Chapter 11
Axiomatic Thinking—Applied to Religion

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Abstract The purpose of the paper is to show that axiomatic thinking can also be applied to religion provided a part of the language used in religion (here called: Religious Discourse) consists of propositions or norms. Although David Hilbert was not concerned with religion when he gave his famous talk "Axiomatisches Denken" in 1917, his published essay (in 1918) treats this topic in such a broad sense that such an application seems appropriate.

This application is done in the following way: The first part discusses the possibility of applying axiomatic thinking to religion by considering the necessary preconditions to be satisfied for a successful application. The second part discusses the specific logical language that will be used in the application. The third part offers two concrete examples of such an application: a short and preliminary axiomatic theory of omniscience and omnipotence.

11.1 On the Possibility of Applying Axiomatic Thinking to Religion

11.1.1 Applying Logic to Religion

Many general problems concerning the application of logic to religion have been discussed in a very detailed way by Bochenski in his book "The Logic of Religion" [4]. Many things that hold for the application of logic to religion hold also for the application of axiomatic thinking to religion.

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Bochenski deals with logic in general and with religion. We shall not go into that or repeat here some of his discussion. However, we have to shortly concentrate on the particular language which is used by members of some religion.

11.1.2 Religious Discourse

The particular language used by members of some religion as a part of their common language is called here (like in Bochenski's book) *Religious Discourse.*

Although the Religious Discourse is a characteristic of every religion, it is especially evident in the three Abraham-Religions: Judaism, Christianity and Islam. This is so because these three religions are based on a specific text which is written by specific people but partially revealed to them by God: Old Testament, New Testament and Koran.

Subsequently, we will be concerned specifically with the Christian Religion which includes besides the New Testament also the Old Testament.

The class of expressions forming the Religious Discourse contain two important subsets which we may call the *Creed*[1] and the *Commands.* The acceptance of both the Creed and the Commands is of such an importance that it can be used for a definition of the believer of a given religion: Person A is a believer of religion R iff A accepts all (or almost all) statements of the Creed of R and all (or almost all) norms of the Commands of R.

The more tolerant expression "almost all" has to be taken with care. For example, if the Creed of the Christian Religion is understood as the core of it expressed in a "profession of faith" then *all* statements of this concentrated Creed have to be accepted. Thus someone who would deny that Christ has resurrected or that there is a Holy Spirit could not be called a Christian believer. Similarly, if the Commands of the Christian Religion are understood as the core of it expressed in the Ten Commandments plus the principle of love and charity then *all* these norms have to be accepted as valid commands. Again, someone who would deny the validity of one of these norms could not be called a Christian believer.

11.1.3 Religious Texts. Example: The Bible of the Christian Religion

Religious Discourse is both oral and written. Concerning the written part the religious texts are very important. The Bible of the Christian Religion consists of the books of the Old Testament and the New Testament as they are determined by councils of

[1] Bochenski stresses only the Creed (Bochenski [4], p. 10). However, in a private conversation with him, where I told him that one has to add the specific Commands as the second important component for demarcating the religious discourse and a religious believer, he agreed.

the Catholic Church (Luther dropped some books of the Old Testament and some of Saint Paul's letters, e.g. that to the Hebrews).

For the possibility of applying logic to religious texts it is a necessary condition that the text contains propositions and norms; i.e. at least some parts of such texts must be formulated in propositions and norms. Since this is an empirical question it can be decided by profane sciences like linguistics with its branch semantics. Such an investigation shows clearly that the Christian Bible contains both propositions and norms. Propositions are understood as being true or false and norms are understood as valid or invalid. Norms can be translated into propositions by so-called "that-clauses" as follows: The norm or command "honor your parents" or "you should not lie" can be translated into "it holds that you should honor your parents" or "it is commanded that you should not lie".

Among the propositions of the Christian Bible some are historical like the report of a certain battle in the Old Testament or that of the crucifixion of Jesus of Nazareth; some others are geographical like that it is *going down* from Jerusalem to Jericho which can even be verified today. Some eventually are genuinely religious.

11.1.3.1 Genuine Religious Propositions

"In the beginning God created the heavens and the earth… And God said: Let there be light. And there was light." (Ge 1, 1–3).

"Nearby stood six stone water jars… each holding from twenty to thirty gallons. Jesus said to the servants 'Fill the jars with water' so they filled them to the brim. Then he told them 'Now draw some out and take it to the master of the banquet'. They did so and the master of the banquet tasted the water that had been turned into wine." (Jn 2, 6–9).

"While they were eating, Jesus took bread and when he had given thanks, he broke it and gave it to his disciples, saying: 'Take and eat, it is my body'. Then he took a cup and when he had given thanks, he gave it to them saying: 'Drink from it, all of you. This is my blood of the new covenant which is poured out for many for the forgiveness of sins.' (Mt 26, 26–28).

In modern theology it is sometimes claimed that such passages of the Bible must be interpreted symbolically, where "symbolically" means that the respective sentences are not propositions that are true or false, but expressions that are neither true nor false. The reason for such a claim may be twofold: (1) to save or defend religion from attacks of non-believers, (2) error. As to the first, such a "defense" has the opposite effect since the claim will be unmasked as swindle or dishonest excuse. God is the creator of the world or he is not; and a religious believer believes that he is, i.e. that this is true. Similarly, for the other two passages. Of course critical exegesis is included, but also critical exegesis cannot turn such propositions into non-committal sentences without truth-value. As to the second, the claim cannot be correct since it leads to absurd consequences. Some are as follows:

(a) *Religious believer* and *disbeliever* loose its meaning and cannot be distinguished anymore. There cannot be belief or disbelief in something that is neither true nor false.
(b) There is no demarcation possible for being or not-being a member of a certain religion if there is no Creed consisting of propositions.
(c) If the claim would be correct for genuine religious statements in the Bible then logic is not applicable to genuine religious texts. But the latter can be refuted by showing that important passages of the Bible have a formal valid logical structure. Two examples are as follows. The first one is a true conditional where antecedent and consequent are both false. The second one is a derivation by contraposition.

    a. "Jesus replied… I know him. If I said I did not I would be a liar like you…" (Ju 8, 55)
    Thomas Aquinas comments this as follows: "But could Christ say these things ["I do not know him" and "I would be a liar"]? He could indeed have spoken the words materially, but not so as to intend expressing a falsehood, because this could be done only by Christ's will inclining to falsehood, which was impossible, just as it was impossible for him to sin. However, the conditional statement is true, although both antecedent and consequent are impossible."[2]
    b. "Then he [Jesus] turned toward the woman and said to Simon: 'Do you see this woman? I came into your house. You did not give me any water for my feet, but she wet my feet with tears and wiped them with her hair… Therefore, I tell you, her many sins have been forgiven because of her great love. But whoever has been forgiven little, loves little.'" (Lk 7, 44, 47)
    The logical inference by contraposition is this:

    Great love $\rightarrow$ many sins forgiven.

    Therefore: not many (little) sins forgiven $\rightarrow$ not great (little) love.

11.1.3.2 Genuine Religious Norms

"You shall have no other gods before me.", "Honour your father and your mother…", "You shall not murder." (Ex 20, 3, 12, 13). "Love the Lord your God with all your heart and with all your soul and with all your strength and with all your mind and love your neighbour as yourself". (Lk 10, 27; cf. Dt 6, 5 and Lev 19, 18).

It is an interesting question whether some of these norms are not specific for a particular religion in the sense that they are invariant w.r.t. more than one religion or underlying religions because of an inborn ability to naturally learn them. Such norms have been summarized under what has been called "natural law" or "natural right" and is described by Saint Paul thus:

[2] Thomas Aquinas (CGJn) 1285 [1].

> "Indeed, when Gentiles, who do not have the law [in the sense of the moral rules of the Old Testament or more specifically the Ten Commandments] do by nature things required by the law, they are a law for themselves, even though they do not have the law. They show that the requirements of the law are written on their hearts." (Ro 2, 14-15)

Thus, "Honour your parents", "You shall not murder" or the Golden Rule "Do to others as you would have them do to you" (Lk 6, 31; Mt 7, 12; Tob 4, 15) are appropriate examples for principles of *natural law* or *natural right*.

The decisive point for our considerations here is that logic and moreover axiomatic thinking can be applied to such norms as systems of Deontic Logic show. As a concrete example it can be shown that the commandments 4–10 of the Ten Commandments (Ex 20, 2–17; Dt 5, 6–21) follow logically from the principle of charity: love your neighbour as yourself, provided that some evident principles of human action are presupposed.[3]

For example, by assuming as valid the following principle of action "If person a loves person b as neighbor then a does not murder b", it follows from the principle of charity: "It is obligatory that a does not murder b." The proof uses two axioms of Deontic Logic: One says that if p is a valid principle then Op holds and the other is the distribution of the deontic operator O (obligatory) to the parts of an implication. Although the first axiom is sometimes only restricted to logically valid principles,[4] as soon as human actions are included into the system, the axiom has to be extended to valid action principles.

11.1.4 The Two Kinds of Belief

Belief can be of two sorts: One satisfies the condition that what is known is also believed in the sense that what is known is also assumed to be true. This sort of belief may be called *knowledge-inclusive belief*. We shall abbreviate it as B-belief ('aBp' stands for: person a B-believes that p). The other satisfies the condition that what is believed is not (yet) known and what is known is (no more) believed. This sort of belief may be called *knowledge-exclusive belief*. We shall abbreviate it as G-belief ('aGp' stands for: person a G-believes that p). Neither of the two sorts means that to believe is the same as not to know. It is easy to see that the first sort of belief, B-belief, violates this equivalence of believing and not knowing since what is known is B-believed. The above equivalence does not either follow from G-belief since the equivalence not only claims that what is believed is not-known but also the opposite, namely what is not-known is believed, and moreover, what is not-believed is known.

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[3] See Weingartner [15], ch. 9, pp.183–202.

[4] If the logically valid principles are not restricted for applying the operator O, this leads to the well-known paradoxes of Deontic Logic. Possible restrictions are by incorporation of action-operators (see Vanderveken [13]) or by restricting the logically valid principles to logically valid and relevant principles (see Weingartner [19]).

And both of the latter are completely untenable if not absurd. Thus, it is wrong to say that to believe is the same as not to know.

11.1.4.1 Examples of *G*-Belief—Scientific Belief

Before the proof of the independence of the Continuum Hypothesis (from the axioms of set theory) was given, v. Neumann *believed* (but didn't know) that the Continuum Hypothesis is independent. After Gödel proved the first part,[5] i.e. that the General Continuum Hypothesis (GCH) can be consistently added to the axioms of Neumann-Bernays-Gödel Set Theory (even if very strong axioms of infinity are used), v. Neumann wrote:

> Two surmised theorems of set theory, or rather two principles, the so-called 'Principle of Choice' and the so-called 'Continuum Hypothesis' resisted for about 50 years all attempts of demonstration. Gödel proved that neither of the two can be disproved with mathematical means. For one of them we know that it cannot be proved either, for the other the same seems likely, although it does not seem likely, that a lesser man than Gödel will be able to prove this.[6]

But after the proof of the second part—that also the negation of GCH can be consistelty added to the axioms of Set-Theory (it holds for both systems: that of Zermelo-Fraenkel and that of Neumann-Bernays-Gödel) —was given by Paul Cohen in 1963,[7] v. Neumann didn't G-*believe* it anymore but *knew* that GCH was independent (from the axioms of Set Theory).

The General Theory of Relativity (completed by Einstein 1915) made three important predictions: (a) the perihelion of Mercury, (b) the deviation if light rays which pass close to big masses, and (c) the red-shift of the light reaching us from distant stars. The first (a) was known as an effect (not explainable by Newton's theory) before Einstein's theory was created. The prediction (and explanation) of stronger gravitation, because Mercury is closer to the sun than other planets, by General Relativity was an immediate success. In this case, Einstein *knew* the positive result of the test of his theory. In the cases of (b) and (c), he *strongly believed* that they were correct and that a positive test would be possible as well. In 1919, the first confirmation of the prediction (b) was given by a British expedition of astronomers who observed a total eclipse of the sun in Africa. They confirmed the effect that light rays from a star, which run very close to the sun, are deviated towards the sun (in general towards great masses). Later, better and more exact confirmations of (b) were obtained. In 1922, the Soviet meteorologist Alexander Friedmann G-*believed* (but did not know) and

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[5] Gödel [7].

[6] v. Neumann [12] in: Bulloff, J.J./Holyoke, Th.C./Hahn, S.W. (1969).
The "Tribute to Dr. Gödel" from which the passage is cited was given by v. Neumann in March 1951 on the occasion of the presentation of the Albert Einstein Award to Gödel. It appeared in print in the volume Foundations of Mathematics (ed. Bulloff et al.), a collection of papers given at a symposium commemorating the sixtieth birthday of Kurt Gödel.

[7] Cohen [5]). Cf. Cohen [6].

predicted (on the basis of Einstein's picture of dynamic space) that the entire universe is in dynamic change. In 1929, the American astronomer Edwin Hubble confirmed this prediction (c). He found out that the light, reaching us from distant stars, is shifted towards red of the spectrum (red-shift) and that this red-shift is proportional to the distance of the emitting star(s) (or galaxy). This was the confirmation of (c) which was later again confirmed many times. Thus, after these positive results of testing predictions, Einstein *knew* that predictions (b) and (c) were correct and were positively confirmed by tests. And this also means that he did not and needed not to believe (G-belief) this anymore since there is sufficient justification to say that he *knows* now.

Both examples also show that before the proof (independence of GCH) has been established or before the observation confirmed the prediction, there was *justified true belief* but not yet knowledge. This also holds in a similar way for other proofs of mathematical conjectures or for other experimental confirmations of predictions of empirical theories. Moreover, this means that such situations are real counterexamples against the claim that justified true belief can be used as a universal definition of knowledge in contradistinction to the artificial straw-examples of Gettier.

11.1.4.2 Examples of B-Belief

No special examples for *B*-belief are necessary since *B*-belief may be interpreted in the following way: To *B*-believe that something (p) is the case means just to think that p is true (valid), to hold that p is true (valid), to strongly assume that p is true (valid) etc. Thus, if someone *knows* that chromosomes duplicate, then he also *B-believes* it. Likewise, if someone *G-believes* that GCH is independent from axioms of Set-Theory then he also *B-believes* it.

11.1.4.3 Religious Belief

Religious belief is always knowledge-exclusive, i.e. it is—like scientific belief—first of all always *G*-belief. If someone believes religiously—for instance that Christ came for the salvation of mankind or that there will be some kind of conscious life after death—one does not know it (and knows that one does not know it). This holds for all religious beliefs, even if not necessarily for all the statements of the creed of some specific religion, since the statements of the creed might not be logically independent of one another: some believer may infer one proposition of the creed from some others (see 11.3 below) and thus *knows* that one proposition is a consequence of the other. Such inferences may also be done by theological argumentation. Still, the derived propositions are not known but believed, and thus they are only known consequences of other propositions which are only believed. What generally holds is that if someone religiously believes something, then he does not know it, but holds it to be true or strongly assumes it to be true.

There are interesting analogies concerning the strengths of beliefs of great scientists w.r.t. scientific belief of their theories on the one hand and religious believers w.r.t religious belief of important propositions of the creed on the other hand.[8] Religious belief extends not only to propositions, like those of the creed of a given religion but also to important norms in the sense that it is believed that they hold; although some of them have been learned naturally belonging to natural law or natural right (see Sect. 11.1.3.2).

11.1.4.4 Relations Between Knowledge and the Two Kinds of Belief

Knowledge-inclusive: $aKp \rightarrow aBp$ (‘aKp’ for ‘a knows that p’)

Knowledge-exclusive:

$$aGp \rightarrow \neg\, aKp$$
$$aKp \rightarrow \neg\, aGp$$
$$aGp \rightarrow aBp$$

11.2 Applying Axiomatic Thinking to Religion: Logical Language

11.2.1 Logic

For the axiomatic study in Chap. 11.3 below we use Classical Two-Valued Propositional Logic which is extended by some operators (see below).

As wffs (short for “well-formed-formulas”) we use propositional variables p, q, r… representing states of affairs. The propositional variables p, q, r… may represent any states of affairs, profane ones or religious ones. Compound propositions can be built up in the usual way by the connectives $\neg$ (negation; not), $\wedge$ (conjunction; and), $\vee$ (disjunction; or), $\rightarrow$ (implication; if…then), and $\leftrightarrow$ (biconditional; if and only if—short: iff). Definitions are understood as valid (true) equivalences. True propositions represent *facts* as a subclass of states of affairs.

11.2.2 Set-Theoretic Elementhood: ε

In this study, the elementhood ε is used explicitly for saying that “the proposition p belongs to the theorems (true propositions) of …”, or short “$p\varepsilon T$”.

[8] Some examples are discussed in ch. 10 of Weingartner [20].

Examples:	$p\varepsilon T(CR)\ldots$	p belongs to the theorems of creation, or of creatures
	$p\varepsilon T(g\text{-}essence)\ldots$	p belongs to the theorems of God's essence

11.2.3 Operators

As operators to be connected to propositional variables we use K, W, C, CN, CS, CW, CC, P for respectively knows, wills, causes, causes as a necessary cause, causes as a sufficient cause, can will, can cause, permits.

In this study, these operators are used only for activities of God in the following sense:

If 'g' is used as a name for God then 'gKp', 'gWp', 'gCp' etc. stands for 'God knows that p (is the case, is true)', 'God wills that p', 'God causes that p' (meant as agent-causality in contradistinction to event-causality) etc.

Since we learn the meanings of these operators from our human activities of knowing, willing, causing etc. we have to admit that we understand the respective activities of God only by some analogy. From this it follows that if we were to use also operators for human activities, we would have to use different signs, for example K^* as corresponding to K etc.

We do not use any individual variables although our logic could be extended to include First Order Predicate Logic or some part of it. In this case, our individual variables could run over things (individuals) of our world (universe) or over creatures in general. However, like in the case of the operators, it would be problematic to let them run over both God and creatures.

11.2.4 Individual Constant

The individual constant 'g' is used as a name for God. To express for example that God is omnipotent, the copula $\in$ (is) will be used: $g \in OM$, $g \in OS$, $g\epsilon AG$, $g \in CT$, $g \in TR$, which means respectively "God is omnipotent", "God is omniscient", "God is allgood", "God is a creator", "God is triune".

11.2.5 Quantifiers

We use the propositional quantifier $\forall p$ and $\exists p$. The extension of Classical Two-valid Propositional Logic by propositional quantifiers is only a conservative extension.[9] However, we use in addition the operators listed above which are attached

[9] For proofs see Kreisel/Krivine [10].

to propositional variables. With these operators the following atomic-wffs can be constructed: gKp, gWp, gCp, $gCSp$, $gCNp$, $gCWp$, $gCCp$, gPp. No other atomic wffs can be constructed; for example no iterations are allowed in the axiomatic system of ch. 11.3. From these, atomic wffs compound wffs can be produced by the connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ in the usual way. For the propositional variable p any wff containing operators K, W, $C\ldots$ cannot be substituted. If these restrictions are satisfied, adding propositional quantifiers will preserve the conservative extension.

Independently of that, it may be mentioned that epistemic operators like *knows*, *beliefs*, or *assumes* are not automatically intensional in the sense of not being reducible to an extensional interpretation, although this seems to be a widespread prejudice. It can be shown that a deductive system with different types of knowledge, two types of belief and one type of assumption can be built up by a decidable ten-valued propositional logic where the epistemic operators are defined by truth tables.[10]

We use the uniqueness existential quantifier $\exists!$ connected with the constant 'g' (as name for God). We could use variables running only over God and the three persons Father, Son and Holy Spirit. However, since the topic of trinity will not be discussed in this essay, we do not use such variables but only the constant 'g'.

11.2.6 Modal Operators

As modal operators we use $\Box$ (necessary) and $\Diamond$ (possible) with the usual interdefinable equivalences $\Box p \leftrightarrow \neg \Diamond \neg p$. The background system of Modal Logic can be the system T or the decidable Modal Logic included in the system RMQ[11] or another system but not as strong as S4 or S5.

Although adding propositional quantifiers to Classical Two-valid Propositional Logic is only a conservative extension (cf. 11.2.5), there is the question (posed by one referee) whether this also holds if modal operators are added. I do not know of any proof concerning this question. However, I have a strong conjecture that the extension is still conservative if the modal logic included in the 6-valued decidable system RMQ is taken. The reasons are the following:

(1) RMQ (and its modal logic) is describable by finite matrices (truth tables).
(2) RMQ (and its modal logic) has 3 values *true* and 3 values *false* and no other value between true and false.
(3) All theorems of RMQ are theorems of Classical Two-valid Propositional Logic.
(4) RMQ (and its modal logic) is consistent and decidable.
(5) RMQ contains a relevance-restriction RC′ which forbids to derive an implication from a conjunction. RC′ also holds in this essay.

[10] See Weingartner [14] and [20] ch. 13.

[11] Weingartner [17].

11.2.7 Interpretation

In order to show that the axiomatic system proposed in ch. 11.3 below is understood as a theory—in this case of omniscience and omnipotence—and not just as a not interpreted and un-committed logical play we cite a discussion between Carnap and Gödel about this question (November 13, 1940; recorded by Carnap; G = Gödel, ich = Carnap):

> G: Man könnte exaktes Postulatensystem aufstellen mit solchen Begriffen, die gewöhnlich für metaphysisch gehalten werden: "Gott", "Seele", "Ideen". Wenn das exakt gemacht würde, wäre nichts dagegen einzuwenden.
>
> Ich: Gewiss nicht, wenn als Kalkül. Oder meinen Sie interpretiert?
>
> G: Nicht blosser Kalkül, sondern Theorie. Aus ihr folgt einiges über Beobachtungen: aber das erschöpft die Theorie nicht.[12]

We understand the proposal made in ch. 11.3 as a theory in Gödel's sense.

11.3 Applying Axiomatic Thinking to Religion: Omniscience and Omnipotence

The following part is a selection of a greater axiomatic study in which all the concepts of A1 (below) are described, defined and explained by definitions, axioms and theorems. Since the composition is simple and transparent it should obey Hilbert's advice in his article "Axiomatisches Denken" that "the principle requirement of the axiomatic set up has to go further, i.e. towards understanding that inside a scientific domain, because of the established axiom system, inconsistencies are impossible at all."[13]

A1 $g \in TR \wedge g \in AC \wedge g \in OS \wedge g \in OM \wedge g \in AG \wedge g \in CT$
(God is triune, actual, omniscient, omnipotent, allgood, and creator.)

A2 $(g \in TR \wedge g \in AC \wedge g \in OS \wedge g \in OM \wedge g \in AG \wedge g \in CT) \rightarrow \exists! g$
(If God is triune, actual, omniscient, omnipotent, allgood, and creator, then there is one God.)

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[12] G: "One could establish an exact postulate-system with concepts that are usually called metaphysical: "God", "Soul", "Ideas". If this is done in an exact way nothing could be said against it." Ich: "Certainly not, as a calculus. Or do you mean with interpretation?" G: Not mere calculus, but theory. From it something follows about observations: but this does not exhaust the theory." (My translation)—This is the first part of the discussion which is recorded in: Köhler et al. (eds.) [9]. *Kurt Gödel. Wahrheit und Beweisbarkeit*, p.127.

[13] "die prinzipielle Forderung der Axiomenlehre muß vielmehr weitergehen, nämlich dahin, zu erkennen, daß jedesmal innerhalb eines Wissensgebietes auf Grund des aufgestellten Axiomensystems Widersprüche überhaupt unmöglich sind". Hilbert [8], p. 411.

T1 $\exists!g$ [A1, A2]

(There is one God.)

T2 $g \in OS$ [A1]

(God is omniscient.)

T3 $\exists!g \wedge g \in OS$ [T1, T2]

(There is one omniscient God.)

11.3.1 Omniscience

D1 $g \in OS \leftrightarrow (\forall p)(gKp \rightarrow p) \wedge (\forall p)[(p\varepsilon T(\mathrm{g}) \vee p\varepsilon T(LM) \vee p\varepsilon T(CR)) \rightarrow gKp] \wedge (\forall p)(gKp \rightarrow \Box gKp)$

(God is omniscient iff (1) whatever God knows is true and (2) God knows everything about himself, about logic and mathematics, as well as about creation, and (3) whatever God knows, he necessarily knows.)

T4 $(\forall p)(gKp \rightarrow p)$ [T2, D1]

T5 $(\forall p)[(p\varepsilon T(g) \vee p\varepsilon T(LM) \vee p\varepsilon T(CR)) \rightarrow gKp]$ [T2, D1]

T6 $(\forall p)(gKp \rightarrow \Box gKp)$ [T2, D1]

T7 $(\forall p)(p\varepsilon T(CR) \rightarrow gKp)$ [T5]

(Whatever belongs to creation is known by God.)

D2 $p\varepsilon T(LM) \leftrightarrow (p\varepsilon T(Lg) \vee p\varepsilon T(Math))$

T8 $(\forall p)(p\varepsilon T(Lg) \rightarrow gKp)$ [T5, D2]

(God knows all theorems of logic.)

D3 $g\epsilon LO \leftrightarrow (\forall p)(p\varepsilon T(Lg) \rightarrow gKp)$

(God is logically omniscient iff God knows all theorems of logic.)

T9 $g \in LO$ [T8, D3]

T10 $(\forall p, q)[(p \rightarrow q)\ \varepsilon T(Lg) \rightarrow gK(p \rightarrow q)]$ [T8]

We assume that the following well-known axiom of Epistemic Logic also holds for God: $(\forall p, q)[gK(p \rightarrow q) \rightarrow (gKp \rightarrow gKq)]$

D4 $g \in LI \leftrightarrow (\forall p, q)[\mathrm{p} \vdash \mathrm{q} \rightarrow (gKp \rightarrow gKq)]$

(God is logically infallible iff for all p, q: if q is derivable from p, then if God knows that p he also knows that q.)

(continued)

D4.1 $p \vdash q \leftrightarrow (p \rightarrow q)\ \varepsilon T(Lg)$

T11 $g \in LI$ [T10, D4, D4.1, Epistemic Logic]

(God is logically infallible.)

D5 $p\varepsilon T(CR) \leftrightarrow (p\varepsilon T(U) \vee p\varepsilon T(OC))$

(p is a fact of creation iff p is a fact of the universe or p is a fact of other creatures.)

D6 $p\varepsilon T(U) \leftrightarrow p\varepsilon T\text{-}Law(U) \vee p\varepsilon T\text{-}State(U) \vee p\varepsilon T\text{-}Init(U) \vee p\varepsilon T\text{-}Const(U) \vee p\varepsilon T\text{-}Event(U)$

(p belongs to the theorems of the universe—representing facts of the universe—iff p belongs to the law-theorems of the universe or p belongs to the theorems about states, initial conditions, constants or events of the universe.)

T12 $(\forall p)(p\varepsilon T(U) \rightarrow gKp)$ [T7, D5]

(God knows all the facts of the universe.)

T13 $(\forall p)[(p\varepsilon T\text{-}Law(U) \vee p\varepsilon T\text{-}State(U) \vee p\varepsilon T\text{-}Init(U) \vee p\varepsilon T\text{-}Const(U) \vee p\varepsilon T\text{-}Event(U)) \rightarrow gKp)$ [T12, D6]

(God knows all laws, states, initial conditions, constants, and events of the universe)

Concerning states and events of the universe, there is the problem whether God knows all future states and events of the universe; especially those which are contingent. This problem can only be handled by introducing time-indices attached to states or events (or to the propositions which describe them). Consequently, new wffs have to be introduced as for example: gKp_t, where 'p_t' says that a certain state or event (i.e. a transition from state s_1 to state s_2) happens at time t, where 't' refers to a reference system of the universe. We will not go into these problems in this article but want just to stress that time-indices cannot be attached to any of the activities of God but only to states or events of the universe.[14] Time has been created by creating a universe with things that change.

T14 $(\forall p)(p \rightarrow \neg gK\neg p)$ [T4]

($\neg p/p$ Contraposition)

T15 $(\forall p)(gKp \rightarrow \neg gK\neg p)$ [T4, T14]

(If God knows that p (is the case) then he does not know that not-p (is the case).)

T15 is a consistency-condition for knowledge which holds also for a rational concept of human knowledge and even for rational concepts of both types of belief distinguished in Sect. 11.1.4.

[14] To this point cf. Weingartner [16] ch. 3 (Whether God knows something at some time).

11.3.2 Problems Concerning Theorem T6: Necessary Knowledge

The three parts of the definiens of God's omniscience (D1 above) may also be expressed as follows:

(1) God has sound knowledge.
(2) God has complete knowledge.
(3) God has necessary knowledge.

Concerning the last part of theorem T6 there could be the following objection: Let us assume T4 holds also with necessity. To simplify matters, we drop the quantifier '$\forall p$' and obtain: $\Box\,(gKp \rightarrow p)$. From this, we obtain by distribution of the modal operator '$\Box$': $\Box gKp \rightarrow \Box p$. With the help of T6 this leads to: $gKp \rightarrow \Box p$, i.e. whatever God knows is necessarily the case. Although this is not a problem concerning God's knowledge about himself or concerning his knowledge about logic and mathematics, it seems to rule out contingent facts of the world and of creation in general.

In order to solve this problem, there are two possible solutions:

(1) The first solution is to restrict the third part of the definiens of omniscience (i.e. T6) to God's knowledge about himself and about logic and mathematics but not about creation. W.r.t this view God's knowledge has a similar relation to creation as God's will. Since God necessarily wills whatever belongs to his essence or to himself, but what he wills concerning creation he does not will necessarily but freely. However, observe that although all creation and creatures are not necessary in the sense of this *external* necessity, there is a different *internal* necessity in the sense that laws of nature are necessary in contradistinction to initial conditions. That means that God wills w.r.t the states of affairs within the universe that some hold with *internal* (natural) necessity, some with *internal* contingency.[15]
(2) The second solution is the following: Assume we instantiate for p in '$gKp \rightarrow \Box p$' some contingent proposition like "the world exists" or "person a freely acts that q (is the case)". Then it follows: If God knows that the world exists (that person a freely acts…) then necessarily the world exists (person a freely acts). Since the latter is false, God does not have knowledge of contingent facts. The solution for this, however, is as follows: God's knowledge is always most complete and comprehensive and it is impossible for him to know only half truths in an incomplete way as it is normal for man's knowledge. Thus God cannot know that the world exists without knowing that it exists contingently. Similarly, he cannot know that person a freely acts that q without also knowing that the action is contingent and that it is free in these or that aspects. Thus, if God's knowledge is at stake, we have to replace "the world exists" by "the world exists contingently", and "person a freely acts that q" by "contingently: person a freely acts that q". In fact, there may be many more properties of these

[15] Cf. Thomas Aquinas (STh) I, 19, 3 and 8 [2]. Weingartner [18], p. 122–128.

two contingent facts of which we are ignorant and which are included in God's knowledge. However, the additional fact that they are contingent is sufficient for the argument here. Since then, we have to replace '$\Box p$' by '$\Box\neg\Box p$' such that the result is: necessarily, if God knows that the world exists contingently then the world exists contingently: $\Box(gK\neg\Box p \rightarrow \neg\Box p)$. By distribution of the modal operator '$\Box$' it follows: if necessarily God knows that the world exists contingently then necessarily the world exists contingently: $\Box gK\neg\Box p \rightarrow \Box\neg\Box p$. And similar for the other example. As a result of this solution to the problem it follows that God can have knowledge of contingent facts in the sense that he necessarily knows that they are contingent and that they necessarily are contingent. We prefer the second solution over the first since we agree with Thomas Aquinas that God's knowledge stands in a different relation to the things than God's will: "As the divine existence is necessary of itself, so is the divine will and the divine knowledge; but the divine knowledge has a necessary relation to the things known; not the divine will to the thing willed."[16] Observe further that the usual axiom used to proceed from modal system T to modal system S5 is to add '$\Diamond\Box p \rightarrow \Box p$' to the system T. Since this axiom is equivalent to '$\neg\Box p \rightarrow \Box\neg\Box p$' it leads directly from "the world exists contingently" to "necessarily: the world exists contingently". For the argumentation above, however, such a strong modal system as S5 is not needed.

11.3.3 Omnipotence

D7 $g \in OM \leftrightarrow (\forall p)(gWp \rightarrow gKp) \wedge (\forall p)[gCCp \leftrightarrow (gCWp \wedge \neg(p\varepsilon T(g\text{-}Essence)) \wedge \neg(p\varepsilon T(LM)))]$

(God is omnipotent iff (1) whatever God wills he knows, (2) God can cause those states of affairs (facts) that he can will and that do not belong to his essence and not to logic or mathematics.)

T16 $g \in OM$ [A1]

(God is omnipotent.)

T17 $(\forall p)(gWp \rightarrow gKp)$ [T16, D7]

T18 $(\forall p)(gWp \rightarrow p)$ [T17, T4]

(God's will is always fulfilled.)

The will of God is understood in such a way that his will is always fulfilled, i.e. never fails. This is expressed in the first part of the definition D7 of omnipotence: $(\forall p)(gWp \rightarrow gKp)$, from which it follows with the help of '$gKp \rightarrow p$' (T4) that $(\forall p)(gWp \rightarrow p)$. Observe, however, that expressions like "God wills that man obeys his ten commandments" are not formulated in a correct way since by the above principle "if God wills that, then man will always obey his ten commandments".

[16] Thomas Aquinas (STh) I, 19, 3 ad 6 [2]. The second solution is described in more detail in Weingartner [16], p. 2f. and p. 15f.

However, this is not the case, as we know. Therefore, if God's will is applied to human action of free will, the correct formulation is "God wills that man *should (ought to)* obey his ten commandments", since God does not destroy the freedom of man. On the other hand, this does not hinder that in some cases God wills that the human person wills something and in these cases this is not a free will decision but may be some inclination (natural right) which is genetically inborn or a result of environment conditions or of education.

T19 $(\forall p)(gCCp \rightarrow gCWp)$ [T16, D7]
(Whatever God *can* cause, he *can* will.)

D8 $gCWp \leftrightarrow [Cons(p) \wedge Cons(\{p\} \cup T(g\text{-}Essence)) \wedge Cons(\{p\} \cup T(g\text{-}Commands))]$
(God *can* will those states of affairs that are consistent and consistent with his essence and with his commands.)

T20 $gCCp \leftrightarrow Cons(p) \wedge Cons(\{p\} \cup T(g\text{-}Essence)) \wedge Cons(\{p\} \cup T(g\text{-}Commands)) \wedge \neg (p \varepsilon T(g\text{-}Essence)) \wedge \neg (p \varepsilon T(LM))$ [D7, D8]
(God *can* cause those states of affairs that are consistent and consistent with his essence and with his commands but are not facts of his essence, or of logic or mathematics.)

A2 $(\forall p)(gCp \rightarrow gCCp)$
(Whatever God causes he *can* cause.)

A3 $(\forall p)(gWp \rightarrow gCWp)$
(Whatever God wills he *can* will.)

T21 $(\forall p)(gCp \rightarrow gCWp)$ [A2, T19]
(Whatever God causes he *can* will.)

T22 $(\forall p)(\neg\, gCWp \rightarrow \neg\, gCCp)$ [T19, Contr.pos.]
(What God cannot will, i.e. what is inconsistent or inconsistent with his essence or his commands (D8) he cannot cause.)

D9 $gCp \leftrightarrow (p \varepsilon T(CR) \wedge gWp)$
(God causes those states of affairs that belong to creation and that he wills.)

God's causation is concerned with creation only and not with himself. Although God wills his existence and his goodness and everything that belongs to his essence he cannot cause them. Therefore, his will is distinguished from his causation w.r.t. himself but they coincide concerning creation. This is expressed in theorem T23:

T23 $(\forall p)(p \varepsilon T(CR) \rightarrow (gWp \leftrightarrow gCp))$ [D9]
(Concerning facts of creation, God 's willing them is equivalent to God 's causing them.)

T24 $(\forall p)(gCp \rightarrow gWp)$ [D9]
(Whatever God causes he wills.)

T25 $(\forall p)(gCp \rightarrow p)$ [T24, T18]
(Whatever God causes obtains.)

(continued)

D10 $gPp \leftrightarrow \neg\, gW\neg p$
(God permits those facts which he does not prevent.)

T26 $(\forall p)(p \rightarrow \neg\, gW\neg p)$ [T18]
(Whatever is the case is not willed not to obtain by God.)

T27 $(\forall p)(p \rightarrow gPp)$ [T26, D11]
(Whatever is the case is permitted by God.)

T28 $(\forall p)(p \rightarrow (gWp \vee gPp))$ [T27]
(Whatever is the case is either willed or permitted by God.)

T28 expresses what Augustine says in his *Enchridion* 95: "Nothing is done, unless the Almighty wills it to be done; either by permitting it or by doing it."[17]

A4 $(\exists p)(gCCp \wedge \neg\, gWp)$
(For some states of affairs it holds: God *can* cause them but does not will them to obtain (or: does not will that they obtain).)

T29 $(\exists p)(gCCp \wedge \neg gCp)$ [T24, Contr.pos. A4]
(For some states of affairs it holds: God *can* cause them but does not cause them.)

T29 expresses what Thomas Aquinas says in his *Summa Theologica*: "In consequence, we should declare quite simply that God can make other things than the things he does make."[18]

T30 $(\exists p)(gCWp \wedge \neg gWp)$ [T19, A4]
(For some states of affairs it holds: God *can* will that they obtain but does not will that they obtain.)

T31 $(\exists p)(gCWp \wedge \neg gCp)$ [T30. T24]
(For some states of affairs it holds: God can will that they obtain but he does not cause them.)

T32 $(\forall p)(p \varepsilon T(CR) \rightarrow p)$ [T4, T7]
(Whatever is a fact of creation is the case.)

T33 $(\forall p)(p \varepsilon T(CR) \rightarrow (gWp \vee gPp)$ [T28, T32]
(Every fact of creation is either willed or permitted by God.)

A5 $(\forall p)(p \varepsilon T(CR) \rightarrow gK\neg\Box p)$
(Every fact if creation is known to be contingent by God.)

Observe, however, what has been said about the difference between external and internal necessity and contingency in Sect. 11.3.2 (1) above. The contingency addressed in A5 is external contingency.

[17] MPL 40, 276.

[18] (STh) I, 25, 5 and ad 1 [2].

T34 $(\forall p)(p \varepsilon T(CR) \rightarrow \neg\Box p)$ [A5, T4]

(Every fact of creation is contingent.)

There are, however, two different definitions of contingency. The first one "what is contingent is not necessary" is used above. The second one includes possibility: "what is contingent is both possible and not necessary", or equivalently "what is contingent is both possible and possible not" (see T36).

T35 $(\forall p)(p \varepsilon T(CR) \rightarrow \Diamond p)$ [T32, Modal Logic]

(Every fact of creation is possible.)

T36 $(\forall p)(p \varepsilon T(CR) \rightarrow (\Diamond p \wedge \Diamond\neg p)$ [T34, T35]

(Every fact of creation is contingent in the sense of being possible and possible not.)

T37 $(\forall p)[(p \varepsilon T(CR) \wedge gWp) \rightarrow (\Diamond p \wedge \Diamond\neg p)]$ [T36]

T38 $(\forall p)(gCp \rightarrow (\Diamond p \wedge \Diamond\neg p))$ [D9, T37]

(Whatever God causes happens contingently.)

T39 $(\forall p)(gCp \rightarrow (p \wedge \neg\Box p))$ [T25, T38]

(Whatever God causes is the case but not necessarily.)

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