The Cost of Flexible Elements of a Rectangular Profile

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Abstract Hinge-supported bending reinforced concrete beams of rectangular profile, loaded with a uniformly distributed load, were considered. The main direction of the work is a theoretical study of the influence of the geometric dimensions of the section of beams on the consumption of materials and their cost in the product. The regulatory documents governing design and development activities in the Russian Federation, existing design solutions for bending elements, as well as actual works of Russian and foreign scientists corresponding to research in this area were used. Structural and analytical analysis was used. The normative, scientific and methodological materials were studied, which made it possible to establish the parameters and factors affecting the strength and cost of bent reinforced concrete elements. This is the basis for proposals for improving the calculation and reinforcement of bending elements. The main factors influencing the cost of concrete and reinforcement in a product are analyzed. Specific proposals for calculation and design were developed to determine the optimal cost structures. The proportions of the cross-sectional dimensions and the variable location of the neutral axis significantly affect the consumption of longitudinal reinforcement and the cost of the beam. The boundary ranges of the section dimensions, heights of the compressed zone and the degree of their influence on the cost of the element have been determined. Recommendations for design are given to ensure the construction of the minimum cost, but meet the requirements of strength and durability.

Keywords Material consumption · Product cost · Reinforcement · Reinforced concrete beams · Section dimensions

1 Introduction

When designing various structural elements, it is important, first of all, to ensure the subsequent safe operation and durability of structures. But in addition to this,

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the economic component is important—minimization of financial costs for the production of elements and structures.

Therefore, a theoretical study of the influence of the geometric dimensions of the section of beams on the consumption of materials and their cost in the product is an important and interesting engineering problem.

It is possible to obtain the calculated amount of reinforcement, providing the strength of the normal section, depending on the specified values of the geometric parameters of the beam and their relationship to each other. You can also set the cost of materials in the beam and in the product as a whole, which will allow you to design structures of a given strength, but the optimal cost. This is what this work is about.

2 Literature Review

Many Russian and foreign scientists dealt with the problems of optimal design of reinforced concrete beams, for example, Karpenko, Baikov, Skladnev, Alekseev, Jensen, Garstecki and many others.

The development of methods for calculating reinforced concrete structures was carried out by Baykov and Karpenko. In the works of Karpenko the general theory of deformation and destruction of reinforced concrete under various types of loading was formulated [1]. Baikov paid great attention to the development of calculation methods for precast concrete structures [2]. Skladnev was engaged in the problems of optimal design of reinforced concrete structures, taking into account reliability and efficiency [3]. Tamrazyan and Alekseytsev studied the problems of optimization of structures taking into account the ratio of production costs and risks of material losses in emergency situations [4, 5].

Chakrabarty studied the relationship between beam cost and unit cost of materials and beam sizes [6]. Jensen and Lapko investigated the design of shear reinforcement in reinforced concrete beams [7]. Coello et al. as well as Lee and Ahn, used genetic algorithms for optimal design beams [8, 9]. Jiin-Po, Guerra, Hare and others have studied the design optimization of reinforced concrete structures, including beams [10–12]. Also Garstecki et al. developed a software package for the optimal design of reinforced concrete beams and columns [13]. Demby addressed the problem of the optimal and safe design of reinforcement structures in reinforced concrete elements [14]. Nemirovsky considers critical characteristics of concrete failure, such as shrinkage, temperature sensitivity, and the influence of production technology [15].

Currently, reinforced concrete structures in the Russian Federation are calculated in accordance with [16, 17]. In Europe, a regulatory document is used [18], as well as various literature, for example [19, 20].

3 Methods

When designing bending elements, the strength of normal sections with single reinforcement is allowed to be determined at values $\xi \leq \xi_R$ ($\alpha_m \leq \alpha_R$). In this case, the limiting state occurs when the design resistance R_s in the tensile reinforcement reaches $\xi = \xi_R$ [13, 14]. However, the condition $\xi \leq \xi_R$ leaves the possibility of underutilizing the strength of concrete or reinforcement, which depends on the strength of concrete and reinforcement, their quantitative ratio, geometric characteristics of the cross-section and the actual location of the neutral axis.

The strength of an element in a normal section M_{sec} with values $\xi \leq \xi_R$ or $\alpha_m \leq \alpha_R$ can be determined equivalently by concrete (1) or reinforcement (2)

$$M \leq M_{sec} = R_b bx(h_0 - 0.5x) \tag{1}$$

$$M \leq M_{sec} = A_S R_S (h_0 - 0.5x) \tag{2}$$

The maximum value of the bearing capacity of an element with a single reinforcement, without the use of compressed reinforcement, $M_{sec}{}^{max}$ and with a coefficient of use of concrete and reinforcement equal to or close to unity, will be achieved at the height of the compressed zone, which is close to or equal to the boundary value, that is, at $\xi = \xi_R$ or $\alpha_m = \alpha_R$.

$$M_{sec}{}^{max} = R_b bx_R (h_0 - 0.5x_R) \tag{3}$$

Taking into account that $x_R = \xi_R h_0$

$$M_{sec}{}^{max} = R_b b\xi_R h_0 (h_0 - 0.5\,\xi_R h_0) \tag{4}$$

or

$$M_{sec}{}^{max} = R_b bh_0^2 \xi_R (1 - 0.5\,\xi_R) \tag{5}$$

or

$$M_{sec}{}^{max} = R_b bh_0^2 \eta_R \tag{6}$$

where

$$\eta_R = \xi_R (1 - 0.5\,\xi_R) \tag{7}$$

The boundary value ξ_R in accordance with [13, 14] for elements without prestressing is allowed to be determined by Formula (8) or tables [13, 14]

$$\xi_R = 560/(700 + R_s). \tag{8}$$

Maximum moment taken by a normal section with full use of concrete strength

$$M_{sec}{}^{max} = \eta_b R_b b h_0^2 \tag{9}$$

Maximum moment taken by a normal section with full use of the strength of the reinforcement

$$\mathrm{M_{sec}{}^{max} = \eta_s A_s R_s h_0} \tag{10}$$

Full use of materials in the section is achieved at $\eta_b = \eta_s$.

The required amount of tensile reinforcement, determined in accordance with Formula (2)

$$A_s = M_{sec}{}^{max}/R\eta_s h_0 = \eta_b R_b b h_0/\eta_s R_s \tag{11}$$

With reinforcement of class A400, the exhaustion of strength is achieved at $\xi_R = 0.531$

$$\eta_s = \xi_R(1 - 0.5\,\xi_R) = 0.531(1 - 0.5 \cdot 0.531) = 0.390 \tag{12}$$

Typically, the section height h depends on the design span l. Let's set the section height as $h = nl$, where the variable coefficient is $n = (0.05 \div 0.1)$. We take the section width b as the product of m by h, that is, $b = mh$, where the coefficient $m = (0.1 \div 0.5)$. We obtain an expression for the section width $b = mnl$. Then expression (11) can be transformed to form (13).

$$A_s = \eta_b R_b n^2 m l^2/(\eta_s R_s) \tag{13}$$

In particular, for elements with A400 class reinforcement at $\eta_s = 0.390$, the expression for determining the area of longitudinal reinforcement in the design section will take the form

$$\mathrm{A_s = \eta_b R_b n^2 m l^2/(0.39 R_s)} \tag{14}$$

Thus, by varying η_b, it is possible to obtain a calculated amount of reinforcement that ensures the strength of the normal section within the specified values of η_b. It is also possible to establish the cost of the materials of the beam and the product as a whole. This will make it possible to design structures of a given strength and optimal cost.

The object of the study was reinforced concrete beams without prestressing—B1, length $l = 3$ m; B2, $l = 6$ m; B3 $l = 9$ m. Rectangular profile beams, made of B20 concrete, reinforced with A400 class reinforcement bars. The section height was taken $h = nl$ for $n = (0.075; 0.1; 0.125)$, the section width $b = mh$ for the values $m = (0.1; 0.2; 0.3; 0.4; 0.5)$.

The influence of the section dimensions on the completeness of the use of materials at various values of the height of the compressed zone (coefficient $\eta_b = 0.05$; 0.1; 0.15; 0.2; 0.3; 0.39), as well as on the cost of materials and products in general, for the Moscow region at average prices for concrete 3500 rubles/m^3 (₽/m^3) and for reinforcement of the A400 class, with a diameter of 14–18 mm, 35,000 rubles/ton (₽/t). The use of other classes of reinforcement and concrete, of various strengths and costs, including in other countries, does not change the outlined approach to calculating the strength and price indicators of bent elements.

The investigated parameters were the costs of longitudinal reinforcement and concrete and their cost in the product with the following initial data. The height and width of the section are variable, the coefficient η_b is constant or the coefficient η_b is variable, the height of the section and the width of the section are constant.

4 Results and Discussion

The research results for the B2 beam are shown in Tables 1 and 2.

The table shows that with a fixed value of $\eta_b = 0.2$, the predominant component in the element price is the cost of reinforcement. The percentage of the cost of reinforcement and concrete does not depend on the height and width of the section. With the accepted class of concrete B20 and reinforcement A400, this is 76.9% for reinforcement and 23.1% for concrete. In Table 2 these data are highlighted in bold.

A graphical interpretation of cost indicators, under these conditions, is shown in Fig. 1.

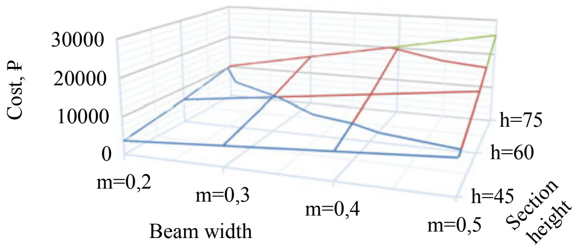
Table 1 Absolute and relative cost of B2 beam materials with variable width and height of the section and a fixed value $\eta_b = 0.2$

Section width b (at m)	$h = 45$ cm ($n = 0.075$)			$h = 60$ cm ($n = 0.1$)			$h = 75$ cm ($n = 0.125$)		
	A_s	B	Σ	A_s	B	Σ	A_s	B	Σ
	₽/%	₽/%	₽/%	₽/%	₽/%	₽/%	₽/%	₽/%	₽/%
$b = mh$ ($m = 0.2$)	2836.9	850.5	3687.4	5043.3	1512.0	6555.3	7880.2	2362.5	10,242.7
	76.9	23.1	100.0	76.9	23.1	100.0	76.9	23.1	100.0
$b = mh$ ($m = 0.3$)	4255.3	1275.8	5531.1	7565.0	2268.0	9833.0	11,820.3	3543.8	15,364.0
	76.9	23.1	100.0	76.9	23.1	100.0	76.9	23.1	100.0
$b = mh$ ($m = 0.4$)	5673.7	1701.0	7374.7	10,086.6	3024.0	13,110.6	15,760.4	4725.0	20,485.4
	76.9	23.1	100.0	76.9	23.1	100.0	76.9	23.1	100.0
$b = mh$ ($m = 0.5$)	7092.2	2126.3	9218.4	12,608.3	3780.0	16,388.3	19,700.5	5906.3	25,606.7
	76.9	23.1	100.0	76.9	23.1	100.0	76.9	23.1	100.0

Table 2 The absolute and relative cost of materials in the B2 beam at constant width and height of the section ($m = 0.3$; $n = 0.1$) and variable values of η_b

η_b	₽			%		
	A_s	B	Σ	A_s	B	Σ
0.05	1891.2	2268.0	4159.2	45.5	54.5	100.0
0.1	3782.5	2268.0	6050.5	62.5	37.5	100.0
0.15	5673.7	2268.0	7941.7	71.4	28.6	100.0
0.2	**7565.0**	**2268.0**	**9833.0**	**76.9**	**23.1**	**100.0**
0.3	11,347.5	2268.0	13,615.5	83.3	16.7	100.0
0.39	14,751.7	2268.0	17,019.7	86.7	13.3	100.0

Fig. 1 The cost of materials for the B2 beam



It should be borne in mind that the results obtained correspond to the full use of the strength of the reinforcement used. The influence of the variable height of the compressed zone, expressed through the coefficient η_b, is shown in Table 2.

The graphs presented in Fig. 2 illustrate the dynamics of changes in the cost of concrete and reinforcement in a product.

The cost indicators of the B2 beam were considered with constant width and height of the section ($m = 0.3$; $n = 0.1$) and variable values of η_b. It is significant

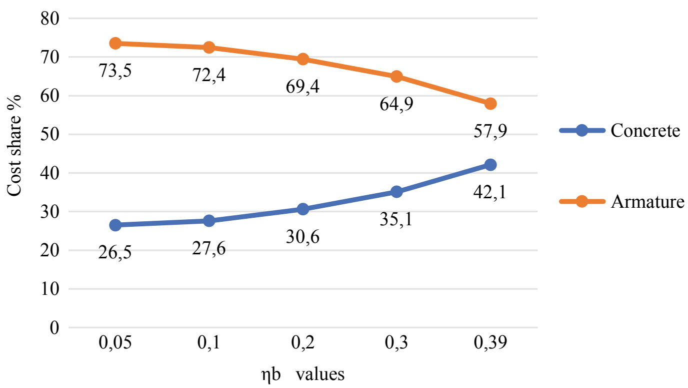


Fig. 2 The ratio of the cost of materials in the B2 beam (n = 0.1, m = 0.3)

that with an increase in η_b, the amount of reinforcement increases. When the strength of concrete and reinforcement is completely exhausted, that is, with $\eta_b = 0.39$, the amount of reinforcement becomes maximum. And this design situation leads to the creation of structures of maximum value.

However, in design practice, when determining the number of reinforcement, the value of the coefficient η_b (α_m) is often much less than the boundary value. This makes it possible to obtain elements of a given bearing capacity, but at a lower cost, when η_b (α_m) changes.

5 Conclusions

The conducted research allows designing flexible elements of a rectangular profile taking into account the real stress state and the cost of the materials used.

The applied technique is universal and applicable for other reinforced concrete structures used in construction practice in various countries.

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