



# Reversibility of Executable Interval Temporal Logic Specifications

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**Abstract.** In this paper the reversibility of executable Interval Temporal Logic (ITL) specifications is investigated. ITL allows for the reasoning about systems in terms of behaviours which are represented as non-empty sequences of states. It allows for the specification of systems at different levels of abstraction. At a high level this specification is in terms of properties, for instance safety and liveness properties. At concrete level one can specify a system in terms of programming constructs. One can execute these concrete specification, i.e., test and simulate the behaviour of the system. In this paper we will formalise this notion of executability of ITL specifications. ITL also has a reflection operator which allows for the reasoning about reversed behaviours. We will investigate the reversibility of executable ITL specifications, i.e., how one can use this reflection operator to reverse the concrete behaviour of a particular system.

**Keywords:** Interval Temporal Logic · Temporal reflection · Program reversion · Reversible computing

## 1 Introduction

Formal methods have been used in computer science to verify desirable and undesirable properties of programs. One type of formalism introduced is temporal logic. A temporal logic allows to reason about properties over time, for example “this resource will eventually be freed”. In this paper, we are dealing with a particular temporal logic, Interval Temporal Logic (ITL).

Another strand of research is reversibility in computing. This is relevant for reversing the effects of operations, for example if, after having performed a number of operations, proceeding in the desired direction is not possible. This could be because a resource is not available or because a result is outside the allowed range of values. In such cases, a potential strategy is to roll back to a safe state and continue operation from there.

Using the ITL notation (details of which will explained in Sect. 3), a program consisting of two parts could be written as `Good;Bad`. The semantics (behaviour)

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of both parts are sequences of states. `Good` and `Bad` are arbitrary names indicating sections of the program which worked as expected respectively did not. We now want to reverse the effect of `Bad`. This would require that we go back to the last state of `Good`. This can be done by using an operator `undo` so that we have `Good ; Bad ; undo(Bad)`. This operator must ensure that the last state of `Good` is the same as the last state of `Good ; Bad ; undo(Bad)`.

We propose a solution for this problem, where we use the reflection operator  $r$ . We show that this reverses the effects of a formula, i.e., reversing the sequence of states of that formula. We also show that it can be applied to any formula that can be specified in ITL. This operator can be used for propositional as well as for first-order logic ITL. Therefore, we have a universal `undo` operation for these formulae. We distinguish *reflection*, which indicates the possibility to reverse the sequences of states, from *reversibility*. Reversibility indicates that an executable formula, a program, can be reversed.

The outline of the paper is as follows: in Sect. 2 we discuss temporal logic in general and compare ITL with other temporal logics. We also discuss reversibility in general. In Sect. 3 we discuss ITL, i.e., intervals, syntax of basic and derived constructs, and the reflection operator and the semantics of these constructs. In Sect. 4, the notion of executability is formalised and show how this notion can be used together with reflection to reverse the effects of bad computations.

An extended version of this paper can be found on arXiv.org at [3].

## 2 Background

Temporal logic can be used to describe reactive systems. After some cursory mentions earlier, a first type of temporal logic was presented by N. Prior [13]. Based on this, other types of temporal logic were devised, including (LTL) [8, 12]. The main operator of LTL is the until ( $\mathcal{U}$ ) while in ITL it is the chop ( $;$ ).  $f \mathcal{U} g$  guarantees that  $g$  will eventually hold at some future state and that  $f$  will continue to hold until then. In ITL, satisfaction of formulas is defined over intervals (sequences of states with at least 1 state) rather than time points which is used in LTL.  $f;g$  denotes that the interval can be split into a prefix and a suffix interval in such a way that  $f$  holds for the prefix interval and  $g$  holds for the suffix interval. So the chop operator corresponds to the sequential composition operator.

There has been increasing interest and research done in reversible computation, for example demonstrated by the COST action IC1405 [14]. We focus on logical reversibility. This is all types of formalisms which allow reversing steps done in order to get back to a previous state of the computation [2].

Any temporal logic models computations over time, and time is generally irreversible, but computations can be reversible, as we have seen. Because of this, it seems a logic extension of temporal logic to introduce a “time reversal”, which undoes computations and therefore seemingly reverses time, whereas actual time is progressing. [10] introduced this for ITL.

Reversibility and reflection of logic is related to reversibility of programming languages. Many works have researched the process of reversing executions of

traditional programming languages, most of which are typically irreversible as information is lost throughout. One approach to reversing such executions is to save this lost information as a program executes forward and later use it to reconstruct previous states (reflection). This includes the Reverse C Compiler [11] and the works of Hoey and Ulidowski [5,6]. Any irreversible step of an execution is made reversible via this saved information. Execution time and memory usage are crucial aspects of these methods, with a forward execution typically being slower and memory requirements being higher as information is recorded. Such approaches including that described here minimise these overheads sufficiently. This differs from checkpointing approaches, where a snapshot of the state is taken at regular intervals and used to restore to previous positions [11]. Depending on the snapshot frequency, large amounts of information must be recorded and forward re-execution is sometimes required.

A second approach is to use reversible languages such as Janus [7], where any valid program written in such a language can be executed both forward and in reverse. This is comparable to ITL programs whose reflections are executable. Janus relies on the use of increment/decrement operators to ensure no old values of variables are lost, as well as post-conditions that allow correct expression evaluation during a reverse execution. However the challenge of converting programs of a traditional language into that of a reversible language may limit its widespread use.

### 3 Interval Temporal Logic

Interval Temporal Logic (ITL) is a flexible notation for both propositional and first-order reasoning about periods of time found in descriptions of hardware and software systems [4,15]. Unlike most temporal logics, ITL can handle both sequential and parallel composition and offers powerful and extensible specification and proof techniques for reasoning about properties involving safety, liveness and projected time. Timing constraints are expressible and furthermore most imperative programming constructs can be viewed as formulas in ITL. AnaTempura (available from [4]) provides an executable framework for developing and experimenting with suitable ITL specifications.

#### 3.1 Interval

In this section we revisit the underlying semantic model of Interval Temporal Logic (albeit restricted to the finite case). The key notion of ITL is an *interval*. An interval  $\sigma$  is considered to be a non-empty, finite sequence of states  $\sigma_0, \sigma_1 \dots, \sigma_n$ . A state is the union of an integer state  $\text{State}^e$  which is a mapping from the set of integer variables  $\text{Var}^e$  to the set of integer values  $\text{Val}$ , and a Boolean state  $\text{State}^b$  which is a mapping from the set of propositional variable  $\text{Var}^b$  to the set of Boolean values  $\text{Bool}$ . Note: the embedding of ITL in Isabelle/HOL is such that one can use any definable type in Isabelle/HOL as type for an ITL variable. We have restricted the types to just integers and

Boolean in this paper. Let  $\Sigma^+$  denote the set of all finite intervals with at least 1 state. The length of an interval  $\sigma$  is denoted by  $|\sigma|$  and is the number of states minus 1, i.e., an interval with one state has length zero. Let  $\sigma = \sigma_0\sigma_1\sigma_2\dots\sigma_{|\sigma|}$  be an interval then  $\sigma_0\dots\sigma_k$  (where  $0 \leq k \leq |\sigma|$ ) denotes a prefix interval of  $\sigma$ ,  $\sigma_k\dots\sigma_{|\sigma|}$  (where  $0 \leq k \leq |\sigma|$ ) denotes a suffix interval of  $\sigma$ ,  $\sigma_k\dots\sigma_l$  (where  $0 \leq k \leq l \leq |\sigma|$ ) denotes a sub interval of  $\sigma$ .

### 3.2 Syntax

We first discuss the basic constructs and then introduce derived constructs.

Syntax of Expressions (Boolean or Integer) in BNF  $e ::= z \mid g(e_1, \dots, e_n) \mid A \mid \text{fin } A \mid \bigcirc A$  where  $z$  is a constant,  $g$  an operator, and  $A, \bigcirc A$  and  $\text{fin } A$  are temporal variables. Syntax of formulae in BNF  $f ::= \text{true} \mid h(e_1, \dots, e_n) \mid \neg f \mid f_1 \wedge f_2 \mid \exists V \cdot f \mid \text{skip} \mid f_1 ; f_2 \mid f^*$  where  $h$  is a Boolean predicate over integer or Boolean expressions, and  $V$  is a Boolean or integer variable. The formula  $\text{skip}$  denotes any interval of exactly two states. The formula  $f_1 ; f_2$ , where  $f_1$  and  $f_2$  are ITL formulae denotes an interval which is the fusion of two intervals,  $f_1$  holds over the first interval and  $f_2$  holds for the second interval. Fusion will concatenate two intervals in such a way that the last state of the first interval and the first state of the second interval are “fused” together. Fusion is only possible when these states are the same. If these states are not the same the resulting interval does not exist, i.e., is **false**. The formula  $f^*$  where  $f$  is an ITL formula denotes the fusion of a finite number of intervals, where for each interval  $f$  holds. Zero times fusion will result in an interval with exactly one state irrespective of  $f$ , i.e., **false\*** is equal to **empty**. Temporal variables  $\bigcirc V$  and  $\text{fin } V$  denote the value of variables at a particular point in an interval and are used to specify assignment constructs. The temporal variable  $\bigcirc A$  denotes the value of  $A$  in the next state. The expression  $\text{fin } A$  denotes the value of  $A$  in the last state. The formula  $\exists V \cdot f$  denotes the introduction of a local variable  $V$ .

**Derived Constructs.** The traditional Linear Temporal Logic (LTL) operators  $\bigcirc, \diamond$  and  $\square$  are defined as follows: The formula  $\bigcirc f \triangleq \text{skip} ; f$  denotes that  $f$  holds from the next state. Note that  $\bigcirc f$  is different from temporal variable  $\bigcirc V$ , although the same  $\bigcirc$  symbol is used,  $\bigcirc f$  is using the  $\bigcirc$  symbol on formula  $f$  whereas  $\bigcirc V$  is using the  $\bigcirc$  on a variable  $V$ .  $\bigcirc f$  itself is a formula whereas  $\bigcirc V$  denotes a value. The formula  $\diamond f \triangleq \text{true} ; f$  (sometimes) denotes that there exists a suffix interval for which  $f$  holds. The formula  $\square f \triangleq \neg \diamond \neg f$  (always) denotes that for each suffix interval  $f$  holds. The formula **more**  $\triangleq \bigcirc \text{true}$  denotes an interval with at least two states. The formula **empty**  $\triangleq \neg \text{more}$  denotes an interval with only one state. Note that no interval will satisfy the formula **false**. The formula  $\otimes f \triangleq \text{empty} \vee \bigcirc f$  (weak next) denotes either an interval of only one state or  $f$  holds from the next state.

**Semantics.** We now define the semantics of ITL which is a mapping from the syntactic constructs of Sect. 3.2 and the semantic model (intervals) defined

in Sect. 3.1 to values (Boolean or integer). Let  $\mathbf{E}[\![\dots]\!](\dots)$  be the “meaning” (semantic) function from  $\mathbf{Expressions} \times \Sigma^+$  to  $\mathbf{Val}$  and let  $\sigma = \sigma_0\sigma_1\dots$  be an interval. Let  $\mathbf{M}[\![\dots]\!](\dots)$  be the “meaning” function from  $\mathbf{Formulae} \times \Sigma^+$  to  $\mathbf{Bool}$ . The detailed semantics of each basic ITL construct is available from [4]. A first order ITL formula  $f$  is satisfiable denoted by  $\models f$  if and only if there exists an interval  $\sigma$  such that  $\mathbf{M}[\![f]\!](\sigma) = \text{tt}$ . A first order ITL formula  $f$  is valid denoted by  $\vdash f$  if and only if for all intervals  $\sigma$ ,  $\mathbf{M}[\![f]\!](\sigma) = \text{tt}$ .

### 3.3 Reflection

We now discuss the notion of temporal reflection for ITL formulae as defined in [10]. We first discuss the semantic notion of the reverse of a sequence of states and then discuss the reflection operator and its corresponding semantics.

Let  $f$  be a formula,  $e$  an expression, and  $\sigma$  be an interval  $\sigma_0\dots\sigma_{|\sigma|}$  then  $\text{rev}(\sigma)$  denotes interval reversal and is defined as  $\text{rev}(\sigma) \triangleq \sigma_{|\sigma|}\dots\sigma_0$ .  $f^r$  denotes temporal reflection of formula  $f$  and is defined as  $\mathbf{M}[\![f^r]\!](\sigma) \triangleq \mathbf{M}[\![f]\!](\text{rev}(\sigma))$ .  $e^r$  denotes temporal reflection of expression  $e$  and is defined as  $\mathbf{E}[\![e^r]\!](\sigma) \triangleq \mathbf{E}[\![e]\!](\text{rev}(\sigma))$ .

The Isabelle/HOL ITL library (available from [4]) has defined reflection laws for all basic ITL operators. We need to add a new temporal variable  $\odot V$  to make ITL closed under reflection. The temporal variable  $\odot V$  is the reflection of  $\circ V$  and denotes the value of  $V$  in the pen-ultimate (previous) state. The formal semantics is as follows: if  $|\sigma| > 0$  then  $\sigma_{|\sigma|-1}(V)$  else any value from the  $\mathbf{Val}$ . This leads to the following theorem

**Theorem 1.** *ITL (extended with  $\odot V$ ) is closed under reflection.*

## 4 Executability, Reflection and Reversibility

In this section we will discuss the notion of executability. It is used to determine whether an ITL formula represents a programming construct. We first formalise the notion of forward executability of a formula which corresponds to generating a sequence of states in a particular fashion: we first generate the first state and then generate the next until the final state is generated. This sequence of state constitutes the behaviour of the system described by the formula. We then investigate the reflection of forward executable formula and this requires the introduction of the notion of backward executability. This notion corresponds to generating a sequence of states but now we first generate the final state and then generate the previous state until we generate the first state. This sequence corresponds to the reversed behaviour of the system described by the formula. Forward and backward executability are related by the reflection operator.

### 4.1 Forward Executability

The intuition of an executable formula (specification) is that it corresponds to a computation, i.e., in our case a sequence of states. Obviously any executable

formula needs to be satisfiable. But not every satisfiable formula is executable because we further require it to be “deterministic”. We will give a formal definition what we mean by this. The executable formula corresponds to programming constructs. The following definitions are used to determine whether a formula is executable or not. First we define the notion of a value trace of a formula wrt a list of variables. These variables are the “free variables” appearing in  $f$ , i.e.,  $f$  constrains the values of these variables. Note: These definitions and all subsequent theorems have been specified and verified in the Isabelle/HOL [1] system (library available from [4]).

**Definition 1.** Let  $\mathbf{s}$  be a state and let  $\bar{v}$  denote a non-empty list of variables  $v_0, \dots, v_n$  and let  $\mathbf{s}(\bar{v})$  denote the corresponding list of values  $\mathbf{s}(v_0), \dots, \mathbf{s}(v_n)$  of  $\bar{v}$  in state  $\mathbf{s}$ . Let  $\text{Spec}$  be a formula and  $\sigma$  be an interval and  $\mathbb{M}[\text{Spec}](\sigma) = tt$  then the value trace of  $\text{Spec}$  wrt  $\bar{v}$  is denoted by  $\text{map}(\lambda \mathbf{s}.\mathbf{s}(\bar{v})) \sigma$  and defined as  $\sigma_0(\bar{v}) \sigma_1(\bar{v}) \dots \sigma_{|\sigma|}(\bar{v})$ .

*Example 1.* The value trace for  $A = 0 \wedge A \text{ gets } A + 1 \wedge \square(B = A * 2)$  wrt  $(A, B)$  is  $(0, 0) (1, 2) (2, 4) (3, 6) (4, 8) \dots$  and it represents how  $A$  and  $B$  change,  $A$  is increased by one and  $B$  equals twice  $A$  in every state.

The following definition is a constraint on the intervals which satisfy a formula. Only intervals that share a common prefix of the value trace are allowed,

**Definition 2.** A formula  $\text{Spec}$  has a common prefix value trace wrt a list of variables  $\bar{v}$  denoted by  $\ddagger[\text{Spec}]_{\bar{v}}$  if and only if for all intervals  $\sigma$  and  $\sigma'$  if  $\mathbb{M}[\text{Spec}](\sigma) = tt$  and  $\mathbb{M}[\text{Spec}](\sigma') = tt$  and  $|\sigma| \leq |\sigma'|$  then  $(\text{map}(\lambda \mathbf{s}.\mathbf{s}(\bar{v})) \sigma) = (\text{map}(\lambda \mathbf{s}.\mathbf{s}(\bar{v})) (\sigma'_0 \dots \sigma'_{|\sigma|}))$ .

In above definition we compare the value trace corresponding to  $\sigma$  with the prefix (of length  $|\sigma|$ ) of the value trace of corresponding to  $\sigma'$ . The intuition is that the latter is a continuation of the first, i.e., the first value trace is a “beginning” of the latter value trace. The following example illustrates this notion.

*Example 2.* The following are some formula that have a common prefix value trace.

- $\ddagger[A = 0 \wedge \text{empty}]_A$ , there is only one possible value trace 0.
- $\ddagger[A = 0 \wedge A \text{ gets } A + 1]_A$ , the possible value traces are  $0; 0, 1; 0, 1, 2$ . Each pair of value traces share a common prefix. The common prefix value trace of pair 0 and 0, 1 is 0 and of pair 0, 1 and 0, 1, 2 is 0, 1. Note that in the latter pair there is another shared prefix 0 but in the definition it states that we are looking for a prefix that has a length equal to the “smallest” of the two. Note we align on the left.

The following are some formula that have no common prefix value trace.

- not  $\ddagger[(A = 0 \vee A = 1) \wedge \text{empty}]_A$ , we have two value traces 0 and 1, but they do not share a common prefix.

- not  $\dagger[A = 0 \wedge \text{skip}]_A$ , we have for instance value traces 0, 0 and 0, 1 but when their length are the same they ought to agree on all values and this does not hold as they disagree in the second state.
- not  $\dagger[\text{skip}]_A$ ,  $A$  does not appear in the formula so values of  $A$  are not constrained at all, one has value trace 0, 0 and 1, 0 and these do not share a common prefix.

The following theorem states that the combination of satisfiability with the notion of common prefix value trace can be used to determine whether a formula is executable or not, i.e., satisfiable and deterministic.

**Theorem 2.** *Let  $\text{Spec}$  be a formula and  $\bar{v}$  be a list of variables. If  $\models \text{Spec}$  and  $\dagger[\text{Spec}]_{\bar{v}}$  then for all  $k \geq 0$   $\#\{(\text{map } (\lambda s. s(\bar{v})) \sigma) \mid M[\text{Spec}](\sigma) = tt \text{ and } |\sigma| = k\} \leq 1$ .*

In above theorem we have that all satisfying intervals of length  $k$  will corresponds to at most one value trace.

The notion of common prefix value trace corresponds to the notion of generating a satisfying interval for a formula but it “limits” how this is achieved, i.e., one proceeds in a forward manner by extending at the right and therefore no backtracking will be used. The following definition introduces the notion of forward executability.

**Definition 3.** *Let  $\text{Spec}$  be a formula and  $\bar{v}$  a list of variables.  $\text{Spec}$  is forward executable wrt  $\bar{v}$  denoted by  $\dagger[\text{Spec}]_{\bar{v}}$  if and only if  $\models \text{Spec}$  and  $\dagger[\text{Spec}]_{\bar{v}}$ .*

In Tempura [9], the executable subset of ITL, a formula  $\text{Spec}$  is rewritten into a normal form  $\text{init } w_0 \wedge^{\circledast} \text{Spec}_0$ . The  $\text{init } w_0$  represents the initial state and  $\circledast \text{Spec}_0$  represents the behaviour of the system from the next state onward but only if there is a next state. This process is repeated for formula  $\text{Spec}_0$ , i.e., it is rewritten to  $\text{init } w_1 \wedge^{\circledast} \text{Spec}_1$ . This process of rewriting into normal form corresponds to our notion of forward executability. This is expressed in the following theorem.

**Theorem 3.** *Given formulae  $w$  and  $\text{Spec}$  and a list of variables  $\bar{v}$ . If  $\dagger[\text{init } w \wedge \text{empty}]_{\bar{v}}$  and  $\dagger[\text{Spec}]_{\bar{v}}$  then  $\dagger[\text{init } w \wedge^{\circledast} \text{Spec}]_{\bar{v}}$ .*

In Example 2 we have seen that one needs to be careful in adding constructs that limit the length of an interval. The following theorem gives conditions for which it is safe to do so.

**Theorem 4.** *Let  $\text{Spec}_0$  and  $\text{Spec}_1$  be formula and  $\bar{v}$  be a list of variables. If  $\models \text{Spec}_0 \wedge \text{Spec}_1$  and  $\dagger[\text{Spec}_0]_{\bar{v}}$  then  $\dagger[\text{Spec}_0 \wedge \text{Spec}_1]_{\bar{v}}$ .*

In this theorem formula  $\text{Spec}_0$  ensures that the values for  $\bar{v}$  are deterministic and formula  $\text{Spec}_1$  is used to put extra constraints on the intervals satisfying  $\text{Spec}_0$ . The  $\models \text{Spec}_0 \wedge \text{Spec}_1$  condition ensures that we have at least one such interval. Examples of such  $\text{Spec}_1$  are  $\text{len}(k)$ ,  $\diamond \text{init } w$  and  $\text{halt } w$ . On their own these formulae are not forward executable but combined with a forward executable one they will be.

## 4.2 Backward Executability

We now investigate reversing executable specifications. Reflection relates the notion of prefix intervals with that of suffix intervals. So we need to introduce the “mirror image” of common prefix value traces, i.e. the notion of common suffix value trace.

The following definition is a constraint on the intervals which satisfy a formula. Only intervals that share a common suffix of the value trace are allowed.

**Definition 4.** *A formula  $\text{Spec}$  has a common suffix value trace wrt a list of variables  $\bar{v}$  denoted by  $\natural[\text{Spec}]_{\bar{v}}$  if and only if for all intervals  $\sigma$  and  $\sigma'$  if  $\mathbb{M}[\text{Spec}](\sigma) = tt$  and  $\mathbb{M}[\text{Spec}](\sigma') = tt$  and  $|\sigma| \leq |\sigma'|$  then  $(\text{map } (\lambda s.s(\bar{v})) \sigma) = (\text{map } (\lambda s.s(\bar{v})) (\sigma'_{|\sigma'| - |\sigma|} \dots \sigma'_{|\sigma'|}))$ .*

The following example illustrates this notion.

*Example 3.* The following are some formula that have a common suffix value trace.

- $\natural[(\text{fin } A) = 0 \wedge \text{empty}]_A$ , there is only one value trace 0.
- $\natural[\Box(A = 0)]_A$ , the possible value traces are 0; 0, 0; 0, 0, 0. Each pair of value traces share a common suffix.

The following are some formula that have no common suffix value trace.

- $\text{not } \natural[(A = 0 \vee A = 1) \wedge \text{empty}]_A$ , we have two value traces 0 and 1, but they do not share a common suffix.
- $\text{not } \natural[(\text{fin } A) = 0 \wedge \text{skip}]_A$ , we have for instance value traces 0, 0 and 1, 0 but when their length are the same they ought to agree on all values and this does not hold as they disagree in the first state.
- $\text{not } \natural[\text{skip}]_A$ ,  $A$  does not appear in the formula so values of  $A$  are not constrained at all, one has value trace 0, 0 and 0, 1 and these do not share a common suffix.

The following lemma states the relationship between common prefix, common suffix and reflection.

**Lemma 1.** *Let  $\text{Spec}$  be formula and  $\bar{v}$  be a list of variables then  $\natural[\text{Spec}^r]_{\bar{v}}$  iff  $\natural[\text{Spec}]_{\bar{v}}$  and  $\natural[\text{Spec}^r]_{\bar{v}}$  iff  $\natural[\text{Spec}]_{\bar{v}}$ .*

For the notion of satisfiability we have the following lemma.

**Lemma 2.** *Let  $\text{Spec}$  be a formula then  $(\models \text{Spec}^r)$  iff  $(\models \text{Spec})$ .*

The following theorem states that the combination of satisfiability with the notion of common suffix value trace can be used to determine whether a formula is deterministic or not, i.e., is backward executable or not.

**Theorem 5.** *Let  $\text{Spec}$  be a formula and  $\bar{v}$  be a list of variables. If  $\models \text{Spec}$  and  $\natural[\text{Spec}]_{\bar{v}}$  then for all  $k \geq 0$   $\#\{(\text{map } (\lambda s.s(\bar{v})) \sigma) \mid \mathbb{M}[\text{Spec}](\sigma) = tt \text{ and } |\sigma| = k\} \leq 1$ .*



The notion of common suffix value trace corresponds to notion of generating a satisfying interval for a formula but it "limits" how this is achieved, i.e., one proceeds in a backward manner. The following definition introduces the notion of backward executability.

**Definition 5.** *Let  $\text{Spec}$  be a formula and  $\bar{v}$  a list of variables.  $\text{Spec}$  is backward executable wrt to  $\bar{v}$  denoted by  $\flat[\text{Spec}]_{\bar{v}}$  if and only if  $\models \text{Spec}$  and  $\sharp[\text{Spec}]_{\bar{v}}$ .*

In Tempura we have unfortunately no rules for backward execution. But we can define a mirror image of Theorem 3, i.e., the normal form would be  $\text{fin } w \wedge \odot \text{Spec}$ . So we first generate the last state of the interval and then proceed to determine the previous state if there is any.

**Theorem 6.** *Given the formulae  $w$  and  $\text{Spec}$  and list of variables  $\bar{v}$ . If  $\flat[\text{fin } w \wedge \odot \text{Spec}]_{\bar{v}}$  and  $\flat[\text{Spec}]_{\bar{v}}$  then  $\flat[\text{fin } w \wedge \odot \text{Spec}]_{\bar{v}}$ .*

The following theorem is similar to Theorem 4.

**Theorem 7.** *Let  $\text{Spec}_0$  and  $\text{Spec}_1$  be formula and  $\bar{v}$  be a list of variables. If  $\models \text{Spec}_0 \wedge \text{Spec}_1$  and  $\sharp[\text{Spec}_0]_{\bar{v}}$  then  $\flat[\text{Spec}_0 \wedge \text{Spec}_1]_{\bar{v}}$ .*

### 4.3 Reversing the Effects of Bad Computations

In the introduction we have seen that we are interested in formulae of the form  $\text{Good}; \text{Bad}; (\text{Bad})^r$ . We now investigate under which conditions can we forward execute  $\text{Bad}; \text{Bad}^r$ . The chop operator is non-deterministic if the length of  $\text{Bad}$  is left unspecified, i.e., generally we have not  $\sharp[\text{Bad}; \text{Bad}^r]_{\bar{v}}$ . However, we can use Theorem 4 to strengthen  $\text{Bad}$  to  $\text{Bad} \wedge \text{len}(k)$ . We similarly strengthen the  $\text{Bad}^r$  to  $\text{Bad}^r \wedge \text{len}(k)$  in order to ensure that we undone that specific bad computation  $\text{Bad} \wedge \text{len}(k)$ . Note that  $(\text{Bad} \wedge \text{len}(k))^r$  is equivalent to  $\text{Bad}^r \wedge \text{len}(k)$ , this follows from the reflection laws. The following theorem gives the conditions necessary to "undo" a bad computation.

**Theorem 8.** *Let  $\text{Spec}$  be a formula and  $\bar{v}$  be a list of variables. If  $\models \text{Spec} \wedge \text{len } k$  and  $\sharp[\text{Spec}]_{\bar{v}}$  and  $\sharp[\text{Spec}]_{\bar{v}}$  then  $\sharp[(\text{Spec} \wedge \text{len}(k)); (\text{Spec}^r \wedge \text{len}(k))]_{\bar{v}}$  and  $\flat[(\text{Spec}^r \wedge \text{len}(k)); (\text{Spec} \wedge \text{len}(k))]_{\bar{v}}$ .*

Notice that  $\text{Spec}$  needs to have both a common prefix value trace and a common suffix value trace. In the first case we proceed in a forward manner while in the second case in a backward manner.

## 5 Conclusion and Future Work

First order ITL is a flexible notation for specifying properties and behaviours of systems. Most imperative programming constructs can be specified by formulae in ITL. We have used the reflection operator for the specification of reversed behaviour of systems. It is shown that ITL is closed under this reflection operator which means that we can specify its reverse for any ITL formula. We have

presented an extensive list of reflection laws that help in the construction of the reverse of an ITL formula. We have shown that when an ITL formula is forward and backward executable then one can indeed reverse its behaviour.

Future work consists of adding the backward execution mechanism to the Tempura tool. The reflection and reversal of event-based programs is another area of interest. In an event-based program, a trigger event causes a chain of reactions by a system. The occurrence of a trigger can not be reversed but the reaction by the system can be reversed. However, this reaction might include other triggers that will set off other chains of reactions. Determining this chain of reactions and reversing its effects are some of the challenges that need to be addressed.

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