Relationships Between Compressive and Splitting Tensile Strengths of Cast and Core High-Strength Concrete Cylinders

Richard Campos[1(✉)], Matias M. Mendez Larrain[1], Musharraf Zaman[2], and Victor Pozadas[1]

[1] Roca Engineering, Inc., 3200 NW 38th Street, Oklahoma City, OK 73112, USA
`richard.campos@ou.edu`, `{mmendez,vpozadas}@rocaengineering.com`
[2] Department of Civil Engineering and Environmental Science, University of Oklahoma, 202 Boyd Street, Rm. 334, Norman, OK 73109, USA
`zaman@ou.edu`

Abstract. Distressed pavements are costly due to frequent maintenance, rehabilitation, and destructive testing. Furthermore, distressed pavements can lead to increased vehicle repair costs, safety concerns, and fatalities. The present study uses the splitting tensile and compressive strengths of 140 cast and core specimens to develop theoretical regression models that can better correlate the indirect splitting tensile and compressive strengths at 28 days for high-strength concrete. The laboratory experiments that were conducted in this study include splitting tensile and compressive strength tests using a universal testing machine. The reliability of the proposed models was verified by comparing their predictive capabilities with the laboratory tests. Statistical regression software was used to analyze the cast and core strength test results and develop regression models. The root mean square error (RMSE) was used to assess the accuracy between the values predicted by the regression models and laboratory values. Existing regression models reported by researchers and institutions were also analyzed with the RMSE method to determine the most accurate model. Based on the analysis, a positive nonlinear correlation between the splitting tensile strength and the compressive strength of high strength concrete was determined. The proposed regression equation exhibits small errors when compared to the laboratory results, which allow for efficient and accurate predictions of the splitting tensile strength.

Keywords: Splitting tensile strength relationship · Compressive strength relationship · Core and cast specimen relationship

1 Introduction

The assessment of a pavement's strength capacity is necessary as the pavement ages and sustains distresses. Pavement distresses can lead to costly vehicle repairs, vehicle crashes, and fatalities (U.S. Department of Transportation 2015; ASCE 2017; Justo-Silva

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and Ferreira 2019). The stress-inducing factors that degrade and crack pavements are temperature, vehicle traffic, and poor subgrade characteristics (Yoder and Witczak 1975). Due to the necessity of evaluating pavement performance by means of Quality Assurance and Quality Control (QA/QC) and ensuring driver safety, Non-Destructive Tests (NDT) and Semi-Destructive Tests (SDT) have been developed (Chen and Won 2007). The most common SDT method for determining the strength of an existing concrete pavement is by core testing, a method by which core specimens are extracted from the pavement and tested in laboratory. Similar to cast cylinder strength tests, core cylinders are also used to determine the compressive strength and splitting tensile strength (ASTM 2018c). Coring is a relatively easy process; however, there are more variables that can affect the strength of a core specimen, such as core size, moisture, drilling direction, and core damages (Chen et al. 2013). Consequentially, there exists no relationship between the concrete strength of core and cast specimens (ASTM 2018c).

Researchers have attempted to relate the compressive and splitting strengths of concrete cast cylinders with regression modeling (Hammitt 1974; Oloukun 1991; Thomas and Ramaswamy 2007; Pul 2008; Xu and Shi 2009; ACI 363R-10 2014; ACI 318-14 2014; Ramadoss 2014); Behnood et al. 2015), however, little literature was found relating the compressive and splitting strengths of concrete cores cylinders. Furthermore, no literature was found that examines the correlation between the compressive and splitting tensile strengths of both cast and core cylinders. Relationships between the compressive and splitting tensile strengths are desirable due to efficiency, similar strength testing procedures, and similar material mechanics. The present study uses the splitting tensile and compressive strengths of cast and core concrete specimens to develop mathematical regression models that can better correlate the splitting tensile and compressive strengths for high-strength concrete. The reliability of the proposed models is verified by comparing their predictive capabilities with the laboratory tests.

2 Background

2.1 Cylindrical Core Specimens

Determining the strength of an existing concrete pavement is crucial for maintenance and rehabilitation and is commonly achieved by extracting cylindrical core specimens from the pavement under consideration and then testing the core specimens in compression. Furthermore, core cylinders may also be necessary when the quality of the concrete during construction is in question (ASTM 2018c). The in-situ compressive strength, determined from testing cylindrical core specimens, is then used to assess the quality of the concrete (ASTM 2018c). Testing core cylinders in compression is attractive for QA/QC purposes due to the similarities between the testing procedures of core and cast cylindrical specimens. Furthermore, the tensile strength of core specimens is estimated in the same way as cast specimens by testing cores in splitting tension. The splitting tensile strength is determined by loading a core specimen in compression along its length until the specimen fails in indirect tension (splitting tension) due to internal tensile stresses.

2.2 Relationship Between Core and Cast Specimens

Generally, the compressive strength of core specimens is lower than that of cast specimens due to factors that potentially decrease the core strength, such as: core location and elevation, drilling orientation, drilling type, drill condition, temperature, moisture, and core damages (ASTM 2018c; Chen et al. 2013). A relationship between the two types of specimens (cast and core) would be ideal however, due to the aforementioned factors that decrease the strength of core cylinders, no such relationship exists between the two specimen types (ASTM 2018c). Instead of relating the two types of specimens, this study aims to combine both core and cast specimen strengths to better predict the compressive or splitting tensile strengths of either core or cast specimens.

2.3 Relationships Between Compressive and Splitting Strengths

Predicting the splitting strength with the compressive strength is efficient, cost effective, and straightforward. Researchers have related the concrete strengths of cast standard cylinders; however, little information was found on the relationships of core concrete strengths and no information was found on the relationships of combined core and cast concrete strengths. The mathematical model proposed by other researchers to relate the compressive and splitting tensile strengths are linear models and power models and are shown in Table 1. The nonlinear regression power model is the most common mathematical model used for relating the compressive and splitting strengths of concrete. This model type is also used to relate the compressive and flexural strengths of concrete. This particular model is attractive for relating concrete strengths due the nonlinear nature of concrete strength gain (Campos et al. 2020). Furthermore, because the compressive and splitting tensile strengths both fail by applying a compressive force to the cylinder, more credence is given to the strength relationship. The relationship between the compressive and splitting tensile strengths can be traced back to Raphael (1984) who proposed a nonlinear power regression model of the following form:

$$f_{st} = af_c^n \tag{1}$$

where, f_{st} is the splitting tensile strength (indirect tensile), a is a regression coefficient, f_c is the compressive strength, and n is a power regression coefficient.

Table 1. Relationship between splitting tensile and compressive strength

Author/Code	Equation
Ramadoss (2014)	$f_{st} = 0.118f_c^{0.84}\ (MPa)$
Xu and Shi (2009)	$f_{st} = 0.21f_c^{0.83}\ (MPa)$
Oluokun (1991)	$f_{st} = 0.294f_c^{0.69}\ (MPa)$
Hammitt (1974)	$f_{st} = 0.08f_c + 0.7014\ (MPa)$

(*continued*)

Table 1. *(continued)*

Author/Code	Equation
Thomas and Ramaswamy (2007)	$f_{st} = 0.57\sqrt{f_c}\ (MPa)$
Behnood et al. (2015)	$f_{st} = 0.219 f_c^{2/3}\ (MPa)$
Pul (2008)	$f_{st} = 0.106 f_c^{0.948}\ (MPa)$
ACI 363R-10 (2010)	$f_{st} = 0.59\sqrt{f_c}\ (MPa)$
ACI 318-14 (2014)	$f_{st} = 0.56\sqrt{f_c}\ (MPa)$

Note: $f_{st} = splitting\ tensile\ strength\ (MPa)$, $f_c = compressive\ strength\ (MPa)$

3 Materials and Methods

3.1 Concrete Mixture Design

Cylindrical core and cast concrete specimens were obtained from two different airport pavement projects. The concrete mix proportions for the specimens were designed to meet the minimum required compressive strength of 34.5 MPa at 28 days, according to US Army Corps of Engineers EM 1110-2-2000 (U.S. Army Corps of Engineers 2001) design specifications. The aggregate size, aggregate moisture, and water to cement ratio were all adjusted until the mix designs met the specified strengths for both pavements at 28 days. The water to cement ratio for both mix designs is 0.43. Tables 2 and 3 show the mix design proportions for both the core and cast specimens. The concrete cast cylinders were fabricated, cured, and tested as per ASTM C31 (2018b), C39 (2018a), and C496 (2017) specifications and the core cylindrical specimens were extracted and tested as per ASTM C42 (2018c).

Table 2. Core concrete mix proportions

Materials	(%)
Cement	10.73
Fly ash	3.58
Coarse aggregate	49.78
Fine aggregate	30.69
Water	5.20
Air entrainment	0.01
Water reducer	0.02
Set retarder	0.00

Table 3. Cast concrete mix proportions

Materials	(%)
Cement	7.97
Fly ash	3.26
Coarse aggregate	44.37
Fine aggregate	24.50
Water	14.31
Air entrainment	0.02
Water reducer	0.06
Air	5.50

3.2 Test Methods

In total, 140 specimens were tested in this study. A portion of these specimens were tested under compression and another portion were tested in indirect tension (splitting tension). A total of 84 cylindrical core specimens each having a diameter of 101.6 mm. and a length of 203.2 mm. were extracted, prepared, and tested. Of these, 42 core cylinders were tested under uniaxial compressive loading as per ASTM C39 (2018a) and C42 (2018c) using a universal testing machine after curing for over 28 days. The remaining 42 core cylinders were tested under indirect tension after curing for over 28 days as per ASTM C496 (2017) and C42 (2018c) using a universal testing machine.

A total of 56 cylindrical cast specimens each having a diameter of 152.4 mm. and a length of 304.8 mm. were fabricated and tested. Of these, 28 cast cylinders were tested under uniaxial compressive loading as per ASTM C39 (2018a) using a universal testing machine after curing for 7 days. The remaining 28 cast cylinders were tested under indirect tension after curing for 7 days as per ASTM C496 (2018c) using a universal testing machine. The average compressive and splitting tensile stresses were then calculated for both the core and cast specimens. A comparison of the average core and cast stresses can be seen in Fig. 1. Due to concrete being mostly nonlinear with high variability, the r-squared values in Fig. 1. are ignored as they only represent the linear trendlines, which are provided to demonstrate the correlation direction. R-squared values are not valid for nonlinear models (Spiess and Neumeyer 2010). Therefore, the Root Mean Square Error (RMSE) will be used to assess the nonlinear regression models instead of the r-squared values.

3.3 Model Development

IBM SPSS Statistics 26, a statistical regression software, was used to analyze and determine mathematical relationships between the compressive and splitting tensile strengths of the concrete core and cast cylinders at 7 and 28 days. Generalized models were developed to predict the splitting tensile strength at potentially any day by simultaneously analyzing both 7-day cast and 28-day core concrete strengths. The compressive and

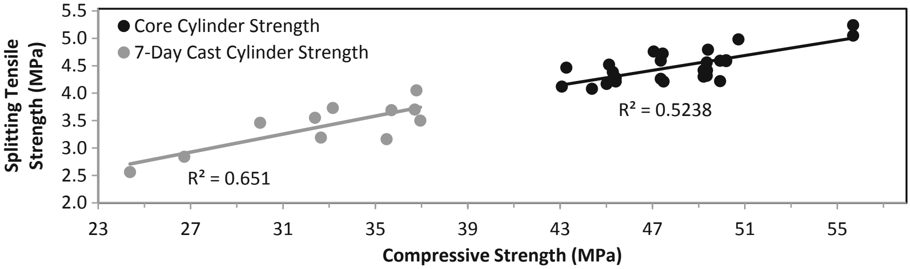


Fig. 1. Strength parameters of cast cylinders at 7 days and core cylinders over 28 days: splitting tensile strength vs. compressive strength.

splitting tensile strengths were related with the power, square root, log-linear, and linear regression models. The error in the prediction equations was assessed by analyzing the differences in the RMSE of each equation to determine the most accurate equation. The RMSE analysis was performed for each regression equation and specimen type (core and cast) to better understand the error between the two types of specimens.

4 Results and Discussion

4.1 Regression Models

The regression equations that were developed with core and cast concrete cylinder strengths are as follows:

$$f_{st} = 0.259{f_c}^{0.735}\ (\mathrm{MPa}) \tag{2}$$

$$f_{st} = 0.753{f_c}^{0.532} - 1.458\ (\mathrm{MPa}) \tag{3}$$

$$f_{st} = 0.094f_c(\ \mathrm{MPa}) \tag{4}$$

$$f_{st} = 0.116\ln(f_c)^{2.695}\ (\mathrm{MPa}) \tag{5}$$

$$f_{st} = 2.801\ln(f_c) - 6.391\ (\mathrm{MPa}) \tag{6}$$

$$f_{st} = 0.631\sqrt{f_c}\ (\mathrm{MPa}) \tag{7}$$

where, f_{st} is the splitting tensile strength (indirect tensile), ln is the natural log, and f_c is the compressive strength.

4.2 Root Mean Square Error Analysis and Model Comparison

Results from the RMSE analysis can be seen in Tables 4 and 5. Table 4 shows the RMSE analysis for equations proposed by researchers/institutions and Table 5 shows the RMSE analysis for the regression equations developed in this study. From Table 5 it is observed that linear Eq. (4) and square root Eq. (7) have the largest RMSEs for cast and core specimens relative to the other regression equations. Consequentially, Eqs. (4) and (7) have the largest overall RMSEs. Log-linear Eq. (6) has the lowest RMSE for cast specimens (0.239 MPa) and a lower RMSE for core specimens (0.206 MPa). Power Eqs. (2), (3), and (5) have nearly identical RMSEs for cast and core specimens. The power equations gave consistent and low RMSEs for each case. The complexity of the power model had almost no effect on the RMSE for each case. From Fig. 2 it is observed that proposed power Eq. (2) and Eq. (5) both trend well with the concrete strengths of cast and core specimens.

Table 4. Root mean square error analysis

Author/Code	RMSE cast 7 day	RMSE core 28 day	Overall RMSE
Ramadoss (2014)	0.294	0.464	0.453
Xu and Shi (2009)	0.505	0.807	0.785
Oluokun (1991)	0.266	0.282	0.298
Hammitt (1974)	0.247	0.224	0.248
Thomas and Ramaswamy (2007)	0.291	0.546	0.521
Behnood et al. (2015)	0.440	0.496	0.516
Pul (2008)	0.523	0.349	0.439
ACI 363R-10 (2010)	0.268	0.422	0.412
ACI 318-14 (2014)	0.317	0.611	0.581

RMSE = Root mean square error (MPa)

Table 5. Regression equation error analysis

Equation	RMSE Cast 7 Day	RMSE Core 28 Day	Overall RMSE
Regression Eq. 2	0.242	0.202	0.231
Regression Eq. 3	0.240	0.202	0.231
Regression Eq. 4	0.373	0.229	0.300
Regression Eq. 5	0.240	0.203	0.231
Regression Eq. 6	0.239	0.206	0.233
Regression Eq. 7	0.358	0.227	0.293

RMSE = Root mean square error (MPa)

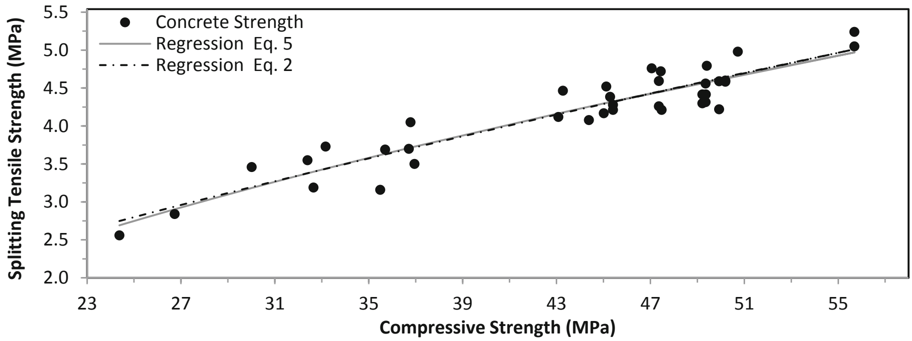
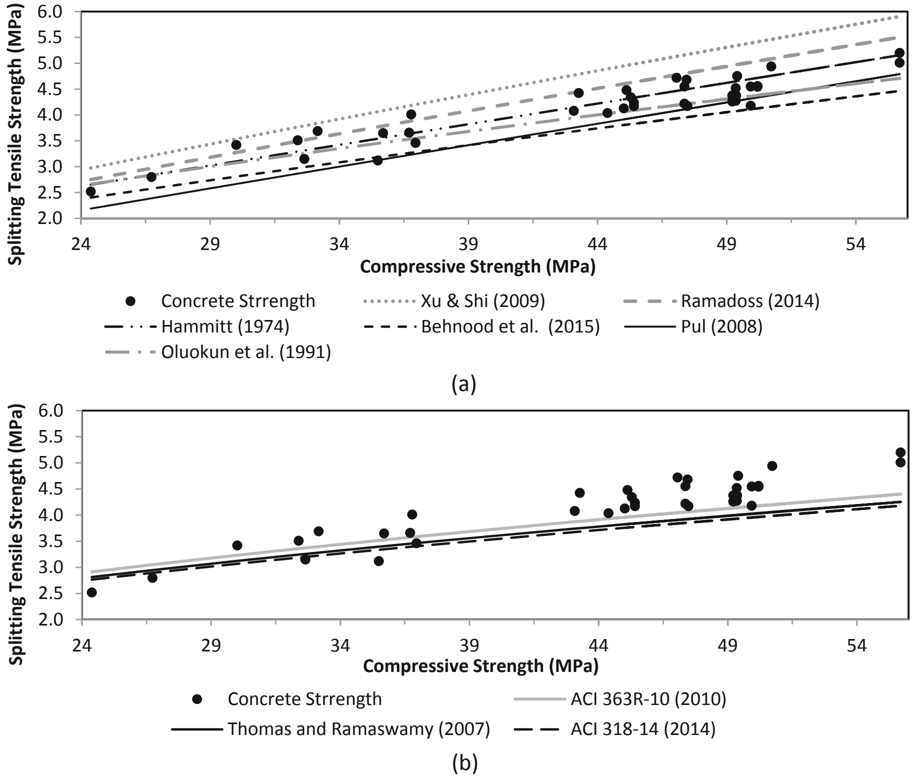


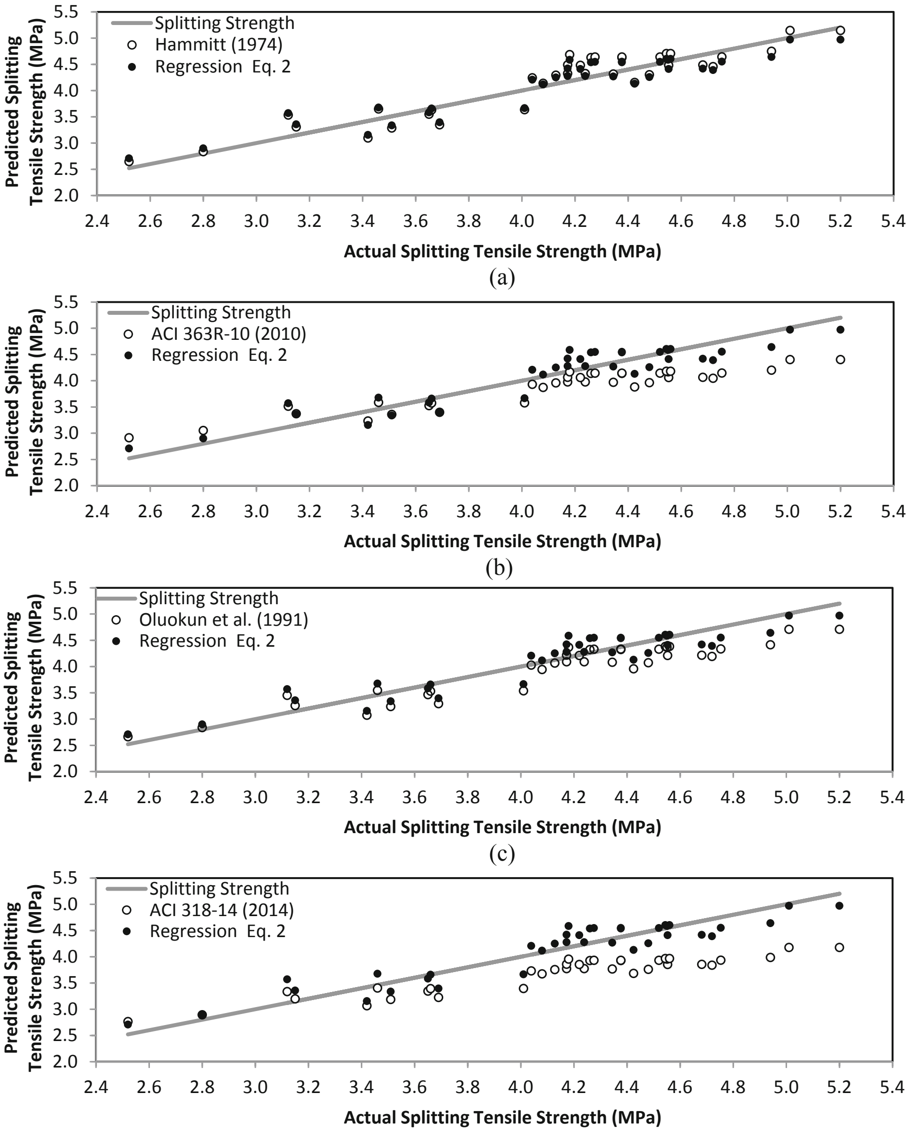
Fig. 2. Proposed splitting tensile strength prediction model capabilities: splitting tensile strength vs. compressive strength.



(a)

(b)

Fig. 3. Splitting tensile strength model prediction capabilities by other researchers: splitting tensile strength vs. compressive strength.



(a)

(b)

(c)

(d)

Fig. 4. Predicted splitting tensile strength vs. actual splitting tensile strength: (a) comparison of splitting tensile strength model by Hammitt (1974) and proposed Eq. (2), (b) comparison of splitting tensile strength model by ACI 363R-10 (2010) and proposed Eq. (2), (c) comparison of splitting tensile strength model by Oluokun (1991) and proposed Eq. (2), comparison of splitting tensile strength model by ACI 318-14 (2014) and Proposed Eq. (2).

From Table 4 it is observed that the equations from the literature generally have lower RMSEs for cast specimens than core specimens. This is most likely due to the exclusion of core specimens in the regression modeling. Furthermore, the overall RMSEs for the equations from the literature are all relatively high. This is most likely due to the lack of core specimens and small sample sizes. Figure 3 shows that the square root model is inadequate for correlating the compressive and splitting tensile strengths. The underperforming predictive capabilities of the square root model can be seen more clearly in Fig. 4. Figure 4 also shows that a linear equation with a regression constant increases its predictive capabilities. Comparatively, proposed Eq. (2) has a lower RMSE than all of the equations from the literature in each case.

5 Conclusions

The present study used the splitting tensile (indirect tensile) and compressive strengths of cast and core specimens to develop mathematical regression models that can better correlate the indirect splitting tensile and compressive strengths at 28 days for high-strength concrete. The reliability of the proposed models was verified by comparing their predictive capabilities with the measured results. Statistical regression software was used to analyze the cast and core strength test results and develop regression models. From the root mean square error and regression model analysis, the following conclusions can be made:

- Regression equations that only utilize the strength parameters from cast specimens leads to larger errors when predicting the strengths of core specimens. Both core and cast specimens should be analyzed together to produce regression equations that can predict either core or cast strengths.
- The nonlinear power model was the most accurate model developed for relating the compressive and splitting tensile strengths for both core and cast specimens. Linear and square root models are not adequate for relating the compressive and splitting tensile strengths. However, including an additional regression constant to the linear model improves its predictive capabilities.
- The compressive strength has a positive nonlinear correlation with the splitting tensile strength for both core and cast specimens. This is likely due to similar specimen geometry and testing methods.
- The proposed model is adequate for evaluating the splitting tensile strength of an existing pavement. Further, the proposed equation is able to predict either the cast or core splitting strengths allowing designers to determine the strength of new and existing pavements. The proposed power model can be used to predict the splitting tensile strength at potentially any day.

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