Vibration Control of Bridge Suspenders Using TMD and RTLD—A Comparative Study

Nihad Mohamed Ali and A. S. Sajith

Abstract Vibration control in civil engineering structures has received much attention recently, specifically in tall buildings and slender, wind-sensitive structures. Suspender cables are fragile and sensitive elements of a suspension bridge. Extreme weather can cause significant vibrations in these cables, which can jeopardize traffic safety. As a consequence, designing effective methods to reduce such vibrations are important. In this paper, study on a relatively new technique of using a Ring Tuned Liquid Damper (R-TLD) to minimize wind induced vibrations of bridge suspenders and its comparison with a Stockbridge Damper is presented. A numerical study is carried out by modeling and analysis in MATLAB, and ideal design parameters are optimized. Also, a simulation of the vibration and forced excitation is created using SIMULINK.

Keywords Structural control · Ring-TLD · TMD · Bridge suspenders

1 Introduction

Suspender cables are the components that are most fragile in a suspension bridge. In general, the vertical loads on the bridge deck coming on to the suspender cable can be catered by adequate design; however, the dynamic loads on the suspender cable are more complicated and critical, necessitating more care. The key causes of a suspender cable's dynamic response are ambient excitation (wind loads, ground motions, etc.) and traffic loads.

Vibration control is mainly incorporated in problems related to aerospace like tracking and pointing and also in flexible space structures. The technology moved quickly into issues related to infrastructure and civil engineering, especially, in the context of protection of bridges and buildings from severe loads of wind and earthquake. Control of structural vibration caused by earthquake and wind can be done by different ways like modifying masses, rigidities, damping or shapes and

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also by providing either active or passive counter forces. A number of factors govern the selection of each type of vibration control device like efficiency, capital cost, operating cost, compactness and weight, requirements of maintenance and safety. The use of passive control devices such as Tuned Mass Dampers (TMD) [1] and Tuned Liquid Dampers (TLD) to control the response of structures has proven to be very effective in structural control. Furthermore, the implementation and monitoring are quite simple for these devices. Another very important aspect is that in an active control system, control action is affected by external power and hence is vulnerable to power failure. This is always a possibility during an earthquake and in such circumstances a passive device such as TMD or TLD is more reliable in attaining the intended purpose.

The idea of TMD was first used by Frahm in 1909 to minimise the ship's rolling motion and hull vibration. Following that, Ormondroyd and Den Hartog introduced a theory of TMD [2], which was accompanied by a thorough examination of optimal tuning and damping parameters in Den Hartog’s book on mechanical vibration. An undamped SDOF device subjected to a sinusoidal excitation was the basis for the initial theory. Numerous researchers have looked into applying the principle to damped SDOF structures [3].

G. W. Housner et al. [4], established a succinct source for researchers and practitioners looking to assess the latest advances in civil engineering structure control and monitoring. They also gave a connection between the structural control and other fields of control theory, mentioning both similarities and variations along with pointing out its future scope of application.

The use of Ring-Shaped Tuned Liquid Dampers (RSTLD) for vibration mitigation was suggested by An et al. [5], who discovered that it has equally strong vibration-mitigation potential in all directions of excitation. RSTLD’s inner and outer diameters, the amount of liquid, the form of liquid, and the inner liquid barriers are the different parameters on which the effect of vibration mitigation was designed.

Di et al. [6], in their paper studied the effectiveness and robustness of Stockbridge Damper (SD) in vibration control of bridge suspenders. Using a simplified design approach based on full-scale suspender experiments, SDs are designed to monitor first and second-mode cable vibrations.

Modi and Seto [7] analysed rectangular TLDs numerically when accounting for nonlinear effects. Wave dispersion, boundary layers at the walls, floating particle interactions at the free surface, and wave breaking were addressed.

2 Modelling and Analysis

Stockbridge Damper is a basic example of dynamic vibration absorber, generally used in stay cables. No significant work has been reported on the problem of lateral vibration of vertical suspenders, which is in fact a critical component of a suspension bridge considering traffic safety and serviceability. The equations governing the system is modelled along with the control force given by the dynamic vibration absorbers.

An abridged numerical model of the chosen suspenders was created in MATLAB to ease the modelling process and reduce the computational workload. The basic assumptions in the modelling ignores non-linear geometry as lateral displacements are assumed to be small and the suspender acts in the linear elastic state obeying Hooke's Law.

2.1 Cable Parameters

Cables are flexible members which can withstand tension effectively. It is helpful to imagine the cable as a piece of rope. Because of its superior flexibility, it is unsuitable for compression, bending, or shear. A cable is an important component for a cable-supported bridge's overall structural stability, such as the main suspension bridge cable, the suspension bridge hanger, and so on.

The relation between frequency and cable forces can be formulated, ignoring cable sag and accounting for bending stiffness. The cable can be treated as a beam under Axial tension. The motion equation is [8]:

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} - T\frac{\partial^2 v(x,t)}{\partial x^2} + m\frac{\partial^2 v(x,t)}{\partial t^2} = 0 \tag{1}$$

where EI is the bending stiffness of the cable. The equation is solved using cable's boundary conditions. When it's simply supported at both ends:

$$\omega_n^2 = \left(\frac{n\pi}{l}\right)^2\frac{T}{m} + \left(\frac{n\pi}{l}\right)^4\frac{EI}{m} \tag{2}$$

where

ω_n	Fundamental frequency of system.
n	Represents mode of vibration.
T	Pretension in cable.
m	Cable's mass per unit length.
l	Length of the cable.

2.2 Parameters of the Model Used

See Table 1 [6].

Table 1 Parameters of the pedestrian bridge model

Span (m)	Diameter (mm)	Mass/length (kg/m)	Pretension (kN)	Modulus of elasticity (GPa)	Natural frequency (rad/s)
36	55	9.6	169	210	11.579

3 Numerical Analysis

The following is the equation of motion governing the system.

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = F_{EXT} + F_{CTRL} \tag{3}$$

where

F_{EXT} External force on the system.
F_{CTRL} Control force given by the Damper.

3.1 State Space Formulation

In this research, the governing equation of motion is modelled and solved through the state space approach. The space whose axes are the state variables is referred to as "state space" within that space, the state of the system can be represented as a vector. The state-space representation of a linear system with u inputs, y outputs, and n state variables is as follows.

System Response = *f* (current state).

Set of State Variables—(Position, Velocity)

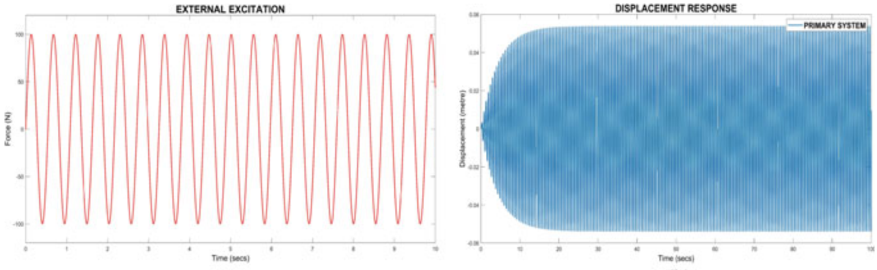
$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4}$$

$$y(t) = Cx(t) + Du(t) \tag{5}$$

4 Results of Numerical Analysis

4.1 Primary System

The present study considers a generalized single degree-of-freedom system (SDOF) model for the suspender cable, the total mass is assumed to be lumped at the central node of the suspender and the structure is taken as steel with damping (ς) as 2% of critical damping. The suspender considered is the longest cable of a pedestrian bridge. The parameters are adopted from the literature [6].



(a) (b)

Fig. 1 **a** Sinusoidal excitation on the structure. **b** Displacement response of the suspender

Analysis is done on the structural model assuming that it is excited by a single component of force at the central node and is coded in MATLAB.

The SDOF model is analysed under sinusoidal excitation of 100 N in resonating condition and the responses without DVA shown in Fig. 1.

4.2 Control System with Stockbridge Damper (SD)

Stockbridge damper-design. As shown in Fig. 2b, A simple SD is made up of two bell- or horseshoe-shaped masses and a messenger cable. Both ends of the messenger cable are connected to the masses, and the damper is clamped to the cable. Same of masses are attached at both ends, and the clamp is located in the middle.

The MATLAB code developed for the SDOF system is amended with a Control system where a Stockbridge Damper (SD) is introduced (Fig. 2b). A two Degree of freedom model with primary system being the suspender and auxiliary system being the Stockbridge Damper attached (Fig. 2c) is analysed. Here the inherent damping of the absorber is neglected. The mass of the damper is fixed as 5% of that of the Suspender. The material of the messenger cable is an aluminium rod of diameter 12 mm. The stiffness has been fine-tuned by choosing a leg length so as to match the fundamental frequency of the suspender. A sinusoidal excitation of amplitude 100 N was applied on the controlled system at resonating condition. The response obtained is shown in Fig. 3a.

From numerical analysis the desired leg length was found to be 283 mm. The reduction in response attained was close to 99% in the ideal condition i.e., the inherent damping of the absorber being neglected. It was observed that the time to reach steady state was around 35–45 s.

Upon optimising the model parameters, a more realistic model was done having a damping ratio of around 3% for the absorber and the response was compared with the model without SD (Fig. 3b), the reduction in response was around 94%. The MATLAB code was proof checked using SIMULINK environment and the percentage error was very minimal.

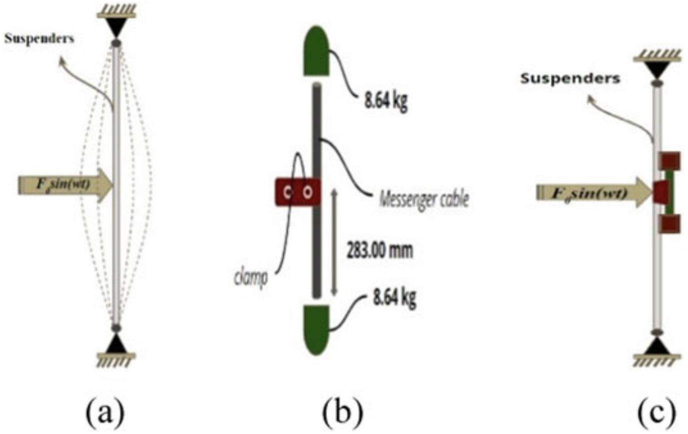


Fig. 2 **a** Primary system. **b** Stockbridge damper. **c** Suspender equipped with SD

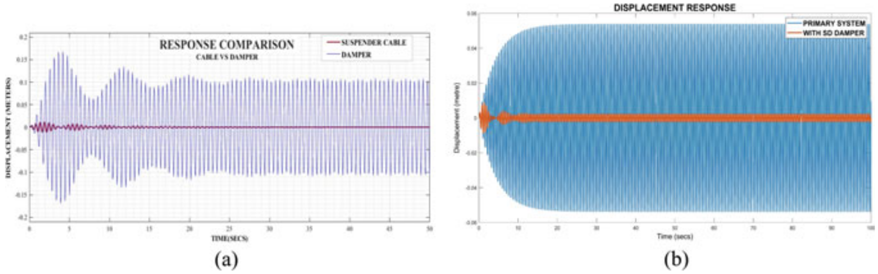


Fig. 3 **a** Ideal tuned response of a SD. **b** Realistic tuned response of a SD

4.3 Control System Annular TLD/R-TLD

RTLD Design. Annular TLD or Ring TLD are special type of Tuned Liquid Dampers which are effective in vibration mitigation of axisymmetric elements. Conventional TLDs adopted are in the form of rectangular or cylindrical sections which actually require a lot of space for installation and operation. This makes it difficult, if not impossible, for applications in tall slender sections owing to lack of space availability. Alternatively, this study presents an annular or ring-shaped TLD which can easily be installed in the cable without disturbing other mechanical devices or components of the structure. The main advantage is the ease of tuning and material cost and maintenance itself. In order to construct an efficient and secure TLD, the liquid's sloshing motion must be well understood. The key design parameters are the mass ratio, the frequency of the liquid sloshing motion, and the intrinsic damping of the TLD [9]. The strength of the sloshing motion is affected by the external excitation used, the tank's layout, the depth of the liquid layer, the properties of the confined liquid, and so on.

The natural sloshing frequency of a Ring-TLD is given by the expression, [10]

$$\omega_n{}^2 = \frac{g}{R}\varepsilon_n tanh\left(\frac{\varepsilon_n h}{R}\right) \tag{6}$$

where

ω_n = natural sloshing frequency, g = acceleration due to gravity, R = Outer radius of the tank, h = height of liquid in the tank, ε_n constant depending on value of (r/R) [11].

The parameters of the RTLD were chosen so as to meet the natural frequency of the suspender cable system. The outer diameter of the cylinder was designed to be 240 mm, height of tank was fixed as 500 mm, the liquid level required was found to be 248 mm, the mass ratio of the system was around 3%, the liquid in the system is assumed to be water. A schematic of the system is shown in Fig. 4.

A MATLAB code is developed to solve the equations of the RTLD provided, then the code was clubbed with SIMULINK to simulate the suspender cable-RTLD system,

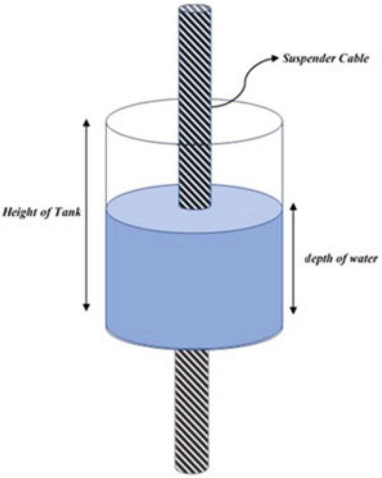
The relevant equations used for the control force are given as follows:

$$F_{RTLD} = mM_L\left\{\ddot{q}_{(t)}F_1 d_1[1 + kR_1 k(\varepsilon_n)] + (1 + k^2)\ddot{x}_{(t)}\right\} \tag{7}$$

$$\ddot{q}_{(t)} + 2\varepsilon_l\omega_l\dot{q}_{(t)} + \omega_l{}^2 q_{(t)} = -\ddot{x}_{(t)} \tag{8}$$

$$d_1 = [2(1 - kR_1)/[(\varepsilon_n^2 - 1) - (k^2\varepsilon_n^2 - 1)R_1^2], \quad F_1 = \tanh(\varepsilon_n c)/\varepsilon_n h_0$$

Fig. 4 Schematic of the RTLD



where,

$$R_1 = \alpha_1 J_1(\,\varepsilon_n k) + \beta_1 Y_1(\varepsilon_n k), \quad \alpha_1 = \frac{1}{J_1(\varepsilon_n)} + \left[\frac{J_1'(\varepsilon_n)}{Y_1'(\varepsilon_n)}\right] Y_1(\varepsilon_n)$$

$$\beta_1 = \left\{\frac{J_1'(\,\varepsilon_n)}{Y_1'(\,\varepsilon_n)}\right\}\alpha_1,$$

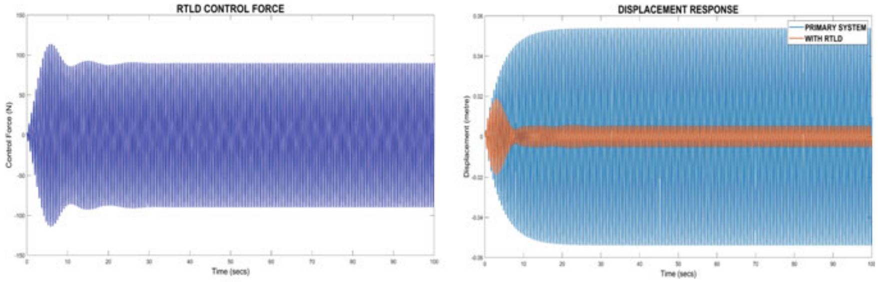
where

m = number of dampers, M_L = mass of liquid, $h_0 = h/R$, h being height of liquid, R being outer radius of the cylinder, k = outer radius to inner radius ratio, ε_l = liquid viscous damping ratio, ω_l = frequency of liquid sloshing, $q_{(t)}$ = generalised liquid motion in the container, $x_{(t)}$ = motion of the suspender cable. R_1, d_1 = relative parameters of the equation, $J_1(\varepsilon_n)$ = first order Bessel's function of first kind, $Y_1(\varepsilon_n)$ = first order Bessel's function of second kind.

Same set of excitation parameters that were used for the previous model are used here also and response is plotted:

The reduction in response was observed around 99%. It may be noted that the system achieved steady state in around 45–55 s. This may be attributed to the fact that SD is a discrete system where as the R-TLD a fluid continuum which require more time to achieve stability. Furthermore, it can be noted that R-TLD possess reduction capability in all directions of excitation.

A more practical model with a damping ratio of around 0.5% was given to the absorber which is water here and the response was compared with the model without control, the reduction in response was around 91% (Fig. 5b).



(a) (b)

Fig. 5 **a** Control force given by the RTLD. **b** Displacement response

5 Conclusions

This paper proposed a comparative study for vibration mitigation methods for slender structures such as the suspender cable in a suspension bridge. The research mainly focused on the design and comparison of a Stock bridge damper and a Ring tuned liquid damper. All the numerical procedures were coded through MATLAB and the validation of the vibration mitigation effect was done through simulation using SIMULINK. Conclusions drawn from the study are summarised below.

- Both the proposed methods dramatically reduced the vibrations in the targeted mode, the response reduction with SD and RTLD employed was comparable.
- As in the case of SD employed, better reduction in peak response was observed in the case of resonant harmonic excitations when compared with the case of R-TLD.
- In the case of Stockbridge Damper, the mitigation can be brought about adjusting the leg length of the messenger cable and cantilever tip mass attached at the ends as well as changing the material which in turn changes the inherent damping of the absorber.
- For the case of RTLD there was very good reduction in peak response but the time to achieve steady state was a bit greater compared to SD, this can be attributed to the fact SD is a discrete system where as RTLD a continuum.
- Compared to SD the designed RTLD has vibration reduction capability in all directions of excitations and thus is more suitable for axisymmetric elements such as the suspender cables. The main advantage is the ease of tuning and material cost and maintenance itself.

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