Large-Scale-Motions and Self-excited Clustering of Coherent Structures in Wall Turbulence

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Abstract We revisit clustering of coherent Reynolds shear stress producing eddies in wall bounded turbulent flows. We question the plausibility of the strict alignment of the packets of vortices that supposedly lead to very-large-scale motions. Attention is later drawn to the self-exciting Hawkes stochastic processes to model and understand the organization of coherent vortices into clusters, and their spatial organization in the logarithmic layer.

1 Introduction

The main aim of this short note is to introduce and propose adequate stochastic point processes to describe and model large-scale (LSM) and very-large scale (VLSM) motions occurring in wall bounded turbulent flows. The LSM originate from the packets of Reynolds (Re) shear stress producing eddies in the buffer layer (quasi-streamwise vortices, QSV), at small-medium and large Re numbers [5, 7]. The origin of very-large-scale motions (VLSM) at large Re is still not clearly understood, but one of the conjectures is the coherent alignment of LSM and the wakes they induce into the subsequent packets [5] and their immersion in the logarithmic layer. Figure 1 shows the top view of the quasi-streamwise vortices populating a turbulent channel flow obtained through direct numerical simulations (DNS) in particularly large computational domains at $Re_\tau = \frac{h\overline{u}_\tau}{\nu} = h^+ = 590$. Here h is the channel half width and $\overline{u}_\tau$ and ν are respectively the shear velocity and the viscosity. Hereafter (+) denotes the quantities scaled by $\overline{u}_\tau$ and ν. Note the large spanwise extend of the cluster. It is also seen that the very-large scale motions have a direct impact on the weighted spectra of the turbulent streamwise velocity intensity in the logarithmic layer.

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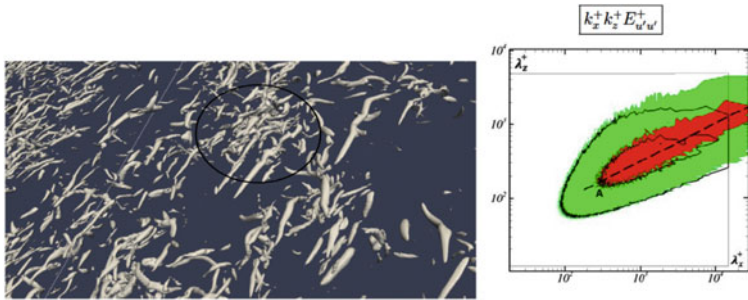e-mail: sedat.tardu@legi.grenoble-inp.fr

Fig. 1 Left: Top view of QSV's in a turbulent channel flow at $Re_\tau = 590$ resulting from our own DNS. Clusters of QSV appear as amalgamation of LSM's at different wall normal positions into a large bulge. The radius of the circle is about $10h$. Right: weighted streamwise velocity fluctuations spectra in the log-layer. Black and colour contours correspond to $h^+ = 395$ and 1100 respectively [10]

2 Coherent Alignment of One-Dimensional (1D) Clusters. Structuration of VLSM

The cumulative probability distribution (cpd) of the QSV interarrival times in the buffer layer shown in Fig. 2-left, is reminiscent of a double Poisson process. The passage of the QSV are detected by single point measurements through different identification schemes combined with a self-similar pattern recognition technique [7]. The results in Fig. 2 have been obtained in a turbulent channel flow at $h^+ = 560$. Two categories of events emerge, those that are close (packets) and the solitary single-events. A Markov chain that contains 3 states is introduced here to model this double Poisson process as in [3] (Fig. 2, right). The state 0 stands for the presence of a QSV within a given interval of time δt. The states 1 and 2 are the waiting states with absence of QSV. The arrows are annotated with their respective probabilities of occurrence. The quantity A is a measure of clustering and t_e is the mean regeneration period of the structures. The Markov chain perfectly estimates the cpd of the interarrival times (squares in Fig. 2 left) as expected. The evaluation of the supposed coherent alignments of the packets that presumably result in VLSM requires the use of a hidden Markov chain. Yet the part of the Markov chain shown by the circle in broken lines in Fig. 2 allows as to estimate the *stricto sensu* alignment probability of the clusters. We found that the probability of having Ξ aligned 1D clusters, which uniformly contain N individual vortices, is proportional to

$$logp(\Xi > 1) \propto (\Xi - 1)(N + 1)logA \tag{1}$$

and since the measure of clustering is $A < 1$, this probability decreases both with N and Ξ. That is, the larger the LSM, the rarer their coherent alignment into VLSM. Coherent alignment of clusters of structures originating from roughly the same buffer
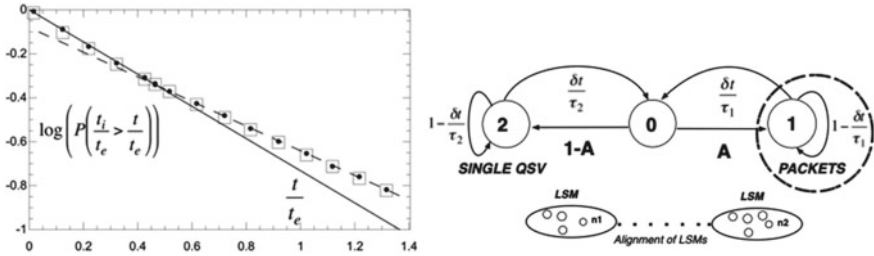
Fig. 2 Left: Cumulative probability distribution (cpd) of the interarrival times of QSV's in the buffer layer. Further details of the methodology can be found in [7]. Circles and squares show the measurements and the model estimations respectively. Right: Markov chain modelling the double Poisson distribution

layer streak, as suggested by [5] is therefore unlikely. This constitutes, at least a formal argument justifying the observations and discussions related to the vortex clusters in the logarithmic region in [2].

3 Self-exciting Processes and Regeneration of Large-Scale-Motions

The clusters of the near wall structures are provoked by a primary structure which is sufficiently intense, and/or close to the wall (Fig. 3 left). This has been shown both experimentally and through direct numerical simulations in [7, 11]. A mother structure with large $s^+ = \Gamma/a^{+2}$, wherein Γ is the circulation of the primary vortex and a^+ is its distance to the wall gives birth to a cascade of active eddies. The characteristic time-scale of the viscous-inviscid response related to the primary structure is $1/s^+$.

We propose to model the buffer layer clusters through a self-exciting non- homogeneous Hawkes Poisson point process [4]. In the Hawkes terminology, the primary intense vortex of the cluster is an immigrant and the provoked subsequent structures are the offsprings. These processes are commonly used in seismology (earthquake
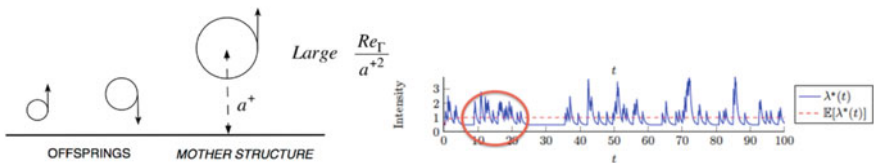

Fig. 3 Left: 'mother' structure with a large enough intensity, and close to the wall regenerates secondary structures according to [7, 11]. Right adapted from [6] with the consent of the authors: The arrival rate in time of a typical Hawkes process [6]. The red circle shows a typical 'packet'

and subsequent aftershocks), neuroscience, epidemiology, insurance and finance. They are long-range dependent. Our aim in this short note is to draw our community's attention to these peculiarities. Consider the process with arrival (occurrence) rate in time:

$$\lambda^*(t) = \lambda + \sum_{t_i < t} \alpha e^{-\beta(t-t_i)} \tag{2}$$

Each arrival in the system increases the arrival intensity by α, then this arrival's effect decays at rate β [6]. This is a non-homogeneous Poisson process in which the intensity explicitly depends on previous events. The excitation function which is exponential, is connected to the regeneration of the QSV packets in the buffer layer. The red circle in Fig. 3b, which is adapted from [6] shows such a self-excited packet. The covariance density of this point process is:

$$R^c(\tau) = \overline{\lambda}^* \delta(t) + \frac{\alpha\beta\lambda(2\beta - \alpha)}{2(\beta - \alpha)^2} e^{-(\beta-\alpha)\tau} \tag{3}$$

The first term of this equation, wherein δ is the Dirac function, stands for a classical non-excited Poisson process and $\overline{\lambda}^* = \frac{\lambda}{1-\alpha/\beta}$. The second term shows that the process is long range dependent, with a typical relaxation time $t_r = (\beta - \alpha)^{-1}$. The original Hawkes's process is one of the self-exciting processes, but other types of excitation processes can also be considered. In the present context, the parameter α is typically the rate of arrivals of the structures into packets. The parameter β is such that at the trailing edge (the end) of a packet the arrival rate α is substantially reduced. We have experimental data to roughly estimate α and β in the buffer layer, but unfortunately only at $h^+ = 560$ [7], and we are not aware of more data analyzing the regeneration process this way. Thus, $\alpha \propto 1/\Delta_P$ where Δt_P is the time interval separating the consecutive events within the packets. Supposing that β dumps the arrival rate by two decades at the leading edge of a cluster, gives $\beta \propto 5/T_P$ where T_P is the typical time duration of the packet. Using the data in [7] one obtains $\alpha^+ \propto 0.05$, $\beta^+ \propto 0.08$ resulting in the memory time of the process $t_r^+ \propto 30$ which is as large as the outer time scale at this Re_τ. Using a convection velocity of 10 wall units in the buffer layer results in the streamwise extends of $l_\alpha^+ = 200$ and $l_\beta^+ = 125$ that are within the range of the lengths of QSV. It has to be emphasized here that the choice (estimation) of α and β is crucial as significantly longer memory periods can result when approaching the excessive case $\beta \to \alpha$. Maximum likelihood estimations are needed to determine these parameters more precisely [6]. The packets form large wakes of typical streamwise lengths $L_w \approx 5h$ in the low buffer [1, 7]. The clusters transport the wakes into the outer region by the lift-up velocity field they induce according to the Biot-Savart law. Hence, the streamwise and spanwise length of the multiple-wakes increase linearly with the wall distance [2]. The wake structure stays self-similar during the long memory time t_r, as the tall self-similar attached clusters grow self-similarly in time in [2]. The lifetime of the wakes is much larger than t_r. Once the wake clusters merge in the log-layer they lose their initial memory and the trails of the streamwise velocity fluctuations they transport adapt themselves to the
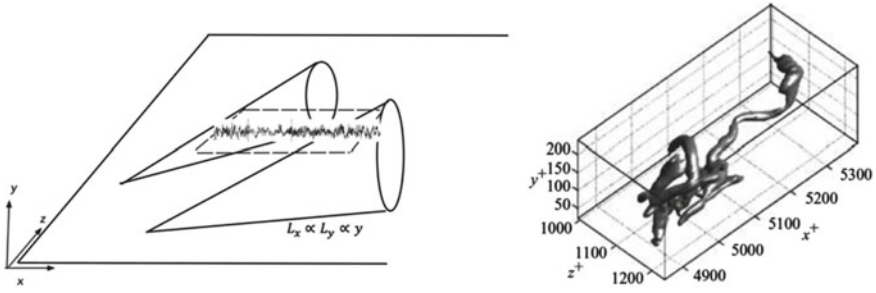
Fig. 4 Conceptual model of the merging of LSM induced wakes into the logarithmic layer (left). Two clusters of wakes coming from different origins shifted in the spanwise direction can lead to persistent streamwise velocity fluctuations in the intersection plane. Right: an attached cluster according to [2] adapted with permission

outer scales. According to our estimations given above, the multiple-wakes reach $y^+ \approx 300$ self-similarly during t_r. Their streamwise and spanwise scales increase by the same amount, attaining roughly $6h$. Although the *stricto sensu* coherent alignment of the clusters is unlikely as discussed before, the alignment of the multiple wakes formed by different buffer-layer clusters with different past stories and that are close in the streamwise direction and (or) shifted in the spanwise one, now becomes plausible. This is schematically shown in Fig. 4, which also shows a typical attached cluster explored in [2] and originating roughly from a region of about 100 wall units spanwise extend. Last but not least, the cluster excitation parameter α and consequently β depends on the Reynolds number. Indeed, the regeneration of offsprings depends on the vorticity intensity of the mother structure and the streamwise vorticity intensity at the wall is Re dependent [9].

4   Conclusion

Self excited processes are good candidates to model the clustering of coherent structures and of their wakes emerging into VLSM in the logarithmic layer. It is unlikely that the vortex clusters originating, say from the same streak in the buffer layer, get coherently aligned to form VLSM in the logarithmic region. The main point is the spanwise expansion of the wakes generated by individual clusters. This consequently makes possible the amalgamation of different wake clusters with different histories, coming from different, but close enough streamwise and spanwise locations. The VLSM result from a self-exciting spatio-temporal Poisson cluster process, in which the spanwise direction plays a capital role in the present context. Existing models such as the Townsend-Perry hierarchy should be modified to take these specific features into account (see [8] for a review). To be short, the contribution of the hierarchies to the shear stresses in the fully developed turbulent region is related to the

probability density for the length scale in the hierarchy. This probability density has to be extended to contain informations on the streamwise and spanwise extends of the clusters. The events in the clusters have to be marked by the wake they induce, which is an additional important variable. The self-exciting feature of the regeneration process has to be taken into account. We are currently working on these key issues.

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