Solid Semantics for Abstract Argumentation Frameworks and the Preservation of Solid Semantic Properties

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Abstract. In this paper, we propose solid admissibility that is a strengthened version of Dung's admissibility to obtain the most acceptable set of arguments. Besides, other solid extensions based on solid admissibility are defined. Such extensions not only include all defenders of its elements but also exclude all arguments indirectly attacked and indirectly defended by some argument(s). Furthermore, we characterize solid extensions with propositional formulas. Using these formulas, we aggregate solid extensions by using approaches from judgment aggregation. Especially, although no quota rule preserves Dung's admissibility for any argumentation framework, we show that there exist quota rules that preserve solid admissibility for any argumentation framework.

Keywords: Abstract argumentation · Solid semantics · Social choice theory

1 Introduction

In Dung's work [12], an argumentation framework (AF) is a directed graph, where nodes represent arguments and edges represent elements of a binary relation. It has been studied widely over the last decades. One of the core notions of AFs is *admissibility*. An *admissible extension* is a set of arguments that contains no internal conflict and defends its elements against any attacker.

In this paper, we mainly focus on obtaining the most acceptable arguments in AFs by strengthening Dung's admissibility. Before discussing this idea, we first illustrate two problems (or drawbacks) observed from the literature. The first one is observed from *graded acceptability* [14] which provides an approach to rank arguments from the most acceptable to the weakest one(s) by parameterizing the numbers of attackers and counter-attackers. Hence, we can require that a set of arguments is graded-acceptable if it contains at least n counter-attackers for each attacker of its elements. Graded acceptability is flexible as we can tune

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the parameter n. What if we want to find out in an AF the sets of arguments such that they exactly contain all counter-attackers for each attacker of their elements? It is impossible to achieve this goal by tuning the parameter n, as different attackers may have different numbers of counter-attackers. Consider the following example.

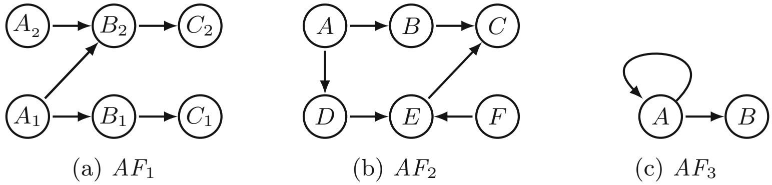


Fig. 1. Three problematic argumentation frameworks

Example 1. In Fig. 1a, $\{A_1, C_1\}, \{A_2, C_2\}$ and $\{A_1, A_2, C_2\}$ are acceptable under graded semantics when we require that a set of arguments is acceptable if it contains at least one counter-attacker for each attacker of its elements. We should notice that although $\{A_1, A_2, C_2\}$ contains all counter-attackers when its elements are attacked, $\{A_2, C_2\}$ fails to contain all counter-attackers whenever its elements are attacked. When the requirement is more demanding in the sense that a set of arguments is acceptable if it contains at least two counter-attackers for each attacker of its elements, $\{A_1, A_2, C_2\}$ is still acceptable under this requirement. Although $\{A_1, C_1\}$ contains all counter-attackers when its elements are attacked, it fails to satisfy this requirement.

The other problem is observed from the situation where some argument indirectly attacks and indirectly defends some argument. There are other semantics [1,7] that also provide approaches to rank arguments. But their approaches rely on conjectures regarding the processing of cycles. However, Dung indicates in [12] that the presence of cycles could be a problem. In this work, an argument A is *controversial* w.r.t. an argument B if A indirectly attacks and indirectly defends B. Such arguments could lead to problematic situations. Consider the following example.

Example 2. In Fig. 1b, A is controversial w.r.t. C as A indirectly attacks and indirectly defends C. From a skeptical view, A and C should not occur in the same set of acceptable arguments. But $\{A, C, F\}$ is admissible in Dung's semantics.

There is no consensus on whether to accept or reject such arguments. Note that any argument in an odd-length cycle is controversial w.r.t. any argument in this cycle. There are many articles aiming to address this problem [3,4,6,11]. For example, Baumann et al. [4] argue that in Fig. 1c, A should be rejected while B should be accepted, since the attack from the self-defeat argument A

is not valid. However, Jakobovits and Vermeir [15] state that A and B can be labeled as "undecided" and "rejected", resp. They argue that since A is "strong enough" to attack A, surely it is strong enough to do the same with B. Compared with these approaches directly facing these disputable situations, our approach is more like bypassing such situations.

Considering the emergence of the problems above, we argue that the most acceptable arguments should satisfy two criteria: (i) they should have defenders as many as possible, and (ii) they should avoid the undesirable interference of some arguments. Dung's admissibility only requires a weak defense in the sense that only one mandatory defender is enough. An interesting fact is that some problematic situations disappear after Dung's admissibility is strengthened. In this paper, we propose *solid admissibility* which satisfies the two criteria. Roughly speaking, a *solidly admissible extension* is a set of arguments that contains no internal conflict, defends its elements against any attackers, and contains all the defenders. We will show that if an argument A is controversial w.r.t. an argument B, then B will never occur in any solid extension based on solid admissibility. To sum up, such extensions not only contain all defenders of their elements, but also avoid the interference of any argument that is indirectly attacked and indirectly defended by some argument. This conforms to the intuition in practical reasoning or real life in the sense that if an argument has more defenders, then surely it has less controversy.

We apply solid semantics in the field of *judgment aggregation*, a branch of social choice theory. When a group of agents evaluates which arguments are acceptable in an AF, each of them may report a different extension under a specific solid semantics that represents a individual viewpoint about which arguments are acceptable. We study whether their collective outcome is also a solid extension under this semantics when quota rules are applied. Especially, we show that there exist quota rules that preserve solid admissibility for any AF.

Contribution. Firstly, we propose a family of new semantics for abstract argumentation. Such semantics provide an approach to circumvent a controversial situation in AFs and also capture a feature that graded semantics fail to capture. Secondly, the new semantics have more possibility results for extension aggregation than Dung's semantics do.

Paper Outline. The rest of this paper is organized as follows. Section 2 reviews the background of abstract argumentation and judgment aggregation. Section 3 defines solid admissibility and shows how the problems are addressed. Section 4 develops more solid semantics and shows the connections among them. Besides, we compare solid semantics with Dung's semantics. Furthermore, we present propositional formulas that characterize the solid semantics and pave the way for solid extensions aggregation. Section 5 shows preservation results for the solid semantics. Section 6 mainly compares solid semantics with other related semantics. Section 7 concludes this paper and points out future work.

2 Preliminary

2.1 Abstract Argumentation

This part reviews some notions of abstract argumentation [12]. Some definitions are adopted from [14].

Definition 1 (Argumentation framework). *An* argumentation framework *is a pair* $AF = \langle Arg, \rightharpoonup \rangle$*, where* Arg *is a finite and non-empty set of arguments, and* $\rightharpoonup$ *is a binary relation on* Arg*.*

For any $A, B \in Arg$, $A \rightharpoonup B$ (or A *attacks* B) denotes that $(A, B) \in \rightharpoonup$. For any $B \in Arg$, $\overline{B} = \{A \in Arg \mid A \rightharpoonup B\}$, namely, $\overline{B}$ denotes the set of the attackers of B. A is an *initial argument* iff $\overline{A} = \emptyset$. For any $\Delta \subseteq Arg$ and any $B \in Arg$, $\Delta \rightharpoonup B$ denotes that there exists an argument $A \in \Delta$ such that $A \rightharpoonup B$. For any $\Delta \subseteq Arg$ and any $A \in Arg$, $A \rightharpoonup \Delta$ denotes that there exists an argument $B \in \Delta$ such that $A \rightharpoonup B$. For any $A, C \in Arg$, A is a *defender* of C iff there exists an argument $B \in Arg$ such that $A \rightharpoonup B$ and $B \rightharpoonup C$.

An argument A *indirectly attacks* an argument B iff there exists a finite sequence $A_0, \ldots, A_{2n+1}$ such that (i) $B = A_0$ and $A = A_{2n+1}$, and (ii) for each i, $0 \leqslant i \leqslant 2n$, $A_{i+1} \rightharpoonup A_i$. An argument A *indirectly defends* an argument B iff there exists a finite sequence $A_0, \ldots, A_{2n}$ such that (i) $B = A_0$ and $A = A_{2n}$, and (ii) for each i, $0 \leqslant i < 2n$, $A_{i+1} \rightharpoonup A_i$. An argument A is *controversial* w.r.t. an argument B iff A indirectly attacks and indirectly defends B. Note that direct attackers (resp. defenders) are also indirect attackers (resp. defenders).

Definition 2 (Dung's defense). *Given* $AF = \langle Arg, \rightharpoonup \rangle$*.* $\Delta \subseteq Arg$ defends $C \in Arg$ *iff for any* $B \in Arg$*, if* $B \rightharpoonup C$ *then* $\Delta \rightharpoonup B$*.*

Definition 3 (Defense function). *Given* $AF = \langle Arg, \rightharpoonup \rangle$*. The* defense function $d\colon 2^{Arg} \longrightarrow 2^{Arg}$ *of* AF *is defined as* $d(\Delta) = \{C \in Arg \mid \Delta \text{ defends } C\}$*.*

Definition 4 (Neutrality function). *Given* $AF = \langle Arg, \rightharpoonup \rangle$*. The* neutrality function $n\colon 2^{Arg} \longrightarrow 2^{Arg}$ *of* AF *is defined as* $n(\Delta) = \{B \in Arg \mid \text{not } \Delta \rightharpoonup B\}$*.*

Definition 5 (Dung's semantics). *Given* $AF = \langle Arg, \rightharpoonup \rangle$*. For any* $\Delta \subseteq Arg$*, (i)* Δ *is a* conflict-free *extension iff* $\Delta \subseteq n(\Delta)$*; (ii)* Δ *is a* self-defending *extension iff* $\Delta \subseteq d(\Delta)$*; (iii)* Δ *is an* admissible *extension iff* $\Delta \subseteq n(\Delta)$ *and* $\Delta \subseteq d(\Delta)$*; (iv)* Δ *is a* complete *extension iff* $\Delta \subseteq n(\Delta)$ *and* $\Delta = d(\Delta)$*; (v)* Δ *is a* preferred *extension iff* Δ *is a maximal admissible extension; (vi)* Δ *is a* stable *extension iff* $\Delta = n(\Delta)$*; (vii)* Δ *is the* grounded *extension iff* Δ *is the least fixed point of the defense function* d*.*

Theorem 1 (Dung, 1995). *Given* $AF = \langle Arg, \rightharpoonup \rangle$*. (i) If* $\Delta \subseteq Arg$ *is a preferred extension, then* Δ *is complete extension, but not vice versa; (ii) If* $\Delta \subseteq Arg$ *is an admissible extension, then there exists a preferred extension* Γ *such that* $\Delta \subseteq \Gamma$*.*

2.2 Integrity Constraints and Judgment Aggregation

Given $AF = \langle Arg, \rightharpoonup \rangle$, Dung's semantics can be captured by propositional language (denoted as $\mathcal{L}_{AF}$), whose literals are arguments in Arg [5]. A model is represented by the set of the literals which it satisfies. In other words, a model of a formula is a subset of Arg: (i) for any $A \in Arg$, $\Delta \vDash A$ iff $A \in \Delta$; (ii) $\Delta \vDash \neg\varphi$ iff $\Delta \vDash \varphi$ does not hold; (iii) $\Delta \vDash \varphi \wedge \psi$ iff $\Delta \vDash \varphi$ and $\Delta \vDash \psi$.

A *property* σ of extensions can be regarded as a subset of 2^{Arg}, namely, $\sigma \subseteq 2^{Arg}$. Then the set of the extensions under a semantics is a property, e.g., completeness is the set of the complete extensions of AF. For any formula φ in $\mathcal{L}_{AF}$, we let $\mathrm{Mod}(\varphi) = \{\Delta \subseteq Arg \mid \Delta \vDash \varphi\}$, namely, $\mathrm{Mod}(\varphi)$ denotes the set of all models of φ. Obviously, $\sigma = \mathrm{Mod}(\varphi)$ is a property. When using a formula φ to characterize such a property, φ is referred to as an *integrity constraint*.

We next introduce a model for the aggregation of extensions [10,13]. Given $AF = \langle Arg, \rightharpoonup \rangle$. Let $N = \{1, \cdots, n\}$ be a finite set of *agents*. Imagining that each agent $i \in N$ reports an extension $\Delta_i \subseteq Arg$. Then $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n)$ is referred to as a *profile*. An *aggregation rule* is a function $F\colon (2^{Arg})^n \longrightarrow 2^{Arg}$, mapping any given profile of extensions to a subset of Arg.

Definition 6 (Quota rules). *Given $AF = \langle Arg, \rightharpoonup \rangle$, let N be a finite set of n agents, and let $q \in \{1, \cdots, n\}$. The* quota rule *with quota q is defined as the aggregation rule mapping any given profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_n) \in (2^{Arg})^n$ of extensions to the set including exactly those arguments accepted by at least q agents: $F_q(\boldsymbol{\Delta}) = \{A \in Arg \mid \#\{i \in N \mid A \in \Delta_i\} \geqslant q\}$.*

The quota rule F_q for n agents with $q = \lceil \frac{n+1}{2} \rceil$ (resp., $q = 1$, $q = n$) for AF is called the *strict majority* (resp., *nomination*, *unanimity*) rule.

Definition 7 (Preservation). *Given $AF = \langle Arg, \rightharpoonup \rangle$. Let $\sigma \subseteq 2^{Arg}$ be a property of extensions. An aggregation rule $F\colon (2^{Arg})^n \longrightarrow 2^{Arg}$ for n agents is said to* preserve *σ if $F(\boldsymbol{\Delta}) \in \sigma$ for every profile $\boldsymbol{\Delta} = (\Delta_1, \cdots, \Delta_1) \in \sigma^n$.*

Lemma 1 (Grandi and Endriss, 2013). *Given $AF = \langle Arg, \rightharpoonup \rangle$. Let φ be a clause (i.e., disjunctions of literals) in $\mathcal{L}_{AF}$ with k_1 positive literals and k_2 negative literals. Then a quota rule F_q for n agents preserves the property* $\mathrm{Mod}(\varphi)$ *for AF iff the following inequality holds: $q \cdot (k_2 - k_1) > n \cdot (k_2 - 1) - k_1$.*

Lemma 2 (Grandi and Endriss, 2013). *Given $AF = \langle Arg, \rightharpoonup \rangle$. Let φ_1 and φ_2 be integrity constraints in $\mathcal{L}_{AF}$. If an aggregation rule F preserves both* $\mathrm{Mod}(\varphi_1)$ *and* $\mathrm{Mod}(\varphi_2)$*, then F preserves* $\mathrm{Mod}(\varphi_1 \wedge \varphi_2)$*, but not vice versa.*

3 Solid Admissibility

To obtain the most acceptable arguments that satisfy the two criteria proposed in the introduction, we formally introduce solid admissibility in this section. Arguments in admissible extensions satisfy the criteria. Firstly, we strengthen Dung's defense. Definition 8 states that a set of arguments *solidly defends* an argument iff this set defends (in Dung's sense) this argument and contains all the defenders of each element of this set.

Definition 8 (Solid defense). *Given $AF = \langle Arg, \rightharpoonup \rangle$. $\Delta \subseteq Arg$* solidly defends *(or* s-defends*) $C \in Arg$ iff for any $B \in Arg$, if $B \rightharpoonup C$, then $\Delta \rightharpoonup B$ and $\overline{B} \subseteq \Delta$.*

Definition 9 (Solid defense function). *Given $AF = \langle Arg, \rightharpoonup \rangle$. The* solid defense function *$d_s \colon 2^{Arg} \longrightarrow 2^{Arg}$ of AF is defined as follows. For any $\Delta \subseteq Arg$,*

$$d_s(\Delta) = \big\{C \in Arg \mid \Delta \text{ s-defends } C\big\} \tag{1}$$

Next we show an important property of the solid defense function. It is easy to see that, if a set of arguments s-defends an argument, then any superset of this set also s-defends this argument by Definition 8.

Theorem 2. *The solid defense function d_s is monotonic.*

Proposition 1. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, $d_s(\Delta) \subseteq d(\Delta)$, but not vice versa.*

Proposition 1 states that solid defense strengthens Dung's defense since, if a set of arguments s-defends an argument, then this set also defends this argument. To show the converse does not hold, consider AF_1 in Fig. 2a. Δ defends C. But the attackers of B are not fully included in Δ. So Δ does not s-defend C.

Definition 10. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, Δ is a* s-self-defending *extension iff $\Delta \subseteq d_s(\Delta)$.*

In graded semantics [14], a set of arguments Δ *mn-defends* an argument C iff there are at most $m-1$ attackers of C that are not counterattacked by at least n arguments in Δ, where m and n are positive integers. A set of arguments is *mn-self-defending* iff it mn-defends each element. We can tune the parameters to obtain defenses with different levels of strength. For example, when $n = 1$, the larger m is, the stronger the defense is. In Fig. 1a, $\{A_1, C_1\}$, $\{A_2, C_2\}$ and $\{A_1, A_2, C_2\}$ are 11-self-defending. But $\{A_2, C_2\}$ fails to contain all defenders of C_2. One might be tempted to tune the parameters to obtain a stronger defense. Then only $\{A_1, A_2, C_2\}$ is 12-self-defending. Although $\{A_1, C_1\}$ contains all defenders of C_1, it is not 12-self-defending. Hence, graded defense can not capture sets of arguments that exactly contain all defenders of their elements by tuning the parameters. However, solid defense can accomplish this, since it is identified that $\{A_2, C_2\}$ is not s-self-defending while $\{A_1, C_1\}$ and $\{A_1, A_2, C_2\}$ are s-self-defending.

Definition 11. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, Δ is a* s-admissible *extension iff $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d_s(\Delta)$.*

Definition 11 states that a set of arguments is a s-admissible extension iff the set is a conflict-free and s-self-defending extension. Next we show that Dung's Fundamental Lemma has a counterpart in our semantics. The following lemma states that whenever we have a s-admissible extension, if we put into this extension an argument that is s-defended by this extension, then the new set is still a s-admissible extension. The proof is similar to Dung's proof [12].

Lemma 3 (s-fundamental lemma). *Given $AF = \langle Arg, \rightharpoonup \rangle$, a s-admissible extension $\Delta \subseteq Arg$, and two arguments C, $C' \in Arg$ which are s-defended by Δ. Then (i) $\Delta' = \Delta \cup \{C\}$ is s-admissible and (ii) Δ' s-defends C'.*

As we have strengthened Dung's admissibility, the second problem mentioned in the introduction can be addressed now. From a skeptical view, it is not cautious to accept an argument that is indirectly attacked and indirectly defended by some argument. The following theorem states that such arguments never occur in s-admissible extensions.

Theorem 3. *Given $AF = \langle Arg, \rightharpoonup \rangle$ and a s-admissible extension $\Delta \subseteq Arg$. If an argument $A \in Arg$ is controversial w.r.t. an argument $B \in Arg$, then $B \notin \Delta$.*

Proof. Assume that A is controversial w.r.t. B. Suppose for the sake of a contradiction that $B \in \Delta$. Since A indirectly defends B and Δ is s-self-defending, we also have $A \in \Delta$. Besides, since A indirectly attacks B, there exists a finite sequence $A_0, \ldots, A_{2n+1}$ such that (i) $B = A_0$ and $A = A_{2n+1}$, and (ii) for each i, $0 \leqslant i \leqslant 2n$, $A_{i+1} \rightharpoonup A_i$. If $n = 0$, then $A_1 \rightharpoonup A_0$, namely, $A \rightharpoonup B$. This contradicts the conflict-freeness of Δ. If $n \neq 0$, then A_{2n} indirectly defends B. Since Δ is s-self-defending, $A_{2n} \in \Delta$. Again, the fact $A_{2n+1} \rightharpoonup A_{2n}$ (i.e., $A \rightharpoonup A_{2n}$) contradicts the conflict-freeness of Δ. So we conclude that $B \notin \Delta$.

Note that in Theorem 3, A is not excluded from Δ, since A could be an initial argument. It is not reasonable to reject an unattacked argument. Consider AF_2 in Fig. 1b. We can see that $\{A, C, F\}$ is admissible. However, since A is controversial w.r.t. C, $\{A, C, F\}$ is not s-admissible. $\{A\}$ is still s-admissible. The problem of odd-length cycles has been widely studied. It is thorny to assign a status to an argument in an odd-length cycle, since any argument in an odd-length cycle is controversial w.r.t. any argument in this cycle. There is no consensus on this problem. Interestingly, the following corollary states that such arguments never occur in s-admissible extensions. Moreover, once there is a path from an odd-length cycle to some argument, this argument will never occur in any s-admissible extension since any argument in the odd-length cycle is controversial w.r.t. it.

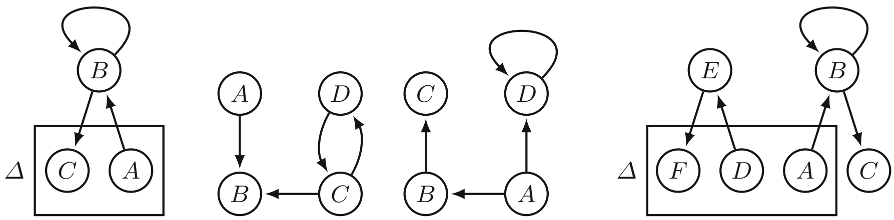
Corollary 1. *Given $AF = \langle Arg, \rightharpoonup \rangle$ and a s-admissible extension $\Delta \subseteq Arg$. If an argument $A \in Arg$ is in an odd-length cycle, then $A \notin \Delta$.*

4 Solid Semantics

We start by developing some solid semantics based on solid admissibility in this section. These semantics strengthen Dung's semantics in the sense that for a *solid extension* Δ, there exists a Dung's extension Γ such that Δ is a subset of Γ. Moreover, we will show connections among solid extensions and compare solid extensions with Dung's extensions. These solid semantics can be characterized by propositional formulas.

Definition 12 (Solid semantics). *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, (i) Δ is a* s-complete *extension iff $\Delta \subseteq n(\Delta)$ and $\Delta = d_s(\Delta)$; (ii) Δ is a* s-preferred *extension iff Δ is a maximal s-admissible extension; (iii) Δ is a* s-stable *extension iff $\Delta = n(\Delta)$ and for any argument $A \notin \Delta$, $\overline{A} \subseteq \Delta$; (iv) Δ is the* s-grounded *extension iff Δ is the least fixed point of d_s.*

Here are some comments for the definition above. We define these solid extensions by using the neutrality function n and the solid defense function d_s, like Dung's extension in Definition 5. A s-complete extension is a fixed point of d_s which is also a conflict-free extension. In other words, a s-complete extension is a s-admissible extension that contains all arguments s-defended by it. A s-preferred extension has maximality and solid admissibility. A set of arguments Δ is a s-stable extension whenever it is a fixed point of n and all attackers of any argument outside of Δ are in Δ. The s-grounded extension is unique.



(a) AF_1 (b) AF_2 (c) AF_3 (d) AF_4

Fig. 2. Four argumentation frameworks

Next we present some connections among solid semantics. A s-stable extension is a s-preferred extension. And a s-preferred extension is a s-complete extension. Besides, the s-grounded extension is the least s-complete extension.

Theorem 4. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, (i) if Δ is a s-preferred extension, then Δ is a s-complete extension, but not vice versa; (ii) if Δ is a s-stable extension, then Δ is a s-preferred extension, but not vice versa; (iii) The s-grounded extension is the least s-complete extension.*

Proof. Take a set of arguments $\Delta \subseteq Arg$. (i) Suppose that Δ is a s-preferred extension. Then according to Definition 11 and the second item of Definition 12, we have $\Delta \subseteq n(\Delta)$ and $\Delta \subseteq d_s(\Delta)$. It suffices to show $d_s(\Delta) \subseteq \Delta$. Suppose that an argument $C \in d_s(\Delta)$, namely, Δ s-defends C. Then $\Delta \cup \{C\}$ is s-admissible by Lemma 3. Suppose for the sake of a contradiction that $C \notin \Delta$. This contradicts the maximality of Δ. Hence, $C \in \Delta$. It follows that $\Delta = d_s(\Delta)$. So Δ is s-complete extension by the first item of Definition 12. Next we show the converse does not hold. Consider AF_2 in Fig. 2b. Take two sets $\Delta = \{A\}$ and $\Gamma = \{A, C\}$. Then Δ is a s-complete extension. However, we can see that Γ is s-admissible and $\Delta \subset \Gamma$. Hence, Δ is not a maximal s-admissible extension (i.e., not s-preferred).

(ii) Suppose that Δ is a s-stable extension. Then applying the third item of Definition 12 yields $\Delta = n(\Delta)$. We next verify $\Delta \subseteq d_s(\Delta)$. Assume that an argument $C \in \Delta$. To demonstrate $C \in d_s(\Delta)$, suppose that an argument $B \in Arg$ attacks C. Then $B \notin \Delta$ as Δ is conflict-free. It follows that $B \notin n(\Delta)$. Thus, $\Delta \rightharpoonup B$ by Definition 4. Again, using the third item of Definition 12 yields $\overline{B} \subseteq \Delta$. Hence, Δ s-defends C by Definition 8. Then $C \in d_s(\Delta)$ by Definition 9. It follows that Δ is a s-admissible extension. At last, we prove the maximality of Δ. Suppose for the sake of a contradiction that there exists a s-admissible set Γ such that $\Delta \subset \Gamma$. Then there exists an argument C' such that $C' \in \Gamma$ but $C' \notin \Delta$. Immediately, we have $\Delta \rightharpoonup C'$ as $C' \notin \Delta$. This contradicts conflict-freeness of Γ. So Δ is a maximal s-admissible extension. To show that the converse does not hold, let us consider AF_3 in Fig. 2c. Let $\Delta = \{A, C\}$. Then Δ is a s-preferred extension. We can see that $D \notin \Delta$. But the attackers of D are not fully included in Δ. Hence, Δ is not a s-stable extension.

(iii) As Arg is finite, the least fixed point of d_s (i.e., the grounded extension) can be computed as $d_s^{i_{min}}(\emptyset)$ where i_{min} is the least integer i such that $d_s^{i+1}(\emptyset) = d_s^{i}(\emptyset)$. Moreover, $d_s^{i}(\emptyset)$ is s-admissible by induction on natural number i and Lemma 3. Hence, the least fixed point of d_s is a s-complete extension according to the first item of Definition 12. Thus, the least fixed point of d_s is a subset of any s-complete extension as as any s-complete extension is a fixed point of d_s.

Recall that any s-admissible extension contains no argument that is indirectly attacked and indirectly defended by some argument. Since we have showed that the solid extensions in Definition 12 are also s-admissible, they contain no such argument either, according to Theorem 3. Next we present an interesting property that the set of arguments outside of a s-stable extension is conflict-free.

Proposition 2. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, if Δ is a s-stable extension, then $Arg \setminus \Delta$ is a conflict-free extension.*

In the following, we show that solid semantics can be interpreted as a class of strengthenings of Dung's semantics. In other words, for any solid extension, there exists a Dung's counterpart such that it is a superset of the solid extension.

Proposition 3. *Given $AF = \langle Arg, \rightharpoonup \rangle$. For any $\Delta \subseteq Arg$, (i) if Δ is a s-self-defending extensions, then Δ is self-defending extension; (ii) if Δ is a s-admissible extension, then Δ is an admissible extension; (iii) if Δ is a s-complete extension, then there exists a complete extension Γ such that $\Delta \subseteq \Gamma$; (iv) if Δ is a s-preferred extension, then there exists a preferred extension Γ such that $\Delta \subseteq \Gamma$; (v) if Δ is a s-stable extension, then Δ is a stable extension; (vi) if Δ is the s-grounded extension, then Δ is a subset of the grounded extension.*

Proof. Take a set of arguments $\Delta \subseteq Arg$. (i) Suppose Δ is s-self-defending. Then from Definition 10 we have $\Delta \subseteq d_s(\Delta)$. We also have $d_s(\Delta) \subseteq d(\Delta)$ by Proposition 1. It follows that $\Delta \subseteq d(\Delta)$. (ii) This item is easily obtained from the first item. (iii) Suppose that Δ is s-complete. Then Δ is s-admissible. Hence, Δ is admissible. Therefore, there exists a preferred extension Γ such that $\Delta \subseteq \Gamma$ by

Theorem 1. Besides, Γ is also a complete extension by Theorem 1. (iv) Suppose that Δ is s-preferred. Then Δ is s-admissible. Hence, Δ is admissible. Therefore, there exists a preferred extension Γ such that $\Delta \subseteq \Gamma$ by Theorem 1. (v) This item follows from the third item of Definition 12 and the fifth item of Definition 5. (vi) Recall that the least fixed point of d_s (resp., d) is found by iterating the application of d_s (resp., d) from the empty set. Besides, we have $d_s^i(\emptyset) \subseteq d^i(\emptyset)$ by induction on natural number i. And we also have $d^i(\emptyset) \subseteq d^{i+1}(\emptyset)$. Therefore, during the process of iteration, the s-grounded extension will be found no later than the grounded extension. So the former is a subset of the latter.

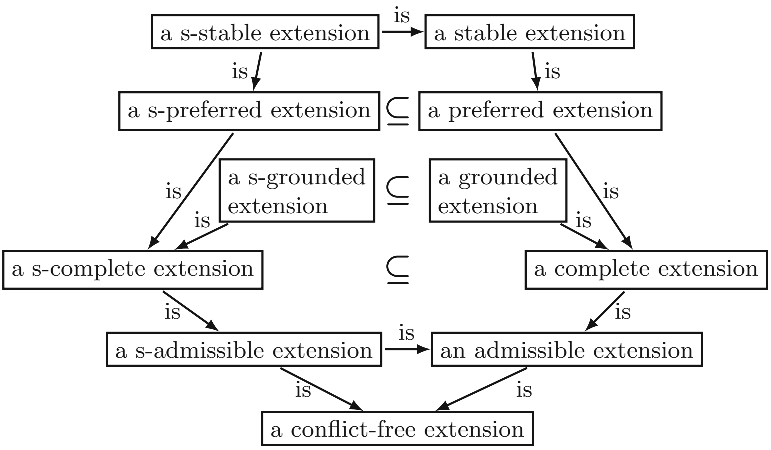


Fig. 3. An overview of solid semantics and Dung's semantics. We can see that for any solid extension, there exists a superset that is also a Dung's extension.

To gain a better understanding of differences between Dung's extensions and solid extensions, we provide two examples. Example 3 shows that if a set of arguments is a self-defending (resp., a complete, a preferred, a stable, or the grounded) extension, then it may fail to be a s-self-defending (resp., s-complete, a s-preferred, a s-stable or the s-grounded) extension. Example 4 shows that if a set of arguments is a s-complete (resp., a s-preferred or the s-grounded) extension, then it may fail to be a complete (resp., a preferred or the grounded) extension. However, it is easy to see that a s-stable extension must be a stable extension by Definition 5 and Definition 12. We illustrate in Fig. 3 an overview of how the solid extensions are related to each other and Dung's extensions.

Example 3. Let us consider AF_1 in Fig. 2a. Take a set of arguments $\Delta = \{A, C\}$. Then it is easy to see that Δ is a self-defending extension but not a s-self-defending extension. Besides, Δ is a complete, a preferred, a stable and the grounded extension. However, according to Theorem 3, C can not be included in any s-admissible extension, since B is controversial w.r.t. C. Hence, Δ is neither a s-complete, a s-preferred, a s-stable, nor the s-grounded extension.

Example 4. Let us consider AF_4 in Fig. 2d. Let $\Delta = \{A, D, F\}$. Then Δ is a s-complete, a s-preferred and the s-grounded extension. However, we can see that argument C which is defended by Δ is not included in Δ. Hence, Δ is neither a complete, a preferred, a stable nor the grounded extension.

Before moving to the next section, we characterize the solid semantics in terms of integrity constraints (i.e., propositional formulas) expressed in $\mathcal{L}_{AF}$. We say that a set of arguments is a *s-reinstating* extension iff each argument s-defended by this set belongs to this set. It is worth mentioning that the integrity constrain of conflict-freeness [5] is $\mathrm{IC}_{CF} \equiv \bigwedge_{\substack{\mathrm{A,B} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \big(\neg \mathrm{A} \vee \neg \mathrm{B}\big)$.

Proposition 4. *Given $AF = \langle Arg, \rightharpoonup \rangle$. $\Delta \subseteq Arg$ is a s-self-defending, s-reinstating, s-stable, s-admissible, s-complete, s-preferred and s-grounded extension, resp. iff*

- $\Delta \models \mathrm{IC}_{SS}$ *where* $\mathrm{IC}_{SS} \equiv \bigwedge_{\mathrm{C} \in Arg} \big[\mathrm{C} \rightarrow \bigwedge_{\substack{\mathrm{B} \in Arg \\ \mathrm{B} \rightharpoonup \mathrm{C}}} \big((\bigvee_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \mathrm{A}) \wedge (\bigwedge_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \mathrm{A})\big)\big]$*;*
- $\Delta \models \mathrm{IC}_{SR}$ *where* $\mathrm{IC}_{SR} \equiv \bigwedge_{\mathrm{C} \in Arg} \big[\bigwedge_{\substack{\mathrm{B} \in Arg \\ \mathrm{B} \rightharpoonup \mathrm{C}}} \big((\bigvee_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \mathrm{A}) \wedge (\bigwedge_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \mathrm{A})\big) \rightarrow \mathrm{C}\big]$*;*
- $\Delta \models \mathrm{IC}_{SST}$ *where* $\mathrm{IC}_{SST} \equiv \bigwedge_{\mathrm{B} \in Arg} \big[\big(\mathrm{B} \leftrightarrow \bigwedge_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \neg \mathrm{A}\big) \wedge \big(\neg \mathrm{B} \rightarrow \bigwedge_{\substack{\mathrm{A} \in Arg \\ \mathrm{A} \rightharpoonup \mathrm{B}}} \mathrm{A}\big)\big]$*;*
- $\Delta \models \mathrm{IC}_{SA}$ *where* $\mathrm{IC}_{SA} \equiv \mathrm{IC}_{CF} \wedge \mathrm{IC}_{SS}$*;*
- $\Delta \models \mathrm{IC}_{SC}$ *where* $\mathrm{IC}_{SC} \equiv \mathrm{IC}_{SA} \wedge \mathrm{IC}_{SR}$*;*
- Δ *is a maximal model of* IC_{SA}*;*
- Δ *is the least model of* IC_{SC}*.*

5 Preservation of Solid Semantic Properties

Quota rules are natural rules to be considered when contemplating mechanisms to perform aggregation. They have low computational complexity and satisfy some appealing properties. For instance, they are monotonic and strategy-proof as studied in judgment aggregation. The problem of aggregating extensions submitted by several agents on a given AF is an important and interesting topic. Especially, no quota rule preserves Dung's admissibility for all AFs [10]. So we wonder what admissible extensions can be preserved. In light of the integrity constraints for solid semantics in Proposition 4, we can investigate the preservation results for solid semantic properties by using the model defined in Sect. 2.2 and quota rules. We analyze that in the scenarios where a set of agents each provides us with a set of arguments that satisfies a specific solid semantics, under what circumstance such solid semantic property will be preserved under aggregation.

5.1 Preserving Solid Self-defence and Solid Admissibility

We start by exploring the circumstances where the s-self-defending property can be preserved. Theorem 5 presents a positive result that every quota rule preserves the s-self-defending property for all AFs.

Theorem 5. *Given $AF = \langle Arg, \rightharpoonup \rangle$. Every quota rule F_q for n agents preserves the property of being s-self-defending for AF.*

Proof. Recall from Proposition 4 that IC_{SS} is a conjunction of formulas of the form: $\varphi \equiv C \rightarrow \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \big((\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A) \wedge (\bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)\big)$. Note that this formula is indexed by argument C, let us study the preservation of such formula. If C is an initial argument (an argument not receiving attacks in AF), then $\varphi \equiv C \rightarrow \top$, which can be simplified to $\varphi \equiv \top$. It follows that in this case, every quota rule F_q for n agents preserves $\mathrm{Mod}(\varphi)$ for AF. If C has at least one attacker, then we need to take into account the following two cases. The first case is that there exists an attacker B of C such that B is an initial argument. In this case, we have $\varphi \equiv \neg C$ (this can be easily obtained as $\bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \big((\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A) \wedge (\bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A)\big)$ is always false if there is an attacker B of C that has no attacker). Then, according to Lemma 1, every quota rule F_q for n agents preserves $\mathrm{Mod}(\varphi)$ for AF, as the inequality $q \cdot (1-0) > n \cdot (1-1) - 0$, which can be simplified to $q > 0$, always holds. The other case is that all attackers of C are not initial. Then, we let $\{B_1, \dots, B_p\}$ be the set of attackers of C, and let $\{A^i_1, \dots, A^i_{\ell_i}\}$ be the set of attackers of B_i where $1 \leqslant i \leqslant p$ and $\ell_i = |\overline{B_i}|$. Then φ can be rewritten as follows: $\varphi \equiv \big((\neg C \vee A^1_1) \wedge \dots \wedge (\neg C \vee A^1_{\ell_1})\big) \wedge \dots \wedge \big((\neg C \vee A^p_1) \wedge \dots \wedge (\neg C \vee A^p_{\ell_p})\big)$. Thus, in this case, φ is a conjunction of clauses with one negative literal and one positive literal. We take one such clause $\psi = \neg C \vee A^i_{l_j}$. According to Lemma 1, every quota rule F_q preserves ψ in this case. Thus, every quota rule for n agents preserves φ. It follows that all clauses of IC_{SS} can be preserved by all quota rules. We conclude that every quota rule F_q for n agents preserves the property $\mathrm{Mod}(\mathrm{IC}_{SS})$ for AF by Lemma 2.

Obviously, the strict majority rule, the nomination rule and the unanimity rule as specific quota rules, will preserve s-self-defense. Note that different from the property of being s-self-defending, Dung's self-defense cannot be preserved by some quota rule. One example is the strict majority rule [10].

We now turn to consider the preservation of solid admissibility. Recall that a set of arguments is s-admissible if it satisfies conflict-freeness and s-self-defense. With Lemma 2, we know that if an aggregation rule preserves both conflict-freeness and s-self-defense, then solid admissibility will be preserved by such rule. The following proposition restates a result for conflict-freeness in [10].

Proposition 5. *Given $AF = \langle Arg, \rightharpoonup \rangle$. A quota rule F_q for n agents preserves conflict-freeness for AF if $q > \frac{n}{2}$.*

Theorem 6. *Given $AF = \langle Arg, \rightharpoonup \rangle$. Any quota rule F_q for n agents with $q > \frac{n}{2}$ preserves solid admissibility for AF.*

Applying Theorem 5 and Proposition 5 immediately yields Theorem 6, which shows that any quota rule higher than or equal to the strict majority rule can preserve solid admissibility for arbitrary AF. Recall that no quota rule preserves Dung's admissibility for all AFs [10]. Thus, we have obtained a positive result, i.e., we know that there exist quota rules that preserve solid admissibility for all AFs. Notably, Chen [9] uses a different model to show that the majority rule guarantees Dung's admissibility on profiles of solid admissible sets during aggregation of extensions. Theorem 6 entails this result, but not vice versa.

5.2 Preserving Solid Reinstatement and Solid Completeness

We turn to explore solid reinstatement. For convenience, we provide three notations. Given $AF = \langle Arg, \rightharpoonup \rangle$. Firstly, for any $C \in Arg$, $\mathscr{D}_{AF}(C)$ denotes the set of C's defenders, i.e., $\mathscr{D}_{AF}(C) = \{A \in Arg \mid A$ is a defender of $C\}$. Secondly, we let $\mathcal{E}(AF)$ denote the set of arguments which are not initial arguments and whose attackers are not initial arguments either, i.e., $\mathcal{E}(AF) = \{C \in Arg \mid \overline{C} \neq \emptyset$ and for any $B \in \overline{C}$, $\overline{B} \neq \emptyset\}$. Thirdly, we let $\mathcal{M}(AF)$ denote the maximal number of the defenders of an argument in $\mathcal{E}(AF)$, i.e., $\mathcal{M}(AF) = \max\limits_{C \in \mathcal{E}(AF)} |\mathscr{D}_{AF}(C)|$.

Theorem 7. *Given $AF = \langle Arg, \rightharpoonup \rangle$. A quota rule F_q for n agents preserves the property of being s-reinstating for AF if $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$.*

Proof. Assume that $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$. IC_{SR} is a conjunction of formulas of the form of $\varphi \equiv \bigwedge_{\substack{B \in Arg \\ B \rightharpoonup C}} \big((\bigvee_{\substack{A \in Arg \\ A \rightharpoonup B}} A) \wedge (\bigwedge_{\substack{A \in Arg \\ A \rightharpoonup B}} A) \big) \rightarrow C$ from Proposition 4. Take an argument $C \in Arg$. If C is an initial argument, then $\varphi \equiv C$, which can be regarded as a 1-clause (with 1 positive literal and no negative literal). Applying Lemma 1, any quota rule F_q for n agents preserves $\text{Mod}(\varphi)$ for AF in this case, as the inequality $q \cdot (0 - 1) > n \cdot (0 - 1) - 1$ (which can be simplified to $q < n + 1$) always holds.

If C is not an initial argument, then we need to consider two cases. The first case is that there exists a C's attacker B such that B is an initial argument, then $\varphi \equiv \top$. It follows that in this case, any quota rule F_q for n agents preserves $\text{Mod}(\varphi)$ for AF. The second case is that any C's attacker B is not an initial argument. Then we can let $\{B_1, \cdots, B_p\}$ be the set of C's attackers, and let $\{A_1^i, \cdots, A_{\ell_i}^i\}$ be the set of B_i's attackers where $1 \leqslant i \leqslant p$ and $\ell_i = |\overline{B_i}|$. we can reformulate φ as follows: $\varphi \equiv \big((\neg A_1^1 \vee \cdots \vee \neg A_{\ell_1}^1) \vee \cdots \vee (\neg A_1^p \vee \cdots \vee \neg A_{\ell_p}^p) \big) \vee C$.

Hence, φ is a $(\sum\limits_{i=1}^{p} \ell_i + 1)$-clause (with $\sum\limits_{i=1}^{p} \ell_i$ negative literals and 1 positive literal). By Lemma 1, a quota rule F_q for n agents preserves $\text{Mod}(\varphi)$ if the following inequality holds: $q \cdot (\sum_{i=1}^{p} \ell_i - 1) > n \cdot (\sum_{i=1}^{p} \ell_i - 1) - 1$. Doing so becomes harder as $\sum_{i=1}^{p} \ell_i$ increases. Note that $C \in \mathcal{E}(AF)$ in this case and $\sum_{i=1}^{p} \ell_i$ is the number of the defenders of C. Recall that the maximal number of defenders of an argument in $\mathcal{E}(AF)$ is $\mathcal{M}(AF)$. Hence, the maximal value of $\sum_{i=1}^{p} \ell_i$ is $\mathcal{M}(AF)$. Thus, by Lemma 1, a quota rule F_q for n agents preserve $\text{Mod}(\varphi)$ for AF in this case, if the inequality holds for the maximal value of $\sum_{i=1}^{p} \ell_i$ (i.e., $\mathcal{M}(AF)$). As we have assumed that $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$, it follows that a quota rule F_q for n agents preserves $\text{Mod}(\varphi)$ for AF. Finally, using Lemma 2, we can conclude that a quota rule F_q for n agents preserves $\text{Mod}(\text{IC}_{SR})$ for AF if $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$.

Recall that the unanimity rule is a quota rule F_q for n agents with $q = n$. It is easy to see that the inequality $n \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$ always holds. Then according to Theorem 7, the unanimity rule preserves the property of being s-reinstating for any AF. The theorem below for s-completeness is a direct consequence of Theorem 6 and Theorem 7.

Theorem 8. *Given $AF = \langle Arg, \rightharpoonup \rangle$. A quota rule F_q for n agents preserves s-completeness for AF if $q > \frac{n}{2}$ and $q \cdot (\mathcal{M}(AF) - 1) > n \cdot (\mathcal{M}(AF) - 1) - 1$.*

5.3 Preserving Solid Groundedness, Solid Preferredness and Solid Stability

As the s-grounded extension is unique in any AF, any quota rule preserves s-groundedness for any AF. We say that a property σ is *inclusion maximal* if for any $\Delta_1,\ \Delta_2 \in \sigma$, if $\Delta_1 \subseteq \Delta_2$ then $\Delta_1 = \Delta_2$. It is easy to see that both the solid preferredness and solid stability are inclusion maximal. Hence, we can investigate these two properties together. Given $AF = \langle Arg, \rightharpoonup \rangle$, let σ be an inclusion maximal property of extensions such that $|\sigma| \geqslant 2$, and let n be the number of agents. If n is even, then no quota rule preserves σ for AF. If n is odd, then no quota rule different from the strict majority rule preserves σ for AF. Such results are highly analogous to Theorem 15 in [10]. We omit the proof for this reason.

6 Related Work

Various notions of admissibility are proposed since Dung's admissibility was introduced in [12]. Baroni and Giacomin introduce the notion of strong admissibility [2] which is stronger than Dung's admissibility. It captures the idea that any argument in a strongly admissible set neither defend itself nor involve in its own defense. Grossi and Modgil propose *Graded admissibility* [14], whereby Dung's admissibility can be strengthened or weakened by parameterizing the numbers of attackers and defenders. Chen [8] proposes *concrete* admissibility. Differently from solid admissibility, concrete admissibility does not require the existence of defenders, although both of them require containing all defenders. Prudent semantics [11] is another semantics that aims at dealing with controversial arguments. Whenever an argument A is controversial w.r.t. an argument B, both prudent semantics and solid semantics can prevent A and B from occurring in the same extension. But there is a difference between these two semantics: both A and B can occur in a prudent extension independently, however, B is excluded from any s-admissible extension, while A might occur in some s-admissible extension (e.g., A is an initial argument).

7 Conclusion and Future Work

This paper mainly makes contributions to the field of abstract argumentation theory. To address the problems observed in ranking-based argumentation and controversial arguments, we develop solid semantics by strengthening Dung's semantics. By applying the technique in [5], we capture solid semantics by using propositional formulas. Finally, in virtue of these formulas, we aggregate solid extensions by using quota rules and obtain positive preservation results.

Recall that solid defense requires that all the defenders of an argument C are included in a set of arguments Δ. It would be interesting to characterize the idea that any percent of the defenders of C are included in a set of arguments Δ. Moreover, we can also allow a part of attackers to be not attacked. For example, we can try to capture the idea that Δ defends C if more than fifty percent of C's attackers have more than fifty percent of their attackers in Δ (i.e., if the majority of C's attackers have the majority of their attackers in Δ). Future work could focus on introducing proportionality to the defense of arguments of abstract argumentation. Furthermore, the complexity of reasoning tasks involving solid semantics should be studied.

References

1. Amgoud, L., Ben-Naim, J.: Ranking-based semantics for argumentation frameworks. In: Liu, W., Subrahmanian, V.S., Wijsen, J. (eds.) SUM 2013. LNCS (LNAI), vol. 8078, pp. 134–147. Springer, Heidelberg (2013). https://doi.org/10.1007/978-3-642-40381-1_11
2. Baroni, P., Giacomin, M.: On principle-based evaluation of extension-based argumentation semantics. Artif. Intell. **171**(10–15), 675–700 (2007)
3. Baroni, P., Giacomin, M., Guida, G.: SCC-recursiveness: a general schema for argumentation semantics. Artif. Intell. **168**(1–2), 162–210 (2005)
4. Baumann, R., Brewka, G., Ulbricht, M.: Revisiting the foundations of abstract argumentation-semantics based on weak admissibility and weak defense. In: Proceedings of the AAAI Conference on Artificial Intelligence, vol. 34, pp. 2742–2749 (2020)
5. Besnard, P., Doutre, S.: Checking the acceptability of a set of arguments. In: NMR, vol. 4, pp. 59–64. Citeseer (2004)
6. Bodanza, G.A., Tohmé, F.A.: Two approaches to the problems of self-attacking arguments and general odd-length cycles of attack. J. Appl. Log. **7**(4), 403–420 (2009)
7. Cayrol, C., Lagasquie-Schiex, M.C.: Graduality in argumentation. J. Artif. Intell. Res. **23**, 245–297 (2005)
8. Chen, W.: Preservation of admissibility with rationality and feasibility constraints. In: Dastani, M., Dong, H., van der Torre, L. (eds.) CLAR 2020. LNCS (LNAI), vol. 12061, pp. 245–258. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-44638-3_15
9. Chen, W.: Guaranteeing admissibility of abstract argumentation frameworks with rationality and feasibility constraints. J. Log. Comput. (2021, forthcoming)
10. Chen, W., Endriss, U.: Aggregating alternative extensions of abstract argumentation frameworks: preservation results for quota rules. In: COMMA, pp. 425–436 (2018)
11. Coste-Marquis, S., Devred, C., Marquis, P.: Prudent semantics for argumentation frameworks. In: 17th IEEE International Conference on Tools with Artificial Intelligence (ICTAI 2005), pp. 568–572. IEEE (2005)

12. Dung, P.M.: On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. Artif. Intell. **77**(2), 321–357 (1995)
13. Grandi, U., Endriss, U.: Lifting integrity constraints in binary aggregation. Artif. Intell. **199**, 45–66 (2013)
14. Grossi, D., Modgil, S.: On the graded acceptability of arguments in abstract and instantiated argumentation. Artif. Intell. **275**, 138–173 (2019)
15. Jakobovits, H., Vermeir, D.: Robust semantics for argumentation frameworks. J. Log. Comput. **9**(2), 215–261 (1999)