Chapter 1
On Dynamic Optimality of Anti-Sandwiches

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Abstract From the viewpoint of structural engineering, natural frequencies and associated eigenmodes of Anti-Sandwiches are crucial points in the context of their dynamic behavior. Here we suggest a general format for dynamic analysis by employing an extended layerwise theory. A finite-element implementation ensures the efficiency of the general solution approach. The set of control variables initially consists of originally 14 geometry and material parameters. The nature of this input enables to bound the space of parameters affecting the eigenbehavior. Due to the lack of any generic measure for optimality, we determine optimal values of the reduced parameters and propose general optimality criteria.

1.1 Prologue

In contrast to sandwich structures, a so-called Anti-Sandwich consists of a three-layered composite structure with thick, shear-rigid skins and a thin, shear-soft core. They are widely applied as structural bearing elements, for example, laminated glasses [16] and photovoltaic modules [21]. In the context of the design, the main challenges are among others

- excellent stiffness properties resulting in small deflections,
- effective boundary conditions [23], and
- eigenbehavior

The first two items are well studied [5, 15, 21, 23]. The optimization of the eigenbehavior is in the focus of the present contribution. The eigenbehavior of Anti-Sandwiches can be modified by the manipulation of the

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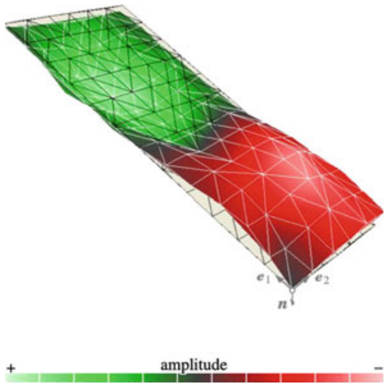

Fig. 1.1 Eigenmode as a result of experimental modal analysis by the aid of Laser Doppler velocimetry

- material parameters,
- geometry parameters,
- damping properties of the core layer, and
- boundary conditions,

where K is a layer index. In the present treatise, the modification of these parameters will be discussed from the point of view that the structural response should be outside of any stimulus spectrum. For this purpose, the modal analysis is a powerful tool. Figure 1.1 is representative of the results of an experimental modal analysis at Anti-Sandwiches. Since experiments are usually very expensive, an alternative procedure is beneficial.

A first analysis is often based on the analysis of the natural frequencies. In this course, it is possible to reduce to geometry and material parameters when restricting to one sample boundary condition. Considering Anti-Sandwiches, the parameters are

- plane length dimensions $L_\alpha \ \forall \ \alpha \in \{1, 2\}$,
- layer thicknesses $h^K \ \forall \ K \in \{t, c, b\}$ (with overall thickness $H = \sum h^K$),
- Young's moduli $E^K \ \forall \ K \in \{t, c, b\}$,
- Poisson's ratios $\nu^K \ \forall \ K \in \{t, c, b\}$, and
- mass densities $\rho^K \ \forall \ K \in \{t, c, b\}$.

Herein, we make use of superscript designators t, c, b for top, core, and bottom layers of the (three-layered) Anti-Sandwich, cf. Figure 1.3. When reviewing the relevant literature on such structural elements, one will find a wide but restricted domain wherein preceding parameters can be located. An overview is given in Fig. 1.2. There, the superscript index s is used representatively for the skin layers (t, b). However, as is typically for Anti-Sandwiches, we can state the following characteristic properties, which are also exploited in the present work.

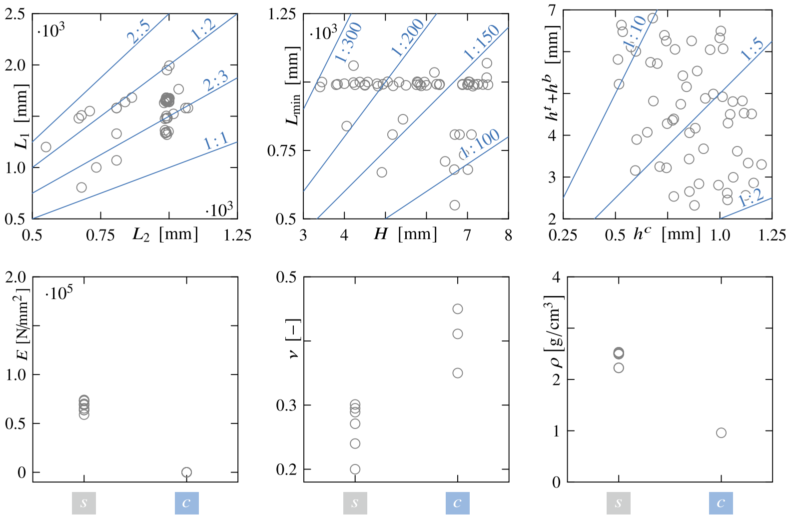

Fig. 1.2 Geometric and material variety of Anti-Sandwiches, data taken from [4]

$$L_1 \approx L_2 \tag{1.1a}$$
$$L_\alpha \gg H \tag{1.1b}$$
$$h^s \gg h^c \tag{1.1c}$$
$$G^s \gg G^c \tag{1.1d}$$
$$Z^s \gg Z^c \tag{1.1e}$$
$$\rho^s \gg \rho^c \tag{1.1f}$$

Herein, the shear modulus $G^K = E^K(2 + 2\nu^K)^{-1}\ \forall K \in \{c, s\}$ is used due to the isotropy assumed for present materials. Furthermore, $Z^K = G^K h^K\ \forall K \in \{c, s\}$ is a simple measure for the transverse shear sensitivity.

1.2 Theoretical Issues

Anti-Sandwiches are complex systems. Let us make the following model assumption allowing a simplified modeling for the analysis of the eigenbehavior of such structures. The assumed cross-section is shown in Fig. 1.3. As it was shown in previous papers [21], in dependence of the material and geometry parameters, different theories can be applied for the analysis. The simplest one is the classical layered plate theory [2]. This is an analogy to Kirchhoff's plate theory. As usual, this approach failed in the case of sandwich structures. A refined approach is related to the so-called first-order shear deformable theory [2]. Now the transverse shear is taken into account which is helpful in the case of classical sandwich structures. But even in

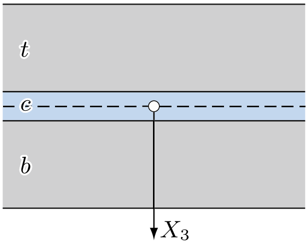

Fig. 1.3 Cross section of the assumed composite structure (t—top layer, c—core layer, b—bottom layer)

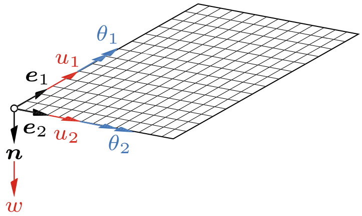

Fig. 1.4 Coordinate system and degrees of freedom of the five-parameter plate continuum ($\boldsymbol{e}_\alpha$, $\boldsymbol{n}$—orthonormal base of the coordinate system, u_α—in-plane displacements, w—deflection, θ_α—rotations)

the case of a non-standard shear correction [1] some ratios of the above-mentioned parameters do not allow a correct analysis of Anti-Sandwiches. Better results one can get applying the extended layerwise theory suggested in [18]. This theory is based on the assumed cross section (Fig. 1.3), but in the first step, each layer of the three-layered composite structure is modeled separately with the help of a five-parameter plate continuum per layer based on the direct approach [3, 13, 24]. Within this theory, five independent degrees of freedom (three translational and two rotational) are used as shown in Fig. 1.4. The cross-sectional kinematic assumptions are similar to Mindlin's theory [17] for every layer separately [5]. In addition, it is assumed that

- geometrical symmetry in transverse direction w.r.t. midplane of the cross section,
- geometrically and physically linear setting,
- isotropic elastic materials,
- decoupled deformation states,
- layers are rigidly connected, and
- layers have a constant thickness.

The following statements are based on previous works performed at the Martin Luther Universität Halle-Wittenberg and the Otto von Guericke Universität Magdeburg during the last decade. The core layer behavior analysis, three-point bending tests and experimental validation to theories were presented in [8–10, 20–23]. The theoretical basics are given in [21, 23] for layerwise beams, in [18] for layerwise plates, and in [19] for layerwise shells. Solutions were presented as closed-form solu-

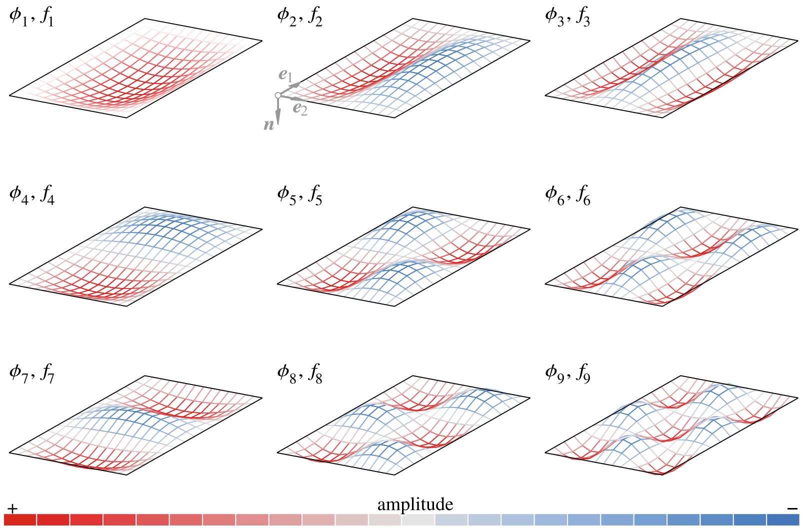

Fig. 1.5 Nine eigenform shapes ϕ_i and corresponding natural frequencies $f_i \ \forall\, i \in \{1, \ldots, 9\}$

tions in [18], computational solution strategies were introduced in [4, 5, 15]. Last but not least, application were shown for coupling with three-dimensional models in [11], for natural loading scenarios in [6] and w.r.t. borders, frontiers and limits in [7, 12, 14].

In the context of the computational solution strategy to analyze the eigenbehavior, [4, 5] introduced a special finite-element which incorporates all the degrees of freedom for all layers. The approach was implemented into the finite-element program system ABAQUS® by using a "user element" (UEL) subroutine. We denominate this computational approach FE-XLWT. The strategy developed proves itself to be particularly efficient. It has emerged as a powerful tool to analyze general three-layered composite structures [12, 14]. Examples of computational results gained are presented in Fig. 1.5.

1.3 Optimal Parameter Basis

For the determination of the structural eigenbehavior of Anti-Sandwiches, it is advantageous to analyze the natural frequencies for different geometrical and material compositions. In general, for three-layered composite structures, the parameter space is defined by a set $\mathcal{S}$ of 14 parameters. To be exact, these are five geometries

$$\mathcal{S}_{\text{geo}} := \{L_1, L_2, h^t, h^c, h^b\}\,, \tag{1.2}$$

and nine material parameters

$$\mathcal{S}_{\text{mat}} := \{E^t, E^c, E^b, \nu^t, \nu^c, \nu^b, \rho^t, \rho^c, \rho^b\}, \tag{1.3}$$

while $\mathcal{S} = \mathcal{S}_{\text{geo}} \cup \mathcal{S}_{\text{mat}}$ holds. Since Anti-Sandwiches can be assigned to the genus of symmetric composite structures, i.e.,

$$h^t = h^b \qquad E^t = E^b \qquad \nu^t = \nu^b \qquad \rho^t = \rho^b \tag{1.4}$$

hold, the number of parameters reduces to 10. The reduced set of parameters is summarized as follows:

$$\mathcal{S}_{\text{red}} := \{L_1, L_2, h^s, h^c, E^s, E^c, \nu^s, \nu^c, \rho^s, \rho^c\}, \tag{1.5}$$

while we make use of the superscript index s for the parameters of the skin layers. Another reduction of this set is based on significant ratios introduced for Anti-Sandwiches, cf. [12]. These are the thickness ratio TR, the length ratio LR, the thickness to length ratio TLR, the shear modulus ratio GR, and the mass density ratio MDR.

$$TR = \frac{h^c}{2h^s} \tag{1.6a}$$

$$LR = \frac{L_2}{L_1} \tag{1.6b}$$

$$TLR = \frac{H}{L_{\min}} \tag{1.6c}$$

$$GR = \frac{G^c}{G^s} \tag{1.6d}$$

$$MDR = \frac{\rho^c}{\rho^s} \tag{1.6e}$$

Obviously, ratios (1.6a), (1.6b), and (1.6c) refer to geometry parameters, while (1.6d) and (1.6e) pertain on material parameters. Following the literature review of [4], we can limit the parameter ranges due to practical applications at products available on the market.

$$TR \approx 0.125\ldots0.45 \qquad LR \approx 0.25\ldots1 \qquad TLR \approx 2\cdot10^{-3}\ldots1.4\cdot10^{-2}$$
$$GR \approx 7\cdot10^{-6}\ldots1.5\cdot10^{-2} \qquad MDR \approx 4\cdot10^{-2}\ldots1$$

These restrictions are naturally based on the possible ranges of the set of original parameters (1.5). As a result, the new set is bounded by five parameters.

$$\mathcal{S}_{\text{bound}} := \{TR, LR, TLR, GR, MDR\}, \tag{1.7}$$

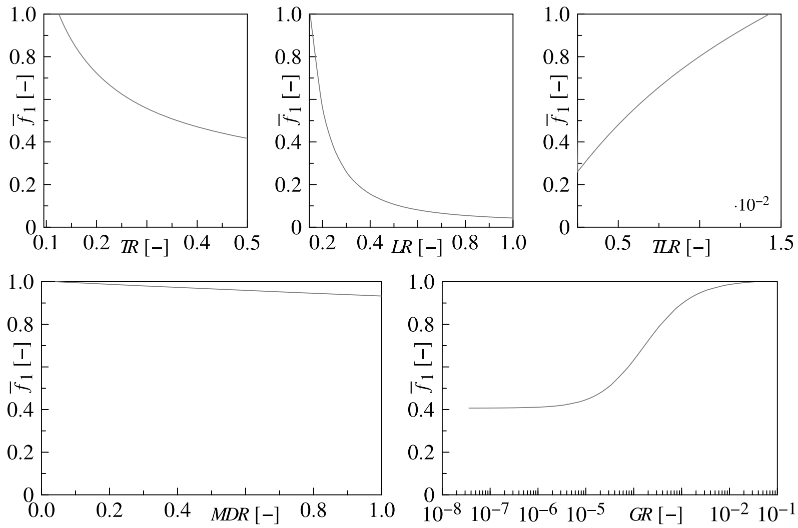

Fig. 1.6 Normalized first natural frequency in dependence of the bounded set of parameters $\mathcal{S}_{\text{bound}}$

In the sequel, these parameters are varied systematically to study their influence on the structural behavior. For the ease of evaluation, we make use of the natural frequency of the first fundamental mode $f_1 = \min(f_i)$ solely. There is a dependency on the set of reduced parameters.

$$f_1 = \mathfrak{F}(\mathcal{S}_{\text{red}}) \tag{1.8}$$

In present case, this dependence is sufficiently described the set of bounded parameters.

$$f_1 = \mathfrak{H}(\mathcal{S}_{\text{bound}}) \tag{1.9}$$

For reasons of comparability, we normalize this criteria.

$$\bar{f}_1(\Box) = \frac{f_1(\Box)}{\max[f_1(\Box)]} \qquad \forall \Box \in \{TR, LR, TLR, GR, MDR\} \tag{1.10}$$

In the present investigations, the parameters were varied individually. Resulting functional relationships are depicted in Fig. 1.6. A thorough discussion of these results is given in [4, 5].

Following the results generated, subsequent universal implications for the five reduced parameters for a low first natural frequency can be drawn.

1. *TR* high 2. *LR* high 3. *TLR* small 4. *MDR* high 5. *GR* small

By the aid of these five statements, engineers are able to estimate and regulate the vibration sensitivity, at least in the context of the product development process.

1.4 A General Optimality Criteria

Instead of working with five parameters and find optima for every parameter separately, it is our aim to establish a generic measure to achieve a general representation of the problem. For this purpose, we introduce a general parameter ratio $0 \leq S \leq 1$.

$$S \hat{=} \overline{\square} \quad \text{with } \overline{\square} = \frac{\square - \min(\square)}{\max(\square) - \min(\square)} \quad \forall \square \in \{TR, LR, TLR, GR, MDR\} \quad (1.11)$$

Herein, S is correlated with the structural stiffness. Based on this measure, we make use of an equally weighted normalized first natural frequency $0 \leq f_g \leq 1$. This results in a regularized function $f_g(S)$.

$$f_g(S) = \frac{1}{n} \sum_{\alpha=1}^{n} \bar{f}_1(\overline{\square}) \qquad \forall \bar{\square} \in \{\overline{TR}, \overline{LR}, \overline{TLR}, \overline{GR}, \overline{MDR}\} \qquad (1.12)$$

With the aid of this generic measure, it is possible to reduce the multidimensional problem.

For the set of results depicted in Fig. 1.6, this yields the function visualized in Fig. 1.7. Due to the normalizations that have been introduced, this graph has general validity, at least in the context of Anti-Sandwiches in the applied boundary conditions. Obviously, we can identify a minimum and a maximum value in the range of S. The maximum is at $S = 0.01$, while the minimum is to be found at $S = 0.36$.

The goal of the generic measure S is to have a single parameter for evaluation. Based on this parameter, it is now possible to find sets of parameters (TR, LR, TLR, MDR, GR as well as E^K, ν^K, ρ^K, L_α, h^K) for optimal S. In the present case, this reads as follows.

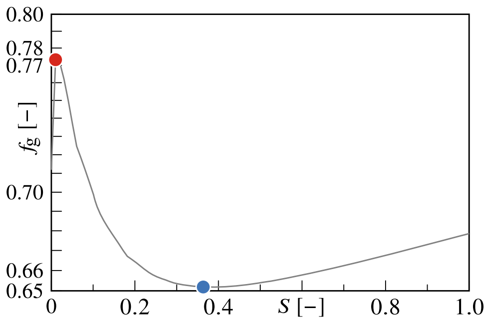

Fig. 1.7 Results for regularized function $f_g(S)$ with minimum and maximum

$$S_{\mathrm{opt}} = S[\min(f_{\mathrm{g}})] \tag{1.13}$$

Such optima, however, can be found by different algorithms. Here, the constraints on the set of parameters (1.5) depicted in Fig. 1.2 are decisive. Ultimately, this results in a variety of combinations of these parameters which fulfill the generalized optimality criteria. Thereby, every combination consists of a concrete specification of the parameters in $\mathcal{S}_{\mathrm{red}}$.

1.5 Epilogue

In the present work, we have proposed general optimality criteria for the dynamic behavior of Anti-Sandwiches by introducing a single measure which incorporates geometry and material properties and thus describes the structural stiffness. The analysis is based on an effective and efficient approach to determine eigenbehavior of Anti-Sandwiches by the utilization of the extended layerwise theory and a computational finite-element implementation based thereon. Significant geometry and material ratios are introduced based on dimensional analysis, which reduced the parameter space considerably. In the present context, the first natural frequency is consulted as a representative sensitivity parameter for the eigenbehavior. Several variant calculations are performed to determine optimal values of the ratios introduced. In sequence, a general representation to assess an optimal eigenbehavior is established. In contrast to the previous procedure, it is thus possible to generate a large number of combinations of the original parameter set. This routine is solely limited by the physical restrictions in the possible parameters for geometry and material. On the other hand, such restrictions help to reduce the number of possible sets.

Finally, the task of selecting and applying an optimization algorithm to generate specific datasets remains open whereby finding the minimum or maximum of a function in many variables is one of the most common problems in numerical computing. Such an algorithm is then to be coupled to FE-XLWT for efficient computational analyses of parameter sets.

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