

# Stochastic Representations for Model Uncertainty in the Ensemble Data Assimilation System



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**Abstract** The ensemble data assimilation system is beneficial to express flow-dependent model errors. Furthermore, the effectiveness of this system depends on the accuracy of the flow-dependent background error covariance. However, the background error covariance is often underestimated due to limited ensemble size, sampling errors and model errors, which causes a filter divergence problem—the analysis state diverges from the nature stage ignoring the observation influence. As one of the remedies to solve this problem, the stochastic representations address the model-related uncertainties by perturbing the model tendency or parameters using a random forcing to replenish the insufficient model errors. In this study, we implemented a stochastic perturbation hybrid tendencies (SPHT) scheme, which perturbs both physical tendency and dynamical tendency using the random forcing, and assessed its impact on the spread of ensemble forecast and ensemble mean error.

## 1 Introduction

Ensemble data assimilation (EnsDA) finds the best initial conditions of the numerical weather prediction (NWP) model using model forecasts and their error covariance as well as observations Evensen (1994). In particular, it describes the flow-dependent forecast error covariance through an ensemble of the model forecasts. Therefore, it contains uncertainties in both the initial conditions and the model. Model uncertainty representations can be distinguished from actual model errors: the former samples model perturbations from some distribution while the latter presents only one realization per model and forecast Leutbecher et al. (2017). In this study, we focus on the model uncertainty in the EnsDA system using the stochastic representations that simulate the errors of model tendencies from random components.

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In EnsDA, the model uncertainty is used in the ensemble background error covariance (BEC) through the 6-h forecasts. However, it is found to be underdispersive due to the limited ensemble size, sampling error, and imperfect model parametrization, resulting in over-confidence in ensemble forecasts Buizza et al. (2005). This problem is usually covered by covariance inflation, e.g., multiplicative inflation Anderson and Anderson (1999), additive inflation Mitchell and Houtekamer (2000), combined multiplicative and additive inflation Whitaker and Hamill (2012), relaxation to the prior Zhang et al. (2004), multischeme ensembles Meng and Zhang (2007), and so on.

In general, the ensemble BEC, composed of an ensemble spread ( $\sigma(x)$ ), i.e., the standard deviation to the ensemble mean ( $\bar{x}$ ), should reflect the ensemble mean error ( $e(\bar{x})$ ) because the ensemble spread distribution determines the analysis status (see Fig. 1). Here, the model error is expressed by the distance between the ensemble mean and the observation while the ensemble spread is represented by the pre-described ensemble distribution. The optimal ensemble spread is expected to have a spread similar to the ensemble mean (Fig. 1a), i.e.,

$$\sigma(x) \approx e(\bar{x}); \quad (1)$$

then, the analysis includes reliable information from the model and observations. The underdispersive ensemble members show a smaller ensemble spread compared to the model error, i.e.,

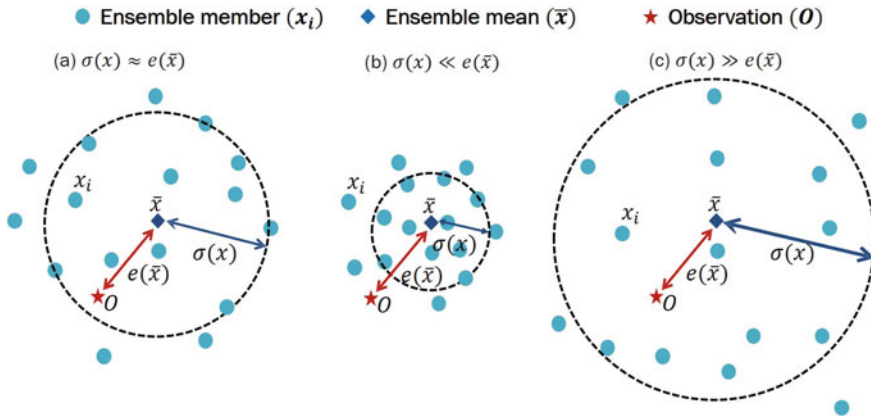
$$\sigma(x) \ll e(\bar{x}), \quad (2)$$

where the analysis ignores the observation and trusts the model more due to small ensemble BEC (Fig. 1b). The overdispersive ensemble members show the larger ensemble spread compared to the model error, i.e.,

$$\sigma(x) \gg e(\bar{x}). \quad (3)$$

In this case, the analysis ignores the model errors and relies more on the observation due to the larger ensemble BEC (Fig. 1c).

To remedy the general problem of the underdispersive model error, various stochastic schemes can be used, e.g., Buizza et al. (1999), Shutts (2005), Palmer et al. (2009). It is based on the fact that the NWP models represent the physical process with simplifications and approximations due to incomplete knowledge and computational costs. The European Centre for Medium-Range Weather Forecasts (ECMWF) developed the original version of the Stochastically Perturbed Parametrization Tendencies (SPPT) scheme—called the Buizza-Miller-Palmer (BMP) scheme Buizza et al. (1999)—for the first time and introduced the stochastic representation of model uncertainty that perturbs the total parametrized physics tendencies using the random forcing. After major revisions on random patterns and distribution of perturbations in 2009 Palmer et al. (2009), the BMP scheme has evolved into the SPPT scheme. Since



**Fig. 1** Schematic diagram of the ensemble spread,  $\sigma(x)$ , and the ensemble mean error,  $e(\bar{x})$ : **a** optimal ensemble spread, **b** underdispersive ensemble spread, and **c** overdispersive ensemble spread

then, the SPPT scheme has been employed by many operational NWP centers, e.g., Environment Canada (EC), Japan Meteorological Agency (JMA), the United States National Centers for Environmental Prediction (NCEP), and the United Kingdom Met Office (UKMO), for their global prediction systems Charron et al. (2010), Leutbecher et al. (2017). It has also been implemented in regional prediction systems, e.g., the Application of Research to Operations at Mesoscale convection-permitting model (AROME) of Meteo-France Bouttier et al. (2012) and the Weather Research and Forecasting (WRF) model Romine et al. (2014), Berner et al. (2015).

Afterward, the Stochastic Kinetic Energy Backscatter (SKEB) scheme was introduced to represent the model uncertainties for scale interactions what is absent in a truncated numerical model by randomly perturbing the stream function and potential temperature tendencies Shutts (2005). The SKEB has also been used for global ensembles in many operational center, e.g., ECMWF, EC, and UKMO Charron et al. (2010), Tennant et al. (2011), Sanchez et al. (2016) as well as regional ensembles (e.g., WRF) Berner et al. (2011), Berner et al. (2015). Recently, a stochastic convective backscatter scheme has been introduced Shutts (2015).

Alternately, the Stochastically Perturbed Dynamical Tendencies (SPDT) scheme, which perturbs the total dynamical tendencies using the random forcing, was introduced: the scheme proved to be effective in global ensemble forecasting Koo and Hong (2014), e.g., in the Global/Regional Integrated Model system (GRIMs) Hong et al. (2013). By combining the SPDT and SPPT schemes, Lim et al. Lim et al. (2020) devised the stochastic perturbation hybrid tendencies (SPHT) scheme to improve the underestimated BEC in the EnsDA system of the Korean Integrated Model (KIM) Hong et al. (2018)—a global model developed at the Korea Institute of Atmospheric Prediction System (KIAPS).

In this study, we introduce the SPHT scheme in the EnsDA system as a covariance inflation method in order to solve the underestimated ensemble BEC by taking into account that model variables are integrated by changes in physical and dynamic tendencies every time. The methodology is described in Sect. 2, and the experimental designs and results are in Sects. 3 and 4, respectively. Section 5 provides the summary and suggests the direction to further development.

## 2 Methodology

### 2.1 Local Ensemble Transform Kalman Filter (LETKF)

We employ the EnsDA system of KIAPS, which is a four-dimensional local ensemble transform Kalman filter (4D-LETKF). The analysis is obtained by assimilating the available observations within a local region Hunt et al. (2007), Shin et al. (2016), Shin et al. (2018). This LETKF also provides ensemble perturbations to the hybrid four-dimensional ensemble-variational (H4DEV) system, another data assimilation system operated by KIAPS. In this study, we use the LETKF results just to distinguish the changes of ensemble BEC.

The control variables are zonal wind, meridional wind, potential temperature, mixing ratio, and surface pressure. The KIM Package for Observation Processing (KPOP) provides quality-controlled real observations to the data assimilation system Kang et al. (2018), including the sonde, surface, aircraft, Global Positioning System-Radio Occultation (GPS-RO), Infrared Atmospheric Sounding Interferometer (IASI), Advanced Microwave Sounding Unit-A (AMSU-A), Cross-track Infrared Sounder (CrIS), Microwave Humidity Sounder (MHS), Advanced Technology Microwave Sounder (ATMS), Atmospheric Motion Vectors (AMVs), and tropical cyclone initialization.

In resolving the filter divergence problems in LETKF, three approaches used to be applied in terms of ensemble size, localization, and inflation method, which are specified below for this study:

1. *Ensemble size*: Increasing the ensemble size is commonly limited due to computational costs. At the early stage of developing LETKF, we used an ensemble size of 30 members, which is now increased to 50 members and is used in this study.
2. *Localization*: We implemented both horizontal and vertical localizations. The horizontal localization is expressed by a Gaussian-like piecewise fifth-order rational function Gaspari and Cohn (1999), Miyoshi (2011) varying from 660 to 1800 km depending on vertical levels Kleist and Ide (2015). The vertical localization varies depending on the observational types (e.g., conventional versus satellite data). For conventional data, it is defined by a Gaussian-like rational function, represented by  $2\sqrt{10/3} \cdot \sigma_v$  where  $\sigma_v$  is chosen to be  $0.2 \ln p$  for wind and surface pressure and  $0.1 \ln p$  for mass variables. For the satellite radiance data, the vertical weight-

ing function is defined by the gradient of transmittance of the measured radiance Thépaut (2003).

3. *Inflation method*: We used two inflation methods in this study. The additive inflation adds the perturbations randomly sampled from the bias-corrected lagged forecast differences to each ensemble member after the analysis step Whitaker et al. (2008). The relaxation to prior spread (RTPS) relaxes the ensemble standard deviation of analysis back to the background Whitaker and Hamill (2012). However, the LETKF still requires additional inflation method to make a sufficient ensemble BEC: we will cover it through the stochastic representation method in this study.

## 2.2 Numerical Weather Prediction (NWP) Model

We employ the KIM, a global NWP model developed at KIAPS Hong et al. (2018), which has recently been operationally implemented in the Korea Meteorological Administration (KMA). It is a non-hydrostatic model on a cubed sphere with state-of-the-art physics parametrization packages—including radiation, gravity wave drag, vertical mixing, convection, cloud physics, and so on.

Since our concern is a BEC in the data assimilation process, we only deal with the 6-h forecast (prior) results. The ensemble size is 50 members and the horizontal resolution for the ensemble forecast is 50 km. The initial conditions of the ensemble forecast have been generated by the lagged forecast difference samples, which are used to generate the static BEC in H4DEV Kwon et al. (2018).

## 2.3 Stochastic Perturbation Hybrid Tendencies (SPHT) Scheme

In this study, we introduce a stochastic perturbation hybrid tendencies (SPHT) scheme that perturbs the dynamic tendency  $\left(\frac{\partial \mathbf{x}}{\partial t}\right)_{dyn}$  and the physical tendency  $\left(\frac{\partial \mathbf{x}}{\partial t}\right)_{phy}$  of the model variables  $\mathbf{x}$  at each time step  $n$  using the multiplicative random forcing ( $r$ ):

$$\begin{aligned} \mathbf{x}^{n*} &= \mathbf{x}^n + (1 + \mu r) \left(\frac{\partial \mathbf{x}^n}{\partial t}\right)_{dyn} \Delta t \\ \mathbf{x}^{n+1} &= \mathbf{x}^{n*} + (1 + \mu r) \left(\frac{\partial \mathbf{x}^{n*}}{\partial t}\right)_{phy} \Delta t \end{aligned} \quad (4)$$

where  $\mu \in \{0, 1\}$  represents the vertical tapering function ( $e^{\eta-1}$ ) in the generalized vertical coordinate  $\eta$ . The model variable  $\mathbf{x}$  consists of temperature and humidity mixing ratio only. Note that in the KIM, physics and dynamics are coupled by time-splitting method; thus, this approach differs from the method of perturbing total model tendency by simply adding up perturbations of two tendencies, i.e.,

$$(1 + \mu r) \left[ \left( \frac{\partial \mathbf{x}}{\partial t} \right)_{dyn} + \left( \frac{\partial \mathbf{x}}{\partial t} \right)_{phy} \right].$$

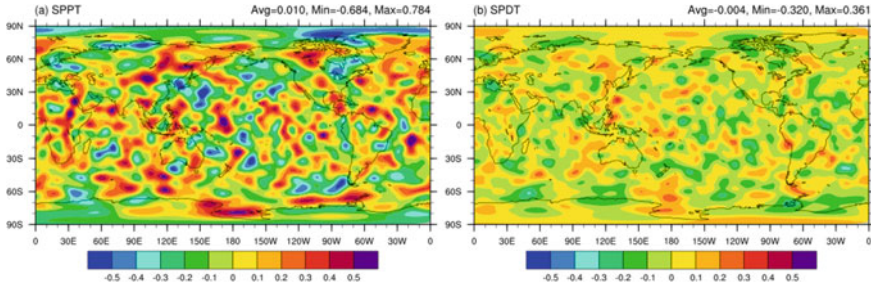
Here,  $r$  is a 2-dimensional value following the Gaussian distributed zero-mean perturbation considering spatial and temporal correlation. Occasionally,  $\mu$  is applied to perturbations for the upper model levels to avoid the instability issue. The amplitude is determined by the standard-deviation ( $\sigma$ ), and the length and time scales are based on the decorrelation lengths ( $L$ ) and times ( $t$ ), respectively.

The SPPT assumes that the model errors from the parametrized physical tendency are proportional to the total physical tendency Buizza et al. (1999), Palmer et al. (2009) while the SPDT assumes that the model errors from the dynamic tendency concern with the computational representations of the underlying partial differential equations Koo and Hong (2014). Since both methods deal with the model tendency, we devised a hybrid stochastic scheme (i.e., SPHT) by combining the two perturbation tendencies based on Eq. (4). The SPHT scheme is applied to the ensemble forecasting in LETKF to obtain an ensemble BEC.

### 3 Experimental Designs

To identify how the SPHT scheme increases the ensemble spread, we designed two experiments: CTRL (representing the *control* run) is without the SPHT scheme and STOC (representing the *stochastic* run) uses the SPHT scheme to perturb the model variables (e.g., temperature and specific humidity). To avoid instability due to excessive inflation, we suppressed perturbation of wind variables. To test the effectiveness of the inflation method, the warm cycle is started from 1200 UTC 22 June 2018 and ended on 1200 UTC 7 July 2018.

The random forcing in both SPPT and SPDT is described in Fig. 2. The SPPT (Fig. 2a) has horizontal correlation scales of 500 km (mesoscale), decorrelation times of 6h, and standard deviations of 1.0. On the other hand, the SPDT (Fig. 2b) has horizontal correlation scales of 500km, decorrelation times of 3h, and a standard deviation of 0.5. The SPDT especially used a tapering function that decreased exponentially with height (e.g., in the upper level) to prevent instability. It is designed to generate a smaller random forcing to ensure stability because the dynamic tendency variability is sensitive to  $r$ .

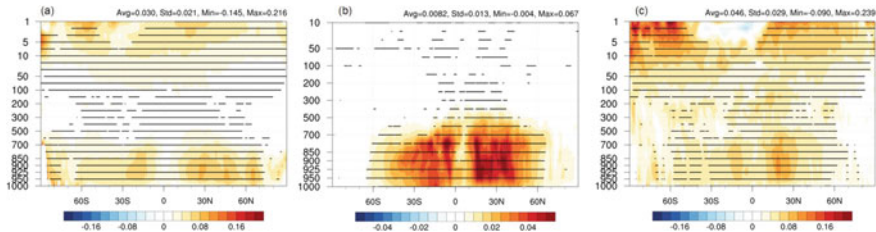


**Fig. 2** Random patterns of (a) SPPT and (b) SPDT at model 90-th level, represented as an ensemble mean of the 6-h forecast initiated from 1200 UTC 6 June 2018

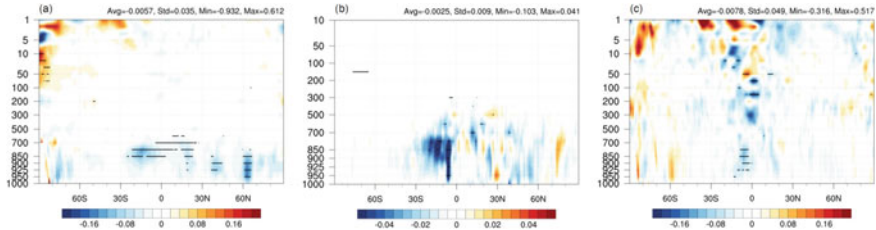
### 4 Results

We have evaluated the SPHT scheme through the root-mean-square difference (RMSD) and ensemble spread. The RMSD represents the model error while the ensemble spread explains the model uncertainty. Here, we assume that the true state is the ECMWF Integrated Forecast System (IFS) analysis, which is well known for high forecast skill. To identify the vertical distribution of ensemble mean spread and ensemble mean error globally, we analyzed the zonal mean during the experiment periods, excluding the spin-up. The STOC, including both SPPT and SPDT, increases the ensemble spread below 700 hPa in the troposphere and above 10 hPa in the stratosphere (Fig. 3).

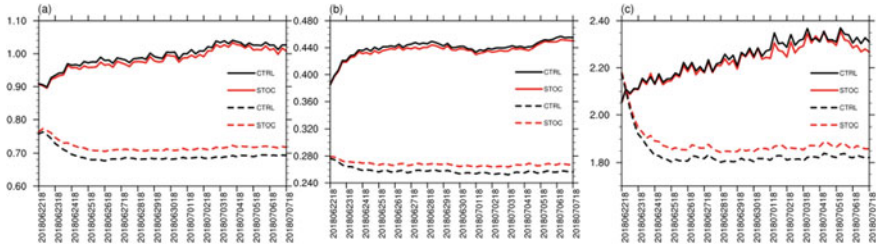
Increase in ensemble spread is evident because the model variables are perturbed by the random forcing. Therefore, it is essential to check if the increase in ensemble spread induces reduction in ensemble mean error: if the ensemble mean error increased, the ensemble spread increase is ineffectual. The augmented ensemble spread reduces the ensemble mean RMSD as well, mostly in the tropical troposphere for wind and in the low to mid-troposphere for temperature and specific humidity (Fig. 4).



**Fig. 3** The difference of zonal mean ensemble spread (STOC – CTRL) for the prior for (a) temperature (in K), (b) specific humidity (in  $\text{g kg}^{-1}$ ), and (c) zonal wind (in  $\text{m s}^{-1}$ ), averaged for the period of 1800 UTC 25 June – 1800 UTC 7 July 2018. Black dots indicate 95% statistical significance based on two-tail  $t$ -test



**Fig. 4** Same as in Fig. 3 but for the difference of zonal mean RMSD against IFS analysis



**Fig. 5** Time series of the globally-averaged ensemble mean spread (dotted line) and the ensemble mean RMSD (solid line) in the prior for STOC (red) and CTRL (black) for (a) temperature (in K), (b) specific humidity (in  $\text{g kg}^{-1}$ ), and (c) zonal wind (in  $\text{m s}^{-1}$ )

We also have assessed the performance of the SPHT scheme, which is applied to the KIM global model, by evaluating the ensemble quality via the globally-averaged RMSD and ensemble spread. The global performance at each prior (the 6-h background) is evaluated with regard to the vertically-averaged RMSD from 1000 hPa to 10 hPa over the globe (see Fig. 5). As shown in Fig. 5, the ensemble spread augmentation obviously brought on the RMSD reduction during the whole experiment period. For temperature, the ensemble mean RMSD decreased by 1% when the ensemble mean spread increased by 3.7%; for specific humidity, the former decreased by 0.65% when the latter increased by 2.0%.

## 5 Summary

We implemented the stochastic perturbation hybrid tendencies (SPHT) scheme perturbing both the physical tendency and the dynamic tendency in a global numerical weather prediction model—the Korean Integrated Model (KIM)—which has recently been operational in the Korea Meteorological Administration. The SPHT scheme inflates the insufficient ensemble background error covariance coupled with the local ensemble transform Kalman filter system: it leads to an increase in ensemble spread as well as a decrease in the ensemble mean errors, thus improving the ensemble background error covariance and ensemble prediction.



The stochastic schemes can be used in various fields as the demand for ensemble systems increases. Recently, Ollinaho et al. (2017) developed the stochastically perturbed parametrizations (SPP) scheme to perturb the parameters and variables in physical parametrizations. For example, many physical processes, including turbulent diffusion, sub-grid orography, convection, cloud, large-scale precipitation, and radiation, used to be perturbed to cover the model uncertainty in the European Centre for Medium-Range Weather Forecasts (ECMWF). As demonstrated, we can extend the use of the stochastic perturbation schemes on demand to overcome underestimation of model uncertainty.

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