Calculation of the Endurance of Reinforced Concrete Bending Elements by the Method of Limit Stresses

Ilizar Mirsayapov(✉)

Kazan State University of Architecture and Engineering, Kazan 420043, Russia

Abstract. In reinforced concrete bent structures under cyclic loading of the stationary regime, inelastic deformations of vibration creep in the concrete of the compressed zone form and develop under connected conditions. For this reason, the conditions for the deformation of concrete in the compressed zone are non-stationary, even when the external load is stationary. Experimental and theoretical studies of the behavior of the reinforced concrete bending element were carried out. The deformation mode of the concrete of the compressed zone of the bending element was established under the stationary mode of cyclic loading. To assess the endurance of concrete compressed zone under such deformation conditions, studies were carried out using methods of fracture mechanics of elastic-plastic materials and equations of endurance of compressed zone concrete for non-stationary deformation conditions were obtained. On the basis of the conducted research, the equation of the endurance of concrete of the compressed zone is developed for practical calculations of reinforced concrete bending elements under stationary conditions of repeated cyclic loading. The proposed method allows the most accurate assessment of the stress-strain state of concrete in the compressed zone and the processes of concrete change from the point of view of fracture mechanics, which is a significant contribution to the theory of fatigue strength and provides concrete savings of up to 25% compared to existing methods.

Keywords: Reinforced concrete · Compressed concrete zone · Endurance · Cyclic loading · Stationary loading · Mechanics of fracture · Vibration creep · Inelastic deformations

1 Introduction

Under the action of repeated cyclic loads of stationary mode in the concrete of the compressed zone of reinforced concrete bending elements, inelastic deformations of vibration creep are manifested and develop. Due to the fact that vibration creep deformations development under cohesive conditions, additional stresses appear in the concrete of the compressed zone and the longitudinal stretched reinforcement as the number of loading cycles increases. In this case, simultaneously with the change in the stresses in the concrete of the compressed zone of the bent element, the coefficient of asymmetry of the loading cycle also changes [1–6]. In the process of cyclic loading in the concrete of the

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compressed zone of the bent element, the stresses decrease and the load cycle asymmetry coefficients increase with an increase in the number of loading cycles [7–14]. In this regard, there is a need to develop a new method for calculating the fatigue strength of concrete in the compressed zone of bent reinforced concrete elements under stationary cyclic loading conditions [14–19, 20].

2 Materials and Methods

When the load is applied repeatedly, the change in the stress-strain state over time of the reinforced concrete bending bar can lead to the fact that the limit state will occur as a result of the exhaustion of the resource of concrete or reinforcement. Therefore, to assess the endurance of a reinforced concrete element, it is necessary to be able to assess the bearing capacity of concrete in a compressed zone.

The current stresses $\sigma_b^{max}(t,\tau)$ in the concrete of the compressed zone are represented as the sum of the initial $\sigma_b^{max}(t_0)$ and additional stresses $\sigma_b^{add}(t)$. Additional stresses in the concrete of the compressed zone due to the vibration creep of the concrete under cohesive conditions are based on (1) and (3). It follows that, with an increase in the number of loading cycles, the stresses in the concrete of the compressed zone decrease.

$$\sigma_b^{max}(t,\tau) = \sigma_b^{max}(t_0) + \sigma_b^{add}(t) \tag{1}$$

$$\sigma_b^{max}(t_0) = \frac{M_{max}}{\omega \cdot b \cdot x(h_0 - \gamma \cdot x)} = \frac{2M_{max}}{\xi\left[(1+\lambda) - 0.33\xi\left(\lambda^2+\lambda+1\right)\right]b{h_0}^2} \tag{2}$$

$$\sigma_b^{add}(t) = \left\{-\frac{h_0}{x} \cdot E_s \int_{t_0}^{t} \sigma_b^{max}(\tau) \frac{\partial}{\partial \tau}\left[\frac{1}{E(t)} + C(t,\tau)\right]dt\right\} \cdot A_s\left[\frac{1}{A_{red}} - \frac{e_0\left(h - x_p\right)}{\mathcal{J}_{red}}\right] \tag{3}$$

$$\xi = \frac{-\mu\alpha \pm \sqrt{\left(\mu\alpha^2\right) + 2\mu\alpha\left(1-\lambda^2\right)}}{1-\lambda^2}$$

Where ξ relative height of the compressed zone, $\mu = \frac{A_s}{bh_0}$, $\alpha = \frac{E_s}{E_b}$.

At the initial loading stage, the stress cycle asymmetry coefficient in the concrete of the compressed zone $\mathcal{P}_{bt_0}$ is equal to cycle asymmetry coefficient of the external load $\mathcal{P}_M$. Under the action of cyclic loads due to the manifestation of vibration creep of concrete in the associated conditions, there is a continuous change in $\mathcal{P}_{bt}$. At the time (t), the stress cycle asymmetry coefficient of the concrete of the compressed zone can be represented as:

$$\mathcal{P}_b(t) = \frac{\sigma_b^{max}(t)\mathcal{P}_M - \frac{h_0}{h}E_sA_s\left[\frac{1}{A_{red}} - \frac{e_0(h-x_p)}{\mathcal{J}_{red}}\right] \cdot \int_{t_0}^{t}\sigma_b^{max}(\tau)\frac{\partial}{\partial\tau}\left[\frac{1}{E_b(t)} + C(t,\tau)\right]dt}{\sigma_b^{max}(t_0) - \frac{h_0}{h}E_sA_s\left[\frac{1}{A_{red}} - \frac{e_0(h_0-x_p)}{\mathcal{J}_{red}}\right] \cdot \int_{t_0}^{t}\sigma_b^{max}(\tau)\frac{\partial}{\partial\tau}\left[\frac{1}{E_b(t)} + C(t,\tau)\right]dt} \tag{4}$$

It follows from (4) that as the number of loading cycles increases, the stress cycle asymmetry coefficient of concrete in the compressed zone decreases.

It is known that the objective strength of concrete under repeated cyclic loads is less than the short-term strength. In the calculations, this circumstance is taken into account by introducing a coefficient $k_{b\mathcal{P}}$ to the short-term strength corresponding to the moment of application of the load τ.

$$k_{b\mathcal{P}} = \frac{R\prime_b(t)}{R_b(\tau_1)} \tag{5}$$

Where $R\prime_b(t)$ is the endurance of concrete at the time *t,* $R_b(\tau_1)$ is the short-term strength of concrete at the time of applying a repeated load (τ_1).

In the elements of reinforced concrete structures, the process of reducing strength is manifested in a more complex form. This is due to the fact that in the concrete of the compressed zone of the bent elements, the maximum stress level of the cycle and the stress cycle asymmetry coefficient $\mathcal{P}_{bt}$ change. The endurance curves of concrete can be constructed experimentally. It is practically impossible to obtain the endurance curve of concrete of the compressed zone in reinforced concrete bending elements by experimental means. There are no proven methods for determining stresses in concrete by direct experiment. The construction of such a curve is associated with the need to use certain additional prerequisites. Data on the fatigue strength of concrete in the compressed zone of reinforced concrete bending elements and, in particular, in over-reinforced elements, can be obtained on the basis of the experimental fatigue resistance curve of the reinforced concrete element and the calculated stress values in the concrete. By comparing the stresses in concrete of samples destroyed by a short-term static load at the time τ_1 and after repeated loading, it is possible to obtain data on the effect of previous variable stresses on the short-term strength of concrete in the compressed zone.

Even under stationary loads under conditions of stress varying in maximum level and in amplitude, it is necessary to establish the concept of endurance under these conditions. There are two possible approaches. The endurance of the concrete of the compressed zone can be found in the following way:

- the value of the stress at the time *t* immediately preceding the destruction (hereinafter - the moment of destruction) - $\sigma_{b*}^{max}(t, \tau_1)$;
- the value of the stress at the time of application of the maximum load of the cycle τ_1, at which the fatigue failure will occur at the time *t* - $\sigma_b^{max}(t, \tau_1)$.

The stress change mode relates the values $\sigma_{b*}^{max}(t, \tau_1)$ and $\sigma_b^{max}(t, \tau_1)$. The set of values make up the curves $R_b^*(t, \tau_1)$ and $R_b(t, \tau_1)$ characterising the endurance of concrete under non-stationary stress conditions. It seems that the definition of endurance in terms of $R_b^*(t, \tau_1)$ is more correct from a physical point of view, because it is associated with stresses at the moment of failure. The value $R_b(t, \tau_1)$ is somewhat conditional, since it is associated with the stresses that acted earlier, i.e. at the time of applying the maximum load of the first cycle, leading to destruction. However, when performing practical endurance calculations, the use of the $R_b(t, \tau_1)$ curve is more preferable, because it does not require determining the stresses formed at the time of failure. For $t \approx N = 2.10^6$

cycles the values of $R_b^*(t, \tau_1)$ and $R_b(t, \tau_1)$ characterize the endurance of the concrete of the compressed zone of the reinforced concrete structure. Coefficients $\eta_\sigma^*(t, \tau_1)$ and $n_\sigma(t, \tau_1)$ characterize the relative endurance of concrete in the compressed zone.

$$\eta_{\sigma_b}^*(t, \tau_1) = \frac{R_b^*(t, \tau_1)}{R_b(\tau_1)} \tag{6}$$

$$\eta_{\sigma_b}(t, \tau_1) = \frac{R_b(t, \tau_1)}{R_b(\tau_1)} \tag{7}$$

Due to the fact that the coefficient $\eta_\sigma^*(t, \tau_1)$ is associated with the stresses formed at the moment of destruction, the decrease in its values under the conditions of cyclic load action is associated with three factors: the accumulation of damages, the decrease of stresses and decrease of stress cycle asymmetry coefficient in the concrete of the compressed zone. For using the coefficient $\eta_\sigma^*(t, \tau_1)$, it is necessary to determine the stresses at the moment of failure, which is associated with certain computational difficulties. For calculations, it is more convenient to use the coefficient $n_\sigma(t, \tau_1)$ because it is related to the stresses at the time of load application. In addition, the decrease in the values of $n_\sigma(t, \tau_1)$ over time is due only to a decrease in the fatigue strength of concrete in the compressed zone under variable stress conditions. From the Eq. (3), it can be seen that $n_\sigma(t, \tau_1)$ shows what proportion of the short-term strength should be the stress in the concrete of the compressed zone by the end of loading. The construction provided that fatigue failure occurs at time $t \approx N = 2.10^6$ cycles. Using the Eqs. (8) and (9) we can obtain an Eq. (10) for determining the relative endurance limit of concrete in a compressed zone.

$$\sigma_b^*(t, \tau_1) = \sigma_b(t, \tau_0) \cdot H_\sigma(t, \tau) \tag{8}$$

$$\mathcal{P}_b(t, \tau) = \mathcal{P}_b(t, \phi_0) \cdot H_{\mathcal{P}b}(t, \tau)$$

$$\sigma_b^*(t, \tau_1) = R_b^*(t, \tau) \tag{9}$$

$$\eta_\sigma(t, \tau) = \frac{R_b^*(t, \tau) \cdot H_{\mathcal{P}b}(t, \tau)}{R(\tau) \cdot H_\sigma(t, \tau)} \tag{10}$$

3 Results

The relative endurance limit of compressed zone concrete in reinforced concrete elements does not coincide with the relative endurance limit of concrete prisms at a constant level of maximum cycle stresses, the stress cycle asymmetry coefficient.

For the analytical analysis of the fatigue strength of concrete in the compressed zone, it is necessary to determine the compressive strength of the loaf at variable values of the maximum stresses and the stress cycle asymmetry coefficient.

To determine the fatigue strength of concrete in the compressed zone of a bent reinforced concrete element under the conditions described above, we use the equation of objective strength under non-stationary conditions. To simplify the calculation process, the endurance evaluation is performed at the final stage of the structure operation, immediately preceding the exhaustion of the bearing capacity.

First we calculate the values $\sigma_b^{max}(t_0)$, $\sigma_b^{max}(t)$, $\mathcal{P}_{bt}$, $H_{\sigma_b}(t,\tau)$. Then, based on the fact that the decrease in $\mathcal{P}_{bt}$ and $\sigma_b^{max}(t)$ depends on the vibration creep of the concrete of the compressed zone, the change is divided into n number of stages (steps). In general, the breakdown into stages can be completely arbitrary.

In this case, we assume that the stresses during the transition from one block to another change by the same value $\Delta\sigma_{bi}(t)$. To determine $\Delta\sigma_b(t)$, we divide the value $H_{\sigma_b}(t,\tau)$ by n (where n is the number of loading stages in the block), and then multiply the initial stresses by the same value.

$$\Delta\sigma_b(t) = \frac{H_{\sigma_b}(t,\tau)}{n} \cdot \sigma_b^{max}(t_0) \tag{11}$$

Consider the following sequence of stress changes in the concrete of the compressed zone (Fig. 1).

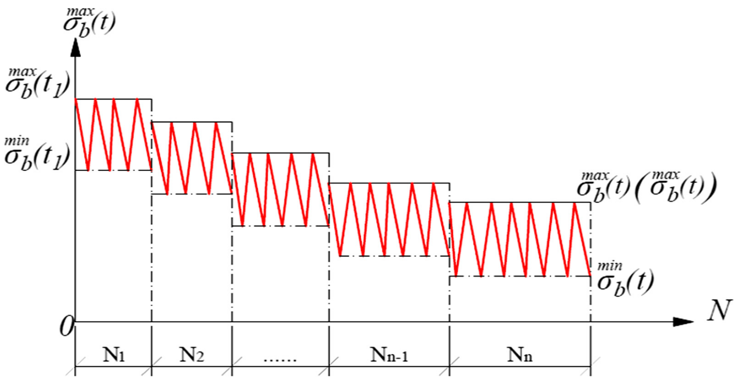

Fig. 1. Diagram of stress changes in compressed zone concrete under repeated loading with M^{max} and $\mathcal{P}_M$ are constant.

In Fig. 1, $\sigma_{b1}^{max} = \sigma_b^{max}(t_0) - \Delta\sigma_b$ – for the number of loading cycles N_1 and the stress cycle asymmetry coefficient P_{b1};

$\sigma_{b2}^{max} = \sigma_b^{max}(t_0) - 2\Delta\sigma_b$ – for the number of loading cycles N_2 and the stress cycle asymmetry coefficient $\mathcal{P}_{b2}$;

$\sigma_{b3}^{max} = \sigma_b^{max}(t_0) - 3\Delta\sigma_b$ – for the number of loading cycles $N3$ and the stress cycle asymmetry coefficient $\mathcal{P}_{b3}$;

$\sigma_{b_{n-1}}^{max} = \sigma_b^{max}(t_0) - (n-1)\Delta\sigma_b$ – for the number of loading cycles $N_{(n\text{-}1)}$ and the stress cycle asymmetry coefficient $\mathcal{P}_{b(n-1)}$;

$\sigma_{bn}^{max} = \sigma_b^{max}(t) - n\Delta\sigma_b$ – for the number of loading cycles N_n and the stress cycle asymmetry coefficient $\mathcal{P}_{bn}$.

With this stress distribution, the number of loading cycles in each block will obey the inequality, $N_1 < N_2 < N_3 < \cdots < N_n$. The stress cycle skewness decrease from block to block, i.e. $\mathcal{P}_{bt_0} > \mathcal{P}_{b_1} > \mathcal{P}_{b_2} > \cdots > \mathcal{P}_{b_n}$.

The objective strength of the concrete of the compressed zone by the time of the end of loading with the above scheme of changes in cyclic stresses is calculated by the formula:

$$R_b(t,\tau) = \frac{2}{b}\cdot\frac{k_{cf}(t)}{\sqrt{\pi\cdot l(t,\tau)\cdot\gamma(l)}}(l_a + 4l_{sh}\cdot tg\alpha\cdot\sin\alpha) =$$

$$= \sqrt{\sqrt{2E_{bt}\int_0^{\varepsilon_R} - \left(\begin{array}{c} \left[\sigma_{ti} + \sum_1^3 min(\varepsilon_R-\varepsilon_{ti})^n\right] d\varepsilon - \\ - \left\{ \begin{array}{c} \sum_1^n\sum_1^{Ni}\left(\left(\begin{array}{c} C_\partial \prod_{k=1}^{k=g} \begin{array}{c} k_k\alpha\psi_{v1i}\sigma_{bt1}^2 + \\ + \mathring{a}_{xi+1}^{nc}\cdot\sigma_{bt1}^{max}(1-\mathcal{P}_{i+1}) - \end{array} \\ -\left(C_\partial\prod_{k=1}^{k=g}k_k\right)_i^1 \cdot\alpha\psi_{vij}(\sigma_{bti}^{max})^2(1-\mathcal{P}_i)^2 * \\ *\left[1+\left(1-\alpha\psi_{vij}\right)^{Ni-1}\right]\end{array}\right)\right) + \\ + \sum_{i=1}^n\sum_1^{Ni}\left(C_\partial\prod_{k=1}^{k=g}k_k\right)_i^1 \alpha\psi_{vij}(\acute{o}_{bti}^{max})^2(1-\mathcal{P}_i)^2 * \\ *\left[1+\left(1-\alpha\psi_{vij}\right)^{Ni-1}\right] + \\ + \sum_{i=1}^n \mathcal{P}_i^2(\sigma_{bti}^{max})^2 C_\infty(t,\tau)\cdot f_i(t,\tau) - \sum_{i=1}^n \varepsilon_{xi}^{nc}\cdot\sigma_{bti}^{max}(1-\mathcal{P}_i)\end{array}\right\} - \\ -\sum_1^{k=1}\Delta W_{npij(j-1)} + \Delta W_c \end{array}\right)} * \\ * k_0^2 \cdot R_{bt,su}\cdot k_{fs}\cdot ds}$$

$$* \frac{2}{\sqrt{\pi}\cdot d_i\,\gamma(l)}\cdot(l_{cl}+4l_{sh}\cdot tg\alpha\cdot\sin\alpha) *$$

$$* \left\{\begin{array}{c} l(t,\tau)+\sum_{i=1}^h l_{MHK} + \\ + \sum_1^n\sum_i^{\text{ж}} \frac{\frac{\langle\langle[k_{\mathrm{I}}\varphi_{11}(\sigma_i)+k_{\mathrm{I}}\varphi_{12}(\acute{o}_i)]\rangle\langle\left[\frac{k_{\mathrm{I}i}\varphi_1(\sigma_i)}{E_{bt}}+\frac{k_{\mathrm{I}i}\varphi_2(\sigma_i)}{E_{bt}}\right]-\int_{t_0}^t[k_{\mathrm{I}i}\varphi_{11}(\sigma_i)+k_{\mathrm{I}}\varphi_{21}(\sigma_i)]\frac{\partial}{\partial\tau}C(t,\tau)d\tau\rangle\rangle^2}{2\pi\left(k_{\mathcal{P}b}R_{bt,\tau}\right)^4 m_j^4(t,\tau)\left[\frac{1}{E_{bt}}C_\partial\prod_{k=1}^{k=g}k_k\alpha\psi_{0ij}\right]^2}}{\frac{\int_0^{\varepsilon_R}\left[\sigma_{ti}+\sum_1^3 min(\varepsilon_R-\varepsilon_{ti})^2 d\varepsilon - \frac{1}{2}\sigma_{ti}\cdot\varepsilon_{ti} - \sum_1^n\sum_1^{\text{ж}}\Delta W_{npij(j-1)}+\sum_2^n\Delta W_c\right]}{[k_{zi}(t)\varphi_{11}(\sigma_i)+k_{zi}(t)\ddot{o}_{21}(\sigma_i)]\left\{\left[\frac{k_{\mathrm{I}i}\varphi_1(\sigma_i)}{E_{bt}}+\frac{k_{\mathrm{I}i}\varphi_2(\sigma_i)}{E_{bt}}\right]-\int_{t_0}^t[k_{\mathrm{I}i}\varphi_1(\sigma_i)+k_{\mathrm{I}}\varphi_2(\sigma_i)]\frac{\partial}{\partial\tau}C(t,\tau)d\tau-A^*\right\}}}\cdot\Delta N_i\end{array}\right\}^{-95} \tag{12}$$

The endurance of structures on the concrete of the compressed zone is evaluated based on the condition:

$$\sigma_b^{max}(t,t_0) \le R_b(t,\tau) \tag{13}$$

4 Discussions

The studies allowed us to establish that the mode of deformation of the concrete of the compressed zone in the composition of the reinforced concrete bending structure is non-stationary even in the stationary mode of a cyclic load.

The equation of the mechanical state of concrete in the compressed zone of a reinforced concrete bending structure under stationary repeated cyclic loads is developed on the basis of the theory of concrete vibration creep and the mechanics of destruction of elastic-plastic materials.

The resulting equation is adequate and sufficiently accurate, from the point of view of the requirements of practical calculations. It allows us to evaluate the endurance of concrete in a compressed zone under stationary cyclic loading conditions and to obtain reliable and at the same time economical solutions.

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