The Stress–Strain State of Reinforced Concrete Arches with a View of Concrete Viscoelasticity

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Abstract The resolving equations for determining the stress–strain state of a reinforced concrete element undergoing the action of a bending moment and a longitudinal force, taking into account the creep of concrete on the basis of a viscoelastic model, are obtained. These equations allow for a known value of internal forces to determine the stress–strain state in arbitrary sections of statically definable arches. Internal forces in the arches are calculated analytically, and a step-by-step calculation is used to determine the stresses. Also, the development of the finite element method for the case of viscoelasticity of concrete for a reinforced concrete element has been carried out. Comparison of the results obtained by means of numerical-analytical calculation and FEM is performed. The calculation by the finite difference method was carried out with the subsequent comparison of the results with the FEM.

Keywords Stress–strain · Reinforced concrete · Concrete viscoelasticity

1 Introduction

Since the arches are small curvature bars, they can be calculated using the formulas for eccentrically compressed reinforced concrete bars. We consider a reinforced concrete element subjected to a bending moment and axial force.

The cross-section, as well as the design scheme are shown in Fig. 1. Tensile stresses are assumed to be positive.

According to the hypothesis of flat sections, the total deformation of concrete is the sum of the axial deformation ε_0 and the deformation due to the change in curvature:

$$\varepsilon_b = \varepsilon_0 - y\chi, \tag{1}$$

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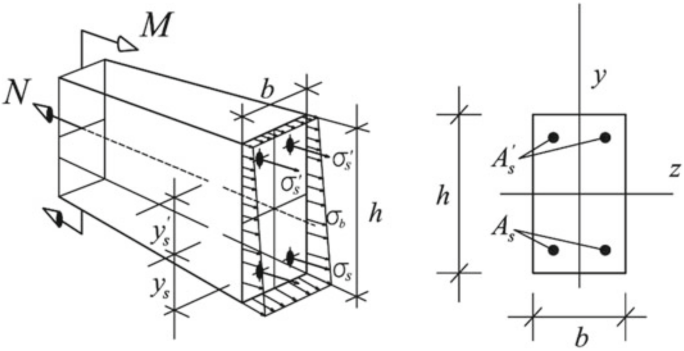

Fig. 1 To the calculation of a reinforced concrete element

where χ—is bar curvature change.

From the condition of the reinforcement and concrete work compatibility, we write down the expressions for reinforcement deformations:

$$\varepsilon_S = \varepsilon_0 + y_S\chi \quad \varepsilon_S' = \varepsilon_0 - y_S'\chi. \tag{2}$$

Distances y_S and y_S' are substituted into the formula (2) by the absolute value.

According to the viscoelastic body model, the total concrete deformation is the sum of elastic deformation ε_b^{el} and creep strain ε_b^* [1]:

$$\varepsilon_b = \frac{\sigma_b}{E_b} + \varepsilon_b^*. \tag{3}$$

From (3), the stresses in concrete are written in the form:

$$\sigma_b = E_b(\varepsilon_b - \varepsilon_b^*) = E_b\left(\varepsilon_0 - y\chi - \varepsilon_b^*\right). \tag{4}$$

Reinforcement stresses are determined as follows:

$$\sigma_S = E_S\varepsilon_S = E_S(\varepsilon_0 + y_S\chi), \quad \sigma_S' = E_S\varepsilon_S' = E_S\left(\varepsilon_0 - y_S'\chi\right). \tag{5}$$

Let us write the equation of the sum of moments about the axis z:

$$-M + \sigma_S A_S y_S - \sigma_S' A_S' y_S' - \int_A \sigma_b y \, dA = 0. \tag{6}$$

Having compiled the sum of the projections of all forces on the longitudinal axis of the bar, we obtain:

$$N = \sigma_S A_S + \sigma_S^{'} A_S^{'} + \int\limits_A \sigma_b \, dA. \tag{7}$$

Substituting (4) and (5) to (6), for the symmetrical reinforcement case. $(A_S = A_S^{'},\ y_S = y_S^{'})$ we get:

$$\chi = \frac{1}{EI_{red}} \left(M - E_b \int\limits_A \varepsilon_b^* y \, dA \right), \tag{8}$$

where $EI_{red} = E_S I_S + E_b I_b$—is the reduced bending stiffness of a cross section; $I_S = E_S \left(A_S y_S^2 + A_S^{'} \left(y_S^{'} \right)^2 \right)$; $I_b = \frac{bh^3}{12}$.

Admeasurement ε_0 is found from the Eqs. (4), (5), (7):

$$\varepsilon_0 = \frac{1}{EA_{red}} \left(N + E_b \int\limits_A \varepsilon_b^* \, dA \right), \tag{9}$$

where $EA_{red} = E_S \left(A_S + A_S^{'} \right) + E_b A_b$—reduced stiffness of a cross-section under axial tension (compression). The Eqs. (4), (5), (8), (9) can be used to calculate the creep of statically definable arches. At the first stage, a static calculation is performed—the internal force factors M and N are determined. In statically definable systems with constant external loads, they do not depend on time. The cross-section in height is divided into m parts Δy, and the time interval is n steps Δt. For the given cross-sections at each point, stresses in concrete are calculated without taking creep into account.

If the creep law is given in differential form, then the calculated stresses can be used to determine the growth rates of creep deformations $\frac{\partial \varepsilon_b^*}{\partial t}$, as well as creep deformation at time $t + \Delta t$ using linear approximation [2–5]:

$$\varepsilon_b^*(t + \Delta t) = \varepsilon_b^*(t) + \frac{\partial \varepsilon_b^*}{\partial t} \Delta t. \tag{10}$$

Time intervals $\Delta \tau_i$ may not be equal to each other. If the section of the arch is rectangular, then the integrals entering into (8) and (9) are also calculated numerically using the trapezoidal method:

$$\int\limits_A \varepsilon_b^* y \, dA = b \int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \varepsilon_b^*(y) y dy = b \Delta y \left(\frac{\varepsilon_{b0}^* y_0 + \varepsilon_{bm}^* y_m}{2} + \sum_{i=1}^{m-1} \varepsilon_{bi}^* y_i \right). \tag{11}$$

2 Methods

Calculation of arches by the finite element method. Derivation of resolving equations.

It is assumed that the behavior of a continuous curved beam is sufficiently accurately characterized by the behavior of a broken bar composed of small rectilinear elements. From physical considerations it follows that with a decrease in the elements' size, the solution should converge and, as experience shows, convergence is observed [6].

At the same time, special attention should be paid to the method of specifying the nodal loads: the distributed load is more correctly represented in the form of statically equivalent concentrated nodal forces [7].

The calculations will use the bar finite element shown in Fig. 2. Each node of this element has 3 degrees of freedom: 2 linear displacements u and v, as well as a rotation angle φ. The vector of nodal displacements will be written as: $\{U\} = \left\{ u_i \; u_j \; v_i \; \varphi_i \; v_j \; \varphi_j \right\}^T$.

The deflection of a finite element will be approximated as follows:

$$\begin{aligned} v(x) &= \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 = \left\{ 1 \; x \; x^2 \; x^3 \right\} \left\{ \alpha_0 \; \alpha_1 \; \alpha_2 \; \alpha_3 \right\}^T \\ &= \left\{ 1 \; x \; x^2 \; x^3 \right\} \{\alpha\} . \end{aligned} \quad (12)$$

The vector $\{\alpha\}$ can be found from the following condition:

$$v(0) = v_i; \quad \varphi(0) = -\left.\frac{dv}{dx}\right|_{x=0} = \varphi_i; \quad v(l) = v_j; \quad \varphi(l) = -\left.\frac{dv}{dx}\right|_{x=l} = \varphi_j .$$

In matrix form, these conditions take the form:

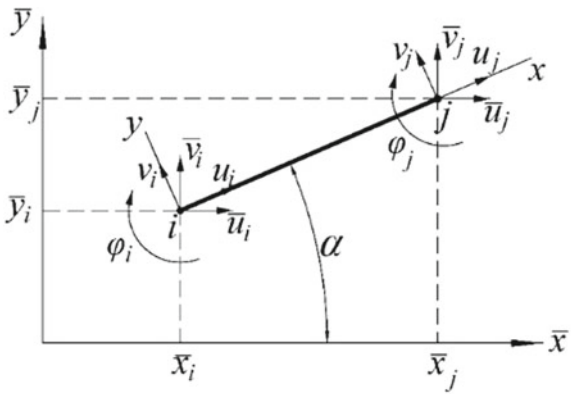

Fig. 2 Bar finite element

$$\begin{Bmatrix} v_i \\ \varphi_i \\ v_j \\ \varphi_j \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & -1 & -2l & -3l^2 \end{bmatrix} \cdot \{\alpha\} = [C] \cdot \{\alpha\}. \quad (13)$$

Let us express the vector $\{\alpha\}$ from (13) by nodal movements:

$$\{\alpha\} = [C]^{-1} \cdot \left\{ v_i \ \varphi_i \ v_j \ \varphi_j \right\}^T$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{3}{l^2} & \frac{2}{l} & \frac{3}{l^2} & \frac{1}{l} \\ 0 & 0 & \frac{2}{l^3} & -\frac{1}{l^2} & -\frac{2}{l^3} & -\frac{1}{l^2} \end{bmatrix} \{U\} = [F]\{U\}. \quad (14)$$

Then the deflection function will be written as:

$$v(x) = \left\{ 1 \ x \ x^2 \ x^3 \right\} [F]\{U\}. \quad (15)$$

And the second derivative of the deflection takes the form:

$$\frac{d^2 v}{dx^2} = \chi = \left\{ 0 \ 0 \ 2 \ 6x \right\} [F]\{U\}. \quad (16)$$

For axial displacements u, we take linear dependence on x:

$$u = \left(1 - \frac{x}{l}\right) u_i + \frac{x}{l} u_j. \quad (17)$$

Then the axial deformation ε_0 will be defined as follows:

$$\varepsilon_0 = \frac{du}{dx} = \left\{ -\frac{1}{l} \ \frac{1}{l} \ 0 \ 0 \ 0 \ 0 \right\} \{U\}. \quad (18)$$

The expressions for the stiffness matrix and the load vector will be obtained based on the Lagrange variational principle. The total energy E is the sum of the deformation potential energy and the external forces work:

$$E = P + A. \quad (19)$$

The potential energy of deformation is the sum of the concrete and reinforcement potential energy:

$$P = P_b + \ P_S + P_S^{'}. \quad (20)$$

The potential energy of concrete is determined by the following expression:

$$P_b = \frac{1}{2} \int\limits_{V_b} \sigma_b \varepsilon_b^{el} dV, \tag{21}$$

where ε_b^{el} denotes elastic deformation of concrete, which is the difference between total and creep deformation:

$$\varepsilon_b^{el} = \varepsilon_b - \varepsilon_b^*. \tag{22}$$

We will assume that the creep strain is independent of x within the element. Substituting (1) in (22) and then (4) and (22) in (21), we get:

$$P_b = \frac{1}{2} E_b \int\limits_{V_b} \left(\varepsilon_0 - y \frac{d^2 v}{dx^2} - \varepsilon_b^* \right)^2 dV = \frac{1}{2} E_b \left[A_b \int\limits_{(l)} \varepsilon_0^2 dx + I_b \int\limits_{(l)} \left(\frac{d^2 v}{dx^2} \right)^2 dx \right.$$
$$\left. + \int\limits_{V_b} (\varepsilon_b^*)^2 dV - 2 \int\limits_{(l)} \varepsilon_0 dx \int\limits_{A} \varepsilon_b^* dA + 2 \int\limits_{(l)} \frac{d^2 v}{dx^2} dx \int\limits_{A} \varepsilon_b^* y dA, \right. \tag{23}$$

where $I_b = \frac{bh^3}{12}$ is the moment of concrete inertia; $A_b = bh$ is the concrete section area.

The potential deformation energy of the reinforcement located at the bottom face can be found as follows:

$$P_S = \frac{1}{2} \int\limits_{V_S} \sigma_S \varepsilon_S dV = \frac{1}{2} E_S A_S \int\limits_{(l)} (\varepsilon_0^2 + 2\varepsilon_0 \, y_S \frac{d^2 v}{dx^2} + y_S^2 \left(\frac{d^2 v}{dx^2} \right)^2) \, dx. \tag{24}$$

For top edge reinforcement similarly:

$$P_S' = \frac{1}{2} \int\limits_{V_S'} \sigma_S' \varepsilon_S' dV = \frac{1}{2} E_S A_S' \int\limits_{(l)} (\varepsilon_0^2 - 2\varepsilon_0 y_S' \frac{d^2 v}{dx^2} + (y_S')^2 \left(\frac{d^2 v}{dx^2} \right)^2) \, dx. \tag{25}$$

In the case of symmetrical reinforcement ($A_S = A_S'$, $y_S = y_S'$) for the potential energy of all reinforcement deformation, we obtain:

$$P_S + P_S' = \frac{1}{2} E_S \left(A_{S,\,gen} \int\limits_{(l)} \varepsilon_0^2 dx + I_S \int\limits_{(l)} \left(\frac{d^2 v}{dx^2} \right)^2 dx \right), \tag{26}$$

where $I_S = A_S y_S^2 + A_S' \left(y_S' \right)^2$ is reinforcement inertia moment.

Substituting (16) and (18) in (24) and (26), we obtain the following expression for the potential energy of a reinforced concrete element:

$$P = \frac{1}{2}\{U\}[K]\{U\} - \{U\}\left\{F_b^*\right\} + \frac{1}{2}E_b\int\limits_V (\varepsilon_b^*)^2\, dV, \tag{27}$$

where $[K] = \begin{bmatrix} [K_c] & \\ & [K_i] \end{bmatrix}$—stiffness matrix, which has a block structure; $\left\{F_b^*\right\}$—contribution of concrete creep deformations to the vector of nodal loads;

$$[K_c] = \frac{EA_{red}}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix};$$

$$[K_i] = EI_{red}\begin{bmatrix} \frac{12}{l^3} & -\frac{6}{l^2} & -\frac{12}{l^3} & -\frac{6}{l^2} \\ -\frac{6}{l^2} & \frac{4}{l} & \frac{6}{l^2} & \frac{2}{l} \\ -\frac{12}{l^3} & \frac{6}{l^2} & \frac{12}{l^3} & \frac{6}{l^2} \\ -\frac{6}{l^2} & \frac{2}{l} & \frac{6}{l^2} & \frac{4}{l} \end{bmatrix};$$

$$\left\{F_b^*\right\} = E_b\left(\int\limits_A \varepsilon_b^* dA\begin{Bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \int\limits_A \varepsilon_b^* y dA\begin{Bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{Bmatrix}\right). \tag{28}$$

The work of external forces is the product of the external nodal loads $\left\{F_q\right\}$ vector on the vector of nodal displacements: $A = \{U\}^T\left\{F_q\right\}$.

From the condition of the total energy functional minimum, we obtain:

$$\frac{\partial E}{\partial\{U\}} = \frac{\partial P}{\partial\{U\}} + \frac{\partial A}{\partial\{U\}} = \frac{\partial}{\partial\{U\}}\Bigg(\frac{1}{2}\{U\}[K]\{U\} - \{U\}^T\left\{F_q\right\} + \frac{1}{2}E_b\int\limits_V(\varepsilon_b^*)^2\, dV\Bigg) - \frac{\partial}{\partial\{U\}}\left(\{U\}^T\left\{F_b^*\right\}\right) = 0.$$

Finally, the problem is reduced to a system of linear algebraic equations of the form:

$$[K]\{U\} = \left\{F_q\right\} + \left\{F_b^*\right\}. \tag{29}$$

The calculation was performed for a three-pivot circular arch loaded with a uniformly distributed load q. The design scheme is shown in Fig. 3.

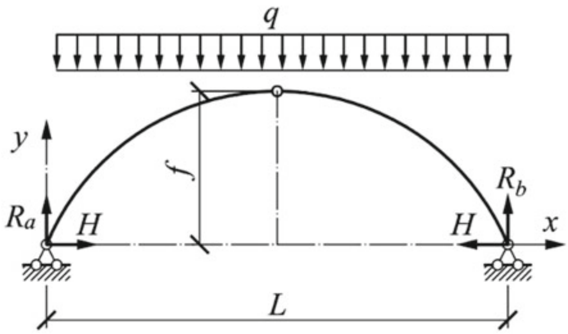

Fig. 3 Design scheme of the arch

The equation for the arch axis outlined along the circular arc is:

$$y = \sqrt{R^2 - \left(\frac{L}{2} - x\right)^2} - R + f;\ R = \frac{f}{2} + \frac{L^2}{8f}; \tag{30}$$

$$\sin\varphi = \frac{L - 2x}{2R};\ \cos\varphi = \frac{y + R - f}{R}.$$

Internal forces in the section K of the arch are calculated by the formulas:

$$k = M_k^\delta - Hy_k; \quad N_k = -\left(Q_k^\delta \sin\ \varphi_k + H\cos\varphi_k\right), \tag{31}$$

where M_k^δ, Q_k^δ denote the moment and shear force in the section K in a beam with a similar span and load. In case of uniformly distributed load: $M_k^\delta = \frac{qx}{2}(L - x);\ Q_k^\delta = \frac{q}{2}(L - 2x);\ H = \frac{qL^2}{8f}$.

The problem was solved with the following initial data: $q = 50\frac{kN}{m}$, arch span $L = 16\,m$, elevation $f = 3.2\,m$, cross-sectional dimensions: $b = 20\,cm, h = 40\,cm$, $\tau_0 = 28\ days$, $E_b(\tau_0) = 3\times10^4$ MP, $E_S = 2\cdot10^5$ MP, reinforcement ratio $\mu = \frac{A_{S,\ gen}}{A_b} = 0.02$, $y_S = y_S^{'} = 15$ cm. Concrete aging was taken into account, i.e., the increase in its elasticity modulus over time. The time dependence of the concrete elasticity modulus was taken as: $E_b(t) = E_b(\tau_0)\cdot\left[b_1 + (1 - b_1)e^{-b_2(t-\tau_0)}\right]$, $b_1 = 1.282$, $b_2 = 0.019$.

The graph of the change in the concrete elasticity modulus is shown in Fig. 4.

The equation of the viscoelastic model of concrete hereditary aging was used in the calculation and has the form:

$$\varepsilon_b(t) = \frac{\sigma_b(t)}{E_b(t)} - \int_{\tau_0}^{t} \sigma_b(\tau)\frac{\partial C(t,\tau)}{\partial\tau}d\tau. \tag{32}$$

The Eq. (32) can also be represented in the form (3), introducing the following notation:

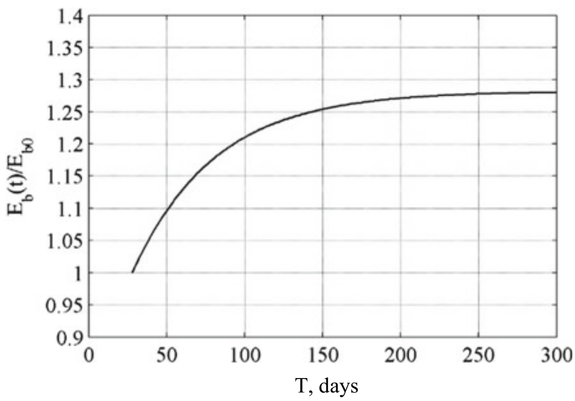

Fig. 4 The graph of the change in the concrete elasticity modulus

$$\varepsilon_b^* = -\int_{\tau_0}^{t} \sigma_b(\tau) \frac{\partial C(t,\tau)}{\partial \tau} d\tau. \tag{33}$$

It is recommended to take the creep measure in [28] in the form:

$$C(t,\tau) = C\frac{e^{\alpha t} - e^{\alpha \tau}}{e^{\alpha t} - 1} + B\left(e^{-\gamma\tau} - e^{-\gamma t}\right). \tag{34}$$

To determine the creep deformations described by the expression (33), it is possible to use the formula (11); however, if the creep measure is a sum of exponentials, it is more convenient to represent the creep law in differential form.

Substituting (34) into (33), we get:

$$\begin{aligned} \varepsilon_b^* &= -\int_{\tau_0}^{t} \sigma_b(\tau) \frac{\partial}{\partial \tau}(C\frac{e^{\alpha t} - e^{\alpha \tau}}{e^{\alpha t} - 1} + B\left(e^{-\gamma\tau} - e^{-\gamma t}\right))d\tau \\ &= \frac{C\alpha}{e^{\alpha t} - 1} \int_{\tau_0}^{t} \sigma_b(\tau) e^{\alpha\tau} d\tau + B\gamma \int_{\tau_0}^{t} \sigma_b(\tau) e^{-\gamma\tau} d\tau. \end{aligned} \tag{35}$$

We represent the concrete creep deformation as the sum of two components:

$$\varepsilon_b^* = \varepsilon_{b1}^* + \varepsilon_{b2}^*;$$

$$\varepsilon_{b1}^* = \frac{C\alpha}{e^{\alpha t} - 1} \int_{\tau_0}^{t} \sigma_b(\tau) e^{\alpha\tau} d\tau; \quad \varepsilon_{b2}^* = B\gamma \int_{\tau_0}^{t} \sigma_b(\tau) e^{-\gamma\tau} d\tau. \tag{36}$$

The component ε_{b1}^* characterizes hereditary creep properties, and ε_{b2}^* characterizes the influence of the growing environment on its deformative properties [28].

Let us find the time derivative of each component:

$$\frac{\partial \varepsilon_{b1}^{*}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{C\alpha}{e^{\alpha t}-1}\right)\int_{\tau_0}^{t}\sigma_b(\tau)e^{\alpha\tau}d\tau + \frac{C\alpha}{e^{\alpha t}-1}\frac{\partial}{\partial t}\left(\int_{\tau_0}^{t}\sigma_b(\tau)e^{\alpha\tau}d\tau\right)$$
$$= -\frac{C\alpha^2 e^{\alpha t}}{(e^{\alpha t}-1)^2}\int_{\tau_0}^{t}\sigma_b(\tau)e^{\alpha\tau}d\tau + \frac{C\alpha}{e^{\alpha t}-1}\sigma_b(t)e^{\alpha t} = \frac{\alpha e^{\alpha t}}{e^{\alpha t}-1}\left(C\sigma_b(t)-\varepsilon_{b1}^{*}\right); \quad (37)$$

$$\frac{\partial \varepsilon_{b2}^{*}}{\partial t} = \frac{\partial}{\partial t}\left(B\gamma\int_{\tau_0}^{t}\sigma_b(\tau)e^{-\gamma\tau}d\tau\right) = B\gamma\sigma_b(t)e^{-\gamma t}.. \quad (38)$$

Using the expressions (37) and (38) together with (10), it is possible to determine the components of creep deformation ε_{b1}^{*} and ε_{b2}^{*} at every moment of time.

The values of the rheological constants in the calculation were taken equal to: $\alpha = 0.032$, $\gamma = 0.062$, $C = 3.77 \cdot 10^{-5}$ MPa^{-1}, $B = 5.68 \cdot 10^{-5}$ MPa^{-1}.

3 Results and Discussion

For the calculations using the FEM, a software package was developed in the Matlab complex. To check the correctness of the program operation, a test problem was solved for a statically definable arch.

Figure 5 shows a graph of the change in stress in the reinforcement depending on x and t.

The upper mesh surface corresponds to the stress σ_S' in the reinforcement at the upper edge. Bottom shaded surface corresponds to the stresses σ_S in the reinforcement at the bottom edge.

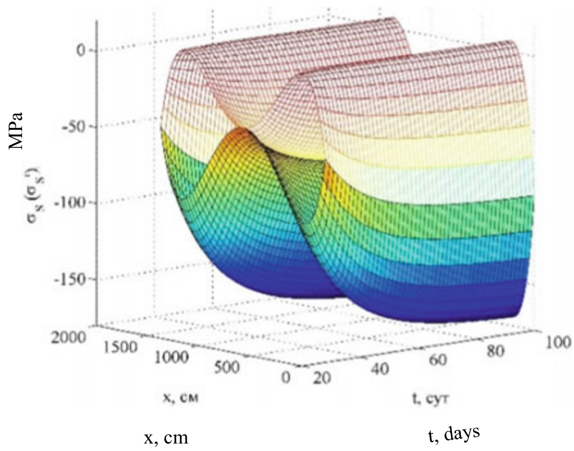

Fig. 5 Reinforcement stress change

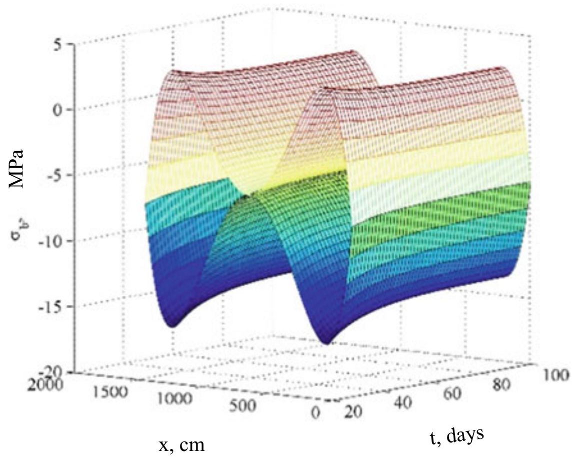

Fig. 6 Change in stresses in concrete at y = h / 2 and y = –h / 2

Figure 6 shows a graph of the change in stresses in concrete depending on x and t.

The upper surface corresponds to the stresses at $y = h/2$, the lower corresponds to the stresses at $y = -h/2$.

Figure 5 shows that, due to concrete creep, the stresses in the reinforcement in absolute value increase. In concrete, stresses, on the contrary, decrease, as evidenced by Fig. 6. The most significant redistribution occurs at the points where the bending moments are maximum ($x \approx 2.1$ m and $x \approx 13.9$ m). Figure 7 shows the distribution of stresses in concrete in the section $x = 2.1$ m at the beginning of the creep process (dashed line) and at the end of the creep process (solid line). Figure 8 shows the change in stresses σ_S in the reinforcement of the lower edge at x = 2.1 m.

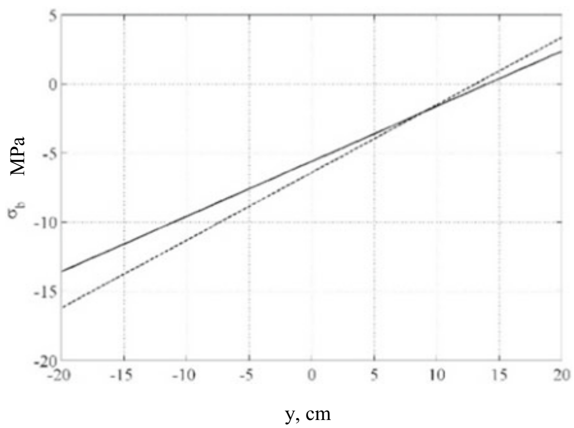

Fig. 7 Stress distribution in concrete in the section x = 2.1 m at the beginning and at the end of the creep process

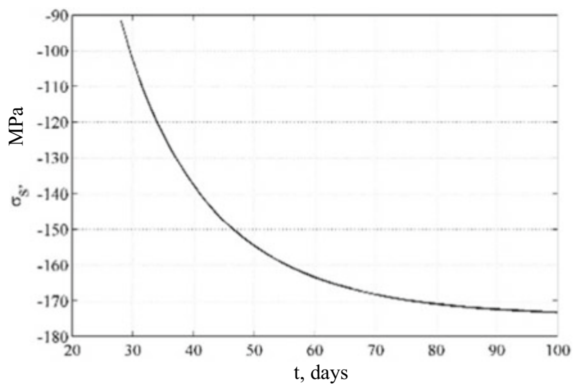

Fig. 8 Change in stresses in the reinforcement of the lower edge with time in the section x = 2.1 m

Due to the creep of concrete, the compressive stresses in the reinforcement of the lower face in the section with the maximum bending moment increased from 91.7 MPa to 173.1 MPa, i.e., 1.9 times.

In concrete, due to creep, the change in stresses is not so significant: for a more compressed face at $x = 2.1$ m, the stresses decreased from 16.2 MPa to 13.6 MPa, i.e., by only 20%. The distribution of stresses in concrete along the section height, both at the beginning and at the end of the creep process, is linear.

Figure 9 shows the distribution of creep deformations ε_b^* depending on x and y at $t = 100$ days. The greatest inelastic deformations are observed in the sections with maximum bending moments.

Table 1 shows a comparison of stresses in concrete and reinforcement at the bottom edge at x = 2.1 m at different points in time, obtained numerically-analytically, as well as numerically using the FEM.

The table shows that the results practically coincide, which indicates the reliability of the technique developed.

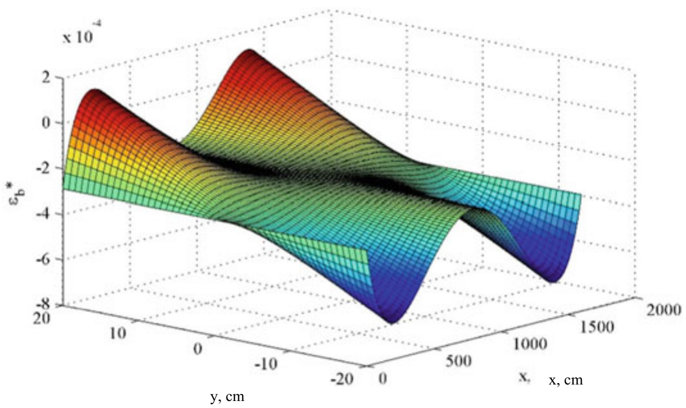

Fig. 9 Distribution of creep deformations depending on x and y at t = 100 days

Table 1 Comparison of the results of numerical-analytical calculation with FEM

t, days		30	40	50	60	70	80	100
$\sigma_b,\ MP$	Numerical-analytical calculation	–15.84	–14.70	–14.16	–13.87	–13.72	–13.64	–13.56
	FEM	–15.81	–14.67	–14.17	–13.87	–13.71	–13.62	–13.54
σ_S, MP	Numerical-analytical calculation	–102.8	–137.7	–154.6	–162.7	–167.8	–170.4	–173.2
	FEM	–102.6	–137.4	–154.3	–163.1	–168.9	–170.3	–172.8

Summarizing the above-said, it can be noted that resolving equations have been obtained to determine the stress–strain state of a reinforced concrete element experiencing a bending moment and longitudinal force, taking into account the creep of concrete on the basis of a viscoelastic model.

These equations allow for a known value of internal forces to determine the stress–strain state in arbitrary sections of statically definable arches. Internal forces in the arches are calculated analytically, and a step-by-step calculation is used to determine the stresses.

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