Pile Rows for Protection from Surface Waves

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Abstract Numerical studies of surface Rayleigh wave interaction with piles using Finite Element Method are presented in this article to show the attenuation effect of such wave barrier along with the possibility to implement pile rows as a method of vibration protection of buildings and underground structures from surface waves of Rayleigh type. Spatial FE models are used to analyze the influence of pile field parameters such as pile length, pile diameter number of rows and pile spacing on vibration reduction effect of the field, with respect to the wavelength that depends on frequency characteristics of vibration loading and soil conditions. Apart from that, it is shown how additional pile rows can decrease internal forces in the piles inside the protected zone which can be important for deep foundations.

Keywords Numerical simulation · Ground vibration · Vibration isolation · Pile wave barrier · Rayleigh wave scattering

1 Introduction

The study of piles as a vibration barrier started from the work of Richart and Woods [1], where the performance of this type of protection is investigated experimentally. In addition to that, the authors suggested initial design guidelines for pile barriers. Later, Woods [2] confirmed screening effect of cylindrical hole barriers on Rayleigh waves using holography. One of the first theoretical studies is performed by Javier Aviles and Sanchez-Sesma [3, 4] who theoretically analysed interaction of pile rows with body waves [3] as well as Rayleigh waves [4] by using planar and spatial

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models. The authors suggested values of pile length, spacing and width of the barrier for effective vibration isolation.

In [5], Kattis et al. adopt Boundary Element Method (BEM) in the frequency domain to analyze vibration isolation by pile rows. Further development of this approach in [6] allowed to model a pile row as an infilled effective trench using the homogenization method which is applied in the mechanics of fibre-reinforced composite materials. In that work, interaction of a pile row with Rayleigh waves is considered accounting for one of the most important factors, which is the volume fraction of piles. It is worth noting, that this simulation method slightly overestimates the reduction effect of a pile row comparatively to the modelling of independent piles. Additionally, the authors show that trench barriers have a better reduction effect than pile rows and the type of a pile cross-section has virtually no effect on the vibration reduction. Afterwards, this solution technique is extended for spatial simulation of row's interaction with Rayleigh waves in frequency domain in [6]. In addition to that, BEM is utilized in the work of Tsai [7] in order to study active vibration protection for different types of piles, pile lengths and spacing.

Another interesting approach based on the periodicity theory and Finite Element Method (FEM) is implemented by Jiankun Huang [8] for the analysis of horizontal vibration attenuation by pile rows. Then, this method is further developed for plane waves in [9] and pile barriers with initial stress in [10]. In these works, the authors propose the concept of dispersion curves and analyse the attenuation zones for pile fields. The waves with the frequencies within the attenuation zone cannot propagate through the periodic pile barriers. It is shown that the reduction ratio of pile rows relates to relative Young's modulus, the relative density of piles (ratio of pile material density to that of the soil) and pile fraction [8]. Meanwhile, initial stress affects [10] the width as well as the lower and upper bounds of the attenuation zone, having practically no effect on the final reduction effect.

Vibration attenuation properties of pile rows in porous media are analysed in the works [11] and [12] of Yuan-Qiang Cai et al. for the cases of surface Rayleigh and body waves respectively using Fourier–Bessel series. In that research, such key factors as pile spacing, relative pile Young's modulus and density are underlined. Moreover, it is shown that vibration isolation from Rayleigh waves in porous media is less effective than that in non-porous elastic media which is not in the agreement with the study carried out by Lu [13] presenting better effectiveness of pile barrier for the case of two phase media.

Multiple body wave scattering by several pile rows is analysed in [14] with the method proposed by the authors. It is shown that an increase in the number of rows improves vibration reduction properties of a pile barrier. At the same time, such method is found to have better screening effect for the case of low-frequency body waves.

Most of the previous researches deal with the parameters of pile field independently, regardless their complex effect on vibration attenuation properties. The parameters of pile fields ensuring maximum reduction effect are difficult to implement in practice accounting for current technology and construction codes. The present work is targeted to the full-scale spatial analysis of Rayleigh wave interaction with piles using FE method implemented in Abaqus software. The energy distribution in the protected zone along with the estimation of the extension to which the parameters of a pile field affect the resulting vibration reduction are analysed.

2 Problem Formulation

Interaction of surface Rayleigh waves with pile fields are considered within the framework of linear elastic constitutive law. This approach can be appropriate for artificial vibrations except blasts as well as for earthquakes and blasts in the areas that are remote from their epicentres. In that case, small strain constitutive equation for homogeneous isotropic media can be written in the following form:

$$c_p^2 \nabla \mathrm{div}\, \mathbf{u}(\mathbf{x}, t) - c_s^2 \,\mathrm{rot}\,\mathrm{rot}\, \mathbf{u}(\mathbf{x}, t) + \frac{1}{\rho}\mathbf{b}(\mathbf{x}, t) = \ddot{\mathbf{u}}(\mathbf{x}, t), \tag{1}$$

where $\mathbf{u}(\mathbf{x}, t)$ and $\mathbf{b}(\mathbf{x}, t)$ are the functions of displacements and body forces respectively c_p, c_s are the compressive and shear wave speeds accordingly:

$$c_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_S = \sqrt{\frac{\mu}{\rho}}, \tag{2}$$

where λ, μ are Lame's parameters.

The following initial conditions are considered:

$$\mathbf{u}(\mathbf{x}, t)|_{t=0} = 0, \quad \partial_t \mathbf{u}(\mathbf{x}, t)|_{t=0} = 0 \tag{3}$$

In that case, the initial stress distribution in the half-space is neglected as it has virtually no effect on the displacements, velocities and accelerations of the points in the observation zone.

For isotropic media on the free surface of the half-space (Fig. 1), the boundary condition of zero stress is used:

$$\mathbf{t}_\xi = \sigma \cdot \xi = 0, \mathbf{x} \in \Pi_\xi, \tag{4}$$

where $\mathbf{I}$ is the unit diagonal matrix, ξ is the unit outward normal to the free surface Π_ξ and $\mathbf{t}_\xi$ is a surface stress vector. In the case of elastic media Eq. (4) takes the following form:

$$\mathbf{t}_\xi \equiv (\lambda tr(\varepsilon)\mathbf{I} + 2\mu\varepsilon) \cdot \xi = 0, \mathbf{x} \in \Pi_\xi, \tag{5}$$

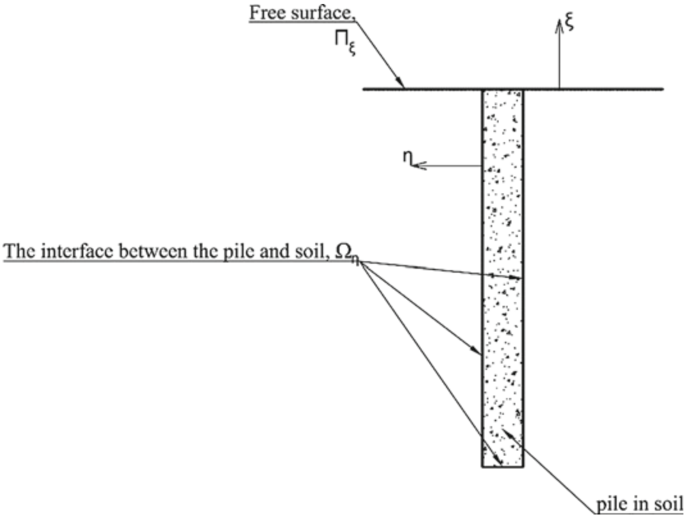

Fig. 1 Boundary conditions and unit normals to the free surface of the half-space and contact interface between the pile and soil respectively

where ε—small strain tensor.

In the case of a pile barrier installed in the medium (Fig. 1), the condition of perfect mechanical contact is applied to the contact surfaces between the piles and the soil:

$$\begin{aligned} \mathbf{t}_{pile}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta} &= \mathbf{t}_{soil}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta} \\ \mathbf{u}_{pile}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta} &= \mathbf{u}_{soil}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta} \end{aligned} \tag{6}$$

where $\mathbf{t}_{pile}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta}, \mathbf{t}_{soil}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta}$ are the stresses on the contact surface from a pile and soil sides respectively; η is the unit normal to the interface between the pile and soil Ω_η; $\mathbf{u}_{pile}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta}, \mathbf{u}_{soil}\big|_{\mathbf{x}\cdot\eta\in\Omega_\eta}$ are the displacement vectors on the contact surface on the pile and soil sides respectively; the indexes $pile$ and $soil$ correspond to the pile and soil accordingly.

3 Simulation Methods and FE Models

3.1 Finite Element Model Configurations

Mathematical formulation including constitutive equations as well as boundary and initial conditions for the considered problem is defined by Eqs. (1–6). The analysis is

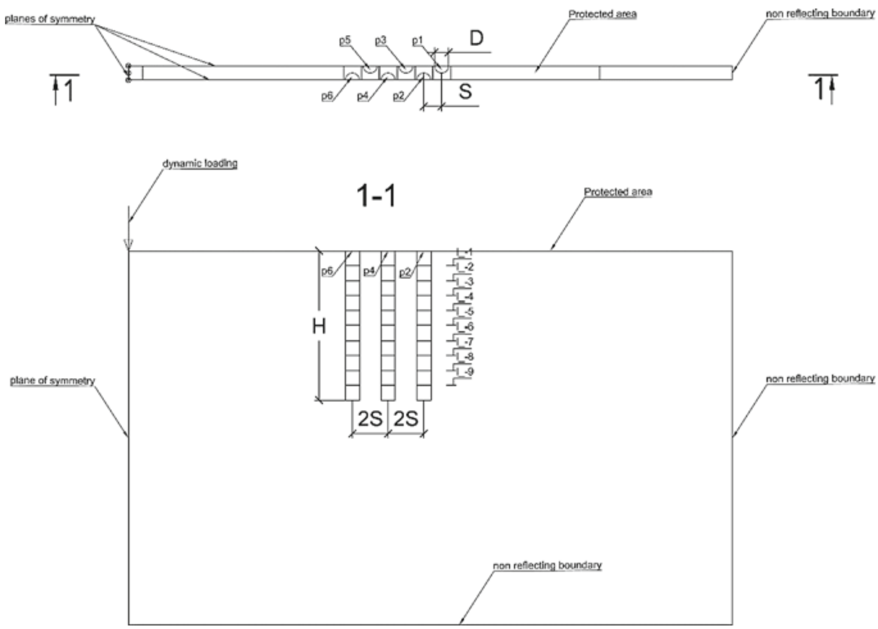

Fig. 2 The scheme of the simplified spatial model of pile field

performed in time domain for surface Rayleigh waves, generated by fully harmonic surface line loading with vibration amplitude A and frequency ω:

$$f(x, t) = A\, e^{i\omega t} \delta(\mathbf{x}) \nu, \tag{6}$$

Time integration is performed using explicit finite-difference procedure which is based on the second order explicit central difference integration scheme involving Lax-Wendroff method [15, 16]. Time increment size is selected automatically by the program satisfying Courant–Friedrichs–Lewy (CFL) condition [15, 16]. Spatial discretization is performed using finite element method (FEM) in Abaqus 2016 software [16]. The standard finite element library of Abaqus/Explicit software package is used in the calculations [16]. The region is meshed with finite elements of the C3D8R type which are eight-node hexahedral elements with a linear shape function with control of deformations. Maximum length ratio is in the range [0.95,1].

In the subsequent analysis two types of 3D models shown in Figs. 2 and 3 are used. The first model represents a piece of a pile field with several rows and three planes of symmetry that are used to decrease the model size (Fig. 2). The first plane of symmetry passes through the wave source perpendicular to the direction of Rayleigh wave propagation and parallel to the pile row. It is assumed that there are several piles in a row located along the same straight line at the same distance from each other(the length of the row can be compared with the dimensions of the wave front or larger than that, so the effect of the row length can be neglected).This allows to

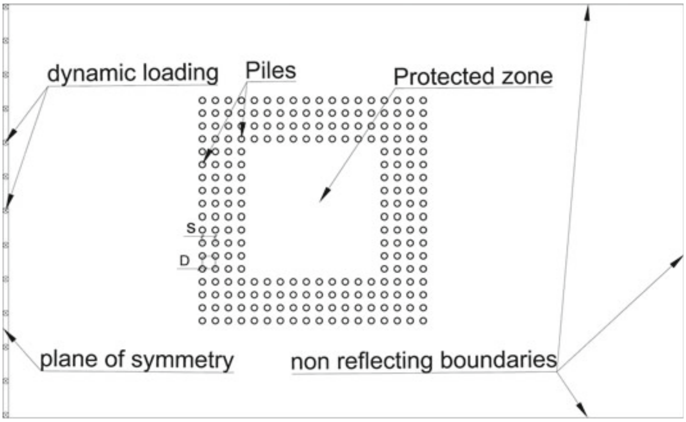

Fig. 3 The scheme of spatial model for real full scale pile field

introduce two additional planes of symmetry passing through the pile axis and middle of the interval between the piles parallel to the direction of propagation of surface waves thus, substantially reducing the number of elements. On the free surface at the top of the symmetry plane fully harmonic line loading defined by Eq. (6) is applied. Meanwhile, the remaining part of the top surface is free. On the bottom and right planes of the model non-reflecting boundaries for P waves are used. This model is used to analyse the influence of pile diameter, spacing and length on the reduction ratio of a pile field as well as to determine the optimal values of these parameters for the following analysis involving the model of a more realistic pile field (Fig. 3).

At the second stage, a full scale spatial model of the regular pile field is adopted to simulate a real finite size pile field which may surround a construction or be the foundation of a structure (Fig. 3). For this model the main parameters are set based on the results obtained from the analysis using the first model (Fig. 2). Basically, the full scale 3d model is used to confirm the main results and trends identified in the first calculation stage. Similarly, to the first model the second one is a three-dimensional with the condition of symmetry applied on the left surface. Top surface of the model is free while non-reflecting boundaries are applied on the other surfaces (Fig. 3).

The dimensions of the models are chosen in a way that the waves reflected from the boundaries of the models should not return to the observation zone with the length equalling to $2l$, where l is the wavelength of Rayleigh wave. The mesh size is less than $0.1l$. Additionally, pile rows are created at a distance from the symmetry plane (left plane) so that the interaction of the waves and piles would occur remote enough from the source with account of symmetry condition.

The models presented in Figs. 2 and 3 allow to analyse the influence of pile field planar configuration (quadratic or triangular cells (Fig. 4) on vibration reduction effect along with the interaction with surface waves. Vertical and horizontal sizes of the first model (Figs. 2) equals to $9l$ and $18l$ respectively (l is the wavelength of Rayleigh wave), while the width of the model varied according to the pile distance.

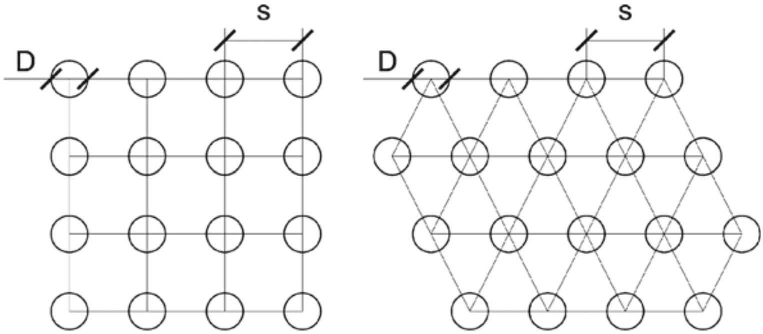

Fig. 4 The types of pile field configuration: square cells (on the left) and triangular cells (on the right)

The sizes of the second model (Fig. 3) along the X,Y and Z axis equals to $9l$, $6l$ and $5l$. The size of the protected zone Δ is l, while the observation zone size equals to $2l$.

Primary calculations show that there is hardly no difference in vibration reduction effect for plane and spherical waves. Therefore, taking into account the requirements to the model size, Rayleigh waves with a plane front are considered in the following text. In addition to that, two main assumptions are also made: (1) the size of the protected zone does not change; (2) the same soil conditions are used for all the calculations.

It is worth noting that a pile field can act as a barrier if the wavelength is comparable or less than the pile length and the dimensions of the pile field in plane. For low frequency range corresponding to earthquakes $f = 2\div10$ Hz the wavelength of Rayleigh wave varies from 100 to 10 m in soft soils, while in rigid soils it can exceed 200 m. At the same time, pile depth which is more than 50 m is difficult to implement in practice. Therefore, the lowest frequency $f = 2$ Hz is chosen as it generates Rayleigh waves with large enough wavelength corresponding to real vibration sources both natural and anthropogenic nature. While, construction of a pile field providing reasonable vibration reduction effect is not possible even in soft soils for lower frequencies as it will require larger pile lengths. At the same time, higher frequencies correspond to shorter wavelengths and require smaller protective pile barriers. The results are shown in relation to the maximum Rayleigh wavelength l equalling to 50 m and corresponding to minimum vibration frequency $f = 2$ Hz.

Young's modulus and density for soft soils are chosen according to the seismic shear wave velocities that are given in Eurocode 8 [16]. The attenuation effect of the field is analysed using the value of the kinetic energy reduction ratio:

$$k_{red,E} = \frac{K^{pile}}{K^{\text{hom}}}, \tag{7}$$

Table 1 Dynamic parameters of materials

Material	Density (kg/m^3)	Poisson's ratio	Young's modulus (MPa)
Soil	1800	0.25	55
Concrete	2450	0.23	30,000

where K^{pile} is the kinetic energy field of the surface layer in the protected zone and K^{hom} is the kinetic energy for the same layer in the model without pile field. The observation layer is placed behind the pile rows at a surface level.

According to the results obtained by Kattis et al. in [6], it is possible to replace a pile row with an effective trench, thus basic qualitative results obtained in the works [1, 18–21] etc. regarding the influence of the depth, width and mechanical material parameters can be extrapolated to pile rows. Which means, the higher the difference in the mechanical parameters of the piles and the soil the better vibration reduction effect can be observed. However, the range of materials for a pile field is quite narrow. Therefore, further analysis is limited by piles made of reinforcement concrete, which are more widely used. Mechanical parameters of concrete and a possible soft soil are shown in Table 1 in agreement with [16].

This work concerns interaction of Rayleigh waves with piles and pile fields outside of the source vicinity. This is primarily due to the fact that the behaviour of waves in the source zone has difficult to predict complex nature which is strongly affected by geological conditions along with the source itself. Additionally, it is possible to distinguish the major waves that carry the energy of vibration source. As a result, the distance between the pile row and the source gives virtually no effect on the final reduction effect in the protected zone.

Hereinafter, if the variable dimension is not explicitly specified, it is presented in the dimensionless form. The main dimensionless complex is given in the section below by default, geometrical variables are given in relation to the Rayleigh wave wavelength.

3.2 Dimensional Analysis

Kinetic energy and displacement fields of the area beyond the pile field can be described by the following group of dimensionless parameters:

$$K^{pile} = f\left(\frac{E_{pile}}{E_{soil}}; \frac{\rho_{pile}}{\rho_{soil}}; \frac{D}{l}; \frac{H}{l}; \frac{S}{l}; \nu_{pile}; \nu_{soil}\right), \tag{8}$$

$$u^{pile} = g\left(\frac{E_{pile}}{E_{soil}}; \frac{\rho_{pile}}{\rho_{soil}}; \frac{D}{l}; \frac{H}{l}; \frac{S}{l}; \nu_{pile}; \nu_{soil}\right) \tag{9}$$

where the index *soil* indicates the soil material of the half-space, while the index *pile* corresponds to the parameters of the pile field; l is the wavelength of the Rayleigh wave in a half-space (this wavelength can be solved from the Bergmann-Viktorov's equation); E_{pile}, E_{soil} correspond respectively to Young's modulus of the piles and soil respectively; ν_{pile}, ν_{soil} are corresponding Poisson's ratios; ρ_{pile}, ρ_{soil} are the corresponding densities; D, H, S are the diameter, length and spacing of the pile field accordingly. A pile field interacts with seismic waves as a uniform composite barrier, thus it is convenient to introduce the value of pile fraction—$\alpha = \frac{\pi D^2}{4S^2}$ showing the density of the pile field. Afterwards, all the geometric values are normalized in relation to the wavelength of Rayleigh's wave.

4 The Computed Results

As a starting point, the influence of pile diameter ($\tilde{d} = \frac{D}{l}$), pile fraction ($\alpha = \frac{\pi D^2}{4S^2}$) and distance between piles ($\tilde{s} = \frac{S}{l}$) is considered. In order to estimate the influence of these parameters, the reduction ratios are calculated at the surface level in the protected zone Δ. Figure 5 represents the reduction ratios for the surface layer.

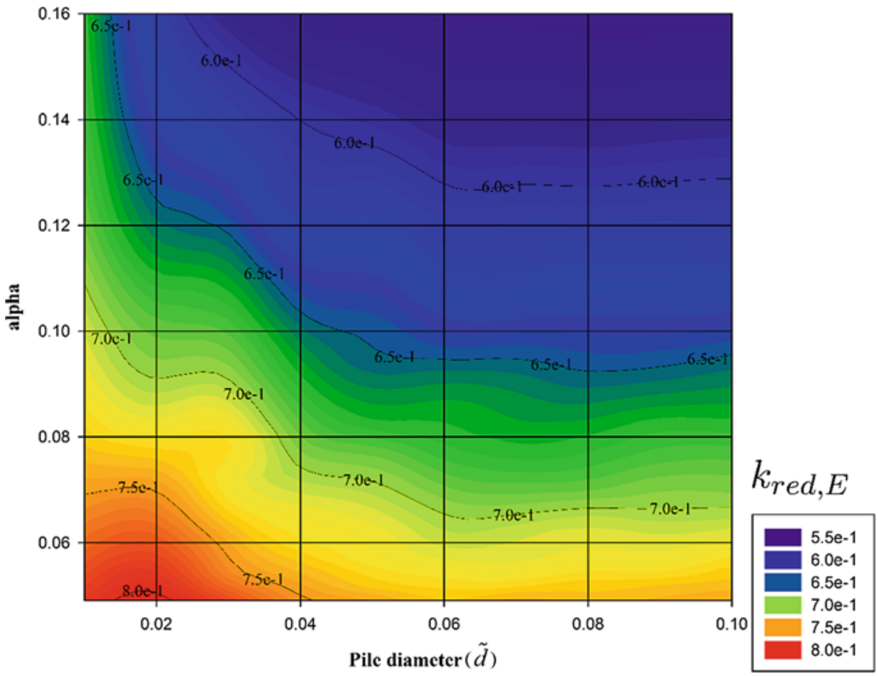

Fig. 5 Reduction ratio for the surface layer $\tilde{E} = 550, \tilde{\rho} = 1.3$ and $\tilde{h} = 1$

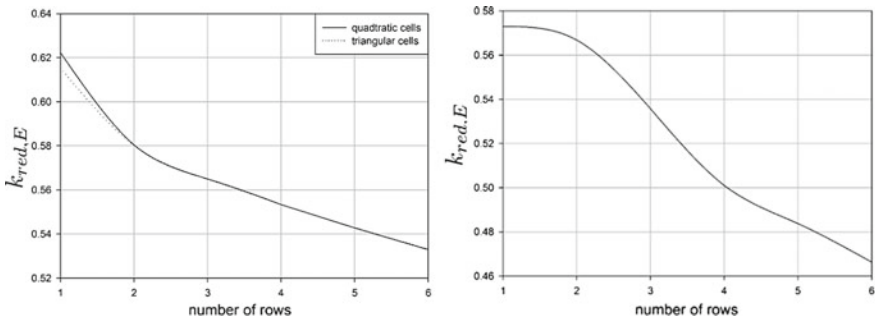

Fig. 6 The influence of row number on the vibration reduction for low diameter piles $\tilde{d} = 0.01$ (left plot) and high diameter piles $\tilde{d} = 0.06$ (right plot) $\tilde{E} = 550, \tilde{\rho} = 1.3, \tilde{h} = 1$ and $\alpha = 0.162$

Contour plots in Fig. 5 are plotted at $\tilde{E} = \frac{E_{pile}}{E_{soil}} = 550, \tilde{\rho} = \frac{\rho_{pile}}{\rho_{soil}} = 1.3$, $\nu_{pile} = 0.2, \nu_{soil} = 0.25$ and $\tilde{h} = \frac{H}{l} = 1$.

The obtained results reveal that for a single row pile barrier, both diameter and pile fraction play and important role as the maximum vibration decrease is observed at the values of pile fraction and diameter equalling to $\alpha = 0.16$ and $\tilde{d} = 0.1$ respectively. However, as it will be shown in the following text, pile diameter is less important if a pile barrier is composed of more than 2 rows (Fig. 6). In addition to that, it can be seen from Fig. 5 that the reduction ratio for the surface layer declines with an increase in the diameter at the constant alpha significantly up to the value of normalized diameter equalling to $\tilde{d} = 0.06$. Then it maintains the same level slightly fluctuating around it. At the same time, pile fraction significantly affects the reduction effect which is growing with an increase of alpha. If $\tilde{d}$ is located in the range $\tilde{d} \in [0, 0.03]$ such one row pile barrier is not effective even if pile fraction is high.

Plots in Fig. 6 show the influence of pile row number on the reduction effect at different pile configurations and two pile diameters—small and large which correspond to $\tilde{d} = 0.01$ and $\tilde{d} = 0.06$ respectively. Curves in the right and left plots in Fig. 6 are plotted at $\tilde{E} = \frac{E_{pile}}{E_{soil}} = 550, \tilde{\rho} = \frac{\rho_{pile}}{\rho_{soil}} = 1.3$, $\nu_{pile} = 0.2, \nu_{soil} = 0.25$, $\tilde{h} = \frac{H}{l} = 1$ and $\alpha = 0.162$.

Figure 6 shows that the pile configurations (triangular and quadratic cells) have virtually no effect on the reduction ratio. Therefore, the curve in the left plot in Fg. 6 is plotted only for the quadratic configuration. Apart from that, an increase in the number of rows leads to a better vibration reduction effect of the pile field and even the barriers designed of low diameter piles but having several rows can give the same reduction effect as a single row pile barrier with high diameter piles. However, high diameter piles give better reduction effect at the same number of rows (Fig. 6, right plot). Therefore, it is important to estimate the optimal configuration of pile field in terms of material volume, designed vibration reduction level and technology for each practical case.

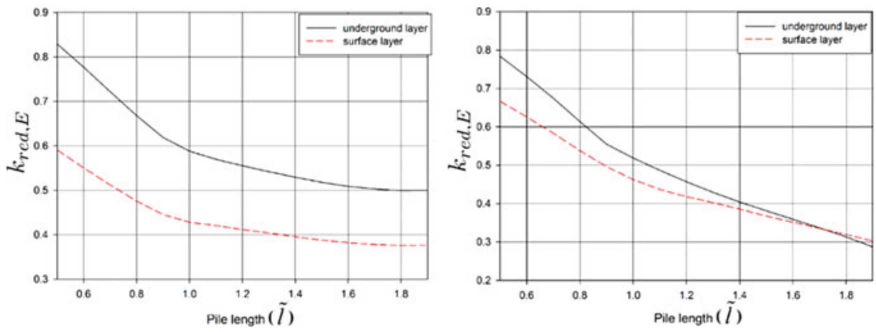

Fig. 7 The change of screening effect with increase in the pile length for low diameter piles $\tilde{d} = 0.01$ (left plot) and high diameter piles $\tilde{d} = 0.06$ (right plot) $\tilde{E} = 550, \tilde{\rho} = 1.3, \tilde{h} = 1$ and $\alpha = 0.162$

It is clear that the pile length should be comparable with the wavelength, otherwise there will be virtually no diffraction and scattering of Rayleigh waves by the piles. Hence, the field itself cannot be used as a vibration barrier. Therefore, it is important to determine the relation between pile length and the attenuation effect. The plots in Fig. 7 show the change in the reduction ratio with the increase in the pile length. The curves in this figure are plotted at $\tilde{E} = \frac{E_{pile}}{E_{soil}} = 550, \tilde{\rho} = \frac{\rho_{pile}}{\rho_{soil}} = 1.3$, $\nu_{pile} = 0.2, \nu_{soil} = 0.25$.

According to the graphs in Fig. 7, reduction effect increases with the pile length significantly resulting in the reduction ratio of $k_{red,E} = 0.4$ reaching an asymptotic limit for low diameter piles although, continue decreasing for high diameter piles. It means that further increase in the pile length will not change the reduction effect noticeably in the case of low diameter piles while it can slightly increase the perfomance of high diameter piles. Additionally, for pile length which is less than the wavelength $\tilde{h} < l$ better reduction is observed at the surface layer while for longer piles underground layer shows better vibration reduction in the case of high diameter piles.

5 Conclusion

Pile field can be an effective measure to protect structures from surface Rayleigh waves as it decreases the transmission of wave energy, that is carried out by the surface waves into the protected region, thus, declining the amplitude of displacements, velocities and accelerations of the points in this zone. Simplified and full scale spatial models are used in the calculations and the results obtained using the both models are in a good agreement. Thus, it is possible to extrapolate the results from the simplified model to the full scale pile field that may surround a structure.

This way of protection shows good effectiveness when the maximum possible wavelength is comparable with the planar dimensions of a protected area along with the geometrical parameters of the pile field. This is the case for seismic waves in soft soils, such as clays with low plasticity index, loose and medium sands etc. as well as high frequency artificial vibration sources generating vibrations in stiffer soils, like clays with high plasticity index, dense sands etc. At the same time, for both cases of application, acoustical density of the pile barrier must be different to that of the soil. In that case, the pile field satisfying this condition can provide up to 50% decrease in the vibration energy transmitted to the protected zone. It is possible to improve vibration reduction effect of a pile field increasing pile diameter, length and fraction. However, further rise of these values may lead to inappropriate cost of the structure along with the additional complexity in the construction technology.

The main parameters that affect vibration reduction are the pile fraction, length, diameter as well as the number of pile rows. It is shown that pile length should be more than half of the wavelength to ensure at least 20% reduction in the kinetic energy, meanwhile the influence of the pile fraction and diameter is strongly affected by the number of rows. It means that for a single row pile barrier, the diameter of piles plays an important role up to the value of diameter equalling to 0.06 l. Then it has virtually no effect on the reduction ratio of the surface layer, while for the underground layer it affects the vibration decrease up to the diameter of 0.08 l (here l is the design wavelength).

In the case of multi row pile barriers, the effect of pile diameter still exists, but becomes less important because the reduction ratio of low diameter piles installed in several rows can be the same as that of high diameter piles but designed as one row barrier. Therefore, there are no strict limitations on pile diameters. However, the volume of the material for the pile field will be equal for a single and multi-rows pile fields if the same vibration reduction is provided. Therefore, it is possible to use lower diameters for the piles which is a better solution from technological point of view.

An additional important result from the use of such barrier is a decrease in bending moments in the inner piles, that can be used as a deep foundation. It is shown that the possible reduction effect in bending moments of the inner piles can reach 80%.

A pile field is a less effective measure than seismic barriers in terms of vibration reduction. Although, they can protect constructions from body waves which, however, is beyond the scope of this research.

As a perspective of this work, the calculations involving current models of elasto-plastic media, that are relevant for soils, will be performed to estimate the effect of pile—soil interaction more precisely.

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