of appropriately qualified teachers of mathematics, distance education became a popular mode of practice-based


**Fig. 11.14** Citation rates of pragmatic paradigm articles by chronological age

professional learning. The second most cited article, with 28 citations, focused on the professional development of mathematics (and science) teachers from a social justice perspective to leverage learners in disadvantaged (farming and township) communities in their struggle for emancipation from the legacy of the past (Ndlovu, 2011a, b, c, d). A key feature of the teacher professional learning model reported was its school-based, in situ, professional practice support, which was in sharp contrast with once off, off school, kick-and-run, workshop approaches that had previously dominated teacher professional development spaces and discourses. The third most cited article, with 23 citations, documented lessons from a mentoring programme for mathematics (and science) teachers in township schools (Fricke et al., 2008). These interventions epitomise valiant attempts by higher education institutions to shake off their ivory tower image and be of relevance to the communities around them.

## Summary and Conclusions

From the data presented and analysed, it can be concluded that qualitative studies in the interpretive paradigm have dominated research in the professional development of mathematics teachers in South Africa making up 64% of the collection in this study. These results are similar to findings by Adler & Ronda (2015) where she investigated mathematics education in general. Mixed-methods studies in the pragmatic paradigm constituted 26% and exhibited an upward growth trend. Quantitative studies in the positivist paradigm were in the minority with a paltry 5%. This answers the first research question in a manner that is consistent with similar observations in mathematics generally. The research question on impact of the research is a major contribution to knowledge in this study as no similar study has investigated this dimension to the best. From the gross citation rates, it was clear that qualitative studies in the interpretivist paradigm were proportionately cited matching the 64% publication rate. However, the few quantitative studies had a much higher average citation rate of 34 citations per article enabling them to achieve a share of 10% of overall Google Scholar citations compared to 5% of publication rates. Research articles in the pragmatic paradigm had surprisingly the least average citation rate of 19.5 citations per article even though they achieved the second highest gross citation rate of 26% by volume. All gross citations had outliers of exceptionally high citation rates. Accepting that Google Scholar citations are the most inclusive measure (metric) of impact, this answers the second research question.

Finally, the most cited articles in each paradigm helped to answer the third and final research question about which issues dominated the research landscape in each paradigm. The language issue in the teaching of mathematics stood out in the interpretative paradigm accompanied by mathematics knowledge for teaching, which later evolved into mathematical discourse in instruction, as well as making connections (Setati, 2008; Adler & Ronda, 2015; Venkat & Adler, 2012). In the pragmatic paradigm, the most cited articles pointed to practice-based, site-based approaches to mathematics teacher professional learning as more socially just, the establishment of professional learning communities as more sustainable, as well as mentoring (Ndlovu, 2011a, b, c, d). The most cited research in the positivist paradigm addressed technology integration into the teaching of mathematics as well as the professional development of teachers of a novel subject in the name of mathematical literacy as a global first of its kind (Julie, 2006; Stols & Kriek, 2011). The conclusion we can draw is that the mathematics teacher professional learning research community has produced invaluable knowledge and insights in all paradigms which future research can build on and prosper.

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# Chapter 12
# Changes in Rwandan Primary Mathematics School Subject Leaders as a Result of a CPD Certificate Program in Coaching and Mentoring

**Jean Francois Maniraho, Sylvain Habimana, Leon R. Mugabo, Clementine Uwamahoro Gafiligi, and Stefaan Vande Walle**

## Introduction

Today's educational systems take teachers under pressure to meet ever-increasing world demands like requirement of second language besides mother tongue, education for all, inclusive education, quality of education, practice-based learning, and so on. Educators must be aware and responsive to these demands and other changes in the educational system. The role of teacher in classroom is to change his behavior in ways that lead to improvement in students' performance. Most of the time, performance in class as result of teaching and learning process is being measured by emphasizing on students, but critically we don't focus on what makes the differences in student performance expectation. One of the reasons is the classroom teacher, so, teacher must be prepared and equipped with knowledge and skills to cope the world education system through students' performance, which involves professional development initiatives (Harwell, 2003).

CPD for teachers is a core part of ensuring the effective teaching in all sectors of formal education. It is arguable that effective teaching and learning is an engine of the country's socioeconomic development. This is emphasized in Rwanda Vision 2020, extended to a new agenda called 2030 SDGs (Sustainable Development Goals) and in country goals and aspirations for socioeconomic development as well as in many other education policy documents. As the most important profession for

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a country's future, teachers need considerable knowledge, skill, and attitudes, which need to be developed as their careers progress.

In this context, UR-CE, in partnership with Rwanda Education Board (REB), and VVOB have developed and implemented a CPD certificate program for mathematics primary school subject leaders.

Within this program, 39 primary school teachers selected from six districts in two provinces (Eastern and Western) of Rwanda took part in the CPD program in 2018. The program aims at equipping them with the competences to accomplish their responsibilities as MSSLs.

Moreover, after the Knowledge, Attitudes and Practices (KAP) survey organized before the CPD course, the training followed, consisting of 8 training days, organized during four weekends. The training days were complemented with practice-based assignments in between sessions and a field visit. The program consists of two modules: one on coaching, mentoring, and CoP and the second on pedagogical content knowledge and gender in mathematics education. Observations from classrooms (videos, note-taking) were used to effectively help those teachers to strengthen their knowledge and skills.

Based on several new ideas in education system in Rwanda, it is important to see how teachers cope with several educational changes very particularly in line with how the received CPD course has helped them, especially MSSLs, to stand in their positions in their different primary schools.

So far, few studies have been conducted on the implementation of CBC in general at macro-level, nationally. However, until now, there is no study at micro-level that specifically studied how colleague teachers can help among themselves on the implementation of CBC at specific subject level and highlight changes caused by that correspondence and very particularly at primary school level.

## Literature Review

### *Continuous Professional Development*

For maintaining and enhancing the quality of teaching and learning in schools, CPDs were acknowledged to be of paramount importance (Harland & Kinder, 1997).

CPDs are increasingly seen primarily as being related to people's professional identities and roles and the goals of the organization they are working for A research has used Day's (1999) definition of CPD. To consider models for evaluating CPD, the research provides an extended conceptual framework:

> *Professional development consists of all-natural learning experiences and those conscious and planned activities which are intended to be of direct or indirect benefit to the individual, group or school, which constitute, through these, to the quality of education in the classroom. It is the process by which, alone and with others, teachers review, renew and extend their commitment as change agents to the moral purposes of teaching; and by which they acquire and develop critically the knowledge, skills and emotional intelligence essential to*

*good professional thinking, planning and practice with children, young - 24 - people and colleagues throughout each phase of their teaching lives. (Day, 1999)*

From the above definition, to evaluate a CPD, we must take into consideration the indirect and direct impact upon different stakeholders and the effects of CPD. This is done by considering the knowledge, skills, as well as the commitment and moral purposes and to its effect upon the thinking and planning, as well as actions of teachers taking account of their life and career phases and the contexts in which they work. However, the evidence on research about evaluation practices in relation to CPD shows that it rarely focuses upon longer-term or indirect benefits; it rarely differentiates between different kinds of benefits in relation to different purposes in the definition, i.e., moral purposes, relevance to phase of development, change, thinking, and emotional intelligence; it is often based upon individual self-report which relates to the quality and relevance of the experience and not its outcomes; it usually occurs simultaneously, after the learning experience, rather than formatively so that it can be used to enhance that experience and that it rarely attempts to chart benefits to the school or department (possibly because these are often not explicitly contained within purposes).

Professional development is the process of ameliorating teacher's skills and ability required to generate distinguished educational outcomes for the students (Hassel, 1999). Guskey (2000) stated that a wanted educational improvement can't take place without professional development initiatives, and we need professional development to meet today's educational demands. Generally, it is related to professionalism reflecting on "cooperative action between teachers and other stakeholders" (Gabriel, 1963).

At school, especially in classroom, the main focus of teachers is students. However, most of the teachers use their preferred methods rather than thinking of which is the best for the students. Similarly, teachers tend to apply the method of the teachers who inspired them while they were students but they don't realize why those teachers used the way they did. So, they have to be aware of different influences in teaching process and to the student's interests. To achieve this, they need opportunities to develop their knowledge skills about teaching as well as learning activities through professional development with workmate and other stakeholders. By the publication of National Commission on Excellence in Education. (1983), educational system has been changed may times to raise the performance of student. However, the consequences observed on students' performance were very little in spite of the big expenses spent. This gap comes from the attention put in classroom. It is crucial to provide teacher professional development that helps educators to change their behaviors that lead to student performance. While expecting students 'performance we must recognize the role of teacher and how teacher may contribute to that performance. In other words, "student achievement is the product of formal study by educators" (Joyce & Showers, 2002).

We can't expect change in students' performance if teachers don't change the way they do. Then, how can we engage teachers in changing their teaching behaviors or how teachers can change their teaching strategies in way that leads to student's performance? The answer is through professional development in which teacher can develop new behavior in classroom (Alexander et al., 1998). For professional development to be successful, there must be support among stakeholders. One of the supporters are administrators. For them, they have first to recognize and value the importance of professional development initiative (McLaughlin & Marsh, 1978).

Another point which strengthens professional development initiative is the contribution or changes that are designed to come up with it. Changes cannot take place if the participants do not have a common understanding about the necessity of that changes; otherwise they come back either voluntary or involuntary to what and how they had been doing (Harwell, 2003). The change in teachers' behaviors occurs mostly when teaching activity is considered as shared activity in which they interact, discus teaching content, and strategize together. They become more skilled as their student improve their performance accordingly (Joyce & Showers, 2002). This is a result of social discussion as a powerful tool to change someone's beliefs as suggested by a number of researchers (Bandura, 1995; Schunk, 1981; Zimmerman & Ringle, 1981). Working together and helping each other by coaching is very important to help teachers adopt to new teaching method in addition to change in behaviors generally (Joyce & Showers, 1988).

Professional development (PD) of teachers has many meanings in its literature. However, its core meaning is explained as how teachers are prepared to teach, their understanding about learning concept, and transferring their knowledge into practice to enhance instruction (Avalos, 2011). PD combines interpersonal professionalism, public professionalism, and intrapersonal professionalism (Camp et al., 2004). Its goals are (Guskey, 2010) to change how teachers normally teach, change in terms of their attitudes and belief, and change brought about by learning outcomes. Although the change within teachers through professional development is not easy, it occurs gradually in an expected way. PD is not only focusing on facilitating change making but also on endurance of change. The contextual factors that impact on learning needs depend on traditions, cultural, environmental, and the experiencing conditions of a school in a particular country. For example, the initiation of professional engagement in Namibian study may not be meaningful to the teachers in Canada or the Netherlands. It means that teachers may move from one stage to another in the same process but within different contexts (Avalos, 2011).

Professional development should extend teachers' knowledge about subject being taught and provide him teaching skills in classroom; it helps individual development as well as professional and increases teacher's capability in student's activity monitoring.

It provides opportunities for teachers to explore new roles, to develop new instructional techniques and methodologies, to refine their practices, and to broaden themselves both as educators and as individuals. In this regard, the content of professional development should not only focus on subject matter but should also be based on research documented facts which help teachers to implement what they are learning (Joyce & Showers, 2002). Successful professional development results in in teachers' behavior change which occurs in classroom (Birman et al., 2000).

Seminal research by Joyce & showers (1988) comes up with decision that levels of teachers' behavior in classroom change when coaching, studying together, and peer support take place.

As explained by Schwartz and Bryan (1998), PD is gained throughout different activities and opportunities encountered in one's jobs. Professional development is thought to be the means for growth and reward. PD is a source of motivation and reward of the staff at all levels. For example, the study carried out in South Africa by Ono and Ferreira (2010) has shown that teachers who were involved in lesson study improved how they deliver lesson. In addition, the rotating cyclical "plan-do-see" is a key technique for professional development research. What is learned from experience and practices should add on improvement of teaching. A lesson study has assisted much in professional development of teachers within the country.

The study conducted by Guskey (2010) showed that there is no change in teachers who participated in trainings but fail to implement practices from trainings. Attitudes to change have occurred only when improved instruction occurred. In fact, how we judge about PD for teachers depends on how one conceptualizes the work they do. Such conception has influence on PD design in three perspectives: (1) When teaching taken as labor, teacher professionalism is viewed as a process of conceiving schemes and then transmitting to teachers in order to implement them. (2) If teaching is taken as profession, PD is viewed as a combination of competencies and skills that teachers use in order to help their students to learn effectively. And (3) then, when teaching is taken as an art, PD is viewed as self and peer judgment that put into consideration the unpredicted and personal nature of learning (Sullivan, 2006).

A number of scholars have identified different ways in which professionalism initiatives can be reached and how they can enhance instruction. For example, Loucks-Horsley and Matsumoto (1999) have identified strategies and structures for professional development, by discussing four strategies such as (a) curriculum, where teachers are taught how they will be using a curriculum; (b) examining practice, where teachers through action research look on how they practice the given task; (c) collaborative work, where learning takes place in class and out of the class in collaboration with students and teachers; and (d) vehicle mechanism, which accommodates any kind of learning. In the studies carried out by Trede (2012), Work Interactive Learning (WIL) was suggested that will boost professional identity building between the university and work through pedagogy. This is because teachers must be able to reflect on their teaching experience and how they taught in order to adopt and come up with acceptance of good ideas and reject poor ones (Sullivan, 2006). Although improving teachers' knowledge and belief which helps to improve learning for PD is challenging (Hochberg & Desimone, 2010), there is a need to respond to three key challenges such as need for aligning instruction with standards, need for adoption of other school-level commitments, and need for meeting odds of the school environment and the district.

In her discussion Malm (2009) suggested five key elements that should be focused on for teachers' PD, such as developing teachers' abilities for being creative and thoughtful, improving teachers to think critically, letting teachers be aware of teaching pedagogy and philosophy, accentuating on teachers' cognition and emotions, and enhancing personal awareness of teaching profession through moral and ethical consideration. In the study carried Maniraho, & Christiansen, (2015), it showed that teachers who realized improvement in teaching have influence on their students' performance. The participation of Local Systemic Change (LSC) through teacher enhancement program has contributed greatly and came up with an improved instructional practice.

The target of PD is to educate students who are able to think critically, a considerate citizen who is fully equipped with lifelong learning capacity (Trede, 2012). Hence, we need to use the current research findings in order to boost conceptualization, decisions, and methods in order to investigate the impacts of teacher's professional development on teachers themselves as well as the students. Once the common conceptual framework is embraced, this will contribute greatly on the quality of professional development and, hence, flash a light on the best practice of how to produce best teachers with opportunities that will allow both teachers and students to benefit afterward (Desimone, 2015).

It was also reiterated by recent research that the key characteristics of successful school improvement are the quality of professional interaction, the focus on staff development, and the relentless pursuit of improved teaching and learning Maniraho, & Christiansen, (2015).

The quality of an education system cannot exceed the quality of its teachers and principals, since student learning is ultimately the product of what goes on in classrooms. What makes a school successful? The quality of education depends on many factors, but teachers and school leaders are two critical factors. Evidence shows that teacher development improves teaching and learning (Hattie, 2009, Maniraho & Christiansen, 2015).

According to Cordingley et al. (2003), in other disciplines which are not education, such as medical research, people are able to justify the impact of a received CPD as checked by easily measuring the differences between outcomes before and after getting CPD and through comparisons with control groups. However, research into CPD in all disciplines is not always able to track inputs or measure outcomes quite so rigorously like in education. Measurement of the effects of CPD not only had to address pupil outcomes but also to embrace the fundamental changes in much less easily evidenced factors such as attitudes, beliefs, knowledge, and behaviors of teachers and their dynamic relationship between these factors and the responses of their students. Classroom observation is a paramount and important tool to do so.

At school level, some continuous professional development is done through coaching and mentoring. In 2003 Cordingley and colleagues suggested the most consistent and main features of the CPD intervention elements. Those are observation, coaching, analyzing efforts at implementing new approaches through professional discussions, peer support, use of outside expertise, teacher ownership of the focus of the CPD, and collaborative planning, experimentation, and implementation by teachers.

Another research has showed that teachers need to be engaged in CPD that promotes inquiry, creativity, and innovation in order to achieve improvements in teaching and better learning outcomes for students. Therefore, different forms of sustaining professional learning such as peer coaching, mentoring, sabbaticals, and others have been identified to have positively affected teaching and learning outcomes (Joyce & showers 1988).

There are many ways that CPD can be delivered and accessed; these include but not limited to training courses and workshops, studying for a qualification or accreditation, online courses/webinars/podcasts, observation (as either observer or person being observed), shadowing a colleague, coaching and mentoring, peer group exchange, visiting other schools/colleges, attending exhibitions and conferences, international visits and exchanges, self-reflection, and personal reading or research.

## Theoretical Framework

For this chapter, analytical framework was used to help researchers do logical thinking in a systematic manner. It has helped researchers to make sense and to well understand the object they were researching on.

It is well known that CPD practice is significantly evaluated when it explores the interrelationship between the impact on teacher, school, and students as illustrated by using Venn diagrams elaborated by Lieberman (1996) and called it "Frameworks for valuating CPD practice" (Fig. 12.1).

Profession development can be seen as a form of learning, a style of getting knowledge or skills for the purpose of keeping or maintaining academic degrees or certification (professional credential) through the formal way of coursework, research, attending different conference, or through informal style of learning opportunities. It is related to the growth and maturation of the knowledge, skills, and attitudes gained throughout the workers' lives, as a result of formal and informal actions of learning at work (Luciana Billett, 2002).

The professional development initiatives can be explained as all those enterprise activities (training, coursework, mentorships, coaching, action research, attending conference, seminal, workshops) (Desmone, 2009). Professional development

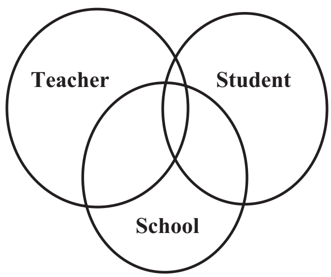


**Fig. 12.1** Framework for evaluating CPD practice by Lieberman (1996)

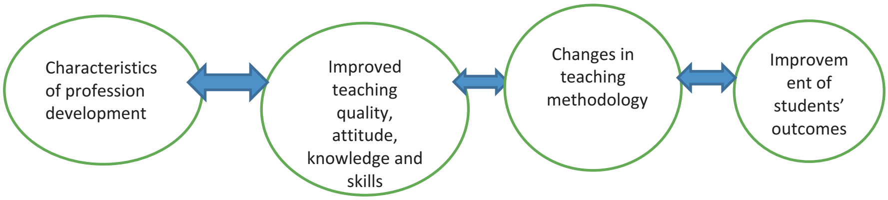


**Fig. 12.2** Teachers' professional development framework. (Adopted from Desimone, 2009)

initiatives are also stressed as all those activities intended to upgrade educators' skills, attitudes, and knowledge toward their work, through certification courses, for the target of enhancing their learners and changing positively their outstanding learning outcomes (Guskey, 2000).

The upgrade of teachers' knowledge is termed as the professional development of teaching staff. This is done mainly because teachers are very important factors in delivering the content of learning. They face difficulties due to a very quick evolving knowledge (Desmone, 2009). The research in the beginning of 1900 showed that most of the jobs at national revel could be performed by low or unskilled workers, but today most of the jobs are professional or technical in nature (technology, business, industry, etc.) and must be carried out by higher educated people (Hammod, 2004).

Profession is characterized by standards for performance such as knowledge, skills, and attitudes, and due to a quick change or growth of the above standards in some area, continuing education in some field most of the time is required for teachers (Bruce Watt, 2013) (Fig. 12.2).

According to Desmone (2009), four steps are to be taken into consideration, which are as follows: (a) educators receive an outstanding professional development program; (b) the program improves educators' competences, knowledge, skills, attitudes, and beliefs; (c) educators use their improved package (skill, knowledge, attitude) to ameliorate the content of the curriculum, their teaching approaches or methodology, their style of testing the learners, and their evaluation of the content; (d) the improved teaching method, then faster, ameliorates or enhances the quality of student learning. The network frame stresses the interactive relationship of educator-learner characteristics, school leader, and the policy related to education in a given area, all of them on the quality of teaching and learning (Desmone, 2009).

## Methodology

As previously informed, in this chapter researchers explored the extent to which changes in implementing CBC by primary school mathematics teachers helped by MSSLs after receiving CPD course organized by VVOB-Rwanda in partnership with REB and UR-CE are impacting on MSSLs' teaching and learning daily activities. This leads to some other research investigative questions like MSSL perceptions about the CPD in Mentoring and Coaching for Mathematics Teachers (MCMT), to what challenges in teaching practices resulted in the CPD in MCMT, and to how challenges associated with the MCMT are dealt with.

The participated MSSLs were from six districts in which learners performed poorly in national mathematics grade six final exam and which had a considerable rate of dropout. For the researchers to be able to get the true picture on the above-mentioned questions, the survey was administered to respondents before and after the training program (pre-test $N = 40$, post-test $N = 39$). Their age of teaching experience ranged between 5 and 30. The data were collected through the administered question papers on mathematics content and pedagogical content knowledge. The answers were marked and analysis was done where different statistical measures were highlighted. Besides, the researchers merged pre- and post-datasets on respondent codes and added relevant variables from post-dataset to pre-dataset (final dataset $N = 39$). Descriptives and testing for differences between pre- and post-tests (paired $t$-test for continuous variables, paired McNemar for dichotomous variables) were used, and the data were enriched by the information obtained during the interview conducted at the trainees' schools.

## Results and Discussions

The mean age of our respondents was (SD) = 35.0 (8.0) and the range was 24–58 years. The mean years of experience as subject leader were (SD) 3.4 (3.2) and range 1–18 years, whereas the mean years of experience as teacher before becoming subject leader were (SD) 8.7 (7.7) and range: 1–38 years.

To address the first research question on skills/confidence of SSL tasks, paired $t$-test shows a significant improvement in self-reported skills/confidence for all topics ($p < 0.001$) (Fig. 12.3).

Indicate to what extent (1–4) you master the following skills (1 = I don't master it at all; 4 = I know it very well) (Fig. 12.4).

Taking a look on the skills/confidence of mathematics teaching, the results of paired $t$-test show a significant improvement ($p < 0.05$) for 5/8 topics. For three topics no change has been observed.

Indicate to what extent (1–4) you master the following skills (1 = I don't master it at all; 4 = I know it very well) (Fig. 12.5).

The above graph shows that one of the major challenges which the MSSLs have is the lack of enough time and resistance to some of their colleagues/teachers who are resisting to attend some community of practice organized by the MSSLs as one of the learned materials. This was emphasized by one of the respondents during the interview: *I have a teaching workload which works as a barrier of conducting some activities like Community of practice.*

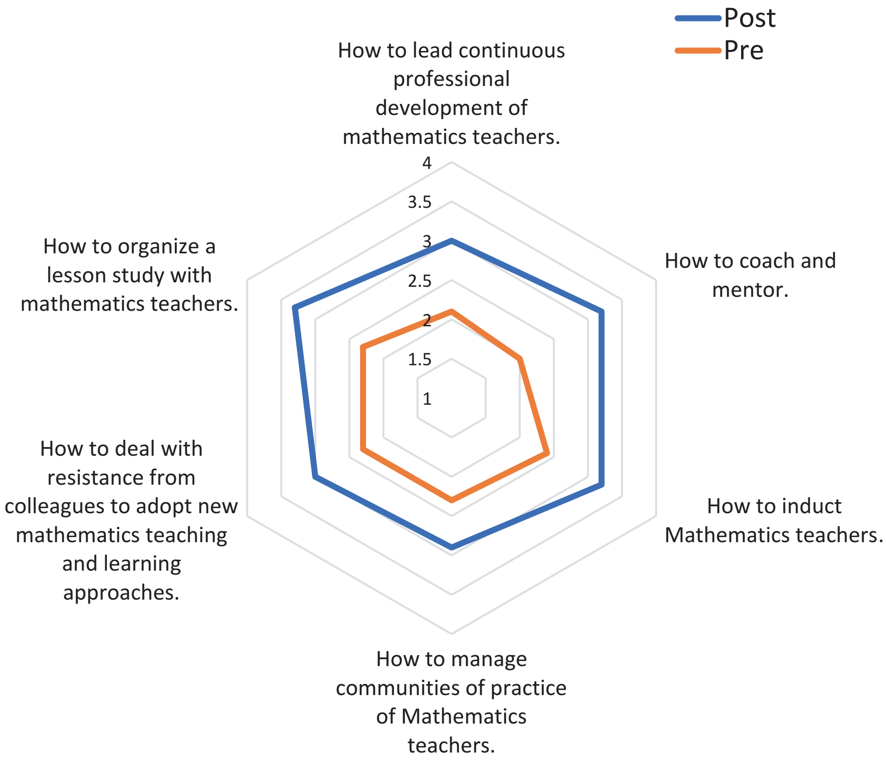


**Fig. 12.3** Skills/confidence of SSL tasks

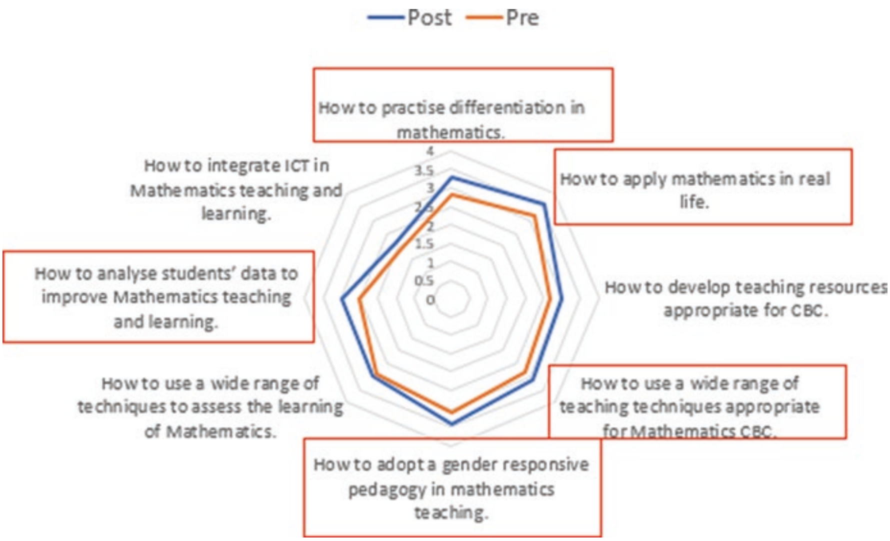


**Fig. 12.4** Skills/confidence of Teaching Mathematics

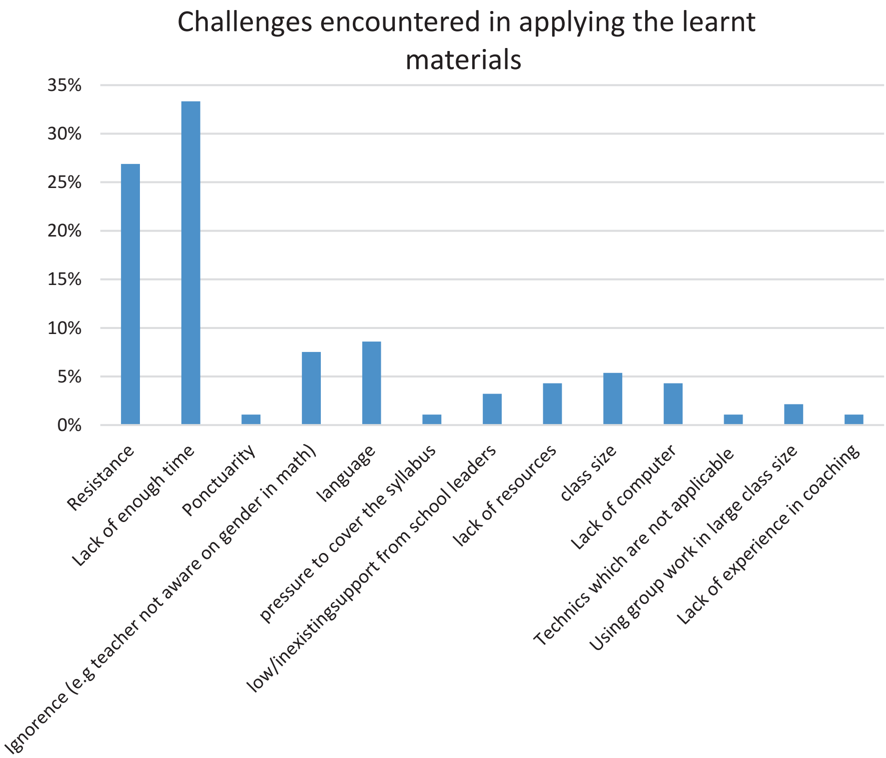


**Fig. 12.5** Challenges encountered in applying learnt materials

## Conclusions and Recommendations

After conducting this study, the researchers posited that the improvements among the trainees were seen in the following: self-reported skills/confidence in SSL tasks, self-reported skills/confidence in mathematics teaching, new teacher mentoring role taken more seriously as frequency of mentoring activities has increased, small increase in competence subscale of the Work-Relatedness Basic Need Satisfaction scale, and small decrease in external motivation for mathematics teaching. Both internal and external motivation were high at pre-test and remain high at post-test.

Issues to still look into include integrating ICT in mathematics which remains difficult for MSSLs. MSSLs struggle with some teaching techniques: practical work, project work, and research work; only 10% of MSSL's time is available for MSSL tasks; what could cause the small decrease in external motivation?

The researchers cannot conclude without informing some limitations that acted on the present work chapter. Those are like positive results which cannot be attributed solely to the training program as external factors may also be influencing the results found. Besides, control group is lacking so we don't know if without the training program the same results would be found (maturation effect). Finally, multivariate analysis was needed to test effects on specific groups (e.g., differences by gender, differences by workload, and differences by district). However, small sample size limits us to some extent and this will be explored further in our next work or in the works of other researchers in the same domain.

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# Chapter 13
# The Challenges of Upgrading Mathematics Teachers' Qualifications: A Case Study from Zimbabwe

**Cathrine Kazunga and Sarah Bansilal**

## Introduction

Zimbabwe, like most countries, considers education as a major tool for sustainable development and this is recognised as a fundamental human right. However, since the advent of democracy, Zimbabwe's education sector has suffered from several challenges that have impacted on the quality of education (Kanyango, 2005). These include poor physical infrastructure, a brain drain, inadequately trained teachers, poorly qualified teachers and declining standards in school performance (Kanyango, 2005). Furthermore, the school curricula for various subjects have been changed a number of times in response to dynamic market needs, similar to both developed and developing countries who have engaged in curriculum revisions to adapt to globalisation demands in the twenty-first century (Gadebe, 2005). This has placed enormous demands on education authorities to provide teachers with the necessary support and training to deal with the new curricula. For developing countries, the challenge is bigger because of the large numbers of underqualified teachers in the system, and they face the twin demands of upgrading teachers while also trying to train new teachers to deliver new curricula.

Recently, the government of Zimbabwe embarked on an intervention in partnership with a large global funding organisation to upgrade the qualifications of practising science and mathematics (amongst other) teachers. The large scale of the intervention necessitated the involvement of many state universities which were contracted to design and administer suitable professional development programmes.

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The mode of learning delivery was open and distance education, the assumption being that the teacher participants would learn as they continued working thereby ensuring that there was no disruption to schools.

Avalos (2011, p. 10) describes the understanding behind the notion of professional development (PD) as being 'about teachers learning, learning how to learn, and transforming their knowledge into practice for the benefit of their students' growth'. Additionally, PD is also seen as a route or path that teachers follow to acquire skills, professional knowledge and other qualities that help them to adjust within the educational system (Vonk, 2011). We are reminded by Steyn (2008) that as teachers become involved and develop further through PD activities, it is ultimately the system that gets transformed, since teachers constitute a significant segment of the system. However, there is strong debate on whether PD programmes produce improvement or significant change in classroom practice (Gore et al., 2017).

Many of the developing countries, however, have neglected teacher professional development because of budget constraints and a heavy emphasis on pre-service education (Leu, 2004). With respect to the Zimbabwean context, many changes were introduced into its educational system without the provision of professional development programmes except for Information, Communication and Technology (ICT) training courses (Mushayikwa & Lubben, 2009). The authors argue that the goal of PD should be to help teachers become effective in all spheres of their work (Mushayikwa & Lubben, 2009). In conducting research on professional development, it is important that we use another lens to conduct the analysis, one that allows us to examine the context of the experiences, the process by which the professional development will occur and the context in which it will take place (Granser, 2000). Accordingly in this study, we address the following research question: What are some challenges faced by the Zimbabwean mathematics teachers while completing their in-service teacher development programme?

There has been little research about the successes and challenges faced by teachers who are enrolled in in-service programmes, especially in developing countries (Bansilal, 2015). We hope that this study can add to knowledge about how in-service programmes are experienced by teachers, so that the design of PD programmes can be improved. We hope that the specific challenges identified in this chapter can alert teacher educators, mentors, government departments and funding agencies on the importance of creating supportive learning environments for in-service teachers who use the intensive block sessions during holidays as their mode of learning.

## Literature Review

Research in professional development of teachers usually focuses on one of four thematic areas: professional learning focusing on the reflection processes, mediations enabled through partnerships involved in the activities such as school-university partnership or teacher co-learning, conditions under which the PD takes place and factors influencing the PD and the effectiveness of professional development which deals with cognitions, beliefs, practices, student learning and teacher satisfaction (Avalos, 2011). Avalos describes the mediation activities as structured or semi-structured processes that facilitate learning and stimulate teachers to alter or reinforce educational practices.

Few studies on PD have made direct links to changes in teacher practice or improved learner outcomes (Gore et al., 2017). The authors found that when positive effects were found, the PD study was usually confined to a small part of the teaching activities, a small group of teachers, or limited to one subject. Some effects have been reported on the outcomes of teacher attitudes and self-efficacy as well as student achievement gains in mathematics or sciences (Gore et al., 2017).

There is debate in the literature about the kind of knowledge mathematics teacher education programmes should actually focus on. Wu (1999) argues that in-service professional development programmes should improve both classroom practice and mathematics knowledge. Furthermore, Zaslavsky and Leikin (2004, p. 29) argued that 'teachers must have a deep and broad understanding of school mathematics for them to be able to offer challenging mathematics to students'. Authors (Ball et al., 2008) point to the important role of horizon content knowledge which is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. In addition, reflective practice also plays an important role in in-service teacher development programmes. Bansilal (2015) asserts that in-service teacher development programmes should provide opportunities for teachers to engage in critical reflection on what they learn and will be teaching. The in-service mathematics teachers' beliefs must be taken into consideration as this will play an important role since a teachers' belief system supports the implementation of change (Chapman, 2002).

Since PD programmes are offered to practising teachers, the modes of delivery may be constrained by the availability of the teachers. Workshops, seminars, conferences or courses are different forms in which teacher professional development programmes are delivered (Schwille & Dembele, 2007). Some research suggests that PD of teachers in developing countries led to no significant change in practice when the teachers returned to their classrooms (Schwille & Dembele, 2007). In contrast, Kelleher (2003, p. 751) pointed out that 'adult pull-out programmes' are likely to result in improvement in teaching. In a similar note, Wadesango et al. (2012) reported that workshops with mathematics teachers in Zimbabwe led to improvements in scheming and planning. Furthermore, Gwekwerere et al. (2013) found that workshops led to improvement in teaching strategies. Therefore, what transpires after PD programmes is uncertain. Leu (2004) states that many developing countries have neglected teachers' PD because of budget constraints and a heavy emphasis on pre-service education.

In South Africa, much research has been focused on the full-time undergraduate students' success rate, and there has been little interest in research about teachers in in-service programmes although with numerous curriculum changes, large numbers of teachers have been funded to enrol in PD programmes (Bansilal, 2015). It is important to acknowledge the context of these teacher participants because they are adults who come to a learning situation with their own set of goals (Bansilal, 2015).

An additional challenge faced by in-service teachers is that their affiliation with university traditions, as adult learners, may be at odds with their own life experiences (Bansilal, 2015). Bachan (2017) highlights that for adults, learning should develop from interests, needs and other life challenges of adult learners. The andragogy approach advocates for the adult learner to be taken at the centre, respected and their experiences should be harnessed in their learning in a manner which upholds their self-esteem and self-actualisation in the lecture. The lecturer should be a facilitator, giving learners sufficient time to engage with and practise concepts thereby allowing self-directed learning which can translate directly to the adult learner's everyday problems of life (Bachan, 2017).

## Details of the In-Service Programme

The teachers were selected by the Ministry of Education and thereafter registered with a university to study a Bachelor of Science Education Honours (B.Sc Hons) degree, relevant to their area(s) of expertise, offered over a period of 3 years. The completion of the programme allows the in-service teacher to be upgraded to teach mathematics at the advanced level. Note that this programme is done by full-time pre-service students over the same time frame of 3 years. For the full-time students, a typical academic semester comprises 12 weeks of lectures, practical and tutorials, 1 week of individual study or revision before examinations and 2 weeks of examinations. The 'block release' mode of learning utilised in the delivery refers to the system where the participants as part-time students attend lectures at the university during school holidays for 2–3 weeks. A full course is one which involves 48 h of the contact time between students and an academic member of staff, while a half course involves 24 contact hours. All mathematics courses are full courses.

The in-service programme comprises five courses per semester for 3 years, as is the case for the full-time B.Sc Hons programme. The timetable for the full-time students is simple: for a full course they have two 2-h lectures per week for the entire semester. These students have 240 h over the semester of 15 weeks to attend classes, complete tutorials, learn and write tests for the five courses. The in-service teachers in the study were expected to fit in the same requirements over their block sessions. One block session of 3-week duration was held in December and 2 weeks in April. They were required to have at least two 2-h lectures and one 4-h lecture a week for a full course. Hence the in-service mathematics teachers had 240 h over 5 weeks to learn, write tests and do tutorials for the five courses. Both full-time students and the in-service mathematics teachers write exams in June for the first semester and in December for the second semester.

## Methodology

In this study, we used the interpretive research paradigm as it acknowledges that individuals with their own varied backgrounds and experiences contribute to the ongoing construction of reality in their context (Guba & Lincoln, 2005). Our study focuses on gaining a deeper understanding of the in-service mathematics teachers' challenges, using a case study design. The most fundamental aspect of the case study is the identification of the individual unit of study and the setting of its demarcations in its casing. In this study, the individual unit was the group of first-year in-service mathematics teachers at a university. The participants in the study were non-traditional students in the sense that they were mature teachers who were categorised as unqualified mathematics teachers because their initial training was no longer considered as sufficient. The design of the upgrading programme was such that it would be completed by the participants in 3 years. Lectures were offered in two intensive block sessions for each semester which took place during the school holidays with classes being held from 08:00 to 18:00 every day.

Of the 116 teachers, 35 completed the given questionnaire with open-ended questions. Purposive sampling was then used to select ten participants for semi-structured interviews which were video-recorded. Ten participants (five men and five women) were selected to participate in the interview. The ten participants were labelled S1, S2, S3, S4, S5, S6, S7, S8, S9 and S10 (where S1 means student 1) without ordering. The analysis of the transcribed data from the ten participants was carried out using an activity systems approach. The questionnaire responses were labelled Q1, Q2, Q3, Q4, … Q35.

## Theoretical Framework

This study was informed by Engstrom's activity theory (Engstrom, 1987, 2001) which assumes that human activity takes place within specific social and historical contexts.

An activity system is composed of the object or goal of the activity, the tools used in the activity, the subjects involved in the activity, the rules governing the activity and the larger community within which the activity takes place and the division of labour agreements between subjects of the community. Figure 13.1 depicts Engstrom's model of 1987 of an activity system.

Figure 13.1 represents the relationships within the activity system. The system consists of the three constructs of subject, object and community together with three processes of mediation (tools, rules and division of labour) which transform the nature of the contexts within which people act. Tools are used by subjects to achieve the object. There should be rules which govern the relationship between the subject and other members of the community which enable the achievement of the object.

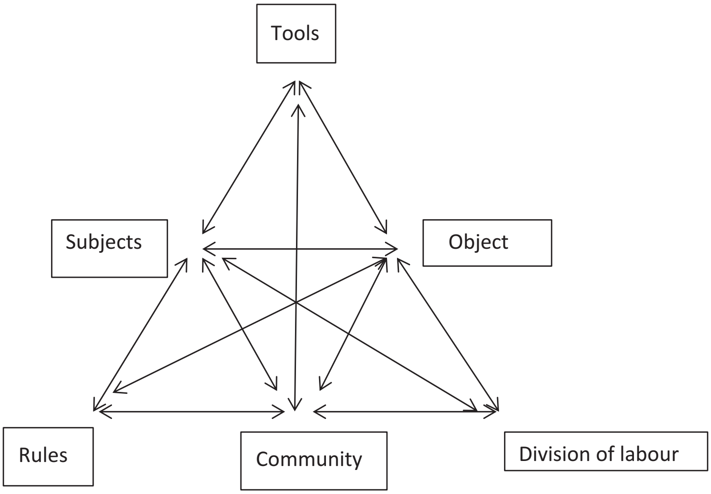


**Fig. 13.1** An activity system. (Adapted from Engstrom, 1987, p. 78)

There should be a division of labour between the members of the community to achieve the goals.

A central feature of activity theory is the identification of the contradictions that emerge. Engstrom (2001) emphasises that contradictions are different from problems or conflicts and can be seen as structural tensions which have accumulated historically. Sheckle (2014) elaborates that the contradictions may initiate a search for their origins in the system, and this may result in shifts and transformation between the elements of an activity system. The contradictions may be resolved or in some cases may lead to the object being redefined.

The activity system is the teachers as students activity system; the subjects are the mathematics teachers who are the students enrolled in the in-service programme. Their goal is to attain fully qualified status so that they can be granted an increase in their salaries.

The 'tools' required by the subjects to fulfil the goals include study material, textbooks, lectures, assignments and other assessments, discussion classes, tutorial sessions, lecture venues and other necessary support.

The subject belongs to a community that is governed and mediated by implicit and explicit rules, and there is also a division of labour agreement between members of the community. In effect members of the community collaborate with each other to achieve the outcome of the activity system. There are three communities that can be identified in this activity system. Firstly, the university 'community' of lecturers, tutors and administrators is depicted as facilitating the teachers' learning in the programme. Secondly, the family 'community' is depicted as providing support and encouragement to the parent or sibling who is studying while also expecting that the person does not neglect the family duties and responsibilities. The third community which intersects in the system is that of the school 'community' which is depicted as providing help and support to the staff member who is studying while also expecting that the school duties and responsibilities are not neglected.

There are 'rules' that govern the relationship between the subject and the different communities. The rules of the university community stipulate conditions about discussion class or lecture attendance, expected behaviour, timetables, fees payable and entrance criteria. It is worth noting that the calendar and support structures of this university are designed around the needs of the typical young full-time student. The part-time in-service programme consisted of courses delivered using the block release mode where teaching was done during contact sessions held over short periods in the holidays. The 'rules' of the schools where teachers work regulate the term times and deadlines for assessments and other duties arising from their teaching responsibilities. The 'rules' of the family regulate the relationship between parents and children and spouses and the extended family.

Division of labour is hierarchical amongst lecturer/tutor/teacher in the university community, where the lecturer is expected to teach and the teacher as learner is expected to learn. In the school community labour is divided according to the relations between principal or school management and the teacher as well as the teacher and pupils. With the family community, the division of labour manifests in the teachers' role as parents to their children. It is expected that parents provide material comforts and emotional support to their child.

## Results and Discussion

In this study, the activity system is made up of the components comprising the mathematics teachers' in-service programme, and Engstrom's advice is that searching for contradictions can help us identify structural tensions which have accumulated historically. What has emerged from the results of this study is that the internal implementation details of the programme have acted in many ways against the achievement of the object, which is to improve the mathematics knowledge for teaching for the group of teachers. By identifying the origins of the contradictions within the system, we can make shifts and transformation between the elements of the activity system, leading to better alignment between the object of the programme and the processes set up to achieve the object.

One of most striking contradictions that emerged from the systems analysis is that the programme was fitted into a system designed for full-time young adult students and not part-time mature adults who have a full-time job already. Hence the needs of the subjects of the activity system were not considered in the planning. Our analysis will now consider in more detail how the different elements of the system contribute to, or act against the functioning of the activity system.

## *The Rules and Regulations Governing the System*

With respect to the rules and regulations of the university, the timetable is designed in favour of full-time students, and the in-service teachers' programme was manipulated to fit into these systems.

The timetabling acted against the teachers' attempts in enabling their learning. The design of the daily timetable during the block session is for 10 h per day, with lectures for modules running for up to 4 h at a stretch as revealed in the comments below:

S1: We start at 8 and finish at 6 pm. There is no allowance for lunch break.
S5: We learn from 8 am to 6 pm without break.

According to comments from many students, the time allocated for each block session was not enough. For most of the participants it was impossible to come to grips with the content, during the sessions as seen in the comments below:

S1: The time is limited. … three-week block is too short a time to cover enough content.
S3: The challenge is that the time we learn the course is too short.
S5: The learning period was too short and I am a little bit slow.
S6; S9: The time is too short.
S8: Another thing the time we spend doing the concepts was too short …
S10: Block harina (don't have) enough time.

Many of the questionnaire responses also support this point as revealed below:

Q1: The university should give students enough time to do self-study, prepare for the exam and to do tutorials.
Q2: Students on block release should be given adequate time to revise as they are afforded little time to prepare.
Q24: Time is limited during the block.
Q30: Students need to be afforded enough time to read (time between semesters) and readily designed modules for the courses.

These responses indicate that the university rules and regulates the timetabling of the contact sessions, assessment and examinations within the mainstream timetabling, which is designed around the needs of the full-time undergraduate student. The full-time student is allocated sufficient time to study, assimilate concepts, write assignments and prepare for tests and examinations while the adult teachers have 1 week of individual study or revision before writing examination. For the in-service teachers, the blocks are packed since they are expected to attend lectures scheduled from 8 am to 6 pm.

## *Division of Labour*

The understanding of the division of labour arrangement in the in-service teacher education programme is that the lecturer is expected to teach and the in-service teacher is expected to learn while complying with the rules and regulations. The lecturers' role is to use the lecture discussion times as well as tutorial support to enable the in-service teacher to optimise their learning. However, the quality of teaching that was offered was compromised because the lecturers rushed, dictated notes, and attempted to cover what they were supposed to do; resorted to teaching only the questions likely to appear in the test or exam; and were caught up with their other university duties which they were still required to fulfil. More details of these issues are discussed below.

Firstly because of the time constraints, the delivery of the lectures was rushed, with an assumption that if the lecturers presented the work, then they had fulfilled their responsibilities. Teacher S10 commented on how lectures were delivered:

> S10: Lecturers are just dictating notes.

It may be that the lecturers themselves do not have sufficient time to explain concepts and resort to dictating notes, hoping to save time for themselves. However, this approach does not offer any benefits in terms of providing opportunities for engagement with the content. Reading out notes on how a problem could be solved offers no participation opportunities for the teachers. Teachers S5, S6 and S8 stated that during lectures many concepts are squeezed into a short time:

> S5: Oh, no the other thing is the speed of the lecturer and the learning period was too short and I am a little bit slow.
>
> S6: We learn a lot of concepts at very short space of time.
>
> S8: Another thing the time we spend doing the concepts was too short and the lecturer is fast when teaching the concepts …

The teachers' comments show that there was no time for inquiry, analysis and reflection, which should be the main goals of learning in higher education programmes (Brown et al., 2001). The development of these higher-order skills such as inquiry, analysis and reflection requires much time and planning and, more crucially, can only take place when students have had time to engage with the concepts. Thereafter, activities can be designed which allow students to reflect about the concepts, by making connections between concepts and thinking about the applicability of the concepts and other issues. For this cohort, however, it was impossible to find time to provide such opportunities since the concepts were covered in a short space of time so that the lecturers could move on to other concepts.

The lack of adequate time compromised the standards of the programme. Lectures lasted for 4 h at a time as noted by S1 below. The lecturer's goal was to finish all the topics included in the course outline (S1), and if that was impossible then he or she resorted to teaching the exam or teaching for the exam as noted by S2:

> S1: Many concepts are squeezed in one lecture of four hours because a lecturer wants to finish his or her course outline leaving us blank.
>
> S2: Block release is very taxing especially when it comes to having lectures because we are given so much to learn in a short period of time, … The lecturers end up teaching for the exam or the exam itself.

As shown in the comment by S2, the lecturers ended up going over the topics that were more likely to be tested in the exams. The teachers felt that the lecturers were impatient with them because they were slow at grasping the concepts as explained by S8 below. S5 also felt inadequate because of the learning struggles:

S8: The lecturer is not patient with us. We have no time to ask questions since lecturer just come to give information and then go. Whether you understand or not they are not worried. Finishing the course outline is there target.

S5: The learning period was too short and I am a little bit slow.

The teachers also felt neglected because the lecturers had other duties they were required to complete as part of their jobs at the university. As S2 explains below, some lecturers did not attend all lectures in the block session because of other urgent duties they needed to complete. Consequently, they were left with less time and then rushed through the explanations of the requisite concepts in the course outline during the shortest space of time, giving much pressure to the teachers. According to teachers S2 and S3 (below), lecturers often did not attend lectures. The comments from teachers S3 and S9 suggest that there were many other university activities that were running at the same time, which had the effect of creating disturbances in the programme.

S2: Some of the lecturers do not attend lectures when we come to block only to give us pressure during the last week of the block and have caused learning as difficult as the course outlines are too long such that they cannot be covered in a week.

S3: The lecturers rarely attend the lectures. They were many things which were combined like registering, orientation and lectures. When we started the block lecturers we busy with something else.

S9: … a lot of inconveniences due to schedule of the University to the fact that some will be having lectures whilst exams will also be running concurrently. Due to shortage of venues and under staffed of lecturer.

It is clear that the lecturers were not granted any relief from their usual commitments and they have had to juggle this additional task in the midst of their usual responsibilities. They have, therefore, not been able to offer learning opportunities which prioritise engagement, reflection and inquiry. Instead they have resorted to reading from their notes or teaching to the test. The teachers in fact require more help since they are mature adult learners. Bachan (2017) highlights that for adults, learning should develop from interests, needs and other life challenges of adult learners. The andragogy approach advocates for the adult learner to be taken at the centre, respected and their experiences should be harnessed in their learning in a manner which upholds their self-esteem and self-actualisation through what has been done in the lecture. Lecturers should be facilitators, giving the learners sufficient time to engage with and practise concepts thereby allowing self-directed learning which can translate directly to the adult learner's everyday problems of life (Slavich & Zimbardo, 2012; Bachan, 2017). Thus, the lecturer's role is to facilitate learning and create an environment conducive to learning where the learner has time for a lunch break and use of technology, and appropriate shelter should be available where safety issues have been considered. However, these principles for adult learning do not seem to have been taken into account in the design and delivery of the in-service programme.

## *Tools*

The 'tools' required by the in-service mathematics teachers to fulfil the goals include study material, textbooks, assignments and other assessments, quality of the learning opportunities, lecture venues, etc. However, these were inadequate for their needs. We discuss the constraints associated with the availability of university study support facilities, the neglect of accommodation provision as well as the poor quality of the learning opportunities provided for the complex mathematics concepts.

One issue that emerged was that because the teachers attended classes from 8 am to 6 pm, they were not able to use the library and other learning support facilities. The library and university computer laboratories close at 4 pm. In this study, the teachers reported that facilities were not available for the adult teachers when they needed them, since the university hours were planned around the full-time student who stays at the university. Hence, the teachers could not access the university facilities because of these contradictory arrangements.

The accommodation requirements were not taken into account especially during examination times, when the usual full-time students were at the university. At these times, there was no suitable accommodation for the teachers, some of whom lived hundreds of km away. During the period when the in-service teachers wrote the examination, the full-time students were still staying on university premises in preparation for their examinations so the teachers could not find accommodation on campus. Some of the teachers ended up sleeping in places such as community halls, churches or school classrooms on top of desks because they could not afford hotel accommodation which was the only other option. Some preferred to live at their homes even if it was far away from the university. For example, one teacher (S10) reported that she travelled 88 km every day to the university to write her examination because of the problem with finding accommodation.

The quality of learning opportunity was severely compromised as shown in the earlier discussion of how the rules and regulations acted against the teachers' interest and how the lecturers did not fulfil their part of the division of labour agreement. A further issue was the content itself. One of the courses that the teachers were involved in during the study was linear algebra which research consistently reports as offering challenges to students throughout the world under normal circumstances (Ozdag & Aygor, 2012; Plaxco & Wawro, 2015; Ndlovu & Brijlall, 2015; Kazunga & Bansilal, 2017, 2018, 2020). Additional support in the form of tutorials, discussion groups and access to technological or text resources from the library would have helped the teachers to develop the necessary conceptual understanding of the abstract concepts that were hurriedly covered. However, access to these resources was not possible during the highly constrained periods of study. The teachers needed much time to develop an understanding of the content as well as to practise under supervised help; however, many teachers could not find the time that was required to carry out the requisite practice for the topics of Gauss-Jordan method and Gauss elimination method which was time-consuming. S1 expressed the struggle:

> S1: The time consumed tichiita (doing) row reduction and Gauss Jordan in matrices that a great challenge.

S7 stated that besides being time-consuming, the algorithm involved in Gauss-Jordan method and Gauss elimination method is confusing. The mathematics in-service teachers felt frustrated at not being able to get to the correct row echelon form which is a key row operation:

> S7: I have faced some challenges with Gauss-Jordan. It is time consuming. Main problem was on fixing these rows. I fill up the whole page doing one row at a time and I end up being confused.

It was impossible to find time to engage with concepts taught because the content was delivered at an inappropriate pace. The teachers (S4 and S5) explained that it took them a lot of time to try to develop conceptual understanding of the Gauss-Jordan method and Gauss elimination method algorithms:

> S4: The procedures like from one stage to another operating row reduction were quite challenging when using Gauss elimination and Gauss-Jordan I was mixing row operations like row 1 minus row 2, row 2 minus row 3, row 3 minus row 2 and so on. I did not grasp the algorithm properly.
>
> S5: Challenges I encounter is when solving system of equations using row reduction and Gauss-Jordan. I have problem with row operation I did not know the algorithm.

Developing conceptual understanding is a gradual process and requires many different activities as described by some of the participants above. Individual attention from instructors is necessary for feedback to the students on how they can improve. Conceptual understanding is enabled by learning environments which prioritise inquiry, analysis, reflection and synthesis.

### *Contradictions Emanating from Division of Labour Demands of the Different Communities*

Another source of contradictions in the system emanated from the three communities that the teachers belong to. Each community had its own division of labour arrangements, and in this case, the demands from the different communities acted against each other. In the interviews, some of the in-service teachers indicated that they found it impossible to juggle their multiple roles and study requirements. Furthermore, they did not find it possible to practise the concepts when they go back home because of the multiple roles they had at home and work.

> S1: We should find time to cover the concepts and time is not available since we have other [things to sort out] back home …

Furthermore, the rule regulating the teachers' membership of the school community did not support the goals of the activity system. The staff development leave allocated by the school community did not include enough time for the teachers to spend studying so that they could be successful at their studies. The teachers felt stressed out with all these demands as shown by the cry from S5 below for more time to practise taught concepts. Teacher S10 was pressurised and found it impossible to find time to complete the tutorials:

S5: We need more time to practice.
S10: Pressure is too much, no time to do tutorials.

Some teachers explained that during the first semester, writing the June examinations was made more stressful because of the demands from their teaching duties. Many of the teachers studying through block sessions were not given a 1-week study period from their principals. They were given staff development leave per the examination timetable which was only for those dates on which they were writing examinations. In June one of the education courses was spaced by 7 days from other courses, and another teacher (S7) was told to come to the university and write the examination. He then had to return to school which was 200 km away, to carry out school duties for the next 7 days. After a week, he then came back to write the rest of the examinations, while other teachers who were granted study leave by their principals spent that week revising for the examinations during the 7 days' break.

For the teachers, it was impossible to fit in the minimum time that was required (notional time per course). Each full course requires 48 h of contact teaching time and in total requires 160 h of notional time to be spent on the course. This means the five courses require the teachers to fit in 800 h in their work schedules to meet the demands, including attending lectures, working on assignments, studying and writing examinations. Most of the teaching and assessment at the university occurs during the 3-week block session in December and 2-week block in April. This arrangement was done so as to fulfil the programme template conditions relating to the minimum number of contact hours required. In the interviews, some of the in-service teachers indicated that they found it impossible to juggle their multiple roles and study requirements. Most participants alluded to the time being too short and it was impossible to create time on their part. They consequently could not find the time to grasp the concepts, prepare for the test, write assignments and prepare for the examination when they returned to their respective workplaces. At school, they needed to do their teaching and assessment duties. In terms of notional hours, five courses required an additional 160 h or more per month (over 5 months) for studying purposes. This translates into an additional 40 h a week, which is frankly impossible for teachers to find considering that they teach on a full-time basis.

## Conclusion

This study set out to explore challenges faced by non-traditional teacher students who want to upgrade their teaching qualifications in the developing country of Zimbabwe.

It is commendable that the government is committed to improving the quality and effectiveness of the unqualified mathematics teachers. Therefore, it is strongly recommended that all the stakeholders should jointly consider the constraints and work together in conceptualising and delivering a programme that is planned around the teachers' needs and availability. This activity analysis has enabled us to identify contradictions in the system which acted against the attainment of the object of improving the teachers' effectiveness. It is important that funders, government and the university reflect on these contradictions so that they could be addressed. The stakeholders should in fact consider whether the design of the programme should be reconceptualised. A major disjuncture was identified between the rules of the university which was set up to enable success of the typical full-time student and the delivery of the in-service programme which could only be offered when the full-time students were on holiday.

The particular planning and implementation of the teacher in-service programme led to constraints from the multiple contradictions across the whole system, which would lead inevitably to low outcomes. The successful completion of the programme requires utmost dedication to the programme and almost complete neglect of other responsibilities. Funders and other education stakeholders need to consider whether in fact it is necessary for practising teachers to participate in programs which have proved so difficult for teachers to participate in. Teachers who are already in the system and are teaching have accrued experience, which needs to be recognised within professional development courses. Other issues that need to be considered include the quality of learning opportunities in the lectures, which was compromised by the rushed approach of the lecturers. The availability of learning facilities was a further constraint. The biggest concern is that it is simply not possible for a person to do justice to a full-time teaching job for 3 years while also completing a university degree (which normally takes 3 years for the full-time student) concurrently. It is hoped that when the programmes are evaluated, the challenges experienced by the teachers will be taken into consideration, so that it may become possible for them to improve their teaching effectiveness in a way that benefits the country.

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# Chapter 14
# Pre-service Teachers' Awareness of Gifted Students' Characteristics

**Lukanda Kalobo and Michael Kainose Mhlolo**

## Introduction

An economics of education and human capital development perspective suggests that society should be concerned about the education of its gifted learners in general and mathematically gifted learners in particular. This is because gifted learners are the component that produces the largest variance on economic growth and consequently provide the best return on investment. The gifted have thus been described as "the world's ultimate capital asset" due to their unique potential to become tomorrow's scientists, inventors, entrepreneurs, engineers, and civic leaders in a knowledge-based economy (Sever, 2011). Terman's *Genetic Studies* (Friedman & Martin, 2011) and the longitudinal Studies of Mathematically Precocious Youth (SMPY) (Lubinski et al., 2014) are arguably the most famous longitudinal studies in psychology to date that tracked mathematically gifted youth for more than five decades with the aim of affirming this thought. Results from these studies confirmed beyond any reasonable doubt that mathematically talented males and females indeed became the critical human capital needed for driving the modern-day conceptual economies.

Consequently, the last few decades worldwide have witnessed countries, particularly developed countries, paying a great attention on studying the phenomenon of giftedness, in general, as well as mathematical giftedness and creativity, in particular, owing to the clarion call of the 4IR and its various aspects. This interest is not accidental because experience has shown that overall progress in all spheres of life has been achieved by great and creative minds and that investing in the development of the gifted is the best investment. A recent discussion among gifted educators has

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been how to improve awareness and instruction of gifted students among the pre-service teachers (Chamberlin & Chamberlin, 2010). Perhaps one of the principal reasons why pre-service teachers have little to no awareness of gifted education is due to their lack of exposure in training.

It is important, in order to understand the need for a special attention to the teaching of gifted students, to know that most of the gifted students in South Africa study mathematics in regular classrooms. Benson (2002) observes that gifted students become more frustrated in a regular classroom in which teachers are mandated to see that all learners reach the standards of the Provincial and National Department of Education. Ironically, South African teachers do not receive any special training on how to identify gifted students' characteristics and address their needs in regular classrooms. Yet an awareness of these characteristics will assist teachers with the early identification process (DoE, 2010). Therefore, it is important for all pre-service teachers to learn about gifted students' characteristics during their training. This investigation on pre-service teachers' awareness of gifted students' characteristics is expected to provide valuable information on how to improve teacher education as well as assist society's think-tanks to reach their full potential. In addition, the study reported in this chapter could serve as a starting point for the development of training programs for pre-service teachers in terms of meeting the needs of mathematically gifted students.

### *Problem Statement*

Although research is unanimous that the quality of gifted education is dependent upon the quality of training that the teachers receive, South African teachers interviewed by Oswald and de Villiers (2013) concurred that they had never received training on how to identify and support gifted students. Thus, less has been done in Africa and various other less developed continents to prepare pre-service teachers so that they become aware of gifted students' characteristics and meet their needs in mathematics. There is general consensus that high education institutions should be the major focus towards improving gifted education. For example, Kokot (1999) posits that teacher colleges and universities in South Africa exert direct influence on the education of gifted children by training (or not training) future teachers. Other authorities in the field of gifted education maintain that all educators working with gifted students should receive adequate training in the characteristics and needs of this special population in order to meet their specific needs (Feldhusen, 1997; Toll, 2000; Gallagher, 2003). Research focusing on this angle seems vital. Hence, this chapter is premised on the view that the much-needed twenty-first-century skills are going to be built on a strong mathematical science foundation.

### *Conceptualization of Giftedness*

There are various definitions of a gifted student. Some put emphasis on the student's current level of achievement based on an overlap and interaction among the three clusters of traits, above average ability, task commitment, and creativity (Renzulli, 1986); whereas for others, the key is the child's potential to perform at a level significantly beyond age peers (Gagné, 2003). There are various influential theories and models in the field of gifted education with Gagné's (1999) model among the top six considered dominant in impacting international classroom practice. The model has received worldwide recognition because it is generally viewed as resolving the controversies that the gifted field has struggled with for years (Pfeiffer, 2013). In 1985 Gagné first conceptualized the theory of talent development, which he first named as the Differentiated Model of Giftedness and Talent (DMGT). Gagné made further refinements to the model over three decades since its inception and these resulted in what he now calls the Comprehensive Model of Talent Development (CMTD) (Gagné, 2015). Essentially, Gagné registers his dissatisfied with the frequent, all-encompassing, and interchangeable use of the terms gifted and talented. He argues that the "one term fits all" use of gifts and talents is inaccurate, misleading, and detrimental to all efforts to identify and nurture talent, because it suggests that talents are inborn and in that way there is no place for systematic training, learning, or practicing. Yet there is ample evidence from elite sport and performing arts programs that have combined the identification of ability with honing of this potential into talents. Gagné therefore argues that there is, and should be, a clear distinction between these two most basic concepts—"gifts" and "talents." In his CMTD model, Gagné (2015) uses the term "giftedness" to refer to the outstanding natural abilities or aptitudes—the emerging form or potential—while the term "talented" is used to refer to the outstanding mastery of systematically developed competencies or performance. An underlying principle of Gagné's view is that while high ability (talent) has some genetic basis (giftedness), learning, practice, and environmental factors are necessary for the emergence and development of such talent. Ultimately, teachers form part of these environmental factors that support or hinder the development of gifted children's potential.

Gagné (2015) was also concerned about treating gifted students as belonging to a homogenous group arguing that there are different levels of giftedness. As an intrinsic component of his model, Gagné then developed a clear and defensible metric-based system (MBS) whose conceptualization posits a five-level system of cutoffs for giftedness as follows: "mildly" 10% (top 1:10), "moderately" 1% (top 1:100), "highly" 0.1% (top 1:1000), "exceptionally" 0.01% (top 1:10,000), and "extremely" 0.001% (top 1:100,000). Using this MBS, he argued that the mildly gifted (1:10) or the top 3 achievers in a regular class of 30 already distance themselves very significantly in terms of ease and speed of learning. He referred to such mildly gifted students as the "garden variety"—a common English expression in the USA that means the "most common group." Thus, this chapter uses the term "mildly gifted" in accordance with the recommendations by Renzulli (2012), and Shayshon et al. (2014) and Gagné (2015) to refer in reference to 1:10 students who attend everyday regular class and demonstrate relatively high mathematical ability.

## *Characteristics of Mathematical Giftedness*

Numerous models of giftedness consider general abilities and domain-specific abilities as important for optimal achievement in a specific field (Subotnik et al., 2012). General abilities in academic domains are often defined by abilities measured by intelligence tests. As noted by the term, domain-specific abilities need to be described in relation to the specific field of giftedness. This chapter reports on a study that focuses on the field of mathematical giftedness based on Krutetskii's 12-year study. According to Krutetskii (1976), "mathematical giftedness" is the name given to a unique aggregate of mathematical abilities that opens the possibility of successful performance in mathematical activity. He defines ability as a personal trait that enables one to perform a given task rapidly and well and contrasts this to a habit or skill, which relates to the qualities or features of the activity a person is carrying out. Following the basic stages of obtaining, processing, and retaining mathematical information, Krutetskii (1976) identifies skills that include the ability to (a) grasp a formal structure of a problem; (b) think logically in spatial, numeric, and symbolic relationships; (c) rapidly and comprehensively generalize mathematical material; (d) curtail mathematical reasoning processes; (e) be flexible with mental processes; (f) strive for clarity, simplicity, and rationality of solutions; (g) reverse and reconstruct mental processes; and (h) memorize schemes, methods, principles, and relationships. These abilities are interrelated and form a general synthetic component, "a distinctive syndrome of mathematical giftedness, the mathematical cast of minds" (Krutetskii, 1976, p. 187). This might be interpreted as a tendency to view the world through a mathematical eye.

It should also be noted that Krutetskii does not equate mathematical giftedness with high achievements in school mathematics. This is also true for other later researchers (Diezmann & Watters, 2002; Pettersson, 2011). Bicknell and Holton (2009) note that mathematical giftedness can manifest in three ways, and these are the analytic type, the geometric type, and the harmonic type. The analytic type is characterized by abstract patterns of thoughts and well-developed verbal-logical skills combined with less well-developed abilities to visualize the subject. The geometric type is characterized with well-developed abilities to visualize the subject that complements less well-developed verbal-logical skills. In this type, gifted students will prefer to use sketches and visual aids to figure problems. The harmonic type combines characteristics of the other two. Krutetskii (1976) argues that students with a harmonic type of mind are most likely to have mathematical aptitude.

## *Teachers' Understanding of the Gifted*

One way in which classroom teachers can broaden understanding of gifted students is through knowledge of the general characteristics that gifted students exhibit (Manning, 2006). Furthermore, characteristics in the cognitive and affective domains most commonly appear in general classroom behavior and, therefore, may be observed by the classroom teacher (Manning, 2006). An awareness of the social and emotional characteristics of gifted students can assist teachers to further understand most of the classroom behaviors they observe in these children. This shows that teachers need to be aware of the unique characteristics possessed by gifted students as these attributes manifest themselves in observable classroom behaviors. According to Manning (2006), some behaviors can be troubling to the classroom teacher, and as such, an awareness of their root causes will assist teachers to meet the gifted students' full needs and build positive relationships vital to meaningful classroom experiences. Finally, lack of knowledge and understanding on giftedness is considered as largely responsible for the mistaken beliefs held by teachers (Collins, 2001; Clark, 2002). Therefore, this chapter explores the pre-service teachers' "awareness of gifted students" characteristics.

## *Pre-service Teachers' Training in Gifted Education*

There is evidence from several empirical studies, which shows that pre-service gifted education training results in greater understandings of giftedness and gifted education and assists teachers to evaluate their own understandings and dispel myths (Cashion & Sullenger, 2000; Goodnough, 2000). The role of pre-service education programs in preparing educators to work effectively with a wide range of learners is critical to student success. There is evidence that providing pre-service teachers with coursework and practicum opportunities to transfer knowledge to practice increases awareness of the differentiated needs of gifted and talented students and confidence in adjusting instruction for them (Bangel et al., 2010). Therefore, pre-service teachers need to be trained on the characteristics of gifted students in order to increase the effectiveness of teachers in working with gifted and talented students. This chapter reports on a study that investigated pre-service teachers' awareness of mathematically gifted students' characteristics. This was done to reinforce the claim on the need to introduce a module/content on gifted education in pre-service teachers training at CUT.

### *Research Questions*

Guided by the literature review the following research questions were raised:

1. How confident are selected pre-service teachers toward being trained to teach gifted students?
2. How cognisant are selected pre-service teachers about attending school with gifted students?
3. How aware are selected pre-service teachers toward the characteristics of gifted students during instructions?

## Methods

### *Research Design*

The research reported in this chapter adopted a qualitative and quantitative approach design in an effort to analyze the pre-service teachers' responses. The theoretical perspective of this investigation is constructivism. Hatch (2002, p. 15) addresses the quest of a constructivist researcher as "individual constructions of reality compose the knowledge of interest to constructivist researcher." In this chapter, the researcher explored pre-service teachers' awareness of the characteristics held by gifted students.

### *Sample and Sampling Technique*

Participants from the study in this chapter were drawn from the fourth-year students at the Central University of Technology (CUT) in the Free State Province of South Africa. A purposive sampling method was used to select 66 pre-service teachers. The researcher selected pre-service teachers who were willing to participate in this research.

### *Research Instrument*

The data collection method included a questionnaire with closed-ended and open-ended questions. The questionnaire was divided into three sections, namely: biographical information, teachers' training, and the awareness of gifted students' characteristics. The main purpose of these closed-ended questions was to explore pre-service teachers' training to teach statistics and attend schools with gifted students. Furthermore, the purpose of the open-ended questionnaire was gathering data about the pre-service teachers' awareness of gifted students' characteristics.

## Results and Data Analysis

In this section presents the results and discussion according to the research question raised in this chapter. Data gathered from the pre-service teachers' responses to the questionnaire was subjected to qualitative and quantitative analysis. The data analysis focused on participants' awareness of the characteristics of gifted students. Tables 14.1 and 14.2 show the pre-service teachers' responses on "teacher training" (Question 1), "attended school with gifted students" (Question 2), and about "the awareness of gifted students" (Question 3).

### *Quantitative Analysis of Pre-service Teachers' Responses to Closed-Ended Questions*

#### Teacher Training

In *Question 1*, the pre-service teachers responded with "agree, neutral and disagree" on training to teach gifted students. Table 14.1 indicates the pre-service teachers receiving training on how to teach mathematically gifted students.

Data in Table 14.1 shows that pre-service teachers (32%) agreed that they are trained to teach gifted students. Data also suggested that pre-service teachers (42%) are neutral about training to teach gifted students. However, only 26% of pre-service teachers disagreed that they are training to teach gifted students (Table 14.2). This analysis indicates that more than two thirds of pre-service teachers (68%) are not sure or disagreed on training to teach mathematically gifted students. Among pre-service teachers who agree, how many of them are aware or know of the characteristics to identify gifted students?

**Table 14.1** Pre-service teachers' training to teach gifted students

| Answer | Percentage (%) | Count (*N*) |
|---|---|---|
| Agree | 32 | 21 |
| Neutral | 42 | 28 |
| Disagree | 26 | 17 |
| Total | 100 | 66 |

**Table 14.2** Attended school with gifted students

| Answer | Percentage (%) | Count (*N*) |
|---|---|---|
| Agree | 73 | 48 |
| Neutral | 20 | 13 |
| Disagree | 8 | 5 |
| Total | 100 | 66 |

### Attended School with Gifted Students

In *Question 2*, pre-service teachers responded with "agree, neutral, and disagree" on whether they had attended school with gifted students in mathematics class. Table 14.2 displays pre-service teachers' responses to this question.

Table 14.2 indicates that pre-service teachers (73%) agreed to Question 1 about attending school with gifted students. Although some pre-service teachers' (20%) responses were neutral, fewer pre-service teachers (8%) disagreed. It is a matter of concern that less than a third of pre-service teachers were not able to identify gifted students or have not attended school with gifted students. It is also not clear how many of the pre-service teachers who agreed are aware or know the characteristics to identify gifted students.

## *Qualitative Analysis of Pre-service Teachers' Responses to Open-Ended Questions*

The research *Question 3* was divided into ten categories: *active participation in the classroom, complete activities very fast, ask different questions, prefer working alone, prepare for class, get bored, challenge the teacher with mathematics, skilful in mathematics, disturbing/misbehaving, and achievement*. The pre-service teachers are referred to as *PST 01*, *PST 02*, etc. In the first category, *PST 01* and *PST 66* mentioned: *students are active in the classroom*. *PST 01* stated: "Students who are gifted are always hyperactive during class." *PST 66* stated: "A gifted student is mostly active in class and able to do problems before the teacher starts the topic." With regard to the second, *students' complete activities very fast*, *PST 07* stated: "A gifted student understands and grasps what the teacher says fast." *PST 18* mentioned: "They are quick in responding with the correct answers." In this third category, *students ask different questions*, *PST 05* revealed that: "Gifted student ask different questions in class." *PST 52* mentioned that: "Gifted students always ask questions for a better understanding." In this fourth category, *students prefer working alone*, *PST 18* mentioned: "They can do things on their own without the teacher's help." *PST 43* was of the view that a gifted student "is able to do well without the help of teacher." *PST 03* and *PST 04* mentioned the fifth category, *students are prepared for class*. *PST 03* related how she/he identify gifted students: "they are always prepared for class and know everything even before the teacher can do them in class." *PST 04* mentioned that: "They can work out the problem way before the teacher has done explaining to everyone in class." In terms of the sixth category, *students get bored in class*. *PST 03* indicated: "They get bored easily if the teacher is incompetent." *PST 13* stated that: "Gifted students give answers beyond my expectations, become easily bored and disrupt lessons." In the seventh category, *students challenge the teacher with mathematics: PST 05* bring out: "They like to challenge teacher with math." *PST 07* mentioned that: "A gifted student identifies the teachers' mistakes." In terms of the eighth and ninth category, *students are skilful in mathematics*, *PST 47* revealed: "there are those learners who find difficult maths problems easy for them." *PST 50* declared: "They can grasp abstract maths concepts faster than other learners." In the ninth category, *students are disturbing/misbehaving in mathematics class*, *PST 34* indicated: "a gifted student always finishes their work first and start misbehaving or interrupting the class." *PST 39* pointed out: "They finish their work very quickly within a given time and start disturbing others." In this last category, *PST 01 and PST 09 mentioned students' achievement*. PST 01 mentioned: "Students who are gifted always got outstanding marks in their tests or exams," while *PST 09* said: "Always score high marks." This section shows that there are various pre-service teachers who have a slight idea on mathematically gifted students' characteristics such as being active in the classroom, ability to complete activities very fast, asking different questions, preferring to working alone, preparing for class, getting bored, challenging the teacher with mathematics, being skilful in mathematics, disturbing/misbehaving, and achievement.

## Discussion

The results from the quantitative analysis indicate that most of the pre-service teachers have not received training on how to teach gifted students. It is concerning that only a third of the pre-service teacher mentioned that they had been trained to teach mathematically gifted students. This controverts the CUT mathematics program where there is no teacher training for gifted students (CUT, 2019).

Results from the quantitative analysis also highlighted that most of the pre-service teachers attended school with gifted students. It is alarming that a quarter of pre-service teachers disagreed or were neutral. This shows that pre-service teachers are not conversant with definitions of giftedness (Renzulli, 1986; Gagné, 2003) and not fully aware of the characteristics of mathematically gifted students (Krutetskii, 1976). While pre-service teacher education has contributed to preparations on teaching for diversity within a mainstream classroom, very few provisions have been made to explicitly cater for pre-service training that is specifically for gifted and talented provisions (Hudson et al., 2010).

The results from the qualitative analysis of the open-ended questions showed that pre-service teachers have a limited knowledge about characteristics of mathematical giftedness (Krutetskii, 1976). Pre-service teachers equate mathematical giftedness to achievement in mathematics, which contradict Krutetskii (1976), Diezmann and Watters (2002), and Pettersson (2011). Educators play an important role in the lives of all their students and act as a variable in the learning environment and the social and emotional development of each student within their classroom. Research has shown the significant and influential role educators play in the education of gifted and talented students (Plunkett, 2002; McCoach, 2007; Lassig, 2009) and the impact they can have on the learning, achievements, and development of these students (Lassig, 2003). Research suggests that gifted and talented students are unlikely to reach their potential on their own and teachers can either have a positive or negative impact on their achievements (Plunkett, 2002).

The teachers who lack awareness of the characteristics and instructional requirements of high ability students are at a disadvantage. Characteristics can manifest themselves in positive ways or in ways that may create problems for gifted students in a classroom (Manning, 2006). Due to the nature of gifted students, both achievers and underachievers, it becomes necessary to recognize that each one of these characteristics may be present in varying degrees. In our discussion, the pre-service teachers need to know that the awareness of characteristics of mathematically gifted students has an impact on their identification (Plunkett & Kronborg, 2011). South African teachers concurred that they had never received training on how to identify and support gifted students (Oswald & de Villiers, 2013). One keyway pre-service teacher can develop understanding of gifted students is through pre-service teachers' training in gifted education. Hence, the inclusion of gifted education modules in the training programs within the mathematics education unit at CUT will provide an opportunity for pre-service teachers to engage with gifted education, understand the characteristics of gifted students, and to identify them.

## Conclusion

Numerous models have been conceptualized on how to improve the education of the mathematically gifted students in South Africa. Nonetheless, questions arise on whether pre-service teachers are aware of the mathematically gifted students' characteristics. If we wish to prosper and grow in the future as a country, it is imperative that our innovators, creators, and inventors get recognized and nurtured through their educational experiences and in that way maximize their opportunities to reach their potential. This can only be done with increased knowledge and understanding of their developmental, learning, and affective needs across the domains of giftedness in our pre-service teachers. This chapter reports on a study that investigated pre-service teachers' awareness of mathematically gifted students' characteristics. Therefore, the report recommends that teacher training courses at CUT include modules that make pre-service teachers aware of gifted students' characteristics.

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# Chapter 15
# Improving Mathematics Education in Malawi Through the Collaboration of Teacher Educators

**Liveness Mwale, Lisnet Mwadzaangati, and Mercy Kazima**

## Introduction

Evidence from literature suggests that collaboration and networking among teachers is essential to developing competence in teaching among teachers within schools. This is because collaboration among teachers can "strengthen the skills of new and struggling teachers, and can make good teachers even better" (Goddard & Goddard, 2007, p. 7). However, opportunities for collaboration are not always available to teachers. In Malawi for example, the government bemoans lack of coordination between different teacher education bodies and the government as follows:

One of the factors contributing to poor quality of education is the lack of Teacher Education Co-ordinating Bodies mandated to link the Ministry of Education, Science and Technology, universities and colleges in order to produce a qualified, dedicated and flexible teaching force (Government of Malawi, 2008, p. 8).

The statement shows that the government recognises the importance of coordination between education institutions including between teacher education colleges to share knowledge and promote the continuous professional development (CPD) initiatives for teacher educators. Despite government's recognition of the importance of CPDs for teacher educators, it does not clearly explain strategies put in place to promote CPDs among teacher educators. Therefore, the coming together of mathematics teacher educators from different teacher colleges for the professional development (PD) workshops is an initiative to promote coordination of mathematics teacher educators within a college and between or among the colleges to share knowledge and improve their quality of teaching.

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The collaboration discussed in this chapter was developed and observed through a PD programme, which was offered by a collaborative project between University of Malawi and University of Stavanger. The project was called *Improving quality and capacity of mathematics teacher education in Malawi* and was funded by the Norwegian programme for capacity building in higher education and research for development (NORHED). It was a large 5-year project which ran from 2014 to 2018 and had five components (for details see Kazima & Jakobsen, 2019). The PD programme was one of the five components of the project and was offered to all mathematics teacher educators in all eight public teacher training colleges (TTCs) for primary school teachers in Malawi. To cover all the eight public teacher colleges which are spread across the country, the PD was offered in 3 years to different sets of teacher colleges each year. It started with two TTCs in the northern region in 2016 and then three TTCs in the southern region in 2017 and finally three TTCs in the central and eastern region in 2018. The PD was facilitated by the project team including the third author of this chapter. The team used a lesson study model and each TTC conducted one cycle of lesson study. The whole PD programme ran from May to November of each year; in May the two or three TTCs were brought together for a 3-day workshop where they were introduced to lesson study and concept study. The concepts studied were multiplication and fractions, which were suggested by the teacher educators during the planning phase of the project.

Lesson study was chosen as a model for the PD because it offers the opportunity for teacher educators to work together and study their own teaching (Fauskanger et al., 2019). At the May workshop the teacher educators worked together to plan their lesson study and started drafting their lesson plans. Between May and November, the teacher educators continued working together to (1) finalise the drafting of their lesson plans, (2) send draft lesson plans to facilitators and get feedback, (3) discuss the feedback and revise their lesson plans, (4) select one of the teacher educators to teach the lesson while the others observe, (5) discuss the lesson and identify what they learnt from it and (6) prepare a report for presentation in the November workshop.

In the November workshop, the teacher educators from two or three TTCs were brought together again for another 3-day workshop where they reported their lesson study and what they had learnt from it. Furthermore, they were introduced to, and discussed, the mathematical discourse within an instruction framework (Adler & Ronda, 2015) which is very useful in studying mathematics teaching and textbooks. The framework also creates opportunities for teachers to improve their practice through better selection of examples and tasks, better explanations and better learner participation.

Fauskanger et al. (2019) studied the lesson planning process from draft lesson plans to final lesson plans for the lesson study. The authors analysed the draft lesson plans that mathematics teacher educators wrote and submitted for feedback and the revised lesson plans the teacher educators wrote after discussing the feedback from the facilitators. They did this for lesson plans of four of the five TTCs that did the PD in 2016 and 2017. Fauskanger et al. (2019) found that all revised lesson plans improved from the draft and especially in terms of focusing the lesson objectives, making their research questions clearer and making students' learning visible. They argue that lesson study is an effective model for professional development of teacher educators and that the support from experienced facilitators is necessary. Similarly, Kazima et al. (2019) analysed lesson plans from the five TTCs that participated in the PD of 2016 and 2017. Kazima et al. (2019) focused on the type of lessons and used the two forms of instrumental and relational teaching of mathematics (Skemp, 1976) as framework for analysing the lesson plans. Instrumental teaching is where mathematics is taught as rules and procedures without reasons, while relational teaching explains the reasons behind the rules and procedures (Skemp, 1976). The findings revealed that four of the five TTCs' draft lesson plans were of instrumental type, while only one was relational type of lesson. After feedback from the facilitators, the revised lesson plans were all of relational type of lesson. Kazima et al. (2019) highlighted the important role of experienced facilitators and similarly argue that lesson study offers a good model for professional development of teacher educators and teachers in Malawi.

The improvements in lesson plans that Fauskanger et al. (2019) and Kazima et al. (2019) observed are encouraging and suggest positive professional growth of the mathematics teacher educators. We agree with the authors that lesson study can be an effective model of professional development and that support from experienced facilitators is crucial. We add to this that the collaboration among the mathematics teacher educators was an important factor in their professional growth. The lesson study model offers the opportunity for mathematics teacher educators at each college to work together collaboratively. The 3-day workshops offered the chance for the different TTCs to work together; hence they experienced further collaboration beyond their own colleges. Fauskanger et al. (2019) and Kazima et al. (2019) focused only on the lesson plans and observed improvements. In this chapter we build on this and discuss the teacher educators' reports and focus group discussions (FGDs) to determine their perceived professional growth that has developed from the collaboration in the PD.

## Literature Review

In this literature review we present the overview of primary teacher education in Malawi and teacher collaboration and networking in education.

### *Teacher Education in Malawi*

Malawi offers 8 years of primary education and 4 years of secondary education. Primary teacher education follows the secondary education and takes 2 years. The primary teacher education programme is offered by the Ministry of Education, Science and Technology through TTCs. Provision of qualified primary school teachers has been challenging for Malawi since the introduction of free primary education in 1994, as a commitment after the declaration of education for all goal of increasing access and quality (Government of Malawi, 2008). However, this was done without proper planning and consequently brought several problems in primary education because within 2 years the student enrolment almost doubled and continued to grow over the years without corresponding growth in school resources (Kazima, 2014). Therefore, primary education has and continues to suffer problems of shortage of teachers, classrooms and resources. In order to cope with the acute shortage of teachers, from 1997 the Ministry of Education recruited about 22,000 unqualified teachers and adopted a school-based model of teacher education called the Malawi Integrated In-service Teacher Education Programme (MIITEP) and reduced the duration of training from 3 to 2 years. The aim was to increase the rate of teacher production and to have the teachers teaching in schools while undergoing a distance mode of teacher education for most of the 2-year duration (Government of Malawi, 2007). This programme ran until 2005 when it was discontinued because it was realised that while the programme alleviated the problem of teacher shortage in schools, the quality of teachers produced was poor.

The poor quality of teachers contradicted the government policies such as the Policy Investment Framework (Government of Malawi, 2000), which highlights the following mission statement for primary teacher education:

Primary teachers play critical roles in the development of primary age children and also in development of the wider community. The overarching aim of a primary teacher development programme in Malawi is to train and continually develop teachers so that they are able to function effectively in the delivery of a quality education to all pupils.

The statement acknowledges the crucial role played by primary school teachers and the need to improve the quality of the primary school teachers through both pre-service and in-service teacher education. In an attempt to improve the quality of primary teacher education, from 2006 the Malawi Government adopted a new model called the Initial Primary Teacher Education, which also had a duration of 2 years but with more face-to-face courses. The first year was for full-time residential courses at the colleges and the second year for full-time school experience in a form of teaching practice (Malawi Institute of Education, 2008). Parallel to this programme, a distance version of the programme was offered to the many unqualified teachers that were still in the school system. In 2017 the programme and curriculum were revised to increase the face-to-face time from half to two thirds of the programme. The structure of the programme now divides the 2 years into six terms, three terms each year that correspond to primary school terms. The programme starts with two terms of residential face-to-face at the colleges, followed by two terms in schools for teaching practice and the final two terms back at the colleges for residential face-to-face and examinations at the end.

Another initiative by the government was the establishment of three new teacher colleges, which increased the number from five in 2008 to eight in 2016. Furthermore, additional students' residential hostels were built at the old colleges to increase their enrolment capacity. The increase in enrolment of student teachers coupled with the changes in the models of teacher education brought pressure on the quantity and capacity of the teacher educators at the colleges. The teacher educators needed to have the capacity to deliver the content and supervise student teachers within the different modes of teacher education. To address the challenge, the Ministry of Education collaborated with the University of Malawi to develop and deliver a special upgrading programme for the teacher educators. The programme was facilitated through a project with Strathclyde University in Scotland (Faculty of Education, 2013). From 2007 a 2-year Bachelor of Education (primary) programme was offered to all teacher educators without a degree qualification to upgrade from diploma to Bachelor degree qualification. By 2016 the programme had ran four cohorts and exhausted the numbers in the teacher colleges (Kazima & Ellis, 2016). From 2014 to 2016, a Master of Education (primary) course was offered to the teacher educators to upgrade from Bachelor degree (Faculty of Education, 2013). The Ministry of Education also recruited more teacher educators to meet the demand (Government of Malawi, 2007). It is however not clear how the Ministry addressed or planned to address CPD of the teacher educators. Regarding mathematics teacher educators, almost all of them reported that they had never before participated in PD focusing on mathematics or mathematics teaching (Kazima & Jakobsen, 2019); thus the PD using lesson study model reported in this chapter was necessary.

## *Teacher Collaboration and Networking*

A growing number of research points out to the two major significance of teacher collaboration and networks, which are building teacher capacity and improving student achievement (Hallinger, 1998; Lima, 2004, 2007; Moolenaar et al., 2012). According to Berry et al. (2009), collaboration provides opportunities for peer learning among teachers to build collective expertise, make teachers more effective in advancing student learning and provide support for ill-prepared teachers in high-needs classrooms. Teachers who collaborate are more likely to learn new approaches of teaching, build their content knowledge, improve their classroom management skills and learn better ways of assessing their students. This in turn advances student learning and achievement. Muijs et al. (2010) argued that teachers who collaborate with their peers are more likely to teach effectively.

Goddard and Goddard (2007) noted that while teachers may have some control over their collaboration at school level, school leaders are instrumental in providing necessary support and focus for collaboration to be effective. When school managers are involved, the degree to which teachers work together to improve outcomes for their students also improves. Principals may foster teacher collaboration by providing instructional leadership and resources ensuring that there is well-structured collaboration among teachers and that they are part of the collaboration. However, the principal cannot be the sole individual to lead a school's instructional programme. The research on principal leadership by Friend and Cook (2009) indicates that principals are most effective when they focus on instructional improvement, share decision-making with teachers and encourage teachers to work together actively towards instructional improvement. Thus, leaders should encourage and model collaboration within their schools to improve teacher performance and student outcomes. For teachers to fully engage collaboratively, they require support from school management to help overcome barriers such as time and structure.

Although research findings indicate the positive impact that teacher collaboration has on their professional growth and development, it is noted that teacher collaboration has its challenges too. Among these challenges are scheduling time to collaborate and coming up with the structure or content of their work together (Darling-Hammond & Richardson, 2009). It may be difficult for teachers of different classes to find adequate and appropriate times for their meetings. Similarly, collaboration may be difficult to plan if the needs of the teachers are different. For example, the structure and content of early grade teachers may be different from that of senior grade teachers. Berry et al. (2009) noted that teacher collaboration may not work if there is no scheduled time for the collaboration, no structured meetings and no mutual trust among the teachers.

Apart from collaboration at school or college level, networking between and among schools proves to be important for teacher professional growth. According to Hadfield (2005), schools can network for the provision of more effective and scalable CPD activities. Network goals include school improvement, enhanced student achievement, broadening opportunities and resource sharing, among others. Collaboration at school level and networking with other schools may help to achieve teacher improvement and student achievement. When schools are connected through a networking programme, they are bound to share resources and expertise directly or indirectly. Teachers from schools in a network are bound to learn from each other and so build their expertise. The PD reported in the chapter initiated collaboration within and among the colleges. It was therefore important to investigate how the teacher educators perceived the PD and its effectiveness.

## Methodology

The study utilised a qualitative study design to explore the professional growth that resulted from the teacher educators' participation in the PD and collaborating with other mathematics teacher educators. The study sample comprised of mathematics teacher educators from all eight TTCs in Malawi.

### *The Sample and Data Collection Methods*

The number of teacher educators from each college ranged from six to ten. In total, 89 mathematics teacher educators participated in the study. Data was collected in two phases. In the first phase, we collected data from presentation reports by mathematics teacher educators from the eight TTCs during PD. In the second phase, we conducted FGDs with three TTCs. We discuss each method in detail in the following subsections.

#### The Presentation Reports at the November PD Workshop

After attending the May PD workshops, teacher educators met several times at their respective TTCs between May and November to discuss their approach in conducting the lesson study. This was the collaboration at their respective TTCs during their lesson study cycle. During the November workshop, the teacher educators presented a report on their lesson study. The report comprised of they worked together to conduct the lesson study, the findings from the lesson study, and the lessons learnt through the lesson study process. We video-recorded these presentations and used them as our data to explore the teacher educators' perceived professional growth that developed from the collaboration in the PD.

Later, after 6 months, we collected data from three TTCs using FGDs. The aim of the FGDs was (1) to find out from the teacher educators the lessons they might have learnt from the PD; (2) to find out if they were continuing to collaborate both internally as a college and externally between or among colleges; (3) to find out the ways, if any, in which they were collaborating; and (4) to find out the benefits, if any, of the collaboration initiatives. We visited the three TTCs and held the FGDs with the mathematics teacher educators. The three TTCs were a convenient sample since they were the closest to Zomba where we are based. Etikan et al. (2016) described convenient sampling as a type of sampling where members of the target population meet certain practical criteria such as easy accessibility, geographical proximity, availability at a given time and the willingness to participate in the study. During the FGDs we asked the teacher educators to explain their experiences from the PD, things that they are doing differently as a result of the PD, whether they continued to collaborate and how they are collaborating after the PD. We captured the FGD data through audio recording.

### *Data Analysis*

Both the video data collected from the teacher educators' report presentations and the audio data collected from the FGDs were transcribed and subjected to thematic analysis where themes were identified upon reading of the data several times. Consequently, the data was coded and interpreted according to five identified themes, namely, (1) lesson planning, (2) teaching and learning, (3) assessment, (4) research in teaching and (5) collaboration and networking.

## Findings and Discussion

In this section we present findings which reveal how teacher collaboration and networking are capable of building teacher capacity. The findings show that teachers perceived the PD workshops, which used the lesson study model, as creating an opportunity for teacher educators to collaborate in a systematic manner. Teachers revealed that their collaboration included planning lessons together and observing lessons as a team. The collaboration continued after the PD and beyond lesson planning. The teacher educators reported that they were planning their terms' work, topics and student assessment together. We first present findings from the PD workshop reports and then from the FGD.

### *Findings from the PD Workshop Reports*

#### Lesson Planning

The teacher educators conducted several meetings during their lesson study cycle. The first meeting was a planning session where they discussed the challenges faced by the teacher educators themselves or their students during the teaching and learning of mathematics education course. They also identified primary school mathematics topics that their students find difficult to understand and to teach. From these topics, they chose one topic and developed research questions to be investigated from their lesson study. Some of the topics that were identified as being difficult for the students were modelling of addition or subtraction of numbers with and without regrouping, modelling multiplication of mixed numbers and modelling division of fractions.

In the second planning meetings, teacher educators discussed how they were going to teach one of the topics that they had identified. They developed a lesson plan and a lesson observation checklist, identified teaching and learning materials to be used and shared some roles like who would teach, who would video-record the lesson and who would observe and write some notes. The teacher educators explained that some of the areas of focus during lesson observation were students' thinking and understanding of the concepts, teacher educators' pedagogical skills for teaching the concept, strategies used to model the concept, the explanations by both the students and teacher educators and the difficulties faced during instruction. After teaching the topic, the teacher educators also met to share their observations and to discuss the strengths and weaknesses of the lesson and to discuss how the lesson could be improved. After the post-lesson evaluation, some colleges taught the lesson again and conducted another post-lesson evaluation, while others were unable to teach the lesson again due to time limitations.

### Teaching and Learning

The teacher educators explained that there were several lessons that they learnt from the teaching and learning process during lesson observation. Some of the issues explained by the teachers included paying attention to students' thinking, reflective teaching, research skills and the importance of doing research on their teaching. They explained that paying attention to students' thinking is a powerful way of finding out some of the causes of students' challenges in understanding some mathematical concepts. During the lesson, they observed that there were instances which required the lecturer to deviate from the lesson plan. This is explained in the following extract:

> *When we were planning we assumed very much about the teachers' content knowledge but in the course of teaching we realised that some of our assumptions were not true. As such the lecturer had to change the plan to suit the needs of the students.*

The teacher educator explained that they were able to learn about their content knowledge because in their plan, they agreed to pay much focus on students' thinking which was one of the new concepts that they learnt during the PD. As such, they agreed to ask students the different types of questions to enlist their thinking. When the lecturer asked some of the questions that were included in the introduction part of the lesson as a way of enlisting prior knowledge, he found that the students did not have enough prior knowledge for the topic. As a result, he decided to address the knowledge gap first before moving into the new topic. It was from this experience that the teacher educators discovered that when a lecturer pays attention to students' thinking, they discover some causes of students' difficulties in learning a particular concept.

Some teacher educators noted that during the lesson presentation, the lecturer distributed the resources before the start of the lesson; the teacher educators learnt that this action might have limited students from coming up with their own ideas of resources and activities to be done during teaching and learning of a particular concept.

### Collaboration and Networking

Regarding teamwork, the teacher educators explained that the post-lesson meetings that they held helped them to develop reflective and autonomous skills through the discussions that they held. Report from one college said:

> *When we were doing the lesson study, we realised that team work is very powerful and it perfects teaching and learning. We discovered that when we plan together and have time to reflect on what we have taught, we check ourselves, our teaching, and share ideas on how best we could have done our teaching. So we have discovered that team work improves teaching and learning.*

The quote shows that the lesson study provided the teacher educators with opportunities to improve in their profession through the different team planning, reflective teaching and sharing of knowledge and skills. They explained that the lesson study has opened their eyes to realise that a teacher is supposed to cultivate a life learning culture and team spirit in order to keep improving. This supports what Muijs et al. (2010) emphasise.

#### Research in Teaching

The teacher educators also explained that the lesson study has helped them to develop and enhance their research skills and to know the importance of research in teaching and learning. Another college explained as follows regarding the importance of the research that they did:

> *For us to be able to improve our teaching, it is always good to collect data regarding how student teachers understand a certain concept. It has helped us to identify and try our new skills and also to come up with some solutions to the problems. The research has offered us a setting for trying out both familiar and unfamiliar ways of teaching different mathematical concepts and how to organise our teaching.*

The teacher educators explained that although it was the first time for them to conduct a lesson study, its process helped them to acquire and enhance different research skills like observation, data analysis and trying out new teaching strategies.

### *Findings from Focus Group Discussions*

The following are the findings from the FDG interactions with mathematics teacher educators in three of the colleges.

#### Lesson Planning

In terms of lesson planning, the FGD revealed that teacher educators had gained knowledge and skills in team planning and careful planning in consideration of the needs of their students. Apart from planning daily lessons as a team, teacher educators were planning the whole term's work together in schemes of work. The introduction of lesson study contributed to their continued working together as a team. Their lesson planning included selection of lesson objectives, teaching and learning resources and methodologies, student and teacher activities, assessment methods and a reflection of the day's lesson. Their team planning enabled them to discuss in detail what mathematics should be learned and how they intend to deliver their lessons. They explained that the teacher educators were not just following the curriculum blindly as they did before, but were now able to analyse the contents for their deeper understanding before teaching. This has helped to build the confidence and strengthened the skills of the educators especially those that were struggling, as observed by Goddard and Goddard (2007). One educator reported that:

> *The PD opened our eyes that we should not always stick to what is in the curriculum but be resourceful in looking for information.*

Lesson planning also considered their students' needs and possible questions. They reported that their students had been trained to ask higher-order questions like *how this* and *why this*? This has encouraged the teacher educators to prepare well for the lessons. One educator observed that:

> *When planning our lessons, we make sure that we get as much information as possible about a topic and we think about possible questions from students.*

From this we observe that the teacher educators have an improved lesson planning to that which considers students' needs and questions. Predicting and planning for students *how* and *why* questions is an important professional growth because it enhances students' understanding of the mathematics.

### Teaching and Learning

In terms of teaching and learning, the teacher educators reported gaining mathematics content knowledge and knowledge for teaching. They explained that they were now able to give justifications for some procedures in mathematics. One educator reported that:

> *I am now able to explain why we follow some rules in mathematics not just telling my students to follow the rules without proper understanding.*

In addition to improved teaching and learning, the teacher educators reported improved use of textbooks and other reference materials; they explained that their use of textbooks and other reference materials was enhanced because of team planning and planning in consideration of the students. They were compelled to find more reading materials including the use of online resources. They were also able to share notes from various books and refer their students to more reading materials. One teacher educator reported that:

> *There is now more collaboration among us and this has helped us to learn how to handle different topics. So it was a good PD since we gained both content and pedagogical knowledge.*

Their teaching methods were also improved since they were planning how to teach together. The teacher educators reported that during their lesson planning phase, they agreed to involve the students more, be attentive to the needs of their students and enable students to carry out many activities within a lesson.

> *Lesson study helped us to work as a team. We agreed to involve students more during our planning meetings, be attentive to them and encourage them to participate in a lesson.*

The improvement in teaching and learning that the teacher educators reported supports the improvements in lesson plans that Fauskanger et al. (2019) and Kazima et al. (2019) observed.

### Assessment

Another aspect that teacher educators reported improvement on is assessment. This includes both assessment of learning and assessment for learning. They reported that they agreed on what to assess and how to assess their students. They reported that their lesson planning involved assessment for learning, which is an integral part of the lesson and informs the teaching process. Assessment for learning is important in the Malawian Outcomes Based Education[1] that emphasises on checking the gains made during the teaching and learning processes. Similarly, the assessment of learning helped them to administer reliable assessments and to measure different skills of their students. The teacher educators were planning and coming up with assessment tasks together and agreed on how to assess their students.

### Collaboration and Networking

The teacher educators also reported an improvement in their teamwork and professionalism where they now aim at working as a team and not as individuals. The teacher educators noted that their participation in lesson study has helped them to maintain their teamwork spirit. This has improved their professionalism and mutual respect and understanding among them.

They pointed out that the PD has helped in building networks between and among the colleges. They indicated that as colleges they share challenges and assist each other on how to address them. They also organise and conduct CPDs together. This has helped them to build their capacity and confidence in teaching.

We note that at the centre of many opportunities that the PD programme offered to the teacher educators was the collaboration and networking within colleges and between colleges. From the discussions, we observed that there was an improved collaboration within the mathematics teacher educators at college level and the inter-college collaboration and networks. The PD created an opportunity for the teacher educators to collaborate and network in a systematic and structured manner. They had scheduled times for their meetings and conducted their collaboration meetings formally. This included recording their discussions, planning lessons together and sharing responsibilities in terms of who will teach during a lesson study and who will be assigned to doing video recordings of the lessons while

[1] The current primary and primary teacher education curriculum in Malawi is called the Outcomes Based Education. The Malawian Outcomes Based Education Curriculum emphasises on the achievement and display of outcomes in terms of knowledge, skills, values and attitudes as evidence that learning has taken place.

others took notes as they observed the lessons. This formal way of collaboration produced evidence of the effectiveness of the PD and lesson study after it was concluded in 2018. Previously, the teacher educators pointed out that they were collaborating in an informal way and there was little or no evidence that demonstrated their collaboration and networking. This suggests that for collaboration to work it has to be made formal and not informal.

Evidence from three colleges, 6 months after the PD, strongly suggested that collaboration and networking among teacher educators is essential in developing their capacity within the colleges. As Goddard and Goddard (2007, p. 7) have emphasised that collaboration and networking “can make good teachers even better”. It was also observed that the colleges were planning and conducting CPDs together apart from their respective college CPDs. For inter-college CPDs, they were identifying challenging topics together and selecting facilitators from within or outside their colleges to assist them. This created opportunities for peer learning among teachers and the building of collective learning opportunities.

## Conclusion

This chapter has discussed the PD programme that was offered to mathematics teacher educators in Malawi from 2016 to 2018 and the influence that it has generated in terms of building teachers’ capacity and professional growth. Drawing from the teacher educators’ presentations at the workshops and focus group discussions at the colleges, we observe that the PD achieved, among other things, improved collaboration and networking within and across the colleges. The teacher educators continue to carry out activities in collaboration within the colleges, including lesson planning, writing terms’ schemes of work, peer learning, structured meetings and planning assessments. The PD has also initiated networking among the colleges. There was evidence of planning and conducting CPDs together, sharing information on how to handle some topics and discussing how to conduct assessment. These findings illustrate that the PD exceeded its main goal of building capacity of the mathematics teacher educators in teaching student teachers and using the lesson study to improve their practice. It has also improved the teacher educators’ capacity to collaborate with each other within the colleges as well as with other colleges. The point here is not to overly celebrate the PD itself, but to highlight the continued and productive teacher collaborations that have resulted from this initiative and to emphasise that it is these forms of collaborations that are opening opportunities for the teacher educators’ continued professional growth.

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# Chapter 16
# Professional Noticing for Mathematics Teacher Growth

**Judah Paul Makonye and Mapula Gertrude Ngoepe**

## Introduction

Many research studies show that while a strong mathematical content knowledge for a teacher is a crucial factor in teaching mathematics, that solid content knowledge does not readily transform to successful teaching of mathematics (Pournara, 2013). Yet other researchers report that questionable teacher mathematics content knowledge factors lead to the South African mathematics low achievement trap (Carnoy et al., 2012). Given that some recent graduate teachers struggle to teach mathematics successfully, what potential can professional development have to improve mathematics teacher quality? We believe that for teachers, learning does not end, and professional development must be continuing given that old teaching problems remain stubborn and teaching situations change and must be addressed with new techniques.

While initial teacher education institutions (ITEIs) can equip pre-service teachers' adequate mathematics disciplinary knowledge, it is crucial to also develop in them a philosophy of pedagogy to teach mathematics so that they have confidence to manage both mathematical knowledge for teaching learners in a robust way. In particular, how to self-reflect to develop professional vision on one's work is important. This helps to develop a mathematics teacher's pedagogical content knowledge (PCK) and mathematics knowledge for teaching (MKfT) (Ball et al., 2008). With this knowledge, a teacher must be able to unpack the mathematics content

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knowledge they have so that it is understandable and comprehensible to young learners who do not yet understand it. The canonical mathematical content knowledge that mathematics teachers have and is represented in texts, constitutes 'concept definition' (Tall & Vinner, 1981). This is contrasted to 'concept image' which is the 'picture' that learners envision has been taught to them. Needless to say, the learners' concept images might actually be preconceptions and alternative concepts quite different to what teachers wish to communicate. This means that good teachers need time to study their learners to observe the patterns of their concept images, which they need to narrow and span the gap between learners' concept image and concept definition.

We argue that for practicing teachers to grow and excel, they need to be ready to learn from and with others in professional teacher learning communities (PTLCs). This is because, often teachers work in isolation in their classes with little help if any from peers. To be better mathematics teachers, it is crucial to set up a professional learning community to maintain professional growth.

## *Professional Teacher Learning Communities*

Cheng and Mun Ling (2013) propose a three-stage learning study cycle. These stages consist of (1) pre-teaching conferences and preparation, (2) one member delivering a lesson under peer observation and (3) evaluation and post-teaching conferences. They regard a learning study as part of PTLCs which are continuing professional processes with the sole purpose of enhancing professional development of all the members (Loughran, 2007). The end goal of the communities is to enhance learners' overall outcomes, in this case, mathematics. One sees PTLCs as helping to inculcate mastery teaching and mastery learning (Bloom, 1971). Bloom argues that given time and adequate processes to diagnose children's learning problems, all children can ultimately achieve lesson objectives. Bloom continues to argue that learners are diverse and have different learning strengths and weaknesses. If teachers have time to analyse and understand learners' weaknesses, then learner's achievement is possible to all. In this, Booth (1997) maintains that for the variation teaching theory the teacher approaches to learners must be variable so that the learners can have time to understand the different notions of an object of learning. An object of learning could be procedural or conceptual—what a learner is required to learn by the teacher. One way teachers can use variation in teaching mathematics is the use of exemplification. Different examples or representations for the same concept need to be used, as well as non-examples of a concept. This way, learners differentiate and integrate the different aspects of either a concept or a procedure and therefore build lasting understanding. Professional learning communities help practicing teachers to alert and awaken each other of such ideas and others. The isolation of being in class in charge of often large classes is addressed as teachers learn best practices from each other. They experiment with new teaching techniques. Furthermore, their colleagues can help them to understand some good and not so good teaching habits that a teacher cannot usually pick by themselves in their busy schedules.

Humans are creatures of habits. Most unproductive habits are so ingrained in teachers. It needs some other teachers to point out others' unproductive habits. Peer observation helps teachers to be able to reflect on themselves and be conscious of what they do. It enables them to describe what they do and therefore be able to consciously change their way of teaching.

Many researchers were impressed by Japanese learners' competitiveness in international comparative tests such as TIMSS and PISA. They discovered that a common feature of the Japanese education system was PTLCs. Therefore, learning studies were benchmarked from the Japanese (see Cheng & Mun Ling, 2013). It was envisaged that PTLCs were responsible for Japan's preeminence in international mathematics comparative tests, hence their adoption worldwide.

Peer agreement to start to work together and have meetings either in one school or grade or across schools is the beginning of PTLCs. They discuss and agree how they will work. They identify a teaching and learning problem, the object of the learning study, one at a time and how it may be tackled. This is referred to as problem scoping. It is a very important part of the PTLC. Literature and strategies are shared, and peers agree on the teaching approach. Sometimes an external resource person such as an official from the district office is called to facilitate this process. Then one teacher does the teaching under observation by others. In some cases, it is important to allow the learners to evaluate the teacher. In other institutions when teachers are applying for tenure or promotion, students' evaluation of the teacher becomes a key requirement. In such schools, when a teacher wants promotion, he is required to produce Head of Department as well as evaluation of teaching by other staff members in the mathematics education department and some from other subjects as well.

Progressive teachers are always ready to learn and always take peer observation as an opportune time to learn to improve teaching based from free advice from colleagues. Participants need to agree that all work done is collegial and that colleagues are critical friends. The aim is to build each other. Participants must agree that in all cases professional language and respect of persons are paramount and that all criticisms are not personal.

The focus must be clear, using an intentional reflective approach to promote quality teaching. Therefore, to reflect and learn becomes paramount, to encourage good practice and discourage not so good practice that could be time-wasting or delirious to learners' long-term disposition to mathematics. In this paper, we report on the insights that came from the reflective approach of professional learning communities in mathematics teaching. Ball (2000) argues that reflective practice needs no special design or conjecture as it is primarily designed to sharpen teachers' foci when deliberating on what is going on in their work. In this study, reflective practice is used as a prism to examine the teacher development in a lesson study.

### *Research Question*

In what ways does an intentional reflective approach in a professional teacher learning community transform mathematics teaching practice?

## Conceptual Framework

NCTM Principles and Standards (2000) of equity, curriculum, learning, assessment, technology and problem solving, communication, connections and representation underpinned the PTLC (see Appendix).

An important way to boost PTLCs is to engage in professional noticing (Sherin et al., 2011). Professional noticing is very important for practitioners. It distinguishes between amateurs in a profession and the experts. When given video analysis of lessons, expert teachers were seen to notice salient aspects of a mathematics lesson (Sherin et al., 2011). Further, in international studies it was found that when teachers watched the same videos of mathematics lessons, American teachers directed their attention more on pedagogical features, whereas the Chinese focussed on mathematical content (Sherin et al., 2011).

Professional noticing has three stages. The first is preparing to notice; what is it that one wants to notice? The second stage is the action of noticing. Needless to say, that sometimes and often good noticing involves going beyond just observation. It also includes further questioning in order to validate and further explore what is noticed. This is the third stage of noticing the unplanned contingent, expedient actions that a teacher takes in the whim of the moment, to grasp an uncanny teaching and learning opportunity that may not occur again. Thus, professional noticing goes beyond evaluating what a learner says, whether it is wrong or right. It goes beyond the visible and digs down to the invisible, in order to ascertain the deeper meaning the learner is thinking so as to engage the learner more productively. Professional noticing is sometimes referred to as professional vision (Sherin et al., 2011).

Stepanek et al. (2007) regard a learning study (LS) as an approach used by teachers to develop their professional competence on the basis of observations of fellow teachers and students' interviews. It is argued that a LS is comprised of two key characteristics. The first is that it is collaborative; secondly, it uses research design principles. The collaboration helps in the co-construction of teacher knowledge by a group of teachers. Teachers jointly build pedagogical content knowledge to improve their teaching capabilities ultimately directed to improved students' performance in mathematics. The principles of research design that are employed in a LS go through four key levels of preparation, teaching, classroom observing and evaluation (Kemmis & McTaggart, 1988). This leads to a new cycle. How a new cycle starts is dependent on whether the identified object of the previous cycle has been accomplished or not. If not, peers discuss why the implemented strategies did not work and what may need to be changed and the cycle starts again. This is unlike in action research where a teacher self-reflects on their practice. In LS, the work done and brainstormed' is always as a group activity. In a LS, lessons are prepared collaboratively but then taught by one group member as the rest observe. Then reflection is done by the whole group and insights shared. In a LS, practitioners agree and establish long-term goals which they must focus on relating to what they want their students to become and achieve (Stepanek et al., 2007). Therefore, the LS identifies the deficits in teacher competencies that need to be overcome so that teachers can effectively help learners. In this study, the LS approach was used by high school teachers with the aim of improving teacher pedagogical competences to instigate and boost students' mathematics outcomes.

Learning studies use video clips of lessons. Good practices are often enhanced through watching good (and bad) video clips of lessons. Proposing that a picture is worth a thousand words, Berk (2009) argues that 'a video can have a strong effect on your mind and senses … it is so powerful that you may download it off the internet, so you can relive the entire experience over and over again' (p. 2). It can be argued that video clips help teachers to draw and infuse powerful cognitive and emotional experiences. Video clips are multimedia and cater for multiple intelligences (Gardner, 2000), as children learn differently via visual, emotional, auditory (musical), spatial and so on. In addition, videos can be replayed repeatedly by a practitioner to study and internalise the essence of a message. The video can sharpen professional noticing skills as they can provide powerful instructional models that can be emulated and shape professional practice. For these reasons, video clips are encouraged as key components of PTLCs. The Reformed Teaching Observation Protocol (RTOP) (Sawada et al., 2000) was used for coding observed lessons.

## *Methodology*

This was a qualitative case study. A qualitative case study aims to generate explanation of phenomenon under given affordances, constraints and boundaries. There were four participants in this study who formed a professional teacher learning community. These mathematics teachers taught at one inner-city Johannesburg high school. Data collected was in the form of observation notes, audio recordings of taught lessons, team reflections and interviews with the teachers. Further learners ($n = 51$) completed a questionnaire to evaluate teacher performance after teaching interventions. The questionnaires were analysed in order to assess whether there were shifts in teacher practices due to the PTLC intervention. There were ten items of which the learners rated their teachers from A as strongly agree (5) to E (1) as strongly disagree with C (3) being neutral. These ratings were also used by other teachers in the PTLC to rate lesson delivery towards achievement of the targeted objectives of the lesson as agreed in the lesson preparation.

In analysing the teaching and learning and delivery of lessons, NCTM Principles and Standards (2000) were used as an analytical framework for the learning study (see Appendix for the categories). Fulfillment of those categories was done as participants filled in the questionnaires with those categories on an A (5) to E (1) basis. The principles were equity, teaching, learning, curriculum, technology and assessment. The curriculum was left out because the learning material was agreed by the group. The standards were problem-solving, communication and connections and representations.

## Teaching Video Clip Comparisons

The teaching video clips were sourced from the Internet. Others were taken from previous teaching observations that the authors took when they visited pre-service teachers on teaching experience. The video clips were watched by all members during pre-teaching conferences. At the beginning, the participants studied the video clips of mathematics lessons and pointed out what needed to be noticed and why. They discussed what they thought was important in the teaching videos which need to be adopted and what was not so good that could be improved in teaching. It was interesting that some of the most educative videos with expensive materials or resources that the teachers could not normally obtain in their schools were not taught.

Participants and one of the authors discussed aspects of the videos in order to share observations. In general, the teachers agreed what they thought were examples of good mathematics teaching and what was regarded as not so good. Teachers shared their reasons for their positions. These videos were an important aspect of the PTLC. They help to sharpen professional noticing, what to notice as well as how to act on what they notice on other teachers and learners.

After video professional training, teachers met and decided to teach the topic of directed numbers at grade 8 level. Teachers agreed that they had problems of teaching operations on directed numbers. They were provided with two readings on how to teach operations on directed numbers through (a) signed discs (Battista, 1983) and (b) number line jumps (Chilvers, 1985). Though quite old these readings were full of good ideas and were discussed. The teachers and the authors discussed the strategies for teaching addition, subtraction and multiplication of directed numbers using the readings.

It was agreed that four lessons were to be taught one for each operation using both devices and the fourth lesson being a revision for all the work done. Each of the lessons was 1-h long. All the lessons were taught in one class by one teacher who volunteered. The other three teachers and the authors observed the teaching.

## Data Analysis and Discussions

Data analysis was based on grounded theory. Data was obtained from peer observation of teaching as well as learners' rating of the teacher's teaching. Learners' questionnaire data were analysed with the help of a computer system dedicated to multi-choice assessments in the institution.

### *Preparatory: Watching of Videos*

The videos were an important part of professional development. Participants analysed each video individually, and then the ideas were compared and contrasted till shared evaluations were consensually agreed.

The PTLC watched five video lessons in which teachers were teaching various mathematical topics. In some of the videos where the teachers just presented the lesson without any learners, they were sort of lectures. In some of the lessons, the teaching was interactive. The merits and demerits of the aspects of lesson delivery were discussed.

Participants reported that the lecture videos in which the teachers exposed mathematical ideas and procedures were limited in that they were more suited for revision purposes and for fast learners. One participant said 'for revision purposes they are very good, but they seem not to be suitable for introducing new concepts'. Another agreed and said, 'learners ought to have some form of productive struggle with the mathematics they are learning, in order to grapple with mathematical ideas and make sense of it by themselves'. It was found that lecture videos seemed to promote rote learning which the teachers were trying to run from. 'They have their limits', one teacher said. On the other hand, participants pointed out that interactive video lessons seemed more difficult to implement. They related to the real situation in the mathematics classrooms in South Africa. Participants indicated that in interactive video lessons, learners participated and could air their views. The teachers in the videos did professional noticing and engaged with learners' errors and misconceptions. Although the progress of the lessons was slower than lecture videos, teacher participants felt that if the lessons were prepared carefully with rich tasks accompanied by models and representations, learners would be more motivated in the class and better achieve learner outcomes in a more lasting way.

### *Analysis of Bernard's Implementing Professionally Agreed Practices*

One teacher, Bernard (pseudonym), volunteered to start to implement PTLC ideas in his grade 8 class on teaching the topic of integers. This study is based on analysing how that one teacher implemented the agreed practices, while others observed him. Four lessons were taught. After each lesson, the merits and demerits of each lesson were shared and agreed upon. Bernard was requested to respond to the friendly critiques to clarify and explain why he did what he did. All this was done in a collegial manner and all critiques were professional and not directed to belittle Bernard's efforts.

### *Lesson Map*

It was observed that it is critical to introduce the lesson by stating what the lesson will cover; this is called lesson objectives. Also raised was the importance of linking what was being currently taught with prior mathematical knowledge that learners had learnt earlier.

### *Relevance of Lesson*

At the beginning of the PTLC, participants observed Bernard. At the first post-teaching conference, one participant Anna commented that it was critical for Bernard to discuss on the relevance of the mathematics concepts which were to be taught. How were that knowledge to be applied? This included discussing examples of how mathematical concepts could be applied to solve daily problems. In later lessons, Bernard was to comment that he saw 'immediate changes in his class … students visibly showed interest in the lessons and were willing to engage more as a result of discussing relevance of the mathematics to be taught'. The participants reported noticing learners saying 'Oh', 'Okay', etc. as they expressed interest in the lesson rather than sitting passively unlike in the past. The students were willing to take risks in the lesson. These observations confirmed what Nyamupangedengu (2015) found that 'students are motivated and show interest when they find personal relevance in the content they are learning' (p. 43). At the post-lesson conference, Bernard was advised to work out mathematics examples in class together with students. Bernard was to exclaim 'I was aware of the importance of these but had begun to forget to implement these in my teaching … in the past used to do it lots!'. This shows that if good teaching habits are taken for granted, they can begin to be extinct; what is required is for fellow practitioners to point them out, to keep them alive and developing by regularly commenting on them. It was later to be discussed that exemplification was to be a key factor in mathematics teaching.

### *Exemplification*

As exemplification (Zhu & Simon, 1987) was used, it was noted that it promoted not only the development of mathematics but also clarified some procedural aspects of doing mathematics. It further increased participation between teacher and students and students among themselves. When given practice questions in class, exemplification helped students to rapidly assess their understanding of taught concepts and to also ask for further explanation from the teacher and peers while the ideas were still fresh in learners' minds.

### *Professional Noticing of Students' Mathematical Thinking*

Participants raised the concern that looking for correct and wrong answers only in learners' responses to questions was evaluative (Davis, 1997), judgemental and therefore behaviouristic. It stifled conversations and communication, which was an important standard for learning mathematics (NCTM, 2000). Participants agreed that any response that a learner gave could be pregnant with meaning. It was noted that evaluative teaching engaged learners only on the surface. Learners were to be given opportunities to explain their thinking to communicate and argue for their ideas. If their ideas were shared in class, other students were interested in what was happening so learning occurred in a better way as learners connected mathematical concepts (NCTM, 2000).

Also noted was the issue of engaging the errors and misconceptions that learners could exhibit. It was agreed that making errors and misconceptions was the expertise of learners and that these were to be welcomed rather than being shunned (Nesher, 1987). Errors and misconceptions were milestones in learning. In particular, it was discussed that making errors and misconceptions provided evidence that learners were grappling with the mathematics they were learning, they were trying to make sense of it and they were trying to connect their prior learning with new concepts in their own way. Inevitably learners' 'concept images' (Tall & Vinner, 1981) of the mathematics they were learning had to be taken advantage of in the moment for learners to see for themselves that their conceptions were not adequate. In Piagetian terms what was required was to instigate 'cognitive conflict' in learners' thinking so that the appropriate cognitive shifts (Makonye, 2016) may occur. These could hardly occur if there was no genuine appreciation of what the learners were thinking. As Freire (1996, p. 37) argued, teaching by telling is a 'banking concept of education' which was oppressive rather than liberating. That form of teaching regards the teacher as the one who knows and that the learners do not know anything. The banking concept of education regards knowledge and pedagogy as static and not to be questioned.

Professional noticing and contingent teaching on unplanned events that emanates from learner ideas were regarded as liberating teaching (Freire, 1996). In this, the construction of mathematical knowledge is shared by the teacher and learners as equal partners in an educational project except that the teacher is a more experienced participant. It is argued that quality teachers learn as much from their learners as their learners learn from them. Professional noticing of learners' ideas in class and acting upon them promotes that.

## *Teaching Intervention*

Five lessons of teaching were done in teaching the topic of integers, comparing integers, as well as the operations of addition, subtraction and multiplication in the inner city high school.

Shifts in Bernard's teaching practices began to occur with regards\to elements of both NCTM Principles and Standards (NCTM, 2000) and elements of RTOP (Sawada et al., 2000). Bernard began to encourage learners to communicate, connect and argue on mathematical concepts. Further problem-solving was encouraged as well as use of different representations of mathematical concepts.

## *Mathematics as a Subject of Many Discourses*

In particular, in teaching the comparison of integers, a vertical number line was used rather than a horizontal one. Such a representation is analogous to labelling floors in a high-rise building which has basements. In teaching, Bernard used the principle, 'the higher the bigger' and 'the lower the smaller'. In this way learners realised that on a vertical number line, −1 was actually higher than −5 hence −1 is larger than −5. This helped to settle the counterintuitive idea that since 5 is larger than 1, so −1 is larger than −5. Because of professional noticing skill, Bernard noted that learners were using the whole number frame of reasoning which they generalised to integers leading to misconceptions in comparing integers. Participants discussed that what teachers and learners required was to realise that there was a change of discourse (Sfard, 2007) from the discourse of whole numbers to that of integers. Sfard did not refer to this as a misconception; rather she referred to this as a change in discourse and which has its own rules. So, when learners use the arithmetic whole number discourse to integers, the discourses are not commensurable; what is required now is to learn a new discourse, which is a discourse of integer numbers and the rules that pertain to it.

## *Learner Evaluation of Teacher*

For teachers to be more responsive to learners, they need to listen to them. Inevitably, this can be very hard as teachers are often very defensive to what learners say about them. Teachers often say learners do not know; therefore, their evaluation is immature and taken from an angle of ignorance. But once the teacher gets courage to listen to the learners and engage with them he often sees a recurring pattern about their own teaching which is liberating. They realise that what learners say of their teachers is often consistent. This is an important part of professional noticing (Sherin et al., 2011). It increases teacher knowledge of students and *of themselves*, helping them to espy which methods are most helpful to learners, rather than what they themselves think are the better methods. When evaluation of teacher by learner questionnaires was analysed, it was found that on average the teacher scored 85%. Teachers on the other hand scored Bernard 95%.

## *Learner Diversity*

Allowing learners to engage differently in class caters for learner diversity. Participants agreed that it is important for social justice as all students' points are valued (NCTM, 2000). As learners were given worksheets in class, they learnt to solve problems immediately in class with plenty of opportunity to collaborate with their peers, rather than during homework or assignments where they were more likely to work alone at home.

## *Interview with Bernard*

When we interviewed Bernard he said:

> I have learnt a lot … never assume anything is easy for learners! … it is very important to always respect the ideas learners produce in class … never rush to judge them right or wrong … they might have something very important to say. Give them time to explain themselves … I have noted that way and learnt a lot about my learners … no learner is really dull, it is only that I did not understand my learners … now I know. I will now be very curious and inquisitive to my learners. For me, teaching mathematics is now so interesting to do I always learn so much from my learners.

PTLCs reminded the teachers in the community what good teaching is and what good teaching is not. Seeing a spirited engagement of students with each other as they worked mathematical tasks was very exciting for the researchers. Professional development is the way to go.

## Conclusion

What is important to note is that long teaching experience is not a guarantee of good teaching. This is because it could be repetition of poor pedagogical techniques year in and year out. The game changer is participating in professional learning communities where teachers learn from each other and where they share problems and solutions.

This report shows Bernard's radical shift to a practice where he became a curious teacher genuinely interested to learn on the contingencies that occur in the classroom when he listens to learners and allow them to air their ideas to the teacher and each other. In particular, learner errors and misconceptions were regarded as old discourses. They were not regarded as wrong. Thus, in fact the modulus of $-5$ is larger than the modulus of $-1$. This is a discourse for whole numbers. In studying integers, a new discourse and its rules needed to be learnt, because an old one was no longer appropriate. This resulted in higher motivation to learners, granted as some psychologists argued 'motivation is the spark-plug of learning' (Redmond, 1989; p. 13). Bernard also increased learners' motivation by emphasising the relevance of the mathematical concepts for application in problem-solving in learners' lives. Bernard moved from being a subject-centred practitioner characterised by strong discipline classification and framing (Bernstein, 2000) to one in which there was weaker framing. As more freedom was given to learners in the classroom, learners began to learn using the 'problem solving concept of education' (Freire, 1996). As a result of participating in the PTLC, Bernard was no longer just interested in dishing out mathematical knowledge to passive learners. He realised the limits of that methodology. By noticing and examining what learners said and thought, he began to engage learners at a deeper level, to bring about authentic learning. Although the lesson pace slowed down, this was a small cost to pay, given the exuberance of students as they began learning the topic of integers more conceptually. We think this is a worthwhile investment as this topic plays out in many high school mathematics topics such as algebra. Directed numbers are also required in studying science, business, social studies and other subjects. So, if students understand them it unlocks many opportunities in learners' school life.

### *Recommendations*

Mathematics teachers are not mathematics dictionaries. Teachers do not teach mathematics, but they teach students. We are entering the 4th Industrial Revolution of Internet and rapid communication. Teachers must learn continually how to teach in this new world lest they are left behind. This is a real challenge for all teachers as they need to participate in PTLCs to work with others to continually adapt and adopt to the ever-changing context of mathematics education.

## Appendix

### *NCTM Principles (2000)*

The Equity Principle: 'Excellence in mathematics education requires equity—high expectations and strong support for all students' (p. 12).

The Curriculum Principle: 'A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well-articulated across the grades' (p. 14).

The Teaching Principle: 'Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well' (p. 16).

The Learning Principle: 'Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge' (p. 20).

The Assessment Principle: 'Assessment should support the learning of important mathematics and furnish useful information to both teachers and students' (p. 22).

The Technology Principle: 'Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning' (p. 24).

### *NCTM Standards (2000)*

**Problem Solving**: 'Teachers play an important role in the development of students' problem-solving dispositions by creating and maintaining classroom environments, from prekindergarten on, in which students are encouraged to explore, take risks, share failures and successes, and question one another' (p. 53).

**Communication**: 'Listening to others' explanations gives students opportunities to develop their own understanding. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections' (p. 60).

**Connections**: 'When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience' (p. 64).

**Representation**: 'The importance of using multiple representations should be emphasized throughout students' mathematical education … As students become mathematically sophisticated, they develop an increasingly large repertoire of mathematical representations as well as a knowledge of how to use them productively' (p. 69).

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# Chapter 17
# Teacher Change and Mathematics/Science Teaching in Kenya: Transformative Attributes in Student Learning

**Samson Madera Nashon, David Anderson, and Peter Okemwa**

## Introduction

Despite numerous attempts to reform education in East Africa, and in particular, Kenya, the question of relevance has always been discussed as part of the reform agenda, but to date careful analysis of the state of education and especially mathematics and science education, relevance is like a mirage (Knamiller, 1984; Yoloye, 1986). Since attaining independence from Britain in 1963, Kenya has had several major educational reforms, each of which has been preceded by a commission of inquiry including Ominde (1964a, b), Gachathi (1976), Mackay (1981), Kamunge (1988), and Koech (2000). All these commission reports have directly or indirectly affected education system in Kenya and at best elicited the unending national debate on the question of relevance in terms of the role of mathematics, science, and technology in national development. Despite the major structural changes in education system over the years, with the question of relevance characterizing the rhetoric for change, there has never been much effective change from traditional curriculum and pedagogy especially in mathematics and science education. The system is still overly examination driven, teacher-centered with colonial as well as foreign-leaning mathematics and science curriculum and pedagogy. This apparent static nature of curriculum and pedagogy is due in part to colonial hangover and influence whereby for a long time foreign experts who had limited knowledge of the local Kenyan

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context dominated high school curriculum development and implementation. Also, those Kenyans positioned to influence change were often trained abroad or trained locally by foreign experts; thus they lacked the skills needed to reform curriculum and pedagogy to reflect the local context. In addition, they often borrowed from foreign instructional models not suited for the Kenyan learner. This has made teachers less receptive to innovative pedagogies despite the fact that there are effective ways to organize student learning experiences that do not violate the curriculum but ensure students' deeper understanding of mathematics or science, the consequence of which could be improved examination performance. Instead, the teachers focus more on getting students to pass examinations. The need to make mathematics and science relevant to the students is regarded as superfluous to examination performance and at best perpetuates the traditional culture where mathematics and science are presented as encapsulated systems that have no relevance to the students in terms of their local contexts whatsoever (Tsuma, 1998). Any attempts to integrate into curriculum visits to authentic mathematics or science learning environments, such as *Jua Kali*,[1] are seen as unnecessary distractions, as had been expressed by many of the teachers prior to their classes' participation in our initial 2006 study that was funded by the Social Sciences and Humanities Research Council (SSHRC) of Canada. But for us, the question of relevance is very important as eloquently expressed by Tsuma (1998): "no nation can develop in any sense of the term, with a population which has not received a thorough and relevant education" (p. i). And, despite the local setting's richness in both mathematical and scientific phenomena that can be readily mediated through curriculum, Kenyan mathematics/science teachers rarely exploit its potential to mediate student learning. But as our 2006 SSHRC funded study demonstrated, contextualizing mathematics/science curriculum and instruction can dramatically transform students' mathematics/science learning with improved understanding of mathematics/science in a canonical sense. Hence, there is the need for Kenyan teachers to change the way mathematics and science curriculum and pedagogy are reformed as a means to making mathematics and science more relevant and meaningful to Kenyan learners in terms of connections to events in their local contexts, such as activities in *Jua Kali*. In other words, whereas teachers may not individually change mathematics or science curriculum content, but they can explore alternative teaching strategies to make mathematics and science meaningful to the students in terms of relevance as was demonstrated in our initial studies. However, our experience with the Kenyan teachers is that they would like their students to understand mathematics and science. It appears that understanding to them equates getting high marks on the examination, which was contrary to what the students in the study had confessed. Because the 2006 study, which involved the teachers in co-developing curricular units that linked curriculum content to *Jua Kali* without violating the official curriculum, had impressive results

[1] "Jua Kali" is a small-scale manufacturing and technology-based service sector where artisans manufacture equipment and other household items such as charcoal stoves, kerosene lamps, and chicken brooders, which are ubiquitous in everyday Kenyan culture while also providing related services to other small-scale producers (UNESCO, 1997).

in terms of student learning of canonical mathematics and science, we saw it as imperative to investigate the effect such student learning has on teachers' professional practices.

## *Teacher Change*

There are many models on how teacher change can be influenced. However, contemporary methods of managing teacher change come in the form of teacher professional development (PD). According to Elmore (2002), there are two formats of PD: traditional and job-embedded. Traditional PD format is a top-down model where policy mandates it and experts hold workshops, seminars, lectures, etc. (Elmore, 2002) on what they consider to be effective pedagogy or curriculum reform. On the other hand, job-embedded PD locates training within the school or local context by utilizing say inquiry groups (collaborative in nature) where teachers participate more closely to their own context in shaping curriculum and pedagogy to the service of student learning (Elmore, 2002). Literature on traditional PD format indicates that it is effective in changing teachers' practices when it is longer in duration (Porter et al., 2000) since teachers need more time (Stein et al., 1999) and variety of activities (Mazzerella, 1980) to learn more about their practice. On the other hand, studies that advocate job-embedded format advise locating PD within the school for purposes of creating ongoing communities (Hord, 1997) and allowing teachers to do the talking, thinking, and learning about their practice and student work (Feiman-Nemser, 2001). Besides, leadership should also focus on instruction and not just accountability (Taylor, 2005). However, the PD activities described in both formats seem to disregard student learning as a change agency. Yet student learning or performance is a very important motive behind any teacher change or any curricular or pedagogical reform. This is the type of change (in instruction or pedagogy, social cultural values, and school culture) our study sought to investigate among Kenyan mathematics/science teachers as mediated through mathematics/science curricular reforms.

According to Anderson-Levitt (2003), teachers have cultural knowledge (or teaching culture) from which they draw when organizing teaching. Cultural knowledge is in this case the knowledge teachers "use to interpret experience and generate social behaviour" (Spradley, 1979, p. 5). Moreover, Anderson-Levitt (2003) sees this behavior to include feelings and values and points out that knowledge is cultural when it has been constructed (learned) including procedural knowledge, which is about knowing how to do things such as organizing student learning experiences and which gets shared during teacher interaction (socialization). Our study was particularly interested in the knowledge that Kenyan mathematics and science teachers who participated in enacting and experiencing student learning in a mathematics/science curriculum reform shared with their colleagues and why. The why question was about the values the teachers attached to the knowledge (successes, failures, and other relevant experiences) they shared with stakeholders including their colleagues. According to Lasky (2005), the interplay between teacher identity, agency, and context affects how they report what and how they experience teaching students. Furthermore, social context plays a role in shaping a teacher's sense of identity and purpose as a teacher (Lasky, 2005) and that individual cognition is the result of social interactions (Lerman, 1996; Harré & Gillet, 1994). Teachers as agency in our study referred to the belief that teachers have ability to influence their lives and environment (Lasky, 2005). This is because individuals as agency to change a context is possible in ways they act to affect their immediate settings (Fullan, 1991) using culturally, socially, and historically developed resources.

## *School Culture*

According to Hinde (2004), school culture surrounds, envelopes, and influences teachers' decisions and actions since they work in a cultural context where all aspects of school life are affected (Peterson & Deal, 1998). It even shapes what they talk about (Kottler, 1997) and how they choose what to emphasize from curriculum (Hargreaves, 1997). In fact, whenever school culture changes, everything else changes (Donahoe, 1993). In short, school *culture is the norms, beliefs, values, traditions, and rituals that pervade a school* (Goodlad, 1984). Thus, school culture changes constantly as it is constructed and shaped through school community member interactions (Finnan, 2000). Inevitably school culture will affect a teacher's teaching behavior including subject matter (content), pedagogy, and context knowledge. Any change in one of the teachers' subject matter, pedagogy and context will affect the overall teacher's teaching, which has become the area of intense research.

Our initial studies revealed important insights about East African (EA) learners. In particular the students who participated in the study understood mathematics and science better in a canonical sense when instruction used local contexts. Through fascinating and contextualized experiences, the students became more critical of their prior learning strategies and habits. Moreover, they showed a clear awareness of the limitations of the learning strategies they were using to be a consequence of the nature of the prevailing curriculum and lack of pedagogical models that made mathematics and science relevant (Anderson & Nashon, 2009; Nashon & Anderson, 2008, 2013b). The potential of contextualized mathematics/science to transform Kenyan students' understanding of mathematics/science concepts meaningfully was demonstrated through this study. What was not understood though is how the teachers' sociocultural values, their pedagogies, and collective school culture were impacted as they enacted and journeyed through and experienced their students' learning in the contextualized mathematics/science curricular reform.

In this chapter, we report the outcomes of our later research that investigated the effect of student learning in a reformed contextualized (integrated classroom-Jua Kali) mathematics/science on their teachers' sociocultural values, their pedagogies, and school culture as they journeyed through and experienced the mathematics/science curricular reforms. We considered the study to be significant for teachers, students, and teacher educators in terms of developing the capacity to make mathematics/science more relevant to the students' local contexts and the future economic and industrial development of Kenya.

We embarked on this study from the platform of already having extensive understanding about the indigenous and EA learner in particular, from our own past research and that of other scholars who have investigated student and teacher learning in the African context. In brief, these understandings take account of dimensions like (1) how indigenous learners including African students struggle to use their natural modes of learning (Anderson & Nashon, 2009) in Western modeled classrooms, (2) the need for meaningful assistance to change from cultural worldviews to canonical science (border crossing) (Aikenhead, 1996), (3) the common use of bodily signs to communicate important information (gestures) (Guerts, 2002), (4) the need to see it as a moral obligation to assist others (importance of group learning) (Gitari, 2006), and (5) the need to have a respect for cultural knowledge as well as canonical mathematics and science and ability to hold both worldviews and only use them relevantly (collateral learning) (Jegede, 1995). This is important knowledge a teacher requires to prepare instructions that recognize these cultural facets with a view to making mathematics and science learning relevant to the EA learner. But this seems not to be the case (cf. Tsuma, 1998). Hence, the drive to study teachers' professional practice in terms of sociocultural values, pedagogy, and school culture are impacted by their students' learning in a contextualized mathematics and science curriculum reform.

## Theoretical Framework

We drew from a derivative framework from Latour's (2005) notion of "agent(s) of change" central to change in student learning and teachers' teaching. According to Latour, the agency for change resides at least in the learning material, the teacher, the curriculum, the student, and so on. Based on the focus and content of the research question, we employed an aspect of a teacher change model that ascribes agency to student learning, to understand and interpret the effect of student learning on teachers' teaching (Nashon & Anderson, 2013a, b; Nashon, 2013). According to Nashon and Anderson (2013a), student learning has the capacity to influence teacher change. This aspect of teacher change agency was employed in analyzing, identifying, and interpreting attributes of student learning that influenced the mathematics/science teachers' teaching.

## Methodology

Given that we were interested in understanding the impact of Kenyan students' learning on their mathematics and science teachers, it was important for us to interrogate the teachers' transformative experiences arising from their development and implementation of a reformed mathematics/science curriculum. We considered it imperative to tease out this from the stories the teachers told about the transformative experiences in terms of sociocultural values, their pedagogy, and school culture. Hence, we considered narrative methodology (Connelly & Clandinin, 1990; Clandinin & Connelly, 1994, 2004; Moen, 2006; Richmond, 2002) to be appropriate at eliciting such information. A narrative according to Moen (2006) is "a story that tells a sequence of events that is significant for the narrator or his/her audience" (p. 4). The study also employed interpretive case study methods (Merriam, 1998; Stake, 1995) because the stories were of particular teachers in a particular context. This being an interpretation of stories about experiences by particular individuals in a particular setting, it required case study methods (Stake, 1995; Erickson, 1986). Case study methods were employed to gain an in-depth understanding of the teachers' experiences of a situation and their meaning (Merriam, 1998). Thus, the study involved interpreting teachers' told experiences within the bounds of only those who participated in developing and implementing student learning experiences. In the study, the 12 Kenyan teachers narrated their experiences of their students' learning as they enacted and journeyed through a curriculum reform and how the students' learning impacted their sociocultural values, pedagogies, and school culture. Their stories (narratives) were interpreted to make sense of the impact of student learning on the teachers.

The 12 participants of the study were all experienced mathematics/science teachers and voluntarily consented and agreed to participate. The teachers represented three core science subjects that include biology, chemistry, and physics and mathematics and taught in five different schools that followed the same national curriculum. Upon agreeing to participate in the study in the first term of the academic year, the teachers, with guidance from the research team, developed and later implemented in the second term of the academic year a 9-week contextualized (classroom-Jua Kali) mathematics/science unit based on the local Jua Kali manufacturing sector—a place where hundreds of artisans and their trainees fabricate metal products including kerosene lamps, chicken brooders, wheelbarrows, charcoal stoves, cooking pots and utensils, and many more, which are sold to local people and represent items common in the households of every Kenyan Student. The Jua Kali (local to the schools) embodies a wide diversity of applied sciences and mathematics including thermodynamics, chemical transformations, and reactions, which students could investigate.

The study commenced by having the research team and the mathematics/science teachers visit a local Jua Kali site, where they selected a variety of production activities and products that could be integrated with school mathematics/science experiences (Nashon & Anderson, 2013a, b). In collaboration with Jua Kali artisans, the teachers and researchers divided the site into ten production stations, clearly labeled them according to the various activities of specialization to ensure that during the later class visits, the students engaged in mathematics/science learning. The teacher and researcher discussions also involved the formation of student groups, identifying topics from the school syllabi related to activities at the Jua Kali. During the discussions the teachers were allowed the flexibility of developing mathematics/

science lessons that capitalized on the richness of Jua Kali and constituted a 9-week-contextualized science unit they later developed. Teachers were encouraged to integrate classroom and Jua Kali experiences, which demanded of the students to use, engage, or understand mathematics/science knowledge holistically as opposed to compartmentalizing it into mathematics, physics, biology, or chemistry. Further, the 12 teachers agreed to teach the units to the Form 3 classes to ensure a coherent integration of subject content that met curricular requirements and in harmony with the Kenyan national curriculum. The unit was implemented in a series of lessons that involved two formal school visits to a local Jua Kali site and numerous independent teacher-sanctioned student visits, given that the Jua Kali site was centrally located in the community where students lived.

One year later, teachers were interviewed at their schools in order to determine the impact of this kind of experience on their classroom pedagogy and practice. For each interview the researchers had conversations with two to three teachers at each school for 2–3 h. The narrative interviews with the mathematics/science teachers were about how their pedagogy, roles, and views about their experience with previously modeled mathematics/science pedagogy were impacted by their students' engagement with learning during the Jua Kali visits and the entire contextualized 9-week science unit experience including any new subsequent units they might have modeled on the 9-week unit. An example of a question that was asked is: "What aspects of the students' learning during participation in the contextualize mathematics/science curriculum unit experiences affected the way you now teach?". The Form 3 mathematics/science teachers' narratives embodying perceptions of how 1 year later their students' learning affected their pedagogy, roles, and views about their experience with previously modeled science pedagogy as they enacted a contextualized science curriculum unit in five select Kenyan high schools were analyzed.

## Data Analysis

All the data were transcribed verbatim for detailed analysis involving examining, categorizing, testing assertions for reliability, and recombining evidence from the different interview transcripts to address the objectives of the study (Miles & Huberman, 1994; Yin, 2003). Consistent with Novak and Gowin's (1984) advice, analysis of different levels of interview data to interpret and understand how the students' learning experiences impacted the mathematics/science teachers' sociocultural values, their pedagogies, and school culture. The analysis was guided by the research question of the study using a thematic approach (Merriam, 1998; Miles & Huberman, 1994). Thick descriptions of interview data were reconstructed to identify evidence of transformations in the mathematics/science teachers' sociocultural values, their pedagogies, and school culture. The research team individually and collectively reviewed interview transcripts as well as videos by reading back and forth (Dahlberg & Drew, 1997) as they looked for emergent themes that cut across case teachers. The research team regularly exchanged their individual insights via email and whole team face-to-face (or via Skype) meetings where the analyses that resulted in the collective interpretation of the data sets were discussed and compared (Stake, 1995).

## Results

The following are the emergent themes that are supported by representative verbatim quotes or excerpts from the participants' narratives (using pseudonyms).

1. *Student ownership of learning and self-teaching:* The teachers told of how they changed the way they taught after their students demonstrated ownership of the learning process, which manifested in self-teaching. Moreover, there seemed to be a sense of relief among the teachers that the students took responsibility for and control of a greater part of what was previously burdensome in terms of teaching responsibilities. It is evident from the select excerpts below that the teachers' attitudes and level of preparedness were transformed as the students took more control of their own learning.

   **Jenny:** … delaying even for a minute, or two to get to that class, you will find a student already there presenting something to the class - meaning everybody is prepared. If I do not show up by chance and have not made plans with a colleague, the students are there ready – Students say to themselves "Why don't we do what we could have done?" So it keeps each and everybody on their toes and they want to move as a group.
   Jenny's comment was in reference to her changed approach to teaching following her students' participation in the 9-week classroom-Jua Kali curriculum unit experience. The first sentence indicates change in terms of what students were capable of doing, and although not explicitly stated, the teacher is talking about punctuality and motivation. In other words, Jenny was saying she had to be punctual. The other message Jenny conveyed was the confidence she now had in her students to teach themselves and learn as a group. She now trusted students for taking responsibility for their own learning as she now did not have to worry so much about informing a colleague to take her class in case of an emergency. Thus, we see evidence that Jenny's teaching was transformed in terms of realization that (1) students could no longer wait for latecomers; therefore she had to be punctual, and (2) she now had confidence in students' ability to take charge of their own learning, something that was previously lacking in her.
2. *Group work and co-learning:* This theme is closely connected to theme 1 above. Although group work was often used in traditional classrooms, it seems the level of engagement was different from that observed in the 9-week classroom-Jua Kali curriculum unit that the students and teachers experienced. From their narratives, some of which are excerpted below, it appears the teachers did not believe mixed ability groups would work since it is often argued that there is often a tendency for the brilliant students to dominate and be more active (Reid

et al., 2006). However, it appeared despite their initial doubt of the efficacy of heterogeneous (mixed) ability groups with regard to learning, the students' engagement with learning through this kind of group work seemed to impact the teachers' attitudes and appreciation of its effectiveness. In addition, an inherent aspect of this observed student engagement was the manifestation of student co-learning within these groups. These new appreciations became critical to the transformation in the teachers' pedagogical values and practices. These insights were discerned from interview transcripts as illustrated in the select excerpts below:

**Wilson:** We never believed mixed ability groups would be affective before Jua Kali. But we noticed at Jua Kali that groups of students from different streams of same grade level were discussing things together. You know we had convinced ourselves that it will not work but when we mixed them the first day they went to the Jua Kali, they were able to work more effectively despite coming from different streams, we got convinced that this method is good and works. I have also been having these groups coming to me to consult and asking very challenging questions.

3. *Intra-class group work and collaboration:* The Jua Kali experience broke the rigid class boundaries that characterized the school culture and was catalytic to producing intra-class collaboration that had not been previously observed. This was characterized by increased teacher collaboration and team-teaching that further influenced intra-class collaboration and the overall school culture.

   **Peter:** Study groups have been enhanced among students. They do not fear each other because after all by the time you (research team) came we had three streams and some tended to limit themselves to their classes and classmates. But, at the end of the Jua Kali contextualized curriculum unit, at least they had made friendships across classes and now you see that there is unity and students are beginning to work together and not just within their own classes. They can now go out to engage with others.

4. *Relating mathematics/science concepts to the real world:* Through the contextualized mathematics/science curriculum and instruction experience, students learned to appreciate classroom mathematics/science knowledge and to relate it to real-world situations. Furthermore, the Jua Kali episodes enabled easy concept formation and knowledge construction in ways that conventional curricular approaches did not afford. These students' transformations (changes) subsequently motivated teachers to change their pedagogical approach to broaden the sources and utilization from the narrow confines of the school laboratories and textbooks to real-world manifestations of the curriculum.

   **Henry:** One thing I discovered from the experiences that the students had, was that they came and appreciated the Chemistry, mathematics, biology and physics we are learning, and I think you heard them talk about the charcoal stoves and their shapes. They could not understand the reactions that take place when we burn carbon or the difference between the conical and cylindrical shaped charcoal stoves. But using those stoves, they could see in the stages where the reactions take place and why the shape matters, and I realized that whatever we do in class, they could see it outside. So, that one was just too good. I felt good as a teacher.

5. *Change in attitudes about mathematics/science and student engagement:* Teachers reported that the Jua Kali experiences have a high level of utility for student learning. In addition, teachers assert that the Jua Kali experiences transformed students' attitudes toward science and application of mathematical principles in product design and improved their academic performance. Consequently, we saw teachers' attitudes toward Jua Kali changed to see it as a valuable resource center to the school and to student learning. In addition, one might typically expect previously poor performance to equate to negative attitude about the mathematics and science subjects. However, teachers reported that student involvement and engagement increased student contentment with the subject content learned. As such, the students' change of attitude toward mathematics and science contributed to the breadth of teachers' sources of mathematics/science teaching resources in the immediate neighborhoods of the school:

   **Agnes:** Whenever something about Jua Kali comes up during discussion my students become very engaged and you could see them using all the mathematics and science knowledge they have learned to try to contribute to the discussion. To me actually the Jua Kali has become a resource center to our school. It was so near yet we did not know that we could learn something from there. We would think of academic trips very far yet we had something very near, which is very resourceful to us. It actually has some impact in changing students' attitudes towards mathematics and the sciences and even improving their academic performance in these subjects and we are very grateful.

## Conclusions

The study's analysis indicates that teacher observations of student empowerment through the way they demonstrated ownership of their own learning were critical to transformations in the mathematics/science teachers' teaching. Further, teachers' experience of the effectiveness of heterogeneous ability groups, manifested co-learning, working together, apparent positive change in academic performance, making connections between classroom and out-of-school mathematics and science, and success of intra-class heterogeneous grouping witnessed at Jua Kali influenced the teachers' mathematics and science teaching. Moreover, as reported elsewhere in Anderson et al. (2015), the teachers were emancipated from seeing the classroom as the only source of knowledge and pedagogical practice to include real-world environments as sources. As well, they appeared emancipated from being disseminators and oracles of content knowledge to a teacher-student collaboration. In this way, they relinquished control manifest in solitary teaching to collaborative team teaching. Further, the teachers felt emancipated from syllabus controlled teaching to student-driven teaching. Thus, they changed from old ways of teacher-centeredness about mathematics/science content knowledge and pedagogy to continued learning in response to active emancipated learning.

The study's findings have potential to influence how the teachers in Kenya and elsewhere to design effective, inclusive learning experiences for contemporary primary, secondary, and post-secondary classrooms (Anderson et al., 2015). The study reveals and models plausible instructional strategies that can assist teachers in Kenya and elsewhere to bridge the divide between classroom knowledge and local settings that have the strong potential to assist students to develop more useful and relevant modes of learning mathematics/science. Moreover, the study has provided new insights about teacher change agents beyond the traditional "Beings" by extending agency to non-beings such as changes in student learning. A key recommendation is for teachers to pay attention to what sparks or inspires students to engage actively with learning and in these moments surrender their control of when, where, and how students learn to the students themselves. Based on the results of this study, yielding such control results in a more powerful and rewarding experience for all. Similar studies could be modeled elsewhere to capture the mathematical and scientific phenomena embedded in students' local contexts where they can experience and appreciate the relevance of what they learn. In this way further studies could be framed to investigate the extent to which the students take responsibility for their own learning as teachers surrender their often tightly held loci of control and become co-learners as well as co-teachers of mathematics and science in their classrooms. The study's results demonstrate the power of student learning at influencing teacher transformation. Moreover, specific features of student learning have the capacity to influence teacher change. Modeling these features by creating opportunities where they can be demonstrated is the hallmark of creating conditions supportive of responsive teacher change.

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# Chapter 18
# Mathematics Through Play: Reflection on Teacher Narratives

**Willy Mwakapenda and Williams Ndlovu**

## Introduction

Issues in mathematics teaching and learning, and debates related to mathematics as a field, have been broad and diverse. Globally, these issues and debates have placed at their centre the need to enable learners to access mathematics. It is being recognised that one cannot succeed in mathematics, and indeed in any field of endeavour, if one does not have access to that field. Therefore, while success appears to be the underlying driver behind the issues and debates in mathematics education and research, it is the notion of access that is clearly key. The centrality of the notion of access became a theme, for the first time since its inception, of the International Group for the Psychology of Mathematics Education (PME) at its 43rd Annual Meeting in 2019. As stated in the introduction to the conference:

> The theme of the conference is *Improving access to the power of mathematics*. Since this is only the second time the conference will be hosted on the African continent, we would like to give the conference a strong African focus – focusing on access, which is very relevant in South Africa as well as in the rest of Africa. However, we would also like to focus on the power of mathematics, thereby giving the conference a strong mathematics flavour. It is hoped that the deliberations at this meeting will affect the course of future mathematics education on all levels worldwide and that this conference will greatly contribute toward closer collaboration between African countries in future (PME43 Conference programme, Graven et al., 2019, p. 1)

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Deliberations at the meeting were diverse, collaborative and all broadly connected to access. They focused on the following themes, in the keynote addresses: (1) mathematics and mathematical literacy, (2) transition zones in mathematics education research for the development of language as resource, (3) representational coherence in instruction as a means of enhancing students' access to mathematics and (4) institutional norms: the assumed, the actual and the possible. The key message from the keynote addresses is that there are foundational structures and norms that define what a classroom looks like. There is therefore a need to 'tear down these norms' (Liljedahl, 2019, p. 1-1) in order to create new possibilities in student behaviours in mathematics classrooms. Key to these norms not only involves the way mathematics is taught but, more critically, how mathematics is conceptualised and what contexts are made available in order to promote new and different interactions among students and teachers. A way of getting to 'tear down these norms' involves revisiting our understanding of the nature of mathematics and the contexts in which it needs to be introduced and experienced by learners and teachers.

In attempts to enable learners to gain access to mathematics, formal and more traditional approaches are still more prevalent. As our topical review of the 2019 PME conference proceedings shows, very few approaches accommodate the more aesthetic dimensions of mathematics that have been widely reported to be effective in earlier years of mathematics teaching. The aesthetic dimension of mathematics, especially play, has become an implicit and often long-forgotten ingredient of high school mathematics instruction. During the 2019 PME conference, very few presentations directly addressed the aesthetic dimensions of mathematics (see, e.g. Antonini, 2019; Bruns & Gasteiger, 2019; Edwards, 2019; Krausel & Salle, 2019; Nakawa et al., 2019). In this chapter, we discuss the performative nature of mathematics, its play and playfulness-related nature. We locate our discussion within a specific play-related activity of mathematics which involved a cohort of high school mathematics teachers. We frame our discussion by using teachers' conceptions of mathematics, after participating in the activity that involved play.

In the next sections, we make some observations about mathematics and the concept of play. Our observations are intended to respond to the question: To what extent should play need to form part of school mathematical activity?

## What Is Play?

Our review of the concept of play has indicated that an understanding of play needs to be considered from several dimensions: cultural, creative, imaginational and developmental. The grounding research work of Bert Van Oers (1998) has been widely acknowledged in the literature related to the interconnection between mathematics and play and is therefore reviewed more closely in this chapter.

## *Play and Culture*

Van Oers and Duijkers (2013) have described play as 'a specific format of cultural activities' and that 'all cultural activities can be formatted in different modes, organizing the actions of the participants in characteristic ways' (p. 515). They also argue that 'learning and play cannot be separated' since cultural activities embed the function of learning. According to Van Oers and Duijkers (2013), 'changes in the playing actions or in the content of the activity of play are demonstrations of learning' (p. 515). Play starts from what is culturally given and what individuals are familiar with. In other words, and according to Van Oers and Duijkers (2013, p. 190):

> … culturally given activities (practices) are made playful in the child's role play. This is, by the way, consistent with the cultural-historical analysis of the relation between culture and play by the famous Dutch cultural-historian Huizinga (1938/1951): it is not play that is the origin of culture, but cultural practices, playfully accomplished, should be seen as the origins of cultural developments.

Play is birthed from activities that are cultural entities. Play is therefore 'essentially a cultural phenomenon dependent on cultural decisions that allow … playfulness in an activity' (Van Oers & Duijkers, 2013, p. 190). Decisions made in play are cultural. They are not self-evident but depend on cultural values and images and relevant norms of society. According to Van Oers and Duijkers (2013, p. 191), play is organised by the following rules (p. 191): social rules (indicating how to interact with each other, with what resources and under what conditions), technical rules, conceptual rules and strategic rules. From this cultural perspective, Van Oers (2008) has indicated that 'play is most certainly the basis for all human culture' and that 'institutions such as the law, constitutions, art, crafts, science, sport, and trade are basically rooted in human playful activities' (Huizinga, 1938, in Van Oers, 2008, p. 192). Van Oers, quoting Huizinga (1938), emphasises that human civilization 'emerges and develops in play, as play'. Also, 'play has a culture-creating function'. This is because:

> In all play activities, "new or modified meanings are attributed to familiar objects and actions, which subsequently become the basis for new objects and actions". (p. 370)

## *Play, Creativity and Imagination*

According to Van Oers and Duijkers (2013), play is also a creative act. This is because individuals 'construct novelty within the constraints and provisions of the situation' (p. 516). Play, conceived from an activity theory perspective, is 'a specific mode of activity defined by a format that includes three basic parameters (rules, degrees of freedom and involvement)' (Van Oers & Duijkers 2013, p. 185). Oers and Duijkers conceive play as 'a form of human activity' whose basic characteristics can be recognised as human behaviour that is repetitive, flexible, affect, non-literal, preferential and pleasurable. These characteristics are essential in creative acts.

A key characteristic of play is that it involves imagination. In play, 'people can free themselves … from conventional meanings, and … are allowed to explore alternatives' (Van Oers, 2008, p. 370). According to Vygotsky (1978), imagination emerges in the context of the activity of play involving the creation of an imaginary situation. The activities of human consciousness and meaning-making capacity are inextricably connected to the development of imagination. According to Van Oers (2005), imagination is defined as the constructive process that 'creates new configurations out of known elements of thought' (p. 5). It is noted that because 'products of imagination can oppose ordinary reality', they are highly useful since they form a strong 'basis for critical thinking that confronts everyday reality with new meanings' (p. 5). Imagination therefore has a creative and transformative power. According to Van Oers (2005):

> When people think of imagination as the act of making images, they are basically talking about a process of constructing mental forms that represent some object, and that presumably could be dealt with as if they were that object. Imagination is not creation out of the blue, but it is based on reconstructions with well-known objects within a familiar activity context: by combining old, conventional objects or ideas into a new construct the [individual] creates new things … Imagination indeed produces new objects (new means, actions, or subthemes). (p. 6)

There is therefore a close relationship between imagination and play: both of these are concerned with the processes of abstraction and divergent thinking intended at producing new objects.

A critical component of imagination involves substitution. In 'object-substitution', a person 'takes an object and treats it as if it is something else' (Van Oers, 2005, p. 6). As an illustration of object substitution, Van Oers (2005) gives the example of a child who takes a stick as a horse. In this case, the child 'does not see a real horse in the stick, but it performs some of the actions that relate to a horse'. Quoting Bodrova and Leong (1996), Van Oers (2005) indicates that:

> for the process of imagination, object-substitution in an activity context is an essential element … The object to be substituted should not necessarily be replaced by another physical object; the substitutes can be symbols, drawings, pictures, mental images, narratives, schemes, models, and so on. (p. 7)

Van Oers (2005) indicates that the 'symbolic character of these substitutions supports the formation of imaginations in children's play activities and—by the same token—provides the psychological means for surpassing the limitations and determinants of the actual setting'. In his emphasis of the substitution act as being 'fundamental for every act of imagination', Van Oers notes that 'by substituting an object for something else, the original object actually gets a new image that can be dealt with instead of the original object' (p. 7). The key aspect of this process of substituting is that it results in the generation of new and unexpected meanings related to common objects. Substitution creates spaces for an individual to go beyond the information given. As Kohl (1994, in Van Oers, 2005, p. 16) argues, 'it is this ability to imagine the world as other than it is, that leads to hope and the belief that even the most oppressive and difficult of conditions are not absolute'.

### *Play and Development*

The significance of play can also be considered from a developmental perspective. Play is a major source of development. It is noted that 'a child's greatest achievements are possible in play' and that these achievements 'become her basic level of real action and morality' (Vygotsky, 1978, in Van Oers, 2008, p. 371). Meaning is also central in play. Meaning (contextualised in an imaginary situation) regulates actions of the individual. 'These actions produce new objects that may be a starting point for new explorations of meanings and the production of new tools' (Van Oers, 2008, p. 371). According to Van Oers, contradictions arise between the meanings in play and the real actions. However, these contradictions are essential because 'contradictions between the meanings in play (emerging from the imaginary situation) and the real actions are the main basis for children's development' (see Vygotsky, 1978, p. 101).

## Connections Between Mathematics and Play

Researchers such as Kuschner (2012) have highlighted that integrating play into the school curriculum is not an easy task. This is because, according to Kuschner, 'play is natural to childhood but school is not' (p. 242). From this perspective, one needs to conceive play as a context that can be recontextualised for school practice. As Van Oers (2013) has noted, there is a value in conceiving play 'as a context for young children's learning' (p. 186). This recontextualisation (through play) can take two forms: horizontal and vertical. Horizontal recontextualisation arises 'when a new situation is recognized as an opportunity for an alternative realization of a well-known activity' (Van Oers, 1998, p. 138). An example of horizontal recontextualisation involves a discovery that one can apply the mathematical notion of area to irregular geometrical forms. On the other hand, vertical recontextualisation is a process of progressive continuous contextualising in which:

> New problems in an activity may arise and become new pivots of action patterns. These action patterns often lead to the invention of new goals, new means for action, and new strategies. These new action patterns develop into new activities and new contexts for acting that, although emerging from a well-known activity, are not directly a new, alternative realization of that activity. (pp. 138–139)

As an example, Van Oers (1998) cites a study that involved a shoe-store play activity in which actions of measuring emerged. During the shoe-store play activity:

> Measuring became a separate activity … including forms of measuring and conversations about measuring that the children never could have heard in a real shoe-store. Measuring as a new activity gradually emerged out of the shoe-store play activity, leading to a new, even more 'abstract' activity and context of acting. (p. 138)

Researchers such as Dowling (1996) have described the practice of using contexts in mathematics as 'recontextualised practice'. Dowling, however, notes that this 'recontextualized practice relating to one activity cannot retain its structure under the gaze of another activity' (p. 410). The esoteric domain of mathematics usually proves difficult to be accessed by many learners because of its highly technical and complex nature and its processes. Textbook authors and teachers then design non-mathematical settings through which the non-arbitrary content can be realised. The arbitrary settings can be transformed and recontextualised to conform to the particular non-arbitrary mathematical content. The aim of this recontextualisation is to enable learners to access the subject matter knowledge of mathematics.

It needs to be noted that aspects of play in mathematics have been widely recognised and utilised in areas of mathematics involving younger learners as they interact with mathematics in primary and elementary mathematics classrooms. Very little activity and discussion of mathematics linked to play has been at the centre of interactions at higher levels of mathematics teaching and learning.

In the next section, we present an activity, a Crossing the River Activity, as an example of play activities that are possible at higher levels of mathematics learning. The participants in the Crossing the River Activity were in-service mathematics teachers engaged in an Education Honours programme at a South African tertiary institution. Thirty-five (35) participants were involved in the activity. The teachers were from schools from different provinces in South Africa and had differing levels of experience in teaching secondary school mathematics.

## An Activity of Mathematics Involving Play: Crossing the River Task Activity

The following activity was given to the teachers:

> Two adults and two children come to a river on their way to a family wedding. The children find a small canoe/boat on the river-bank. They discover that the boat will hold one or two children, but only one adult and no children. How must they cross the river? Mr. Shinghaih, a businessman, the owner of the boat, charges R50.00 per trip for making use of the canoe/boat. How much will the group of 8 adults and two children pay?

As can be seen, at face value, this activity seems to have no obvious link to traditional school mathematics. As such, it has the potential to enable participation of all. Coming to a river on the way to a wedding should be fun. Children finding a small boat should also be fun and demonstrates curiosity that is inherent in children! The constraints on the use of the boat make crossing the river appeal to problem-solving abilities of children! However, having to pay for unnecessary trips should not be much fun; it can be expensive, especially if the family is from a poor background! In any case, they would need to reserve as much money for upcoming transactions at the wedding!

Central to this activity is imagination. This can be seen in the following excerpts arising from the dialogue among the teachers while engaged in the activity.

1 T1: Who is going to be a mother and father?

2 T5: You are going to be …

3 T3: Anyone can be a parent here … [laughter …]

4 T2: A girl cannot be a father or a boy a mom.

5 T7: For us to follow and understand eeesh! One lady here will be a mother, … I volunteer to be a father in this activity.

6 T10: Then I will be the mom in the activity

7 T9: Lets be serious and think about the problem mathematically. … not wasting time.

In mathematics anyone can represent anything like x and y variables. In drama a boy can act as girl but does not mean it's impossible. …

8 T4: We need to keep record of our steps so that we can check if there is any mistakes.

9 T9: Since we start with a family of four people, dad, mom, daughter and son, … Thandile and James[not real names] are the children. [The class claps hands agreeing.]

10 T8: But I think, … we need to read the instructions and questions carefully. Why are they giving us the mass of each member in this house. …?

11 T5: I am lost. The problem is talking about money, weight of people, a family of four people, someone is a businessman, eeish! Yaaah ne.

12 T6: Aaah!!, I see now, the boat as well can only carry so much. … Becoz, if too much it will definitely sink and people will die. …

13 T9: That's why we can't be wasting time, we all are in it and must think and work as one. Let's allow the boy to go first and …

14 T2: No no no! it will be expensive, … and who will bring back the boat?

15 T9: Oooh! I see why T8 was talking about weights of each person.

16 T1: Lets make dad and mom go first. Aah no, but we can try first.

17 T3: From the conditions, a dad and mom cannot go, you see dad weighs 70 kilogram and mom is 78 kilogram. If we add 70 and 78 we get what? … it's 148 kg. This will be too much for the boat. The owner needs R15 for one-way trip.

….

The group members agree to now physically attempt to demonstrate their strategies. They agreed to draw a line on the floor with a white chalk to represent a river and used a loose desktop plank as a boat, where two members (as agreed in their earlier conversations) will hold the boat side by side as they move past the line marking. All of these actions involved imagination, improvisation and creativity. This kind of dialogue is rare in typical mathematics lessons:

26 T7: Recalls, we are starting with children only right? [the members echoed yaaah]

27 T8: Boy and girl can now go, … and stop there for now …, that's one trip you see. Now let's choose who should bring back the boat ne, that's second trip.

28 T2: I think it does not matter who brings back the boat between the children. … but if a child and parent go together, yes it does. So a boy can bring back the boat.

29 T9: Are you recording that in a table?

30 T4 & T5: Yes yes yes.

31 T8: Lets see the table … No this is not a table. Make it like table of values in graphs of functions in algebra.

32 T9: We can do that afterwards when putting together like presenting our findings.

33 T8: Ooh!! I see.
34 T3: Let us continue the activity …
35 T1: I will now go with mom.
36 T6: That's a third trip [the members agree …]
37 T7: Its becoming tricky now, as you can see Thandile and Mom cannot go back.

## A Formal Solution to the Activity

In describing a formal solution to the activity, one could denote the movements as follows: Since there are two children and two parents, the letter C could indicate child, P as parent, F as going forward and B as going backward. The first trip would consist of both children going forward to the other side of the river (i.e. 2CF). In the second trip, one child would return (go back) with the boat to the original side (i.e. 1CB), and then one parent would get into the boat to get to the other side (i.e. 1PF). Proceeding in this way, we have a representation of the trips from one side of the river to the other side: 2CF, 1CB, 1PF, 1CB, 2CF, 1CB, 1PF, 1CB and 2CF. This gives a total of nine trips when we have two children and two parents/adults. [Therefore the family would pay 9 × R50 = R450 for the use of the boat.] When an uncle joins the family, how many trips would be required? Upon observation, one finds that we need an extra four trips. This means there would be 13 trips when there are three adults and two children. Proceeding in this way, and when we increase the number of adults, we come up with a pattern that can be summarised in Table 18.1.

It can be seen in Table 18.1 that since there is a common difference of four trips as we move from one column to the next, we obtain the following as a general pattern (formula) for describing the number of trips ($T$) when we change the number of adults but keep the number of children to two. We obtain the formula $T = 4n + 1$ (where $n$ is the number of adults (parents)).

As observed from the dialogue, as an illustration of modelling, participants pretended to be adults and children. They had to cross a river using a small boat that could hold either one or two children or one adult only. After working out how to do it, they discovered that the number of crossings required was four times the number of adults, plus one, giving $Tn = 4n + 1$ as a generalisation that represents the mathematical essence of the activity.

**Table 18.1** Total number of trips ($T$) when no. of adults increases but no. of children is constant

| Number of parents ($n$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Number of children[a] | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Number of trips ($T$) | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 | 37 |

[a]We keep the number of children constant. This is a critical principle of mathematical thinking and problem-solving

## Teachers' Comments Linked to the Crossing the River Activity

We now present voices from what the mathematics teachers said after taking part in the performance of the activity. After the performance, the teachers were asked to respond to a questionnaire that had five close-ended and open-ended questions. The instructions given to the teachers were as follows:

> You are required to reflect on the Crossing-the-River activity and write detailed comments based on what you observed and experienced in today's maths class. Write down your reflections using the following leads:
>
> (i) According to my observations from the Crossing-the-River activity, mathematics is …
>
> (ii) According to my observations from the Crossing-the-River activity, mathematics is about …
>
> (iii) I learned the following mathematics ideas from my participation in the Crossing-the-River activity, …
>
> (iv) I liked the Crossing-the-River activity because …
>
> (v) My other comments on the Crossing-the-River activity are as follows …

### *Comments Linked to the Nature of Mathematics*

Key comments from the teachers involved perceptions of mathematics that considered mathematics as a subject that 'does not come from another planet' and that mathematics can be role-played or demonstrated through everyday life situations. Comments from teacher T1 below illustrate this observation:

> I have learnt that mathematics problems are interrelated with our daily lives so, in order to solve problems, we must demonstrate the problems or give the problems live; we can do this by either <u>role playing</u> or demonstrations. And also I learnt that when it's easier not to rush to formulas right away but to solve the problems manually where applicable, because by doing this you will understand how the formula came about or how it was derived … Mathematics is not abstract or from another planet but this activity showed me that things that we do in our daily lives have mathematics. Again it showed me that most of <u>mathematics problems can be demonstrated or role-played</u> in order to make someone understand better.

This activity made T1 to see mathematics in a different way from before. He noted:

> My mind has shifted from thinking that mathematics is about equations and formulas; rather it showed me that mathematics is interrelated with our daily lives … I think as teachers we also should stop teaching mathematics as an abstract subject but try to make demonstrations and role plays that learners would understand. And one thing for sure, learners enjoy a subject when they realise that they interrelate with it. I think this will make learners enjoy the subject compared to before. We should make examples that they see in their daily lives. This kind of activities they make you think, they make you participate and I think if you took part in something you will never forget it in your life. I now know that <u>when I see a mathematics problem I should give it life by demonstrations or role-plays</u>.

The above comments seem to indicate that, for mathematics to be enjoyable and readily relevant to learners, it needs to be rooted and find its application in modes of engagement that incorporate the aesthetic and artistic forms of thinking and acting that are embedded and promoted through play.

Teacher T2's comments highlighted the communicative, conversational and observational aspects of mathematics. She noted:

> Mathematics is a subject that **communicates** with individuals based on practical observations. It is the subject that requires full attention of the student and gives more of the attention to what is being observed. Mathematics in this activity is an eye-opening and mind-opening subject that enables one to come up with their own ideas and extend further in visualising from observation so that one may not forget the next time they solve problems. Mathematics is about paying attention and giving full focus on scenarios, and also making notes and in every observation. It is about understanding **what is needed and what is not needed** or allowed in the process of solving problems. It is also about understanding patterns.

## *Comments Linked to 'Performing' Mathematics*

Teacher T3 now viewed mathematics as a subject that needs to be performed. One needs to perform mathematics, rather than stick to the textbook:

> I learned that you can teach maths without using any tool such as a textbook because maths is a practical-based learning area and not a theoretical one. It is very interesting and more enjoyable. I have also learned that as a mathematics teacher you can use your learners as teaching aids in class in order to <u>perform practical examples</u> so that your learners can understand your lesson far more better and enjoy it. I liked the crossing river activity because it has taught me a lesson that as a mathematics teacher you need to make the subject to be more interesting by ensuring that you <u>don't always stick to the textbook</u> whenever you are teaching the kids, but <u>perform some of the activities practically</u> so that the learners can see the nature and the importance of mathematics.

## *Comments Linked to Culture and Everyday Experiences*

Teachers T3 and T4 made comments that highlighted cultural aspects involved in a mathematical activity:

> T3: Mathematics is <u>part of nature</u>. In this activity there was a family that needed to cross the river so that they can attend a wedding. They were using a boat to get to the other side of the river … The number of adults kept on increasing until we got a linear number pattern.
>
> Mathematics is formulated from what is really happening in our life. From this activity I think <u>people who invented mathematics observed what is happening around us</u>, then came up with possible solutions for the problems that human beings may come across. I am saying this because in this activity <u>what the family went through in order for them to reach their destination might happen to a real family who live in rural places where there are rivers</u>.
>
> T4: Mathematics is doable. It is difficult yet doable. From the river crossing activity I realised that mathematics is <u>what we do every day</u>, the way [we] think and the decisions we

have to make. Mathematics is about dealing with the logic of numbers, shape, quantity and arrangement. Maths is all around us, in everything we do. It is the building block of everything in our daily lives, including art, as well as the arrangement of objects.

## Comments Linked to Pedagogy

T5: It was interesting, enjoyable and a little bit challenging. At first it was challenging to figure out the solution on my own, but after the first trip I was able to see what was going on. The way it was enjoyable, I couldn't stop to add more adult relatives in order to see how many trip I would have. It also made me see that every problem that one may come across have solutions … learning is more interesting when you learn looking/using what is happening around you … one figure[s] out solutions more wisely when he put himself in the situation.

T6: You can teach maths without using any tool such as a textbook because maths is a practical-based learning area and not a theoretical one. It is very interesting and more enjoyable. I have also learned that as a mathematics teacher you can use your learners as teaching aids in class in order to perform practical examples so that your learners can understand your lesson far more better and enjoy it. I liked the crossing river activity because it has taught me a lesson that as a mathematics teacher you need to make the subject to be more interesting by ensuring that you don't always stick to the textbook whenever you are teaching the kids, but perform some of the activities practically so that the learners can see the nature and the importance of mathematics.

## Comments Linked to 'Creative and Critical Thinking'

T7: The activity involves creativity and imagination to discover the solutions. When crossing the river the family had to consider money they were supposed to pay. Therefore creativity was also needed.

T8: I liked the cross river activity because help me acquire problem-solving skills, to be a critical thinker or think out of the situation. It also help to understand maths in real life situations, reminds me why I am doing mathematics as how to teach it such as creating scenarios in class that can help learners to understand mathematics better.

It is important to think out of the box as to understand certain principles in order to solve a problem. The Crossing the River activity was reminding us the qualities that are nurtured by mathematics which are power of reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability and effective communication skills.

## Comments Linked to Change of Mindsets

T9: Most learners will realise that mathematics is not a monster that most of them think it is. Through this activity my reflection in mathematics is that, mathematics is practical and relevant to all our daily activities, we just need to realise it and embrace this great subject called mathematics. We eat maths, walk maths and talk maths.

> T10: Mathematics is a subject derived from a common activity. It is concepts that can be derived from any activity that forms a pattern, from that activity of Crossing the River, a pattern was formed while it was just a common thing that can involve one capability of thinking. Taking down all results and observations made it easy for us to come up with a sequence or pattern for any given number of kids or adults. So I viewed mathematics different from that activity, it makes me to note everything that I do every day, something that is a routine to think a mathematical concept for it and come up with something or pattern for it. It also opened my mind business-wise, that when I see someone doing something over and over again, what can I do to actually improve the situation. So the Crossing over River activity was a mind-blowing for me.
>
> The activity also made me to actually be conscious about the results in any activity you are doing, because in that activity you were supposed to always have a child moving with the boat. I mean the restrictions were in such a way that you think beyond the current trip in order to sustain them.

The voices of these teachers highlight the importance of 'bringing learners to mathematics'. We claim that the underperformance that often characterises the problematic classroom experiences of many learners arises from pedagogies that strive to 'bring mathematics to learners'. Such pedagogies pay little recognition to the need to bring learners to mathematics. To bring learners to mathematics means that we need to know not only what mathematics is and is about, but also who the people are. We argue that play is the stage that levels the playing field for learners and teachers of mathematics. Play is the stage that unveils the inner stage that bonds mathematics and learners. Through play, learners become engaged and sufficiently glued to that stage.

## Reflections on Teacher's Comments

At the centre of this Crossing the River play activity was imagination. The activity involved imagining a family of four going to attend a wedding, possibly on a Saturday morning, after a rainy night. They possibly needed to cross a flooded river that connects their home to a neighbouring village. The activity also involved imagining having to pay for using a boat to cross the river. This imagination, though a product of the mind, has a strong appeal to the everyday world and contexts in which the teachers who participated in this study are located. Being adults themselves, the teachers are more likely to have had the experience of attending an actual wedding (possibly of their own!). They are likely to appreciate the significance of this story activity as a context for everyday living. However, there is an immediate gap that is inherent in this story context. Although the story immediately appeals to the every day, it does not readily connect itself to any aspect of formal mathematics as we may know it. It is this gap that is critical in this chapter.

In our opinion, a gap like this one is critically and pedagogically necessary for mathematics teaching and learning. In the context of this chapter, we propose that play fills the gap between the everyday and the mathematical worlds. Play plugs in the gap! We argue here, in agreement with Popkewitz (1988), that there is a gap between school mathematics and mathematics. He observes that 'School Mathematics is shaped and fashioned by social and historical conditions that have little to do with the meaning of mathematics as a discipline of knowledge' (p. 221). School mathematics is a recontextualised practice. It is a recontextualisation of mathematics. There is a gap, a space, between school mathematics and mathematics, between the everyday world and the world of mathematics (the esoteric domain of mathematics). We are arguing here that play or playfulness in the mathematics classroom makes visible the aesthetics of the gap or spaces between school mathematics and esoteric mathematics. In other words, play creates aesthetic spaces between the everyday world and the mathematical world.

In this chapter, we argue that there are gaps that are always present in any mathematical experience or activity in a classroom or curriculum. As long as mathematics teaching is intended to move students from one state of knowing or experience to a different or possibly 'better' state of functioning in the mathematical world, there will always be gaps to contend with. There will be gaps involving several aspects such as gaps between routine and non-routine procedures in mathematical problem-solving, gaps between the so-called 'easy' and 'difficult' content, gaps between the concrete and abstract, gaps between method and content, gaps between known and unknown and gaps between one mathematics content area and another content area. Considered in this way, we argue that mathematics and mathematics education generally are practices that are 'pregnant' with gaps and that play gives birth and creates aesthetic spaces from these gaps. The aesthetic spaces so created, and as can be seen from the data in this chapter, have a strong potential to 'fill' the gap between the everyday and the mathematical world that we are intending to get students closer to as they attend mathematics lessons. Teaching mathematics should therefore be more concerned with recognising and bringing into lessons the aesthetic spaces and experiences that are relevant to the lesson topic. Learning mathematics should in turn be about productively responding to the aesthetic spaces or experiences that are made available. Where relevant aesthetic experiences may be unavailable or non-existent, the teacher and his/her students need to recognise that they have a collective responsibility to search for or create the aesthetic spaces that may be critical for the mathematical experiences that are intended to be gained.

## Conclusion: Mathematics and the Aesthetic

The Crossing the River Activity reflected on here is an attempt to align school mathematics activity with aesthetic approaches currently being proposed in mathematics education reforms. A key aspect of the activity analysed in this chapter involves foundational elements of play, creativity and imagination that have been advocated particularly in the seminal work of von Oers. The Crossing the River Activity is an illustration of the creative and transformative power of play when linked to key mathematics concepts and learning:

> Imagination is not creation out of the blue, but it is based on reconstructions with well-known objects within a familiar activity context: by combining old, conventional objects or ideas into a new construct the [individual] creates new things … Imagination indeed produces new objects (new means, actions, or subthemes). (Van Oers, 2005, p. 6)

We have illustrated in this chapter the imaginative and playful aspects of mathematics that lead to learning, recontextualising and development of new ideas. The recontextualisation aspects involved in play not only lead to new ideas and meanings but also give rise a kind of mathematics described by Gadanidis et al. (2016, p. 225) as:

> Mathematics worthy of attention, worthy of conversation, worthy of children's incredible minds, which thirst for knowledge and for opportunities to explore, question, flex their imagination, discover, discuss and share their learning.

There is a notable thirst for aesthetic mathematics experiences that need quenching, arising from a recognition that school mathematics 'commonly lacks an aesthetic quality' (p. 226). Learners have 'never experienced the aesthetic quality of quenching a thirst for mathematics'. Quoting Papert (1978), Gadanidis et al. (2016) argue that 'common views of mathematics exaggerate its logical face and devalue all connection with everything else in human experience', thus missing 'mathematical pleasure and beauty' (p. 226). In agreement with Brown et al. (1989), Gadanidis et al. also argue that 'many of the activities students undertake are simply not the activities of practitioners and would not make sense or be endorsed by the cultures to which they are attributed' (p. 34). In this connection, Root-Bernstein (1996) has noted that 'students rarely, if ever, are given any notion whatever of the aesthetic dimension or multiplicity of imagining possibilities of the sciences' (p. 62). Our reflections of the Crossing the River Activity illustrates the aesthetic experiences that result from involving learners in play. In their research, Gadanidis et al. (2016) used a number of empirical examples to suggest that, in order to enhance the aesthetic quality of (school) mathematics, educators need to learn from the arts and from artists. In doing so, they first recognise the need to distinguish between mathematics and school mathematics and express a need to be 'cautious about suggesting that mathematics or the work of mathematicians might be devoid of "art"' (p. 227). According to Root-Bernstein (1996), the sciences (mathematics) and the arts are 'very similar, certainly complimentary, and sometimes even overlapping ways of understanding the world' (p. 49). Gadanidis et al. (2016) have argued in agreement with Root-Bernstein and have identified that the problem may not be with mathematics, but with school mathematics:

> The problem with school mathematics is not that it lacks the arts, but rather that it lacks the aesthetic that is common to mathematics, the arts, and other disciplines: the aesthetic that makes the experience of these disciplines human. (p. 227)

In fact, Popkewitz (1988) has argued that there are limitations with school mathematics. He observes that 'school Mathematics is shaped and fashioned by social and historical conditions that have little to do with the meaning of mathematics as a discipline of knowledge' (p. 221). Gadanidis et al. (2016) have proposed a model for designing aesthetic mathematics experiences that draw on 'important connections among elements of narrative (what makes for a good story) and mathematics (what makes for a good math experience)' (p. 228). The activities mentioned in Gadanidis et al. (2016) present opportunities for students to experience mathematics worthy of 'attention' and 'conversation' and present opportunities to 'explore, question, flex their imagination, discover, discuss and share their learning' (p. 238).

We propose that the Crossing the River Activity that we have reflected upon in this chapter represents an attempt towards capturing the story-based nature (Gadanidis & Hoogland, 2003) of human cognition that needs to form part of mainstream school mathematics education. The comments from the teachers indicate the aesthetic spaces that the activity attempted to create in order to enable them to access the mathematical concepts intended to be developed through the activity. The positive experiences that this activity generated among teachers suggest that activities such as these need not only form an integral part of the mainstream higher level of school mathematics but also part of mathematics teacher professional development. This chapter demonstrates that play, and mathematics through play, creates an embodied stage for simultaneously showcasing the nature of mathematics and enabling access to mathematics.

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