

Modelling of Non-linear Multi-objective Programming and TOPSIS in Software Quality Assessment Under Picture Fuzzy Framework



Sameer Anand, Ritu Bibyan, and Aakash

Abstract The open-source software (OSS) market comprises of thousands of products and applications with different quality. The primary concern of the individuals and organizations is to evaluate the quality OSS products and packages. In this chapter, we have demonstrated a MCDM based model to assess the quality of OSS by taking performance and cost-based criteria related with OSS to analyze its quality on the basis of feedback gathered from the users and the experts. To avoid the uncertainty attached with the opinion of the expert, the Maximum-Entropy-Minimum-Variance-Ordered-Weighted-Aggregation (MEMV-OWA) operator has been incorporated. The criteria weights are calculated by solving a non-linear multi-objective programming problem proposed in the MEMV-OWA operator. The picture fuzzy set information has been used which is an addendum of intuitionistic fuzzy set, representing the human opinion more precisely. In this chapter, we have proposed a model named MEMV-OWA-PF-TOPSIS (Maximum Entropy Minimum Variance-Ordered Weighted Aggregation-Picture-Fuzzy-TOPSIS). A step by step procedure has been exhibited to show its implementation in real-life problems. A numerical illustration related to the software quality assessment of OSS on the basis of their performance and cost-based criteria has also been provided in this study.

Keywords Software quality assessment · Uncertainty · TOPSIS · MEMV-OWA · Picture fuzzy set

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1 Introduction

An Open source software (OSS) is distributed with specific kind of license which permits end-users to access the source code legally. The programmers can modify the software in any way they want with the grant of the license. Many licenses exist in the software market, but, the software is open source if it can be purposed again, which implies that source code can be taken by anyone and can deliver their code or program. And also, It is accessible in the form of source code with no supplementary cost, means users can access the code and makes changes to it easily.

There are thousands of projects and millions of registered developers in the software market sector. Due to which the quality of the software is questioned and at times becomes a matter of fuss Karakoidas et al. (2007). The ample variety among the available OSS products offering similar functionalities makes it a challenging task to rank software for experts and customers it becomes challenging to choose which software to use. This decision-making problem has grabbed considerable attention in the software market and academia.

There are various MCDM methods and techniques which have been studied so far by researchers, but the essential elements remain the same in MCDM problems. In MCDM for each technique, a fixed or absolute number of trails are considered, at least one decision-maker and two criteria. These elements help in decision building by sorting, selecting, rating or ranking of trails. Therefore it is consequently said that MCDM is just not a collection of theories, technologies, methods but it is even a specific way to handle the decision-making problems. It has been implemented on an increasing number of domains from the last few decades. It has a feature that it can efficiently deal with conflicting criteria.

In this research work, we have proposed a model to rank the OSS based on its quality and cost by using expert opinion. Various criteria are taken into consideration which are relevant to OSS quality. The MEMV-OWA operator has used the uncertainty attached with the response of expert. This has been done by maximizing the information and minimizing the variation in their point of view or response. A non-linear multi-objective programming problem is constructed to extract the weights using MEMV-OWA technique.

We have also used the notion of Picture Fuzzy sets (PFS) introduced by Zadeh (1965), which is the extension of Intuitionistic fuzzy sets (IFS). It presents human opinions more precisely as compared to IFS as it considers options like acceptance, neutral, rejection and refusal/desist. So far, in the literature, no work has been done to handle the unknown criteria weights using PFS in the respective field of software. To fill this research gap, we have attempted to introduce an MCDM model with Picture Fuzzy settings. Therefore, our objective in this research study is to:

1. To identify the most important criteria for assessing the quality of software
2. To construct an effective software selection model by applying integrated MEMV-OWA-PF-TOPSIS.

The rest of the chapter is structured as follow: In the Sect. 2, we have briefly provided related work in the field of MCDM, Software Quality and Picture Fuzzy.

Later, a picture fuzzy sets based weighted distance-based similarity measure is used to rank software explained in Sect. 3 by step by step process. The study in this chapter intends to establish a complex MCDM problem using TOPSIS technique: a similarity index based technique. So far, in the literature, no work has been done to handle the unknown criteria weights by using information based on PFS in the field of software. We have presented an example with specific criteria to rank OSS in Sect. 4. At the end of the chapter in Sect. 5, the entire conclusion of the study with the results is discussed and future scope has also been provided.

2 Research Background

The concern for various researchers belonging to different areas has a common interest in the field of selection of software which results in many approaches. These approaches are generally a combination of different MCDM techniques, making it simple to deal with the complexity of the research problem. Reliability of software depends on various factors; thus, MCDM problem can easily be used to evaluate the software. The approaches which come under MCDM are: “Analytic Hierarchy Process (AHP)”, “Fuzzy Analytic Hierarchy Process (FAHP)”, “Compromise programming (CP)”, “Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)”, “Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)”, “Artificial Network Process (ANP)” (Coyle 2004; Karayalcin 1982; Charnes and Cooper 1961; Brans et al. 1986; Deng 1999; Chen 2000; Saaty 1996).

In general, the quality of a software depends on various factors, one of the factors is reliability which is considered to learn and understand the reason behind a software failure (Yadav and Khan 2012). There are traditional models which analysis qualities of software which can be used to evaluate a software (Boehm 1978; Grady 1992; McCall 1977; Jacobson et al. 1999; Dromey 1996; Linda and Shaw 1998). In this chapter, we have incorporated MEMV-OWA operator with picture fuzzy TOPSIS to handle the uncertainty in the point of view of an expert. The OWA operator was initially by Yager (1988). Later, an OWA operator for maximizing variance (MV-OWA) was introduced by Fullér and Majlender (2001).

The fuzzy set theory was first given by Zadeh (1965) and he defined it as a class of objects with a sequence of grades membership. This idea opened a new area of research for the researchers. Some of the researchers worked on its extension and the application of a fuzzy set. One of the significant and essential extensions of the fuzzy set is intuitionistic fuzzy sets, and it was given by Atanassov (1999). The theory was focused on the extension of definitions fuzzy set objects, new objects and their characteristics. Cuong and Kreinovich (2013) introduced Picture fuzzy set (PFS) which take into consideration human’s opinion. It has more than two options like yes, no, neutral, and refusal. Later, Cường (2014) also proposed some operations based on PFS.

The PFSs has three essential components, namely the degree of belongingness, non-belongingness, and neutral. The property of PFS is that aggregation of these components must not exceed 1. In literature, the aggregation operators for PFSs and its application on MCDM is presented by Garg (2017). Some of the significant developments are: Singh (2015) brought the concept of finding the correlation coefficient of PFSs. A generalized picture distance measure was developed by Son (2016) and used it to picture fuzzy clustering. Van Viet and Van Hai (2017) introduced the system based on picture fuzzy set and named it as picture inference system (PIS) in. Hwang and Yoon (1981) bought a technique named TOPSIS, and used crisp values to handle MCDM problems. Later, it was extended by different makers to utilize it using an extension of fuzzy sets. Kuo (2017) modified the TOPSIS method with different ranking indexes. Tian et al. (2018) used TOPSIS by calculating weights using the best–worst method and to evaluate MCGDM problem with intuitionistic fuzzy information.

Wei (2016) used the concept of cross-entropy measure of PFSs in MCDM problems. Later in 2017, he utilized arithmetic and geometric operation of PFSs on MADM method. Wu and Wei (2017) used pictures of fuzzy aggregation operators in MADM problems like enterprise resource planning (ERP) selection. Furthermore, Ashraf et al. (2018) introduced the concept of cubic PFS which is also an extended form of PFSs. Dombi aggregation operators were introduced by Jana et al. (2019) for PFSs situations and applied them on MADM problems. Wang et al. (2018) ranked the characteristics of energy performance contracting projects (EPC) using MCDM technique and information from picture fuzzy. Peng (2017) introduced an operator called “Picture Fuzzy Ordered Weighted Geometric” Operator to deal with the MADM problems. He also introduced “Picture Fuzzy Induces Ordered Weighted Geometric” operator. Wang and Li (2018) used a hesitant picture fuzzy set and used it in MCDM.

Wang et al. (2019) worked on the MCDM problem by incorporating maximum deviation technique to calculate the weights. He developed a method to compare PLTSs (probabilistic linguistic term sets) based distance measure. Zhang et al. (2019) focused on “picture 2-tuple linguistic numbers” (P2TLNs) and created a score, action rules and accuracy functions, And later used them to solve MCDM problem established on the distance from the average solution.

Zaidan et al. (2015) presented a comparative study on Open Source of Electronic Medical Records by using MCDM techniques. To rank these software systems Weighted Sum Method (WSM), Weighted Product Method (WPM), Simple Additive Weighting (SAW), and TOPSIS were used. The aggregation of AHP and TOPIS is frequently used for ranking of different alternatives. One such research was done for selection of ETL (Extract, Transform and Load) software by Hanine et al. (2016) whereas Kara and Cheikhrouhou (2014) used the fuzzy AHP with TOPSIS for the ranking of alternatives. A hybrid approach was given by (Efe (2016)) using fuzzy AHP and Fuzzy TOPSIS. Yazgan et al. (2009) addressed the issue of prioritizing ERP software by creating Artificial Network Process (ANP) model. The results of the ANP model were used to train an artificial neural network (ANN), model. Lee et al. (2014) proposed an AHP application to solve the issue of evaluation of Open

Source Customer Relationship Management software. A model for software selection was formed with the combination of “Stepwise Weight Assessment Ratio Analysis” (SWARA) and the PROMETHEE by (Shukla et al. 2016). Wei et al. (2019) proposed the VIKOR method to evaluate multi-criteria group decision making problem having 2-TLNNs (2-tuple linguistic neutrosophic numbers).

3 Methodology

In this chapter, we have demonstrated a picture fuzzy-based TOPSIS model by incorporating non-linear multi-objective MEMV-OWA operator to rank the Open Source Software. The MEMV-OWA operator is adopted to evaluate the unknown criteria weights for the selection of software. We have also emerged the Picture Fuzzy information while constructing a TOPSIS model.

Preliminaries

Definition 1 The operator OWA with dimension “n” is a function $F : R_n \rightarrow R$ associated with weight vector say $W = (w_1, w_2, w_3, \dots, w_n)$ such that.

$$0 \leq w_i \leq 1; i = 1, 2, \dots, n$$

$$w_1 + w_2 + w_3 + \dots + w_n = 1$$

Further, it has a property that 1

$$F(a_1 + a_2 + a_3 + \dots + a_n) = \sum_i^n w_i b_i \tag{1}$$

In the assemblage of $(a_1, a_2, a_3, \dots, a_n)$ arranged in descending order, we can say that b_i is one of the largest i th element.

Definition 2 Measure of Orness exemplify the location of an OWA operator, which is nearer to either orlike or andlike level. The weights of the OWA operator are near to one another concealed by a specific degree of orness. Therefore orness is defined as.

$$orness(W) = \sum_{i=1}^n \frac{n-i}{n-1} w_i = \alpha \in [0, 1] \tag{2}$$

- (i) If α is closer to zero, then we can say that the interconnection between the various attributes has high andlike value. This implements that decision-maker is utmost noncommittal.

- (ii) If α is closer to one, then we can say that the interconnection between the various attributes has high orlike value. This implements that decision-maker is utmost optimistic.
- (iii) If $w_i = \frac{1}{n}$, then $\alpha = 0.5$, which implements that the decision-maker faces reasonable assessment.

Definition 3 For a specific level of orness, the variability in the weighting vector is determined by a measure of variance. The measure of variance is defined as:

$$D^2(W) = \frac{1}{n} \sum_{i=1}^n [w_i - E(w)]^2 = \frac{1}{n} \sum_{i=1}^n w_i^2 - \frac{1}{n^2} \tag{3}$$

While considering multiple attribute decision making, the variability in the weighting vector should be controlled to ignore the overestimation of a single attribute

Definition 4 The measure of entropy discovers that to what extent the information is exploited or utilized under an uncertain environment and conditions. The other term used for this is measure of dispersion and is given as:

$$Disp(W) = - \sum_{i=1}^n w_i \ln(w_i) \tag{4}$$

- (i) If $w_i = 1$ and $w_j = 0$ ($j \neq i$), then $Disp(W)$ is minimum i.e. 0. It implies that only a single attribute is studied during the course of aggregation.
- (ii) If $w_i = \frac{1}{n}$, then $Disp(W)$ is maximum i.e. $Disp(W) = \ln(n)$. It implies that all attributes are studied during the course of aggregation.

Definition 5 The MEMV-OWA is a basically a bi-objective non -linear programming model which is constructed to evaluate the weighting vector. In this, the uncertain information based on the experience of decision makers is exploited by maximizing the entropy and on the other hand to ignore the over estimation of the preferences by decision-maker, we minimize variance of the weighting vector. The mathematical non-linear programming model is represented as:

Model:

$$\begin{aligned} & \text{Maximize: } \sum_{i=1}^n w_i \ln(w_i) \\ & \text{Minimize: } \frac{1}{n} \sum_{i=1}^n [w_i - E(w)]^2 \end{aligned}$$

$$\text{subject to: } \sum_{i=1}^n \frac{n-i}{n-1} w_i = \alpha; 0 \leq \alpha \leq 1$$

$$\sum_{i=1}^n w_i = 1, 0 \leq w_i \leq 1, i = 1, 2, \dots, n$$

Definition 6 A set whose all elements have degree of membership of belongingness in them is said to be Fuzzy Set. Suppose, $X = \{x_1, x_2, \dots, x_n\}$ is a universal set, then fuzzy set Z defined on X is written as.

$$Z = \{(x, \mu_A(x)) | x \in X\}$$

where membership function is $\mu_Z(x) : X \rightarrow [0, 1]$ such that $x \in X$ to the set Z .

Definition 7 An intuitionistic fuzzy set I on universal set X is defined as:

$$I = \{(x, \mu_I(x), \vartheta_I(x)) | x \in X\}.$$

where $\mu_I(x) \in [0, 1]$, is the membership degree and $\vartheta_I(x) \in [0, 1]$ is the non-membership degree of x in I with the following condition:

$$0 \leq \mu_I(x) + \vartheta_I(x) \leq 1$$

For all $x \in X$, the lack of uncertainty is reflected by the amount $\pi_I(x) = 1 - (\mu_I(x) + \vartheta_I(x))$. This amount $\pi_I(x)$ is the degree of the hesitancy of $x \in X$ to the set I .

Definition 8 A picture fuzzy set (PFS) P on X is defined as:

$$P = \{(x, \mu_P(x), \eta_P(x), \vartheta_P(x)) | x \in X\}$$

where, $\mu_P(x)$ = positive membership degree, $\eta_P(x)$ = neutral membership degree, $\vartheta_P(x)$ = negative membership degree with the following condition:

$$0 \leq \mu_P(x) + \eta_P(x) + \vartheta_P(x) \leq 1$$

For all $x \in X$, the amount $\pi_P(x) = 1 - (\mu_P(x) + \eta_P(x) + \vartheta_P(x))$ is called the refusal degree of $x \in X$ to the set P .

Let M and S be two PFSs defined on X , then some operators are represented below:

- (i) $M \subseteq S$ iff $\mu_M(x) \leq \mu_S(x), \eta_M(x) \leq \eta_S(x)$ and $\vartheta_M(x) \geq \vartheta_S(x)$ for all $x \in X$.

- (ii) $M = \text{Siff } M \subseteq S \text{ and } S \subseteq M.$
- (iii) $M \cap S = \{x, \min(\mu_M(x), \mu_S(x)), \max(\eta_M(x), \eta_S(x)), \max(\vartheta_M(x)\vartheta_S(x)) | x \in X\}$
- (iv) $M \cup S = \{x, \max(\mu_M(x), \mu_S(x)), \min(\eta_M(x), \eta_S(x)), \min(\vartheta_M(x)\vartheta_S(x)) | x \in X\}$
- (v) $M^C = \{x, \vartheta_S(x), \eta_S(x), \mu_S(x) | x \in X\}$

Distance Measures

The picture fuzzy-based distance measures with picture fuzzy information are given as:

(a) Distance Measure between Two Picture Fuzzy Numbers

Let M and S be two PFSs defined on $X = \{x_1, x_2, x_3, \dots, x_n\}$, then the distance between M and N is given by:

$$D_P(M, S) = \frac{1}{3n} \sum_{i=1}^n (|\mu_M(x_i) - \mu_S(x_i)| + |\eta_M(x_i) - \eta_S(x_i)| + |\vartheta_M(x_i) - \vartheta_S(x_i)| + \max[|\mu_M(x_i) - \mu_S(x_i)|, |\eta_M(x_i) - \eta_S(x_i)|, |\vartheta_M(x_i) - \vartheta_S(x_i)|]) \tag{5}$$

The $D_P(M, S)$ is distance measure if it holds the following conditions:

- (i) $0 \leq D_P(M, S) \leq 1$
- (ii) $D_P(M, S) = 0$ iff $M = S$
- (iii) $D_P(M, S) = D_P(S, M)$
- (iv) For any $M, S, O \in PFSs(X)$, we have $D_P(M, O) \geq D_P(M, S)$ and $D_P(M, O) \geq D_P(S, O)$.

(b) Weighted Distance Measure

Let M and S be two PFSs defined on $X = \{x_1, x_2, x_3, \dots, x_n\}$ and the weights of the “m” criteria be w_j holding condition that $\sum_{j=1}^m w_j = 1$. Then, the weighted distance measure is given by:

$$D_P^w(M, S) = \frac{1}{3n} \sum_{i=1}^n w_j (|\mu_M(x_i) - \mu_S(x_i)| + |\eta_M(x_i) - \eta_S(x_i)| + |\vartheta_M(x_i) - \vartheta_S(x_i)| + \max[|\mu_M(x_i) - \mu_S(x_i)|, |\eta_M(x_i) - \eta_S(x_i)|, |\vartheta_M(x_i) - \vartheta_S(x_i)|]) \tag{6}$$

The $D_P^w(M, S)$ is weighted distance measure if it holds the following conditions:

- (i) $0 \leq D_P^w(M, S) \leq 1$
- (ii) $D_P^w(M, S) = 0$ iff $M = S$
- (iii) $D_P^w(M, S) = D_P^w(S, M)$
- (iv) For any $M, S, O \in PFSs(X)$ we have $D_P^w(M, O) \geq D_P^w(M, S)$ and $D_P^w(M, O) \geq D_P^w(S, O)$.

(c) Similarity Index Measure

Now, using above distance measures between two PFSs defined on $X = \{x_1, x_2, x_3, \dots, x_n\}$, we can define a similarity index measure as:

$$I_P(M, S) = 1 - \frac{1}{3} \sum_{i=1}^n w_j (|\mu_M(x_i) - \mu_S(x_i)| + |\eta_M(x_i) - \eta_S(x_i)| + |\vartheta_M(x_i) - \vartheta_S(x_i)| + \max[|\mu_M(x_i) - \mu_S(x_i)|, |\eta_M(x_i) - \eta_S(x_i)|, |\vartheta_M(x_i) - \vartheta_S(x_i)|]) \quad (7)$$

Here, the weights of the “m” criteria are w_j holding condition that $\sum_{j=1}^m w_j = 1$. The $I_P(M, S)$ is a similarity measure if it holds the following conditions:

- (v) $0 \leq I_P(M, S) \leq 1$
- (vi) $I_P(M, S) = 0$ iff $M = S$
- (vii) $I_P(M, S) = I_P(S, M)$
- (viii) For any $M, S, O \in PFSs(X)$, we have $I_P(M, O) \geq I_P(M, S)$ and $I_P(M, O) \geq I_P(S, O)$.

Step by Step Process of Maximum Entropy Minimum Variance OWA-Picture Fuzzy TOPSIS (MEMV-OWA-PF-TOPSIS).

Now, we have demonstrated a Multi-criteria decision-making approach, i.e. TOPSIS with picture fuzzy information. And the maximum entropy minimum variance OWA approach is used to find out the criteria weights. Consider, a discrete set of alternatives say $A = \{A_1, A_2, A_3, \dots, A_n\}$ and “m” criteria $C = \{C_1, C_2, C_3, \dots, C_m\}$ having weights say $W = \{w_1, w_2, w_3, \dots, w_m\}$ such that $\sum_{j=1}^m w_j = 1$.

We also define Picture Fuzzy decision matrix say $M = [\Delta_{ij}]_{m \times n} = [(\mu_{ij}, \eta_{ij}, \vartheta_{ij})]_{m \times n}$, where $\mu_{ij}, \eta_{ij}, \vartheta_{ij}$ is the degree of acceptance, neutral and rejection respectively that alternatives A_i fulfils. The MCDM procedure is briefly explained below in order to make the best decision:

Step 1: Firstly build the hierarchical model for the selection of software.

Step 2: Obtain the MEMV-OWA weights $W = \{w_1, w_2, w_3, \dots, w_m\}$ for the given set of criteria by solving non-linear programming problem given in **Definition 5**.

Step 3: Develop the matrix $M = [\Delta_{ij}]_{m \times n}$ which is the picture fuzzy decision matrix using decision maker’s information.

Step 4: Identify the benefit criteria (B_1) and cost criteria (B_2). Later, we need to determine the Picture Fuzzy Positive Ideal Solution (Δ_p^+) and Picture Fuzzy Negative Ideal Solution (Δ_p^-) given as:

$$\Delta_p^+ = \begin{cases} [(\max_j(\mu_{ij}), \min_j(\eta_{ij}), \min_j(\vartheta_{ij}))], & C_j \in B_1 \\ [(\max_j(\mu_{ij}), \max_j(\eta_{ij}), \min_j(\vartheta_{ij}))], & C_j \in B_2, \end{cases} \quad (8)$$

$$\Delta_p^- = \begin{cases} [(\min_j(\mu_{ij}), \min_j(\eta_{ij}), \max_j(\vartheta_{ij})], & C_j \in B_1 \\ [(\min_j(\mu_{ij}), \max_j(\eta_{ij}), \max_j(\vartheta_{ij})], & C_j \in B_2, \end{cases} \tag{9}$$

Step 5: Find out the degree of weighted similarity index $(I_{p_i}^+)$ and $(I_{p_i}^-)$ between Δ_p^+ and Δ_p^- respectively where $1 \leq i \leq n$.

$$I_I^+(A_i, \Delta_p^+) = 1 - \frac{1}{3} \sum_{j=1}^m w_j (|\mu_A(x_i) - \mu_{ij}^+| + |\eta_M(x_i) - \eta_{ij}^+| + |\vartheta_M(x_i) - \vartheta_{ij}^+| + \max[|\mu_M(x_i) - \mu_{ij}^+|, |\eta_M(x_i) - \eta_{ij}^+|, |\vartheta_M(x_i) - \vartheta_{ij}^+|]) \tag{10}$$

$$I_I^-(A_i, \Delta_p^-) = 1 - \frac{1}{3} \sum_{j=1}^m w_j (|\mu_A(x_i) - \mu_{ij}^-| + |\eta_M(x_i) - \eta_{ij}^-| + |\vartheta_M(x_i) - \vartheta_{ij}^-| + \max[|\mu_M(x_i) - \mu_{ij}^-|, |\eta_M(x_i) - \eta_{ij}^-|, |\vartheta_M(x_i) - \vartheta_{ij}^-|]) \tag{1.11}$$

Step 6: Using the above equations, the degree of weighted similarity index $(I_{p_i}^+)$ and $(I_{p_i}^-)$ is calculated and later evaluate the Relative Closeness measure for a given set of alternatives with respect to Δ_p^+ .

$$RelativeCloseness(RC_i) = \frac{I_{p_i}^+}{I_{p_i}^+ + I_{p_i}^-} \tag{12}$$

The higher the value of RC_i of the given alternatives with respect to Δ_p^+ which picture fuzzy positive ideal solution corresponds to best alternatives from A_i .

4 Numerical Illustrations

In this chapter, a numerical illustration related to software quality assessment of open source software (OSS) is provided in order to validate the application of MEMV-OWA-PF-TOPSIS. Several OSS is freely available online. The quality of OSS is questioned and has become the primary concern because OSS makes a significant influence on commercial marketing sector (Karakoidas et al. 2007).

In this research, four open-source-software (OSS) are assessed, which are freely available online. The assessment information is collected through a decision-maker who is the users of the software. The set of alternatives for OSS is denoted by

$S = \{S_1, S_2, S_3, S_4\}$. All these OSS software are assessed on the basis six criteria which comprise a set denoted by

$$C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$$
$$= \{\text{TechnicalAspects}, \text{Cost}, \text{SystemReliability}, \text{compatibility}, \text{ImplementationTime}, \text{Functionality}\}$$

where *Cost* and *Implementation Time* are the cost criteria and others are the benefit criteria. The steps for software quality assessment of four OSS on the basis of six criteria through MEMV-OWA-PF-TOPSIS are given as follows:

Step 1: The hierarchal model for software quality assessment is provided in Fig. 1.

Step 2: The orness (α) level is selected by the uncertain preferences of experts. If they are in moderating state, then $\alpha = 0.5$ and if they are maximally optimistic then $\alpha = 1$. In our study, OSS software experts have given moderate optimistic preferences; therefore, the level of orness (α) will be equal to 0.8. The weight vector (W_i) of MEMV-OWA averaging operator with respect to $n = 6$ and the particular level of orness (α) is provided in Table 1.

These W_i can be used to solve the information of performance and cost-related criteria of open source software (OSS). So, the weight vector concerning $\alpha = 0.8$ from Table 1 are as follows:

$$W_1 = 0.4352; W_2 = 0.2492; W_3 = 0.1441; W_4 = 0.0974;$$
$$W_5 = 0.0471; W_6 = 0.0270.$$

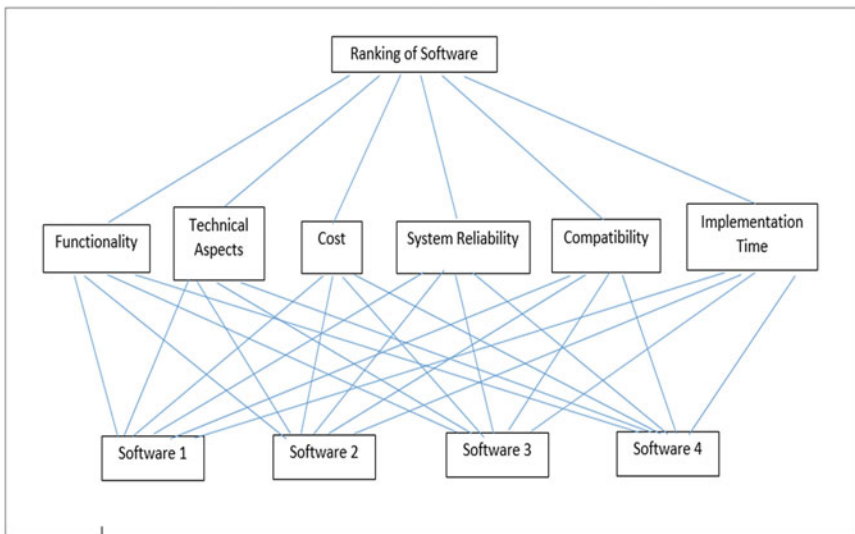


Fig. 1 Hierarchical model for quality assessment of different OSS

Table 1 Weight vector of MEMV-OWA averaging operator

W_i	Orness (α)					
	1	0.9	0.8	0.7	0.6	0.5
W_1	1	0.6227	0.4352	0.3096	0.2158	0.1667
W_2	0	0.2345	0.2492	0.2236	0.1852	0.1667
W_3	0	0.0894	0.1441	0.1614	0.1589	0.1667
W_4	0	0.0337	0.0974	0.1165	0.1364	0.1667
W_5	0	0.0129	0.0471	0.0842	0.1171	0.1667
W_6	0	0.0068	0.0270	0.1047	0.1866	0.1667

Step 3: The matrix $M = [\Delta_{ij}]_{m \times n}$ which is the picture fuzzy decision matrix using decision maker’s information is in Table 2.

Step 4: Now, we calculate the Picture Fuzzy Positive Ideal Solution (Δ_p^+) and Picture Fuzzy Negative Ideal Solution (Δ_p^-) on the basis of Eqs. (8) and (9).

$$\Delta_p^+ = [(0.9, 0.0, 0.05), (0.75, 0.1, 0.1), (0.6, 0.0, 0.30), (0.75, 0.0, 0.1), (0.5, 0.1, 0.4), (0.6, 0.0, 0.3)]$$

$$\Delta_p^- = [(0.5, 0.0, 0.4), (0.5, 0.1, 0.4), (0.3, 0.0, 0.6), (0.3, 0.0, 0.6), (0.25, 0.1, 0.6), (0.25, 0.0, 0.6)]$$

Step 5: In this step, we calculate the weighted similarity index ($I_{p_i}^+$) and ($I_{p_i}^-$) by putting the criteria weights calculated in Step 2 in the Eqs. (1.10) and (1.11) where $1 \leq i \leq n$.

$$I_{p_1}^+ = 0.9490; I_{p_2}^+ = 0.6399; I_{p_3}^+ = 0.7542; I = 0.6352.$$

$$I_{p_1}^- = 0.6819; I_{p_2}^- = 0.9028; I_{p_3}^- = 0.8347; I_{p_4}^- = 0.9167.$$

Step 6: On the basis of Eq. (1.12), we calculate the value of RC_i of the i th software, where $1 \leq i \leq 4$, such that: $RC_1 = 0.5819$; $RC_2 = 0.4148$; $RC_3 = 0.4746$; $RC_4 = 0.4093$ that provides the assessment order as: $S_1 > S_3 > S_2 > S_4$, represents that first software (S_1) is the best alternative.

To analyze the effectiveness of MCDM based MEMV-OWA-PF-TOPSIS method proposed in this study, we have used this model to basically assess the quality of four Open Source Softwares based on six performance and cost-related criteria. Now, we can observe from Fig. 2 that the first software S_1 is relatively more effective than other OSS software because the value of its relative closeness is much larger than the rest of the software.

Table 2 Picture fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
S_1	(0.90,0.00,0.0)	(0.75,0.05,0.10)	(0.60,0.00,0.30)	(0.50,0.10,0.40)	(0.25,0.05,0.60)	(0.60,0.00,0.30)
S_2	(0.60,0.00,0.3)	(0.50,0.10,0.40)	(0.30,0.00,0.60)	(0.75,0.05,0.10)	(0.30,0.00,0.60)	(0.30,0.00,0.60)
S_3	(0.75,0.05,0.1)	(0.50,0.10,0.40)	(0.30,0.00,0.60)	(0.50,0.10,0.40)	(0.50,0.10,0.40)	(0.30,0.00,0.60)
S_4	(0.50,0.10,0.4)	(0.60,0.00,0.30)	(0.30,0.00,0.60)	(0.30,0.00,0.60)	(0.25,0.05,0.60)	(0.25,0.05,0.60)

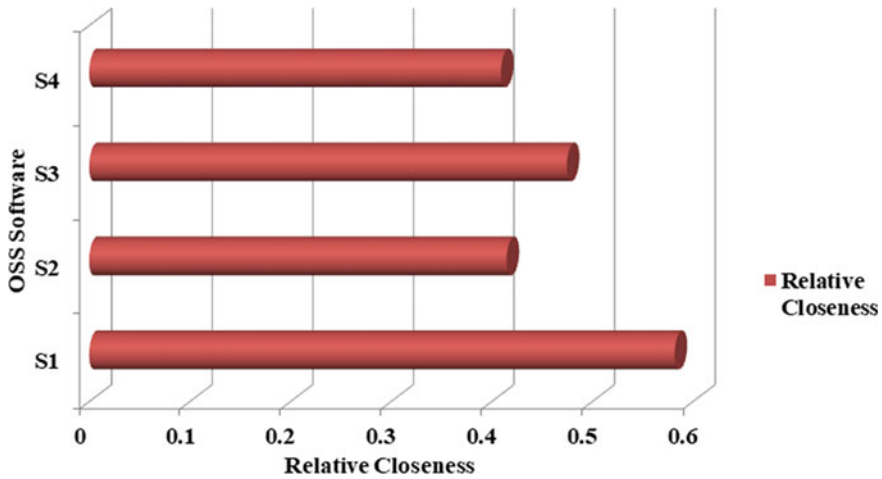


Fig. 2 OSS quality assessment by applying MEMV-OWA-PF-TOPSIS method

5 Conclusion and Future Scope

The current software environment has become active and influential and software customers' needs to think about the quality of the software before buying the license. The selection of software has become a serious activity as there various criteria for the quality. The software selection can influence various software companies, customers, developers in various aspects. Therefore, it is significant to select software for every user or customer before using any software. This selection process is quiet challenging task as it involves multiple criteria. As we know, MCDM approach is well known for ranking, selecting, evaluating these multiple criteria. So, in this chapter, the quality of open-source software (OSS) are assessed using the MCDM based approach: Maximum Entropy Minimum Variance OWA-Picture Fuzzy TOPSIS (MEMV-OWA-PF-TOPSIS). The criteria considered in this study to rank or select software are based on quality and cost. The following are the criteria taken: Technical Aspects, Cost, System Reliability, Compatibility, Implementation Time, and Functionality. The basic approach of TOPSIS technique is to first divide the criteria into cost and benefit criteria. So in our model, Cost and Implementation Time are the cost criteria as we need to minimize these whereas Technical Aspects, System Reliability, Compatibility, and Functionality are the benefit criteria which need to be maximized. The unknown weights of the criteria are evaluated using MEMV-OWA method which is a bi-objective non-linear programming approach. Another reason to use MEMV-OWA is that it handles the uncertainty while finding the weights of the criteria.

In the past studies, most researchers have applied Intuitionistic Fuzzy Sets (IFS) based MCDM technique for assessing the software reliability, but, IFSs cannot incorporate all the cases efficiently. For example, in the case of election voting, people

thoughts consider more degrees as, refusal, no, neutral, yes. To overcome this, in this research, we have applied PFSs based MCDM technique which is the extension of IFSs. The Picture fuzzy decision matrix has been created to remove uncertainty to another level that is present in the real-life decision-making problem like software quality assessment. Hence, the uncertainty factor is handled twice in this model.

This chapter also adds to the literature on software quality assessment by providing an advanced Picture Fuzzy based MEMV-OWA operator technique which includes the decision maker's uncertain preferences. The proposed method has been demonstrated with a numerical illustration for validating their reliability and effectiveness. The MEMV-OWA-PF-TOPSIS method has been implemented in a numerical example and we have reported the results obtained graphically as well.

The future research direction should focus on the techniques that can be extended in the field of software using MCDM approach under the environment of multi-granular fuzzy linguistic, polygonal fuzzy sets and other unclear situations.

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