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Communications in Computer and Information Science 1308

Information and Communication Technologies in Education, Research, and Industrial Applications

16th International Conference, ICTERI 2020
Kharkiv, Ukraine, October 6–10, 2020
Revised Selected Papers

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Preface

This volume contains a number of selected refined and extended contributions to ICTERI 2020, the 16th International Conference on Information and Communication Technologies (ICT) in Education, Research, and Industrial Applications. The conference was held in Kharkiv, Ukraine, during October 6–10, 2020, with a focus on research advances in ICT, business or academic applications of ICT, and design and deployment of ICT infrastructures. To our regret, the COVID-19 pandemic situation did not allow us to organize the conference as a face-to-face gathering. Therefore, the conference proceeded in the virtual mode[1].

ICTERI 2020 continued the tradition of hosting co-located events this year by offering six workshops and a PhD Symposium. The workshops addressed the following:

(1) 3L-Person 2020: new and emerging technologies in education, learning environments, and methods that aimed at satisfying the life-long learning needs of a person based on the use of a person-oriented approach.
(2) CoSinE 2020: theory and practice of computer simulation in education, with special emphasis on real-world applications and settings.
(3) ITER 2020: research advances, business and academic applications of ICT related to solving practical economic problems, and pushing forward economic research.
(4) MRQL 2020: new and emerging applications in the open systems of higher education, including the use of open education and research tools, resources, and methods for forming a creative and ICT competent person in view of European Research Area development.
(5) RMSEBT 2020: rigorous methods used in different fields of software engineering, in particular the specification, verification, and optimization of software; software analysis, modelling, and business rule extraction; software testing; re-engineering problems; and modeling and verification of token economies.
(6) The RMIT 2020: building an efficient and effective bridge between the mathematical reliability and engineering practices in safety-critical industries and domains such as energy grids, aerospace, rail and automotive systems, and health systems.

The PhD Symposium provided the opportunity for PhD candidates to present, listen to, and discuss the research on the topics relevant for ICTERI.

The proceedings of ICTERI 2020 have been published at CEUR-WS as three separate volumes: 2740 for the main ICTERI conference (http://ceur-ws.org/Vol-2740/); 2732 for the workshops (http://ceur-ws.org/Vol-2732/); and 2791 for the PhD Symposium (http://ceur-ws.org/Vol-2791/). These three volumes contained a total of 149

[1] The YouTube channel containing the video records of all ICTERI 2020 talks has been madavailable at https://www.youtube.com/channel/UCeAQKbvvphKaSpy23RimLLA/playlists

papers selected from 347 submissions. The authors of the 20 best papers, nominated by the program chairs, workshop chairs, and ICTERI steering committee, were invited to submit substantially extended and revised versions for this proceedings volume. Out of these best papers, 14 extended and refined submissions were received and reviewed by at least three experts. Finally, the Proceedings Review Panel selected the 8 most mature and interesting papers for publication. The acceptance rate was thus 2.3% in terms of the overall number of ICTERI 2020 submissions.

The papers in this volume were grouped into two topical parts.

Part I collected the contributions that further elaborate effective and efficient use of ICT in teaching, learning, and professional training. These contributions discussed the use of mobile learning approaches in the digital instrumentation of the didactics, identifying the gaps between the current educational standards and approaches versus the evolving requirements of the labour market, the specifics of mobile teaching and learning under the impact of COVID-19 pandemic constraints, and implementation and use of immersive technologies and tools for professional training.

Part II dealt with the development, application, and use of ICT-based techniques and approaches in research and industrial uptake. The contributions elaborated on novel approaches and solutions for evaluating the quality business process models, recognizing brain tumors on MRI images using artificial neural networks, studying discrete systems with outputs using a coalgebraic approach, and introducing induction principles for diamond-free partial orders.

This volume would not have been possible without the support of many people. First, we are very grateful to all the authors for their commitment and intensive work. Second, we would like to thank the Program Committee members and additional reviewers for providing timely and thorough assessments. Furthermore, we would like to thank all the people who contributed to the organization of ICTERI 2020. Without their efforts, there would have been no substance for this volume.

April 2021

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Contents

ICT in Education

Instrumental Digital Didactics of Physics Study in the Aspect of M-learning

Iryna Slipukhina[1(✉)], Ihor Chernetckiy[1(✉)], Nataliia Kurylenko[2(✉)], Sergii Mieniailov[3(✉)], and Serhii Podlasov[4(✉)]

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Abstract. The experience of distance and combined learning during the pandemic COVID-19 has demonstrated the complexity of organizing experimental research in natural sciences, especially physics. In conditions of limited access to full-sized laboratory equipment, smartphones have significant potential: they contain sensitive elements (distance, lighting, magnetic field sensors, accelerometer, GPS and others). Their use in the educational process of teaching physics is effective for the formation of key and subject skills of students in terms of the digital age. The use of digital means of obtaining and processing experimental data during the educational process in natural sciences enriches pedagogical science with methods that can be described by the general concept of «instrumental digital didactics» (IDD). The authors explain a definition of IDD, identify the types and functions of IDD tools. It is shown that the use of smartphone sensors for educational research in physics demonstrates the intersection of the conceptual fields of IDD and m-learning. Foreign and Ukrainian experience of the use of information technology capabilities of smartphones at the physics study is analyzed; in this context, the classification of mobile applications, in particular, designed to process data from sensitive elements is considered. An anonymous survey in June and December 2020 concerning the distance learning experience revealed, on the one hand, the presence of earlier experience and interest of students in use of smartphone sensors for physics research, on the other hand, the need for clear instructions. The methods proposed by the authors, which are demonstrated on the resource STEM-laboratory MANLab (stemua.science), do not require an experiment in a laboratory and are based on the use of only sensitive elements of a smartphone and PC. The design features of sensors can also be studied during physics lessons. The author's technological map of use of the mobile application Phyphox for laboratory work on the topic «Mechanics» is proposed.

Keywords: M-learning · Smartphone sensor · Mobile application · Physics · Instrumental digital didactics

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1 Introduction

The practice of distance and combined learning in the conditions of COVID-19 demonstrated the complexity of organizing experimental research in the field of natural sciences, in particular, the performance of laboratory works. One of the possible and affordable solutions may be the use of virtual models. However, as it is known, a significant disadvantage of virtual simulations is the idealization of the considered processes.

Assumed the limited access to full-size laboratory equipment, the technical and technological capabilities of gadgets, especially smartphones, have significant potential. These powerful and complex devices combine various subsystems such as communication module (calls and Internet, Bluetooth, Wi-Fi), navigation system (GPS, QZSS and others), a set of sensors, I/O device (display with keyboard, speakers).

Anderson M. and Jiang J. research [1] found that 95% of teenagers had a smartphone in constant use and 45% permanent access to the World Wide Web already in 2018 year. According to G. Hendeby [2], the smartphone is now an inexpensive, portable and reliable research equipment for educational purposes, flexible in setup and suitable for use in the school laboratory and at home. This explains the fast extent of BYOD policy and the exponential growth in the number of relevant pedagogical researches worldwide and in Ukraine [3, 4]. Models of learning using smartphones, which are combined in the common term m-learning, have developed especially over the last 5 years. For that reason, on request for words m-learning, smartphone, mobile device on the platform Research Gate you can get about 100 articles (the earliest, apparently, in 2014).

The importance of developing skills of processing experimental data using digital means, one of which is a smartphone, is now protected in regulations. For example, in the section «Natural education» of the new standard of Ukraine basic secondary education [5] in the competence potential among the key competencies are the following skills:

- «interpret the information presented in infographics, tables, diagrams, graphs, etc.» (using the state language);
- «solve problems of natural content with the help of mathematical methods and mathematical models, natural objects, phenomena and processes, graphs, tables, diagrams, etc.»;
- «use digital resources to obtain new scientific knowledge, search, process, store information of natural content, convert it from one types to another using Internet resources and digital devices, explore the environment using modern digital technologies and devices» (mathematical competence)»;
- «use scientific knowledge, achievements of engineering and technology to solve problems» (competence in the field of natural sciences, engineering and technology);
- «find, process, store information of natural content, transform it from one type to another using information and communication technologies»;
- «use and create digital content of natural sense»;
- «study the environment with the help of modern information and communication technologies (information and communication competence)» [5].

It is well known that the analysis of experimental data requires a significant amount of time and requires the skills to use the tools of appropriate software. Therefore, as pedagogical experience shows, there is no stable logical connection between the essence of the physics phenomenon and its mathematical description in the minds of students, in particular, ability for the graphical representation. A real breakthrough in solving this problem could be the use of specialized mobile applications (MA) for physical research focused on processing data from sensitive elements of smartphones. An example of a modern open platform for executing, processing, and analyzing the results of experimental data is Phyphox (phyphox.org).

2 Analysis of Related Research

Our study directly relates to the use of mobile phones for didactic purposes, as defined by the term «m-learning». The founder of this concept is A. Kay, who in the 1970s he created a team to develop Dynabook, a portable personal computer [6]. This gave impulse to the development of the «smartphone» and provided a platform for mobile learning (m-learning). However, the active use of the concept of «m-learning», as noted by Mike Sharples [7], began in the 2000s in parallel with the rapid technical and technological development of smartphones. Today, there is no single definition of «m-learning» among scientists. For example, in the Cambridge Dictionary m-learning is defined as «…the use of electronic devices such as smartphones, laptops, and tablets as teaching devices» [8].

Discovering the essence of the concept of «m-learning» and its place in education, the use of mobile technologies during physics study is the subject of research of many foreign and Ukrainian scientists. In particular, theoretical aspects of mobile learning were considered by S. Lytvynova, O. Melnyk et al. [9], N. Rashevska [10], A. Korucu and A. Alkan [11]; the possibilities of using mobile technologies in the educational process were investigated by S. Semerikov [12], A. Sönmez et al. [13], M. Al-Emran et al. [14]; V. Sipiy [15], O. Slobodyanyk, N. Myslitskaya and V. Zabolotny [16], G. Skrypka [17], S. Tereshchuk [18]. The work of J. Traxler [19] is devoted to the study of certain applied methods of using m-learning. It demonstrates the ability of mobile devices to expand the range of time frames of information perception. The use of mobile applications in university education of future professionals has been studied by D. Parsons [20], N. Keskin and A. Kuzu [21], A. Maha and M. Heba [22], M. R. Bice, et al. [23]. M. Oprea and K. Miron [24] dealt with the problems of introduction and use of mobile devices in teaching physics. Summarizing these scientific sources, m-learning will mean the interaction of teacher and student, where access to educational resources is carried out using mobile gadgets.

The situation related to the need for distance and combined learning in 2020–2021 brought the study of pedagogical practices of m-learning to a new level. Authors A. Naciri et al. [25] and G. Basilaia and D. Kvavadze [26] believe that mobile learning is an inevitable alternative during COVID-19. Very interesting is the study of B. Biswas et al. [27] of students' perceptions of the use of mobile learning devices during COVID-19 in Bangladesh. They found that most students were positive about mobile learning and m-learning was useful for linking knowledge gaps, educational institutions were stimulated to integrate mobile technology into the entire education system.

2.1 Classification of MA According to the Didactic Purpose

Obviously, the significant didactic potential of m-learning, despite certain shortcomings [28], can be fully realized only using different types of content, which is now divided into the following types: mobile applications, mobile site, social networks and user content, unique mobile content [28]. In this context, our study required the analysis of mobile applications (MA), which can be used to create methods for field experiments in physics.

A previous study [29] showed that all MA can be divided into four classes according to the didactic purpose: application-directories, applications for monitoring and evaluation of knowledge, virtual laboratories, and sensors (Fig. 1).

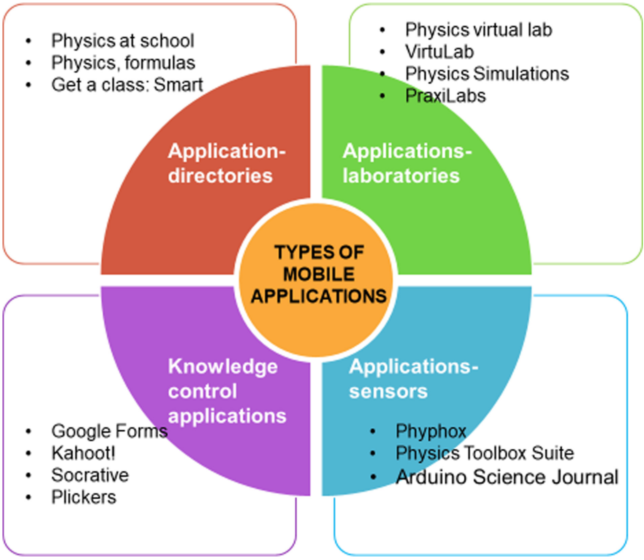


Fig. 1. Classification of MA according to the didactic purpose

Detailed features and capabilities of such mobile applications as «Google Forms», «Google Classroom», «Kahoot!», «Socrative», «Plickers», «Get a class: Smart», «Physics virtual lab», «Science experiments in physics lab», Physics at school in the educational process are presented by the authors T. Goncharenko, N. Yermakova-Cherchenko and Y. Anedchenko [30].

In this article, we consider m-learning using smartphone sensors as one of the tools of innovative didactics – instrumental digital didactics (IDD). Therefore, we briefly reveal the meaning of this concept. R. Krumsvik, A. G. Almås [31] and a team of authors led by I. Jahnke [32–34] were among the first to use the term «digital didactics». In these works it is defined as a means of managing the stream of knowledge (R. Krumsvik, A. G. Almås) and the development of new knowledge (I. Jahnke et al.). A feature of the Digital Didactic structure, in addition to the aims of learning and its evaluation, socio-personal interaction in the digital society and the digitalization of education, is the presence of different levels of technology integration. Very successful for the description of Digital Didactic is the heuristic model of the tetrahedron of pedagogical interaction proposed by K. Ruthven [35]; each face of the tetrahedron reflects a certain educational interaction between teacher, content, technology and student. M. Tchoshanov [36] using the concept of «Digital Age Didactics» and F. Perri [37] showed that Digital Didactical Designs significantly changes the structure of teachers activity; they perform a new function of educational process engineer. S. Pedrosa and T. Tortori [38] were among the first to use the concept of «digital didactics» to describe the study of natural sciences on the basis of creativity, interactivity and interdisciplinary interaction.

An important part of digital didactics is the part, which includes methods of using digital tools and technologies to obtain, provide, process and interpret data from natural experiment. Note that such studies may include real-time measurements (the use of a digital speedometer to directly measure speed) and studies of stored digital data (video analysis of moving objects). This creates grounds for distinguishing the concept of instrumental digital didactics as a component of digital didactics, which reflects the learning activities using digital means of obtaining and processing empirical data obtained during a scientific experiment. IDD tools perform the following functions (Fig. 2):

– information function (search of reference materials, description of researches, modeling of real tasks, monitoring of educational achievements);
– device for obtaining empirical data or measuring instrument: digital measuring systems (DMS), digital cameras, digital telescopes, digital microscopes, etc.;
– means of processing the numerical results of the experiment (software for various calculations, data visualization, model building, etc.).

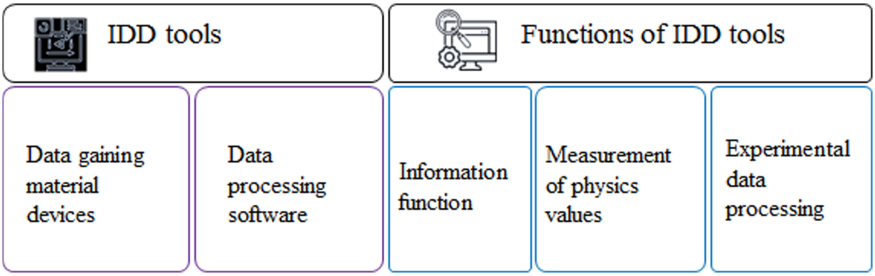


Fig. 2. Types and functions of IDD tools

The smartphone is an IDD tool and combines in one device technological segments of the digital measuring system: the perception of external information by the sensor, its conversion into a digital signal, further processing by information technology and presentation in a user-friendly cognitive form adapted to his previous experience - visual, tabular or graphical (Fig. 3).

Thus, in the context of our investigation, the methodology of using sensitive elements of the smartphone as tools for obtaining empirical data and software for their visualization and data processing is a practical example of IDD and interconnects with m-learning through the use of MA sensors (Fig. 4).

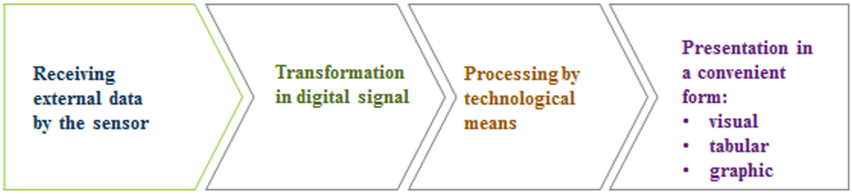


Fig. 3. Technological parts of data processing by smartphone sensors

One of the first examples of development of a smartphone application based on the Sensor Fusion App for Android was proposed by G. Hendeby et al. [2]. This MA is designed for easy registration, visualization and transmission of sensor data, as well as full integration of streaming data with Matlab and Matematica.

T. Pierratos and H. Polatoglou considers an example of using MA Phyphox (https://phyphox.org/) to study uniform motion using an optical stopwatch, which is based on the action of a smartphone photosensor. There is also attention to the convenience of its use in lectures through the projection of the smartphone screen of the teacher [39].

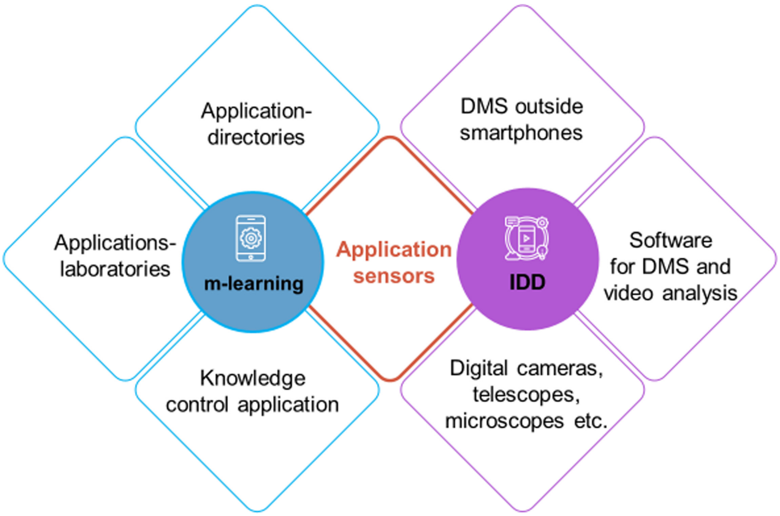


Fig. 4. Applications sensors at the intersection of m-learning and IDD

The original training method for measuring the coefficient of friction using an accelerometer and protractor of a smartphone and using MA Physics Toolbox Sensor Suite (https://www.vieyrasoftware.net/) was proposed by the authors A. Çoban and M. Erol [40].

Original examples of using a smartphone accelerometer developed by Ch. I. Larnder [41]. Interesting teaching experiments using the MA Physics Toolbox Sensor Suite aimed at overcoming problems in studying the relationship between the action of force and acceleration of the body are offered by R. Vieyra et al. [42]. An extensive study of the development of MA in data processing from smartphone sensors (more than 30), as well as some examples of methods of using the most popular of them (Physics Toolbox Sensor Suite and Phyphox), are presented in L. Sukariasih et al. [43].

Our preliminary analysis showed that the number of scientific reports that cover specific techniques with clear instructions (this is especially important for high school (K-12) and junior students) and the practice of their implementation, which are focused on the use of smartphone sensors to collect, analyze and make interpretation of data during educational research in natural sciences, is evidently insufficient.

3 The Purpose of the Article and Method

The aim of the study is to demonstrate the capabilities of smartphone sensors as a means of IDD. This aim have several directions: to clarify the experience gained by students in the process of distance learning of physics; identification of technical capabilities of smartphones available to students; analysis of the didactic possibilities of popular MA used for teaching physics; proposals for the application of IDD to educational experiments on the school course of physics (K-12).

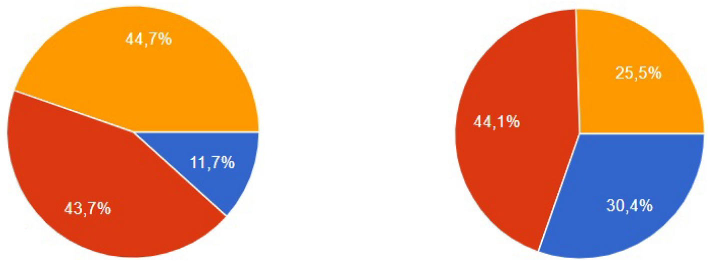
The authors conducted several anonymous surveys of students using Google Forms throughout 2020 with the aim of clarifying certain aspects of IDD and m-learning. We also analyzed publicly available data sources through a systematic review. At this stage, data from several databases (Scopus, IEEE Explore, Google Scholar, and Research Gate) were used. The keywords «m-learning in physics learning/teaching», «the smartphone sensor for physics learning», «m-learning with smartphone sensor», etc. were used for the search. In this article, we have used only data of the use of smartphone sensors and software in free access for data streaming, which can be easily used by teachers as a means of teaching physics. Methods of performing an education experiment in physics using a smartphone were developed by the authors in the «STEM-laboratory MANLab» in 2020.

4 Results and Discussion

To analyze the challenges of distance learning in the context of the COVID-19 pandemic, we conducted an anonymous survey of students at the end of June 2020. 198 students of higher education institutions in Kyiv took part in the survey. Respondents filled out a questionnaire created by Google Forms, which asked questions with incompatible alternatives. For some of the questions there was an open answer option «Other». For comparison, a similar survey was conducted in late December 2020. This time the respondents were 204 graduates of the National Aviation University (Kyiv), Kherson State University, Kherson Maritime College of Fish Industry, National Technical University of Ukraine «Kyiv Polytechnic Institute named after Igor Sikorsky». The vast majority of respondents (93% in both cases) were aged 17 to 21 years.

One of the questions was: «What data sources do you use in preparation for classes?». The distribution of respondents' answers (Fig. 5) showed that almost half of students in educational activities prefer the recommended literature in digitized form (approximately 44% in both surveys). Instead, the percentage of those who used random materials on online networks decreased significantly (from 45% in June to 26% in December 2020). The number of students who use lecture materials, abstracts, recommended textbooks and manuals in paper form has increased (from 12% in June to 30% in December 2020).

The changes can be explained by the improvement of organizational, pedagogical and psychological conditions of distance and combined learning, the growth of trust of students to the recommended online resources and their quality, increasing the efficiency of all participants in the educational process.



June 2020 December 2020

- Lecture materials, abstracts, recommended textbooks and manuals in paper form
- Recommended literature in digitized form: textbooks, manuals, recommendations
- Focus on control questions using random materials in online networks

Fig. 5. Distribution of answers to the question «What data sources do you mainly use in preparation for classes?»

The analysis of the distribution of respondents' answers to the question: «Would you like to perform an independent study using smartphone sensors instead of scheduled laboratory work in physics?» showed considerable interest in using gadgets to conduct physics education experiments (Fig. 6). As can be seen from the data, the total percentage of students interested in performing laboratory work using smartphone sensors during distance learning and teamwork increased from 75.6% in June to 88.7% in December. Instead, the number of responses such as «No, this is difficult for me» and «Other», which we considered as «No» in implicit form, decreased from 24.4% in June 2020 to 11.3% in December 2020. The result can be explained by the awareness of the necessity of using only the available opportunities provided by the gadget and the development of appropriate skills during the forced self-isolation of 2020.

Another question of the survey was «What conditions, in your opinion, are necessary for the effective implementation of distance learning?». Respondents were asked to highlight the three most important factors among the list, which included: clear instructions (1), convenient feedback and a flexible rating system (2), the use of learning tools adapted to smartphones (3), online interactive workbooks (4), simple «input» (5), providing support in the use of digital devices (6), providing psychological and pedagogical support (7). It was also possible to express your own answer in the «Other» version. The distribution of respondents by answers in the June and December 2020 questionnaires is presented in Fig. 7.

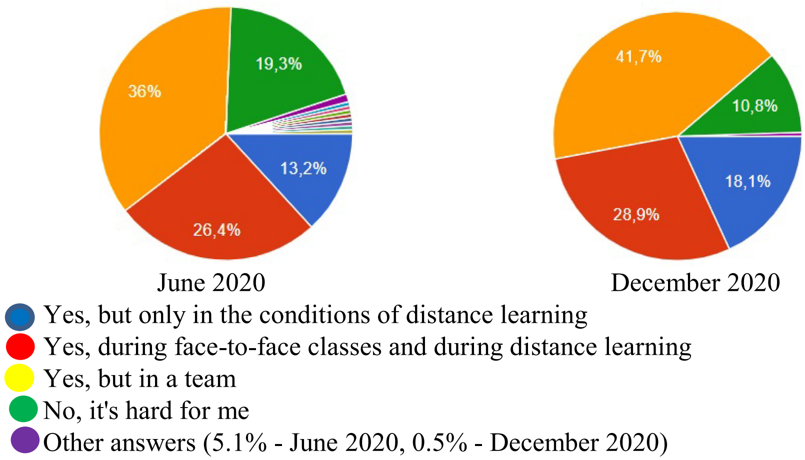


Fig. 6. Distribution of answers to the question «Would you like to perform an independent study using smartphone sensors instead of scheduled laboratory work in physics?»

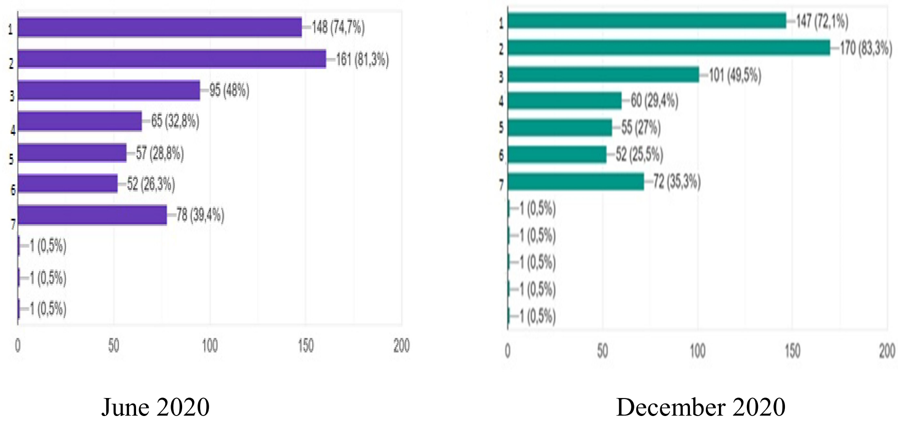


Fig. 7. Distribution of answers to the question «What conditions, in your opinion, are necessary for the effective implementation of distance learning?»

Comparison of answers showed that the rating of factors remains unchanged: the most important for respondents are convenient feedback and a flexible assessment system (81%/83%), clear instructions (75%/72%), the use of learning tools adapted to smartphones (48%/50%) and providing psychological and pedagogical support (39%/35%).

Among the answers «Other» were provided the following «adequate amount of work», «competence of teachers in the use of digital technologies», «convenient online learning resources» (1 answer or 0.5%), «fast and stable Internet connection» (3 answers or 1.5%), «electronic diary» (2 answers or 1%).

Thus, respondents are aware that a specific educational trajectory, which involves the use of recommended educational resources, the use of tools available to them personally, can create a much greater educational effect than finding information in random sources. In addition, students appreciate the importance of convenient feedback, a flexible rating system and clear instructions.

Features of Smartphone Sensors. Descriptions of the design and operation of smartphone sensors can be found in a large number of online sources: popular science articles (for example, the message D. Nield [44]), data from smartphone manufacturers (for example, https://consumer.huawei.com/eg-en/support/content/en-us00685236/), engineering research (for example, A. Elhattab et al. [45]).

Important physical elements of the smartphone are the sensitive elements that provide the measurement process. In general, the sensors of modern mobile phones can be divided into four categories (Fig. 8): motion sensors, such as accelerometer; position sensors, such as magnetometer, GPS, proximity sensor; ambient sensors, such as light sensor and barometer; unique sensors, such as a thermal imager, fingerprint scanner, or Face Unlock, which are more commonly used as security features.

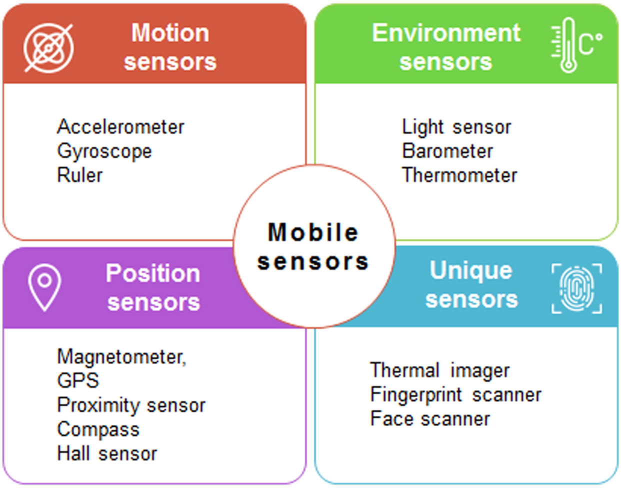


Fig. 8. Types of mobile sensors

4.1 Research on the Hardware and Operating Parts of the Smartphone

Note that the capabilities of the smartphone to measure physical quantities and their analysis are determined by two main factors: the presence of sensitive elements - sensors, as well as the type and generation of the operating system (OS).

The OS version also defines the MA that can be installed on the smartphone. In this context, we previously conducted a comparative analysis of the main characteristics of popular brands of these gadgets, the results of which were summarized in the table presented in [29]. Much larger is the database of sensors collected by users of the Phyphox resource [46] using the «Submit to sensor database» experiment.

To find out the types of OS and sensors as well as the skills of using them as research tools, we conducted a survey using Google Forms with the participation of teachers of natural sciences and students studying natural sciences and engineering at the National Pedagogical University after M.P. Drahomanov, Kherson State University and the National Aviation University (136 people in total). One of the issues was to determine the characteristics of smartphones using the system menu of gadgets (Fig. 9). It was found that most of the respondents use the Android operating system (Fig. 9, a).

Note that we can check which sensors are in the smartphone with the help of many MA available for free download such as Sensors MultiTool, Sensor Kinetics, Sensor (for Android) and Sensors (for iOS).

The study showed that the full use of MA sensors is possible if the OS is not lower than Android 5. The survey also found that users of gadgets with Android OS have new versions 5 and above (99%), which creates optimal opportunities for the use of MA (Fig. 9, b).

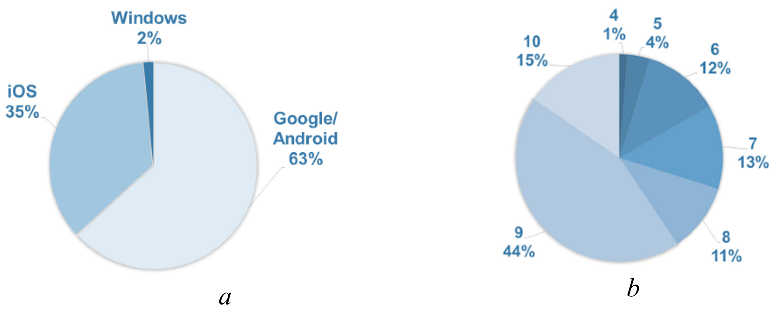


Fig. 9. Distribution of answers to the questions: OS type (a); Android version (b)

In the same questionnaire it was found that the vast majority of respondents are aware of the presence of sensors (Fig. 10, a), (especially popular applications for GPS navigation), but did not use them for educational activities (Fig. 10, b) and want to learn more about these opportunities (Fig. 10, c).

Review and testing of the most promising MA sensors that can be installed and run on the studied smartphones allowed singling out the Physics Toolbox Sensor Suite, Phyphox and Arduino Science Journal. Note that additional selection criteria were free of charge and cross-platform.

These MA automatically detect the presence of sensitive elements and can store data and transmit them in a format that uses mathematical tables (Excel). Note that the use of each of these applications is determined by the task. For example, MA Physics Toolbox registers and transmits data to other digital devices but the analytical capabilities of this application are insignificant.

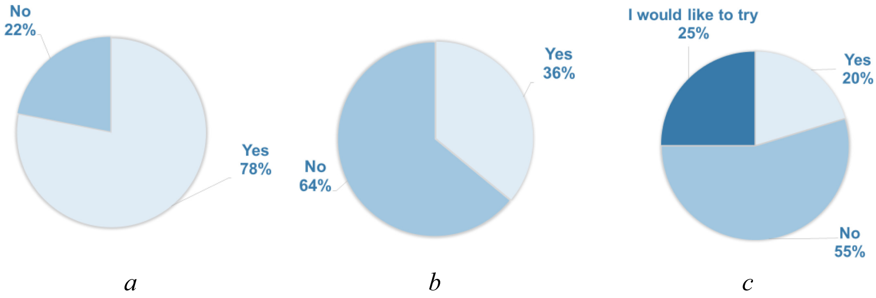


Fig. 10. 3. Sensor usage data

MA Phyphox contains ready-made methodological solutions for practical work and is an open platform with the ability to add your own techniques, record sensor data and the results of data processing. Using the provided user interface, the teacher can arrange a new experiment with appropriate processing and presentation of measurements. In addition, the application allows you to share the screen of a smartphone with another device (desktop computer with a video projector) as well as control the smartphone through a connected device.

The MA Arduino Science Journal, unlike the previous two, does not always identify available sensitive elements but allows to create a full-length study with comments and illustrations and save it to Google Drive.

4.2 Application Example. Methods of Carrying Out Educational Research Using a Smartphone

A study of existing teaching methods has shown that the most commonly used for didactic purposes are the proximity sensor and accelerometer. Details of their design can also be a topic of physics study. For example, the smartphone proximity sensor monitors its distance to a specific object and turns off the display, for example, when you hold the phone to your ear to prevent accidental touching of the touch screen during a call, to save battery power during long conversations, to recognize gestures or focus the camera. The distance is most often determined by the principle of radar: the smartphone emits infrared rays, which are reflected on an obstacle and generate a signal in a special receiver. There are now sensors for smartphones («Ellipticlabs»), which use ultrasound. Note that this option does not require radical changes in the design of the smartphone, as the radiation and reception of ultrasound is carried out by the speaker and microphone of the device.

The principle of operation of the accelerometer, otherwise known as the G-sensor, is based on the inert properties of bodies during their motion with acceleration: a load of a certain mass is attached between the spring and the damper, which are connected to the frame. When moving with acceleration, the force of inertia acts on the load, which leads to deformation of the spring. The amount of deformation determines the readings of the device. The damper dampens the load vibrations. The G-sensor for a smartphone has the design of an electromechanical chip [40]. Accelerometers are most widely used in aircraft navigation devices, in car DVR and speedometers, in industrial vibration control systems, in information systems for hard disk protection, etc.

The original applications of the accelerometer, as mentioned at the beginning of this article, were developed by Ch. I. Larnder [41] to measure the «critical angle» at which a smartphone rotates from portrait to landscape views, as well as to determine the actual location of the accelerometer sensor in the mobile phone case. We also described a method of laboratory work for determination of the acceleration of free fall where the accelerometer was used to compare with the «key data» obtained from the proximity sensor [29].

In this context, we have developed a method of laboratory work «Determination of the coefficient of sliding friction». Its implementation requires a smartphone with an installed program for processing streaming data sensors (in this case, the accelerometer) as well as simple improvised means (spring or rubber band, plastic bag, binder, etc.). The measurement is based on a mechanical description of the movement of the system during braking under the action of friction. The dynamic equation of motion for such a model will look like: $\mu mg = ma$, after simple transformations we get $\mu = a/g$. Therefore, to find the coefficient of friction [47, pp. 281–293] of sliding, it is sufficient to measure the braking acceleration of the body. We can use a smartphone (a braking object) and its accelerometer (measuring device). To display data in a convenient graphical form, it is advisable to use MA Phyphox [46].

The technological map of the proposed laboratory work consists of the following main stages: data receiving (acceleration measurement), data processing (determination of the friction coefficient), error estimation and presentation of results. Let's consider them briefly.

Measurement of Acceleration of a Mobile System. A support with a long spring is placed on a horizontal surface. The transparent (polyethylene) package with the smartphone by means of a binder and a thread is connected to the spring (Fig. 11). The length of the thread allows the phone to move to a complete stop. In the open MA Phyphox select the tab «Acceleration (without g)». Place the smartphone in a bag and move for some distance while stretching the spring. Turn on data recording and release the phone. Repeat the procedure several times.

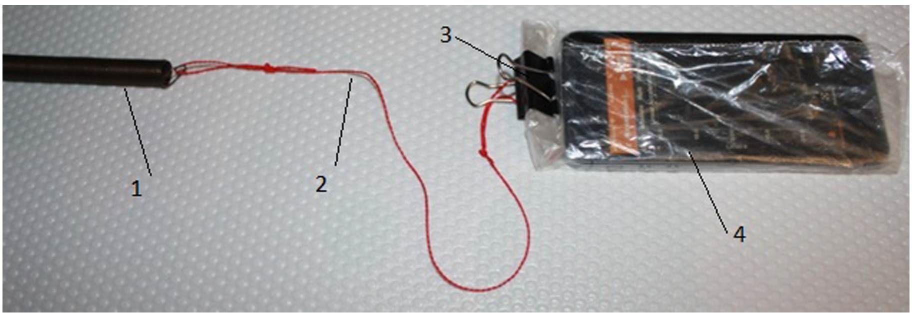


Fig. 11. The experimental setup: spring (1), thread (2), binder (3), smartphone with MA Phyphox enabled in a transparent package (4)

Determination of the Coefficient of Friction. Gadget acceleration values displayed on the «Acceleration (without g)» tab are used for calculations. The graph is displayed on the «Acceleration (without g)» tab. Find the axis for which the graph of the acceleration change look similar to Fig. 12. The horizontal sections of the graph for the values of a < 0 in certain time intervals correspond to the dynamic motion described above. Fluctuations in the values of acceleration in these areas are explained by the peculiarities of the structure of the Micro Electrical Mechanical System and their relatively low sensitivity and high output noise density, thermal noise, Johnson-Nyquist noise [45].

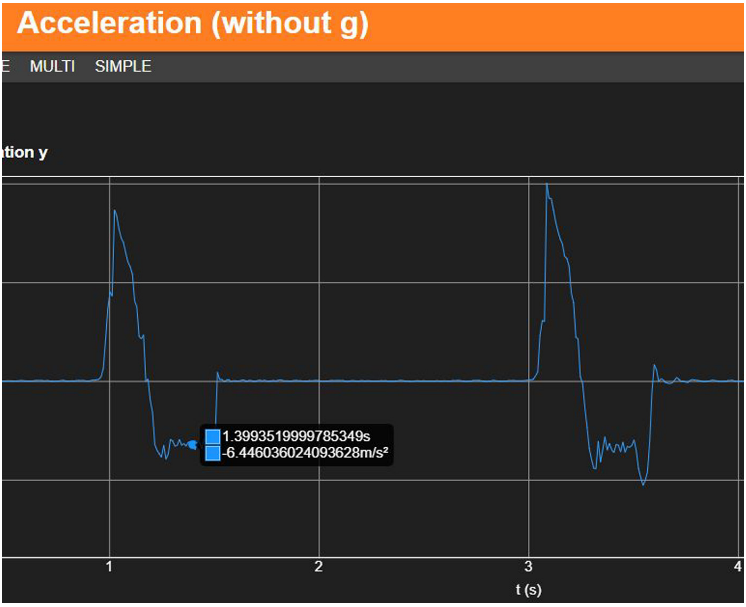


Fig. 12. The value of the braking acceleration a (m/s^2) for a certain time t(s), on the graph $a = f(t)$ in the tab «Acceleration (without g)» MA Phyphox

The MA Phyphox data extraction tool can be used to select several sections, estimate and calculate the average value of the acceleration modulus during braking in each experiment. We can transfer data from a mobile phone to a PC for more accurately calculation of the acceleration. This can be done in two ways: by transferring the data file indirectly via e-mail or messengers, or by remotely controlling the application from a PC via wi-fi.

These brake acceleration values downloaded to the PC should be processed in Excel mathematical tables (Table 1), which should be used to find the average acceleration value a (m/s^2) for each individual measurement (it is enough to use three such values). Calculate the coefficient of friction according to the equation $\mu = a/g$ and enter it in the tables.

Error Estimation and Presentation of Results. Using the data table, determine the absolute $\Delta\mu$ and the relative error $\varepsilon\mu$ of the direct measurement, [47] the result is presented as: $\mu = \mu av \pm \Delta\mu$, $\varepsilon\mu$ (%).

Table 1. Experimental data table. Source: own construction

№	Acceleration of the system a, (m/s^2)	The average value of the acceleration of the system aav, (m/s^2)	Coefficient of friction μ
1			
2			
3			
4			
5			

In the absence of a spring, we can use a piece of rubber, which allows to work at home with a limited amount of laboratory equipment.

Research Development. To obtain conclusions about the dependence of the coefficient of friction on the weight of the braking system, additional loads are placed on the package with the phone and the experiment is repeated. We can also find the coefficient of friction for other surfaces.

We recall also that in MA Phyphox gives an opportunity to create and publish own experiments [46].

5 Conclusions

Smartphone sensors and MA streaming data processing are effective means of IDD of physics study; the use of their technical and technological capabilities has opened an innovative direction in m-learning. Natural science experiments with MA stream processing (Phyphox, Physics Tool Box Suite and others) eliminate routine stages, optimize and facilitate the process of analysis and interpretation and, above all, provide students with individual tools for real educational research based on scientific method and engineering design. Learning technologies using smartphone sensors complement the traditional «classroom» methods of teaching natural sciences, individualize the process of acquiring new knowledge, and promote the development of students' key skills such as critical thinking and independent work. Techniques based on the use of smartphone sensors also demonstrate the significant potential of IDD in developing such important research skills as understanding and interpreting the graphical representations of experimental data.

Our survey credibly demonstrated the convenience and accepting of the use of IDD tools by students as well as the lack of their skills of purposeful use of data obtained from smartphone sensors. Obviously, this problem can be solved, first of all, through the formation of appropriate skills and abilities of teachers, which was also discussed in our work [29].

The proposed methods for processing experimental data are based only on the use of a smartphone and PC and do not require an experiment in a laboratory. This approach greatly simplifies the organization of learning, especially in the context of distance learning during forced self-isolation. The method was tested by the authors of this article in the «STEM laboratory MANLab» [49] with the participation of members of the National Center «Junior Academy of Sciences of Ukraine».

Note that the methods of laboratory research using smartphone sensors, which we create in the «STEM laboratory MANLab» [49], are focused on the curriculum in physics for secondary education (similar to K-12 in the US). Therefore, creating experimental tasks, we are guided by the need for their visibility and the use of simple models according to the level of readiness of pupils. For example, the proposed approaches do not involve the simultaneous use of data «merger of sensor data» as demonstrated in a very interesting proposal [41] for the university course of physics.

IDD capabilities have a wide range of applications not only in teaching physics. We hope that this article will be useful for teachers of all natural sciences in the context of raising awareness about the possibilities of m-learning to solve educational problems.

It should also take into account optimality of methods of teaching physics on the basis of IDD when developing individual educational trajectories for learners. Some of the methods proposed by the authors are posted on the resource STEM-laboratory MAN-Lab (stemua.science) in the form of labs, practical works and research using available equipment such as a smartphone.

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Bridging Business Analysts Competence Gaps: Labor Market Needs Versus Education Standards

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Abstract. Efficient human resource management needs accurate assessment and representation of available competencies as well as effective mapping of required competencies for specific jobs and positions. Thus, the definition and identification of competence gaps express differences between acquired and required competencies using a mathematical approach to support accurate competence analytics. Lack of skills and insufficient qualifications of employees are cited as major barriers to the adoption of new technologies in the context of Industry 4.0. These changes require skills for data analytics tasks. The purpose of this paper is to investigate gaps in the preparation of a business analyst between the requirements of the labor market and the standards of study programs. For the IT and other industries, the most important competencies from the study program of Ukrainian High Educational Institutes, which correspond to labor market requirements, were revealed using RStudio. Formation of business analysts' competencies required in the labor market is created both higher education (47%) and experience of applicants on the labor market (53%).

Keywords: IT education · Business analyst · Generic and specific subject competencies · Soft and hard skills · Competencies · Educational standards · Complementarities of requirements

1 Introduction

The European Higher Education Area promotes the design of curricula focused on the acquisition of competencies. Efficient human resource management needs accurate assessment and representation of available competencies as well as effective mapping of required competencies for specific jobs and positions. Thus, the definition and identification of competence gaps express differences between acquired and required competencies using a mathematical approach to support accurate competence analytics. Lack of skills and insufficient qualifications of employees are cited as major barriers to the

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adoption of new technologies in the context of Industry 4.0. These changes require skills for data analytics tasks. Business analytics (BA) becomes increasingly important under a rapidly changing business environment. It requires a conceptual model for the professional profile of a Data Scientist in the field of Information and Communications Technology (ICT), namely in the European e-Competence (e-CF) framework and the Skills Framework for the Information Age (SFIA) are related with ICT competencies/skills, including programming, machine learning, and databases. The Data Scientist professional profile combining contributions from different areas, such as computer science, statistics, and mathematics. To analyze the impact of competencies on employment we subcategorized competencies into generic and specific subject competencies for different job types.

The results show that data management capability fully mediates between IT competence and BA use. The paper analyses how individual job competencies requirements impact wage changes.

The purpose of this paper is to investigate gaps and complementarities in the preparation of a business analyst between the requirements of the labor market and the educational standards.

The remainder of our paper is organized as follows: in Sect. 2 we analyze the Ukrainian IT market and the imbalance of quality in the IT labor market in Ukraine. In Sect. 3, we present and discuss the experimental model where we investigate specific subject competencies from high education institutes of Ukraine which significantly impact an average wage of a business analyst. Finally, the last section concludes.

2 Related Works

2.1 The Imbalance of Quality in the IT Labor Market in Ukraine

Despite the importance of higher education, IT specialists seem to prefer non- or informal education to receive updated professional skills and build a successful well-paid career. Based on a survey of 8,638 questionnaires of IT sector workers, a portrait of a modern Ukrainian IT specialist was drawn up. Thus, 87% of women and 82% of men working in this field, have higher education. It is interesting to note that 58% of women and 64% of men, who hold technical positions, have higher field-specific education [1]. Moreover, IT majors are also popular among second higher education students. “Information Systems” is becoming more and more popular in recent years [2]. However, the interest in higher education within technical majors (especially at the second level degree) gradually declines. The reasons are the irrelevance of some educational components and neglect of the diploma’s importance in the real labor market. Consequently, formal education hands to work experience and non-formal/informal education guaranteeing certificates, which are usually more significant for employers to compare with a prestigious university diploma. In long-term orientation, this tendency hides the risk to tear down fundamental basics of training of specialists within technical majors at classical universities, in favor of professional or informal training. As a result, universities can transform into a networking platform rather than being educational providers. For instance, a common report of the Western NIS Enterprise Fund and UNIT.City on the topic of IT ecosystem in Ukraine profiles 7 leading universities within technical majors and 18 IT courses and schools. Moreover, the general number of students in IT courses and schools is 46.5 times bigger than at universities (Fig. 1).

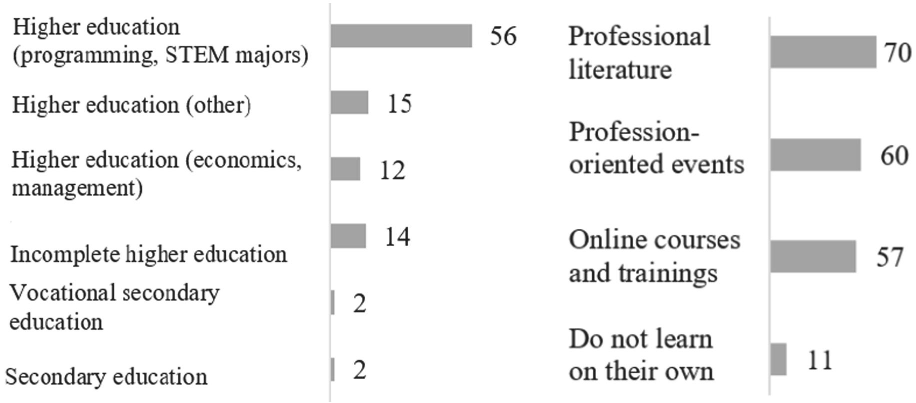

Fig. 1. Types of formal and non-formal education of IT specialists, % [1]

In 2018, 10% of employed and hired people in the Ukrainian IT sphere did not have a university diploma [3]. According to HeadHunter, around 56% and ITUkraine Association - 36% of domestic IT professionals do not have a university diploma and this indicator is gradually increasing both in Ukraine and all over the world [4]. Apple CEO Tim Cook has officially stated that half of the 2018 hires do not have a college degree because most colleges do not teach their students skills that are required by large corporations [4]. Despite old mathematical and IT schools, historically formed in Ukraine, modern systems of secondary and higher education in Ukraine are focused more on the acquisition and reproduction of fundamental knowledge, rather than skills and competencies: professional (ability to solve case studies), as well as soft skills (communication, presentation, organizational, teamwork), which are priorities for employers when being hired. Consequently, the government spends resources irrationally to teach future specialists, while business has been preparing their corporate roster for 3–6 months before hiring (Table 1).

Thus, to meet the current challenges of the labor market, IT representatives are forced to actively engage in self-education mostly aimed at learning foreign languages (often English) and developing soft skills through psychological and management training (most often in time management, team-building, leadership, project management, promotion, marketing) rarely in hard skills (different programming languages) [2]. A sufficient system of motivation for the IT work based on an assessment of their competence level should be a tool for solving the problems of IT education development [5].

In 2015, the reform of higher education in Ukraine began. It was aimed at acquisition competencies - Generic Competencies, closely connected with soft skills and Subject Specific Competencies.

To investigate the gap between the quality of higher education and the requirements of employers, we have selected a job position “Business Analyst”. Both IT specialists

Table 1. Institutional support for IT education in Ukraine [6]

	Formal education			Nonformal education				
#	Institutions	Students	#	Institutions	Students	#	Institutions	Students
1	Kharkiv National University of Radio Electronics	2,968	1	Blockchain-Hub Academy	30	10	Main Academy	6,000
2	Lviv Polytechnic National University	2,675	2	BrainBasket Foundation	6,000	11	Projector	1,500
3	National Technical University of Ukraine “Ihor Sikorsky Kyiv Polytechnic Institute”	4,314	3	CyberBionic Systematics	3,000	12	Prome-theus	700,00
			4	GoIT	2,000	13	QALight	8,000
4	National Technical University “Kharkiv Polytechnic Institute”	2,105	5	“STEP” computer academy	95,000	14	Sigma Software University	600
5	National University “Kyiv-Mohyla Academy”	3,500	6	UNIT Factory	900	15	Ukrainian IT School	1,600
6	Taras Shevchenko National University in Kyiv	1,324	7	ITEA	11,000	16	SkillUP	18,850
			8	uData Schoo	90	17	iTalent	6,000
7	Zhukovsky National Aerospace University “Kharkiv Aviation Institute”	1,692	9	LITS (Lviv IT School)	2,000	18	SoftServe IT Academy	1,600
	SUM	*18,578*		*SUM*				*864,170*

and economists with analytical thinking and basic knowledge of programming can apply for this position (Fig. 2).

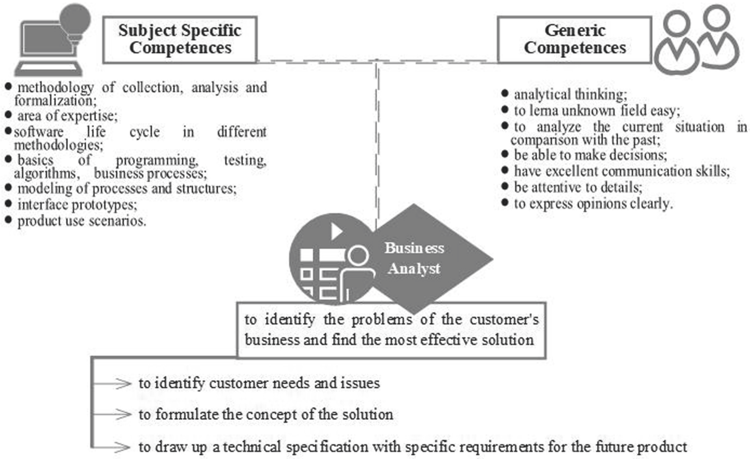

Fig. 2. Basic characteristics of a business analyst

As a consequence, graduates of three majors can work as business analysts: systems analysis (124), economics (051, specialization: economic cybernetics), and information systems and technologies (126).

2.2 Compare Analysis of BA Skills by IIBA and Ukrainian Education System

The importance of BA specialists, within IT and Industry 4.0 development, is difficult to overestimate. However, even developed countries [7, 8] and most corporations [9, 10] feel unmet demand from highly skilled specialists, especially in the field of STEM. As a result, the term gap skills have even emerged, which the American Society for Training & Development (ASTD) describes as “a significant gap between an organization’s current capabilities and the skills it needs to achieve its goals” [9]. The scale of this problem is underlined by the fact that gap skills have become inherent both for the micro-level and the labor market, which is a significant barrier to the new technologies [10] and thus can create a threat to the national economy. One of the reasons for the gap in skills is the gap in education and the real requirements of the labor market, which is especially evident in the rapidly growing IT sector [11, 12].

International Institute of Business Analysis (IIBA) identifies 6 roles of BA naming various skills, knowledge, and experience of each of them [13]. Among soft skills in demand, almost all of them are in Generic Competencies of higher education standards of Business Analyst in Ukraine and Ukrainian labor market except facilitation skill and ability to participate on multi-disciplinary work teams, which should be considered in detail (Table 2).

Comparing hard skills by IIBA, Ukrainian labor market requirements, and Specific Subject Competencies (Table 3), we can mention that in Ukraine BA focuses mainly on IT skills and knowledge, whereas abroad it includes strong business skills (Business rules analysis, Costs benefit analysis, SME, KPI, etc.).

Table 2. Compare analysis of BA soft skills by IIBA and Ukrainian education system [13].

<table>
<tr><th>Requirements</th><th>Business Requirements Analyst</th><th>Business Systems Analyst (BSA)</th><th>Systems Analyst</th><th>Functional Analyst</th><th>Service Request Analyst</th><th>Agile Analyst</th></tr>
<tr><td rowspan="5">Education</td><td colspan="2">Undergraduate Degree</td><td></td><td>Undergraduate Degree</td><td></td><td>Undergraduate Degree</td></tr>
<tr><td colspan="2">College Diploma</td><td></td><td colspan="2">College Diploma</td><td></td></tr>
<tr><td></td><td>Certificate</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="2">Degree</td><td></td><td colspan="2">Degree</td></tr>
<tr><td></td><td></td><td></td><td></td><td colspan="2">Work experience</td></tr>
<tr><td rowspan="6">Soft skills</td><td colspan="2">Good communications skills (oral and written)*</td><td rowspan="6"></td><td>Effective work with diverse groups of people</td><td rowspan="6"></td><td rowspan="6"></td></tr>
<tr><td colspan="2">Good interpersonal and consultative skills</td><td rowspan="5">Ability to participate on multi-disciplinary work teams effectively</td></tr>
<tr><td colspan="2">Facilitation skills</td></tr>
<tr><td colspan="2">Analytical thinking and problem solving</td></tr>
<tr><td colspan="2">Detail oriented and committed to a high level of accuracy</td></tr>
<tr><td colspan="2">Organizational skills</td></tr>
</table>

* marked skills are common both for Ukrainian high education standards, Ukrainian labor market and research of IIBA

This is confirmed by many studies as well: C. Bullen and et al. [14] identified project and business skills as more important than technical; A. Aken and M. Michalisin established 4 skills types Soft, Technical, Programming, and Business [15]; T. Chakabuda said employers deem business interpersonal competency as one of the most important [16]. That is why the educational reform in Ukraine should take into account foreign experience of the business analysis sphere including the economic part.

There are different ways to limit the skills gap: certification of employees, skills certificates, on-the-job training, apprenticeship programs, training, and post-secondary education, etc. [9]. However, the last option is not highly effective, especially in the IT sector compare with work experience [7]. Instead, the improvement of BA skills should include two components: basic – update educational program at universities jointly with business, public sector, other stakeholders, and variable – practical courses provided by employers at the workplace, since “82% of organizations are increasing investments in reskilling their workforce” [10]. It requires common efforts from all parties, because “Businesses are beginning to realize the importance of collaborating with local education institutions, and vice versa” [9]. For instance, the Business analysis competency model by IIBA includes 3 components “knowledge, experience and choice of behavior, these together consistently create success” [13]. However, these items can not be developed independently, but only in cooperation (Fig. 3).

Table 3. Compare analysis of BA soft skills by IIBA and Ukrainian education system [13].

Business Requirements Analyst	Business Systems Analyst	Systems Analyst	Functional Analyst	Service Request Analyst	Agile Analyst
Knowledge of the BABOKR Guide					+ IIBA Agile Extension
Knowledge of business structure	Modeling notation (UML, Structured, BPMN)		Modeling: process, data, systems (UML, BPMN)	System enhancement requests elicitation, analysis, documentation and implementation	Modeling: data, processes
Business rules analysis	Knowledge of software development lifecycle (SDLC)		Business rules analysis		Time boxing, Estimating
Modeling processes using various notation (BPMN, Workflows) *	Models systems and data to a logical level, ...	Models systems and to a physical level, …	Experience in requirements elicitation, fit/ gap analysis, and preparation of functional design documents	System evaluation studies utilizing data analysis to identify quality/ performance improvements to system	Knowledge of Scrum and/or XP, Lean and Kanban Software Development
	user interface design, and prototyping, decision tables/ trees, data flows				
Stakeholder analysis	Writing unambiguous, verified requirements to a testable level		Subject matter expertise (SME) in the area served by the software (e.g. finance, HR, supply chain)		DSDM
Knowledge of technology structures: networks, database, internet, communication	Knowledge of technology system interfaces and data integration; networks, operating systems	Data extraction, ELT DBMS, output (report) design, data queries	Experience identifying opportunities for process and system improvements either in existing designs or by adopting new functionalities in the software		Feature Driven Development (FDD)
Data analysis		Knowledge of technology system interfaces, application architecture and data integration; networks, operating systems, commercial-off-the-shelf software, web architecture	Facilitating organizational change management plans/ strategies, tactics and approaches with different business units		User Stories
Decision analysis			Measures and Key Performance Indicators (KPI)		Prototyping
Costs benefit analysis			Provide SME content documentation for training material		
Organization modeling			Organizational readiness assessment		
Requirements workshops/ interviews/ observation					

* marked skills are common both for Ukrainian high education standards, Ukrainian labor market and research of IIBA

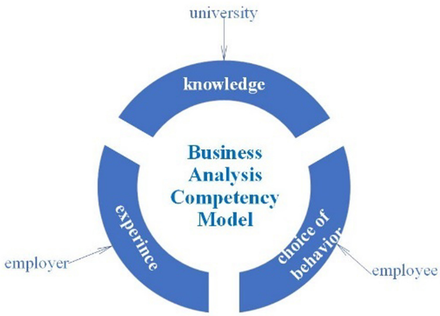

Fig. 3. Model of BA skills improvement

3 Experimental Model

3.1 The Gaps Between the Requirements of the Labor Market and the Educational Standards

Domestic higher education institutions in business analysts training should develop an educational program and frame educational process to form generic (GC) and subject specific competencies (SC) of graduates in a line with standards of the Ministry of Education and Science of Ukraine in System Analysis (124), Economics (051, economic cybernetics), Information Systems and Technologies (126) [17].

We have carried out a comparative analysis of generic and subject specific competencies according to the Ministry of Education and Science of Ukraine for majors 051, 124, 126 essentials to become a business analyst. As a result, we obtained 18 common generic (Table 4) and 22 common subject specific competencies (Table 5).

Table 4. Generic Competencies of business analyst by higher education institutions

№	Generic competencies for business analyst
1	Ability to be critical and self-critical
2	Ability to learn and become proficient in modern knowledge
3	Ability to generate new ideas (creativity)
4	Ability to act socially responsible and consciously
5	Ability to abstract thinking, analysis and design
6	Ability to adapt and act in a new situation
7	Ability to search, process and analyze information from various sources
8	Ability to apply knowledge in practical situations
9	Ability to evaluate and ensure the quality of work performed
10	Ability to plan and manage time

(*continued*)

Table 4. (*continued*)

№	Generic competencies for business analyst
11	Ability to work independently
12	Ability to work in a team
13	Ability to make informed decisions
14	Ability to develop and manage projects
15	Ability to communicate in the official language orally and in writing
16	Ability to communicate in a foreign language and work in an international context
17	Information and communication technology skills
18	Interpersonal skills

Table 5. Subject specific competencies of business analyst by higher education institutions

№	Subject specific competencies of a business analyst
1	Ability to identify knowledge and understand the problems of the subject area, the basics of the modern economy at the micro, meso, macro and international levels
2	Ability to explain economic and social processes and phenomena through theoretical models, to analyze and interpret the results
3	Ability to formalize problems described in natural language, through mathematical methods as well, to apply common approaches to mathematical modeling of specific processes
4	Ability to build correct models of static and dynamic processes, and systems with distributed and lumped parameters, taking into account the uncertainty of external and internal factors
5	Ability to use modern information technology to implement machine-assisted realization of mathematical models and predict behavior of specific systems, namely: object-oriented approach in the design of complex systems of different types, applied mathematical packages, use of databases and knowledge
6	Ability to identify the main impact factors of the development of physical, economic, social processes, pick out stochastic and indeterminate indicators, formulate them in random or fuzzy quantities, vectors, processes and to study the dependencies between them
7	Ability to analyze and design complex systems, crate relevant information technologies and software
8	Ability to design experimental and observational studies and analyze the results
9	Ability to analyze, synthesize and optimize information systems and technologies using mathematical models and methods

(*continued*)

Table 5. (*continued*)

№	Subject specific competencies of a business analyst
10	The ability to perform simulation experiments, to compare the results of experimental data and the solutions obtained
11	Ability to use computer technology and data processing software to solve economic problems, analyze information, and prepare analytical reports
12	Ability to analyze and solve problems in the field of economic and social-labor relations
13	Ability to predict socio-economic processes based on standard theoretical and econometric models
14	Ability to use modern sources of economic, social, management, accounting information to prepare official documents and analytical reports
15	Ability to conduct economic analysis of the operation and development of business entities, assess their competitiveness
16	Ability to identify economic problems in the analysis of specific situations, to offer ways to solve them independently
17	Ability to formulate optimization problems in the design of systems of management and decision making, namely: mathematical models, optimality criteria, constraints, management goals, choose rational methods and algorithms for solving optimization and optimal management
18	Ability to apply information technologies to create, implement and utilize quality management system and estimate the costs of its development and maintenance
19	The ability to manage the quality of products and services of information systems and technologies throughout their life cycle
20	Ability to develop business decisions and evaluate new technology offers
21	Ability to manage and use modern information and communication systems and technologies (including Internet based)
22	Ability to create new competitive ideas and implement them in projects (startups)

We have analyzed the general requirements of employers for the competencies of applicants for the position of business analyst on the sites for job search: work.ua, rabota.ua, djinni.co, linkedin.com, hh.ua, it-stars.ua, jobs.ua. Then, we have compared them with standards of the Ministry of Education and Science of Ukraine (Tables 4 and 5). As a result, we identified generic (Table 6) and subject specific (Table 7) competencies common both for the labor market and higher education institutions.

Table 6. Common generic competencies of a business analyst for higher education institutions and labor market

GC	GC by higher education institutions	GC by labor market
GC1	Ability to learn and become proficient in modern knowledge	Desire to learn
GC2	Ability to generate new ideas (creativity)	Creativity
GC3	The ability to act socially responsible and consciously	Responsibility
GC4	Ability to abstract thinking, analysis and design	Analytical and logical thinking, systems thinking
GC5	Ability to search, process and analyze information from various sources	Attention to detail
GC6	Ability to apply knowledge in practical situations	Problem solving
GC7	Ability to plan and manage time	Time management
GC8	Ability to work in a team	Teamwork
GC9	Ability to develop and manage projects	Organizational skills
GC10	Ability to communicate in a foreign language and work in an international context	English skills
GC11	Information and communication technology skills	Presentation skills
GC12	Interpersonal skills	Communication
GC13	NA*	Self-motivation

*NA – not announced

Based on open Internet sources of websites for job search, a data set of 118 vacancies for the position of the business analyst was created. This data set includes the name of the employer, vacant position the average salary, and required generic and specific subject competencies (Table 8), and wage distribution of business analysts (Fig. 4).

All vacancies can be classified by the following industries: IT consulting (48), Banks (23), Retail (12), Government Institution (6), Others (29) (include mobile operators, agriculture, oil, etc.).

To determine the relevance of the impact of generic and specific subject competencies on the average wage, we consider a multiple regression model:

$$w_i = b_0 + \sum_{j=1}^{13} GC_j + \sum_{k=1}^{15} SC_k + u_i \tag{1}$$

Table 7. Common subject specific competencies of a business analyst for higher education institutions and labor market

SC	SC by higher education institutions	SC by labor market
SC1	Ability to explain economic and social processes and phenomena through theoretical models, to analyze and interpret the results	Development of use-cases and user-stories
SC2	Ability to formalize problems described in natural language, through mathematical methods as well, to apply common approaches to mathematical modeling of specific processes	Requirements collection, negotiations with stakeholders, UML/BPMN
SC3	Ability to use modern information technology to implement machine-assisted realization of mathematical models and predict behavior of specific systems, namely: object-oriented approach in the design of complex systems of different types, applied mathematical packages, use of databases and knowledge	Hands-on experience with data visualization via reports and dashboards, Flow charts, Lucidchart MS Access, MS SQL Server, Oracle
SC4	Ability to analyze and design complex systems, crate relevant information technologies and software	Experience in the development of technical documentation, requirements, software development processes (UML, Use Cases, Business Rules, Functional Non-Functional Specifications, User Interface Design Specifications, User Stories, Backlogs)
SC5	Ability to analyze, synthesize and optimize information systems and technologies using mathematical models and methods	Information systems
SC6	The ability to perform simulation experiments, to compare the results of experimental data and the solutions obtained	Data analysis, Database Management System, DBMS
SC7	Ability to use computer technology and data processing software to solve economic problems, analyze information, and prepare analytical reports	BA technics (interview, workshop, document analysis, estimation, mind mapping, etc.)
SC8	Ability to use modern sources of economic, social, management, accounting information to prepare official documents and analytical reports	Management accounting, corporate finance and financial statements/1C

(*continued*)

Table 7. (*continued*)

SC	SC by higher education institutions	SC by labor market
SC9	Ability to formulate optimization problems in the design of systems of management and decision making, namely: mathematical models, optimality criteria, constraints, management goals, choose rational methods and algorithms for solving optimization and optimal management	Algorithms, data structures, client-server application architecture, web application architecture, service-oriented architecture (SOA)
SC10	Ability to apply information technologies to create, implement and utilize quality management system and estimate the costs of its development and maintenance	CRM
SC11	The ability to manage the quality of products and services of information systems and technologies throughout their life cycle	Project management, Software Development, Software Development Life Cycle and MVP
SC12	Ability to develop business decisions and evaluate new technology offers	Business processes modeling, MS Visio
SC13	Ability to manage and use modern information and communication systems and technologies (including Internet based)	MS Access, MS Excel, MS Power Point
SC14	Ability to create new competitive ideas and implement them in projects (startups)	JIRA Confluence, MS Project
SC15	NA*	Understanding of agile development processes (e.g. Scrum, SDLC, Kanban)

*NA – not announced

where w_i – the average wage for a job i, GC_j – generic competence j, SC_k – specific subject competence k, u_i – error term. The following code snippets in R are used to calculate the regression parameters:

```
f <- read.csv("HEI and LM.txt", sep="\t", header=TRUE,
dec=".")
model1 <- lm(data=f, Wage~.)
summary(model_1)
f2<- read.csv("IT consulting.txt", sep="\t",
header=TRUE, dec=".")
model2 <- lm(data=f2, Wage~.)
summary(model2)
```

It has been obtained that by the sampling of all positions of business analysts:

1) use of information and communication technologies or Presentation Skills (GC11) can increase the average monthly wage by +UAH9361.9;

Table 8. Vacant position profile of business analyst competencies on the labor market

№	Employer	Position	Monthly wage, UAH	GC1	…	GC13	SC1	…	SC15
1	IT Specialist, Ltd	System analyst	35000	0	…	0	0	…	0
2	INNOWARE	Junior Business Analyst	35000	0	…	1	0	…	0
3	Proxima International	Business Analyst	50225	0	…	0	0	…	0
4	Betinvest Ltd	Business Analyst	18000	0	…	0	0	…	1
5	Linkos Group	System analyst	18000	0	…	0	0	…	0
6	Deep Consulting Solutions	Business Analyst	36000	1	…	1	0	…	0
7	PMLAB	Data/Business Analyst	44000	0	…	0	0	…	0
8	Paymentwall	Business Intelli-gence Analyst	35000	0	…	0	0	…	0
9	EPAM/Epam Systems	Senior Business Analyst	31000	1	…	0	0	…	0
…	…	…	…	…	…	…	…	…	…
118	Lifecell	System analyst	30000	0	…	0	0	…	0

2) the ability to apply knowledge in practical situations or Problem Solving (SC6) causes an average wage increasing by UAH 10009 (Table 9).

Among BA job applicants with GC11 competence; only 10% have SC6 specific subject competence simultaneously, and vice versa: if a candidate has SC6 competence, only every tenth has GC11 competence using library ggplot2 and following function in R (Fig. 5):

```
mosaic(data=f, ~GC11+SC6, shade=TRUE)
```

For the IT industry, the most important competencies were revealed as follow:

1) SC8 Management Accounting, Corporate Finance, and Financial Statements/1C determined an average monthly wage increase of +UAH 23376;

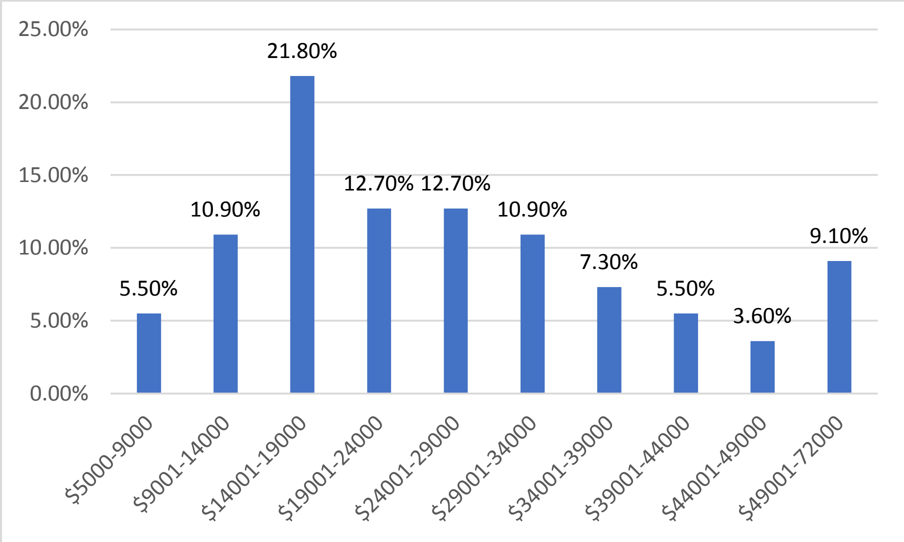

Fig. 4. Wage distribution of business analysts, UAH (2020)

Table 9. Statistically significant Competencies for the labor market

Model	Explanatory competencies	Marginal effect parameters (UAH)	$R^2(\%)$
All	GC11 SC6	+9361.9 +10009	22,56
IT consulting	SC8 SC14	+23376 −9062	23,83
Banks	NA	NA	NA
Retail	NA	NA	NA
Others	SC13 SC14	+29896 +36683	87,3

2) SC14 The ability to form new competitive ideas and implement them in projects (start-ups) for graduates does not meet the requirements of the IT industry (JIRA Confluence, MS Project), and therefore leads to a decrease in salary by UAH 9062, which confirms the lack of competence in the HEA.

Among BA job applicants with SC8 competence, only 20% have SC14 specific subject competence at the same time. If the applicant has SC14 competence, only 10% have SC8 competence. Among BA job applicants with GC11 competence, only 10% have SC6 specific subject competence, and vice versa: if a candidate has SC6 competence, only every tenth has GC11 competence (Fig. 6):

```
mosaic(data=f2, ~SC8+SC14, shade=TRUE)
```

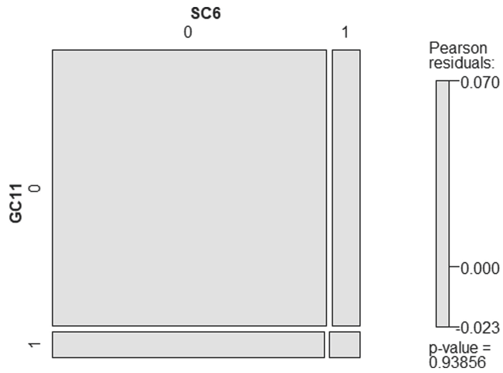

Fig. 5. Relationship between GC11 and SC6 competencies

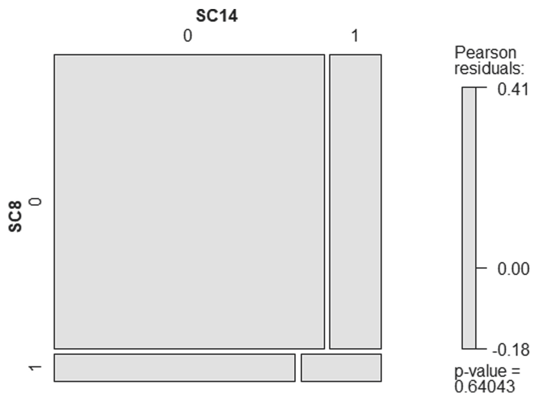

Fig. 6. Relationship between SC8 and SC14 competencies

As well as if the applicant has SC8 competency for the BA vacancy, his/her average wage remains lower than in the case of other competencies required:

`g2 + facet_grid(SC8~SC14)` (fig. 7)
`gg0 + stat_smooth(method="lm") + facet_grid(~SC14)` (fig. 12)

SC8's competence slightly increases the average wage, while in its absence the wage increases much faster. Therefore, the skills of an accountant in the labor market are worth much less than the skills of a business analyst (Fig. 8).

For other positions not covered by IT consulting, Banks Government Institutions, the most required competencies were revealed:

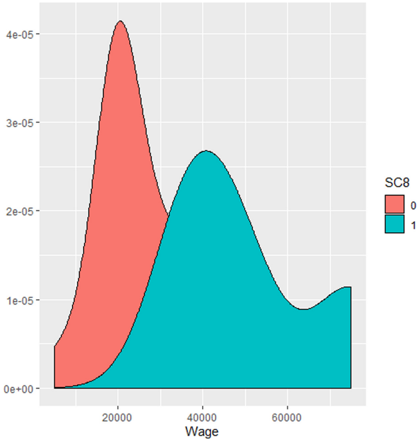

Fig. 7. Wage distribution for SC8 competence

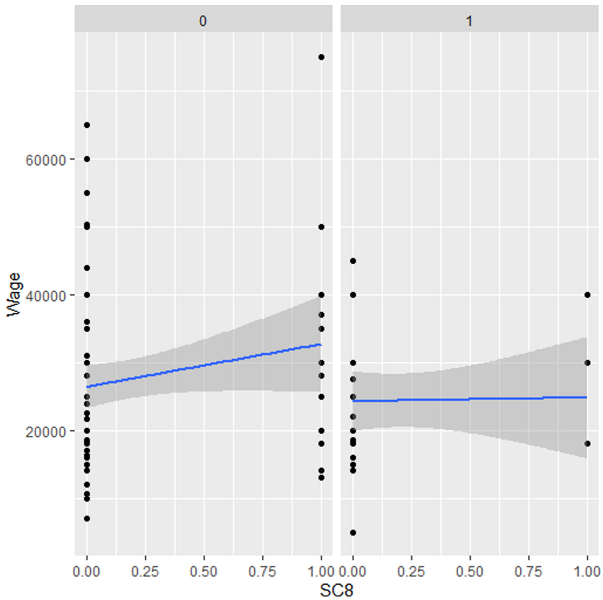

Fig. 8. Wage dynamics in the absence and availability of SC8 competence

1) SC13 Ability to manage and use state-of-the-art information and communication systems and technologies (including Internet-based ones, MS Access, MS Excel, MS PowerPoint) adds +UAH 29996 each month;

2) SC14 The ability to form new competitive ideas and implement them in projects (startups), JIRA Confluence, MS Project proves that this competence is formed for other industries and raises wages by an average of UAH 36683.

Thus, the wage distribution is set out as follows (Fig. 9), where the average wage per year is UAH 20,000 and has a significant potential to increase when additional required specific subject competencies in the labor market are presented.

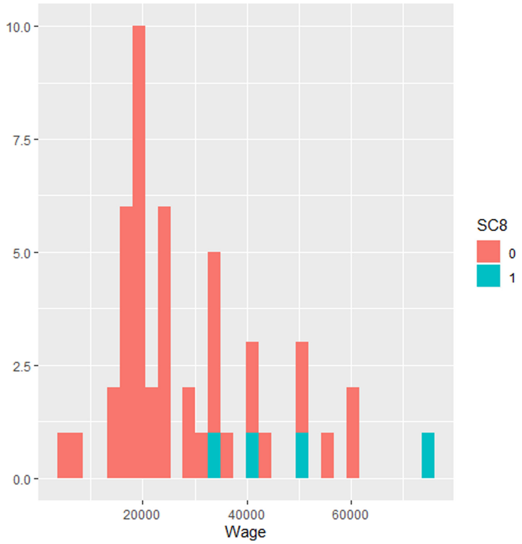

Fig. 9. Wage distribution for BA position

A system of certificates can be an effective tool to overcome a gap between educational programs [18–21] and employers' requirements [22–25]. This system should be developed jointly by university representatives and companies and will certify the students' competence in certain highly specialized fields of knowledge and confirm the quality of the education obtained through professional qualifications (Fig. 10).

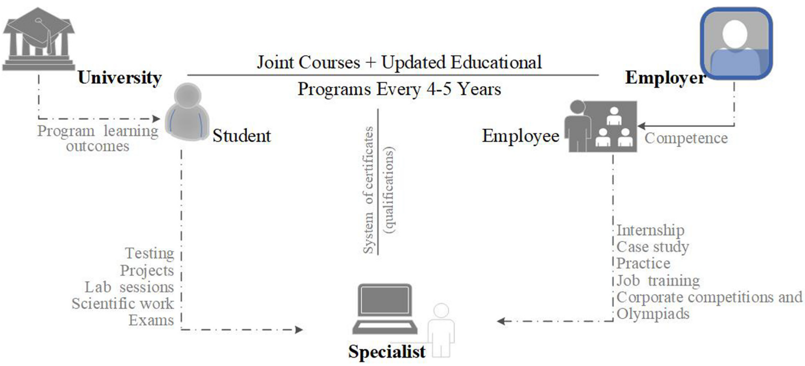

Fig. 10. Scientific-educational-practical complex of students teaching at IT majors

3.2 The Complementarities Between the Requirements of the Labor Market and the Educational Standards

Due to the results of the analysis of 325 vacancies from 107 companies in open Internet sources in the field of business intelligence (IT-consulting/Services/Equipment Manufacturing) (45%), Banks (20%), Retail (11%), Governmental institutions (6%) and Others (19%)) demand for soft skills (general competencies) in percentage is determined by the following distribution (Fig. 11).

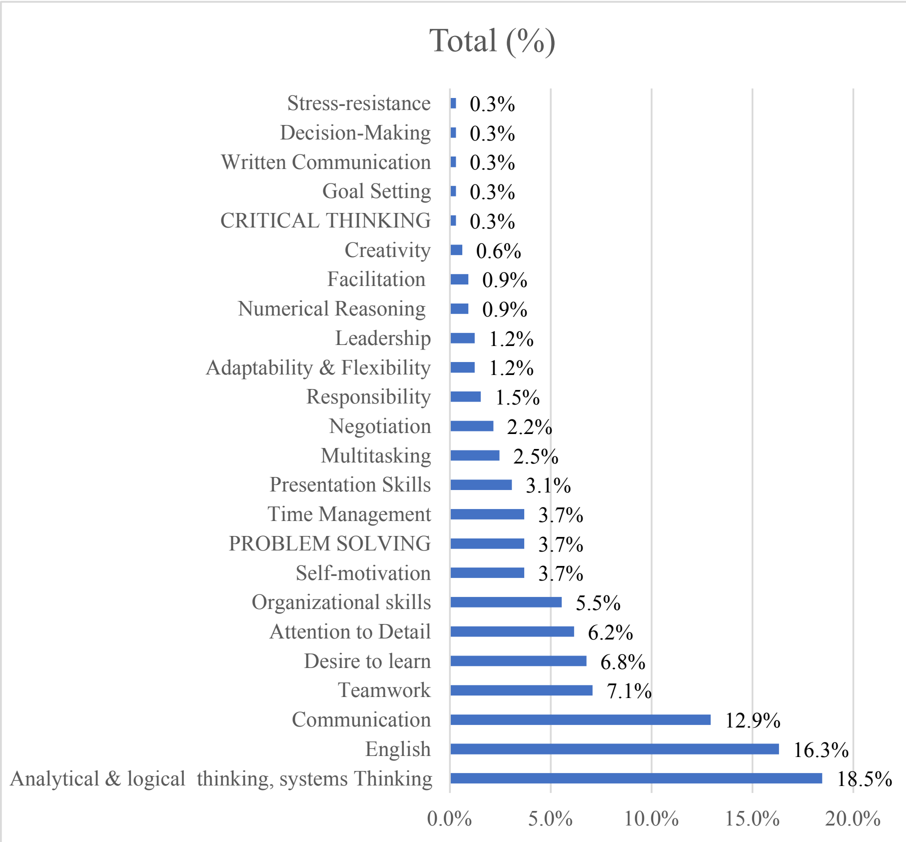

Fig. 11. The share of required soft skills for business analysts (%)

The most demanded soft skills (general competences) are: GC_4 Analytical and logical thinking, systems thinking (18,5%), GC_{10} English (16,3%), GC_{12} Communication (12,9%), GC_8 Teamwork (7,1%), GC_1 Desire to learn (6,8%), GC_5 Attention to Detail (6,2%).

At the same time, more than half of vacancies for business analysts (55%) are in demand in the field of IT-consulting/Services/Equipment Manufacturing (Fig. 12). The most important soft skills are English (17.9%), Analytical and logical thinking, systems thinking (12.8%), Communication (12.3%), Teamwork (7.3%), Attention to Detail (7, 3%), Desire to learn (7.3%).

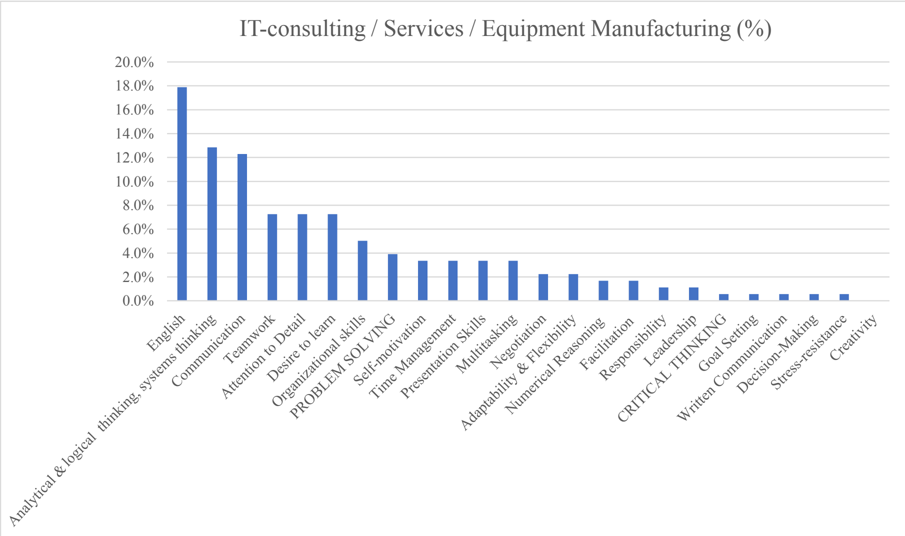

Fig. 12. Distribution of demanded soft skills for business analysts in the field of IT-consulting/Services/Equipment Manufacturing (%)

Some competencies are formed in students during higher education, others during full or part-time employment in the firms. With experience, employees develop more specific subject competencies in demand in the labor market. Combinations of these competencies create vacancies in the labor market. Formation of some competencies requires both higher education and experience in the firms.

According to the results of data analysis revealed statistically significant regression parameters between the demand for competence and the average years of schooling in higher education institutions ($y = b_0 + b_1 \cdot x_1$) and between the demand for competence and work experience (applicant's experience) ($y = c_0 + c_1 \cdot x_2$) based on open data (Table 10), where b_1 shows how the number of vacancies with this competence will change with the increasing length of studies by one year; c_1 shows how the number of vacancies with this competence will change with increasing experience by one year. In parentheses near to each competence included in the vacancy, specific subject competencies in educational programs (standards) are indicated, if any.

Simple regression models in Table 10 show that the applicant is more likely to get a job due to work experience, rather than higher education. Simple regression is more relevant in this case than multiple one due to forming of some competences only under impact of higher education or only under work experience. Thus, the formation of the competence ‘Experience in the development of technical documentation, requirements, software development processes’ due to the additional year of study at the HEI allows to increase the number of vacancies by 4, while the formation of this competence due

Table 10. Formation of competencies under the influence of higher education and work experience

Specific subject competences (y), numbers required competences for different job positions	**Formed under the impact of higher education (x_1), years**	**Formed under the impact of work experience (x_2), years**
UML/BPMN (SC_2)	☑ $y = 1.3 + 2.8 \cdot x_1$ ($R^2 = 86\%$)	☑ $y = -13.3 + 9.6 \cdot x_2$ ($R^2 = 74\%$)
Experience in the development of technical documentation, requirements, software development processes (SC_4)	☑ $y = 3.9 + 4.0 \cdot x_1$ ($R^2 = 60\%$)	☑ $y = -19.2 + 14.8 \cdot x_2$ ($R^2 = 61\%$)
Understanding of agile development processes (e.g. Scrum, SDLC, kanban) (SC_5)	☑ $y = -0.1 + 3.4 \cdot x_1$ ($R^2 = 80\%$)	☑ $y = -19.5 + 12.4 \cdot x_2$ ($R^2 = 82\%$)
Business Process Improvement, Business Process Reengineering	☑ $y = -0.6 + 1.1 \cdot x_1$ ($R^2 = 58\%$)	☑ $y = -8.4 + 4.8 \cdot x_2$ ($R^2 = 77\%$)
Hands-on experience with data visualization via reports and dashboards, Flow charts, Lucidchart (SC_5)	☑ $y = 0.1 + 0.9 \cdot x_1$ ($R^2 = 70\%$)	☑ $y = -5.0 + 3.3 \cdot x_2$ ($R^2 = 68\%$)
JIRA Confluence (SC_{14})	☑ $y = 1.1 + 1.6 \cdot x_1$ ($R^2 = 86\%$)	
Development of use-cases and user-stories (SC_1)	☑ $y = -0.9 + 1.7 \cdot x_1$ ($R^2 = 62\%$)	☑ $y = -11.5 + 6.7 \cdot x_2$ ($R^2 = 71\%$)
Working with database, database management systems (SC_6)	☑ $y = -0.6 + 1.1 \cdot x_1$ ($R^2 = 58\%$)	☑ $y = -8.4 + 4.8 \cdot x_2$ ($R^2 = 76.9\%$)
Algorithms, data structures, client-server application architecture, web application architecture, service-oriented architecture (SC_9)	☑ $y = -0.02 + 1.6 \cdot x_1$ ($R^2 = 78\%$)	☑ $y = -8.4 + 5.5 \cdot x_2$ ($R^2 = 71\%$)
Software Development, Software Development Life Cycle and MVP (SC_{11})		☑ $y = -9.1 + 6.1 \cdot x_2$ ($R^2 = 76\%$)
Requirements collection, negotiations with stakeholders (SC_2)	☑ $y = -0.5 + 1.7 \cdot x_1$ ($R^2 = 66\%$)	
BA technics (interview, workshop, document analysis, estimation, mind mapping, etc.) (SC_7)		☑ $y = -5.4 + 3.9 \cdot x_2$ ($R^2 = 81\%$)
Web application architecture for IOS, Android (SC_9)	☑ $y = -0.3 + 1.1 \cdot x_1$ ($R^2 = 65{,}8\%$)	☑ $y = -7.5 + 4.4 \cdot x_2$ ($R^2 = 86\%$)
API, Rest Api, Corezoid	☑ $y = -0.3 + 0.8 \cdot x_1$ ($R^2 = 78\%$)	☑ $y = -4.8 + 2.9 \cdot x_2$ ($R^2 = 73\%$)
Managerial Accounting, corporate finance and financial statements / 1C (SC_8)		☑ $y = -6.0 + 4.7 \cdot x_2$ ($R^2 = 81\%$)
Data analysis, Database Management System, DBMS (SC_6)		☑ $y = -5.7 + 3.9 \cdot x_2$ ($R^2 = 95\%$)
MS Visio (SC_{12})	☑ $y = 2.2 + 0.6 \cdot x_1$ ($R^2 = 55\%$)	
Testing, including SOAP UI	☑ $y = 0.7 + 1.0 \cdot x_1$ ($R^2 = 67\%$)	
Waterfall philosophy		☑ $y = -2.9 + 1.9 \cdot x_2$ ($R^2 = 55\%$)

to the additional year of experience increases the number of vacancies on average by 15. The chances to fill a vacancy with the formed competence 'Understanding of agile development processes in higher education and due to work experience is for each additional year 3.4 against 12.4, respectively.

If the formation of competencies requires both higher education and work experience, then according to Table 10, greater opportunities for the applicant open up through work experience than through higher education by an average of 3.4 times. The calculations are made based on coefficients of explanatory variables for both regressions of the Table 10 (x_i, $i = 1, 2$).

According to the results of the analysis of the relevance of higher education and work experience, the following results were obtained:

Table 11. Relevant factors that form the competencies of applicants

Specific subject competences	Number (%) of competencies
Competences that are formed both under the impact of higher education and work experience	10 (52.6%)
Competences that require higher education and are not affected by experience	4 (21.1%)
Competences that require experience and are not affected by higher education	5 (26.3%)

Table 11 demonstrates that the formation of business analysts' competencies required in the labor market is due to higher education (47%) and the experience of applicants (53%).

4 Conclusions

For the IT industry, the most important competencies were revealed as follow: Management Accounting, Corporate Finance, and Financial Statements determined an average monthly wage increase of +UAH 23376; the ability to form new competitive ideas and implement them in projects (start-ups) for graduates does not meet the requirements of the IT industry (JIRA Confluence, MS Project), and therefore leads to a decrease in salary by UAH 9062, which confirms the lack of competence in the HEA.

For other positions not covered by IT consulting, Banks Government Institutions, the most required competencies were revealed: the ability to manage and use state-of-the-art information and communication systems and technologies (including Internet-based ones, MS Access, MS Excel, MS PowerPoint) adds +UAH 29996 each month; the ability to form new competitive ideas and implement them in projects (startups), JIRA Confluence, MS Project prove that this competence is formed for other industries and raises wages by an average of UAH 36683. Formation of business analysts competencies required in the labor market is created both higher education (47%) and experience of applicants on the labor market (53%).

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Using Mobile ICT for Online Learning During COVID-19 Lockdown

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Abstract. The research is aimed at theoretical substantiation, development and experimental verification of methods of applying mobile technologies to university students training during the COVID-19 lockdown. The research objective implies adapting mobile ICT to online learning during the COVID-19 lockdown. There are analyzed Ukrainian and foreign researches into the issues of mobile ICT application to the University students training. The authors have developed methods of applying Audience Response Systems by taking Plickers as an example; Mobile Multimedia Authoring Tools development through using augmented reality; Mobile Learning Management Systems by taking Moodle; Mobile Modeling and Programming Environments through using Pydroid; Mobile Database Management Systems by taking Google Cloud SQL. There is comparative assessment of functionality of five systems of mobile ICT for online learning during the COVID-19 lockdown. Efficiency of the developed technology is experimentally verified and confirmed.

Keywords: Online learning · Mobile ICT · Mobile Learning Management System · Mobile Modeling and Programming Environments · Mobile Database Management Systems · Mobile Multimedia Authoring Tools · Audience Response Systems · COVID-19 lockdown

1 Introduction

There are developed and introduced new strategies of online teaching and learning, support and service structures, which often call for system-related changes at universities. Relying on central elements of distance learning in practice and theory, the authors [6] apply new opportunities of ICT – from multimedia training objects to social, combined media and the virtual environment – to online learning. L. M. Blaschke and S. Bedenlier

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elaborated a teaching guide and practical instructions for teachers to develop and carry out educational software and courses online.

Online tools [33] are a means of improving qualifications of university teachers and instructors of higher vocational education. The tools are aimed at improving teachers' abilities to use innovative teaching methods and develop syllabuses for online learning on the basis of digital training materials and tools including open educational resources.

L. Mishra, T. Gupta and A. Shree [31] note that the educational system, from the primary to the higher one, changed during the COVID-19 lockdown. The research presents qualitative and qualitative approaches to studying teachers' and students' perception of online learning. The authors focus on overcoming obstacles in learning and enhancing online academic activity as a standard procedure within the educational system.

The authors [11] accentuate the fact that wrong ergonomics of a working place including that at home hides long-term consequences for health. During the COVID-19 lockdown, students stayed at home, this leading to their dependence on online classes, workshops and meetings. The research conducted evaluates impacts of ergonomic conditions on students who have to spend more time with gadgets. The research reveals that both students and teachers were not aware of ergonomics and did not observe ergonomic rules during the two-month lockdown. As a result, many of them developed serious problems that might cause a variety of health complications in future. There arises a necessity to include the issues of ergonomics and working place arrangement into the curricula.

- Many countries have issued the documents to regulate academic activity during the COVID-19 lockdown, including:
- Guidance for all schools in England "Restricting attendance during the national lockdown: schools" [34];
- Guidance on distance learning modalities to reach all children and youth during school closures by UNICEF Regional Office for South Asia [19];
- Helping children and young people while they are learning at home by New Zealand Ministry of Education [22];
- Teaching and learning during the quarantine: changes in the educational system by the Ministry of Education and Science of Ukraine [48];
- Coronavirus: online learning resources: Discover a selection of online resources and tools for learners, teachers and educators during the outbreak of COVID-19 by European Commission [14];
- Colleges, Universities, and Higher Learning: Plan, Prepare and Respond – COVID-19 by U.S. Department of Health & Human Services [12];
- Coronavirus (COVID-19) information for higher education providers by Department of Education, Skills and Employment of Australian Government [13].

Application of mobile technologies in online education was the issue for investigation in works by M. E. Jantjies [24] and F. Fotouhi-Ghazvini [17] (in teaching languages); S. O. Semerikov [40] (in teaching Computer Sciences); A. Abu-Al-Aish [1], S. S. Baharom [3], P. W. Bird [5], C. Billington [4], M. Hepburn [23], J. J. Trinder [47] and E. A. Wali [49] (in the tertiary education system); Y. Shao [41] (in open education); V. Jotham [25] and N. N. Chan [8] (in everyday life).

ICT alters both teaching and learning by adding active elements to the academic environment and is considered as a powerful tool for providing academic services. Nowadays, researchers think that the educational system is capable of improving only when ICT is implemented. At schools, colleges and universities, the staff use technology to execute their duties including technical, office, managerial, administrative, etc. Computer literacy is one of the basic skills when applying for a lecturer's position besides teaching qualifications and skills. Thus, we can state that ICT has contributed much to improving general conditions of the academic environment [26].

ICT cover all aspects of social life and play an important part in education by motivating, encouraging acquisition of basic skills and changing the academic environment, thus improving education quality. ICT possesses a great educational potential through making teachers updated and dynamic. It also enriches students' educational experience and promotes their future career in the technological world [43].

In the paper [10] presents the results of comparative analysis of the academic content designed for mobile devices, special attention being paid to the issues affecting mobile learning, i.e. its duration, systemacy, complexity, aims and structure. It is proven that the mobile content is better structured than the content of other educational media and a critical evaluation is offered for the content type to be developed.

Research [39] aims to provide quantitative analysis of ICT's impacts on students' mobility and motivation in the professional context by various criteria: the level of implementation and practicality of mobile technologies; an organizational system; a degree of mobility, motivation and application of ICT to the professional context, individual efficiency. The results reveal that creation of the organizational system with ICT enables increase of mobility. Besides, the organizational system that enhances mobility at work also increases motivation.

In paper [45] we have analyzed the latest publications on the following tools:

- Mobile Learning Management Systems (L. Sanz-Martínez et al. [38], V. Bykov [7], I. Han and W. S. Shin [21]);
- Mobile Modeling and Programming Environments (B. Altayeb and K. Damevski [2], F.-J. Lordan Gomis [28]);
- Mobile Database Management Systems (D.M. Gezgin [18], M. Guo et al. [20], M. Eric et al. [16]);
- Mobile Multimedia Authoring Tools (Yu. Yechkalo et al. [50], D. Tjondronegoro [44], A. Striuk et al. [42]);
- Audience Response Systems (T.F.I. Chan et al. [9], D.M. Moorleghen et al. [32]).

2 Application of Mobile ICT to Training

2.1 Methods of Applying Audience Response Systems

Tools of monitoring, controlling and assessing academic results are some of mobile software ICT types.

While monitoring the maturity level of students' ICT competences, there are applied various monitoring tools with the testing control of knowledge being a priority. Audience Response Systems are tools for assessing students' academic results [36, 37, 39].

Audience Response Systems are a variety of mobile software support aimed to measure students' academic results that enables automatizing the process of current and final control through applying modern testing tools and intensifying students' learning due to:

- ensuring mobility, cost effectiveness (efficiency) and privacy of testing through developing and implementing the technology of storing and using a short-term session of transmitting test assignments from the Internet server by wireless connection means;
- solving the problem of constraints in terms of technical characteristics and distance differentiation of teachers' and students' computers during a testing session arrangement [35].

Audience Response Systems aligning with this definition include ClassMarker, EasyTestMaker, Google Forms, iSpring QuizMaker, Kahoot!, MyTestXpro, Plickers, ProProfs, etc.

In our research, we distinguish the Audience Response System Plickers as the one providing an opportunity to arrange a rapid feedback between a teacher and an academic group including individual students; conducting a mobile survey, in-class general questioning and instant control of students' attendance. This system is beneficial in terms of high efficiency as it is very time-consuming and availability of students' smartphones or computers is not obligatory as a teacher's mobile device will do.

Plickers can be used in various operating systems of fixed computers and mobile devices. The system is meant for up to 63 students tested simultaneously. The system is noted for instant scanning of students' responses by a teacher's mobile device. The system also enables viewing students' testing results (both of a group as a whole and each student separately) in the form of a table.

To work with Plickers, a University teacher should get registered on the site (https://www.plickers.com/) and compile a library of tests on a variety of subjects.

During testing, students are provided with cards with QR-codes (https://www.plickers.com/PlickersCards_2up.pdf) containing response options A, B, C, and D (Fig. 1). After reading a question, students raise a card with a chosen response scanned by a teacher's mobile device.

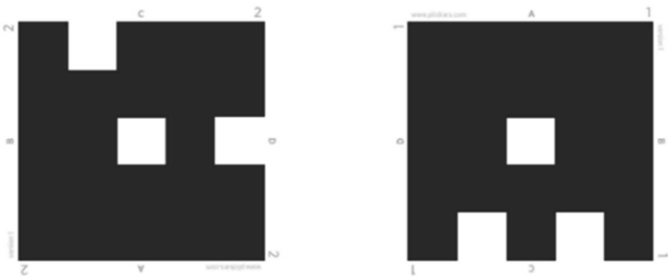

Fig. 1. Plickers cards with QR-codes.

After scanning QR-codes of the cards, the data from the teacher's mobile device is transmitted to the Plickers cloud where they are processed and stored. Plickers enables either analysis of individual students' results or a general characteristic of a student group's statistics. Students' use Plickers cards in Informatics class at Kryvyi Rih National University (Fig. 2).

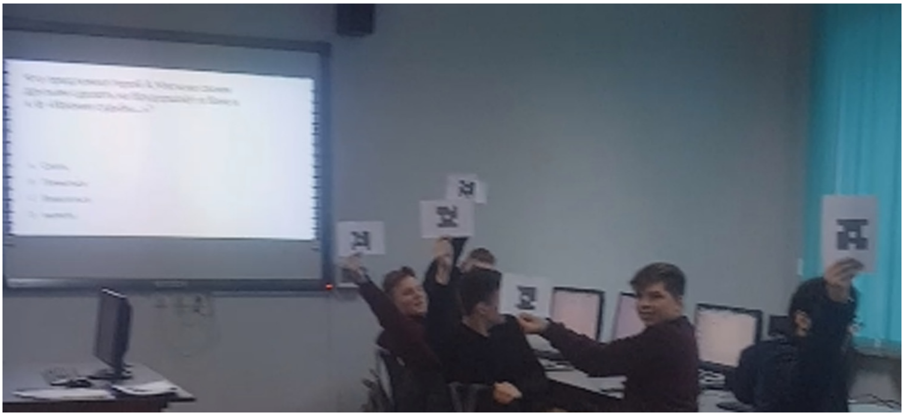

Fig. 2. Students' Plickers cards in class.

When implementing online learning in the pandemic period, methods of Plickers application have not changed much: students presents electronic pictures on their mobile Internet-devices in addition to printed ones. With that, the speed of responding has increased due to scanning pictures of similar sizes in Zoom or Google Meet.

Table 1 gives comparison of functionality of mobile tools with Audience Response Systems. Among various tools under analysis, attention should be paid to Plickers that enables combining of mobile device and augmented reality into a single multimedia environment.

2.2 Methods of Applying Mobile Multimedia Authoring Tools

Combination of various ways of data presentation is the core of the multimedia learning theory developed by R.E. Mayer who distinguishes four cognitive processes – choice, arrangement, transformation and integration of data [30, p. 118]. Selected text and graphical data are processed separately first. Next, selected data are arranged into two separate models for word and graphical data. While being processed, word data can be transformed into graphical ones (for example, by building mental images) and vice versa (by using internal verbalization of images). To successfully accomplish multimedia learning, both models should be integrated and associated with previously acquired knowledge [30].

Table 1. Assessment of functionality of Audience Response Systems.

Characteristics	Audience Response System							
	Class-Marker	Easy-Test-Maker	Google Forms	iSpring Quiz-Maker	Kahoot!	My-Test-Xpro	Plickers	ProProfs
Assignment types								
Choosing one of two contrary	+	+	+	+	+	+	+	+
Choosing one of many	+	+	+	+	+	+	+	+
Multiple-choice	+	+	+	+	+	+	–	+
Relevance	+	+	+	+	+	+	+	+
Open response	+	+	+	+	+	+	+	+
Others								
Availability of a web-version	+	–	+	–	+	–	+	–
Possible independent work	+	–	–	–	–	–	+	–
Ukrainian localization	–	–	+	–	–	–	–	–
iPhone OS and Android	–	+	–	+	–	+	+	+
Availability of full-scale free version	–	–	+	–	+	+	+	+
Cloud storage	+	–	+	–	+	–	+	–
Minimal requirements to a mobile device	+	–	–	–	+	–	+	–
Rating	**9**	**6**	**9**	**6**	**9**	**7**	**10**	**7**

According to R.E. Mayer [29], there are three basic approaches to presenting multimedia materials:

- according to transmitting channels – by two or more devices (for example, a screen and speakers);
- according to transmitting modes – text- and graphics-based (screen texts and animation);
- according to perceptive modality – audial and visual (animation accompanied by narration).

Each of these approaches is relevant to a separate class of multimedia development tools: the first class (tools of video-data designing), the second class (presentation designing tools), and the third one (augmented reality tools including Augment, Blippar, Amazon Sumerian, Anatomy 4D, AR Flashcards Space Lite, AR Freedom Stories, AR-3D Science, Chromville, Elements 4D, HP Reveal, and Google Lens. With any approach applied, Mayer insists on the following principles of multimedia development to be guided by [29, pp. 59–60]:

1. The multimedia principle: people learn by words and images better than by words only.
2. The space vicinity principle: people learn better when corresponding words and images go together and not far from each other on a page or screen.
3. The time adjacency principle: people learn better, when corresponding words and images go simultaneously and not in succession.
4. The coherence principle: people learn better, when irrelevant words, images and sounds are off and not on.
5. The modality principle: people learn better by using animation and narration than by animation and a screen text.
6. The excessiveness principle: people learn better by using animation and narration than by animation, narration and a screen text.
7. The personalization principle: people learn better, when words are presented in spoken language than in formal one.
8. The interactivity principle: people learn better, when they control the presentation pace.
9. The signalization principle: people learn better, when words contain markers on presentation arrangement.
10. The principle of individual distinctions: multimedia effects affect low-level students better than high-level ones. Multimedia effects are more powerful for highly professional students than for low-professional ones.

Observance of these principles enables us to declare that any system in compliance with them is a mobile tools of multimedia development.

Application of Mobile Multimedia Authoring Tools allows increasing efficiency of controlling students' attention span and motivation.

Considering the fact that methods of using tools of designing videos and presentations are revealed in [27, 50, 51], it is more reasonable to deal with Mobile Multimedia Authoring Tools in this paper.

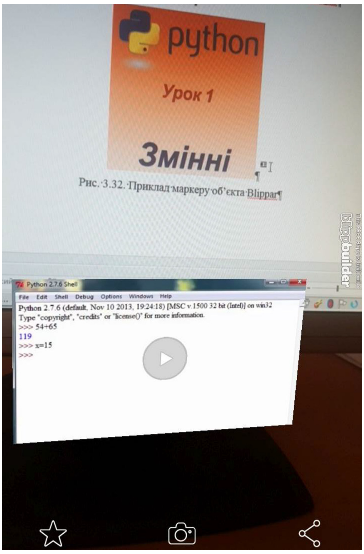

Fig. 3. The Blippar object in the AR browser Blippar.

To arrange students' activity while studying Computer Technologies in Educational Process, we apply the system Blippar [15], which enables multimedia projects of augmented reality to be implemented.

For perform a multimedia project with augmented reality in Blippar, one should get registered on their official site – https://accounts.blippar.com/signup/free.

Figure 3 shows the marker associated with the video-lesson *Variables in Python language* in Informatics class at Kryvyi Rih National University. After scanning the Blippar object, students download a video-lesson from YouTube.

In online learning, efficiency of applying AR-based multimedia tools has improved due to 'going beyond the screen' that creates conditions for increasing students' motion activity restricted by the current pandemic.

Table 2 gives comparison of functionality of mobile tools with augmented reality. Among various tools under analysis, attention should be paid to Amazon Sumerian that enables combining tools of virtual and augmented reality into a single multimedia environment based on the web-browser supporting WebGL 2.0 and WebXR 1.0.

2.3 Methods of Applying Mobile Learning Management Systems

Mobile Learning Management Systems are scientific, informational, reference materials and tools developed in the digital form and disposed in computer networks. They are reproduced by using mobile tools and required to arrange the effective educational process and use high-quality training and methodological materials. These systems facilitate learning and content management and provide for reporting without continuous Internet access.

Table 2. Assessment of functionality of mobile tools of Mobile Multimedia Authoring Tools.

Characteristics	Mobile Multimedia Authoring Tools							
	Amazon Sumerian	AR Flashcards Space	AR-3D Science	Augment	Blippar	Chromville	Elements 4D	HP Reveal
Free dissemination	±	–	±	±	+	+	+	±
Development of one's own objects	+	+	–	+	+	–	–	+
Ukrainian localization	–	–	–	–	–	–	–	–
Support of different platforms	+	–	–	+	+	+	+	+
Support of visual editing of objects	+	+	+	+	+	+	+	+
Support of various scientific fields	+	+	–	+	+	–	–	+
Rating	**4,5**	**3**	**1,5**	**4,5**	**5**	**3**	**3**	**4,5**

Moodle, Google Classroom, ATutor, CourseSites, EdApp, GoSkills, Sakai, Schoology, etc. are the most wide-spread free learning management systems that are actively engaged into learning support through e-academic resources.

To work with Moodle, a University teacher should get registered on the site (https://www.moodle.com/) and create a learning course (Fig. 4 and Fig. 5).

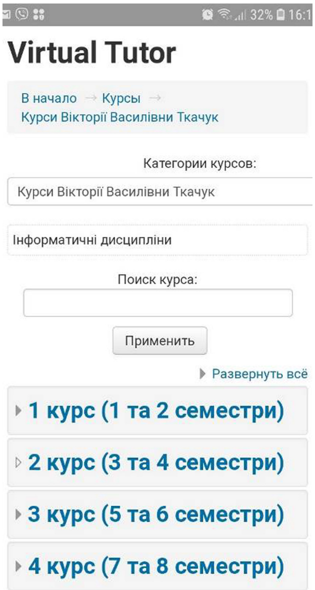

Fig. 4. Mobile courses in informatics subjects.

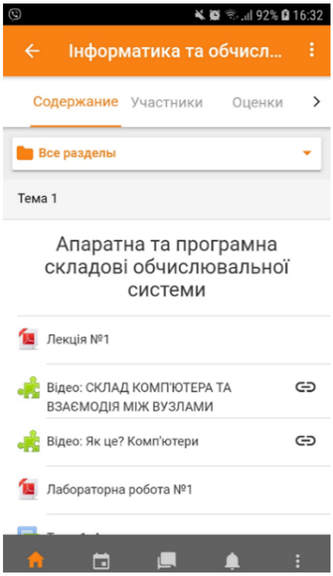

Fig. 5. The course *Informatics and Computer Practical Training* in Moodle.

The year that has gone since the beginning of the pandemic reveals that Mobile Learning Management Systems such as Moodle and Google Classroom have become a golden standard for creating a new mobile learning environment through uniting university teachers, students and administration to prevent destruction of the education-al process.

Table 3 gives assessment of opportunities of the most wide-spread learning management systems.

2.4 Methods of Applying Mobile Modelling and Programming Environments

The Mobile Modeling and Programming Environments is a software complex adapted to various operating systems and mobile devices that combines basic tools required for designing and adjusting software. The environment includes a word processor supporting syntax of a selected programming language, a compiler or an interpreter of a programming language, input and output tools, adjustment and profiling of a programming code, tools for teamwork designing, etc.

Table 3. Assessment of functionality of mobile LMS.

Characteristics	Mobile LMS							
	Moodle	Google classroom	ATutor	CourseSites	EdApp	GoSkills	Sakai	Schoology
Feedback	+	±	+	+	+	+	+	+
Testing, quiz	+	+	+	+	+	+	+	+
Ukrainian localization	+	+	–	–	–	–	–	–
Video-conference	±	+	±	–	±	–	–	–
Calendar	+	+	+	+	+	+	+	+
Assessment, register	+	+	±	+	+	+	±	+
Various task types	+	±	+	+	+	+	+	+
Rating	**6,5**	**6**	**5**	**5**	**5,5**	**5**	**4,5**	**5**

Mobile Modeling and Programming Environments can be integrated, modular or composed of separate components (a code editor, a compiler, an operator), each of them being installed on a mobile device. With the visualized environment, it can be accessed by one of cloud access models (usually SaaS). To support students specializing in *Informatics and Computer Practical Training* (the content module *Fundamentals of algorithmization and programming by Python*), it is reasonable to use the mobile environment *Pydroid 3 – Educational IDE for Python 3*.

Basic advantages of using this environment for teaching informatics-related subjects include:

– no need to have permanent Internet access;
– a built-in manager of pip-packages;
– a full-scale terminal emulator;
– built-in compilers of the languages C, C++ and Fortran;
– support of the operator PDV;
– a code editor with autcompletion and syntax highlighting;
– exchange of code fragments through Pastebin.

Pydroid can be adjusted to a user's needs by using corresponding menu elements. Then according to the laboratory work task, students input a code, compile it and see the results (Fig. 6).

The pandemic period activates the potential of these environments for organizing communication, distance activity and mutual editing, this accentuating web-based environments based on Jupyter Notebok such as CoCalc and Google Colab.

Table 4 presents general assessment of functionality of Mobile Modeling and Programming Environments.

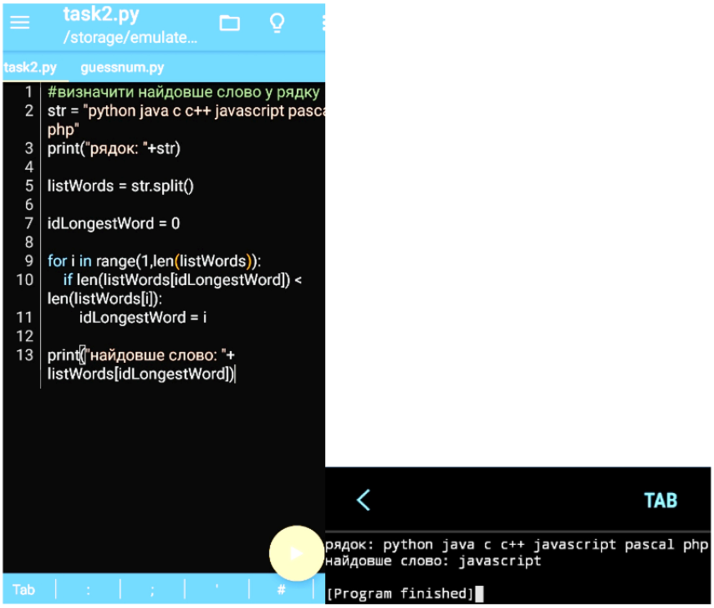

Fig. 6. Solving a problem in Pydroid.

2.5 Methods of Applying Mobile Database Management Systems

The DBMS (Database Management System) is a system software for designing and controlling databases that enable users and programmers to design, create, obtain, update and control data systematically [43].

The mobile database management system is the DBMS with the client access to the server provided by mobile interfaces and/or mobile Internet devices.

To access the mobile DBMS via Google Cloud SQL, it is necessary to use a version with a yearly free access. This service is provided to both relational (Amazon Aurora, Google Cloud SQL, Oracle Database Cloud Service, Microsoft Azure SQL Database) and nonrelational (Amazon DynamoDB, Azure Cosmos DB, Google Cloud Datastore, Oracle NoSQL Database Cloud Service) DBMS.

While studying the DBMS in the content module *Software of Computing Systems*, students work in the mobile client environment MySQL. To start MySQL, a teacher adjusts a cloud server and downloads templates of databases. The earlier downloaded database *classic models* are composed of the following tables (Fig. 7):

- customers: stores a client's data;
- products: stores a list of car makes;
- productlines: stores a list of product lines;
- orders: stores a client's orders;
- orderdetails: stores lines of sale orders for each client's order;
- payments: stores payments from clients on the basis of calculations;

Table 4. Assessment of functionality of Mobile Modeling and Programming Environments.

Characteristics	Mobile Modeling and Programming Environments																
	Pydroid	Visual Studio Code	Vim	Sublime Text	Jupyter Notebook	Spacemacs	Spyder	Atom	Wing Python IDE	Geany	Kdevelop	Eric Python IDE	PyScripter	Emacs	Spacemacs	Codenvy	Eclipse
Support of various programming languages	+	+	+	+	+	+	–	+	–	+	+	–	+	+	+	+	+
Support of various operating systems	+	+	+	+	+	+	+	+	+	+	+	+	–	+	+	+	+
Built-in operator	+	+	–	–	–	+	+	+	+	+	+	+	+	–	–	+	+
Error highlighting	+	–	+	–	+	+	+	–	+	–	–	+	+	+	+	+	–
Version check	+	+	+	+	+	+	+	±	+	–	+	+	+	–	+	+	+
Code template	+	–	+	+	±	–	–	+	+	–	–	+	+	–	–	+	+
Modular testing	+	–	–	–	+	–	+	–	+	–	–	+	+	–	–	+	+
Extra management	+	+	+	+	–	+	–	+	+	–	–	–	–	+	+	–	–
Distance editing	±	–	+	+	+	–	–	+	+	–	–	–	–	+	+	–	–
Joint editing	–	–	–	–	+	+	–	+	–	–	–	–	–	+	+	–	–
Rating	**8.5**	**5**	**7**	**6**	**7.5**	**7**	**5**	**7.5**	**8**	**3**	**4**	**6**	**6**	**6**	**7**	**7**	**6**

– employees: stores all the data on employees and a company's structure;
– offices: stores data of the sales department.

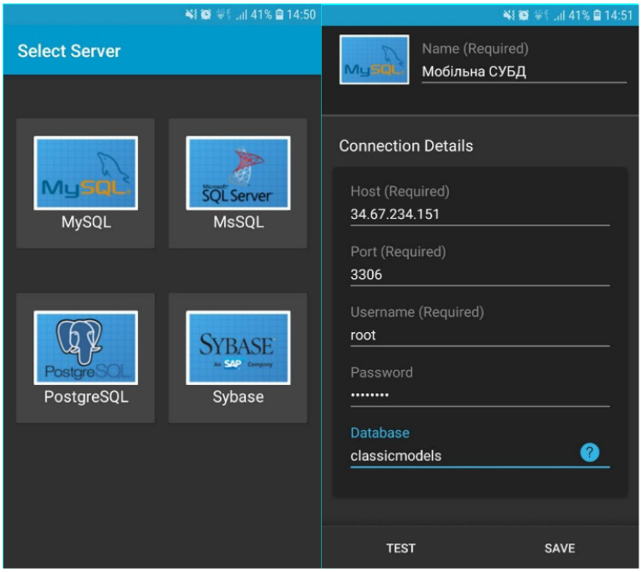

Fig. 7. The initial screen of Google Cloud SQL.

Considering the fact that the work with the server part of cloud DBMS is performed via mobile clients, functionality of mobile systems of database management is assessed by comparing client characteristics (Table 5).

3 Results of Experimental Examination of the Suggested Methods

The first-fourth year students of Kryvyi Rih National University (the Faculty of Information Technologies), Kryvyi Rih State Pedagogical University (the Faculty of Physics and Mathematics) and the Separated Structural Subdivision "Kryvyi Rih Professional College of National Aviation University" (the Faculty of Information Technologies) took part in the experiment in the 2020–2021 academic year during the COVID-19 lockdown with 176 17–21 year old students engaged. Engineering students were involved in the experiment including 87 students of the experiment group and 89 students of the control group. The knowledge level defined either by examinations or the external entrance tests (the first year students) became a criterion for selecting students for the control and experiment groups.

The experiment aimed to determine efficiency of applying mobile ICT to online learning during the COVID-19 lockdown by assessing the formation level of IC-competences.

Table 5. Assessment of functionality of Mobile Database Management Systems.

Characteristics	Mobile Database Management Systems												
	SQL Editor	SQL Client	RemoDB SQL Client My SQL	SQL Tool Pro Database Editor	DB Client - Database Client	SQL Server Mobile Manager	Connect2SQL – SQL Client	SQL Mobile Developer	SQL Query Developer	SQLApp	SQL Widget	SQL Studio	Google Cloud SQL
Coloring syntax errors	+	+	–	–	+	–	+	–	–	–	–	–	+
Visualization of results	–	+	+	+	+	+	+	+	+	+	+	+	+
Data export	–	+	+	+		+	–	–	–	–	–	+	+
Support of complex enquiries	+	+	+	+	+	+	+	+	+	+	+	+	+
History of enquiries	–	–	+	–	+	+	+	+	+	–	+	+	+
Storage of enquiries	–	–	+	+	+	–	+	–	–	–	–	+	+
Potential to execute SQL commands	–	–	+	+		+	–	–	–	+	–	–	+
Editing of lines in enquiry results	–	–	+	+	–	–	–	–	–	–	–	–	+
SSH-tunneling to access protected databases	–	–	–	+	–	+	+	–	–	–	–	–	+
Connection to several databases	+	–	+	+	+	+		–	–	+	+	+	+
SQL-keyboard	–	–	–	+	+	+	–	–	–	–	–	–	+
Rating	**3**	**4**	**8**	**9**	**7**	**8**	**6**	**3**	**3**	**4**	**4**	**6**	**11**

Table 6. Efficiency criteria of applying mobile technologies in university classrooms.

Criterion		Levels of formation	Research methods
1st	Competencies of using Mobile Learning Management Systems	High; medium; low	Matrix of Competencies
2nd	Competencies of using Mobile Modeling and Programming Environments	High; medium; low	
3rd	Competencies of using Mobile Database Management Systems	High; medium; low	
4th	Competencies of using Mobile Multimedia Authoring Tools	High; medium; low	
5th	Competencies of using Audience Response Systems	High; medium; low	

Table 7. Experimental results.

Criterion	Levels	Control group		Experiment group	
		Number of students	%	Number of students	%
Competencies of using Mobile Learning Management Systems	High	14	15,73	18	20,69
	Medium	50	56,18	55	63,22
	Low	25	28,09	14	16,09
	Total	**89**	**100**	**87**	**100**
Competencies of using Mobile Modeling and Programming Environments	High	11	12,36	19	21,84
	Medium	55	61,80	55	63,22
	Low	23	25,84	13	14,94
	Total	**89**	**100**	**87**	**100**
Competencies of using Mobile Database Management Systems	High	14	15,73	22	25,29
	Medium	54	60,67	50	57,47
	Low	21	23,60	15	17,24
	Total	**89**	**100**	**87**	**100**
Competencies of using Mobile Multimedia Authoring Tools	High	10	11,24	16	18,39
	Medium	57	64,04	56	64,37
	Low	22	24,72	15	17,24
	Total	**89**	**100**	**87**	**100**
Competencies of using Audience Response Systems	High	12	13,48	20	22,99
	Medium	55	61,80	53	60,92
	Low	22	24,72	14	16,09
	Total	**89**	**100**	**87**	**100**

The experiment group used mobile ICT (Audience Response Systems, Mobile Learning Management Systems, Mobile Modeling and Programming Environments, Mobile Database Management Systems, Mobile Multimedia Authoring Tools), while the control one used various tools depending on teachers' IC-competences.

The mobile ICT tools were used in the following way: students of the experiment group used Mobile Learning Management Systems, Audience Response Systems, Mobile Multimedia Authoring Tools as these systems were of general application designated for studying any subjects. Yet, Mobile Modeling and Programming Environments, Mobile Database Management Systems were applied only to studying specific vocation-related subjects for the third-fourth year students.

To examine the method, we determine efficiency criteria of applying mobile technologies in university classrooms, their indices and research methods (Table 6). The research is based on experimental studies.

To monitor competency formation and diagnostics of its formation level, competency matrices were used. To assess the formation level of each competency, the content of relevant IC-competences was set and a corresponding matrix was designed. Assessment methods of the formation level of competences were described in our study [46].

The number of students of the experiment group with high and medium levels is larger than that of the control group by the 1st criteria – 12,00%, by the 2nd criteria – 10,90%, by the 3rd criteria – 6,36%, by the 4th criteria – 7,48%, by the 5th criteria – 8,63% (Table 7).

After generalizing the results of the pedagogical experiment, we can conclude that the developed methods of applying mobile technologies to online learning are quite efficient, especially during the COVID-19 lockdown. The total experimental results are given in Fig. 8.

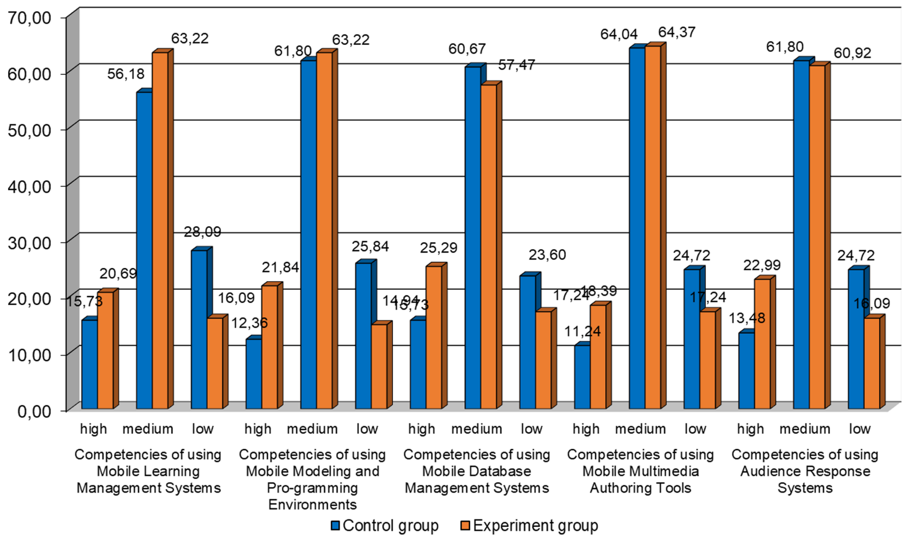

Fig. 8. Generalized results of the experiment.

4 Conclusions

While investigating into potentials of mobile technologies used by students in online learning during the COVID-19 lockdown, we obtained the following results:

– it is determined that online learning is a leading technology for providing education in Ukraine and abroad during the lockdown;
– Ukrainian and foreign researches into the issues of mobile technology application to online learning are analyzed;
– to select the most effective mobile ICT, their functionality is assessed. Five leading groups of mobile ICT that cover all the aspects of distance learning are selected. In each group, mobile ICT tools are compared by several criteria and the most efficient are selected (Plickers, Moodle, Pydroid, Google Cloud SQL, Blippar);
– the following methods of teaching informatics subjects by using Mobile ICT tools are implemented: Audience Response Systems (using Plickers as an example), Mobile Learning Management Systems (using Moodle as an example), Mobile Modeling and Programming Environments (using Pydroid as an example), Mobile Database Management Systems (using Google Cloud SQL as an example), Mobile Multimedia Authoring Tools (using Blippar as an example);
– efficiency of the developed methods is experimentally verified and confirmed, namely the results of the “Competencies of using Mobile Learning Management Systems” criterion increased by 4,96% at the high level and by 7,04% on average; the results of the “Competencies of using Mobile Modeling and Programming Environments” criterion increased by 9,48% at the high level and by 1,42% on average; the results of the “Competencies of using Mobile Database Management Systems” criterion increased by 9,56% at the high level and by 3,20% on average; the results of the “Competencies of using Mobile Multimedia Authoring Tools” criterion increased by 7,15% at the high level and by 0,32% on average; the results of the “Competencies of using Audience Response Systems” criterion increased by 9,51% at the high level and by 0,88% on average.

Online learning is essential for both students and lecturers, so universities should exert every effort to discover the optimal way of reaching the best learning outcomes especially during the COVID-19 lockdown. To do this, we suggest a wider application of:

– AR tools that enable increasing students’ motion activity by combining various learning stages: this helps mitigate consequences of movement constraints;
– communication tools and joint activity on ICT-projects in modeling, programming and database design: this helps mitigate consequences of social communication constraints;
– machine learning tools in Learning Management Systems aimed at the algorithm forecasting of learning results through monitoring students’ activity: this eliminates destruction of the educational process.

The current pandemic actualizes the university's function of being the students' socialization environment in the educational process. Thus, developing the immersive cloud-based learning environment as an open multidimensional technological and pedagogical system which comprises mobile ICT, VR/AR technologies and ensures interaction, cooperation, development of university teachers, students and administration while solving educational, technological and scientific problems anytime and anyplace.

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Implementation of Immersive Technologies in Professional Training of Teachers

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Abstract. The inclusion of immersive technologies such as augmented reality, virtual reality and mixed reality in the curriculum provides potential pedagogical benefits: providing a platform to increase student engagement, interaction, enjoyment, and a managed reflective learning experience. Research goal: theoretical justification for the use of immersive technologies in the educational process and the development of a blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for retraining teachers. Research objectives: to determine the role and place of augmented and virtual reality technologies in the educational process and their use in teacher retraining for the formation of professional competence. Object of research: the formation of professional competencies of teachers in the retraining process. Subject of research: immersive technologies as a component of the school educational environment. Used research methods: theoretical methods containing analysis of scientific sources; empirical methods of interviewing and questioning teachers. Research results: analysis of scientific publications allows us to define the concept of augmented and virtual reality, its types, directions of using augmented and virtual reality in education, examples of its application in the educational process. The developed course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for the retraining of teachers allows the formation of professional information and communication technologies competencies of teachers. Main conclusions: the introduction of immersive technologies in the educational process increases the effectiveness of training, promotes the development of logical thinking of students and increases the level of motivation of participants in the educational process. The identified gaps indicate the shortcomings of technical and methodological support for teachers to implement immersive technologies for education, which may motivate further work in the field.

Keywords: Immersive technologies · Augmented reality · Virtual reality · AR · VR · Blended course · Technology · Teacher retraining · Professional competences

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1 Introduction

Technology has been growing fast and noticeably influencing different aspects of life such as education. Studies have revealed that immersive technologies, such as Augmented Reality (AR) and Virtual Reality (VR) have strong potentials for helping students to improve their skills and knowledge. In fact, bridging AR/VR and education can bring teaching and learning experiences in an attractive and effective way [1].

Augmented reality is an interactive experience of a real-world environment where the objects that reside in the real-world are "augmented" by computer generated perceptual information, sometimes across multiple sensory modalities, including visual, auditory, haptic, somatosensory, and olfactory [2].

AR allows combining and superimposing real objects with information and with virtual objects. Azuma defines an AR as a system to have the following properties:

- combines real and virtual objects in a real environment;
- runs interactively, and in real time; and
- registers (aligns) real and virtual objects with each other.

At the same time, the augmented information may not be limited to the sense of sight but may also be applied to all senses, such as hearing, smell and touch [3].

Supporting a variety of ways of presenting, acting, and many ways to engage students in the learning process makes AR a promising educational strategy.

Virtual reality is a popular information technology (IT) area that provides an indirect experience by creating a virtual space that interacts with the human sensory systems and overcomes spatial and physical constraints of the real world (Electronics and Telecommunications Research Institute (ETRI), 2001).

According to research in USA almost 80% of teachers have access to virtual reality devices, but only 6.87% use them regularly in the educational process. At the same time, the virtual reality course is interesting for 97% of students [4].

Mixed reality (MR) is the merging of real and virtual worlds to produce new environments and visualizations, where physical and digital objects co-exist and interact in real time. Mixed reality does not exclusively take place in either the physical or virtual world, but is a hybrid of reality and virtual reality, encompassing both augmented reality and augmented virtuality via immersive technology [5].

What is XR? XR – often referred to as "extended reality" – includes virtual reality, augmented reality, mixed reality, and similar means of immersion.

Areas of AR and VR application in education are depicted in Fig. 1.

In Europe in 2020, the total production value of the XR industry is expected to reach 15–34 billion euros, and the number of direct and indirect jobs will reach 225–480 thousand [7].

The vast majority of respondents (86%) to the 2019 Augmented and Virtual Reality Survey Report agreed that XR technologies will be as ubiquitous as mobile devices by 2025 [8].

The 2020 Augmented and Virtual Reality Survey Report found growing momentum for nearly every area of immersive technology's use, such as healthcare, education,

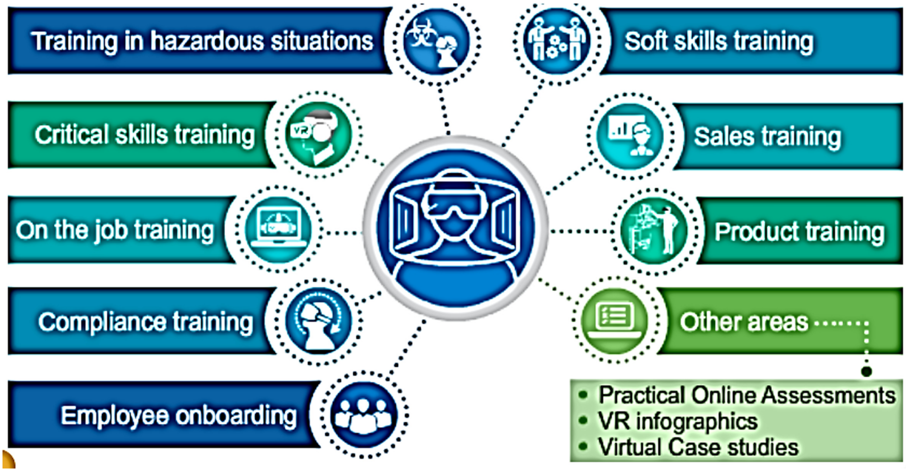

Fig. 1. Applications of AR and VR in E-learning [6]

workforce training, manufacturing and retailand, importantly, expanding avenues for monetization [9].

2 Related Work

A brief overview of the history of the development of VR and AR technologies and their use in education is presented in [10]. In these works the authors explore the concept of Virtual and Augmented reality, its history and set out the ontological difference between virtuality, possibility and actuality.

A worldwide survey of the use of AR and VR in education has been published by Pellas, Fotaris, Kazanidis [11].

Research has shown that with the rapid adoption of technological innovation and increased investment from large companies such as Apple, Facebook, Google, Samsung, and Magic Leap to increase the availability of AR and VR technologies, educational institutions will be able to deliver learning in virtual environments. The possibilities of virtual technologies to overcome the boundaries of formal education are demonstrated [12].

Jaime Donally pays attention, that the most important part of selecting what immersive technology to deploy is "defining goals and expectations for how mixed reality integration will benefit classroom instruction" [13].

Stephanie Chard in "Four ways extended reality (XR) will influence higher education in 2020 and beyond" [14] analyzes methods, technologies and the level of use of extended reality in higher education settings at present. She notes that XR offers educators a new, seductive interplay of visualization technology and human perception that has the potential to transform both teaching and learning.

However, despite the advantages of using XR technology in the educational process, there is a slowdown in the rate of implementation of these technologies. With this in mind, the author suggests four main ways immersive technology can transform higher education:

- Immersion in virtual worlds. Virtual reality improves the learning process by providing more natural presentation of information, interacting with the simulated environment, and significantly reducing the student's cognitive load.
- The future of AR ubiquity. Where VR creates new, remote and imaginary worlds, AR technology augments the physical world; adding layers of data onto what we see with the naked eye. It augments and contextualises our vision with information, sound, video and graphics.
- Training "Anytime, anywhere". In a pandemic, the accessibility of laboratories is an ongoing challenge for universities, and the ability to teach remotely significantly reduces the barriers that students may face when attending classes in person.
- Personalization of training. Immersive learning offers a powerful combination of motivating the learner and implementing an individual learning path. Personalization of learning promises to supercharge current educational standards and learning delivery.

XR technologies can help present a more detailed picture of exactly how an individual student learns, allowing teachers to create a personalized experience optimized for each student.

Important issues of the impact of augmented reality technologies on increasing the level of student motivation are considered in [15].

Ukrainian researches Mariya P. Shyshkina, Serhiy O. Semerikov, Viacheslav V. Osadchyi, Kateryna P. Osadcha, Oleksandr Yu. Burov, Svitlana H. Lytvynova, Andrii M. Striuk and others address issues of the AR implementation in science education and applying AR in professional training and retraining [16–18].

EU Research and Innovation programmes aimed at providing breakthroughs, discoveries, and world-firsts pay particular attention to projects related to AR and VR technologies.

Erasmus+ project entitled VR@School - Future schools using the power of Virtual and Augmented Reality for education and training in the classroom was written and coordinated by Liceul Teoretic de Informatica "Grigore Moisil" (Romania) and involves Pixel in the transnational partnership. The project was funded by the European Commission in the framework of the Erasmus+ Programme, KA2 - Strategic Partnership for School Education. The VR@School project aims at supporting Virtual Reality as a teaching methodology, which helps students feel immersed in an experience, gripping their imagination and stimulating thought in ways not possible with traditional books, pictures or videos, and facilitates a far higher level of knowledge retention [21].

The main objectives XR4ALL as an initiative of the European Commission to strengthen the European XR industry are to create a pan-European (XR-tech) community, to discover existing EU XR technology, to develop a research agenda, to award grants for innovative technology projects, to increase investments and tech transfers to help products reach market [7].

A large number of international scientific and practical conferences (AR & VR World 2020, VRDAYS EUROPE 2020, EuroVR 2019, IEEE VR, XRDC, AREdu and etc.) confirms the relevance of the introduction of AR and VR technologies in education and the interest of the scientific community in these innovative problems.

The Kherson State University and the Communal Higher Educational Establishment "Kherson Academy of Continuing Education" of Kherson Regional Council have extensive experience in teacher training [22–24].

3 Problem Setting

In modern conditions of the development of technologies and changes in public life, one of the most important components of education is the development of information competence.

The issues of development of information competency of teachers are of special importance and urgency because the problem of development of information educational environment and the possibility of its use in professional activity is very important.

The purpose of the article is the theoretical justification for the use of immersive technologies in the educational process and the development of a blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for retraining teachers.

Tasks:

- Analysis of the main features and advantages of AR and VR technologies;
- Find the attitudes of teachers to use augmented and virtual reality in education;
- Development of a blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for retraining teachers;
- Implementation of the blended course in order to form the professional competencies of teachers in the field of information and communication technologies;
- Analysis of the results of teachers' implementation of immersive technologies after studying course.

4 Development of a Blended Course "Using the Technologies of Augmented and Virtual Reality in the Practice of Modern Educational Institutions" for Retraining Teachers

In order to increase the level of professional information and communication technologies competencies of teachers at the Communal Higher Educational Establishment "Kherson Academy of Continuing Education" of Kherson Regional Council and at Kherson State University the blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" was developed.

The objective of the blended course is to prepare teachers in the system of continuous pedagogical education for the use of augmented and virtual reality technologies in everyday and professional activities, generalization, systematization and ex-change of experience, raising the level of their professional mastery using digital technologies.

The target group of the course is teachers of different categories and specialties who see the potential of AR and VR technologies in education, striving by modern technology to minimize the digital divide between teacher and student, to improve the quality of education and the motivation of objects of educational activity.

The blended course "Using the technologies of virtual and augmented reality in the practice of modern educational institutions" is aimed at solving the following problems:

- disclosing the educational capabilities of augmented and virtual reality technologies;
- assistance to the quality development of the information educational environment of teachers;
- stimulation of the productive activities of teachers and students, oriented toward creative self-realization by means of augmented and virtual reality technologies;
- popularization of augmented reality as a factor that contributes to the generalization of experience, the dissemination of copyright developments, professional growth and development of teachers;
- effective use of modern digital technologies for educational purposes;
- increasing the interest of teachers in the active use of augmented and virtual reality to solve the problems of modernization of education;
- improving the effectiveness and efficiency of professional teaching activities;
- dissemination of advanced pedagogical experience.

As a result of studying the course, the teacher should know:

- the concept, types and possibilities of using augmented and virtual reality technologies in the professional activities of teachers;
- normative documents defining state policy in the field of information competence of teachers and students, as well as the basics of information security;
- types and features of the functioning of finished projects of augmented reality (Futurio, Quiver, Elements 4D, Octagon series cards, etc.);
- the purpose and basic functionality of the platforms for creating augmented reality Futurio, Quiver, BlippAr, Wallame;
- programs and algorithms for creating marker augmented reality;
- methodological features of using augmented reality technology in the context of ordinary, flipped, blended and distance learning;
- methodology for introducing elements of augmented reality in the professional activity of a teacher.

As a result of studying the course, the teacher should be able to:

- apply ready-made augmented and virtual reality projects in education (Futurio, Quiver, Elements 4D, Octagon series cards, etc.);
- create augmented reality with the Wallame application tools and exchange messages in the real world with the help of augmented reality;
- create interactive maps of the area using the technology of augmented reality;
- create augmented reality scenes using the Blippar$ application;
- apply augmented and virtual reality technologies in the organization of educational and extracurricular activities.

The program is based on the principles of scientific, systematic, accessibility; practical orientation, taking into account the experience of introducing courses on the study of digital technologies in educational institutions of various types, as well as the needs of students of continuing education courses, taking into account their professional and digital competencies.

Schema of blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" is depicted in Fig. 2.

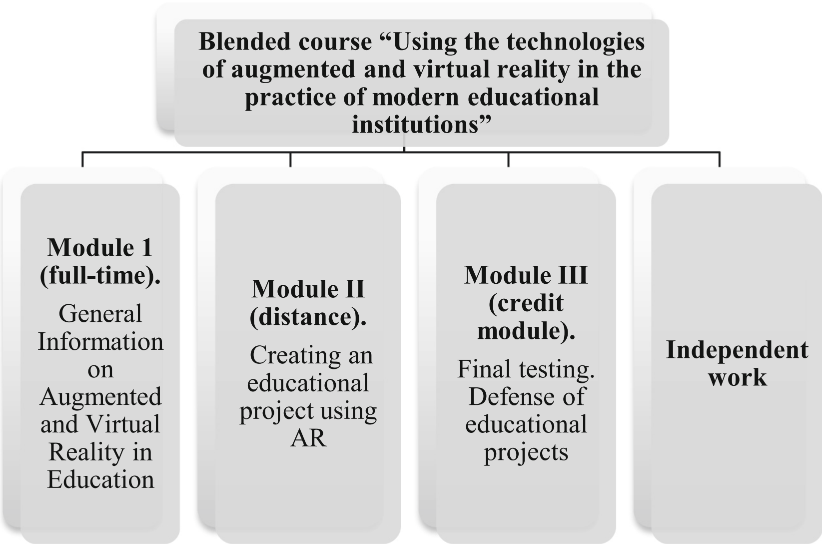

Fig. 2. Schema of blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions"

The blended course program is designed for 36 h, of which 4 h are lecture classes, 14 - practical classes, 18 - independent work.

The program includes the following training modules: full-time module "General Information on Augmented and Virtual Reality in Education"; distance module "Creating an educational project using AR"; independent work; credit module (individual project of each teacher).

The program provides for training sessions with various forms of cooperation, involving the use of active innovative organizational forms, interactive methods and based on the use of digital technologies with a professional focus. This creates the conditions for the manifestation of initiativities for students, creativity, independent search, ensuring the appropriate level of professionalism of students, the ability to introduce the knowledge into the educational process of modern educational institutions.

The distribution hours of classroom work to lecture and practical is approximate and can be changed while maintaining the total number of hours.

A significant part of the study time is devoted to independent work.

The educational strategy of the course is also implemented by distance learning through online consultations and the implementation of individual practical tasks; independent study by students of the training material recommended by the teachers of the course.

The training course is practice-oriented, and therefore, the predominant part of the time is devoted to the practical work of teachers with augmented reality applications: modeling and designing of a training project, creation, design, filling, optimization, etc.

Tasks for independent work are focused on self-study of the course; adding your own information resources; search for ways to implement educational projects using augmented and virtual reality technologies in the context of the traditional educational process or the introduction of flipped learning and etc.

The task of the final control is to check the teachers' understanding of the material of the course as a whole, the logic and relationships between the individual sections, the ability to creatively use the accumulated knowledge, the ability to form their own attitude to a particular problem of academic discipline and apply augmented and virtual reality technologies in the educational process.

The final test provides for 2 groups of questions: questions that verify the degree to which teachers master the course material; questions that test the practical skills of teachers in using augmented and virtual reality applications.

The thematic plan for studying the blended course is presented in Table 1.

Table 1. Thematic plan for studying the blended course "Using the technologies of virtual and augmented reality in the practice of modern educational institutions"

No п/п	Lesson topic	Total hours	Of them		
			Lecture	Practice	Independent work
Module 1 (full-time) General Information on Augmented and Virtual Reality in Education		20	4	6	10
1	Introductory questionnaire. The history of the emergence and development of augmented and virtual reality		1	1	
2	An introduction to augmented and virtual reality. *Practical work No 1. Applications for Augmented and Virtual Reality*		1	1	2

(*continued*)

Table 1. (*continued*)

No п/п	Lesson topic	Total hours	Of them		
			Lecture	Practice	Independent work
3	Application of augmented reality technology in education. *Practical work No 2. AR and VR in education. Work with ready-made augmented reality projects*		1	2	4
4	Introduction to the applications of creation of augmented reality in education. *Practical work No 3. Creation of AR and VR projects*		1	2	4
Module II (distance) Creating an educational project using AR		14		6	8
1	Wallame application and its fields of using in education. *Practical work No 4. Create an interactive map with the Wallame app*			2	4
2	BlippAr app. *Practical work No 5 Creating cards with elements of augmented reality using the BlippAr application*			4	4
Module III (credit module)		2		2	
Total		36	4	14	18

The curriculum of the blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" provides for the creation of individual and collective creative projects that have prospects for implementation in Ukrainian schools based on their own pedagogical experience.

5 Setting up the Pedagogical Experiment

The course was held for 198 students of continuing education courses (teachers of the Kherson region, Ukraine). There were 43 (21.7%) teachers of mathematics and computer science, 35 (17.7%) primary school teachers, 28 (14.1%) Ukrainian language and literature teachers, 22 (11.1%) biology teachers, 21 (10.6%) physics teachers, 19 (9.6%) chemistry teachers, 17 (8.6%) history teachers. The remaining categories did not exceed 2%.

To identify teachers' readiness to support and implement augmented and virtual reality technologies, the questionnaire was developed in Google Forms (Fig. 3).

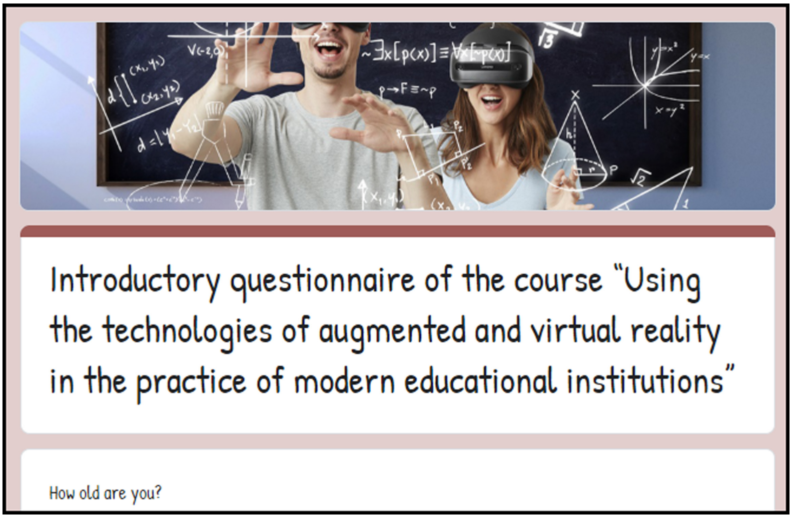

Fig. 3. Introductory questionnaire of the course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions"

The questionnaire survey showed that the overwhelming majority of teachers uses digital technologies at the basic (35.4%) and higher than the basic (30.3%) levels. However, it should be noted that there were 12 teachers (6.1%) who did not have the skills to work with digital technologies. These are mainly teachers of philological disciplines of the older age.

Figure 4 shows the responses of teachers about the level of digital technology using.

A sufficiently large number of teachers have skills of using information and communication technologies in the educational process, as were show answers presented in Fig. 4.

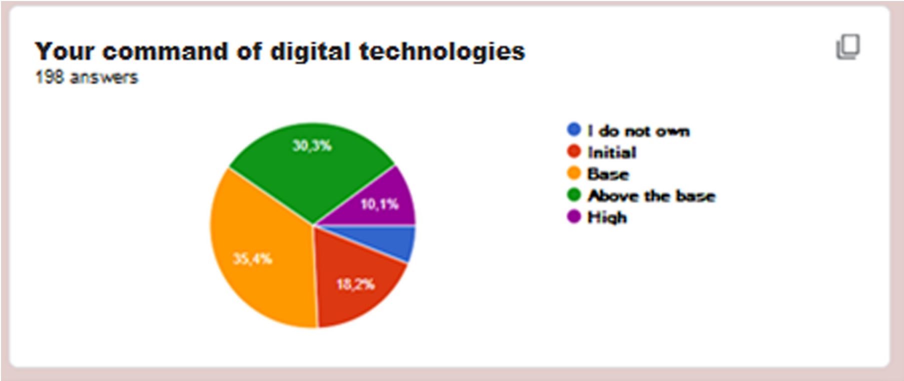

Fig. 4. Responses of teachers about the level of digital technology using

The level of awareness of teachers about immersive technologies turned out to be quite high. This is evidenced by the answers presented in Fig. 5.

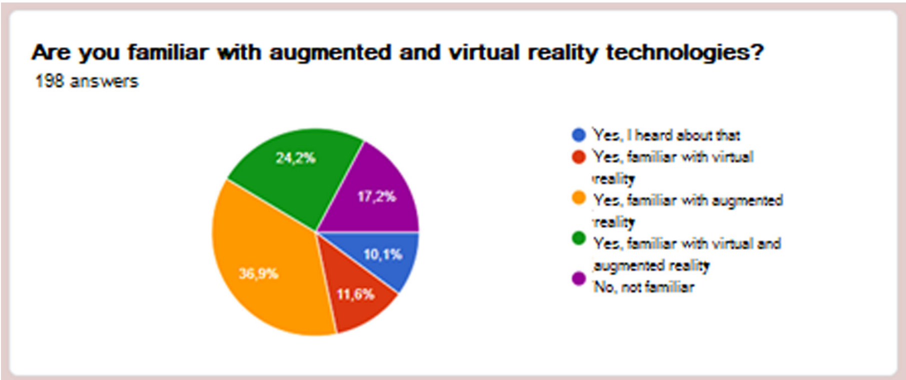

Fig. 5. Answers to teacher awareness of immersive technologies question

24.2% of respondents have an idea of both AR and VR technologies, and also 48.5% are familiar with at least one of them. It should be noted that augmented reality technology is more popular among teachers than virtual reality technology. The dominance of smartphones and tablets, the absence of the need for additional equipment, the growing distribution of software with support for augmented reality make augmented reality technologies more preferable than virtual reality technologies for use in the educational process.

The results of a survey of teachers showed a high level of motivation to study the course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for the development of digital competencies with the purpose the application of innovative technologies in educational process.

For different groups of teachers, topics of projects using augmented reality were selected that correspond to the disciplines they teach.

So, for teachers of elementary school, environmental studies, astronomy, it was proposed to use ready-made augmented reality projects based on Octagon Studio's live instruction cards. Flash cards Space 4D+ allow users to study the properties of space objects. The application scans printed space maps and allows users to watch how space comes to life. The collection of cards includes the Solar system, space objects, planets, satellites, etc.

Examples of the results of practical work of teachers with educational cards Space 4D+ with augmented reality depicted in Fig. 6.

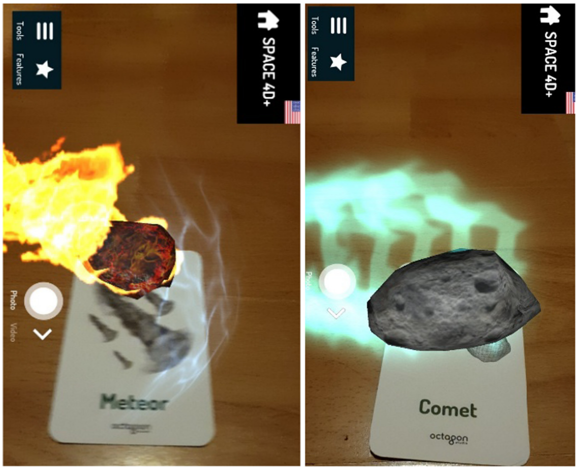

Fig. 6. Examples of using Space 4D+ training augmented reality cards to study the properties of space objects

The use of such cards in the lessons can significantly increase the level of students' motivation and the degree of perception of the material.

Among similar applications, Octagon Studio also includes Humanoid 4D+ for studying the structure of the human body, Octaland 4D (for studying professions), Animal 4D+ (for studying animals), etc.

Practical work on studying the tool for developing educational objects of augmented reality of the BlippAR mobile application and the Blipp-Builder web service involves the creation of interactive cards.

An example of using augmented reality based on the development of interactive cards by teachers of mathematics is shown in Fig. 7.

When creating a didactic card, the model of organizing access to educational objects of augmented reality using markers, mobile devices and augmented reality browsers, proposed in [21], is used.

A card with a math assignment is marked with a special marker. To recognize the marker, a mobile device with a camera and an augmented reality browser installed is used.

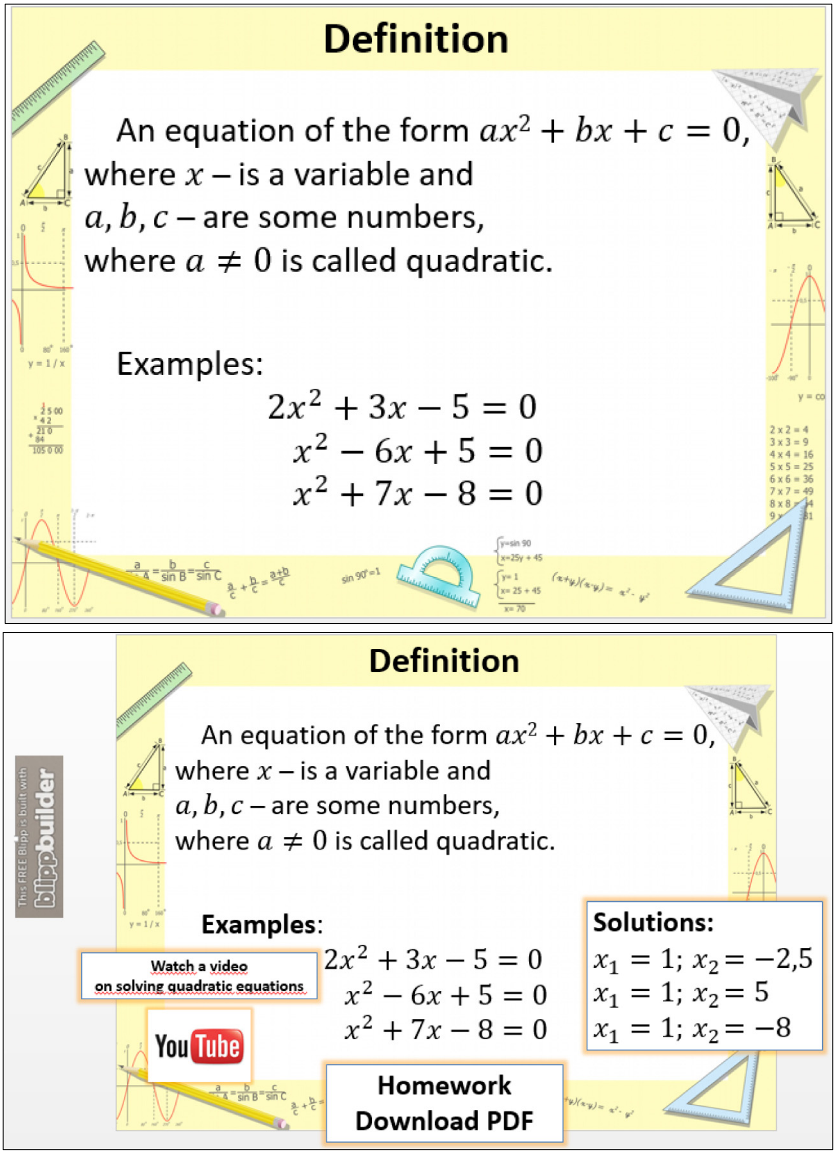

Fig. 7. An example of a project with elements of augmented reality, completed by teachers of mathematics, created using the BlippAr application

Once in the camera field, the marker is recognized by the program and the augmented reality scene is displayed on the device screen.

In this scene, elements such as an image, text (for example, with answers for self-testing), video (with an explanation of theoretical material or an algorithm for solving problems of this type), a three-dimensional model, etc. are superimposed on the image received from the device's camera. The interactive scene suggests the possibility of interacting with the user based on the script [25].

6 Study of Services for Creating and Using Augmented Reality, Taking into Account the Specialization of Teachers

It should be noted that the study of services for creating and using augmented reality is carried out taking into account the specialization of teachers.

So, for example, specialized applications AR Math and Geogebra AR are considered separately for mathematics teachers [26].

AR Math allows students to establish connections between objects around them and geometric shapes. The student can virtually manipulate geometric objects, as well as use the AR protractor available in the application. The capabilities of the AR Math application include the implementation of a virtual mathematical situation; recognition of geometric objects in everyday life, classification of detected objects by classes; interactive interaction of students with a virtual character who helps to solve a mathematical problem or write a solution in the form of a mathematical expression based on understanding the semantics (or to compare the solution found to one of the proposed mathematical expressions). The application contains a virtual assistant that promotes the involvement of the student in the augmented reality environment, thereby increasing the motivation to study mathematics [27].

The Geogebra AR application is also especially relevant for mathematics teachers. Developed for iOS, it allows you to explore a second-order surface, a spiral staircase, a Klein bottle in a real environment using this object in any part of the room in which it is used. When using the application on the street, pointing a smart-background or tablet at architectural structures, you can see the corresponding mathematical objects [27, 28].

For physics teachers, applications Electricity AR, cg-physics AR, e-Pathshala AR, Physics Lab AR are considered [29, 30]. Figure 8 shows the results of teachers' work with the Electricity AR app. This application allows you to determine the division price of measuring devices, as well as independently make measurements using augmented reality technology. To work with the application, you need to print a set of special cards, which includes eight 3D models of electrical circuit elements and various measuring instruments (amperemeters and voltmeters) [30].

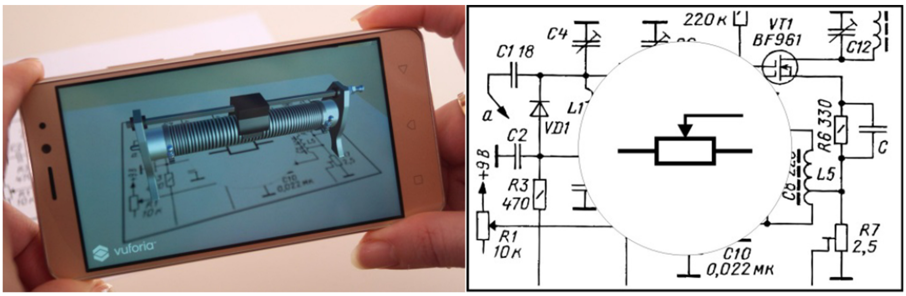

Fig. 8. An example of the Electricity AR application for studying physics (theme "Electricity")

For teachers of geography and biology, the applications LandscapAR, Anatomy 4D, Animal 4D+, Augmented Reality Dinosaurs, etc. are relevant [31, 32].

The most popular AR applications, which are considered, among other things, as part of a special course, are applications that simulate viruses, the solar system, a biological cell and a model of the human body [29].

For example, the Galactic Explorer application (Fig. 9) allows students to observe the solar system, the rotation of planets around the Sun, explore the surfaces of different planets (color, texture), discover interesting facts while moving around the universe [33].

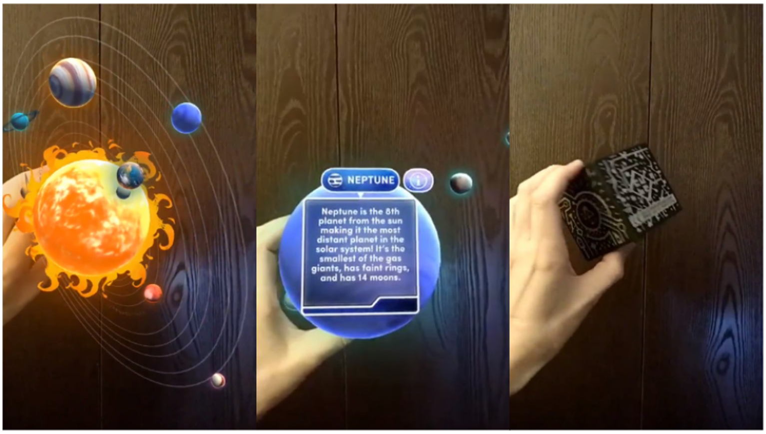

Fig. 9. Examples of using Galactic Explorer apps to explore the solar system

Another application, Merge Explorer (Fig. 10), allows students to familiarize themselves with the objects of the world and to engage students in active, hands-on learning based on research [34].

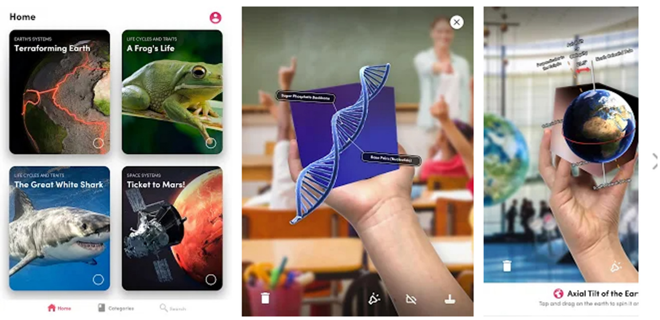

Fig. 10. Examples of using Merge Explorer apps to explore objects in the surrounding world

The new generation Merge Cube toy, which is also considered as part of the special course, is of particular interest to the listeners. It is but a STEM toy that allows you to hold digital objects in your hands, opening up a new way for students to explore and interact with the world around them. This unusual toy is made of soft durable foam and has edges with 6 different holograms. The cube can be used with a smartphone, on which you must first download the corresponding application from PlayMarket or AppStore, thanks to which the effect of a "new world" is created in the palm of your hand [34].

In addition to the smartphone, the Merge Cube can also be used with virtual reality glasses for a fully immersive experience. The use of a cube paired with a headset causes the user to have the effect of replacing an object (cube) that imitation. The principle of operation of the application is that due to the drawings on the cube, similar to QR codes, it understands the pattern and its orientation in space and loads the corresponding position and design, and creates a real hologram. The learning capabilities of the device have been highly appreciated at the world level by numerous awards from educational institutions, including the American Association of Librarians, ISTE (International Society for Technology in Education) [35].

The unique properties of the device allow teachers to diversify the learning environment with new content, because such augmented reality perfectly complements traditional teaching methods, allows students to effectively learn natural sciences and STEM using three-dimensional objects and simulations, brings interactivity and develops spatial intelligence.

As a final work of the course, students are invited to develop a project that provides for the creation of an interactive laptop with elements of augmented reality and to develop a lesson summary using augmented reality technology within the chosen discipline.

In combination with augmented reality technology, lepbook allows you to easily understand theoretical material, illustrate and detail it, which will contribute to the development of creative thinking and increase cognitive activity.

Fig. 11. The project of an English teacher, completed as part of the study of a special course: "Lepbook with elements of augmented reality "My House" for learning English in grade 5"

Figure 11 shows a lepbook with elements of augmented reality "My House" for learning English in grade 5, created as part of the study of a special course. Lepbook contained 5 active pages with different content to study this topic.

As part of the presentation of the topic, augmented reality markers were placed on all pages, which comprehensively supplemented the lepbook material with interactive content: video, audio, pop-up images and text. Such a laptop using the BlippAr application allows the student to easily and interestingly gain new knowledge, and the teacher to plan an interesting STEM lesson [36].

The created works were presented at an exhibition organized as part of the study of the special course.

7 Ergonomic Requirements for the Use of Immersive Technologies in the Educational Process

The relevance of the issue of ergonomic requirements for the use of immersive technologies in the educational process is determined by their widespread use both in the system of higher and secondary education.

By the ergonomic requirements for the pedagogical system, which uses immersive learning technologies, we mean the rationing in relation to this system of the values of indicators of the ergonomic characteristics of the educational process, means and conditions for optimal learning activity. Virtual visualization technologies and psychophysical laws of human perception should be applied to achieve effective and safe use of modern electronic learning systems [37].

General ergonomic requirements are defined and specified by the international standard EN ISO 9241 Ergonomics of Human System Interaction [38].

To describe and assess the quality of teaching systems based on immersive technologies, along with methodological and pedagogical, it is necessary to use ergonomic characteristics. These characteristics constitute a specific system of quality indicators. Ergonomic characteristics describe the degree to which the software and hardware components and conditions of the learning environment correspond to the potential capabilities of the teacher and student to create favorable conditions for effective learning activities, preserving health and personal development - solving the triune task of ergonomics. The characteristics of external technical and instrumental means, sanitary and hygienic conditions and social environment, peculiarities of the organization of individual and joint educational activities are subject to evaluation and optimization. The most general criteria for ergonomic optimization are safety and comfort, the level of which determines the effectiveness of the educational process and the functioning of immersion systems.

It should be noted that for ergonomic assessment, indicators are used that are similar to those in the assessment of electronic educational resources [39]. The main ergonomic indicators of the qualities of training systems based on immersive technologies include:

1. technical characteristics of learning tools: smartphones, tablets, stereo glasses and VR helmets, manipulator controllers, video cameras;
2. qualitative characteristics of software: simplicity and ease of use of software, quality parameters of graphic, multimedia software objects, ease of navigation;

3. aesthetic factors: the harmony of colors, sound environment, equipment, the completeness of immersion in the learning environment;
4. organizational conditions: physical activity, threats of injury, fatigue, excessive information load;
5. psychophysiological conditions: load on the organs of hearing and vision, muscle activity, emotional load, mental and physical stress.

To analyze the influence of ergonomic requirements on the quality of the learning process when using immersive technologies in the educational process, an expert method was used, in which teachers were the experts.

As part of the study of the special course, teachers paid special attention to the ergonomic requirements of using mobile devices in the classroom. Problem discussions were proposed on the topics: "How to preserve the health of modern children when using ICT and immersive technologies in particular?", "Lesson with a gadget: what to fear and how to act", "The influence of blue glow on the human body", in the framework of which teachers based provided materials with the use of active forms of work discussed the problem and presented the results in the form of posters, interactive whiteboards, etc.

Of particular interest among teachers of all specialties was the topic of the influence of blue glow on the human body, because the use of gadgets is directly related to taking into account their influence.

Harvard Health research shows that not all colors affect people the same. It is believed that the cause of such common diseases as insomnia, blurred vision, depression, obesity, diabetes, headaches and even cancer is the blue spectrum of the display of a gadget or PC. It is also known that the highest permeability to blue is in a ten-year-old child. Children at this age are already actively using gadgets, but they do not yet have formed natural filters. Therefore, such students are at risk. These also include those who use gadgets with increased light sensitivity. It is not unequivocally established which blue glow is harmful. Some sources talk about a spectrum from 415 to 455 nm, others - up to 510 nm. It is also indicated that the blue glow is especially harmful in a dimly lit room [40–42].

The new Sanitary Regulations for general secondary education institutions of Ukraine, introduced in 2021, determine how to safely organize the educational process and work with technical means of learning. When using immersion technology during the lesson it is necessary to alternate types of learning activities. The use of virtual and augmented reality hardware is allowed no more than 15 min during the lesson and no more than 10% of the time during the week. During the lesson, after working with technical means, students must do exercises to prevent visual and static fatigue [43].

8 Results of Implementation Immersive Technologies by the Teachers

In order to study the degree of teachers' application of the knowledge received at the blended course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions", after six months from the beginning of the introduction of the course in postgraduate education, a survey was conducted. The main objectives of the questionnaire were: to find out whether teachers use the knowledge gained during the study of the special course and what difficulties they encountered in their work when using technologies of augmented and virtual reality.

72 teachers of Kherson and the Kherson region took part in the survey, who studied the blended course "Using the technologies of virtual and augmented reality in the practice of modern educational institutions" as part of professional development. There were 17 (23.6%) teachers of Ukrainian language and literature teachers, 15 (20.8%) mathematics and computer science, 11 (15.3%) primary school teachers, 10 (13.9%) biology teachers, 7 (9.7%) physics teachers, 7 (9.7%) chemistry teachers, 5 (6.9%) history teachers.

To the question "Do you use augmented reality technology in your professional activities?" more than half of the teachers (45 teachers, 62.5%) answered that they use it, among them 18 (25%) answered that they often use augmented reality applications in the classroom, 27 (37.5%) - occasionally. When asked why teachers do not use augmented reality technologies in class, 23 out of 72 (31.9%) answered that they do not have the technical ability to implement this technology in the educational institution in which they work. Let us denote that these were mainly teachers from rural schools (Fig. 12).

In the questionnaire, the teachers noted the following most common impressions from the use of augmented reality technologies in the classroom:

- "Augmented reality always makes the lessons come alive and helps children gain new knowledge, evoking vivid positive emotions";
- "The use of augmented reality is a feature of my lessons, which helps to make any lesson interesting and unforgettable";
- "I often use quests that I do with the WallaMe application. Children love them very much";
- "I make augmented reality lepbooks with children. Students look forward to each new lepbook";
- "Geogebra AR always helps to clearly explain even the most complex aspects of geometry".

It should be noted that among the contingent of teachers who took part in the survey, the number of those who used augmented reality technologies after the introduction of the course was distributed as follows: physics teachers (6 people out of 7; 85.7%), primary education teachers (9 people out of 11, 81.8%), teachers of mathematics and computer science (12 people out of 15; 80.0%), biology (7 out of 10; 70%), chemistry (4 out of 7; 57%), Ukrainian language and literature (7 out of 17; 41.2%). Thus, we see that the largest number of teachers using the studied special course were teachers of physics, elementary education, mathematics and computer science.

The results obtained allowed us to conclude that it is necessary to modernize the special course in the direction of the obligatory division of teachers into groups, taking into account the specialization (the subject taught), as well as an increase in the number of hours for the use of augmented and virtual reality technologies in the classroom. For this purpose, a topic was added to the course: "Using the technology of virtual and augmented reality at different stages of the lesson".

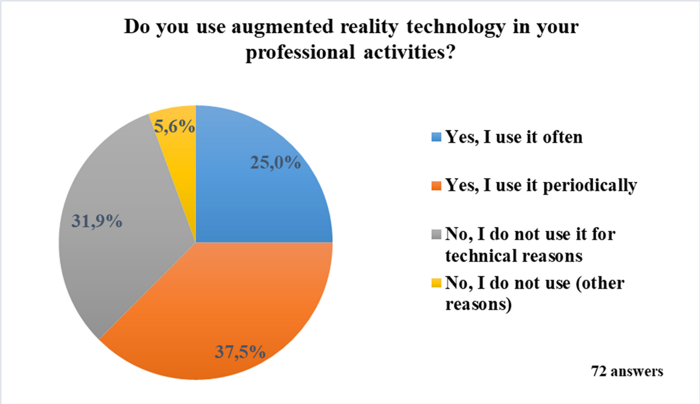

Fig. 12. Results of a questionnaire survey of teachers on the introduction of augmented reality technologies (after studying the blended course "Using the technologies of virtual and augmented reality in the practice of modern educational institutions")

So, at the stage of updating basic knowledge, immersive technologies contribute to an increase in the level of motivation and involvement. For example, augmented reality flashcards can help increase student attention.

At the stage of explaining new material, augmented reality can be used to visualize and provide a better understanding of the topic.

At the stage of formation of skills and abilities, immersive technologies make it possible to provide a research learning environment, create a safe space for conducting virtual experiments, etc.

At the stage of consolidation, elements of augmented reality can be used to animate the studied phenomenon and create creative projects.

Analysis of the feasibility of using immersive technologies at a certain stage of the lesson contributes to an increase in the number of teachers using augmented and virtual reality in the learning process.

9 Conclusions and Outlook

Analysis of scientific publications allows us to define the concept of augmented and virtual reality, its types, directions of using immersive technologies in education, examples of its application in the learning process. The developed course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" for the retraining of teachers allows the formation of professional information and communication technologies competencies of teachers.

The course provides examples of the use of augmented reality applications in the study of mathematics, physics, astronomy, biology, English and others.

The study of services for the creation and use of augmented reality as part of the course "Using augmented and virtual reality technologies in the practice of modern educational institutions" was carried out taking into account the specialization of teachers.

It is necessary to take into account the ergonomic requirements when using immersive technologies in order to ensure the safe organization of the educational process.

The results of the pedagogical experiment showed a high level of motivation of teachers of elementary and secondary schools to study and use augmented reality technologies in the educational process.

Implementation of the blended course in professional training of teachers has proven its relevance. 198 teachers of the Kherson region studied on the course. The result of studying the course at the initial stage was the creation of a didactic project in the form of augmented reality cards, a Lepbook or a lesson outline.

The introduction of a special course and further application of immersive technologies in teaching practice at school made it possible to draw a conclusion about increasing the effectiveness of teaching through the use, stimulating the development of logical thinking of students and increasing their level of motivation.

According to the results of the research, most teachers after studying the course "Using the technologies of augmented and virtual reality in the practice of modern educational institutions" use augmented reality technologies in the educational process.

The introduction of immersive technologies in the educational process increases the effectiveness of training, promotes the development of logical thinking of students and increases the level of motivation of participants in the educational process.

The research results can be used in the practice of retraining of teaching staff, as well as serve as a basis for developing a methodology for introducing immersive technologies into the educational process of school.

The prospects for further research are the search for mechanisms for training teachers of various categories to work on the use of immersive technologies in learning process; development of modern methods, techniques and technologies for training students in various subjects using immersive technologies, studying foreign experience in the implementation of augmented and virtual reality in the classroom.

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ICT in Research and Industry

Towards the Method and Information Technology for Evaluation of Business Process Model Quality

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Abstract. Business process management has become the most widely-used and reliable approach to organizational management over the last decades. It is also considered as a part of quality management system in an organization. Business process modeling is the core of business process management, which is used for visualization, analysis, and improvement of organizational activities. Moreover, business process modeling plays an important role in the context of business process management maturity of an overall enterprise. Therefore, this paper is focused on the problem of business process model quality evaluation. Existing approaches based on business process modeling guidelines, measures, and their thresholds are considered. Refined business process modeling rules, measures, quality criteria of numerical and linguistic values, and a method for evaluation of business process model quality are proposed. The corresponding information technology is designed and implemented, and results of its usage are outlined.

Keywords: Business process model · Process model quality · Quality measure · Quality criteria · Information technology

1 Introduction

Business process modeling is considered as a key tool of Business Process Management (BPM). In paper [1] BPM is considered as both art and science of monitoring organizational activities in order to provide quality of produced products or services and find ways to improve these activities. According to Dumas et al. [1], BPM is about managing entire chains of events, activities and decisions that ultimately add value to the organization and its customers. Whereas, business processes are considered as "chains of events, activities and decisions" that seems quite understandable and clear [1]. The BPM lifecycle itself starts with the modeling stage. More formally BPM is considered as the managerial discipline that uses the technologies for the process oriented management [2]. According to paper [3], a high-level overview of the BPM lifecycle includes the four key activities: model (create a business process model to be used for analysis or enactment), enact (use a process model to control and support concrete cases), analyze (analyze a process using a process model and/or event log), and manage (all other activities, e.g., adjusting the process, reallocating resources, or managing large

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collections of related process models). Business process models are mostly used in the design and analysis of information systems and are considered as a good mechanism for communication among the stakeholders [4]. Therefore, it is important to ensure high quality of created business process models. A business process model of poor quality can disturb business process implementation and execution, as well as its performance [5]. While a high-quality model is expected to be accepted by stakeholders and thus prevents problems of business process implementation, deployment, execution, and continuous improvement [6].

Business process modeling is an essential part of understanding and improving the activities that an enterprise uses to achieve its business goals, however there was no generally accepted framework of process model quality [7]. To the best knowledge of the authors there is still no such quality framework nowadays. Therefore, this paper proposes evaluation criteria of business process model quality, which are based on existing best practices and guidelines together with measures and their corresponding thresholds. The objective of such criteria is to calculate a numerical degree to which a business process model fulfills requirements of process modeling guidelines. This definition of business process model quality is derived from ISO 9001's definition of quality "degree to which a set of inherent characteristics fulfills requirements" [8].

Nowadays the Business Process Model and Notation (BPMN) is the leading standard for modeling business processes. It is provided by the Object Management Group (OMG) and it is used by business professionals as a standard notation allowing not only internal communication of the business procedures, but business-IT alignment and collaboration between business partners as well [9]. According to the latest survey in the domain of business process modeling [10], BPMN diagrams are used by 64% of respondents. After the BPMN 2.0 notation came up, and original name "Business Process Modeling Notation" was changed to "Business Process Model and Notation", the graphic notation had been extended with the metamodel, the XML-based (eXtensible Markup Language) exchange file format, and the execution semantics.

Section 2 and its subsections describe related work in the field of business process quality management, BPM maturity models, and business process model quality evaluation. Business process modeling guidelines, measures, derived quality criteria, generalized criteria, weights calculation for these generalized criteria, translation of crisp quality values into linguistic values, and a method for evaluation of business process model quality are demonstrated in Sect. 3. Section 4 includes software requirements capturing, details of information technology design, and implementation aspects. This section also describes the software architecture and a metamodel of business process data. Section 5 briefly describes software usage, as well as outlines statistical analysis of business process model quality with results discussion and given recommendations. Conclusion is provided and the future work in this research field is formulated.

2 Related Work

2.1 Business Process Quality Management and Maturity Models

According to Tobias and Kern [12], BPM has its roots in Total Quality Management (TQM) appeared in late 1980s and in Business Process Reengineering (BPR) introduced in early 1990s by Hammer, Champy, and Davenport. One of BPM meanings is related to a four-step Plan-Do-Check-Act (PDCA) quality management cycle by Deming and Shewhart [12]. PDCA is a continuous cycle that consists of the following four successive stages: planning for change (plan), execution of the plan (do), evaluation of results (check) and standardization of the new, improved process (act) [13]. While TQM takes a scientific, statistical, and evidence-based approach for detecting and reducing quality anomalies through incremental quality improvement, BPR focuses on systematically reducing process anomalies through visualization and radical quality performance improvement [14]. Early BPM concept was focused largely on the technical aspects of process control, especially those related to production. Under the TQM paradigm, BPM gradually evolved from the narrow focus on technical issues toward a more management direction [13]. Authors of [12] define BPM as the management approach that focuses on business processes. A business process is defined as a horizontal sequence of activities that transform an input (need) to an output (result) to meet the needs of customers or stakeholders [12].

BPM took a more concrete form during the major revision of the ISO 9001 standard in 2000, and then in 2008 and 2015. These versions of the standard made the shift of focus from individual quality management requirements to a more holistic process-oriented approach [13]. The ISO 9001:2015 standard considers process approach as the one of its quality management principles. A process approach means an organization manages their business as a system of processes. A process is described as a set of interrelated or interacting activities that use inputs to deliver an intended result. Process inputs and outputs might be tangible (e.g. materials, components or equipment) or intangible (e.g. data, information or knowledge) [8]. Since the revision of ISO 9001, BPM has had a central role in the implementation of all standards. Now it has a role of an integrated mechanism of managing the processes [13].

Since BPM is heterogeneously defined in the literature, a need of a standardized BPM framework has inspired appearance of BPM Maturity Models (BPMMM) [12]. Authors of review [15] refer to a maturity model as to the sequence of discrete maturity levels used to assess processes in one or multiple business domains. Maturity levels represent expected or typical evolution of these processes [15]. For example, Capability Maturity Model Integration (CMMI) in the area of software engineering emerged at the beginning of 1990s as a means to improve software development processes to achieve higher quality, and has been used since then by hundreds of organizations worldwide. The success of CMMI inspired the development of several maturity models in other domains, including BPM [15]. In paper [16], the importance and impact of an organization's business process maturity on overall performance has been presented. BPMMM presented in this study is based on the maturity model that was defined for the maturity of software development capability [2]. According to [16], with the increasing level of an organization's BPM maturity, the importance and impact of BPMN models (since BPMN is considered as de-facto standard in business process modeling) is increasing and becoming critical for overall BPM success.

Table 1 describes BPM maturity levels with respect to the dimensions of awareness of business processes (ABP), documentation of business processes (DBP), monitoring of business processes (MBP), and refinement of business processes (RBP).

Table 1. BPM maturity levels from a business process modeling viewpoint.

Maturity level	ABP	DBP	MBP	RBP	Business process modeling viewpoint
(0) Non-existent	None	None	None	None	None
(1) Ad-Hoc	Some	None	None	None	Top-level "process landscape"
(2) Repeatable	All	Some	None	None	Non-standardized process diagrams
(3) Defined	All	All	Some	None	BPMN models at the descriptive level
(4) Managed and Measurable	All	All	All	Some	BPMN models with defined process monitoring activities
(5) Optimized	All	All	All	All	BPMN models accessible over a repository or cloud-based solution

Therefore, to the best our knowledge, the maturity model presented in [16] is the only maturity model that, unlike BPMMMs reviewed in [15], presents the role of business process modeling and supporting tools in different process maturity levels.

2.2 Business Process Model Quality

The goal of business process modeling is the representation of organizational activities, so that current processes may be analyzed and improved. Business process modeling is not only a requirement for ISO 9001 quality standards or BPM maturity models; it plays an important role in the implementation of work-flow management [17].

Modeling helps visualize the important steps in a business process, how they are related to each other, which actors and information systems are involved in carrying out various activities, and where communication takes place with external parties. Business process models are usually described in a visual way, using figures that are connected to each other and supported by textual annotations [17]. Graphical notation of the BPMN 2.0 specification includes following elements to describe business processes (see Fig. 1) [18].

BPMN business process diagrams basically describe business processes in terms of events and actions connected through control flows that indicate valid sequences in the process execution. Gateways are special nodes connected through control flows that indicate whether the process executes in parallel (AND), alternatively (XOR) or optionally (OR). The beginning of the process is denoted by a start event and its conclusion by

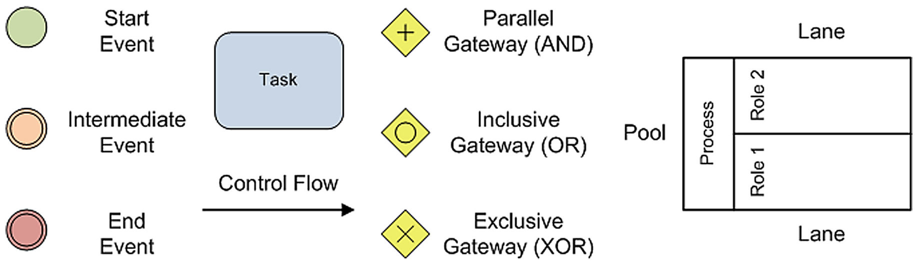

Fig. 1. Core elements of BPMN graphical notation.

a set of end event nodes. Each pool represents a process itself, while each lane represents a human participant in the activity [18].

Authors of [17] proposed a business process model quality framework called SIQ (it is "Simple enough to be practically applicable, Integrates the most relevant insights from the BPM field, and deals with Quality" [17]). SIQ defines three quality categories of the business process model: syntactic quality (conformance to the rules of a modeling notation), semantic quality (conformance to a captured process), and pragmatic quality (understandability by readers) [17].

While syntactic quality of designed business process models is checked by modeling software tools and checking semantic quality is hardly possible without human involvement (it requires understanding the domain in question and the exact purpose of the process model [17]), checking pragmatic quality is of interest for research in terms of its automation. Mendling, Reijers, and van der Aalst defined Seven Process Modeling Guidelines (7PMG) framework for creating understandable models that are less error-prone or improving the quality of existing models [17, 19].

As for quantitative process model quality indicators, various metrics and thresholds for such metrics were proposed for BPMN process models [19], such as:

- Size (number of events, gateways, tasks, and sequence flows).
- Gateway mismatch (sum of gateway pairs that do not match each other).
- Connectivity coefficient (ratio of the number of arcs to the number of nodes).
- Control flow complexity (sum over all gateways weighted by their potential combinations of states after the split).

2.3 Conclusion on Related Work Review

Performed review demonstrates that, despite the lack of clear definition and standards, BPM is the dynamically growing management approach adopted by many organizations and considered as the one of quality management principles of ISO 9001 standard. Business process modeling, being the core technique of BPM, plays an important role in business process analysis and improvement, and also facilitates increasing of BPM maturity of an overall organization. Present State-of-the-Art shows there are heterogeneous metrics and corresponding thresholds used to measure business process model quality. The generalized measure that could combine viewpoints of size, gateway mismatch, nodes connectivity, and control flow complexity is not revealed yet.

3 Evaluation of Business Process Model Quality

3.1 Business Process Model Measures

Out of the seven process modeling guidelines provided by the 7PMG framework, we have left only five by merging G1 and G7, and avoiding the guideline G6 [20]. It is suggested, that G7 "Decompose a model with more than 31 elements" might be applied as the consequence of G1 "Do not use more than 31 nodes", so that is why these guidelines were merged into the single rule R1 "Do not use more than 31 nodes or decompose a model with more than 31 elements if possible". The guideline G6 "Use verb-object activity labels" is avoided, since it belongs to the semantic process model quality, while the pragmatic quality is considered in this study. Remaining four guidelines G2–G5 are adopted as rules R2–R5 respectively (see Table 2). For the selected five BPMN modeling rules based on the 7PMG guidelines we need to define criteria that might quantifiably evaluate the degree of fulfillment of each of the modeling rules R1–R5 by the analyzed BPMN process model. Such criteria might be derived from the following measures.

Total Number of Nodes (TNN). The size of the collection of business process model nodes, $TNN = |N|$.

Number of Invalid Elements (NIE). Number of nodes with invalid inputs or outputs (G2 recommendation should be applied for all types of nodes, not only gateways; also it is important to distinguish intermediate events from start and end events, since they have different nature and, therefore, connection rules):

$$\begin{aligned} NIE = & \sum\nolimits_{t \in T} \left(\left| t^{in} \right| \neq 1 \vee \left| t^{out} \right| \neq 1 \right) + \sum\nolimits_{e \in E \setminus (E_s \cup E_e)} \left(\left| e^{in} \right| \neq 1 \vee \left| e^{out} \right| \neq 1 \right) \\ & + \sum\nolimits_{e \in E_s} \neg \left(\left| e^{in} \right| = 0 \wedge \left| e^{out} \right| = 1 \right) + \sum\nolimits_{e \in E_e} \left(\left| e^{in} \right| = 1 \wedge \left| e^{out} \right| = 0 \right) \\ & + \sum\nolimits_{g \in G} \neg \left[\left(\left| g^{in} \right| = 1 \wedge \left| g^{out} \right| > 1 \right) \vee \left(\left| g^{in} \right| > 1 \wedge \left| g^{out} \right| = 1 \right) \right]. \end{aligned} \tag{1}$$

Where:

- T is the collection of tasks $t \in T$, E is the set of events $e \in E$, G is the collection of gateways $g \in G$;
- E_s is the collection of start events $E_s \subseteq E$, while E_e is the collection of end events $E_e \subseteq E$;
- $\left| n^{in} \right|$ is the size of the set of inputs of the node, $\left| n^{out} \right|$ is the size of the set of outputs of the node, $n \in N = T \cup E \cup G$.

Number of Start Events (NSE). The size of the set of start events, which is the subset of the collection of events, $NSE = |E_s \subseteq E|$.

Number of End Events (NEE). The size of the set of end events, which is the subset of the collection of events, $NEE = |E_e \subseteq E|$.

Number of Mismatched Gateways (NMG). The count of gateways that do not have matching gateways. It is calculated as the total difference between numbers of split gateways (with 1 input and more than 1 output) and join gateways (with more than 1 input and 1 output) of each type $c \in C$, $C = \{and, xor, or\}$:

$$NMG = \sum_{c \in C} \left| \sum_{g \in G_c} \left(\left| g^{in} \right| = 1 \wedge \left| g^{out} \right| > 1 \right) - \sum_{g \in G_c} \left(\left| g^{in} \right| > 1 \wedge \left| g^{out} \right| = 1 \right) \right|. \tag{2}$$

Where G_c is the collection of gateways of type $c \in C$.

Total Number of Gateways (TNG). The size of the set of gateways, $TNG = |G|$.

Total Number of Inclusive (OR) Gateways (TNI). The size of the collection of inclusive (OR) gateways, which is the subset of the set of gateways, $TNI = |G_{or} \subseteq G|$.

3.2 Criteria of Business Process Model Quality

Continuous Quality Criteria. Using business process model measures proposed in the previous subsection, we have formulated criteria of business process model quality $r_i, i = \overline{1, 5}$ (see Table 2). These criteria are based on the set of modeling rules R1–R5 derived from the 7PMG recommendations for business process modeling [20].

Table 2. Modeling rules and corresponding continuous criteria based on the 7PMG framework.

BPMN modeling rule	Criteria equation
R1: Do not use more than 31 nodes or decompose a model with more than 31 elements if possible (merged G1 and G7)	$r_1 = \begin{cases} 1, TNN \le 31, \\ \frac{31}{TNN}, TNN > 31 \end{cases}$
R2: Avoid nodes with invalid inputs or outputs (1) (refined G2)	$r_2 = \begin{cases} 1, TNN = 0, \\ 1 - \frac{NIE}{TNN}, TNN > 0 \end{cases}$
R3: Avoid usage of multiple start or multiple end events or missing events (G3)	$r_3 = \min\left\{ \frac{1}{1+(NSE-1)^2}, \frac{1}{1+(NEE-1)^2} \right\}$
R4: Avoid gateways mismatch (2) (G4)	$r_4 = \begin{cases} 1, TNG = 0, \\ 1 - \frac{NMG}{TNG}, TNG > 0 \end{cases}$
R5: Avoid inclusive (OR) gateways (G5)	$r_5 = \begin{cases} 1, TNG = 0, \\ 1 - \frac{TNI}{TNG}, TNG > 0 \end{cases}$

Each of these criteria $r_i, i = \overline{1, 5}$ serves to quantifiably evaluate the degree to which the analyzed business process model fulfills corresponding rules R1–R5. Therefore, calculated values of these criteria belong to the interval $r_i \in [0, 1], i = \overline{1, 5}$.

Discrete Quality Criteria. In case if discrete criteria are required in order to strictly define if the business process model under evaluation does or does not correspond to the introduced modeling rules R1–R5, previously mentioned criteria $r_i, i = \overline{1,5}$ could be calculated using the following equations (see Table 3). Unlike the equations outlined in Table 2, proposed discrete criteria are binary values $r_i \in \{0, 1\}, i = \overline{1,5}$ that do not describe the degree to which the analyzed business process model fulfills the rules R1–R5, but measure exact match or mismatch of the model under evaluation to each of R1–R5 rules.

Table 3. Modeling rules and corresponding discrete criteria based on the 7PMG framework.

BPMN modeling rule	Criteria equation
R1: Do not use more than 31 nodes or decompose a model with more than 31 elements if possible (merged G1 and G7)	$r_1 = \begin{cases} 1, TNN \leq 31, \\ 0, TNN > 31 \end{cases}$
R2: Avoid nodes with invalid inputs or outputs (1) (refined G2)	$r_2 = \begin{cases} 1, NIE = 0, \\ 0, NIE > 0 \end{cases}$
R3: Avoid usage of multiple start or multiple end events or missing events (G3)	$r_3 = \min\left\{\begin{cases} 1, NSE = 1, \\ 0, NSE \neq 1 \end{cases}, \begin{cases} 1, NEE = 1, \\ 0, NEE \neq 1 \end{cases}\right\}$
R4: Avoid gateways mismatch (2) (G4)	$r_4 = \begin{cases} 1, NMG = 0, \\ 0, NMG > 0 \end{cases}$
R5: Avoid inclusive (OR) gateways (G5)	$r_5 = \begin{cases} 1, TNI = 0, \\ 0, TNI > 0 \end{cases}$

By introducing these criteria $r_i, i = \overline{1,5}$ (both continuous and discrete) we did contribute to the State-of-the-Art, however, the need for the generalized quality criterion of business process model still exists.

3.3 Generalized Criteria of Business Process Model Quality

Weighted Sum Model. In order to define the generalized criterion for BPMN process model quality evaluation, we used the weighted sum model (WSM) [21]. It is applicable in this case, since all the criteria are expressed in the same unit. Hence the generalized criterion of business process model quality might be defined as following:

$$PMQ = \sum\nolimits_{i=1}^{5} w_i \cdot r_i. \tag{3}$$

Where $w_i, i = \overline{1,5}$ are the relative weights of importance of the criteria $r_i, i = \overline{1,5}$.

Values of the relative weights are based on the ranks of 7PMG recommendations defined in [20] (see Table 4). Therefore, the generalized quality criterion takes values in the interval $PMQ \in [0, 1]$ (3), where 0 stands for the poor quality, while 1 indicates the best quality of a business process model.

Table 4. Ranks of process modeling rules [20] and weights of corresponding criteria.

Business process modeling rules	R1	R2	R3	R4	R5
7PMG ranks	80.5	86.5	101	58.5	104
Introduced quality criteria of process modeling	r_1	r_2	r_3	r_4	r_5
Weights of the quality criteria	0.21	0.19	0.16	0.28	0.16

Pessimistic Decision Attitude. The WSM (3) considered above is simple and suitable for the generalized criterion of business process model quality. But the WSM still has limitations: some criteria of poor quality $r_i, i = \overline{1,5}$ (which are closer to 0) may be compensated by other criteria $r_i, i = \overline{1,5}$ of high quality (which are closer to 1). If this is not allowed for generalized criterion of business process model quality, another one aggregate function should be introduced.

Therefore, instead of the WSM, there can be used a pessimistic decision attitude and the corresponding aggregate function [22]:

$$PMQ = \min_{i=\overline{1,5}} \left\{ (w_i \cdot r_i) \cdot \left(\max_{i=\overline{1,5}} \{w_i \cdot r_i\} \right)^{-1} \right\}. \quad (4)$$

According to the pessimistic strategy, the worst possible outcome $r_i, i = \overline{1,5}$ with the lowest weight $w_i, i = \overline{1,5}$ should be selected [22]. Then BPMN models should be compared to each other in order to select the "best worst" model of a considered business process. The division by max is used here to obtain values of compared products $w_i \cdot r_i$ between 0 and 1, in order the generalized quality criterion could also take values in the interval $PMQ \in [0, 1]$. Negative exponentiation is used here instead of explicit division only to have formula font size large enough to read.

In general, proposed aggregate function (4) of pessimistic decision attitude may help to detect violations of the least important process modeling rules (those with the lowest corresponding weights $w_i, i = \overline{1,5}$) in order to achieve the best quality of evaluated business process models.

Weights Calculation. As it is mentioned in [20], these 7PMG ranks were obtained as the result of questioning of 21 professionals in business process modeling. All the participants had to estimate the potential of each 7PMG recommendation to impact the business process model understandability. The questionnaire used a scale of 1 to 7, where a rank of 1 represents the highest impact, while a rank of 7 indicates the lowest impact. Final ranks of each 7PMG recommendation were obtained by accumulation of all ranks given by participants [20].

For the rule R1 we used the highest rank of 80.5 among two merged guidelines: G1 (80.5) and G7 (73.5). Ranks of the remaining rules R2–R5 correspond to respective 7PMG elements G2–G5, while G6 is avoided.

Considering the scale used in questionnaire, each quality criteria weight $w_i, i = \overline{1,5}$ was calculated as following:

$$w_i = \tilde{v}_i \cdot \left(\sum\nolimits_{j=1}^{5} \tilde{v}_j\right)^{-1}, i = \overline{1,5}. \quad (5)$$

Where $\tilde{v}_j, j = \overline{1,5}$ is the relative value of the rank of j business process modeling rule. Negative exponentiation is used here also only to have readable formula font size.

Relative ranks of business process modeling rules R1–R5 are obtained using the following equation:

$$\tilde{v}_i = {}^{1}\!/_{v_i} \cdot \min_{j=\overline{1,5}}\{v_j\}, i = \overline{1,5}. \quad (6)$$

Where $v_j, j = \overline{1,5}$ is the value of the rank of j business process modeling rule.

Equations (5) and (6) were used to find corresponding weights $w_i, i = \overline{1,5}$ (see Table 4) for accumulated ranks $v_j, j = \overline{1,5}$ of 7PMG presented in [20]. Since lower ranks represent higher impact on process model understandability and vice versa, we had to obtain higher weights for lower ranks and lower weights for higher ranks. This is why we use min in (6) to calculate the relative ranks of business process modeling rules.

Quality Measurement Beyond 7PMG. Previously mentioned business process modeling rules R1–R5, corresponding quality criteria, and their weights are based on the 7PMG guidelines and their respective ranks [20]. However, these weights are based on ranks given by certain group of specialists with their own preferences and viewpoints on business process model quality. Also, specific quality measures may change over the time making the ranks of 7PMG guidelines, which were given once upon a time, could be found irrelevant at the moment they are used in future. Whereas 7PMG ranks could be used as the starting point, the technique of their evolution depending on the analyzed business process models should be considered.

Therefore, we would like to propose a model to describe changes of process modeling rule weights $w_i, i = \overline{1,5}$ over time, taking into account 7PMG ranks and derived weights (5) as the starting point. We assume that ranks of business process modeling rules R1–R5 may be revised over time (in compare to original ones [20]), taking into account frequency and severity of violations of such rules in the sequence of evaluated business process models. Therefore, respective weights $w_i, i = \overline{1,5}$ within the generalized quality criteria (3) and (4) should be changed as well.

Proposed model considers that rank of a certain rule among R1–R5 should be directly proportional to the frequency of such rule have been violated in evaluated business process models. This simple assumption could be described using the following differential equations, each of which corresponds to the respective rules of R1–R5:

$$\frac{\partial y_i(x_i)}{\partial x_i} = x_i, y_i(0) = w_i, i = \overline{1,5}. \quad (7)$$

Where x_i is the frequency (number of times) the i-th business process modeling rule has been violated, which means $r_i < 1, i = \overline{1, 5}$, while y_i describes changes of the i-th rule weight, assuming the initial conditions $y_i(0) = w_i$ as weights of process modeling rules obtained in (5) and based on the ranks of 7PMG guidelines [20].

These differential Eqs. (7) could be solved by integrating their left and right sides with respect to $x_i, i = \overline{1, 5}$, so the following solutions could be obtained:

$$\int \frac{\partial y_i(x_i)}{\partial x_i} dx_i = \int x_i dx_i, i = \overline{1, 5}$$
$$\Rightarrow y_i(x_i) = \frac{x_i^2}{2} + C_i, i = \overline{1, 5}. \tag{8}$$

Where C_i are the arbitrary constants, which could be found by substituting the initial conditions $y_i(0) = w_i$ into corresponding equations $y_i(x_i)$ obtained after integrating their both sides (8); it is found that $C_i = w_i, i = \overline{1, 5}$.

Hence, changing of business process modeling rule weights could be described using the following equation:

$$y_i(x_i) = 0.5 \cdot x_i^2 + w_i, i = \overline{1, 5}. \tag{9}$$

This means when a certain BPMN process model is evaluated and respective quality criteria $r_i, i = \overline{1, 5}$ are calculated (either continuous or discrete ones), the frequency values $x_i, i = \overline{1, 5}$ should be updated, changes of weights $w_i, i = \overline{1, 5}$ should be calculated according to (9), and final re-arranged weights should be obtained after calculating relative ranks as following:

$$w_i = y_i(x_i) \cdot \left(\sum\nolimits_{j=1}^{5} y_i(x_i)\right)^{-1}, i = \overline{1, 5}. \tag{10}$$

Even though proposed business process modeling rules R1–R5 and corresponding quality criteria $r_i, i = \overline{1, 5}$ are derived from 7PMG guidelines, the modeling rule ranks now could be changed over time – the more BPMN process models will be evaluated, the more substantially initial weights (see Table 4) will be affected.

This initial proposal demonstrates how the future software system for quality evaluation of business process models may "learn" over time in order to remain relevant to the state-of-the-art of business process modeling (for example, some rules may not be violated for a long time but still have significant weights that "hide" more relevant rules violated frequently within a sequence of analyzed business process models).

3.4 Translation of Quality Criterion Values into Linguistic Values

Inspired by the approach Shown in [6], we propose the procedure of translation from crisp quality values represented by the *PMQ* criterion into linguistic values (Table 5). While [6] proposes fuzzification of multiple business process model measures and inference procedure in order to obtain the linguistic values of process model understandability and modifiability, our approach is based on the translation of the generalized quality criterion into the single linguistic value according to the harrington desirability scale (see Table 5) [23], which describes the quality of the business process model based on multiple criteria (see Tables 2 and 3). The last row in Table 5 shows that it is not possible to assess the quality level of business process model if it does not contain any nodes $|N| = 0$ or even if it does not contain any tasks $|T| = 0$.

Table 5. Quality criterion translation into linguistic values based on the Harrington scale [23].

Quality level	Thresholds	Quality level	Thresholds
Good	$0.8 \le PMQ \le 1$	Well	$0.64 \le PMQ < 0.8$
Satisfied	$0.37 \le PMQ < 0.64$	Poor	$0.2 \le PMQ < 0.37$
Bad	$0 \le PMQ < 0.2$	Indefinable	$\lvert N\rvert = 0 \vee \lvert T\rvert = 0$

Therefore, considered *PMQ* thresholds and quality levels, described in Table 5 above, could be used to formulate the following quality assessment function:

$$d(PMQ) = \begin{cases} Good, & 0.8 \le PMQ \le 1, \\ Well, & 0.64 \le PMQ < 0.8, \\ Satisfied, & 0.37 \le PMQ < 0.64, \\ Poor, & 0.2 \le PMQ < 0.37, \\ Bad, & 0 \le PMQ < 0.2, \\ Indefinable, & |N| = 0 \vee |T| = 0. \end{cases} \tag{11}$$

This function takes *PMQ* numerical values calculated using the WSM (3) or pessimistic decision (4) models, and translates it into the linguistic values defined by the Harrington desirability scale (see Table 5).

3.5 Method for Evaluation of Business Process Model Quality

Evaluation of business process model quality is based on materials outlined in Subsects. 3.1–3.4. There were proposed:

- Business process model measures: *TNN*, *NIE*, *NSE*, *NEE*, *NMG*, *TNG*, and *TNI*.
- Business process modeling rules R1–R5 based on 7PMG and corresponding quality criteria $r_i, i = \overline{1,5}$ (continuous and discrete, see Tables 2 and 3).
- Generalized criterion of business process model quality *PMQ*, which may be calculated using the weighted sum model (3) or pessimistic decision model (4).
- Quality assessment function d (11) to translate values of generalized quality criterion *PMQ* into linguistic values based on the Harrington scale (Table 5).

Now these summarized results obtained in Subsects. 3.1–3.4 could be arranged into the following sequence of steps of the proposed method of business process model quality evaluation:

1. Process a BPMN business process model taken as input in order to extract nodes of different types: tasks T, events E, and gateways G.
2. Calculate measures TNN, NIE, NSE, NEE, NMG, TNG, and TNI of the business process model under evaluation using extracted nodes T, E, G and their properties (numbers of incoming and outgoing flows).
3. Select the nature (continuous or discrete) of quality criteria $r_i, i = \overline{1,5}$ that assess conformance of analyzed business process model to corresponding business process modeling rules R1–R5.
4. Re-calculate weights $w_i, i = \overline{1,5}$ that correspond to the business process modeling rules R1–R5, taking into account violations frequencies of each of these rules, using Eqs. (9) and (10).
5. Select the weighted sum (3) or pessimistic decision model (4) to calculate generalized criterion of business process model quality PMQ.
6. Use the quality assessment function (11) to obtain the linguistic quality value based on the previously calculated PMQ value.

Described steps of the proposed method are demonstrated in Fig. 2.

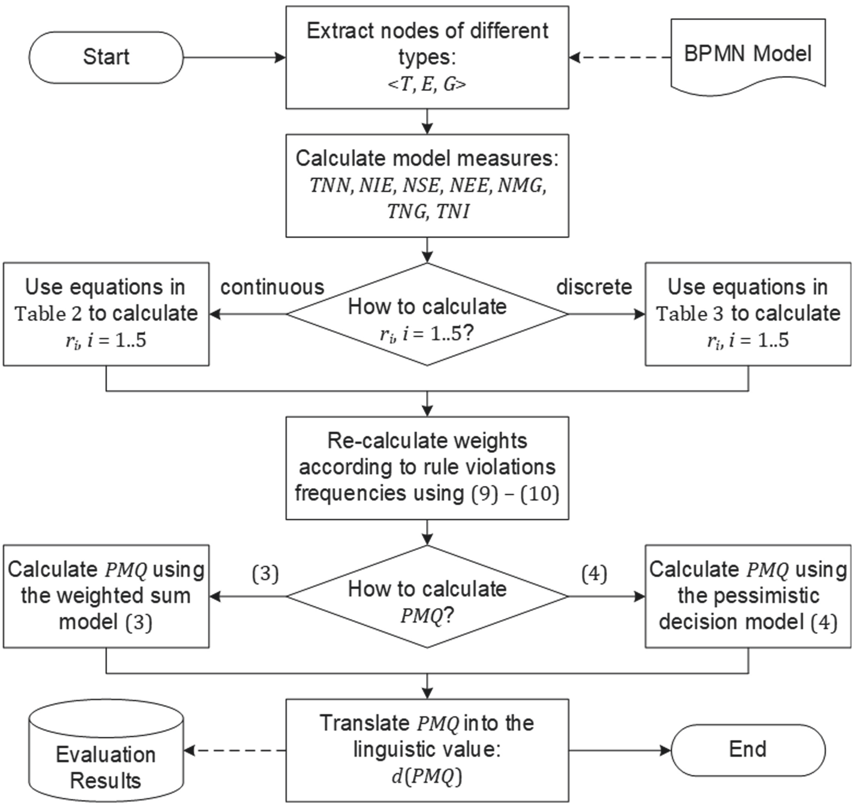

Fig. 2. Flowchart of the proposed method.

As it is shown in Fig. 2, business process model evaluation results should be stored in order to compare and select the best models of a business process, detect the most common modeling errors, query and reuse evaluated business process models. All of these capabilities require design and development of an information technology that implements proposed method and provides storage functions.

4 Information Technology for Evaluation of Business Process Model Quality

4.1 Information Technology Requirements and Design

Requirements Capturing. In general software requirements are usually divided into three groups [24]:

- Functional Requirements (FR). FRs include services that should be provided by the system and how the system should react to user inputs.
- Nonfunctional Requirements (NFR). NFRs describe how the system should behave in the context of its quality attributes (security, reliability, usability, maintainability etc.). Besides quality constraints, NFRs could include design constraints (architecture designs, programming languages, frameworks etc.), economic constraints (development costs, maintenance costs etc.), operating constraints (system availability, accessibility etc.), and political constraints (laws and policies to be applied).
- Domain Requirements (DR). DRs mostly are used to specify how particular computations must be performed with respect to the application domain. They usually complement FRs with clarifications and constraints.

According to the introduced classification of software requirements, we have captured the following FRs (see Table 6) provided in the form of user stories, which now are widely used in Agile software development methodologies [25].

Most of the proposed FRs could be also considered as domain requirements:

- FR02 constraints business process model querying by process features (related to particular domain) and quality features (quality criteria, generalized criterion, and linguistic quality level values).
- FR05 elaborates uploading of BPMN files with detection of modeling flaws (violations of modeling rules R1–R5).
- FR06 elaborates evaluation of business process model quality with quality criteria $r_i, i = \overline{1, 5}$, which should be calculated in both continuous and discrete forms, and the generalized criterion *PMQ* that should be calculated also using both weighted sum (3) and pessimistic (4) decision models.

Nonfunctional requirements include the following constraints:

- Quality constraints. No function may exceed 200 LOC (Lines of Code) and source code that violates consistent coding style may not exceed 20% of total LOC in order to achieve better maintainability and readability.
- Design constraints. Microservice architecture should be implemented with NodeJS platform, JavaScript programming language, and Express web framework.

Table 6. Captured FRs for the information technology.

FR number	User	Task	Goal
FR01	Guest	Log in	Access user workspace
FR02	Model reviewer	View model collection	Search models by file name
			Query models by process features
			Query models by quality features
FR03		Log out	Leave user workspace
FR04	Model designer	Invite user	Provide team members with the access
FR05		Upload BPMN file	Detect modeling flaws
FR06		Evaluate model quality	Calculate quality criteria
			Calculate generalized quality criterion
FR07		Create results report	Print report (or save as a file)

– Economic constraints. Uncommented code may not exceed 30% of all units (modules, classes, functions etc.) and unit tests must ensure at least 80% coverage in order to reduce maintenance expenses.
– Operating constraints. Automatic source code deployment from the version control repository to cloud platform should be configured.

System Design. Microservice software architecture (MSA) has been chosen to design the information technology for evaluation of business process model quality. MSA is based on single responsibility principle, which considers granularity of microservices based on their functional capabilities. Unlike the monolithic architecture, MSA offers less coupled modules, which could be frequently deployed, easily managed and maintained, and which are more scalable and reliable [26].

While MSA is implemented using the NodeJS platform, Express web framework, and JavaScript (JS) programming language, web client is implemented using HTML (HyperText Markup Language) markup, CSS (Cascading Style Sheets) styles, and JS scripts. Detailed system architecture is presented in Fig. 3 using UML software components diagram. This diagram demonstrates main components of client web application, server web application, and the database.

Client web application is described with all of its components (JS scripts, HTML pages, and CSS styles), except JS libraries: jQuery that implements interaction with the server using AJAX (Asynchronous JavaScript and XML) approach, bpmn-viewer that serves to display BPMN business process diagrams, bootstrap.js that implements responsive behavior of web pages; and CSS library Bootstrap that provides attractive user interface of web pages.

At the current stage of software development, it is deployed on the single node (see Fig. 3) using Heroku cloud PaaS (Platform as a Service) platform. Therefore, services

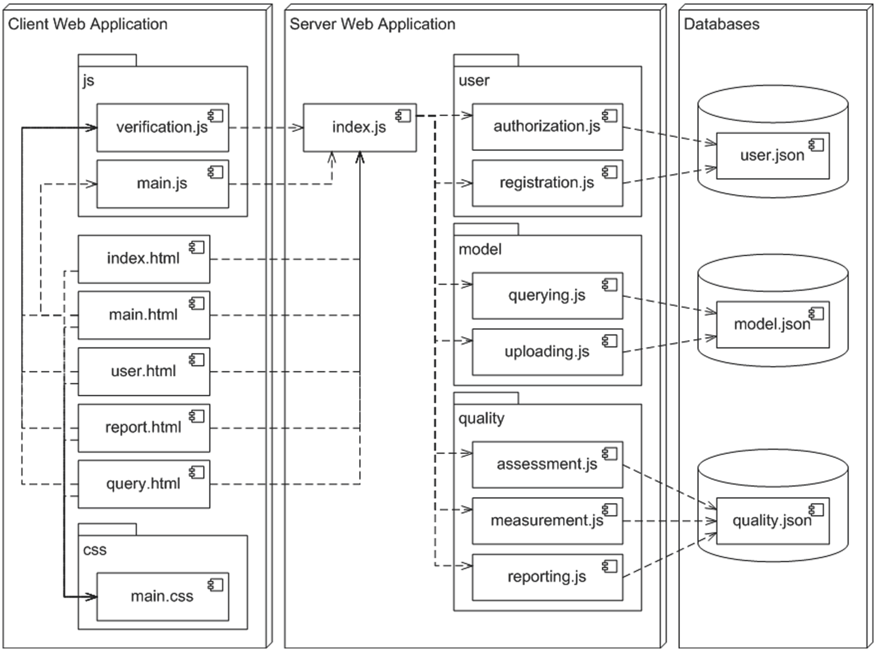

Fig. 3. Software components diagram.

that manage user accounts, upload and query business process models, perform quality assessment and store result reports are implemented as NodeJS modules, however further iterations of the software development may include decomposition into separate nodes responsible for each of services.

According to the diagram in Fig. 3, there are three independent modules:

– User. It is responsible for user registration and authorization.
– Model. It is responsible for business process model uploading and querying.
– Quality. It is responsible for evaluation of business process model quality, results reporting and saving to the database.

Also at the current stage the tiny and lightweight database tool StormDB, based on JSON-like (JavaScript Object Notation) object collections, is used. Further development of the information technology may include transfer from StormDB to production database management systems such as MongoDB, CouchDB, and Cassandra, or other NoSQL solutions. More details about business process data storing and querying are outlined in following subsection.

4.2 Storing and Querying Business Process Data

In order to store analyzed BPMN models in a way they can be queried and reused, we have chosen the ontology defined in Zachman Framework and elaborated by authors of

[11], since its Business Model perspective describes aspects used by business process owners and other involved stakeholders.

We also used practitioners experience in categorization of BPMN artifacts according to Zachman Framework [27]. This categorization was made because of the nature of BPMN elements [28]:

- What. Data stores and data objects represent systems and documentations (reports, forms, guides etc.) respectively that process stakeholders and other involved people are familiar with. These elements show what data is used by process activities.
- Where and Who. Pools represent process boundaries, while lanes represent process participants. These elements show where and by whom process activities are executed.
- When. Events indicate when processes start, end, or when something happen when processes are executed. These elements show when processes begin, end or certain activities within processes are triggered.
- Why. Gateways represent decisions driven by asked questions and various options (minimum two). These elements show why different process paths are executed.
- How. Activities represent tasks and subprocesses to which business processes may be broken down. These elements show how processes are carried out.

Such categorization by Zachman Framework (ZF) aspects (What, Where, When, Why, Who, and How [11]) is demonstrated in Table 7.

Table 7. Business process model deconstruction using Zachman Framework [27].

ZF aspect	ZF aspect description	BPMN element
What	Data	Data store, data object
Where	Location	Pool
When	Time	Event (start, intermediate, end)
Why	Motivation	Gateway (exclusive, inclusive)
Who	People	Lane
How	Process	Activity (task, subprocess)

While "Business Model" perspective of the Zachman Framework ontology [11] and business process model deconstruction using Zachman Framework [27] describe process features (activities, people, events, data etc.), there are also process model quality features expressed as numerical measures, criteria, and linguistic quality levels, which should be also included into the metamodel of business process data described using BPMN models (see Fig. 4).

Using proposed metamodel, insights about analyzed BPMN models and business processes they describe could be inferred. For example, there could be asked:

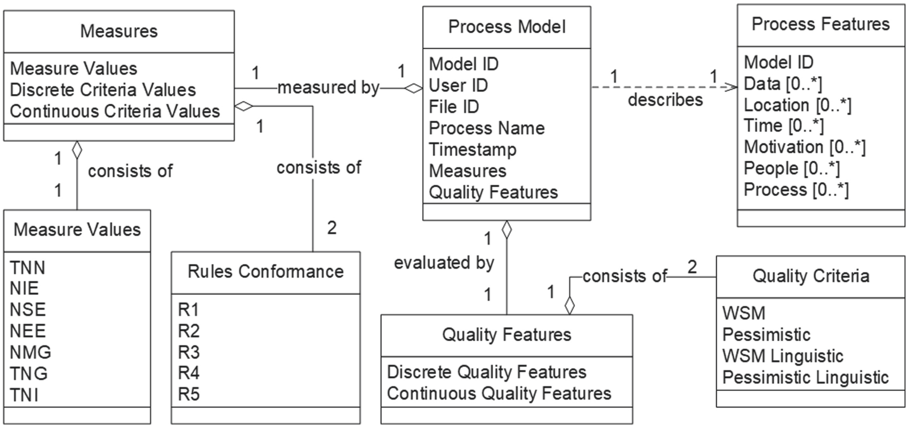

Fig. 4. Metamodel of business process data described using BPMN models.

– Which tasks are associated with violations of the rule R2?
– Which events are associated with violations of the rule R3?
– Which motivations are associated with violations of the rule R4?
– Which elements are associated with business process models of good quality?

And many other queries that may be used to define dependency between business process features and correctness of business process models.

The metamodel shown in Fig. 4 is used to build the data model for storing and querying business process data. Since the NoSQL database tool StormDB is used for storing and querying business process data, we are dealing with collections of JSON objects [29] that should reflect the metamodel.

5 Results and Discussion

5.1 Software Usage Examples

This section demonstrates basic usage examples of developed information technology. As it was shown in Table 6, users are able to login, upload BPMN files, measure quality, and explore collections of processed models. BPMN files could be dragged and dropped to the web page or selected from user's disk through the dialog window. After BPMN file is uploaded, business process models are extracted and measures *TNN*, *NIE*, *NSE*, *NEE*, *NMG*, *TNG*, *TNI* are calculated.

Then, according to the introduced method (see Fig. 2), user may select the quality measure type (discrete or continuous) and the quality measurement model (WSM or pessimistic) in order to calculate aggregate quality measure of a given business process model and its linguistic quality value (see Fig. 5).

Quality measurement reports contain evaluations of business process model correspondence to process modeling rules R1–R5. Measurements are demonstrated using both binary (to provide simple yes/no estimations) and continuous (to provide estimations of conformance degree) criteria (see Fig. 6).

General estimation: incorrect

Select quality measure type

- ○ Hard (0 if a model corresponds to a rule, or 1 otherwise)
- ◉ Soft (correspondence is measured within the 0-1 range)

Select quality model

- ◉ Allow compensation of poor measures by good ones (Weighted Sum)
- ○ Forbid compensation of poor measures by good ones (Min)

Quality: 0.56 (Satisfied)

Fig. 5. Form to select quality measure type and quality model.

Do not use a lot of elements or decompose a model with a lot of elements if possible: Yes (1.00)

Avoid nodes with invalid inputs or outputs: No (0.80)

Avoid usage of multiple start or multiple end events or missing events: Yes (1.00)

Avoid gateways mismatch: No (0.00)

Avoid inclusive (OR) gateways: Yes (1.00)

Fig. 6. Form to check business process model correspondence to modeling rules.

For example, there were obtained following suggestions for improvement of business process model demonstrated in figure above (see Fig. 7):

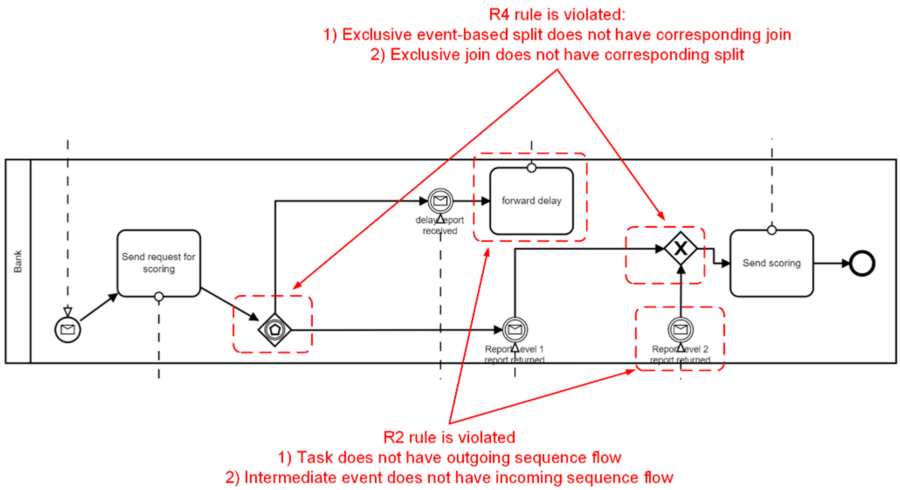

Fig. 7. Examples of detected violations of business process modeling rules.

– The model contains elements with invalid inputs or outputs. Task “forward delay” has only outgoing message flow that is used for business process collaboration and does not affect current workflow, while outgoing sequence flow is missing for this tasks. Also intermediate event “Report level 2 report returned” has only incoming message flow, while incoming sequence flow is missing. Such drawbacks may signalize about modeling errors or even business process shortcoming when workflow is interrupted with task “forward delay” or event “Report level 2 report returned”.
– The model contains gateways do not have corresponding splits or joins respectively (gateways mismatch). Exclusive event-based split-gateway produces two workflow scenarios, which never merged later using respective exclusive join-gateway. Also exclusive join-gateway does not have respective exclusive choice that should be then merged. Such design flaws bring uncertainty to the business process workflow, by making it hard to understand, follow, and maintain. These are also signals for revision of BPMN model or even a business process itself.

Uploaded and evaluated business process models are stored in the system and then could be used for querying to get insights which business process elements (according to the Zachman ontology model) affect conformance to modeling rules R1–R5. Such insights may help to detect and resolve issues in real business processes, since modeling mistakes usually occur when business activities or business events are understudied, or engaged stakeholders are not identified properly [30]. Therefore, these insights may help not only to bring business process models to required quality level, but also to achieve adequacy of business process models to actual processes, understand, and, if necessary, improve business processes themselves. It is also possible to filter business elements by general model quality or BPMN file name (see Fig. 8).

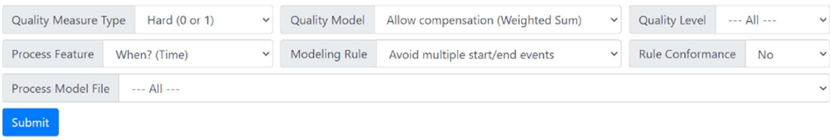

Fig. 8. Form to query collected business process models.

The information technology is still under development, however all captured FRs (see Table 6) are implemented in the software prototype, which is accessible at [31]. There should be used “test” as login and password to access the sandbox account.

5.2 Statistical Analysis of Business Process Model Quality

In order to perform statistical analysis of business process model quality, the software was complemented with batch processing of BPMN models and public GitHub repository of Camunda [32] was used. In total there were processed 6137 business process models. Using discrete quality criteria, we found which modeling rules were violated the most: R4 (3112), R2 (2918), and R3 (2294). Examples of rules R4 and R2 violations were demonstrated in Fig. 7 of previous subsection.

The statistical analysis (Fig. 9) of continuous quality criteria has shown that R4 is the most "vulnerable" modeling rule:

- About 25% of process models (see Max. in Table 8) show absolute violation of the rule R4 ($r_4 = 0$), and about 25% more (see Q1 and Median in Table 8) demonstrate bad, poor, satisfied, or sometimes well structuredness ($0 < r_4 \leq 0.8571$) in terms of R4 process modeling rule.
- About 50% of models (see Median in Table 8) are of good structure in terms of R4 rule ($0.8571 < r_4 \leq 1$), among which about 25% models (see Q3 in Table 8) show absolute conformance to the rule R4 ($r_4 = 1$).

As for rules R2 and R3, the statistical analysis has shown the following results:

- About 50% of models (see Median in Table 8) show absolute conformance to rules R2 and R3 ($r_2 = 1$ and $r_3 = 1$).
- About 25% of models (see Min. in Table 8) show better borderline conformance to R2 than R3 ($r_2 \geq 0.6875$ and $r_3 \geq 0.027$), but there are still a lot of outliers (see Fig. 9) that demonstrate severe violations of R2.

Statistical characteristics of continuous quality criteria are outlined in Table 8.

Table 8. Results of statistical analysis of continuous quality criteria.

Statistical measure	R1	R2	R3	R4	R5
Min	1.0000	0.6875	0.0270	0.0000	1.0000
Q1	1.0000	0.8750	0.5000	0.0000	1.0000
Median	1.0000	1.0000	1.0000	0.8571	1.0000
Q3	1.0000	1.0000	1.0000	1.0000	1.0000
Max	1.0000	1.0000	1.0000	1.0000	1.0000

Violations of rules R1 and R5 are represented only by outliers (for most of models $r_1 = 1$ and $r_5 = 1$). The boxplot chart that demonstrates results of statistical analysis of aggregate quality criteria is also shown in Fig. 9.

Statistical characteristics of aggregate quality criteria are outlined in Table 9.

According to these results, discrete and continuous WSM-based criteria estimated that about 50% of models correspond to all of modeling rules by more than 70% (see Median in Table 9, $PMQ > 0.72$) and only about 25% of evaluated process models correspond to all of the rules by less than 70% (see Q1 in Table 9, $PMQ < 0.7041$).

As for pessimistic criteria, about 50% of models correspond to all of the modeling rules by less than 50% (see Median in Table 9, $PMQ < 0.5$) and about 25% of models do not correspond to all of modeling rules (see Q1 in Table 9, $PMQ = 0$) according to the continuous model. Whereas, according to the discrete model, about 75% of models do not correspond to all of modeling rules (see Q3 in Table 9, $PMQ = 0$).

Analysis of derived linguistic quality measures has shown that the discrete WSM and the continuous pessimistic criterion brought almost the same number of models to

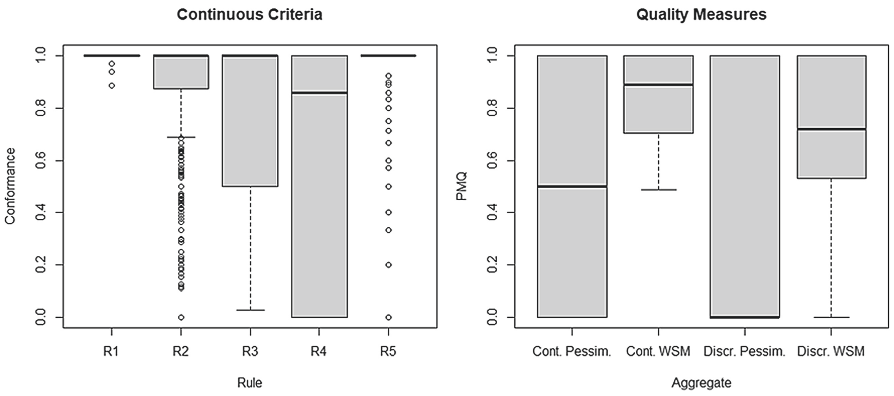

Fig. 9. Statistical analysis of quality criteria.

Table 9. Results of statistical analysis of generalized quality criteria.

Statistical measure	Cont. Opt.	Cont. Pessim.	Discr. Opt.	Discr. Pessim.
Min	0.4887	0.0000	0.0000	0.0000
Q1	0.7041	0.0000	0.5300	0.0000
Median	0.8893	0.5000	0.7200	0.0000
Q3	1.0000	1.0000	1.0000	1.0000
Max	1.0000	1.0000	1.0000	1.0000

"Good" ones (see Fig. 10). While the number of "Good" models evaluated using the continuous criterion based on WSM is twice greater than the number of "Good" models evaluated using the discrete pessimistic criteria (see Fig. 10).

Thus, the discrete WSM-based and continuous pessimistic models may be used to detect "Good" process models with minimum mistakes, while the discrete pessimistic measure better suites when BPMN diagrams with any number and severity of errors must be found. The continuous WSM-based criterion may mislead users with overestimated quality of evaluated process models because WSM tend to compensate criteria of poor quality by high-quality criteria and, therefore, should be used very carefully when the business process model quality is critical.

In order to overcome such disadvantage of WSM and to keep weights of business process modeling rules relevant to evaluated BPMN models, ranks of process modeling rules could be re-arranges using Eqs. (9) and (10).

After all the BPMN models were evaluated, some of initial weights w_i, $i = \overline{1, 5}$ (see Table 4) have changed significantly over time (see Fig. 11).

As it is demonstrated in Fig. 11 above, the ranks of the following business process modeling rules were re-considered:

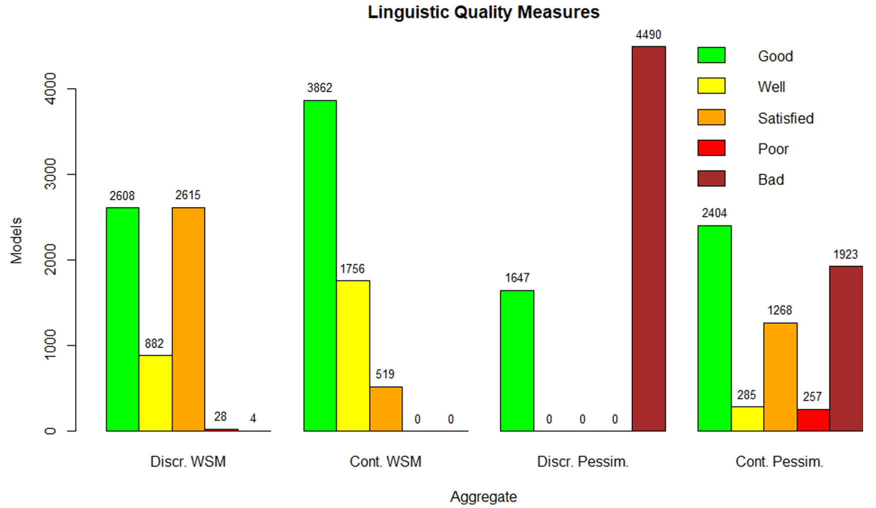

Fig. 10. Statistical analysis of linguistic quality criteria using the stacked bar chart.

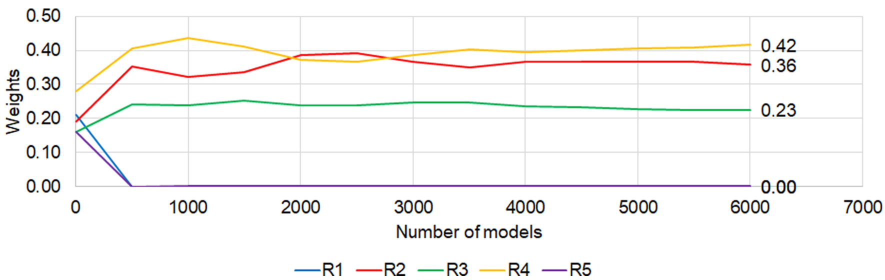

Fig. 11. Changes of process modeling rule weights during processing period.

– Most of evaluated BPMN models were demonstrating conformance to the rules R1 and R5, whereas relatively high corresponding weights $w_1 = 0.21$ and $w_5 = 0.16$ were "hiding" more urgent issues. That is why weights of the rules R1 and R5 have been decreased to $w_1 = 0$ and $w_5 = 0$ respectively.
– The weight of rule R2 has been increased from $w_2 = 0.19$ to $w_2 = 0.36$.
– The weight of rule R3 has been increased from $w_3 = 0.16$ to $w_3 = 0.23$.
– The weight of rule R4 has been increased from $w_4 = 0.28$ to $w_4 = 0.42$.

Considered changes of weights were captured after the whole collection of BPMN models was processed, however, during the processing weights of each modeling rule R1–R5 were changing within following ranges: $w_1 \in [0, 0.21]$, $w_2 \in [0.19, 0.39]$, $w_3 \in [0.16, 0.25]$, $w_4 \in [0.28, 0.44]$, and $w_5 \in [0, 0.16]$. These changes were keeping the system relevant to vital problems of business process modeling – frequent design flaws were attracting more attention in compare to the original priorities [20].

6 Conclusion and Future Work

In this paper we have introduced the set of business process model measures used to evaluate the degree to which BPMN business process models fulfill business process modeling rules. In total there were formulated five process modeling rules R1–R5 based on the state-of-the-art of business process model quality analysis. Then we have proposed five quality criteria of business process modeling, which are based on introduced BPMN model measures and formulated in two variants – as discrete (take values of 0 or 1) and continuous (take values between 0 and 1), and which are used to evaluate conformance of analyzed BPMN models to formulated rules R1–R5. Based on these criteria we have proposed generalized quality criteria based on weighted sum and pessimistic decision models, which crisp values could be translated into linguistic values in order to obtain generalized user-friendly decision on business process model quality. We have also outlined how ranks of process modeling rules R1–R5 could be re-arranged in order to keep them relevant to ongoing issues of business process modeling. In order to summarize theoretical proposals, such as measures and quality criteria, we have formulated the method of business process model quality evaluation.

The information technology is based on the proposed method and was implemented using the microservices architectural approach. Its users are able to upload BPMN files, select types of partial quality criteria (discrete or continuous), as well as types of generalized quality criteria (WSM-based or pessimistic attitude), review quality evaluation reports, and query previously evaluated business process models in order to get insights about their "weak spots" and define whether these are modeling drawbacks or reflections of workflow issues. Nevertheless, detected shortcomings of BPMN models signalize about wrong models that do not reflect real business processes adequately or about problems within real business processes that have led to poor models.

Future research in this field should be devoted to development of models, methods, and software tools to evaluate adequacy of business process models to real processes, how business process efficiency is affected by poor models and vice versa. Directions of future research also include elaboration of methods and models that can be used to increase the quality of both models and depicted business processes.

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Artificial Neural Networks for Recognition of Brain Tumors on MRI Images

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Abstract. In this paper, artificial neural networks for recognition of brain tumors on MRI images are analyzed. This analysis allows to choose the most appropriate neural network architecture and various preprocessing techniques to increase the precision of tumor instance recognition. Understanding the image and extracting information from it to accomplish some result is an important area of application in digital image technology. Image recognition has quickly found its use in medicine and specifically oncology. Precise recognition masks may not be critical in other cases, but marginal recognition errors in medical images may render the results unreliable for clinical use. Therefore, biomedical problems require much higher boundary detection precision to improve further analysis. Various methods and algorithms for image recognition and segmentation are considered. The advantages and disadvantages of neural network architectures (ResNet, U-Net, SegNet, YOLO v3) are considered and analyzed in more detail. Additionally, the analysis of data preprocessing methods was carried out, as well as a study of the input data. Comparison of different artificial neural networks algorithms and architectures will achieve the highest accuracy of recognition. During the comparison, a system of the U-Net architecture with additional processing methods was selected as the final model. Its accuracy reached 94%, which is a significant result compared to manual image recognition.

Keywords: Neural network · Instance recognition · Deep learning · Biomedicine · Neoplasm · MRI image

1 Introduction

In recent years, the field of machine learning (ML) [1] has undergone significant development, and advances in this field have influenced the development of other areas of our lives [2]. Machine learning algorithms are used everywhere: to analyze financial markets, search engines, personalize online advertising, recognize speech or handwriting, and detect online fraud - these are only a small part of what they are capable of [3, 4]. According to scientific publications of the last two decades, artificial neural networks (ANNs) are increasingly used in medicine. They are used in medical diagnostics, image

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recognition in radiology and photomicrographs of damaged cells, interpretation of data in intensive care units and analysis of waveform in the variable signal. In medicine, machine learning methods related to image recognition and processing are most commonly used. These are work with X-rays of the breast for the presence of malignant tumors, the cardiovascular system in order to detect defects in the passage of blood through the vessels, lungs to diagnose lung diseases and more [1, 2]. One of the important areas of ML is computer vision (CV), the technology of machine development that can detect, track, and classify objects. The task of recognition (identification) is to distinguish a specific object among its peers. Segmentation is one of the subtasks of recognition [2]. Segmentation in CV is the process of dividing a digital image into several segments [5–8].

2 Related Works and Problem Statement

Machine learning has found its way into medicine: for example, regression systems based on patient's data can classify them as potentially ill, and pattern recognition systems can analyze medical images and extract certain features for further analysis [9, 10]. One of the most important applications of segmentation is the field of biomedicine. In most cases, video data is obtained by microscopy, tomography, and the like. An example of the information obtained is the measurement of organ size or even blood circulation. This application area also promotes research and provides new information such as the structure of the brain or the effects of certain medical devices.

To detect tumors in the tomographic images, the image is segmented. Computer vision segmentation is the process of dividing an image into several segments. The result of segmenting an image is a set of pixels that together form segments and cover the entire image. All pixels in a segment are similar regarding one or more properties, such as color. Neighboring segments differ significantly in such factors. In the case of this problem, an example of the result of the finished model is the contour of the tumor on the image of a slice of the brain. Certain universal algorithms and methods have been developed for image segmentation, but some classical machine learning methods have also been used to solve segmentation problems. Since there is no common solution to the problem of image segmentation, often such methodologies have to be combined with knowledge of the subject area to find an effective solution to the problem.

The segmentation problem is being considered on a set containing data on 110 patients from The Cancer Genome Atlas, created by National Cancer Institute and National Human Genome Research Institute, USA. The purpose of the project is to systematize data on genetic mutations that contribute to the development of cancer. This project started in 2005 and has been designed for only 3 years. As of 2017, the TCGA project was finalized, but the investigated data has been made open. The TCGA includes samples from more than 11,000 patients for 33 types of cancer. It is by far the largest collection of cancer data [3, 5].

A systematic set of brain MRI images of only one type of cancer, glioma, was selected for the task [6, 7]. Glioma is a very common type of brain tumor: they are diagnosed in 60% of cases. Glioma malignancies are classified according to the World Health Organization classification. In this case, low-grade gliomas (or LGGs) are considered -

these tumors usually have signs of benignity, but can sometimes rise in order, so they are classified as malignant. The set of data with such neoplasms is very relevant because low-grade gliomas are treatable, so the accurate detection and delineation of such a disease are of the utmost importance.

3 Machine Learning Methods for Brain MRI Neoplasm Segmentation

The artificial intelligence industry has been developing since the 1960s and has many different algorithms and approaches for solving computer vision. Quite often, developers have relied on elementary representations of human brain processes (for example, the process of transmitting nerve impulses has become the basis of the perceptron), combining them with approaches of statistics [11]. Consider some methods for solving segmentation problems.

The thresholding algorithm is the simplest method of image recognition and segmentation. It is to replace each pixel of the image with a black pixel if the image intensity is less than some constant T (threshold), or a white pixel if the image intensity is greater than T. Thus, the input black and white image (in shades gray) is converted to binary, i.e. one in which the pixels can take only two values (in this case, black and white). An example of using the threshold algorithm is shown below: Fig. 1 shows the input data for processing, and the result of the thresholding algorithm [12].

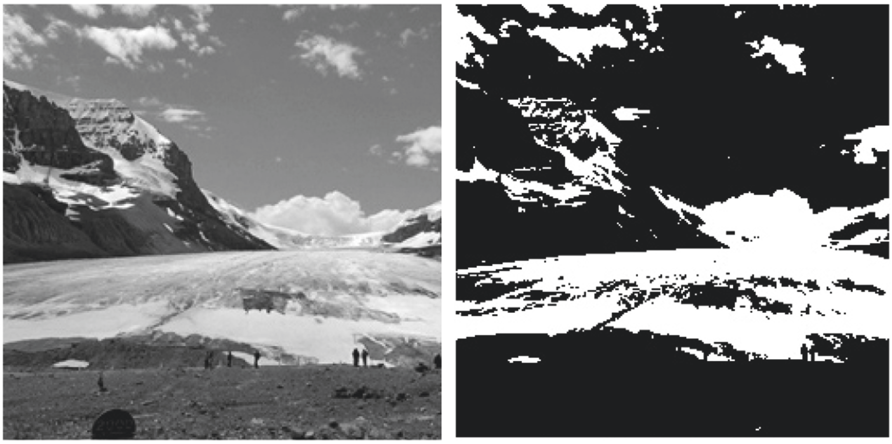


Fig. 1. Input data for processing and the result of the thresholding algorithm.

This method is very simple and can be used in systems with or without a teacher. However, the simplicity of this algorithm affects the accuracy of segment localization: this method should be used in problems where extremely high accuracy of area separation is not required. It should be noted that this algorithm can also be used not as the main algorithm for segmentation, but also for image pre-processing.

Clustering is the task of dividing a sample of data into subsets, or clusters. In this case, each cluster must consist of similar objects, and objects from different clusters must be significantly different. The task of clustering refers to the tasks of statistical processing, and to the class of learning tasks "without a teacher". In our case, the problem can be formulated as a process of clustering pixels in the image. K-means clustering is a popular method of clustering for the ordering of many objects into relatively homogeneous groups, the number of which is indicated by the value of k. The purpose of the method is to divide n observations into k clusters, so that each observation belongs to the cluster with the closest average value. The method is based on minimizing the sum of the squares of the distances between each observation and the center of its cluster, i.e. the function [13]:

$$f(x) = \sum_{i=1}^{k} \sum_{x \in S_i} (x - \mu_i)^2 \tag{1}$$

where k is the number of clusters; S_i are the obtained clusters; μ_i are the centers of mass of all vectors.

The result of segmentation by the k-means method is shown in Fig. 2.

Fig. 2. Input image for processing and the result of segmentation by the k-means method.

The main advantages of the k-means method are its simplicity and speed of execution. The k-means method is quite simple and transparent, so it is successfully used in various fields - marketing segmentation, astronomy, agriculture, etc. [13].

Despite the obvious advantages of the method, it has significant disadvantages:

1. The result of the classification strongly depends on the random initial positions of the centers of each cluster;
2. The algorithm is sensitive to emissions - results that stand out significantly from the total sample of input data; emissions cause distortion of the average value;
3. The number of clusters must be set in advance by the researcher.

Deep learning is a field of artificial intelligence of machine learning based on a list of methods that seek to create high-level abstractions in visual data [14, 15]. Research in this area is trying to make better representations and create models for learning these representations from large-scale data without professional markup [16–18].

Deep learning algorithms are based on certain principles. In this case, the observed data are generated by the interaction of certain factors that have formed at a deep level in the networks. Deep learning assumes that these levels of factors correspond to different levels of abstraction. Advances in deep learning have contributed to significant progress in optimizing optical image recognition tasks. Nowadays, architectures have become standard to solve computer vision problems, including segmentation. Usually, greedy optimization algorithms are quite common for deep learning problems [19].

Convolutional neural network (CNN) is a class of deep artificial neural networks that have been successfully applied to the analysis of visual images. CNNs use a variety of multilayer perceptrons designed to require minimal preprocessing [20, 21]. To explain the operation of this type of model, the main building blocks of CNN should be explained in detail.

Convolutional layer. The convolutional layers apply a convolution operation to the input, passing the result to the next layer. The convolution simulates the response of an individual neuron to a visual stimulus [3].

The Input is an array of input data, such as an image. The convolution process itself is performed using a filter. Filter sizes and values are usually determined by an expert who designs the CNN model, and they are smaller than the input data because they are used as some sort of "sliding windows" [2].

During the convolution process, the filter is "overlaid" on the input image. During the overlay, the corresponding values from both arrays are multiplied and the results are summed [3, 4].

After that, the filter starts to move in the image with a certain step and forms an output data layer whose dimension is smaller than the dimension of the input data. For a 5×5 image, a 3×3 filter, and a 1-pixel step, a 3×3 output array is obtained when minimized [21].

Pooling. After the convolution process, the resulting feature map undergoes a pooling process. During aggregation, data is "compacted" and shrinks in size, highlighting the most important features from a model perspective. Aggregation is also one way to prevent model retraining [3, 14, 21].

During aggregation, the size and pitch of the filter are chosen, which is similar to the convolution filter, but when aggregating, the filter only separates the receptive field regions for the values of the resulting array [4].

There are different ways to aggregate a feature map. The most common is max pooling. The operation of max-pooling selects the maximum value of the receptive field for each of the cells in the output matrix. This approach preserves important information in the image and eliminates noise information when the model is processing large images [2–4, 21].

Now let's take a look at some popular CNN architectures.

U-Net is also one of the implementations of CNN [22, 23]. It was designed to segment biomedical images at the Faculty of Computer Science, University of Freiburg, Germany. The network is based on a fully convolutional network whose architecture has been modified and expanded to work with fewer images for training and to provide more accurate segmentation results. The basic idea is to complement the conventional network of sequential layers where pooling operations are reduced (dimension reduction) the objects are replaced by the operators of transposed convolutions or deconvolutions. Therefore, these layers increase the output resolution. Moreover, the next convolutional layer can learn to build an accurate output based on this information. One of the important changes in U-Net is that in the part where the dimension increases, there are a large number of feature channels that allow the network to distribute contextual information to the layers of higher resolution. As a result, the part in which the expansion occurs is more or less symmetrical to the part of the narrowing, this gives a U-shaped architecture, as shown in Fig. 3 [22].

Understanding the building blocks of this architecture is enough to understand the whole architecture in general, as well as to begin to highlight the benefits of using this architecture [23].

U-Net is characterized by [22]:

1. Achieving high results in various real problems by maintaining the accuracy of localization of segments (the technique of “copy and crop” allows this approach);
2. Using a small amount of data to achieve optimal results.

Among the disadvantages of using the U-Net model can only be noted the need to use a relatively modern graphics processor: for efficient operation used Nvidia GTX Titan with 6 GB of graphics memory. But usually cloud technologies bypass this shortcoming.

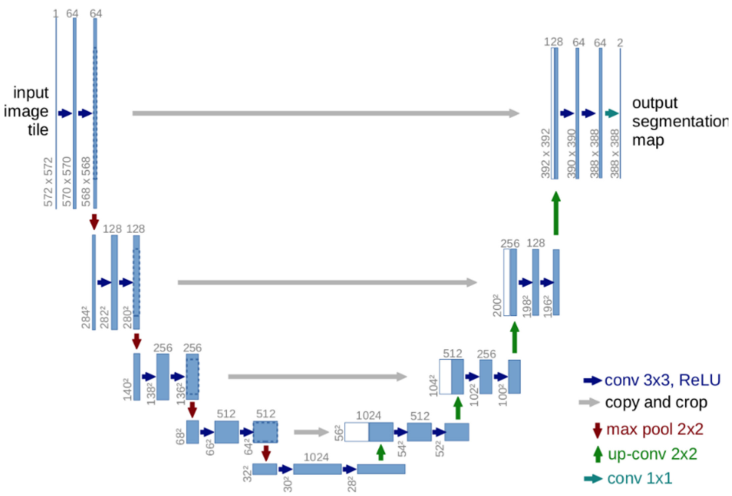


Fig. 3. An example of the U-Net model that was used in this implementation.

The specified image (Fig. 3) shows the following symbols [2, 22]:

– conv 3 × 3 - convolutional layer, which has a filter size 3 × 3;
– ReLU (Rectified linear unit) - rectifier, this function is used in the activation layer and corrects all negative values in the sent data;
– max pool 2 × 2 - aggregation layer, which has a filter size 2 × 2;

– up-conv 2 × 2 - this layer is also called a scan, or a layer of transposed convolution, its purpose - to reverse the convolution process; has a filter size 2 × 2;
– copy and crop - the technique of copying and adding parallel layers of the convolutional part of the architecture to the scan, this allows you to raise the quality of the image size, because the expansion loses quality.

ResNet refers to one modification of convolutional neural networks [24]. When the deeper network begins to collapse, there is a problem: as the depth of the network increases, accuracy first increases and then decreases rapidly. Reduced learning precision shows that not all networks are easy to optimize. To overcome this problem, a "residual" learning structure was introduced. It uses shortcut connections. Shortcut connections skip one or more layers and perform ID mappings. ResNet models consist of serial shortcut connections that are built from basic convolutional layers.

Using ResNet, you can solve many problems, such as [24]:

– ResNet is relatively easy to optimize: "simple" networks (which simply form layers) show a large learning error as depth increases;
– ResNet makes it relatively easy to increase accuracy by increasing the depth, which is more difficult to achieve with other networks.

The architecture of the SegNet model was published in the journal by IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI 2017) [25]. This post has over 2600 references. The paper [25] proposed an architecture, which is shown in Fig. 4.

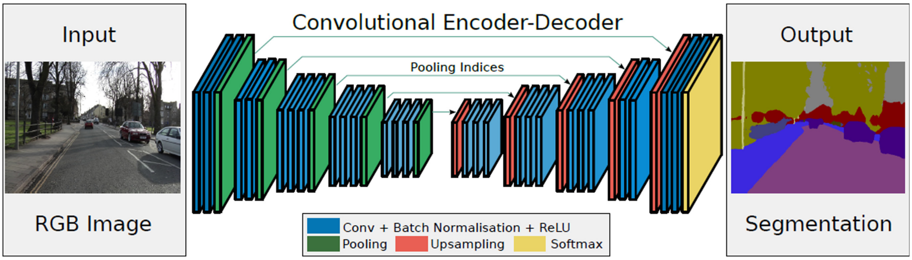


Fig. 4. SegNet architecture.

This architecture is reminiscent of the U-Net form: the basic structures are the collapsing and expanding layers, they will process the input images in parallel. But, unlike U-Net, SegNet migrates aggregation layer indexes to maintain dimension instead of fully copying and pasting data to source layers. Thus, SegNet requires less computing power than U-Net.

To improve the accuracy of the model and unify the format of data storage, various methods of data processing are used. Some of them are used in the work and are described below [26].

Image cropping is used to reduce the size and remove unnecessary information at the edges, leaving very important information. Very often, other machine learning algorithms are used to crop images, but usually just statistical methodologies are used. An example of cropping an image is shown in Fig. 5 [26].

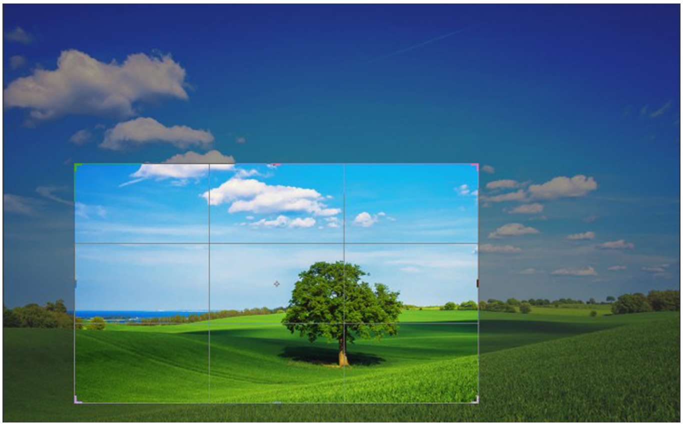


Fig. 5. Image cropping.

Sometimes it is necessary to increase the size of the image and give it internal indentations. This is what padding does, as shown in Fig. 6.

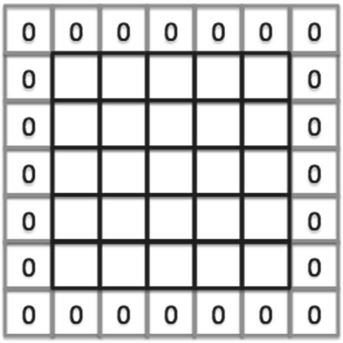


Fig. 6. Adding internal indents (padding).

The figure shows the creation of internal indents with zeros, but sometimes other methods are used to synthesize indents, for example, based on information near the edge (autocomplete indents).

Resizing is a simple operation that needs to be described because of the need to unify image sizes for an in-depth learning model. Different data interpolation methods are used to change the size, at the choice of the expert [26].

As you know, all color images consist of pixels. The values of which are controlled by 3 color RGB channels (red, green, blue). In addition, the channel of each pixel can take values from 0 to 255. This is important to note, because this range of values gives some problems in optimizing the model: optimizers (usually based on gradient descent) can reach local lows more often due to lack of normalization. The normalization process aims to convert the values of the input data, but to preserve the relationship between the data itself. During normalization, the array of the input image can take values from 0 to 1. Very often during normalization, the data array is processed by the formula of standardized evaluation:

$$f(x) = \frac{x - \mu}{\sigma} \tag{2}$$

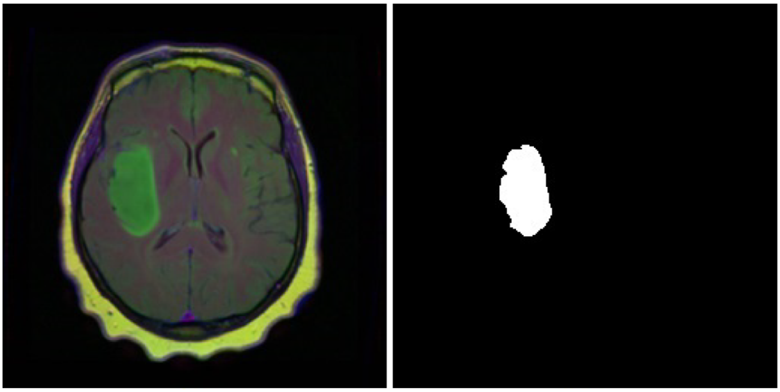
where x is the input image, μ is the mean value of the image array, σ is the standard deviation of the image array.

Very often in the process of solving the problem in the field of computer vision there is a problem of insufficient number of copies for training. To increase accuracy, various upsampling techniques are often used. One of the methods of upsampling is the synthetic creation of new images [26, 27].

Combining such data creation techniques can significantly increase the number of instances in a data set, spending only a small amount of time compared to manual data collection.

4 Training and Selecting Brain MRI Segmentation Model

Therefore, a low-order glioma dataset distributed by the TCGA project was selected for the task. An example of such images is shown in Fig. 7. Images are saved in TIF format and have a size of 256 × 256 pixels. Each of the 110 patients has a separate folder with a complete set of brain scans at all levels.



a) b)

Fig. 7. MRI scan of the brain (a) and manually selected area of the tumor (b).

Each image of the slice of the brain is provided with the original segmentation mask, an example of which is shown in Fig. 7(b). These masks were approved by a certified radiologist from Duke University. The pixels that make up the area of the tumor selected by the radiologist are identified in white.

Based on a preliminary analysis of the input data and architectures of neural networks, we choose algorithms and methods to solve the problem of segmentation on MRI images of the brain.

Recall that for each section is provided a tumor mask. This means that, in addition to the input data for training, we have the expected output. This type of learning is called "supervised learning". Accordingly, you should choose algorithms that specialize in teaching with a teacher. To solve the problem of tumor segmentation, different variations of ANNs will be used to achieve the highest accuracy: ResNet, SegNet and U-Net.

Data processing methods are also used to increase accuracy. For example, all images in an array are transformed to a uniform size for the model. Because the data set is very well documented, you do not need to use such methods to change and correct images. However, the input pre-processing methods listed in Sect. 3 will be used during the practical part of this paper.

It should be noted that some of the studied machine learning algorithms are also used as methods of pre-processing of input data. For example, the thresholding method allows binaryization of certain images to guide them to the model, and clustering algorithms are sometimes used to train other systems with the teacher. Some such techniques will be considered in practice.

A certain technology stack was selected to solve this problem.

The low order glossary dataset was reviewed on Kaggle, a platform for machine learning and analytics competitions. Kaggle proposes to use Kaggle Kernels, a cloud-based technology that allows code to be run in a deployed Linux mini-environment, to process competition data. This is what Docker technology can do. Such notepads include the following technology points and make it possible to interact with the code in real-time. Such technologies are based on Jupyter systems.

Jupyter allows you to deploy multiple *Python* interactive viral environments and work with data analysis or neural network training. The *IPython* (Interactive Python) system simplifies the process of analyzing and visualizing data.

Other technologies are used for visualization and data processing. The *pandas* library is used for statistical data analysis - it makes it easy to analyze a large and documented array of data. *Matplotlib* and seaborn libraries are usually used to display graphs. Finally, the *scikit-image* library will be used for image processing.

A set of technologies to work with different machine learning methods should also be identified. For more classic algorithms, the *scikit-learn* library is used. However, a second technology stack was selected for deep learning. The popular *Pytorch* library has been selected for its work and has found its purpose in solving computer vision problems.

Modeling was done in the *Python* library using *Pytorch* building blocks. Because all other mechanisms and algorithms (such as data processing or result analysis) are unified, the program requires manual model selection, as all Pytorch library models inherit the *nn.Module* class.

Most of the operation is performed at Kaggle. The main reason for this is the ability to use the GPU for free to train models in the cloud (providing 30 h of GPU per week).

The following models use the same settings: one of the variations of the stochastic gradient descent *Adam* is used to write off.

This section the creation of U-Net, SegNet, and ResNet models and the selection of a shared accuracy metric. In this case, the Dice coefficient, or DSC, was used - the statistical coefficient used to compare two statistical samples, in this case, the sample is the pixels of the image assigned to the class. It allows you to compare the manual mask around the tumor with the created mask.

To create the ResNet architecture, certain classes have been developed that implement the elementary building blocks of a residual network: the classes *ResidualBlock*, *ResNetEncoder*, *ResNetDecode*, and *ResNet* itself. Also, according to research materials, several types of ResNet were created that differ only in settings: from *resnet18* to *resnet152*.

The ResNet model was then trained. This model architecture shows a good result in 79%. The ResNet training process is shown in Fig. 8.

```
epoch 47 | loss: 0.04930392333439185
epoch 47 | val_loss: 0.548101031117969
epoch 47 | val_dsc: 0.6590788001889422
epoch 48 | loss: 0.04698622013841357
epoch 48 | val_loss: 0.5462475981977251
epoch 48 | val_dsc: 0.6565815171985613
epoch 49 | loss: 0.04524667348180498 6
epoch 49 | val_loss: 0.5514573918448554
epoch 49 | val_dsc: 0.6474720114761051
epoch 50 | loss: 0.04397666880062648 5
epoch 50 | val_loss: 0.5241848793294694
epoch 50 | val_dsc: 0.682728758625622

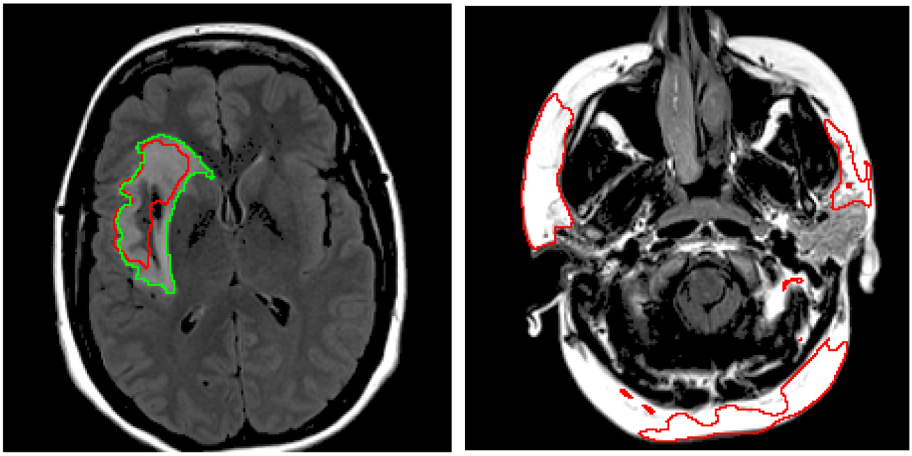
Best validation mean DSC: 0.705724
```

Fig. 8. The ResNet training process.

An example of a segmented tumor and an error are shown in Fig. 9. If you look at the original model and the predictions in the images, you can see that the model gives an error on the slides where the tumor is not present. In Fig. 9(b), the model highlights the tumor on foreign slides where it is not present, so it recognizes illogical abstract features there.

After this training, the SegNet model passed. This model architecture shows a result of 70.57%. At the moment, the ResNet model prevails over SegNet accuracy. The SegNet training process is shown in Fig. 10.

To visualize the validation results, 10 validation instances are used, the values of which are shown by the histogram in Fig. 11.



a) b)

Fig. 9. Example of tumor recognition (a) and an error (b) by ResNet.

```
epoch 47 | loss: 0.049303923334394185
epoch 47 | val_loss: 0.548101031117969
epoch 47 | val_dsc: 0.6590788001889422
epoch 48 | loss: 0.04698622013841357
epoch 48 | val_loss: 0.5462475981977251
epoch 48 | val_dsc: 0.6565815171985613
epoch 49 | loss: 0.045246673481804986
epoch 49 | val_loss: 0.5514573918448554
epoch 49 | val_dsc: 0.6474720114761051
epoch 50 | loss: 0.043976668800626485
epoch 50 | val_loss: 0.5241848793294694
epoch 50 | val_dsc: 0.682728758625622

Best validation mean DSC: 0.705724
```

Fig. 10. The SegNet training process.

Red indicates the average value of the coefficient, and green - the median. It should be noted that the statistics are calculated from a common set of validation instances, and only the first 10 are used for visualization. At the moment, the ResNet model is more accurate than SegNet.

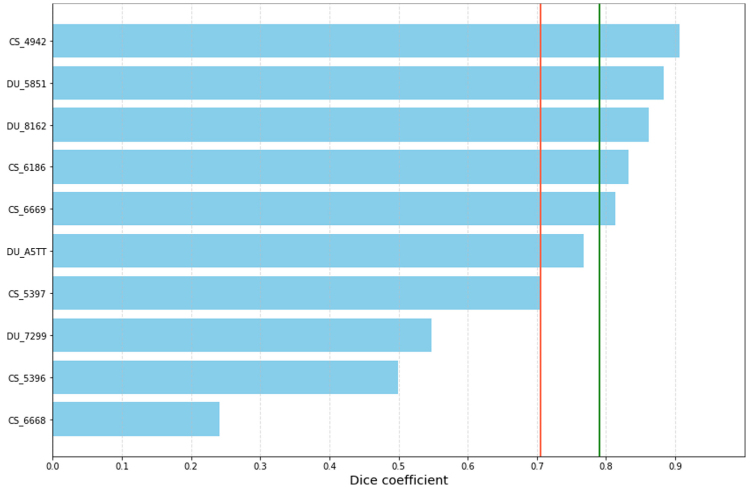


Fig. 11. Distribution of the Sorensen coefficient by ResNet. (Color figure online)

Now we compare these models with U-Net [28]. A UNet class was created to follow the nn.Module class. The standard U-Net building block is a set of convolution, normalization, and activation. Such blocks are combined into a double structure, where data is first encoded and collapsed, and then decoded and expanded. The architecture of the U-Net model shows a result of 79%. The model shows better results than the previous two models. The training process of U-Net is shown in Fig. 12.

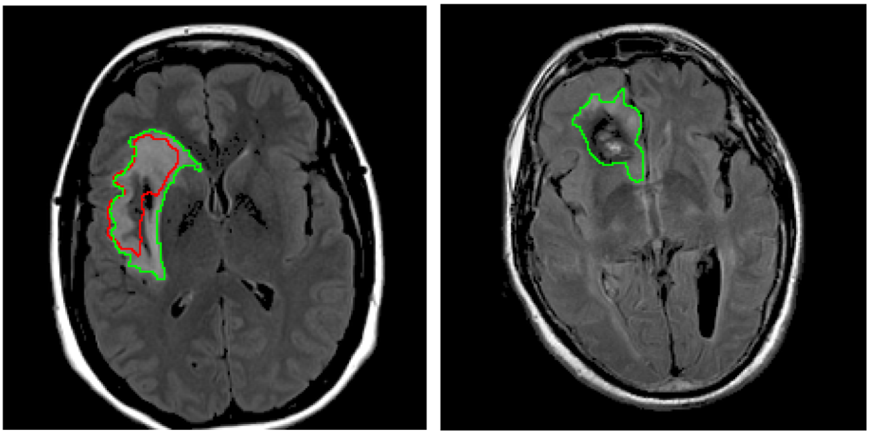
```
epoch 47 | loss: 0.03771830209026253
epoch 47 | val_loss: 0.41676533222198486
epoch 47 | val_dsc: 0.7273059945566407
epoch 48 | loss: 0.03729282418011569
epoch 48 | val_loss: 0.4059857726097107
epoch 48 | val_dsc: 0.7367154885728437
epoch 49 | loss: 0.03706725263385206
epoch 49 | val_loss: 0.40996408462524414
epoch 49 | val_dsc: 0.7323191760759673
epoch 50 | loss: 0.0365457629317229
epoch 50 | val_loss: 0.42175281047821045
epoch 50 | val_dsc: 0.7193032282495503

Best validation mean DSC: 0.794821
```

Fig. 12. The U-Net training process.

An example of a segmented tumor and an error are shown in Fig. 13.

If we consider the original model and predictions in the images, we can see that the model gives an error on the slides where the tumor is present (Fig. 13(b)).



a) b)

Fig. 13. Example of tumor recognition (a) and an error (b) by U-Net.

In the future, we will apply pre-processing methods to improve the accuracy of the models. To simplify the work and improve the results, we apply the statistical technique of "ceiling analysis". During operation, the model will consistently train without processing methods and then train on these methods, which will be added sequentially. This process is repeated for each architecture. This approach allows you to check the progress of model training and the impact of each part of the system on accuracy: taking into account changes in accuracy in different parts will allow us to highlight priority areas for further work. The results of such a ceiling analysis are shown in Table 1.

Table 1. Ceiling analysis of CNN models used for the segmentation of brain MRI scans on neoplasms.

	No processing	Cropping	Resizing	Rotation	Thresholding
ResNet	0.7948	0.8011	0.8114	0.7915	0.8674
SegNet	0.7057	0.7544	0.7915	0.8114	0.8456
U-Net	0.7948	0.8023	0.9012	0.9115	0.9498

As you can see from the table, the best result is to use the U-Net model with additional data processing to unify the data, and then to increase the localization accuracy. The result of 94% is a significant improvement and satisfies the needs of the task, so the problem can be considered solved.

In Fig. 14 the predictions of the final optimized model on the validation data are indicated. Model predictions are indicated using red color, while ground truths (input segmentations from a radiologist) are colored green.

However, let us point out the advantages of such a representation approach. The table has a key place for the U-Net architecture: with the use of resizing techniques the

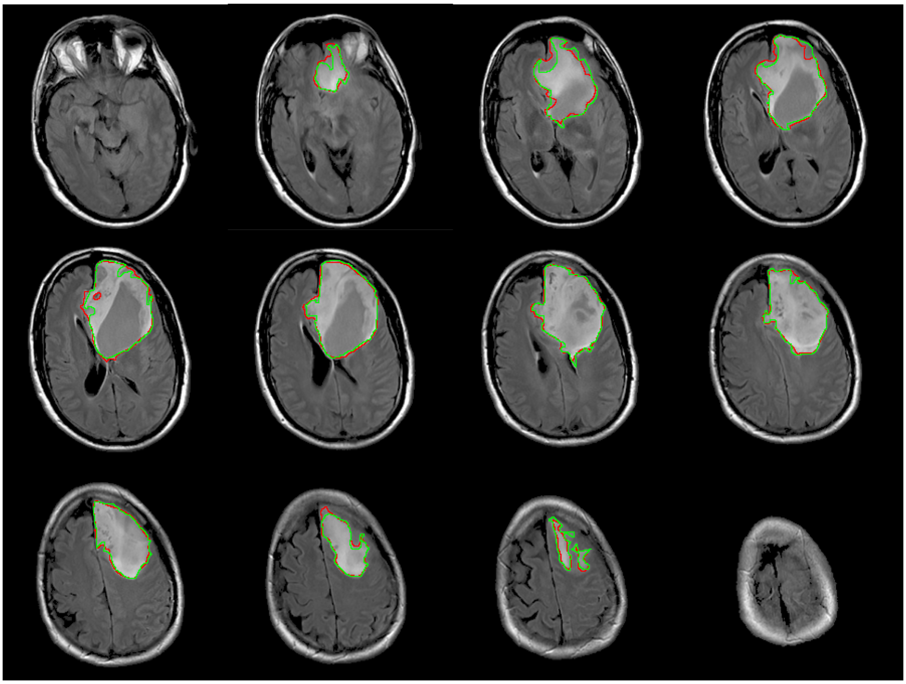


Fig. 14. Validation predictions - U-Net (final version).

accuracy of the U-Net model has increased by 10%. This shows that image resizing technology has significantly affected the accuracy of the model as a whole.

As the results of the segmentation of brain MRI scans must be verified by a doctor it makes sense to consider using a detection neural network that would just indicate the presence of a glioma. Thus, the neural network architecture YOLO v3 is considered. The total number of MRI scans is 8123 from 110 patients.

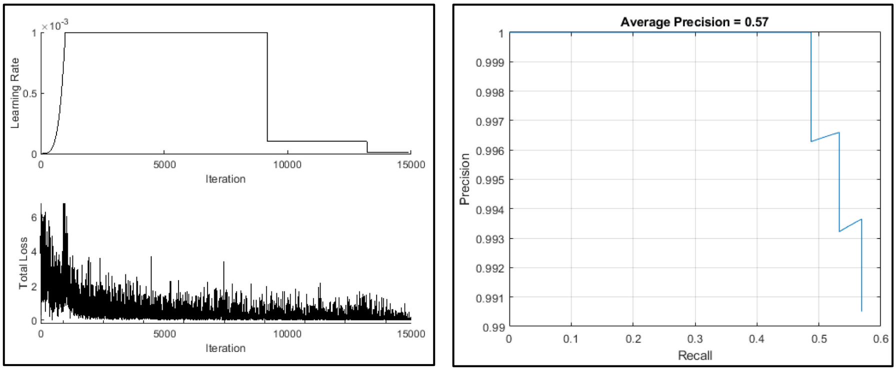
For training YOLO v3, 1374 scans were used, namely those containing a glioma, since only labeled data can be used for training. Of these, 60% was used for training and the remaining 40% for testing [29–33].

To increase the sample, the following modifications were applied to the original data: color jitter, horizontal flipping, and scaling.

A pretrained SqueezeNet v1.1 network with an input size [227 227 3] was used as a basic neural network. In it, the layers following fire9-concat and responsible for the initial classification were replaced with new ones, forming two outputs.

YOLO trained for 70 epochs of 206 iterations. Learning rate and total loss during the training are shown in Fig. 15(a) and the precision-recall curve of the resulting neural network on the test data is shown in Fig. 15(b).

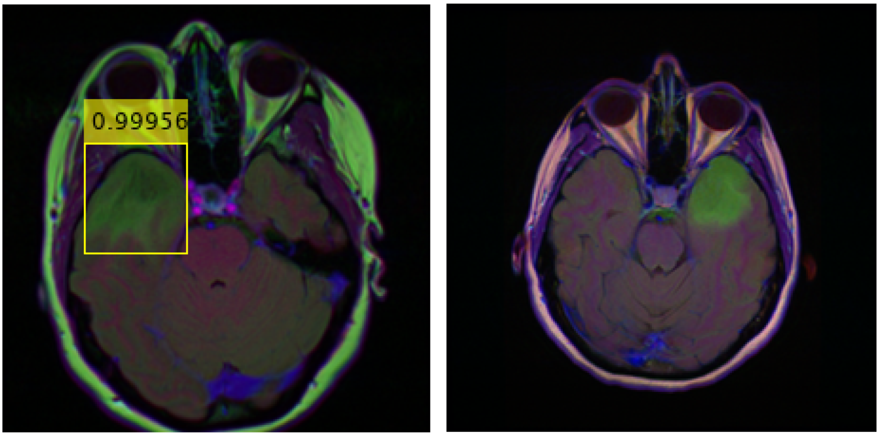
The network has a precision of over 99% but only 57% of gliomas were recognized and this value is unacceptable low. It would be much better to have high recognition with moderate precision to avoid false-negative results.



a) b)

Fig. 15. Learning rate and total loss during the training (a), the precision-recall curve of the resulting neural network on the test data (b)

An example of a segmented tumor and an error are shown in Fig. 16. If we consider the original model and predictions in the images, we can see that the model gives an error on the slides where the tumor is present (Fig. 16(b)).



a) b)

Fig. 16. Example of tumor recognition (a) and an error (b) by YOLOv3.

An additional study of the input data was also conducted using a neural network UNet in combination with ResNet34.

The problem of inaccuracy of the original algorithm for creating a mask. Since the test mask was made by an unknown algorithm, which can make mistakes and make inaccuracies, in some cases there are options when the created network turns out to be more accurate. Its predictions are more accurate and sharper than the original algorithm.

Therefore, checking the accuracy of a network on a test dataset cannot guarantee accuracy. In addition, networks that have received low accuracy on test data can better show the accuracy and general level of generalization in real practice (Fig. 17).

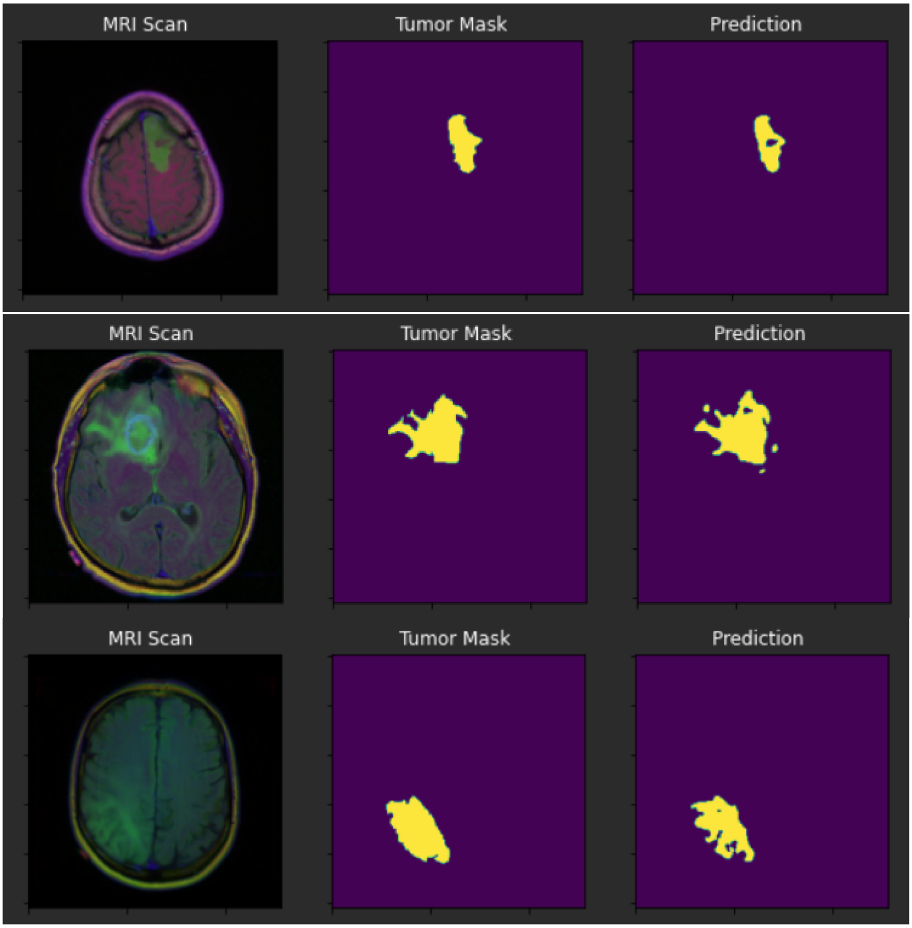


Fig. 17. The problem of inaccuracy of the original algorithm for creating a mask.

Usually, when a tumor is detected in a patient, it is also found in the subsequent pictures. In Fig. 18 shows that the original algorithm showed a random definition of a tumor, the accuracy of which is questionable. Indeed, in subsequent and previous images, the tumor is no longer there. In addition, the developed network showed correctly.

The problem of very small tumors. As can be seen in the following images, if the tumor is very small, then the likelihood of finding it is also low (Fig. 19). The reason for this problem lies in the fact that a tumor is responsible not just for a certain color, but a whole sequence of pixels of different colors. As can be seen in the example, the tumor can be of different colors. The network has learned to identify large tumors with high

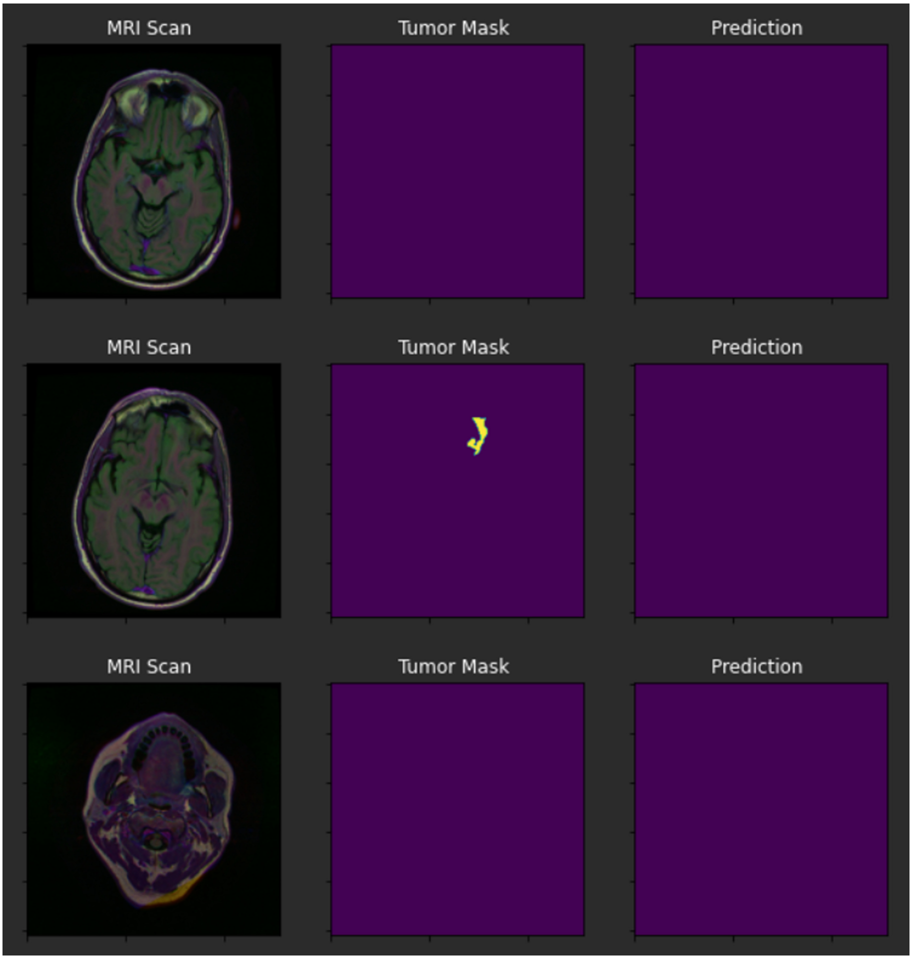


Fig. 18. Erroneous definition of a tumor by the original algorithm.

accuracy by examining pixel sequences, but for short ones, the network simply does not have enough confidence in the presence of a tumor. The color problem is also a cause of network uncertainty.

The figure below (Fig. 20) shows the problem of the variety of images and colors.

Usually the light green areas in the images represent a tumor. However, there are images (Fig. 20) are made by the device in green tones, so the network is difficult to determine and it is mistaken in the early stages.

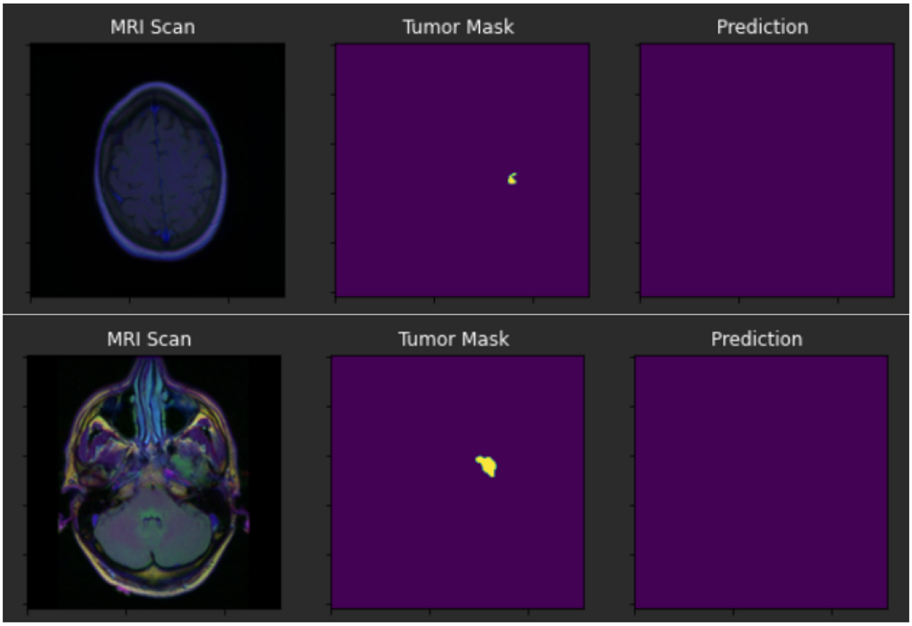


Fig. 19. The problem of very small tumors.

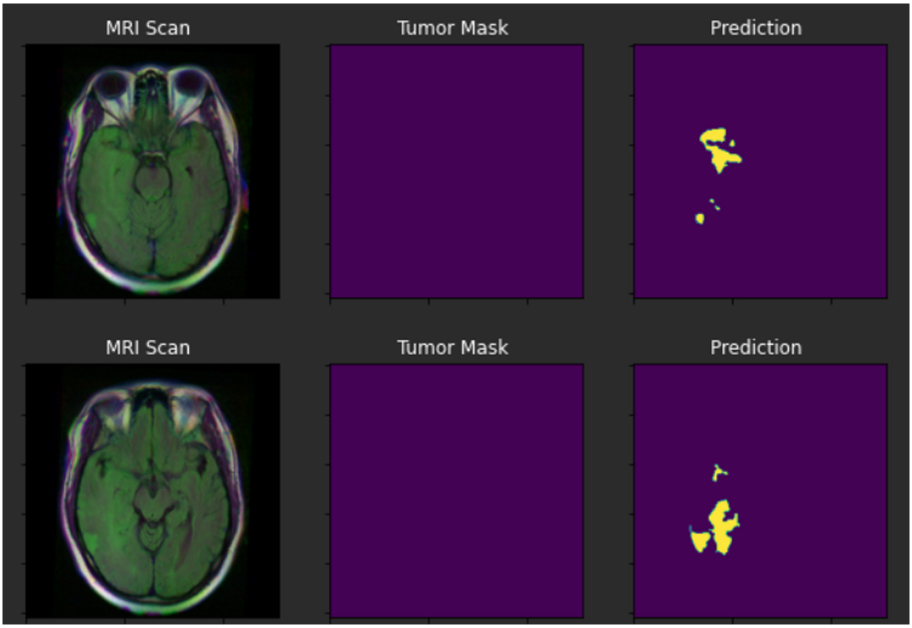


Fig. 20. The problem of the variety of images and colors. (Color figure online)

5 Conclusions

This paper focuses on the study of machine learning algorithms for the automated segmentation of neoplasms in brain MRI images.

The first section provides a general description of the field of computer vision, its application in biomedicine, and the importance of pre-diagnosis of neoplasms.

The second section analyzes the selected task and the selected dataset from The Cancer Genome Atlas project. The steps required for the study were also formulated, namely the study and selection of available libraries for machine learning, testing of various machine learning and depth learning algorithms, optimizing tumor segmentation while improving the performance of existing architectures.

In the third section, the selected machine learning algorithms were used and developed to solve the problem of segmentation of tumors on brain MRI images. After all, during the comparison, a system of the U-Net architecture with additional processing methods was selected as the final model. Its accuracy reached 94%, which is a significant result compared to manual image segmentation.

Additionally, the neural network architecture YOLO v3 is considered. To increase the sample, the following modifications were applied to the original data: color jitter, horizontal flipping, and scaling. An additional study of the input data (the problem of inaccuracy of the original algorithm for creating a mask, the problem of very small tumors) was also conducted.

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Coalgebraic Approach to Studying Discrete Systems with Output
The General and Distributed Cases

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Abstract. Abstract discrete systems with output are widely used in Theoretical Computer Science. They are also used as design patterns in Software Engineering. These are reasons to the issue the challenge to develop a theoretical framework for specifying and analysis behavioural constraints for systems of such a kind. In the paper, authors propose to use the theory of coalgebras to answer the challenge. The paper introduces the needed preliminaries, demonstrates the general approach, establishes principal results in the general case. In particular, authors show that safeness is naturally formulated in coalgebraic and category-theoretic terms. Further, this general approach is applied to study causality constraints in distributed systems. Authors prove the existence of a universal language for specifying causality constraints as clock constraints. Also, the practically important class of constraints has been studied. Authors show rigorously that this class is not sufficient for specifying all clock constraints. The paper demonstrates how to specify constraints belonging to the class using Diophantine equations. The constraint verifying algorithms for the class are discussed too. It is shown the complexity of verifying a linear Diophantine constraint.

Keywords: Discrete system · Discrete system with output · Category · Endofunctor · Coalgebra · Coalgebraic morphism · Final coalgebra · Anamorphism · Detector · Safety constraint · Clock constraint · Counter-detector · Diophantine equation

1 Introduction

In Computer Science, abstract discrete systems are widely used as a tool for specifying and analysing both pure software systems and embedded systems or, more generally, cyber-physical systems. In the case, the use of models with discrete nature is motivated by the observation formulated, for example, by G. Plotkin in [12] that for digital a computer system, its behaviour consists of the sequence of elementary steps.

The generally accepted approach distinguishes between the behaviour of a system and its trajectory. This is because of the behaviour completely determines

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the correctness of system operating. In contrast, the system trajectory depends as well on the implementation details of this behaviour. In other words, we are in a situation where every step of the system's behaviour triggers a notification. And we can observe streams (see Subsect. 2.1) of these notifications only.

Thus, the problem is to propose a behavioural constraint specification method ensuring distinguishing correct and incorrect system behaviours. This class of constraint was introduced by L. Lamport in [6] and refined by B. Alpern and F.B. Schneider in [2].

Due to works of J. Rutten [13] and B. Jacobs [5], it is generally accepted that the suitable mathematical background for studying the behaviour of discrete systems is the theory of coalgebras in the category of sets.

The paper proposes to apply this general approach to the study of a discrete systems with output. Section 3 of the paper contains the main results demonstrating that the class of safety constraints is naturally defined using category-theoretical concepts.

In the case of studying or developing distributed systems, verifying causality relationships is an important specific problem. These relationships are used for providing the required behaviour of the system as a whole by orchestrating only individual behaviours of system components.

Section 4 explains how the general approach, which is developed in the above sections, can be used for this specific problem. Considering that a more specific class of constraints is used in practice, this class is studied in Sect. 5. In particular, we demonstrate with the general technique how to indicate constraints, which cannot be specified using this class.

2 Basic Concepts and Notation

In this section, principal concepts and notation are presented used in the paper.

2.1 Sequences

Firstly, let us remind that a partial mapping from a set X into a set Y is a subset $f \subset X \times Y$ holding the condition: $(x, y') \in f$ and $(x, y'') \in f$ where $x \in X$, and $y', y'' \in Y$ ensure $y' = y''$. The fact $(x, y) \in f$ is usually denoted by $fx \downarrow= y$. The fact $\forall\, y \in Y\,.\,\neg fx \downarrow= y$ is denoted by $fx =\uparrow$.

Let X be a set then a partial mapping $\boldsymbol{s} : \mathbb{N} \dashrightarrow X$ is called a sequence if for any $n, m \in \mathbb{N}$, $n < m$ and $\boldsymbol{s}n =\uparrow$ ensure $\boldsymbol{s}m =\uparrow$.

A sequence $\boldsymbol{s}$ is called a stream if $\boldsymbol{s}n =\uparrow$ is wrong for all $n \in \mathbb{N}$ otherwise $\boldsymbol{s}$ is called a word.

In the context, for any $n \in \mathbb{N}$ and sequence $\boldsymbol{s}$ we usually denote by s_n the value of $\boldsymbol{s}n$ if $\boldsymbol{s}n =\uparrow$ is wrong.

Below we use the notation

X^∞ for referring to the set of all sequences;
$X^{\mathbb{N}}$ for referring to the set of all streams;
X^* for referring to the set of all words.

Taking into account that the totally undefined function represents the sequence called empty and denoted by $\boldsymbol{\epsilon}$, we use the denotation X^+ for referring to the set of all nonempty words.
Also, we associate the word $\boldsymbol{x} = \{(0, x)\}$ with each element $x \in X$. Usually, we identify x and $\boldsymbol{x}$.

The length of a word $\boldsymbol{u}$ is the natural number $\#\boldsymbol{u}$ defined as follows

$$\#\boldsymbol{u} = \min(n \in \mathbb{N} \mid \boldsymbol{u}n =\uparrow) .$$

For any word $\boldsymbol{u}$ and sequence $\boldsymbol{s}$, one can define the sequence $\boldsymbol{us}$ as follows

$$\boldsymbol{us} = \boldsymbol{u} \cup \{(n + \#\boldsymbol{u}, x) \mid \boldsymbol{s}n \downarrow= x\} .$$

For any $A \subset X^\infty$ and $\boldsymbol{u} \in X^*$ one can define the next sets of sequences

$$\boldsymbol{u} \cdot A = \{\boldsymbol{us} \mid \boldsymbol{s} \in A\}$$
$$\boldsymbol{u}^{-1} \cdot A = \{\boldsymbol{s} \in X^\infty \mid \boldsymbol{us} \in A\} .$$

For any $\boldsymbol{s} \in X^\infty$ and $m, l \in \mathbb{N}$, we denote by $\boldsymbol{s}_{m..}$ the sequence such that

$$\boldsymbol{s}_{m..} = \{(n - m, x) \mid m \le n \wedge \boldsymbol{s}n \downarrow= x\}$$

and by $\boldsymbol{s}_{m..l}$ the word such that

$$\boldsymbol{s}_{m..l} = \{(n - m, x) \mid m \le n < l \wedge \boldsymbol{s}n \downarrow= x\} .$$

2.2 Properties of Sequence Sets

The following concepts are crucial for our research

Definition 1. *A subset $P \subset X^*$ is called* ***prefix-free*** *if $\boldsymbol{u}_{0..m} \notin P$ whenever $\boldsymbol{u} \in P$ and $0 \le m < \#\boldsymbol{u}$.*

Remark 1. If a prefix-free subset of X^* contains $\boldsymbol{\epsilon}$ then this subset is exactly $\{\boldsymbol{\epsilon}\}$.

Definition 2 (see [2]). A ***safety constraint*** is a subset $S \subset X^{\mathbb{N}}$ satisfying the condition[1]

$$\boldsymbol{s} \in S \quad \text{iff} \quad \text{for any } m \in \mathbb{N}, \text{ there exists } \boldsymbol{s}' \in S \text{ such that } \boldsymbol{s}_{0..m} = \boldsymbol{s}'_{0..m} .$$

[1] Here and below "iff" means "if and only if".

2.3 Coalgebraic Preliminaries

In this section we consider in general the fundamental to our research category-theoretical notion of a coalgebra and additionally some important facts related to this concept. Moreover, we examine specifically the significant to the further comprehension case, namely additional features arising when dealing with the concept of a coalgebra in category **Set**.

Thus, from now on we assume that some category $\mathbb{C}$ and some endofunctor $\mathcal{F}$ of $\mathbb{C}$ are given and held fixed in the section (the required concepts of basic category theory are covered, for example, in [4,8]).

Definition 3. *A morphism* $\mathfrak{a}$ *of* $\mathbb{C}$ *is called an* $\mathcal{F}$-***coalgebra*** *if the equation* $\operatorname{cod}\mathfrak{a} = \mathcal{F}(\operatorname{dom}\mathfrak{a})$ *is fulfilled.*

Remark 2. In the case, endofunctor $\mathcal{F}$ establishes the ***type*** of $\mathfrak{a}$. We call $\operatorname{dom}\mathfrak{a}$ the ***carrier*** of $\mathfrak{a}$ and denote it below by $\underline{\mathfrak{a}}$.

Definition 4. *A* ***coalgebraic morphism*** *from coalgebra* $\mathfrak{a}$ *into coalgebra* $\mathfrak{b}$ *is a morphism* $f\colon \underline{\mathfrak{a}} \to \underline{\mathfrak{b}}$ *ensuring commutativity of the diagram*

$$\begin{array}{ccc} \underline{\mathfrak{a}} & \xrightarrow{\;f\;} & \underline{\mathfrak{b}} \\ {\scriptstyle\mathfrak{a}}\big\downarrow & & \big\downarrow{\scriptstyle\mathfrak{b}} \\ \mathcal{F}\underline{\mathfrak{a}} & \xrightarrow{\;\mathcal{F}f\;} & \mathcal{F}\underline{\mathfrak{b}} \end{array}$$

or, equivalently, validity of the equation

$$(\mathcal{F}f)\mathfrak{a} = \mathfrak{b}f\,.$$

Remark 3. The definition of a coalgebraic morphism is given in the context of a certain endofuctor, in our case with regard to endofunctor $\mathcal{F}$. One can specify this detail clearly by using the notion of a $\mathcal{F}$-coalgebra morphism instead.

Proposition 1. $\mathcal{F}$*-coalgebras together with* $\mathcal{F}$*-coalgebra morphisms constitute a category.*

Remark 4. The corresponding category is usually denoted by $\mathbf{Coalg}_{\mathcal{F}}$ (or $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$ if the underlying category requires explicit referring) and called the category of coalgebras of type $\mathcal{F}$ over category $\mathbb{C}$.

Terminal objects of such categories of coalgebras are crucial for constructing semantic models of both programming and specification languages and some of them play a key role in our study.

Definition 5. *A terminal object in a category of coalgebras is called a* ***final coalgebra****. Each final coalgebra generates the family of* ***anamorphisms****, which consists of the corresponding unique morphisms into this final coalgebra from every coalgebra in the category.*

Remark 5. We usually denote a select final coalgebra in category $\mathbf{Coalg}_{\mathcal{F}}$ by $\nu\mathcal{F}$ and, subsequently, for any coalgebra $\mathfrak{a}$ in this category the corresponding anamorphism into $\nu\mathcal{F}$ by $[\![\mathfrak{a}]\!]$.

The next important notion for studying the theory of universal coalgebra is the concept of bisimulation. You can familiarize yourself with a category-theoretical representation of a bisimulation in [1], but for the sake of simplicity we provide one of several possible definitions here.

Definition 6. *A span* $\mathfrak{a} \xleftarrow{\mathfrak{r}_{\mathfrak{a}}} \mathfrak{r} \xrightarrow{\mathfrak{r}_{\mathfrak{b}}} \mathfrak{b}$ *in category* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$ *establishes a bisimulation of coalgebras* $\mathfrak{a}$ *and* $\mathfrak{b}$ *if the corresponding span* $\underline{\mathfrak{a}} \xleftarrow{\mathfrak{r}_{\mathfrak{a}}} \underline{\mathfrak{r}} \xrightarrow{\mathfrak{r}_{\mathfrak{b}}} \underline{\mathfrak{b}}$ *in the underlying category* $\mathbb{C}$ *is a mono span i.e. the fulfilment of the equations* $\mathfrak{r}_{\mathfrak{a}}h' = \mathfrak{r}_{\mathfrak{a}}h''$ *and* $\mathfrak{r}_{\mathfrak{b}}h' = \mathfrak{r}_{\mathfrak{b}}h''$ *guarantees the validity of* $h' = h''$ *for any morphisms* h', h'' *in* $\mathbb{C}$ *such that* $\operatorname{dom} h' = \operatorname{dom} h''$ *and* $\operatorname{cod} h' = \underline{\mathfrak{r}} = \operatorname{cod} h''$.

Next, we are going to consider some propositions related to the concept of bisimulation which take place in the case of finitely complete underlying category.

Proposition 2. *Suppose category* $\mathbb{C}$ *is finitely complete and there is a final coalgebra* $\nu\mathcal{F}$ *in* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$*. For any coalgebras* $\mathfrak{a}$ *and* $\mathfrak{b}$ *in* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$ *one can consider the cospan* $\underline{\mathfrak{a}} \xrightarrow{[\![\mathfrak{a}]\!]} \underline{\nu\mathcal{F}} \xleftarrow{[\![\mathfrak{b}]\!]} \underline{\mathfrak{b}}$ *in* $\mathbb{C}$*. Let* $\underline{\mathfrak{a}} \xleftarrow{p_{\mathfrak{a}}} P \xrightarrow{p_{\mathfrak{b}}} \underline{\mathfrak{b}}$ *be a pullback of this cospan. Then, for any bisimulation* $\mathfrak{a} \xleftarrow{\mathfrak{r}_{\mathfrak{a}}} \mathfrak{r} \xrightarrow{\mathfrak{r}_{\mathfrak{b}}} \mathfrak{b}$ *there exists a unique monomorphism* $f\colon \underline{\mathfrak{r}} \to P$ *such that* $\mathfrak{r}_{\mathfrak{a}} = p_{\mathfrak{a}}f$ *and* $\mathfrak{r}_{\mathfrak{b}} = p_{\mathfrak{b}}f$.

The next proposition in turn allows us to obtain the greatest bisimulation of coalgebras on some additional condition.

Proposition 3. *Assume category* $\mathbb{C}$ *is finitely complete, endofunctor* $\mathcal{F}$ *preserves pullbacks, and there is a final coalgebra* $\nu\mathcal{F}$ *in* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$*. Then, for any coalgebras* $\mathfrak{a}$ *and* $\mathfrak{b}$ *in* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$ *the pullback of the cospan* $\underline{\mathfrak{a}} \xrightarrow{[\![\mathfrak{a}]\!]} \underline{\nu\mathcal{F}} \xleftarrow{[\![\mathfrak{b}]\!]} \underline{\mathfrak{b}}$ *lifts uniquely to the greatest bisimulation of these coalgebras.*

One can find proofs of these two propositions in [14].

Another useful concept to the further research is given by the notion of a subcoalgebra.

Definition 7. *Let* $\mathfrak{a}$ *be a coalgebra in* $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$*. Then, an* $\mathcal{F}$*-coalgebra* $\mathfrak{c}$ *is called a* ***subcoalgebra*** *of* $\mathfrak{a}$ *if there exists a morphism* $j\colon \mathfrak{c} \to \mathfrak{a}$ *such that* j *is a monomorphism in* $\mathbb{C}$.

In the current study we are mainly interested in coalgebras over category **Set**. So at the end of this section we are going to emphasise several properties category **Set** and some of its endofunctors possess.

If $\mathbb{C} = \mathbf{Set}$ then $\mathcal{F}$-coalgebras are called ***$\mathcal{F}$-systems*** according to J. Rutten [13] and the corresponding category of coalgebras $\mathbf{Coalg}_{\mathcal{F}}(\mathbb{C})$ is denoted below simply by $\mathbf{Sys}(\mathcal{F})$.

The first feature we should stress is that category **Set** is finitely complete. In our research we deal with so-called polynomial endofunctors of category **Set** (see, for example, [5]). These endofunctors are defined inductively from some basic functors by applying different category-theoretical operations to them. The important subcollection of polynomial endofunctors comprise exponent polynomial endofunctors. We begin with the definition of a simple polynomial endofunctor that allows us to determine exponent polynomial endofunctors. The following affirmations establish the corresponding collection of simple polynomial endofunctors

1. the identity functor i.e. the endofunctor $\mathsf{Id}_{\mathbf{Set}}$ such that $\mathsf{Id}_{\mathbf{Set}} X = X$ and $\mathsf{Id}_{\mathbf{Set}} f = f$ is a simple polynomial endofunctor;
2. a constant endofunctor i.e. an endofunctor $\mathcal{C}$ such that $\mathcal{C}X = \mathsf{C}$ and $\mathcal{C}f = \mathrm{id}_{\mathsf{C}}$ for some set C is a simple polynomial endofunctor;
3. if $\mathcal{G}_1$ and $\mathcal{G}_2$ are simple polynomial endofunctors then the endofunctor $\mathcal{G}$ such that $\mathcal{G}X = \mathcal{G}_1 X \times \mathcal{G}_2 X$ and $\mathcal{G}f = \mathcal{G}_1 f \times \mathcal{G}_2 f$ is a simple polynomial endofunctor;
4. if $\mathcal{G}_1$ and $\mathcal{G}_2$ are simple polynomial endofunctors then the endofunctor $\mathcal{G}$ such that $\mathcal{G}X = \mathcal{G}_1 X + \mathcal{G}_2 X$ and $\mathcal{G}f = \mathcal{G}_1 f + \mathcal{G}_2 f$ is a simple polynomial endofunctor (moreover, a set-indexed coproduct of simple polynomial endofunctors is a simple polynomial endofunctor).

The collection of exponent polynomial endofunctors includes simple polynomial endofunctors and additionally for any set C and simple polynomial endofunctor $\mathcal{G}$ the endofunctor $\mathcal{G}^{\mathsf{C}}$ such that $\mathcal{G}^{\mathsf{C}} X = \mathcal{G}(X)^{\mathsf{C}}$ and $\mathcal{G}^{\mathsf{C}} f$ maps $h\colon \mathsf{C} \to \mathcal{G}(X)$ to $\mathcal{G}(f) \circ h$ is an exponent polynomial endofunctor.

The crucial property of exponent polynomial endofunctors we are interested in is that such endofunctors preserve pullbacks. The systems we will explore in the course of our article actually correspond to exponent polynomial endofunctors.

3 Safety Behavioural Constraints in General

First of all, we consider several systems that will allow us to understand safety constraints by means of universal coalgebra. In this section we will concentrate on safety constraints overall, not imposing some additional structure or origin on them. All systems we will encounter have type of exponent polynomial endofunctor.

Henceforth, $1 = \{\Downarrow\}$, where "$\Downarrow$" is used for referring to a violation, "$\times$" denotes the sign of the product in category **Set** i.e. the sign of the Cartesian product of sets specifically in our case, "$+$" denotes the sign of the coproduct in category **Set** i.e. the sign of the standard disjoint union of sets, and N is some finite set of elements interpreted as system notifications.

3.1 Systems with Termination

At first, we explore the properties of systems with termination. Systems of this type are used for modelling dynamical systems that tend to terminate. This feature is captured by the endofunctor which establishes the type of such systems.

Definition 8. *A **system with termination** is a* T*-type system, where the endofunctor* T *of category* **Set** *is defined as follows:*

$$\mathsf{T}X = 1 + X \qquad \textit{for any set } X$$

$$\mathsf{T}f = \lambda\, z \in (1 + X) \,.\, \begin{cases} \Downarrow & \textit{if } z = \Downarrow \\ fx & \textit{otherwise} \end{cases} \qquad \textit{for any sets } X \textit{ and } Y \textit{ and } f\colon X \to Y.$$

One can consider the carrier of such a system as the state space of a dynamical system, while the coalgebra itself set the dynamics of this dynamical system.

One can make sure that the following specification of a T-morphism agrees with the general definition of a coalgebraic morphism.

Proposition 4. *Let* $\mathfrak{a}$ *and* $\mathfrak{b}$ *be systems with termination. A mapping* $f\colon \underline{\mathfrak{a}} \to \underline{\mathfrak{b}}$ *is a* T*-morphism if and only if for any* $x \in \underline{\mathfrak{a}}$ *the following conditions hold*

1. *equations* $\mathfrak{a}x = \Downarrow$ *and* $\mathfrak{b}(fx) = \Downarrow$ *both are fulfilled or not;*
2. $\mathfrak{a}x \neq \Downarrow$ *implies* $f(\mathfrak{a}x) = \mathfrak{b}(fx)$.

There is a terminal object in category $\mathbf{Sys}(\mathsf{T})$. The next proposition demonstrates the structure of a final T-type system we denote by $\nu\mathsf{T}$ and the corresponding family of anamorphisms.

Proposition 5 (see [14] Subsect. 4.1).

$$\underline{\nu\mathsf{T}} = \mathbb{N}_\infty \qquad \text{where } \mathbb{N}_\infty = \mathbb{N} + \{\infty\}\,,$$

$$\nu\mathsf{T} = \lambda\, m \in \mathbb{N}_\infty \,.\, \begin{cases} \infty & \text{if } m = \infty \\ \Downarrow & \text{if } m = 0 \\ m - 1 & \text{otherwise}\,, \end{cases}$$

and for any T-type system $\mathfrak{c}$ and $x \in \underline{\mathfrak{c}}$,

$$[\![\mathfrak{c}]\!]x = \begin{cases} \min\{m \in \mathbb{N} \mid \mathfrak{c}^{(m+1)}x = \Downarrow\} & \text{if } \{m \in \mathbb{N} \mid \mathfrak{c}^{(m+1)}x = \Downarrow\} \neq \emptyset \\ \infty & \text{otherwise} \end{cases}$$

where $\mathfrak{c}^{(0)}x = x$ and for $k > 0$, $\mathfrak{c}^{(k)}x = \begin{cases} \Downarrow & \text{if } \mathfrak{c}^{(k-1)}x = \Downarrow \\ \mathfrak{c}\big(\mathfrak{c}^{(k-1)}x\big) & \text{otherwise}\,. \end{cases}$

Taking into account the specificity of category **Set** and that T preserves pullbacks, one can easily verify that a bisimulation of systems with termination $\mathfrak{a}$ and $\mathfrak{b}$ is defined by a relation $R \subset \underline{\mathfrak{a}} \times \underline{\mathfrak{b}}$ such that $\langle x, y\rangle \in R$ ensures $[\![\mathfrak{a}]\!]x = [\![\mathfrak{b}]\!]y$.

3.2 Systems with Output

Further, we are going to consider systems with output. The feature of systems of such a kind is that they can represent a dynamical system that provides an external tool for monitoring the system functioning. While operating such a dynamical system produces a notification stream. Assessing the arising output stream one can determine if the system is working correctly.

Of course, the set N which is used for referring to the finite set of system notifications contains at least two elements.

Definition 9. *A* ***system with output*** *is an* $\mathbf{S_N}$*-type system, where the endofunctor* $\mathbf{S_N}$ *of category* **Set** *is defined as follows:*

$$\begin{aligned} \mathbf{S_N} X &= \mathsf{N} \times X && \textit{for any set } X \\ \mathbf{S_N} f &= \mathrm{id}_{\mathsf{N}} \times f && \textit{for any sets } X \textit{ and } Y \textit{ and } f\colon X \to Y\,. \end{aligned}$$

Remark 6. Again, the definition of a system with output is given in the context of the set N.

The mappings $\mathfrak{c}^{\mathrm{out}} = \mathrm{pr}_{\mathsf{N}}\,\mathfrak{c}\colon \underline{\mathfrak{c}} \to \mathsf{N}$ and $\mathfrak{c}^{\mathrm{tr}} = \mathrm{pr}_{\underline{\mathfrak{c}}}\,\mathfrak{c}\colon \underline{\mathfrak{c}} \to \underline{\mathfrak{c}}$ can be associated with a system with output $\mathfrak{c}$.

Below, we use the following systems with output.

Example 1. Let $\boldsymbol{s} \in \mathsf{N}^{\mathbb{N}}$ then the denotation $[\boldsymbol{s}]$ is used for referring to the system with output defined as follows

$$\begin{aligned} \underline{[\boldsymbol{s}]} &= \{\boldsymbol{s}_{k..} \mid k \in \mathbb{N}\} \\ [\boldsymbol{s}]\,\boldsymbol{s}_{k..} &= \langle s_k, \boldsymbol{s}_{k+1..}\rangle && \text{for } k \in \mathbb{N}\,. \end{aligned}$$

The following proposition characterises $\mathbf{S_N}$-morphisms.

Proposition 6 (see [14] Subsect. 4.2). Let $\mathfrak{a}$ and $\mathfrak{b}$ be systems with output. A mapping $f\colon \underline{\mathfrak{a}} \to \underline{\mathfrak{b}}$ is a $\mathbf{S_N}$-morphism if and only if for any $x \in \underline{\mathfrak{a}}$ the next equations hold

1. $\mathfrak{a}^{\mathrm{out}} x = \mathfrak{b}^{\mathrm{out}}(fx)$;
2. $f(\mathfrak{a}^{\mathrm{tr}} x) = \mathfrak{b}^{\mathrm{tr}}(fx)$.

There is a terminal object in category $\mathbf{Sys}(\mathbf{S_N})$. The next proposition demonstrates the structure of a final $\mathbf{S_N}$-type system we denote by $\boldsymbol{\nu}\mathbf{S_N}$ and the corresponding family of anamorphisms.

Proposition 7 (see [14] Subsect. 4.2).

$$\begin{aligned} \underline{\boldsymbol{\nu}\mathbf{S_N}} &= \mathsf{N}^{\mathbb{N}}\,, \\ \boldsymbol{\nu}\mathbf{S_N} &= \lambda\, \boldsymbol{s} \in \mathsf{N}^{\mathbb{N}}\,.\, \langle s_0, \boldsymbol{s}_{1..}\rangle\,, \end{aligned}$$

and for any $\mathbf{S_N}$-type system $\mathfrak{c}$, $x \in \underline{\mathfrak{c}}$, and $k \in \mathbb{N}$

$$\Big(\llbracket \mathfrak{c} \rrbracket x\Big)_k = \mathfrak{c}^{\mathrm{out}}\Big(\big(\mathfrak{c}^{\mathrm{tr}}\big)^k x\Big)\,.$$

Thus, one can consider a point of the carrier of $\boldsymbol{\nu}\mathbf{S_N}$ as an observed behaviour of the $\mathbf{S_N}$-type system being studied. So our next objective is to provide and explore a mechanism for distinguishing between admissible and inadmissible system behaviour.

Taking into account the specificity of category **Set** and that $\mathbf{S_N}$ preserves pullbacks, one can easily verify that a bisimulation of systems with output $\mathfrak{a}$ and $\mathfrak{b}$ is defined by a relation $R \subset \underline{\mathfrak{a}} \times \underline{\mathfrak{b}}$ such that $\langle x, y \rangle \in R$ ensures $\mathfrak{a}^{\mathrm{out}} x = \mathfrak{b}^{\mathrm{out}} y$ and also $\langle \mathfrak{a}^{\mathrm{tr}} x, \mathfrak{b}^{\mathrm{tr}} y \rangle \in R$.

3.3 Detectors

Now we are ready to introduce the next kind of system namely the detector. Systems of this type exactly provide the required tool for analysing and specifying behaviour of systems with output. Detectors allow to distinguish between admissible and inadmissible system behaviour by only observing the output stream of the system being studied. Thus, the detector is also referred to as the detector of behavioural violations.

Definition 10. *A **detector** is a $\mathbf{D_N}$-type system, where the endofunctor $\mathbf{D_N}$ of category* **Set** *is defined as follows:*

$$\mathbf{D_N} X = (1 + X)^{\mathsf{N}} \qquad \text{for any set } X$$

$$\mathbf{D_N} f = \lambda\, \psi \in (1 + X)^{\mathsf{N}} \,.\, \lambda\, n \in \mathsf{N} \,.\, (\mathbf{T} f)(\psi n) \qquad \text{for any sets } X \text{ and } Y \text{ and } f \colon X \to Y\,.$$

Remark 7. The definition of a detector is given in the context of the set N.

The following proposition characterises $\mathbf{D_N}$-morphisms.

Proposition 8. *Let $\mathfrak{a}$ and $\mathfrak{b}$ be detectors. A mapping $f \colon \underline{\mathfrak{a}} \to \underline{\mathfrak{b}}$ is a $\mathbf{D_N}$-morphism if and only if for any $x \in \underline{\mathfrak{a}}$ and $n \in \mathsf{N}$ the next conditions hold*

1. $(\mathfrak{a} x) n = \Downarrow$ *and* $(\mathfrak{b}(f x)) n = \Downarrow$ *both are fulfilled or not;*
2. $(\mathfrak{a} x) n \neq \Downarrow$ *implies* $f\big((\mathfrak{a} x) n\big) = \big(\mathfrak{b}(f x)\big) n\,.$

Most of the proofs omitted hereafter in the section can be found in [14].

There is a terminal object in category $\mathbf{Sys}(\mathbf{D_N})$. The next proposition demonstrates the structure of a final $\mathbf{D_N}$-type system we denote by $\boldsymbol{\nu}\mathbf{D_N}$ and the corresponding family of anamorphisms.

Proposition 9.

$$\underline{\boldsymbol{\nu}\mathbf{D_N}} = \{P \subset \mathsf{N}^+ \mid P \text{ is prefix-free}\}\,,$$

$$\boldsymbol{\nu}\mathbf{D_N} = \lambda\, P \in \underline{\boldsymbol{\nu}\mathbf{D_N}} \,.\, \lambda\, n \in \mathsf{N} \,.\, \begin{cases} \Downarrow & \text{if } n \in P \\ n^{-1} \cdot P & \text{otherwise}\,, \end{cases}$$

and for any $\mathbf{D_N}$-type system $\mathfrak{c}$ and $x \in \underline{\mathfrak{c}}$,

$$[\![\mathfrak{c}]\!] x = \{\boldsymbol{u} \in \mathsf{N}^+ \mid \mathfrak{c}^+(x, \boldsymbol{u}) = \Downarrow \text{ and } \mathfrak{c}^+(x, \boldsymbol{u}_{0..k}) \neq \Downarrow \text{ whenever } 0 < k < \#\boldsymbol{u}\}$$

where $\mathfrak{c}^+ \colon \underline{\mathfrak{c}} \times \mathsf{N}^+ \to 1 + \underline{\mathfrak{c}}$ *defined as follows*

$$\mathfrak{c}^+(x, \boldsymbol{n}) = (\mathfrak{c}x)n \qquad x \in \underline{\mathfrak{c}},\ n \in \mathsf{N}\,,$$

$$\mathfrak{c}^+(x, \boldsymbol{un}) = \begin{cases} \Downarrow & \text{if } \mathfrak{c}^+(x, \boldsymbol{u}) = \Downarrow \\ \big(\mathfrak{c}\mathfrak{c}^+(x, \boldsymbol{u})\big)n & \text{otherwise} \end{cases} \qquad x \in \underline{\mathfrak{c}},\ n \in \mathsf{N},\ \boldsymbol{u} \in \mathsf{N}^+.$$

Proving the proposition based on the next facts, which are useful for studying detectors.

Lemma 1. *For any* $\mathbf{D}_\mathbf{N}$*-type system* $\mathfrak{c}$*,* $x \in \underline{\mathfrak{c}}$*, and* $\boldsymbol{u}, \boldsymbol{v} \in \mathsf{N}^+$*, the equation*

$$\mathfrak{c}^+(x, \boldsymbol{uv}) = \begin{cases} \Downarrow & \text{if } \mathfrak{c}^+(x, \boldsymbol{u}) = \Downarrow \\ \mathfrak{c}^+\big(\mathfrak{c}^+(x, \boldsymbol{u}), \boldsymbol{v}\big) & \text{otherwise} \end{cases}$$

holds.

Lemma 2. *Let* $P \subset \mathsf{N}^+$ *be prefix-free. Consider* $n \in \mathsf{N}$ *such that* $\boldsymbol{n} \notin P$*. Then* $n^{-1} \cdot P \subset \mathsf{N}^+$ *and it is prefix-free.*

Lemma 3. *Let* $P \subset \mathsf{N}^+$ *be prefix-free. Then, one can represent* P *as the next disjunctive union*

$$P = \mathsf{N}_P + \sum_{n \in \mathsf{N} \setminus \mathsf{N}_P} n \cdot P_n$$

where $\mathsf{N}_P = \{n \in \mathsf{N} \mid \boldsymbol{n} \in P\}$ *and* $P_n = n^{-1} \cdot P$*.*

Remark 8. In the previous lemma, when providing such a representation, we regard the elements of N_P as words.

The next proposition gives a characterisation of the subsets of $\underline{\nu\mathbf{D}_\mathbf{N}}$ that are carriers of subsystems of $\nu\mathbf{D}_\mathbf{N}$.

Proposition 10. *Let* C *be a subset of* $\underline{\nu\mathbf{D}_\mathbf{N}}$*. Then, there exists a* $\mathbf{D}_\mathbf{N}$*-type system* $\mathfrak{c}$ *such that* $\underline{\mathfrak{c}} = C$*, and the natural embedding* $j_C \colon C \hookrightarrow \underline{\nu\mathbf{D}_\mathbf{N}}$ *lifts to the coalgebraic morphism* $j_C \colon \mathfrak{c} \to \nu\mathbf{D}_\mathbf{N}$ *if and only if the condition*

$$\text{for any } P \in C \text{ and } n \in \mathsf{N},\ \text{either } \boldsymbol{n} \in P \text{ or } n^{-1} \cdot P \in C$$

is satisfied.
In the case, $\mathfrak{c}$ *is a subsystem of* $\nu\mathbf{D}_\mathbf{N}$ *defined by the natural embedding.*

3.4 Monitoring Outputs Using Detectors

Now we are going to demonstrate a general coalgebraic framework that allows us to combine a detector and a system with output and obtain a system with termination. It must be emphasised that this combination is functorial i.e. it is given by a special bifunctor, for which the domain is the product of the category of systems with output and the category of detectors, and the codomain is the category of systems with termination. This framework exactly provides a tool for recognising behavioural violations of a system with output.

As above, N is a notification set.

Let $\mathfrak{d}\colon \mathbf{Sys}(\mathbf{D_N})$ then for any system with output $\mathfrak{s}$, one can define

the set $\underline{\mathbf{M}_{\mathfrak{d}}\mathfrak{s}} = \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}$,

the mapping

$$\mathbf{M}_{\mathfrak{d}}\mathfrak{s} = \lambda\,\langle x, y\rangle \in \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}\ .\ \texttt{let}\ z = (\mathfrak{d}y)\,(\mathfrak{s}^{\mathrm{out}}x)\ \texttt{in} \begin{cases} \Downarrow & \text{if } z = \Downarrow \\ \langle \mathfrak{s}^{\mathrm{tr}}x, z\rangle & \text{otherwise}, \end{cases}$$

and also, for any $\mathbf{S_N}$-morphism $f\colon \mathfrak{s}_1 \to \mathfrak{s}_2$, the mapping

$$\mathbf{M}_{\mathfrak{d}}f = f \times \mathrm{id}_{\underline{\mathfrak{d}}}\ .$$

Lemma 4. $\mathbf{M}_{\mathfrak{d}}$ *is a functor from* $\mathbf{Sys}(\mathbf{S_N})$ *into* $\mathbf{Sys}(\mathbf{T})$.

Proof. Evidently, we need only to prove that $\mathbf{M}_{\mathfrak{d}}f$ is a $\mathbf{T}$-morphism whenever $f\colon \underline{\mathfrak{s}_1} \to \underline{\mathfrak{s}_2}$ is an $\mathbf{S_N}$-morphism from $\mathfrak{s}_1$ into $\mathfrak{s}_2$.
Assume that $x \in \underline{\mathfrak{s}_1}$ and $y \in \underline{\mathfrak{d}}$ then

$$(\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_1)\langle x, y\rangle = \Downarrow \quad \text{iff} \quad (\mathfrak{d}y)\,(\mathfrak{s}_1^{\mathrm{out}}x) = \Downarrow .$$

Similarly,

$$(\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_2)\big((\mathbf{M}_{\mathfrak{d}}f)\langle x, y\rangle\big) = \Downarrow \quad \text{iff} \quad (\mathfrak{d}y)\,(\mathfrak{s}_2^{\mathrm{out}}(fx)) = \Downarrow .$$

Taking into account f is an $\mathbf{S_N}$-morphism, we conclude that $\mathfrak{s}_1^{\mathrm{out}}x = \mathfrak{s}_2^{\mathrm{out}}(fx)$ and, therefore, $(\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_1)\langle x, y\rangle = \Downarrow$ is equivalent to $(\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_2)\big((\mathbf{M}_{\mathfrak{d}}f)\langle x, y\rangle\big) = \Downarrow$.
If $(\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_1)\langle x, y\rangle \neq \Downarrow$ i.e. $(\mathfrak{d}y)\,(\mathfrak{s}_1^{\mathrm{out}}x) \neq \Downarrow$ then

$$\begin{aligned}(\mathbf{M}_{\mathfrak{d}}f)\big((\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_1)\langle x, y\rangle\big) &= (\mathbf{M}_{\mathfrak{d}}f)\langle \mathfrak{s}_1^{\mathrm{tr}}x, (\mathfrak{d}y)\,(\mathfrak{s}_1^{\mathrm{out}}x)\rangle \\ &= \langle f(\mathfrak{s}_1^{\mathrm{tr}}x), (\mathfrak{d}y)\,(\mathfrak{s}_1^{\mathrm{out}}x)\rangle = \langle (\mathfrak{s}_2^{\mathrm{tr}}(fx)), (\mathfrak{d}y)\,(\mathfrak{s}_2^{\mathrm{out}}(fx))\rangle \\ &= (\mathbf{M}_{\mathfrak{d}}\mathfrak{s}_2)\big((\mathbf{M}_{\mathfrak{d}}f)\langle x, y\rangle\big)\ .\end{aligned}$$

Note this calculation uses the assumption that f is an $\mathbf{S_N}$-morphism. □

Similarly, let $\mathfrak{s}\colon \mathbf{Sys}(\mathbf{S_N})$ then for any detector $\mathfrak{d}$, we can define

the set $\underline{\mathbf{C}_{\mathfrak{s}}\mathfrak{d}} = \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}$,

the mapping

$$\mathbf{C}_{\mathfrak{s}}\mathfrak{d} = \lambda\,\langle x, y\rangle \in \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}\ .\ \texttt{let}\ z = (\mathfrak{d}y)\,(\mathfrak{s}^{\mathrm{out}}x)\ \texttt{in} \begin{cases} \Downarrow & \text{if } z = \Downarrow \\ \langle \mathfrak{s}^{\mathrm{tr}}x, z\rangle & \text{otherwise}, \end{cases}$$

and also, for any $\mathbf{D_N}$-morphism $f\colon \mathfrak{d}_1 \to \mathfrak{d}_2$, the mapping

$$\mathbf{C}_{\mathfrak{s}}f = \mathrm{id}_{\underline{\mathfrak{s}}} \times f\ .$$

Lemma 5. $\mathbf{C}_{\mathfrak{s}}$ *is a functor from* $\mathbf{Sys}(\mathbf{D_N})$ *into* $\mathbf{Sys}(\mathbf{T})$*.*

Proof. Evidently, we need only to prove that $\mathbf{C}_{\mathfrak{s}} f$ is a $\mathbf{T}$-morphism whenever $f\colon \underline{\mathfrak{d}_1} \to \underline{\mathfrak{d}_2}$ is a $\mathbf{D_N}$-morphism from $\mathfrak{d}_1$ into $\mathfrak{d}_2$.
The corresponding reasoning is quite similar to the reasoning used for proving the previous lemma and, therefore, we omit it. □

Theorem 1. *For* $\mathfrak{s}\colon \mathbf{Sys}(\mathbf{S_N})$ *and* $\mathfrak{d}\colon \mathbf{Sys}(\mathbf{D_N})$*, let us define*

the set $\underline{\mathbf{J}(\mathfrak{s},\mathfrak{d})} = \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}\,,$

the mapping

$$\mathbf{J}(\mathfrak{s},\mathfrak{d}) = \lambda\,\langle x,y\rangle \in \underline{\mathfrak{s}} \times \underline{\mathfrak{d}}\;.\;\texttt{let}\; z = (\mathfrak{d}y)\,(\mathfrak{s}^{\mathrm{out}}x)\;\texttt{in}\begin{cases}\Downarrow & \text{if } z = \Downarrow\\ \langle \mathfrak{s}^{\mathrm{tr}}x, z\rangle & \text{otherwise}\,,\end{cases}$$

and also, for any morphisms $f\colon \mathfrak{s}_1 \to \mathfrak{s}_2$ *and* $g\colon \mathfrak{d}_1 \to \mathfrak{d}_2$ *of systems with output and detectors respectively, the mapping*

$$\mathbf{J}(f,g) = f \times g\,.$$

then $\mathbf{J}$ *is a functor from* $\mathbf{Sys}(\mathbf{S_N}) \times \mathbf{Sys}(\mathbf{D_N})$ *into* $\mathbf{Sys}(\mathbf{T})$*.*

Proof. Note that for any $\mathfrak{s}\colon \mathbf{Sys}(\mathbf{S_N})$ and $\mathfrak{d}\colon \mathbf{Sys}(\mathbf{D_N})$, $\mathbf{M}_{\mathfrak{d}}\mathfrak{s} = \mathbf{C}_{\mathfrak{s}}\mathfrak{d}$.
Further, for $\mathfrak{s}_1, \mathfrak{s}_2\colon \mathbf{Sys}(\mathbf{S_N})$, $\mathfrak{d}_1, \mathfrak{d}_2\colon \mathbf{Sys}(\mathbf{D_N})$, and the corresponding morphisms $f\colon \mathfrak{s}_1 \to \mathfrak{s}_2$ and $g\colon \mathfrak{d}_1 \to \mathfrak{d}_2$,

$$\begin{aligned}(\mathbf{C}_{\mathfrak{s}_2} g)(\mathbf{M}_{\mathfrak{d}_1} f) &= (\mathrm{id}_{\underline{\mathfrak{s}_2}} \times g)(f \times \mathrm{id}_{\underline{\mathfrak{d}_1}}) = f \times g\\ &= (f \times \mathrm{id}_{\underline{\mathfrak{d}_2}})(\mathrm{id}_{\underline{\mathfrak{s}_1}} \times g) = (\mathbf{M}_{\mathfrak{d}_2} f)(\mathbf{C}_{\mathfrak{s}_1} g)\,.\end{aligned}$$

Using Proposition 1 of [8, p. 37] and Lemmas 4 and 5, one can conclude the existence of a unique functor $\mathbf{B}\colon \mathbf{Sys}(\mathbf{S_N}) \times \mathbf{Sys}(\mathbf{D_N}) \to \mathbf{Sys}(\mathbf{T})$ such that $\mathbf{B}(\mathfrak{s},\mathfrak{d}) = \mathbf{M}_{\mathfrak{d}}\mathfrak{s} = \mathbf{C}_{\mathfrak{s}}\mathfrak{d}$ and $\mathbf{B}(f,g) = (\mathbf{C}_{\mathfrak{s}_2} g)(\mathbf{M}_{\mathfrak{d}_1} f) = (\mathbf{M}_{\mathfrak{d}_2} f)(\mathbf{C}_{\mathfrak{s}_1} g)$. But the direct calculation shows that $\mathbf{B}(\mathfrak{s},\mathfrak{d}) = \mathbf{J}(\mathfrak{s},\mathfrak{d})$ and $\mathbf{B}(f,g) = \mathbf{J}(f,g)$.
Thus, $\mathbf{J}$ is a functor. □

Theorem 2. *For* $\mathfrak{s}_k\colon \mathbf{Sys}(\mathbf{S_N})$*,* $\mathfrak{d}_k\colon \mathbf{Sys}(\mathbf{D_N})$ $(k = 1, 2)$ *and bisimulations* $\mathfrak{s}_1 \xleftarrow{p_1} \mathfrak{s}_{rel} \xrightarrow{p_2} \mathfrak{s}_2$ *and* $\mathfrak{d}_1 \xleftarrow{q_1} \mathfrak{d}_{rel} \xrightarrow{q_2} \mathfrak{d}_2$*, the span*

$$\mathbf{J}(\mathfrak{s}_1,\mathfrak{d}_1) \xleftarrow{\mathbf{J}(p_1,q_1)} \mathbf{J}(\mathfrak{s}_{rel},\mathfrak{d}_{rel}) \xrightarrow{\mathbf{J}(p_2,q_2)} \mathbf{J}(\mathfrak{s}_2,\mathfrak{d}_2)$$

is a bisimulation.

Proof. See [14, Subsect. 5.1, Theorem 3]. □

The next lemma is useful below.

Lemma 6 (see [14] Subsect. 5.2, Lemma 4). For $\boldsymbol{s} \in \mathsf{N}^{\mathbb{N}}$, $\mathfrak{c}\colon \mathbf{Sys}(\mathbf{D_N})$, and $x \in \underline{\mathfrak{c}}$,

$$\mathbf{J}([\boldsymbol{s}], \mathfrak{c})^{(m)}\langle \boldsymbol{s}, x\rangle = \begin{cases} \Downarrow & \text{if } \mathfrak{c}^{+}(x, \boldsymbol{s}_{0..m}) = \Downarrow \\ \langle \boldsymbol{s}_{m..}, \mathfrak{c}^{+}(x, \boldsymbol{s}_{0..m})\rangle & \text{otherwise} \end{cases} \quad \text{for } m > 0.$$

Let $\mathfrak{s}\colon \mathbf{Sys}(\mathbf{S_N})$ and $\mathfrak{d}\colon \mathbf{Sys}(\mathbf{D_N})$ then one can define a relation "$\mathfrak{s} \bowtie \mathfrak{d}$" on $\underline{\mathfrak{s}} \times \underline{\mathfrak{d}}$ as follows

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \textit{if and only if} \quad [\![\mathbf{J}(\mathfrak{s}, \mathfrak{d})]\!]\langle x, y\rangle = \infty\,.$$

This relation, informally speaking, establishes the fact whether we can determine the behavioural violation of system $\mathfrak{s}$ using detector $\mathfrak{d}$ if the initial state of $\mathfrak{s}$ is x, and the initial setting of $\mathfrak{d}$ is y.

In the case when the relation is fulfilled, such a determination is impossible, and system state x and detector setting y are called below ***consistent***.

Proposition 11. *For* $\mathfrak{s}\colon \mathbf{Sys}(\mathbf{S_N})$ *and* $\mathfrak{d}\colon \mathbf{Sys}(\mathbf{D_N})$*,* $x \in \underline{\mathfrak{s}}$ *and* $y \in \underline{\mathfrak{d}}$*,*

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \textit{if and only if} \quad [\![\mathfrak{s}]\!]x \; (\nu\mathbf{S_N} \bowtie \nu\mathbf{D_N}) \; [\![\mathfrak{d}]\!]y\,.$$

Proof. First of all, let us consider the morphism

$$\mathbf{J}(\,[\![\mathfrak{s}]\!],\, [\![\mathfrak{d}]\!]\,)\colon \mathbf{J}(\mathfrak{s}, \mathfrak{d}) \to \mathbf{J}(\nu\mathbf{S_N}, \nu\mathbf{D_N})\,.$$

Taking into account the property of final system with termination, the following equation is fulfilled

$$[\![\mathbf{J}(\mathfrak{s}, \mathfrak{d})]\!] = [\![\mathbf{J}(\nu\mathbf{S_N}, \nu\mathbf{D_N})]\!] \circ \mathbf{J}(\,[\![\mathfrak{s}]\!],\, [\![\mathfrak{d}]\!]\,)\,.$$

Due to the definition of $\mathfrak{s} \bowtie \mathfrak{d}$, we have

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \text{iff} \quad [\![\mathbf{J}(\mathfrak{s}, \mathfrak{d})]\!]\langle x, y\rangle = \infty\,.$$

Therefore,

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \text{iff} \quad \Big(\,[\![\mathbf{J}(\nu\mathbf{S_N}, \nu\mathbf{D_N})]\!] \circ \mathbf{J}(\,[\![\mathfrak{s}]\!],\, [\![\mathfrak{d}]\!]\,)\Big)\langle x, y\rangle = \infty\,,$$

or equivalently

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \text{iff} \quad [\![\mathbf{J}(\nu\mathbf{S_N}, \nu\mathbf{D_N})]\!]\Big(\langle\,[\![\mathfrak{s}]\!]x,\, [\![\mathfrak{d}]\!]y\rangle\Big) = \infty\,.$$

Hence,

$$x \; \mathfrak{s} \bowtie \mathfrak{d} \; y \quad \text{iff} \quad [\![\mathfrak{s}]\!]x \; (\nu\mathbf{S_N} \bowtie \nu\mathbf{D_N}) \; [\![\mathfrak{d}]\!]y$$

and proof is complete. □

We now establish an association between detectors and safety constraints. For any $\mathfrak{c}\colon \mathbf{Sys}(\mathsf{D_N})$ and $x \in \underline{\mathfrak{c}}$, let us define the following set

$$[\![\mathfrak{c}]\!]_x = \{\boldsymbol{s} \in \mathsf{N}^{\mathbb{N}} \mid \boldsymbol{s}\ [\boldsymbol{s}] \bowtie \mathfrak{c}\ x\}.$$

Remark 9. It must be emphasised that in the case the following representation is equivalent to the one above

$$[\![\mathfrak{c}]\!]_x = \{\boldsymbol{s} \in \mathsf{N}^{\mathbb{N}} \mid \boldsymbol{s}\ \nu\mathsf{S_N} \bowtie \mathfrak{c}\ x\}\,.$$

This is related to the fact that $[\boldsymbol{s}]$ is a subsystem of $\nu\mathsf{S_N}$ and the corresponding anamorphism $[\!(\,[\boldsymbol{s}]\,)\!]$ is the inclusion map.

Lemma 7 (see [14] Subsect. 5.2, Lemma 5). For any $\mathfrak{c}\colon \mathbf{Sys}(\mathsf{D_N})$ and $x \in \underline{\mathfrak{c}}$, the set $[\![\mathfrak{c}]\!]_x$ is a safety constraint.

Lemma 8 (see [14] Subsect. 5.2, Lemma 6). For any safety constraint $S \subset \mathsf{N}^{\mathbb{N}}$, there exist a $\mathsf{D_N}$-type system $\mathfrak{c}_S$ and $x \in \underline{\mathfrak{c}_S}$ such that $[\![\mathfrak{c}_S]\!]_x = S$.

Theorem 3 (about universal detector). *A subset $S \subset \mathsf{N}^{\mathbb{N}}$ is a safety constraint if and only if there exists $P \in \underline{\nu\mathsf{D_N}}$ such that $S = [\![\nu\mathsf{D_N}]\!]_P$.*

Proof. Lemmas 7 and 8 ensure that a subset $S \subset \mathsf{N}^{\mathbb{N}}$ is a safety constraint if and only if there exist $\mathfrak{c}\colon \mathbf{Sys}(\mathsf{D_N})$ and $x \in \underline{\mathfrak{c}}$ such that $S = [\![\mathfrak{c}]\!]_x$. Let us take $P = [\!(\mathfrak{c})\!]x$. Then, by virtue of remark 9 and proposition 11 one can conclude that $S = [\![\mathfrak{c}]\!]_x = [\![\nu\mathsf{D_N}]\!]_P$. ☐

Corollary 1. *A subset $S \subset \mathsf{N}^{\mathbb{N}}$ is a safety constraint if and only if it is equal to $\{\boldsymbol{s} \in \mathsf{N}^{\mathbb{N}} \mid \boldsymbol{s}_{0..m} \notin P$ for any $m > 0\}$ for some prefix-free subset P of N^{+}.*

Corollary 2. *For any $\mathsf{D_N}$-type systems $\mathfrak{a}$ and $\mathfrak{b}$ and $x \in \underline{\mathfrak{a}}$ and $y \in \underline{\mathfrak{b}}$, the equation $[\!(\mathfrak{a})\!]x = [\!(\mathfrak{b})\!]y$ ensures $[\![\mathfrak{a}]\!]_x = [\![\mathfrak{b}]\!]_y$.*

Theorem 3 guarantees that for the family of all safety constraints, there is a universal detector, i.e. a detector that recognises any safety constraint when the detector configured appropriately.

Definition 11. *Let $\mathfrak{F}$ be a family of safety constraints. $\mathfrak{F}$ is called a **family with a universal detector** if there exists $\mathfrak{c}\colon \mathbf{Sys}(\mathsf{D_N})$ such that $\mathfrak{F} = \{[\![\mathfrak{c}]\!]_x \mid x \in \underline{\mathfrak{c}}\}$. In the case, we call $\mathfrak{c}$ a **universal detector** for $\mathfrak{F}$.*

Theorem 4 (see [14] Subsect. 5.3, Theorem 5). A family of safety constraints $\mathfrak{F}$ is a family with a universal detector if and only if

$$\{P \in \underline{\nu\mathsf{D_N}} \mid [\![\nu\mathsf{D_N}]\!]_P \in \mathfrak{F}\}$$

is the carrier of a subsystem of $\nu\mathsf{D_N}$ defined by the natural embedding.

An interesting fact can be derived using this theorem.

Theorem 5. *There exists a universal set for the class of prefix-free recursively enumerable sets.*

Proof. Firstly, let us demonstrate for any prefix-free recursively enumerable set $P \in \underline{\boldsymbol{\nu}\mathbf{D}_{\mathsf{N}}}$ and $n \in \mathsf{N}$, there is an algorithm that recognises $\boldsymbol{n} \in P$ and is halted iff $\boldsymbol{nu} \in P$ where $\boldsymbol{u} \in P$.

Indeed, let us take A_P be a semi-decision procedure for P then the required algorithm is the next

1. **let** $\boldsymbol{w}_k = (\boldsymbol{nu})_{0..k+1}$ for $k = 0, \ldots, \#\boldsymbol{u} + 1$;
2. **launch parallelly** A_P for $\boldsymbol{w}_0, \ldots, \boldsymbol{w}_{\#\boldsymbol{u}+1}$;
3. **wait** for one of the runs to halt;
4. **let** k be number of the halted run;
5. **if** $k = 0$ **then return** "$\boldsymbol{n} \in P$";
6. **if** $0 < k \leq \#\boldsymbol{u}$ **then repeat infinitely** an empty action;
7. **if** $k = \#\boldsymbol{u} + 1$ **then return** $\boldsymbol{nu} \in P$.

Note the precondition P is prefix-free ensures at most one run to halt. Hence, only one of items 5, 6, 7 is realised. Endless waiting for the completion of point 3 is possible only if neither $\boldsymbol{n}$, nor each non-empty prefix of $\boldsymbol{nu}$ lies in P.

This algorithm guarantees the hypothesis of Theorem 4 is fulfilled. Thus, due to the theorem a universal set for the class of prefix-free recursively enumerable sets exists. ☐

4 Safeness and Causality in Case of Distributed System

In this section we provide some reasoning about applying the constructed coalgebraic approach to the problem of specifying causality relationships in distributed systems.

Anywhere in this section, C is an arbitrary but fixed finite set of clocks and $\mathcal{N}\mathsf{C}$ is the corresponding set of clock notifications that is the set of all non-empty subset of C. Informally, each message contains a list of clocks ticked at the same time as the message was sent. Thus, notifications are no longer atomic entities or, in other words, first-class citizens. Now, clock ticks are atomic entities or, in other words, first-class citizens, whose combinations are notifications.

So we are going to adjust the results obtained above for this more specific situation.

The logical clock model considers clock tick notification streams as the only information available about temporal order between events. These streams are called schedules in the model context. The coalgebraic meaning of schedules is related to the system with output with the only difference that in the case the definition of the system is given with regard to the set of clock notifications $\mathcal{N}\mathsf{C}$. In other words, we will consider a distributed system with the clock set C as a $\mathbf{S}_{\mathcal{N}\mathsf{C}}$-type system. Thus, our capacity to formulate causality constraints for any system equipped with the clock set C is limited by statements in schedules terms only.

Similarly, the detectors we will be using to deal with causality constraints are, in fact, common detectors, but defined in the context of the set $\mathcal{N}\mathsf{C}$, namely these detectors are $\mathbf{D}_{\mathcal{N}\mathsf{C}}$-type systems and called detectors of causality constraint violations.

Overall, the defined in the previous section systems, their properties, and related concepts remain unchanged, but are adapted for $\mathcal{N}\mathsf{C}$ as a system notification set. Thus, in our situation causality constraints specification is a special yet extremely important case of safety constraints specification.

Theorem 5, completing the previous section, gives the positive answer to the question of whether there exists a universal language for specifying all clock constraints being effectively verified. Unfortunately, the corresponding algorithm has uncontrolled complexity and, therefore, it cannot be applied for practical issues.

This fact is a reason to search for more specific classes of constraints hold a suitable complexity. The next section explores the class of such a kind underlying the Clock Constraint Specification Language (CCSL) [3] used in the UML Profile for MARTE [11].

5 Counter-Detectors and Their Diophantine Properties

In this section, we consider a special class of systems of type $\mathbf{D}_{\mathcal{N}\mathsf{C}}$, which is used to define the semantics of CCSL.

In this section, we define the function $\mathrm{occ}_{\mathsf{c}} \colon \mathcal{N}\mathsf{C}^* \to \mathbb{N}$ associated with $\mathsf{c} \in \mathsf{C}$ as follows

$$\begin{aligned} &\mathrm{occ}_{\mathsf{c}} \\ &= \lambda\, \boldsymbol{u} \in \mathcal{N}\mathsf{C}^* \,.\, \begin{cases} 0 & \text{if } \boldsymbol{u} = \boldsymbol{\epsilon} \\ \mathrm{occ}_{\mathsf{c}}\, \boldsymbol{u}' & \text{if } \boldsymbol{u} = \boldsymbol{u}'\boldsymbol{n} \text{ for } \boldsymbol{u}' \in \mathcal{N}\mathsf{C}^*,\ n \in \mathcal{N}\mathsf{C}, \text{ and } \mathsf{c} \notin n \\ \mathrm{occ}_{\mathsf{c}}\, \boldsymbol{u}' + 1 & \text{if } \boldsymbol{u} = \boldsymbol{u}'\boldsymbol{n} \text{ for } \boldsymbol{u}' \in \mathcal{N}\mathsf{C}^*,\ n \in \mathcal{N}\mathsf{C}, \text{ and } \mathsf{c} \in n\,. \end{cases} \end{aligned}$$

This function counts how many times the corresponding clock ticked in a given word of clock notifications.

5.1 Counter-Detectors

Let $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^{\mathsf{C}}$ then an $\mathbf{D}_{\mathcal{N}\mathsf{C}}$-type system $\mathfrak{c}_{\mathsf{V}}$ is called a $\mathbf{D}_{\mathcal{N}\mathsf{C}}$-type counter-detector if it is defined as follows

$$\begin{aligned} &\underline{\mathfrak{c}_{\mathsf{V}}} = \mathbb{N}^{\mathsf{C}} \\ &\mathfrak{c}_{\mathsf{V}} = \lambda\, x \in \mathbb{N}^{\mathsf{C}} \,.\, \lambda\, n \in \mathcal{N}\mathsf{C} \,.\, \begin{cases} \Downarrow & \text{if } \langle n, x\rangle \in \mathsf{V} \\ x + \boldsymbol{n} & \text{otherwise} \end{cases} \end{aligned}$$

where $x + \boldsymbol{n} = \lambda\, \mathsf{c} \in \mathsf{C} \,.\, x\,\mathsf{c} + \mathrm{occ}_{\mathsf{c}}\, \boldsymbol{n}\,.$

Remark 10. Given the fact that such $\boldsymbol{n}$ is actually $n \in \mathcal{N}\mathsf{C}$ one can refine the above operation to the following representation

$$x + \boldsymbol{n} = \lambda\,\mathsf{c} \in \mathsf{C}\,.\begin{cases}(x\,\mathsf{c}) + 1 & \text{if } \mathsf{c} \in n\\ x\,\mathsf{c} & \text{otherwise}\,.\end{cases}$$

Our nearest aim is to establish whether the family of causality constraints recognised by counter-detectors is a family with a universal detector.

The first step for achieving the claimed aim is given by the next theorem.

Theorem 6. *Let P be a prefix-free subset of $\mathcal{N}\mathsf{C}^+$ then there exists $\mathsf{A} \subset \mathcal{N}\mathsf{C}\times\mathbb{N}^{\mathsf{C}}$ such that $[\![\mathfrak{c}_{\mathsf{A}}]\!]x = P$ for some $x \in \underline{\mathfrak{c}_{\mathsf{A}}}$ if and only if P meets the following condition*

for any $\boldsymbol{v} \in \mathcal{N}\mathsf{C}^+$,
$\boldsymbol{u}'\boldsymbol{v} \in P$ is equivalent to $\boldsymbol{u}''\boldsymbol{v} \in P$ whenever $\boldsymbol{u}', \boldsymbol{u}'' \in \mathcal{N}\mathsf{C}^+$ are such that (2)
$\boldsymbol{u}'_{0..m} \notin P$ and $\boldsymbol{u}''_{0..k} \notin P$ for all $0 < m \leq \#\boldsymbol{u}'$, $0 < k \leq \#\boldsymbol{u}''$ and $\operatorname{occ}_{\mathsf{c}} \boldsymbol{u}' = \operatorname{occ}_{\mathsf{c}} \boldsymbol{u}''$ for each $\mathsf{c} \in \mathsf{C}$.

For proving the theorem, we need some auxiliary results.

Lemma 9. *Let $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^{\mathsf{C}}$ and $x \in \underline{\mathfrak{c}_{\mathsf{V}}}$. Then $[\![\mathfrak{c}_{\mathsf{V}}]\!]x$ meets condition (2).*

Proof. Let us assume $P = [\![\mathfrak{c}_{\mathsf{V}}]\!]x$ for some $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^{\mathsf{C}}$ and $x \in \underline{\mathfrak{c}_{\mathsf{V}}}$ and take $\boldsymbol{u}', \boldsymbol{u}'' \in \mathcal{N}\mathsf{C}^+$ such that $\boldsymbol{u}'_{0..m} \notin P$ and $\boldsymbol{u}''_{0..k} \notin P$ for all $0 < m \leq \#\boldsymbol{u}'$, $0 < k \leq \#\boldsymbol{u}''$ and $\operatorname{occ}_{\mathsf{c}} \boldsymbol{u}' = \operatorname{occ}_{\mathsf{c}} \boldsymbol{u}''$ for each $\mathsf{c} \in \mathsf{C}$.

Then the first group of conditions ensures $\mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}'), \mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}'') \in \underline{\mathfrak{c}_{\mathsf{V}}}$ and the condition $\operatorname{occ}_{\mathsf{c}} \boldsymbol{u}' = \operatorname{occ}_{\mathsf{c}} \boldsymbol{u}''$ for each $\mathsf{c} \in \mathsf{C}$ ensures $\mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}') = \mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}'')$.

Now, for an arbitrary $\boldsymbol{v} \in \mathcal{N}\mathsf{C}^+$ and $0 < l \leq \#\boldsymbol{v}$, we have (see 1)

$$\begin{aligned}\mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}'\boldsymbol{v}_{0..l}) &= \mathfrak{c}^+_{\mathsf{V}}(\mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}'), \boldsymbol{v}_{0..l})\\ &= \mathfrak{c}^+_{\mathsf{V}}(\mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}''), \boldsymbol{v}_{0..l}) = \mathfrak{c}^+_{\mathsf{V}}(x, \boldsymbol{u}''\boldsymbol{v}_{0..l})\end{aligned}$$

Thus, one can conclude $\boldsymbol{u}'\boldsymbol{v} \in P$ is equivalent to $\boldsymbol{u}''\boldsymbol{v} \in P$. □

Proof (Proof of Theorem 6). Taking into account Lemma 9, we need only to prove that for any prefix-free $P \subset \mathcal{N}\mathsf{C}^+$ satisfying condition (2), there exists $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^{\mathsf{C}}$ such that $P = [\![\mathfrak{c}_{\mathsf{V}}]\!]x$ for some $x \in \underline{\mathfrak{c}_{\mathsf{V}}}$.

To do this let us take

$$\begin{aligned}\mathsf{V} = \{\langle n, x\rangle \in \mathcal{N}\mathsf{C} \times \mathbb{N}^{\mathsf{C}} \mid\\ \boldsymbol{v}\boldsymbol{n} \in P \text{ for some } \boldsymbol{v} \in \mathcal{N}\mathsf{C}^* \text{ such that } x = \lambda\,\mathsf{c} \in \mathsf{C}\,.\operatorname{occ}_{\mathsf{c}} \boldsymbol{v}\}\end{aligned}$$

and check that $[\![\mathfrak{c}_{\mathsf{V}}]\!]z = P$ where $z = \lambda\,\mathsf{c} \in \mathsf{C}\,.\,0$ i.e. that for any $\boldsymbol{u} \in \mathcal{N}\mathsf{C}^+$, $\boldsymbol{u} \in [\![\mathfrak{c}_{\mathsf{V}}]\!]z$ iff $\boldsymbol{u} \in P$.

Let us assume that $\boldsymbol{u} \in [\![\mathfrak{c}_\mathsf{V}]\!] z$.

If $\boldsymbol{u} = \boldsymbol{n}$ for some $n \in \mathcal{N}\mathsf{C}$ then $\boldsymbol{n} \in [\![\mathfrak{c}_\mathsf{V}]\!] z$ means $(\mathfrak{c}_\mathsf{V} z) n = \Downarrow$ i.e. $\boldsymbol{n} \in P$ because of $\lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{v} = z$ iff $\boldsymbol{v} = \boldsymbol{\epsilon}$.

If $\boldsymbol{u} \in [\![\mathfrak{c}_\mathsf{V}]\!] z$ and $m = \#\boldsymbol{u} > 1$ then for $x_0 = z$, we have $x_{k+1} = (\mathfrak{c}_\mathsf{V} x_k) u_k \in \underline{\mathfrak{c}_\mathsf{V}}$ whenever $k = 0, \ldots, m-2$ and $(\mathfrak{c}_\mathsf{V} x_{m-1}) u_{m-1} = \Downarrow$. In other words, $\langle u_k, x_k \rangle \notin \mathsf{V}$ where $k = 0, \ldots, m-2$ and $\langle u_{m-1}, x_{m-1} \rangle \in \mathsf{V}$.

Thus, $\boldsymbol{u}_{0..k} \notin P$ for any $0 < k \le m-1$ but there exists $\boldsymbol{v} \in \mathcal{N}\mathsf{C}^*$ such that $\boldsymbol{v}\boldsymbol{u}_{\boldsymbol{m-1}} \in P$ and $\lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{v} = \lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{u}_{0..m-1}$. But the condition $\boldsymbol{v}\boldsymbol{u}_{\boldsymbol{m-1}} \in P$ implies $\boldsymbol{v}_{0..l} \notin P$ for any $0 < l \le \#\boldsymbol{v}$ due to P is a prefix free set. Thus, condition (2) ensures $\boldsymbol{u} = \boldsymbol{u}_{0..m-1}\boldsymbol{u}_{\boldsymbol{m-1}} \in P$.

Now assume $\boldsymbol{u} \in P$ and $m = \#\boldsymbol{u}$.

Then we have $\boldsymbol{u}_{0..k} \notin P$ for any k such that $0 < k < m$ and $\langle u_{m-1}, \lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{u}_{0..m-1} \rangle \in \mathsf{V}$ due to the definition of V. One can conclude that $\langle u_k, \lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{u}_{0..k} \rangle \notin \mathsf{V}$ for any $0 \le k < m-1$. Indeed, otherwise there exists $0 \le k < m-1$ and $\boldsymbol{v} \in \mathcal{N}\mathsf{C}^*$ such that $\boldsymbol{v}\boldsymbol{u}_{\boldsymbol{k}} \in P$ and $\lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{v} = \lambda\, \mathsf{c} \in \mathsf{C}$. $\mathrm{occ}_\mathsf{c}\, \boldsymbol{u}_{0..k}$. But condition (2) ensures $\boldsymbol{u}_{0..k}\boldsymbol{u}_{\boldsymbol{k}} = \boldsymbol{u}_{0..k+1} \in P$ for some $0 \le k < m-1$.

Thus $\mathfrak{c}_\mathsf{V}^+(z, \boldsymbol{u}_{0..k}) \in \underline{\mathfrak{c}_\mathsf{V}}$ for $0 < k < m$ and $\mathfrak{c}_\mathsf{V}^+(z, \boldsymbol{u}) = \Downarrow$. Therefore we have $\boldsymbol{u} \in [\![\mathfrak{c}_\mathsf{V}]\!] z$. □

Corollary 3. *If* $P = [\![\mathfrak{c}_\mathsf{V}]\!] x$ *for some* $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ *and* $x \in \underline{\mathfrak{c}_\mathsf{V}}$ *then* $P = [\![\mathfrak{c}_{\mathsf{V}'}]\!] z$ *for some* $\mathsf{V}' \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ *and* $z = \lambda\, \mathsf{c} \in \mathsf{C}$. 0.

Corollary 4. *The family of clock constraints being detected by counter-detectors is a constraint family with a universal detector.*

Proof. To prove this fact, we use Theorem 4 and Proposition 10. Assume $P = [\![\mathfrak{c}_\mathsf{V}]\!] z$ for some $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$.

Let $n \in \mathcal{N}\mathsf{C}$ such that $\boldsymbol{n} \notin P$, $\boldsymbol{u}'_{0..m}, \boldsymbol{u}''_{0..k} \notin n^{-1} \cdot P$ for all $0 < m \le \#\boldsymbol{u}'$ and $0 < k \le \#\boldsymbol{u}''$, and $\mathrm{occ}_\mathsf{c}\, \boldsymbol{u}' = \mathrm{occ}_\mathsf{c}\, \boldsymbol{u}''$ for any $\mathsf{c} \in \mathsf{C}$ then $\boldsymbol{n}\boldsymbol{u}'_{0..m} \notin P$ and $\boldsymbol{n}\boldsymbol{u}''_{0..k} \notin P$. Theorem 6 ensures $(\boldsymbol{n}\boldsymbol{u}')\boldsymbol{v} \in P$ and $(\boldsymbol{n}\boldsymbol{u}'')\boldsymbol{v} \in P$ are equivalent for any $\boldsymbol{v} \in \mathcal{N}\mathsf{C}^+$ i.e. $\boldsymbol{u}'\boldsymbol{v} \in n^{-1} \cdot P$ and $\boldsymbol{u}''\boldsymbol{v} \in n^{-1} \cdot P$ are equivalent. Therefore, $n^{-1} \cdot P$ also satisfies condition (2) and this fact completes the proof. □

Theorem 6 allows us to show the impossibility of confining ourselves to counter-detectors only, as shown in the next example and the remark following after it.

Example 2. Let us take $\mathsf{C} = \{\mathsf{c}_1^+, \mathsf{c}_1^-, \ldots, \mathsf{c}_m^+, \mathsf{c}_m^-\}$ for some $m > 1$ and define the next $\mathcal{N}\mathsf{C}$-type system $\mathfrak{q}$

$$\underline{\mathfrak{q}} = \{c_1, \ldots, c_m\}^*$$

$$\mathfrak{q} = \lambda\, \boldsymbol{u} \in \{c_1, \ldots, c_m\}^* \,.\, \lambda\, n \in \mathcal{N}\mathsf{C} \,.\, \begin{cases} \boldsymbol{u}\boldsymbol{c}_{\boldsymbol{k}} & \text{if } n = \{\mathsf{c}_k^+\} \\ & \quad \text{for some } k = 1, \ldots, m \\ \boldsymbol{u}' & \text{if } \boldsymbol{u} = \boldsymbol{c}_{\boldsymbol{k}}\boldsymbol{u}' \text{ and } n = \{\mathsf{c}_k^-\} \\ & \quad \text{for some } k = 1, \ldots, m \\ \Downarrow & \text{otherwise} \end{cases}$$

Also, let us take $P = [\![\mathfrak{q}]\!]\boldsymbol{\epsilon}$, and $\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\}, \{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\} \in \mathcal{N}\mathsf{C}^+$ for any $1 \leq i \neq j \leq m$. Then it is evident $\mathfrak{q}^+(\boldsymbol{\epsilon}, \{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\}) \neq \Downarrow$, $\mathfrak{q}^+(\boldsymbol{\epsilon}, \{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\}) \neq \Downarrow$, and $\mathrm{occ}_{\mathsf{c}_k^\pm}\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\} = \mathrm{occ}_{\mathsf{c}_k^\pm}\{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\}$ for all $k = 1, \ldots, m$. In other words, $\{\mathsf{c}_i^+\} \notin P$, $\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\} \notin P$, $\{\mathsf{c}_j^+\} \notin P$, $\{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\} \notin P$, and $\mathrm{occ}_{\mathsf{c}_k^\pm}\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\} = \mathrm{occ}_{\mathsf{c}_k^\pm}\{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\}$ for all $k = 1, \ldots, m$.

The assumption $P = [\![\mathfrak{c}_\mathsf{V}]\!]x$ for some $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ and $x \in \underline{\mathfrak{c}_\mathsf{V}}$ leads to the equivalence of the statements $\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\}\{\mathsf{c}_i^-\} \in P$ and $\{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\}\{\mathsf{c}_i^-\} \in P$ due to Lemma 9.

But the definition of $\mathfrak{q}$ ensures $\{\mathsf{c}_i^+\}\{\mathsf{c}_j^+\}\{\mathsf{c}_i^-\} \notin P$ and $\{\mathsf{c}_j^+\}\{\mathsf{c}_i^+\}\{\mathsf{c}_i^-\} \in P$. This contradiction proves that for any $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ and $x \in \underline{\mathfrak{c}_\mathsf{V}}$, $P = [\![\mathfrak{c}_\mathsf{V}]\!]x$ is false.

Remark 11. This abstract example has practical value. Indeed, let we have m servers (here $m > 1$) each of them provides access to the queue being stored by it. Then c_k^+ and c_k^- can be interpreted as append and pop operations respectively for the k^{th} queue.

Example 2 guarantees the behaviour like to a queue for the system of the servers cannot be ensured by a counter-detector.

Remark 12. Reasoning similar to the reasoning given in Example 2 leads to the conclusion that the behaviour like to a stack for the system of the servers cannot be ensured by a counter-detector.

5.2 Diophantine Representation of a Counter-Detector

The definition of a $\mathbf{D}_{\mathcal{N}\mathsf{C}}$-type counter-detector ensures that such a detector $\mathfrak{c}_\mathsf{V}$ is uniquely defined by the corresponding set $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$, which below is referred as the ***defining set*** of a counter-detector. Hence, to specify $\mathfrak{c}_\mathsf{V}$ we need to specify V. Taking into account that any tool for specifying a set can specify a recursively enumerable set only, we concentrate on counter-detectors with the recursively enumerable defining set. Such a counter-detector is below called a ***recursively defined counter-detector***. Everywhere below we consider counter-detectors with the recursively enumerable defining set only.

Note that a set $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ is recursively enumerable if and only if the sets $\mathsf{V}_n = \{x \in \mathbb{N}^\mathsf{C} \mid \langle n, x\rangle \in \mathsf{V}\}$ are recursively enumerable for all $n \in \mathcal{N}\mathsf{C}$. This claim follows directly from the finiteness of $\mathcal{N}\mathsf{C}$. In other words, to specify a recursively enumerable set $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ is equivalent to specify the set $\{\mathsf{V}_n \mid n \in \mathcal{N}\mathsf{C}\}$ of recursively enumerable subsets of $\mathbb{N}^\mathsf{C}$.

Due to Matiyasevich-Davis-Robinson-Putnam theorem (see, [9]), the last is equivalent to specifying for any $n \in \mathcal{N}\mathsf{C}$ a polynomial P_n such that

$$\mathsf{V}_n = \{x \in \mathbb{N}^\mathsf{C} \mid P_n(x, y_1, \ldots, y_s) = 0 \text{ for some } y_1, \ldots, y_s \in \mathbb{N}\}.$$

Thus, any family of the form $\{P_n \in \mathbb{Z}[x_1, \ldots, x_m, y_1, \ldots, y_s] \mid n \in \mathcal{N}\mathsf{C}\}$ where m is the cardinal number of C and s is some positive natural number specifies the set $\mathsf{V} \subset \mathcal{N}\mathsf{C} \times \mathbb{N}^\mathsf{C}$ as follows

Table 1. An example of a diophantine representation

c_1	c_2	$P(x_1, x_2, y_1)$	Diophantine set
0	1	$x_1 - x_2 + y_1$	$\{(x_1, x_2) \in \mathbb{N}^2 \mid x_1 \leq x_2\}$
1	0	$1 + x_1 - x_2 + y_1$	$\{(x_1, x_2) \in \mathbb{N}^2 \mid x_1 < x_2\}$
1	1	$x_1 - x_2 + y_1$	$\{(x_1, x_2) \in \mathbb{N}^2 \mid x_1 \leq x_2\}$

1. let us select some enumeration $\{\mathsf{c}_1, \ldots, \mathsf{c}_m\}$ of clocks from C;
2. let us associate natural variable x_i with clock c_i $(1 \leq i \leq m)$;
3. for each $n \in \mathcal{N}\mathsf{C}$, let us specify polynomials $P_n \in \mathbb{Z}[x_1, \ldots, x_m, y_1, \ldots, y_s]$;
4. let us assume

$$\langle n, x\rangle \in \mathsf{V} \quad \text{iff} \quad \exists\, y_1 \in \mathbb{N}; \ldots; y_s \in \mathbb{N},\ P_n(x, y_1, \ldots, y_s) = 0\,.$$

Below we refer to this manner of specifying the defining set of a counter-detector as to a ***Diophantine Representation*** of the detector.

Example 3 (of a Diophantine Representation). Let us assume $\mathsf{C} = \{\mathsf{c}_1, \mathsf{c}_2\}$ and consider the clock constraints specification defined as in Table 1.

This specification determines that clock c_1 is strictly faster than clock c_2 and can be considered as a specification of the Lamport's relation "happen before" (see [7]).

Note that the rows corresponding to the situation c_1 does not tick but c_2 ticks and also to the situation c_1 and c_2 tick at the same time are coincide in columns 3 and 4. Hence, we can represent Table 1 as follows

$$\begin{aligned} &\mathsf{c}_2 && \Rightarrow x_1 - x_2 + y_1 = 0 \\ &\mathsf{c}_1 \wedge \neg\mathsf{c}_2 && \Rightarrow 1 + x_1 - x_2 + y_1 = 0 \end{aligned}$$

5.3 Universal Diophantine Simulator of Recursively Defined Counter-Detectors

Now we use Diophantine Representation for describing a universal algorithm simulating any counter-detector if its defining set is recursive. The main advantage of this result is that it gives a metric for estimating the complexity of the detector and the detection problem. Namely, the number of additional variables and the power of the polynomial representing the corresponding Diophantine set can be used for defining such a metric. The detailed discussion of this problem one can find in [10].

Let us assume the universal Diophantine simulator is set up for simulating the counter-detector with the defining set specified by polynomials $\{P_n(x_1, \ldots, x_m, y_1, \ldots, y_s) \mid n \in \mathcal{N}\mathsf{C}\}$ under assumption that the clock set is enumerated as follows $\mathsf{C} = \{\mathsf{c}_1, \ldots, \mathsf{c}_m\}$. Also, we choose some algorithm α enumerating the set $\mathbb{N}^s$ without repetitions. We mean below that αi is the i-th member of this enumeration.

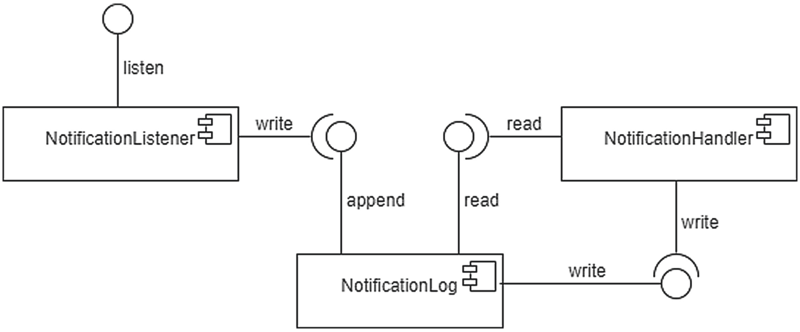

Fig. 1. Composite structure of the universal Diophantine simulator of recursively defined counter-detectors

We begin describing the universal Diophantine simulator of recursively defined counter-detectors with specifying its composite structure (see Fig. 1).

The simulator consists of one passive component (`NotificationLog`) and two active ones (`NotificationListener` and `NotificationHandler`).

Component `NotificationLog` is a data store that allows to write data items in append and direct access modes and to read ones in direct access mode.

Component `NotificationListener` provides listening the notifications channel and writing the corresponding data items using the interface of component `NotificationLog` in accordance with the next algorithm.

`NotificationListener`:

1. $x_k \leftarrow 0$ for $k = 1, \ldots, m$
2. **wait on** receiving a notification
3. **let** the received notification be n **then** append to `NotificationLog` the item $(x_1, \ldots, x_m, n, 0)$
4. $x_k \leftarrow x_k + 1$ for $k = 1, \ldots, m$ such that $\mathsf{c}_k \in n$
5. **go to** 2

Component `NotificationHandler` provides handling data items stored in `NotificationLog` for recognising behavioural violation in accordance with the following algorithm.

`NotificationHandler`:

1. $i \leftarrow 0$
2. $j \leftarrow 0$
3. **try** to read j-th element from `NotificationLog`
4. **if** the attempt is successful **then**
5. **let** $(x_1, \ldots, x_m, n, t)$ be the reading result
6. $(y_1, \ldots, y_s) \leftarrow \alpha t$
7. **if** $P_n(x_1, \ldots, x_m, y_1, \ldots y_s) = 0$ **then**
   signal about the recognised violation and **terminate** the handling
8. rewrite $(x_1, \ldots, x_m, n, t + 1)$ at j-th entry of `NotificationLog`

```
9.      j ← j + 1
10.     if j > i then i ← i + 1 and j ← 0
11.     go to 3
12.   else go to 2
```

Of course, the proposed universal Diophantine simulator is not efficient. For providing the efficiency of a counter-detector, we need to require the decidability of Diophantine problems related to the detector. For example, we can consider detectors with linear in $y_1, \ldots, y_s$ the related to them Diophantine problems.

5.4 Linear Case of Diophantine Representation

Now we are going to provide some simplifications that take place in case of all equations in the Diophantine Representation of a recursively defined counter-detector being linear in $y_1, \ldots, y_s$. Given a recursively defined counter-detector assume its Diophantine Representation has the following form

$$P_n(x, y_1, \ldots, y_s) = Q_{n,0}(x) + Q_{n,1}(x)y_1 + \ldots + Q_{n,s}(x)y_s$$

where $n \in \mathcal{N}\mathsf{C}$ and $Q_{n,0}, Q_{n,1}, \ldots, Q_{n,s}$ are polynomials with integer coefficients.

As usual we are interested in existence of natural solutions of Diophantine problems $P_n(x, y) = 0$ under some values of x for different $n \in \mathcal{N}\mathsf{C}$. Let $n \in \mathcal{N}\mathsf{C}$ and $z \in \mathbb{N}^{\mathsf{C}}$. Then one can substitute x for z in $P_n(x, y)$. Then the problem boils down to determination of existence of natural solutions of

$$a_1y_1 + \ldots + a_sy_s = b \tag{3}$$

where $a_k = Q_{n,k}(z)$ for $1 \leq k \leq s$ and $b = -Q_{n,0}(z)$. Note that $b, a_k \in \mathbb{Z}$. Without loss of generality, we assume that $b \geq 0$. Now we consider the main cases.

Case 1: $b = 0$. Then there exists an evident trivial solution namely $y_i = 0$ for all $0 \leq i \leq s$.

Case 2: $a_1, \ldots, a_s$ are negative. In this case, the required solution does not exist.

Let $g = \gcd(a_1, \ldots, a_s)$ be the greatest common divisor of the coefficients.

Case 3: g does not divide b. Then the required solution is evidently absent.

Otherwise, let $a_i := \frac{a_i}{g}$ for $i = 1, \ldots, s$ and $b := \frac{b}{g}$. The obtained equation with new coefficients is equivalent to the original one, but now $\gcd(a_1, \ldots, a_s) = 1$. So, let us proceed with consideration of cases, but now with the new equation (though we will still refer to it by (3) below).

Case 4: $a_i = 1$ for some $0 \leq i \leq s$. Then $y_i = b$ and $y_j = 0$ for all $0 \leq j \leq s$ such that $j \neq i$ constitute the required solution. The condition $\gcd(a_1, \ldots, a_s) = 1$ ensures that there exist $z_1, \ldots, z_s \in \mathbb{Z}$ such that (Bezout's identity)

$$a_1z_1 + \ldots + a_sz_s = 1\,. \tag{4}$$

Case 5: $a_i < 0$ for some $0 \leq i \leq s$. Without loss of generality assume that $a_1, \ldots, a_k > 0$ and $a_{k+1}, \ldots, a_s < 0$ for some $1 \leq k < s$. Let $n = a_1 + \ldots + a_k$ and $m = -(a_{k+1} + \ldots + a_s)$. Consider $y'_i = m$ for $i = 1, \ldots, k$ and $y'_i = n$ for $i = k+1, \ldots, s$. Then one can check the following equation holds

$$a_1 y'_1 + \ldots + a_s y'_s = 0\,. \tag{5}$$

Let us set $y_i = bz_i + Ky'_i$ where $K \in \mathbb{N}$ is such a value, that $y_i \in \mathbb{N}$ for all $i = 1, \ldots, s$. Then Eq. (3) holds under these values y_i by virtue of Eqs. (4), (5) and therefore we have a solution to the initial problem.

Note if for some $1 \leq i < j \leq s$, the equality $a_i = a_j$ is fulfilled then, evidently, the solution of the equation exists if and only there exists solution of the equation $a_1 y_1 + \ldots + a_{i-1} y_{i-1} + a_{i+1} y_{i+1} + \ldots + a_{j-1} y_{j-1} + a_{j+1} y_{j+1} + \ldots + a_s y_s + ay = b$ where $a = a_i = a_j$. This remark leads us to the last case.

Case 6: $1 < a_1 < a_2 < \ldots < a_s$. One can see that if $s \geq 2$ (the opposite case has been certainly handled up to and including Case 4) then Eq. (3) has a solution if and only if the equation $a_1 y_1 + \ldots a_{s-1} y_{s-1} = b - a_s n$ has a solution at least for one value n such that $0 \leq n \leq \left\lfloor \frac{b}{a_s} \right\rfloor$.

Thus, one can roughly assess the upper limit for the number of search options in the worst case. The number of trivial Diophantine problems the original equation comes down to in the worst case is roughly limited from above by

$$\left(\left\lfloor \frac{b}{\min\limits_{i=1,\ldots,s} a_i} \right\rfloor + 1 \right)^{s-1}.$$

This value is exponential in the number of coefficients and polynomial in the constant term b.

Now let us modify our universal Diophantine simulator to deal with this specific situation in a more appropriate way. Taking into account that the corresponding Diophantine problems are decidable with reasonable complexity we can handle incoming notifications completely one at a time. Therefore, we only need to require `NotificationLog` to provide a queue-consistent interface.

`NotificationListener`:

1. $x_k \leftarrow 0$ for $k = 1, \ldots, m$
2. **wait on** receiving a notification
3. **let** the received notification be n **then** enqueue to `NotificationLog` the item $(x_1, \ldots, x_m, n)$
4. $x_k \leftarrow x_k + 1$ for $k = 1, \ldots, m$ such that $\mathsf{c}_k \in n$
5. **go to** 2

Adjust the underlying algorithm of the simulator in accordance with the reasoning above; the changes mainly concern `NotificationHandler`.

`NotificationHandler`:

1. **try** to dequeue element from `NotificationLog`
2. **if** the attempt is successful **then**
3. **let** *problem* $= (x_1, \ldots, x_m, n)$ be the reading result
4. **if** `hasSolutions`(*problem*) **then**
   signal about the recognised violation and **terminate** the handling
5. **go to** 1

Given a notification (x, n), procedure `hasSolutions` checks whether there exists an s-tuple y of natural numbers such that $P_n(x, y) = 0$ in a manner based on the cases described earlier (**Case 1**–**Case 6**): it builds the corresponding Diophantine equation and involves a recursive kernel (**Case 6**) that determines the existence of natural solutions to this equation.

6 Conclusions

Summing up the above we can claim the next.

1. The general coalgebraic approach under applying to the study of systems with output provides a unified mathematical framework for reasoning about systems of this type.
2. The bifunctor $\mathbf{J}$ (see Subsect. 3.4) gives the formal mechanism for combining the system being studied with the detector providing control of the system behaviour correctness.
3. Combining using bifunctor $\mathbf{J}$ of a system with output with a detector is bisimulation stable.
4. The demonstrated coalgebraic technique ensures the specification of safety behavioural constraints with detectors only (see Theorem 3).
5. The developed technique provides proof of the existence of a universal prefix-free recursive set (see Theorem 5). Such a set is a universal language for setting the behavioural constraints of systems with an exit.
6. Applying the developed technique for studying distributed systems demonstrates causality relationships can be specified with the corresponding class of detectors (see Sect. 4). In particular, there exists a universal language for specifying clock constraints.
7. A class of special detectors, which is used to determine the semantics of CCSL constructs, has been introduced (see Subsect. 5.1). Detectors of this class are called counter-detectors.
8. The class of counter-detectors is not sufficient for specifying all possible causality constraints (see Example 2 and Remark 12).
9. Each counter-detector can be specified with a Diophantine equation (see Subsect. 5.2).
10. The complexity of the algorithm for verifying the correctness of a causal relationship determined by a linear counter-detector (i.e., a counter-detector is given by a linear Diophantine equation) is exponential in the worst case (see Subsect. 5.4).

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On Induction Principles for Diamond-Free Partial Orders

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Abstract. We propose a formulation of an induction principle for diamond-free partial orders, which can be considered as a generalization of one of the variants of the real induction principle. This principle may be useful for specification and verification of non-discrete systems using interactive proof assistant software. As an example, a formalization in Isabelle proof assistant is presented.

Keywords: Formal methods · Partial order · Real induction · Open induction

1 Introduction

Induction principles are widely used in proofs in mathematics and computer science. Examples include the ordinary induction by a natural number parameter, formalized in Peano arithmetic, transfinite induction, structural induction, well-founded induction.

Most commonly, inductive proofs are used to prove statements of the form $\forall x\, P(x)$, where x takes values which are, in a certain sense, discrete. However, in the literature, one can find discussions of induction principles which are applicable to the cases, where x takes values in the set of real numbers (with the usual ordering), or other non-well-founded orders.

One of such principles, real induction [1, Theorem 2], has been recently used in logics intended for specification and verification of mathematical models of cyber-physical systems [2,3]. It states that a subset $S \subseteq [a, b]$ ($a < b$ are real numbers) is inductive if and only if $S = [a, b]$. Here a subset $S \subseteq [a, b]$ is called inductive, if it satisfies the following properties [1]: (1) $a \in S$; (2) if $a \leq x < b$, then $x \in S$ implies $[x, y] \subseteq S$ for some $y > x$; (3) if $a < x \leq b$ and $[a, x) \subseteq S$, then $x \in S$. A more general theorem which characterizes Dedekind-complete total orders, can also be found in [1,4].

For example, this principle can be used to prove that the solution of

$$\frac{dy}{dx} = 1 - xy, \qquad y(0) = 1 \ ,$$

satisfies $y(x) \geq 1$ for all $x \in [0, 1]$ without using an explicit analytic solution, which is $y(x) = e^{-x^2/2}\left(1 + \int_0^{x/\sqrt{2}} e^{t^2} dt\right)$ in this case, or relying on numerical

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simulations. Instead, one can study the local behavior of a solution and obtain a global result ($y(x) \geq 1$ for all $x \in [0, 1]$) from it using the induction principle. Some related examples and discussions can be found in [2].

Related induction principles applicable to partial orders can also be found in the literature, e.g., Raoult's open induction principle [5] and induction principles for complete lattices and partial orders with some completess requirements which can be found in [6].

However, the mentioned principles either lack a convenient and straightforward first-order formulation (analogous to the induction schema in the case of Peano arithmetic), or do not cover some cases of interest for formal methods and verification in computer science. Such cases of interest include posets or preorders related to representation of the behavior of nondeterminstic discrete-continuous (hybrid) dynamics systems, e.g., the poset of partial trajectories of a nondeterministic system, defined on time intervals of the forms $[0, t)$, $[0, t]$, ordered by the extension relation ($s_1 \leq s_2$, if s_2 extends s_1), where diverging trajectories (incomparable elements) have no joins.

But in this example and in some other, a poset of interest belongs to the class of diamond-free posets, i.e. posets such that there is no tuple (a, b, c, d) in which $a \leq b \leq d$, $a \leq c \leq d$, and b, c are incomparable.

In this paper we study induction principles for diamond-free partial orders, and, also, present a formalization of one of such principles in Isabelle proof assistant. These results may be useful for specification and verification of non-discrete systems using interactive proof assistant software.

2 Preliminaries

Let $(X, \leq)$ be a subset and $A \subseteq X$ be a subset. We will use the following notation:

- $<$ is the strict order which corresponds to $\leq$;
- $[a, b]$ is the set $\{x \in X \mid a \leq x \wedge x \leq b\}$;
- $[a, b)$ is the set $\{x \in X \mid a \leq x \wedge x < b\}$;
- $x = \sup_{\leq} A$ denotes that x is the least upper bound of A in $(X, \leq)$.

We will assume that the axiom of choice holds.

3 Diamond-Free Induction for Partial Orders

Let us consider the following auxiliary statement.

Theorem 1 (Converse open induction principle [7]). *Let $(X, \leq)$ be a poset. Assume that X is the only directed open subset $S \subseteq X$ which satisfies the following condition:*

(1) for each $x \in X$, if $\forall y \in X (x < y \Rightarrow y \in S)$ holds, then $x \in S$.

Then $(X, \leq)$ is a dcpo.

Proof. Suppose that $(X, \leq)$ is not directed complete (so $(X, \leq)$ is not chain complete). Then there exists a nonempty $\leq$-chain $C \subseteq X$ which has no supremum in $(X, \leq)$. Let C' be the directed closure of C. Let $S = X \backslash C'$. Let us show that S satisfies the condition (1).

Let $x \in X$. Assume that $\forall y \in X (x < y \Rightarrow y \in S)$ holds. Suppose that $x \notin S$. Then $x \in C'$. The set $\{\sup_{\leq} C'' \mid C'' \subseteq C, C'' \neq \emptyset, \text{ and } C'' \text{ has a supremum}\}$ is directed closed. Then $x = \sup_{\leq} C''$ for some nonempty $\leq$-chain $C'' \subseteq C$. Let us show that C and C'' are cofinal. Let $c \in C$. Suppose that for each $c'' \in C''$, $c < c''$ does not hold. Since $C'' \subseteq C$, each element of C'' is comparable with c. Then c is an upper bound of C''. Then $x \leq c$. The relation $x < c$ cannot hold, because it implies $c \in S = X \backslash C'$, but $c \in C \subseteq C'$. Hence $x = c \in C$. Then each $y \in C$ such that $x < y$ belongs to S. This implies that x is the largest element of C. Then $x = \sup_{\leq} C$. This contradicts the assumption that C has no supremum. Thus there exists $c'' \in C''$ such that $c < c''$ holds. Since $c \in C$ is arbitrary, we conclude that C and C'' are cofinal. Then $x = \sup_{\leq} C$ and we get a contradiction with the assumption that C has no supremum. Thus $x \in S$.

Then S satisfies the condition (1). Note that S is directed open. Then $S = X$. Then $C' = \emptyset$ and we get a contradiction with the fact that C is nonempty.

Thus $(X, \leq)$ is directed complete. □

The induction principle is given by the following theorem.

Definition 1. *A subset $S \subseteq X$ in a poset $(X, \leq)$ is called df-inductive, if*

(1) for each $x \in X$, if $\forall y \in X (x < y \Rightarrow y \in S)$ holds, then $x \in S$;

(2) for each $x \in X$ and $z \in S$ such that $x < z$ and $\sup_{\leq}[x, z) = z$, there exists $y \in [x, z)$ such that $[y, z) \subseteq S$.

Theorem 2 ([7]). *Let $(X, \leq)$ be a diamond-free poset. Then $(X, \leq)$ is directed complete if and only if the only df-inductive subset of X is X itself.*

Proof. "If". Assume that X is the only df-inductive subset of X.

Let us show that each directed open set S satisfies the condition (2) of Definition 1. Let $S \subseteq X$ be a directed open set, and $x \in X$ and $z \in S$ be such that $x < z$ and $\sup_{\leq}[x, z) = z$. Suppose that for each $y \in [x, z)$ we have $[y, z) \backslash S \neq \emptyset$. Then the sets $[x, z)$ and $[x, z) \backslash S$ are cofinal. Denote $C = [x, z) \backslash S$. Then $\sup_{\leq} C = z \in S$. Moreover, C is a $\leq$-chain, since $[x, z) \backslash S \subseteq [x, z]$ and $(X, \leq)$ is diamond-free. Since S is directed open, $C \cap S \neq \emptyset$. We have a contradiction with the definition $C = [x, z) \backslash S$. Thus there exists $y \in [x, z)$ such that $[y, z) \subseteq S$.

Thus X is the only directed open subset of X which satisfies the condition (1) of Definition 1. Then $(X, \leq)$ is a dcpo by Theorem 1.

"Only if". Assume that $(X, \leq)$ is a diamond-free dcpo. Obviously, $S = X$ satisfies the conditions (1)–(2) of Definition 1, so it is df-inductive. Let $S \subseteq X$. Assume that S is df-inductive. Let us show that $S = X$.

Firstly, let us show that S is a directed open set.

Let C be a nonempty $\leq$-chain such that $\sup_{\leq} C = z \in S$.

Let us show that $C \cap S \neq \emptyset$. Without loss of generality, assume that $z \notin C$. Let x be some element of C. Note that $x < z$.

Let $C_1 = \{y \in C \mid x \leq y\}$. Then C_1 is a nonempty $\leq$-chain. Moreover, C and C_1 are cofinal, so $\sup_{\leq} C_1 = z$. Since $z \notin C$, we have $C_1 \subseteq [x, z)$.

Let $y \in [x, z)$. Since $\sup_{\leq} C_1 = z$, y is not an upper bound of C_1. Then exists $c \in C_1$ such that $c \leq y$ does not hold. Note that c and y are not incomparable, since $c, y \in [x, z]$ and $(X, \leq)$ is diamond-free. Then $y < c$.

Since y is arbitrary, C_1 and $[x, z)$ are cofinal. Then $\sup_{\leq}[x, z) = z$.

Then from the condition (2) of Definition 1, it follows that there exists $y \in [x, z)$ such that $[y, z] \subseteq S$. Since $x \leq y < z$ and $\sup_{\leq} C = z$, there exists $c \in C \cap [x, z]$ such that $c \leq y$ does not hold. Note that c and y are not incomparable, because $c, y \in [x, z]$ and $(X, \leq)$ is diamond-free. Then $y < c$, so $c \in [y, z] \subseteq S$ and $C \cap S \neq \emptyset$.

Since C is arbitrary, this implies that S is a directed open set in $(X, \leq)$. From the condition (1) of Definition 1, it follows that S is the set of elements which satisfy an inductive property in the sense of [5, Proposition 1.2].

Then $S = X$ by the open induction principle [5]. □

Note that the conditions (1)–(2) of Definition 1 can be formulated as a first-order formula in a signature that has symbols for the predicate of membership in S, the relation $\leq$, and equality.

The condition (1) corresponds to the formulation of Noetherian induction schema. The condition (2) is analogous, but somewhat weaker than the formulation of the condition [6, (POI2')] for the opposite order relation ($\leq^{-1}$).

Theorem 2 is applicable to total orders. In the special case when X is the real interval $[0, 1]$ and $\leq$ is a restriction of the order, opposite to the standard order on real numbers, $(X, \leq)$ is directed complete and the "only if" part of the statement of the theorem reduces to the statement that $[0, 1]$ is the only subset $S \subseteq [0, 1]$ which satisfies the conditions (1)–(2) of Definition 1, where (1)–(2) are equivalent to statement that S is an inductive subset of $[0, 1]$ in terms of [1].

Theorem 2 can also be applied to sets of partial trajectories of nondeterministic continuous-time dynamical systems, defined on time intervals of the form $[0, t)$, $[0, t]$, ordered by the extension relation ($s_1 \leq s_2$, if s_2 extends s_1), or the opposite of this relation.

4 Formalization in Isabelle and Generalization to Preorders

We have used Isabelle proof assistant to formalize an induction principle, similar to the one given by Definition 1 and Theorem 2, but for diamond-free downward-complete posets (every nonempty chain has a greatest lower bound) instead of diamond-free dcpos (this change is not so essential, since one can obtain the a variant of the induction principle for dcpos by considering an opposite poset). The formalization builds on the existing formalization of Raoult's open induction principle given in [9] (which also uses downward-complete posets).

More specifically, we have obtained a formal proof of the following statement `dfinduct` in Isabelle/HOL (where "`lb P B x`" means that a set B has a lower bound x w.r.t. a binary relation P, as defined in [9], "`glbrel`" formalizes the notion of a greatest lower bound, "`dc_onrel`" formalizes downward-completeness, and "`df_on`" expresses the diamond-free condition):

```
definition glbrel where
  "glbrel P A B x ≡ lb P B x ∧ (∀y ∈ A. lb P B y ⟶ P y x)"


definition dc_onrel where
  "dc_onrel P A ≡ ∀C. chain_on P C A ∧ C ≠ {} ⟶
                                            (∃x∈A. glbrel P A C x)"

definition df_on where
  "df_on P A ≡ ∀a b c d. (a∈A ∧ b∈A ∧ c∈A ∧ d∈A ∧ P a b ∧ P b d
                                     ∧ P a c ∧ P c d ⟶ P b c ∨ P c b)"

lemma (in order) dfinduct:
  assumes dccond: "dc_onrel (op ≤) UNIV"
    and dfcond: "df_on (op ≤) UNIV"
    and cond: "⋀z x. (Q z ∧ z<x ∧ (∀ e . (z<e ∧ e≤x ⟶
                         (∃ e' . z<e'∧e'<e)))) ⟹
                 (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ Q t))"
    and ind: "⋀x. (⋀y. y < x ⟹ Q y) ⟹ Q x"
  shows "Q x"
```

Note that here `order` refers to partial orders, `dccond` expresses the downward-compleness assumption, `dfcond` means that a poset is diamond-free, `ind` corresponds to the condition (1) of Definition 1, and `cond` corresponds to the condition (2) of Definition 1, reformulated without using a notation for intervals in a poset ($[x, y]$, etc.).

We have also obtained a generalization of this statement to preorders (the proof is based on considering a corresponding partial order on $X/\sim$, where $x \sim y$ if and only if $a \leq b$ and $b \leq a$ and reducing the question to the above mentioned `dfinduct` lemma):

```
lemma (in preorder) dfinduct2:
  assumes dccond: "dc_onrel (op ≤) UNIV"
    and dfcond: "df_on (op ≤) UNIV"
    and ind1: "⋀x. (⋀y. y < x ⟹ Q y) ⟹ Q x"
    and ind2: "⋀z x. (Q z ∧ z<x ∧ (∀ e . (z<e ∧ e≤x ⟶
                                        (∃ e' . z<e'∧e'<e)))) ⟹
                  (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ Q t))"
  shows "Q x"
```

A more explicit formulation is given below (where `dcorder` means downward complete preorders, as defined in [9]):

```
theorem (in dcorder) df_induction:
assumes
diamond_free: "∀ a b c d . (a ≤ b ∧ b ≤ d) ∧ (a ≤ c ∧ c ≤ d) ⟶
                                        (b ≤ c) ∨ (c ≤ b)"
    and ind1: "∀ x.   (∀ y. y < x ⟶ P y) ⟶ P x"
    and ind2: "∀ z x. (P z ∧ z < x ∧ (∀ e . (z<e ∧ e≤x ⟶
                                          (∃ e' . z<e'∧e'<e))) ) ⟶
                      (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ P t))"
shows "∀ x . P x"
```

5 Completion

One of the conditions of applicability of Definition 1 and Theorem 2 as a method for proving statements of the form $\forall x\ P(x)$ in a poset is that a poset is a dcpo.

If a poset of interest is not directed complete (e.g. the real interval $[0, 1)$ with the usual order), one may try to consider a related poset, e.g. the opposite poset or a suitably completed poset which is directed complete, and use the induction principle for it.

However, some well-known poset completion constructions, such as MacNeille completion, may not produce a diamond-free poset from an original (incomplete) diamond-free poset.

In this section we propose a method for constructing a diamond-free directed complete preorder from a given diamond-free poset which satisfies the least-upper-bound property (e.g. the real line, or some analogous partial orders), but which is not necessarily a directed complete partial order.

The opposite poset to the result of this construction can be used, e.g., with the induction principle `dfinduct2` for diamond-free downward complete preorders discussed in the previous section.

Let $X \neq \emptyset$ and $(X, \leq)$ be a poset.

We say that a poset $(X, \leq)$ has *the least-upper-bound property*, if every non-empty subset of X which has an upper bound in $(X, \leq)$ has a supremum in $(X, \leq)$.

Let $C^*_{\leq}(X)$ be the set of all maximal chains in $(X, \leq)$ which have no maximal element.

Let $\mathcal{X} = (\{0\} \times X) \cup (\{1\} \times C^*_{\leq}(X))$ and $\leq_{\mathcal{X}}$ be a binary relation on $\mathcal{X}$ such that $(i, x) \leq_{\mathcal{X}} (j, y)$ if and only if one of the following conditions holds:

- $i = j = 0$ and $x \leq y$;
- $i = 0$, $j = 1$ and there exists $y' \in y$ such that $x \leq y'$;
- $i = j = 1$ and x and y are cofinal subsets of $(X, \leq)$.

Lemma 1. *If $(X, \leq)$ is a diamond-free poset, then $(\mathcal{X}, \leq_{\mathcal{X}})$ is a diamond-free preorder.*

Proof. Assume that $(X, \leq)$ is diamond-free. It is easy to see that $\leq_{\mathcal{X}}$ is reflexive. Moreover, if $(i, x) \leq_{\mathcal{X}} (j, y) \leq_{\mathcal{X}} (k, z)$, where $i, j, k \in \{0, 1\}$, $x, y, z \in X \cup C^*_{\leq}(X)$, then one of the following cases holds:

– $i = j = k = 0$. Then $(i, x) \leq_{\mathcal{X}} (k, z)$, since $\leq$ is transitive.
– $i = j = k = 1$. Then x, y and y, z are cofinal sets, so x, z are cofinal sets.
– $i = 0$, $j = k = 1$. Then $x \in X$, y, z are cofinal sets, and there exists $y' \in y$ such that $x \leq y'$. Then there exists $z' \in z$ such that $y' \leq z'$, so $x \leq z'$. Thus $(i, x) \leq_{\mathcal{X}} (k, z)$ holds.
– $i = j = 0$ and $k = 1$. Then $x \leq y$ and there exists $z' \in z$ such that $y \leq z'$, so $x \leq z'$ and $(i, x) \leq_{\mathcal{X}} (k, z)$ holds.

Thus $\leq_{\mathcal{X}}$ is transitive. Then $(\mathcal{X}, \leq_{\mathcal{X}})$ is a preorder.
Let $(i, a), (j, b), (k, c), (l, d) \in \mathcal{X}$, where $i, j, k, l \in \{0, 1\}$, $a, b, c, d \in X \cup C^*_{\leq}(X)$. Assume that $(i, a) \leq_{\mathcal{X}} (j, b)$, $(i, a) \leq_{\mathcal{X}} (k, c)$ and $(j, b) \leq_{\mathcal{X}} (l, d)$, $(k, c) \leq_{\mathcal{X}} (l, d)$. The following cases are possible:

– $l = 0$. Then $i = j = k = 0$ and $a \leq b$, $a \leq c$, $b \leq d$, $c \leq d$, whence $b \leq c$ or $c \leq d$, since $(X, \leq)$ is diamond-free, so $(j, b) \leq_{\mathcal{X}} (k, c)$ or $(k, c) \leq_{\mathcal{X}} (j, b)$.
– $i = 1$. Then $j = k = l = 1$ and b, d are cofinal sets and c, d are cofinal sets. Then b, c are cofinal sets, whence $(j, b) \leq_{\mathcal{X}} (k, c)$.
– $i = 0$ and $l = 1$. Then the following subcases are possible:
  • $j = 1$. Then b, d are cofinal sets, so $(l, d) \leq_{\mathcal{X}} (j, b)$, so $(k, c) \leq_{\mathcal{X}} (j, b)$ by transitivity of $\leq_{\mathcal{X}}$ (shown above).
  • $k = 1$. Then c, d are cofinal sets, so $(l, d) \leq_{\mathcal{X}} (k, c)$, so $(j, b) \leq_{\mathcal{X}} (k, c)$ by transitivity of $\leq_{\mathcal{X}}$ (shown above).
  • $j = k = 0$. Then $a \leq b$, $a \leq c$, $d \in C^*_{\leq}(X)$, and there exist $d', d'' \in d$ such that $b \leq d'$ and $c \leq d''$. Moreover, $d' \leq d''$ or $d'' \leq d'$, because d is a $\leq$-chain. Then by transitivity of $\leq$, either $a \leq b \leq d''$ and $a \leq c \leq d''$, or $a \leq b \leq d'$ and $a \leq c \leq d'$. Note that $a, b, c, d', d'' \in X$ and $(X, \leq)$ is diamond-free, so b and c are $\leq$-comparable. Then $(j, b) \leq_{\mathcal{X}} (k, c)$ or $(k, c) \leq_{\mathcal{X}} (j, b)$.

We conclude that $(\mathcal{X}, \leq_{\mathcal{X}})$ is a diamond-free preorder. □

Lemma 2. *Assume that $(X, \leq)$ is a diamond-free poset. Let C_1, C_2 be chains in $(X, \leq)$ and for each $x \in C_1$ there exists $y \in C_2$ such that $x \leq y$.*

Then for each $x_0 \in X$, $\{x \in C_1 \cup C_2 \mid x_0 \leq x\}$ is a chain in $(X, \leq)$.

Proof. Let $x_0 \in X$. Let $a, b \in C_1 \cup C_2$ be elements such that $x_0 \leq a$ and $x_0 \leq b$. Let us show that a, b are $\leq$-comparable. Without loss of generality assume that $a \in C_1$ and $b \in C_2$. By the assumption, for a there exists $y \in C_2$ such that $a \leq y$. Let y' be the $\leq$-greatest element of $\{b, y\}$ (which exists since C_2 is a chain). Then $x_0 \leq b \leq y'$ and $x_0 \leq a \leq y \leq y'$, so a and b are $\leq$-comparable, since $(X, \leq)$ is diamond-free. Thus $\{x \in C_1 \cup C_2 \mid x_0 \leq x\}$ is a chain in $(X, \leq)$. □

The completion construction is given by the following theorem.

Theorem 3. *If $(X, \leq)$ is a diamond-free poset which has the least-upper-bound property, then $(\mathcal{X}, \leq_{\mathcal{X}})$ is a directed complete diamond-free preorder.*

Proof. Assume that $(X, \leq)$ is a diamond-free poset which has the least-upper-bound property. By Lemma 1, $(X, \leq)$ is a diamond-free preorder.

Let $\mathcal{C} \subseteq \mathcal{X}$ be a nonempty $\leq_{\mathcal{X}}$-chain. Let us show that $\mathcal{C}$ has a supremum in $(\mathcal{X}, \leq_{\mathcal{X}})$. The following two cases are possible:

- there exists $c \in C^*_{\leq}(X)$ such that $(1, c) \in \mathcal{C}$. Then, from the definition of $\leq_{\mathcal{X}}$, it is easy to see that $(1, c)$ is a $\leq_{\mathcal{X}}$-greatest element in $\mathcal{C}$, since $\mathcal{C}$ is a $\leq_{\mathcal{X}}$-chain. Then $\mathcal{C}$ has a supremum in $(\mathcal{X}, \leq_{\mathcal{X}})$.
- $\mathcal{C} \subseteq \{0\} \times X$. Let $c = \{x \in X \mid (0, x) \in \mathcal{C})$. Then c is a nonempty $\leq$-chain, since $\mathcal{C}$ is a nonempty $\leq_{\mathcal{X}}$-chain. Consider the following subcases:
  - c has an upper bound in $(X, \leq)$. Then c is a nonempty upper bounded subset in $(X, \leq)$, so c has a supremum in $(X, \leq)$ by the condition (1). Denote this supremum as x^*. Then $(0, x^*) \in \mathcal{X}$ is an upper bound of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$, since $\mathcal{C} \subseteq \{0\} \times X$.

    Let us show that $(0, x^*)$ is a supremum of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$.
    Let (i, x) be an upper bound of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$ and let us show that $(0, x^*) \leq_{\mathcal{X}} (i, x)$.
    If $i = 0$, then $x \in X$ and $(0, x') \leq_{\mathcal{X}} (0, x)$ for all $x' \in c$, whence x is an upper bound of c in $(X, \leq)$, so $x^* \leq x$ and $(0, x^*) \leq_{\mathcal{X}} (i, x)$.
    Consider the case when $i = 1$.

    Firstly, suppose that there is no pair (a_0, x_0) such that $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$. Let a'_0 be some element of c (which exists since $c \neq \emptyset$). We have $x \in C^*_{\leq}(X)$ and $(0, a) \leq_{\mathcal{X}} (1, x)$ for all $a \in c$, whence for each $a \in c$ there exists $x' \in x$ such that $a \leq x'$. Then by Lemma 2,
    $\{t \in c \cup x \mid a'_0 \leq t\}$ is a chain in $(X, \leq)$. Let $x'_0 \in x$ be an element such that $a'_0 \leq x'_0$ (which exists as mentioned above). Then
    $x'_0 \in \{t \in c \cup x \mid a'_0 \leq t\}$. Then x'_0 is an upper bound of $\{t \in c \mid a'_0 \leq t\}$, since, otherwise, there exists $t \in c$ such that $a'_0 \leq t$ and $t \not\leq x'_0$, whence $x'_0 \leq t$ (since here x'_0 and t belong to a $\leq$-chain), which contradicts the assumption that there is no pair (a_0, x_0) with $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$. Note that x^* is a supremum of $\{t \in c \mid a'_0 \leq t\}$ (since c and $\{t \in c \mid a'_0 \leq t\}$ are cofinal) in $(X, \leq)$, so $x^* \leq x'_0$. Moreover, $x'_0 \in x$. Thus $(0, x^*) \leq_{\mathcal{X}} (1, x) = (i, x)$.

    Now, suppose there exist $a_0 \in c$ and $x_0 \in x$ such that $x_0 \leq a_0$. We have $x \in C^*_{\leq}(X)$ and $(0, a) \leq_{\mathcal{X}} (1, x)$ for all $a \in c$, whence for each $a \in c$ there exists $x' \in x$ such that $a \leq x'$. Then by Lemma 2, $\{t \in c \cup x \mid x_0 \leq t\}$ is a chain in $(X, \leq)$. Suppose that for each $x' \in x$ there exists $a \in c$ such that $x' \leq a$. Then the chain x is bounded from above by x^*. Then x^* is a maximal element of x (since $x \in C^*_{\leq}(X)$ and x is maximal), and we have a contradiction with the fact that $x \in C^*_{\leq}(X)$. Thus there exists $x' \in x$ such that for all $a \in c$, $x' \not\leq a$. Let x'' be the $\leq$-greatest element of $\{x_0, x'\}$ (which exists since x is a $\leq$-chain). Then $x'' \in \{t \in c \cup x \mid x_0 \leq t\}$ and for all $a \in c$, $x'' \not\leq a$ (since, otherwise, $x' \leq x'' \leq a$ for some $a \in c$). Then for each $a \in c$, if $x_0 \leq a$, then $a \leq x''$ (since, in this case, a and x''

are $\leq$-comparable). In particular, for each $a \in c$, $a_0 \leq a$ implies $a \leq x''$ (since $x_0 \leq a_0$). Then x'' is an upper bound of $\{a \in c \mid a_0 \leq a\}$. Note that x^* is a supremum of $\{a \in c \mid a_0 \leq a\}$ (since c and $\{a \in c \mid a_0 \leq a\}$ are cofinal), so $x^* \leq x''$. Moreover, $x'' \in x$, so $(0, x^*) \leq_{\mathcal{X}} (1, x) = (i, x)$. We conclude that $\mathcal{C}$ has a supremum in $(\mathcal{X}, \leq_{\mathcal{X}})$.

- c has no upper bound in $(X, \leq)$. By Zorn's lemma, there exists some maximal by inclusion $\leq$-chain c_m in $(X, \leq)$ such that $c \subseteq c_m$.
  Note that c_m has no $\leq$-maximal element (since c has no upper bound), so $c_m \in C^*_{\leq}(X)$. Then $(1, c_m) \in \mathcal{X}$ and $(0, a) \leq_{\mathcal{X}} (1, c_m)$ for each $a \in c \subseteq c_m$. Then $(1, c_m)$ is an upper bound of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$, since $\mathcal{C} \subseteq \{0\} \times X$.
  Let us show that $(1, c_m)$ is a supremum of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$.
  Let (i, x) be an upper bound of $\mathcal{C}$ in $(\mathcal{X}, \leq_{\mathcal{X}})$ and let us show that $(1, c_m) \leq_{\mathcal{X}} (i, x)$. Note that $i = 1$, since, otherwise, $x \in X$ and $(0, a) \leq_{\mathcal{X}} (0, x)$ for all $a \in c$, whence $a \leq x$ for all $a \in c$, which contradicts the assumption that c has no upper bound in $(X, \leq)$. Then $x \in C^*_{\leq}(X)$.
  Let us show that c and x are cofinal sets in $(X, \leq)$.
  Since (i, x) is an upper bound of $\mathcal{C}$, we have $(0, a) \leq_{\mathcal{X}} (1, x)$ for all $a \in c$. Then for each $a \in c$ there exists $x' \in x$ such that $a \leq x'$. Then by Lemma 2, for any $x_0 \in X$, $\{t \in c \cup x \mid x_0 \leq t\}$ is a chain in $(X, \leq)$. Then $\{a \in c \mid x_0 \leq a\} \subseteq x$ for any $x_0 \in x$, since, otherwise, there exists $a \in \{t \in c \cup x \mid x_0 \leq t\}$ such that $a \notin x$ and a is $\leq$-comparable with all elements t of x such that $x_0 \leq t$, so $x \cup \{a\}$ is a $\leq$-chain and a proper superset of x, which contradicts the fact that $x \in C^*_{\leq}(X)$.
  Let us show that for each $x' \in x$ there exists $a \in c$ such that $x' \leq a$.
  Firstly, let us show that there exists a pair (a_0, x_0) such that $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$. Suppose that there is no pair (a_0, x_0) such that $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$. Let a'_0 be some element of c (which exists since c is nonempty). As mentioned above, $\{t \in c \cup x \mid a'_0 \leq t\}$ is a chain in $(X, \leq)$. Let $x'_0 \in x$ be an element such that $a'_0 \leq x'_0$ (which exists as mentioned above). Then $x'_0 \in \{t \in c \cup x \mid a'_0 \leq t\}$. Then x'_0 is an upper bound of $\{t \in c \mid a'_0 \leq t\}$, since, otherwise, there exists $t \in c$ such that $a'_0 \leq t$ and $t \not\leq x'_0$, whence $x'_0 \leq t$ (since here x'_0 and t belong to a $\leq$-chain), which contradicts the assumption that there is no pair (a_0, x_0) with $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$. Then x'_0 is an upper bound of c (since $a'_0 \in c$ and c is a $\leq$-chain), which contradicts the assumption that c has no upper bound in $(X, \leq)$.
  Thus there exists a pair (a_0, x_0) such that $a_0 \in c$, $x_0 \in x$, and $x_0 \leq a_0$.
  Let $x' \in x$. Suppose that for all $a \in c$, $x' \not\leq a$.
  Note that $\{a \in c \mid a_0 \leq a\} \subseteq \{a \in c \mid x_0 \leq a\} \subseteq x$. Then for all $a \in c$ such that $a_0 \leq a$, a and x' belong to a $\leq$-chain, so $a \leq x'$. Then, since $a_0 \in c$ and c is a $\leq$-chain, x' is an upper bound of c, which contradicts the assumption that c has no upper bound in $(X, \leq)$. Thus there exists $a \in c$ such that $x' \leq a$.
  Taking into account that for each $a \in c$ there exists $x' \in x$ such that $a \leq x'$ (as mentioned above), we conclude that c and x are cofinal sets in $(X, \leq)$. Note that c is a cofinal subset of c_m, since, otherwise, there

exists $a \in c_m$ such that $a \not\leq y$ for all $y \in c$, so $y \leq a$ for all $y \in c$, which contradicts the assumption that c has no upper bound in $(X, \leq)$. Then c_m and x are cofinal sets in $(X, \leq)$. Thus $(1, c_m) \leq_{\mathcal{X}} (i, x)$.

We conclude that $\mathcal{C}$ has a supremum in $(\mathcal{X}, \leq_{\mathcal{X}})$.

We conclude that $(\mathcal{X}, \leq_{\mathcal{X}})$ is a directed complete preorder [8]. □

6 Conclusions

We have proposed a formulation of an induction principle for diamond-free partial orders, which can be considered as a generalization of one of the variants of the real induction principle, and presented a formalization of a variant of this principle in Isabelle proof assistant. This principle may be useful for specification and verification of non-discrete systems using interactive proof assistant software.

APPENDIX: Isabelle Formalization

```
theory DF_Induction
imports Main Open_Induction OIRel
begin

definition above where "above P A B ≡ ∀ a ∈ A . ∃ b ∈ B . P b a"
definition coinitial where "coinitial P A B ≡ above P A B ∧ above P B
A"

lemma (in preorder) aboveglb: "y ∈ U ⟹ above (op ≤) A B ⟹ glbrel
(op ≤) U A x ⟹ glbrel (op ≤) U B y ⟹ y ≤ x"
  apply(unfold above_def glbrel_def lb_def)
  by (meson local.order_trans)

lemma (in preorder) coinitrefl: "coinitial (op ≤) A A"
  using above_def coinitial_def by fastforce

lemma (in order) coinitglb1: "coinitial (op ≤) A B ⟹ glbrel (op ≤)
U A a ⟹ glbrel (op ≤) U B a"
  apply(simp add: glbrel_def lb_def above_def coinitial_def)
  by (metis local.dual_order.trans)

lemma (in order) coinitglb2: "a ∈ U ⟹ b ∈ U ⟹ coinitial (op ≤) A
B ⟹ glbrel (op ≤) U A a ⟹ glbrel (op ≤) U B b ⟹ a = b"
  by (meson coinitial_def local.aboveglb local.eq_iff)

definition df_on where
  "df_on P A ≡ ∀a b c d. (a∈A ∧ b∈A ∧ c∈A ∧ d∈A ∧ P a b ∧ P b d ∧ P
a c ∧ P c d ⟶ P b c ∨ P c b)"
```

```
lemma (in order) dfinduct:
  assumes dccond: "dc_onrel (op ≤) UNIV"
    and dfcond: "df_on (op ≤) UNIV"
    and cond: "⋀z x. (Q z ∧ z<x ∧ (∀ e . (z<e ∧ e≤x ⟶ (∃ e' .
z<e'∧e'<e))) ⟹
                              (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶
Q t))"
    and ind: "⋀x. (⋀y. y < x ⟹ Q y) ⟹ Q x"
  shows "Q x"
proof -
  have "open_onrel (op ≤) Q UNIV"
  proof (unfold open_onrel_def, rule allI, rule impI, erule conjE, erule
conjE)
    fix C
    assume a1: "chain_on op ≤ C UNIV"
    assume a2: "C ≠ {}"
    then obtain x where b2: "x ∈ C" by auto
    assume a3: "∃x∈UNIV. glbrel (op ≤) UNIV C x ∧ Q x"
    then obtain z where b3: "glbrel (op ≤) UNIV C z" and b4: "Q z" by
blast
    then have c1:"z ≤ x" using b2 by (unfold glbrel_def lb_def, blast)
    have "¬ z ∈ C ⟹ ∃y∈C. Q y"
    proof -
      assume d1:"¬ z ∈ C"
      then have "¬ z = x" using b2 by blast
      then have c2: "z < x" using c1 local.dual_order.order_iff_strict
by blast
      let ?C1 = "{y ∈ C . y ≤ x}"
      let ?zx = "{ t . z < t ∧ t ≤ x }"
      from a1 have "chain_on (op ≤) ?C1 UNIV" by (unfold chain_on_def,
blast)
      have d2: "glbrel (op ≤) UNIV ?C1 z"
      proof -
        from a1 b2 have "coinitial (op ≤) C ?C1"
          by (simp only: coinitial_def above_def chain_on_def, blast)
        thus "glbrel (op ≤) UNIV ?C1 z" using b3 coinitglb1 by blast
      qed
      then have incl1: "?C1 ⊆ ?zx" using d1
        apply(simp only: glbrel_def lb_def)
        using local.dual_order.order_iff_strict by blast
      have d3: "coinitial (op ≤) ?C1 ?zx"
        apply(unfold coinitial_def)
        apply(rule conjI)
        apply(unfold above_def)
        using incl1 apply blast
        apply(simp)
        apply(rule allI)
        apply(rule impI)
        proof -
          fix y
```

```
        assume v1: "z < y ∧ y ≤ x"
        then have "∃ c ∈ ?C1 . ¬ (y ≤ c)" using d2 by (auto simp
add: glbrel_def lb_def)
        then obtain c where v2: "c ∈ ?C1 ∧ ¬ (y ≤ c)" by blast
        have v3: "z ≤ c ∧ c ≤ x" using v2 incl1
local.order.strict_implies_order by blast
        have "c ≤ y ∨ y ≤ c"
        proof -
          from v1 have "z ≤ y ∧ y ≤ x"
            by (simp only: local.dual_order.order_iff_strict, blast)
          moreover have  "z ≤ c ∧ c ≤ x" using v3 by blast
          ultimately show ?thesis using dfcond v3 by (unfold
df_on_def, blast)
        qed
        thus "∃b. b ∈ C ∧ b ≤ x ∧ b ≤ y" using v2 v3 by blast
      qed
    then have d4: "glbrel (op ≤) UNIV ?zx z" using d2 coinitglb1 by
blast
    have "∀ e . (z<e ∧ e≤x ⟶ (∃ e' . z<e'∧e'<e))"
    proof(rule allI, rule impI, rule ccontr)
      fix e
      assume u1: "z<e ∧ e≤x"
      assume u2: "¬ (∃e'>z. e' < e)"
      from u1 u2 have "∀ e' ∈ ?zx . e ≤ e'"
        by (smt UNIV_I df_on_def dfcond
local.dual_order.not_eq_order_implies_strict
local.dual_order.strict_implies_order mem_Collect_eq)
      then have "glbrel (op ≤) UNIV ?zx e" using u1 by (auto simp
add: glbrel_def lb_def)
      then have "e = z" using d3 d4 coinitglb2 coinitrefl by blast
      then show False using u1 by simp
    qed
    then have "∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ Q t)" using
cond b4 c2 by blast
    then obtain y where d5:"z<y ∧ y≤x" and "∀ t . z<t ∧ t≤y ⟶ Q
t" by blast
    then have d6:"∀ t . z≤t ∧ t≤y ⟶ Q t" using b4 using
local.dual_order.order_iff_strict by blast
    have "∃ c ∈ C . z ≤ c ∧ c ≤ x ∧ ¬ (y ≤ c)" using d5 b3
      apply(simp add: glbrel_def lb_def)
      by (metis (full_types) a1 b2 chain_on_def
local.order.strict_iff_order local.order.strict_trans2)
    then obtain c where d7: "c ∈ C ∧ z ≤ c ∧ c ≤ x ∧ ¬ (y ≤ c)"
by blast
    have "c ≤ y ∨ y ≤ c"
    proof -
      have "z ≤ c ∧ c ≤ x" using d7 by blast
      moreover have "z ≤ y ∧ y ≤ x" using d5 by auto
      ultimately show ?thesis using dfcond by (unfold df_on_def,
blast)
```

```
      qed
      then show "∃y∈C. Q y" using d6 d7 by blast
    qed
    moreover have "z ∈ C ⟶ (∃y∈C. Q y)" using b4 by blast
    ultimately show "∃y∈C. Q y" by blast
  qed
  then show ?thesis using dccond ind local.dc_open_inductrel by blast
qed

class dfdcorderrel = order + dcorderrel +
  assumes diamond_free: "(a ≤ b ∧ b ≤ d) ∧ (a ≤ c ∧ c ≤ d) ⟹ (b
≤ c) ∨ (c ≤ b)"
begin
  theorem df_induction:
  assumes
        ind1: "⋀x.   (⋀y. y < x ⟹ P y) ⟹ P x"
    and ind2: "⋀z x. (P z ∧ z < x ∧ (∀ e . (z<e ∧ e≤x ⟶ (∃ e' .
z<e'∧e'<e))) ) ⟹
                    (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ P t))"
  shows "P x"
  proof -
    have "dc_onrel (op ≤) UNIV" using dc_on_UNIV by blast
    moreover have "df_on (op ≤) UNIV" using diamond_free df_on_def by
blast
    ultimately show ?thesis using dfinduct ind1 ind2 by blast
  qed
end


definition eqrel where "eqrel P A a ≡ (λ b . b ∈ A ∧ P a b ∧ P b a)"

lemma eqreltr: "transp_on P A ⟹ transp_on (eqrel P A) A"
  by (unfold transp_on_def eqrel_def, blast)

lemma eqreln: "reflp_on P A ⟹ a ∈ A ⟹ eqrel P A (Eps (eqrel P A
a)) a"
  apply(unfold eqrel_def reflp_on_def)
  by (metis (no_types, lifting) someI_ex)

lemma eqreldom: "reflp_on P A ⟹ a ∈ A ⟹ Eps(eqrel P A a) ∈ A"
  apply(unfold eqrel_def reflp_on_def)
  by (metis (mono_tags, lifting) someI2)

definition eqcl where "eqcl P A ≡ { S . ∃ a ∈ A . S = eqrel P A a }"
definition eqcrel where "eqcrel P u v ≡ P (Eps u) (Eps v)"
definition unpr where "unpr U Q S ≡ ∀ x ∈ U . (S x ⟶ Q x)"

lemma eqcrelqo: "qo_on P A ⟹ qo_on (eqcrel P) (eqcl P A)"
proof -
 assume a1: "qo_on P A"
```

```
 have "reflp_on P A ⟹ reflp_on (eqcrel P) (eqcl P A)"
  by (auto simp add: reflp_on_def eqcrel_def eqcl_def eqreldom)
 moreover have "qo_on P A ⟹ transp_on (eqcrel P) (eqcl P A)"
  apply(auto simp only: qo_on_def transp_on_def eqcrel_def eqcl_def)
  by (meson eqreldom)
 ultimately show "qo_on (eqcrel P) (eqcl P A)" by (meson a1 qo_on_def)
qed

lemma eqcrelantisymp: "qo_on P A ⟹ antisymp_on (eqcrel P) (eqcl P A)"
proof (auto simp only: antisymp_on_def eqcrel_def eqcl_def)
  fix aa ab
  assume a1: "qo_on P A" and a2: "aa ∈ A" and a3: "ab ∈ A"
  let ?u = "eqrel P A aa"
  let ?v = "eqrel P A ab"
  assume a4: "P (Eps ?u) (Eps ?v)" and a5: "P (Eps ?v) (Eps ?u)"
  have "(Eps ?u) ∈ A" by (meson a1 a2 eqreldom qo_on_def)
  moreover have "(Eps ?v) ∈ A" by (meson a1 a3 eqreldom qo_on_def)
  ultimately have "eqrel P A (Eps ?u) (Eps ?v)" using a4 a5 by (simp
only: eqrel_def)
  moreover have "eqrel P A aa (Eps ?u)" by (metis a1 a2 eqrel_def
qo_on_def reflp_on_def someI)
  moreover have "eqrel P A (Eps ?v) ab" by (metis a1 a3 eqrel_def
qo_on_def reflp_on_def someI)
  ultimately have "eqrel P A aa ab" using eqreltr a1 a2 qo_on_def
eqrel_def transp_on_def by smt
  then have "⋀ x . ?u x = ?v x" using qo_on_def transp_on_def by
(metis a1 a2 eqrel_def)
  thus "?u = ?v" by blast
qed

lemma eqcreldf: "reflp_on P A ⟹ df_on P A ⟹ df_on (eqcrel P) (eqcl
P A)"
  apply (simp only: df_on_def eqcrel_def eqcl_def)
  by (smt CollectD eqreldom)

lemma eqcrelchain: "qo_on P A ⟹ chain_on (eqcrel P) C (eqcl P A) ⟹
chain_on P { b ∈ A . ∃ S ∈ C . S b } A"
proof (simp add: chain_on_def eqcrel_def Collect_restrict)
  assume a1: "qo_on P A"
  assume a2: "C ⊆ eqcl P A ∧ (∀x∈C. ∀y∈C. P (Eps x) (Eps y) ∨ P (Eps
y) (Eps x))"
  show "∀x. x ∈ A ∧ (∃S∈C. S x) ⟶ (∀y. y ∈ A ∧ (∃S∈C. S y) ⟶ P
x y ∨ P y x)"
  apply(rule allI, rule impI, rule allI, rule impI)
  proof -
    fix x y
    assume b1: "x ∈ A ∧ (∃S∈C. S x)" and b2: "y ∈ A ∧ (∃S∈C. S y)"
    from b1 obtain S1 where b3: "S1 ∈ C ∧ S1 x" by blast
    from b2 obtain S2 where b4: "S2 ∈ C ∧ S2 y" by blast
```

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    from a2 b3 b4 have b5: "P (Eps S1) (Eps S2) ∨ P (Eps S2) (Eps S1)"
by blast
    from a2 b3 obtain a1 where b6: "a1 ∈ A ∧ S1 = eqrel P A a1" by
(smt CollectD eqcl_def subset_eq)
    from a2 b4 obtain a2 where b7: "a2 ∈ A ∧ S2 = eqrel P A a2" by
(smt CollectD eqcl_def subset_eq)
    have "eqrel P A (Eps S1) a1" using a1 b6 by (simp only: eqreln
qo_on_def)
    moreover have "eqrel P A (Eps S2) a2" using a1 b7 by (simp only:
eqreln qo_on_def)
    moreover from b3 b6 have "eqrel P A a1 x" by blast
    moreover from b7 b4 have "eqrel P A a2 y" by blast
    ultimately show "P x y ∨ P y x" using a1
      apply(simp only: eqrel_def qo_on_def transp_on_def)
      by (metis b5 b6 b7 eqreldom)
  qed
qed

lemma eqcreldc: "qo_on P A ⟹ dc_onrel P A ⟹ dc_onrel (eqcrel P)
(eqcl P A)"
proof (simp only: dc_onrel_def eqcrel_def, rule allI, rule impI)
  fix C
  assume a1: "qo_on P A"
  assume a2: "∀C. chain_on P C A ∧ C ≠ {} ⟶ (∃x∈A. glbrel P A C
x)"
  assume a3: "chain_on (eqcrel P) C (eqcl P A) ∧ C ≠ {}"
  let ?C1 = "{ b ∈ A . ∃ S ∈ C . S b }"
  from a1 a3 have "chain_on P ?C1 A" using eqcrelchain by blast
  moreover have "?C1 ≠ {}"
  proof -
    from a3 obtain S where b1: "S ∈ C" by blast
    then have "S ∈ eqcl P A" by (metis a3 chain_on_def insert_absorb
insert_subset)
    then obtain b where "b ∈ A ∧ S = eqrel P A b" using eqcl_def by
(smt mem_Collect_eq)
    then have "∃ b ∈ A . S b" by (metis a1 eqrel_def
qo_on_imp_reflp_on reflp_on_def)
    thus ?thesis using b1 by blast
  qed
  ultimately obtain x where b1: "x∈A ∧ glbrel P A ?C1 x" using a2 by
blast
  let ?S = "eqrel P A x"
  from a3 have a4: "C ⊆ eqcl P A" using chain_on_def by metis
  from b1 have "?S ∈ eqcl P A" using eqcl_def by blast
  moreover have "glbrel (eqcrel P) (eqcl P A) C ?S"
  proof (unfold glbrel_def lb_def eqcrel_def, rule conjI)
    show "∀Y∈C. P (Eps (eqrel P A x)) (Eps Y)"
    proof
      fix Y
      assume c1:"Y ∈ C"
```

```
      then obtain y where c2:"y ∈ A ∧ Y = eqrel P A y" using a4 by
(smt CollectD eqcl_def subset_iff)
      then have "y ∈ ?C1" by (metis (no_types, lifting) CollectI a1 c1
eqrel_def qo_on_def reflp_on_def)
      moreover have "lb P ?C1 x" using b1 by (simp only: glbrel_def)
      ultimately have "P x y" by (simp only: lb_def)
      then show "P (Eps (eqrel P A x)) (Eps Y)" by (smt c2 a1 b1
eqrel_def eqreldom eqreln qo_on_imp_reflp_on qo_on_imp_transp_on
transp_on_def)
    qed
    show "∀Y∈eqcl P A. (∀ya∈C. P (Eps Y) (Eps ya)) ⟶ P (Eps Y) (Eps
(eqrel P A x))"
    proof(rule ballI, rule impI)
      fix Y
      assume c0: "Y ∈ eqcl P A"
      assume c1: "∀ya∈C. P (Eps Y) (Eps ya)"
      have c2: "(Eps Y) ∈ A" by (smt a1 c0 eqcl_def eqreldom
mem_Collect_eq qo_on_def)
      have "∀y ∈ ?C1 . P (Eps Y) y"
      proof
        fix y
        assume d1: "y ∈ ?C1"
        then obtain ya where d2: "ya ∈ C ∧ ya y" by blast
        then have "P (Eps Y) (Eps ya)" using c1 by blast
        moreover have "P (Eps ya) y"
        proof -
          from a4 d2 obtain a where "a ∈ A ∧ ya = eqrel P A a" by
(unfold eqcl_def, blast)
          then show ?thesis using d2
             by (simp, smt a1 eqrel_def qo_on_imp_transp_on tfl_some
transp_on_def)
        qed
        moreover have "(Eps ya) ∈ A" by (smt CollectD d2 a4 eqcl_def
eqrel_def set_rev_mp tfl_some)
        moreover have "y ∈ A" using d1 by blast
        ultimately show "P (Eps Y) y" using a1 c2 qo_on_def
transp_on_def by metis
      qed
      then have "lb P ?C1 (Eps Y)" by (unfold lb_def, blast)
      then have "P (Eps Y) x" using c2 b1 by (simp only: glbrel_def)

      moreover then have "P x (Eps (eqrel P A x))" by (metis a1 b1
eqrel_def eqreln qo_on_def)
      ultimately show "P (Eps Y) (Eps (eqrel P A x))" by (meson a1 b1
c2 eqreldom qo_on_def transp_on_def)
    qed
  qed
  ultimately show "∃x∈eqcl P A. glbrel (eqcrel P) (eqcl P A) C x" by
blast
qed
```

```
definition qinduct where
"qinduct P A Q ≡ ∀x. (x ∈ A ∧ (∀ y. y ∈ A ∧ strict P y x ⟶ Q y)
⟶ Q x)"

lemma eqcrelqind: "qo_on P U ⟹ qinduct P U Q ⟹ qinduct (eqcrel P)
(eqcl P U) (unpr U Q)"
proof (unfold qinduct_def eqcrel_def unpr_def, rule allI, rule impI, rule
ballI, rule impI)
  fix S u
  assume a1: "qo_on P U" and
         a2: "∀x. x ∈ U ∧ (∀y. y ∈ U ∧ strict P y x ⟶ Q y) ⟶ Q x"
and
         a3: "S ∈ eqcl P U ∧
              (∀Y. Y ∈ eqcl P U ∧ strict P (Eps Y) (Eps S) ⟶ (∀x∈U.
Y x ⟶ Q x))" and
         a4: "u ∈ U" and a5: "S u"
  from a3 obtain s where a6: "s ∈ U ∧ S = eqrel P U s" by (smt
eqcl_def mem_Collect_eq)
  have "∀y. y ∈ U ∧ strict P y u ⟶ Q y"
  proof (rule allI, rule impI)
    fix y
    assume b1: "y ∈ U ∧ strict P y u"
    obtain Y where b2:"Y = eqrel P U y" by blast
    then have "Y ∈ eqcl P U" using b1 eqcl_def by fastforce
    moreover have "Y y" using b2 by (metis a1 b1 eqrel_def
qo_on_imp_reflp_on reflp_on_def)
    moreover have "strict P (Eps Y) (Eps S)" by (smt a1 a5 a6 b1 b2
calculation(2) eqrel_def qo_on_imp_transp_on tfl_some transp_on_def)
    ultimately show "Q y" using a3 b1 by blast
  qed
  thus "Q u" using a2 a4 by blast
qed

definition qinduct2 where
"qinduct2 P A Q ≡ ∀ z ∈ A . ∀ x ∈ A. (Q z ∧ strict P z x ∧ (∀ e ∈ A.
(strict P z e ∧ P e x ⟶ (∃ e' ∈ A . strict P z e'∧ strict P e' e))))
⟶
                       (∃ y ∈ A . strict P z y ∧ P y x ∧ (∀ t ∈
A . strict P z t ∧ P t y ⟶ Q t))"

lemma eqcrelqind2: "qo_on P U ⟹ qinduct2 P U Q ⟹ qinduct2 (eqcrel
P) (eqcl P U) (unpr U Q)"
proof (unfold qinduct2_def eqcrel_def unpr_def, intro ballI impI conjI)
  fix Z X
  assume a1: "qo_on P U"
  assume a2: "∀z∈U. ∀x∈U. Q z ∧ strict P z x ∧
           (∀e∈U. strict P z e ∧ P e x ⟶ (∃e'∈U. strict P z e' ∧
strict P e' e)) ⟶
           (∃y∈U. strict P z y ∧ P y x ∧ (∀t∈U. strict P z t ∧ P t
y ⟶ Q t))"
```

```
  assume a3: "Z ∈ eqcl P U" and a4: "X ∈ eqcl P U"
  assume a5: "(∀x∈U. Z x ⟶ Q x) ∧ strict P (Eps Z) (Eps X) ∧
              (∀E∈eqcl P U. strict P (Eps Z) (Eps E) ∧ P (Eps E) (Eps
X) ⟶
                 (∃E'∈eqcl P U. strict P (Eps Z) (Eps E') ∧ strict P
(Eps E') (Eps E)))"
  obtain z where b1: "z ∈ U ∧ Z = eqrel P U z" by (smt a3 eqcl_def
mem_Collect_eq)
  obtain x where b2: "x ∈ U ∧ X = eqrel P U x" by (smt a4 eqcl_def
mem_Collect_eq)
  have c1: "Z z" using b1 by (metis a1 eqrel_def qo_on_imp_reflp_on
reflp_on_def)
  have c2: "X x" using b2 by (metis a1 eqrel_def qo_on_imp_reflp_on
reflp_on_def)
  have "Q z ∧ strict P z x"
  proof
    have c0: "reflp_on P U" by (simp only: a1 qo_on_imp_reflp_on)
    show "Q z" using a5 b1 c1 by blast
    have "strict P (Eps Z) (Eps X)" using a5 by blast
    moreover have "P z (Eps Z)" by (metis b1 c1 eqrel_def tfl_some)
    moreover have "P (Eps X) x" by (metis a1 b2 eqrel_def eqreln
qo_on_imp_reflp_on)
    ultimately show "strict P z x" by (metis (full_types) a1 c0 b1 b2
eqreldom qo_on_imp_transp_on transp_on_def)
  qed
  moreover have "∀e∈U. strict P z e ∧ P e x ⟶ (∃e'∈U. strict P z
e' ∧ strict P e' e)"
  proof (intro ballI impI)
    fix e
    assume d1: "e ∈ U" and d2: "strict P z e ∧ P e x"
    obtain E where d3: "E = eqrel P U e" by blast
    have "E ∈ eqcl P U" using d1 d3 eqcl_def by blast
    moreover have "strict P (Eps Z) (Eps E)"
      by (smt a1 b1 d1 d2 d3 eqrel_def eqreldom eqreln qo_on_imp_reflp_on
qo_on_imp_transp_on transp_on_def)
    moreover have "P (Eps E) (Eps X)"
      by (smt a1 b2 d1 d2 d3 eqrel_def eqreldom eqreln qo_on_imp_reflp_on
qo_on_imp_transp_on transp_on_def)
    ultimately obtain E' where d4: "E'∈eqcl P U ∧ strict P (Eps Z)
(Eps E') ∧ strict P (Eps E') (Eps E)"
      using a5 by blast
    obtain e' where d5: "e' = Eps E'" by blast
    have "e' ∈ U" by (smt a1 d4 d5 eqcl_def eqreldom mem_Collect_eq
qo_on_imp_reflp_on)
    moreover have "strict P z e'"
      by (smt a1 b1 calculation d4 d5 eqrel_def eqreldom eqreln
qo_on_imp_reflp_on qo_on_imp_transp_on transp_on_def)
    moreover have "strict P e' e"
      by (smt a1 calculation(1) d1 d3 d4 d5 eqrel_def eqreldom eqreln
qo_on_imp_reflp_on qo_on_imp_transp_on transp_on_def)
```

```
    ultimately show "∃e'∈U. strict P z e' ∧ strict P e' e" by blast
  qed
  ultimately obtain y where b3: "y∈U ∧ strict P z y ∧ P y x ∧ (∀t∈U.
strict P z t ∧ P t y ⟶ Q t)"
    using a2 b1 b2 by blast
  then obtain Y where b4: "Y = eqrel P U y" by blast
  have c3: "Y y" using b4 by (metis a1 b3 eqrel_def qo_on_imp_reflp_on
reflp_on_def)
  from b4 have "Y ∈ eqcl P U" using b3 eqcl_def by blast
  moreover have "strict P (Eps Z) (Eps Y) ∧ P (Eps Y) (Eps X)"
  proof
    have "P (Eps Z) z" by (metis b1 c1 eqrel_def tfl_some)
    moreover have "strict P z y" using b3 by blast
    moreover have "P y (Eps Y)" by (metis b4 c3 eqrel_def tfl_some)
    ultimately show "strict P (Eps Z) (Eps Y)"
      by (smt a1 b1 b3 b4 eqreldom qo_on_imp_reflp_on qo_on_imp_transp_on
transp_on_def)
    have "P (Eps Y) y" by (metis b4 c3 eqrel_def tfl_some)
    moreover have "P y x" by (simp only: b3)
    moreover have "P x (Eps X)" by (metis b2 c2 eqrel_def tfl_some)
    ultimately show "P (Eps Y) (Eps X)"
      by (metis a1 b2 b3 b4 eqreldom qo_on_imp_reflp_on
qo_on_imp_transp_on transp_on_def)
  qed
  moreover have "(∀T∈eqcl P U. strict P (Eps Z) (Eps T) ∧ P (Eps T)
(Eps Y) ⟶ (∀x∈U. T x ⟶ Q x))"
  proof (intro ballI impI)
    fix T x
    assume d1: "T ∈ eqcl P U"
    assume d2: "strict P (Eps Z) (Eps T) ∧ P (Eps T) (Eps Y)"
    assume d3: "x ∈ U" and d4: "T x"
    obtain t where d5: "t ∈ U ∧ t = Eps T" by (smt a1 d1 d4 eqcl_def
eqreldom mem_Collect_eq qo_on_imp_reflp_on)
    moreover have "strict P z x"
    proof -
      have "P z (Eps Z)" by (metis b1 c1 eqrel_def tfl_some)
      moreover have "strict P (Eps Z) t" using d2 d5 by blast
      moreover have "P t x"
      proof -
        obtain a where "a ∈ U ∧ T = eqrel P U a" by (smt d1 eqcl_def
mem_Collect_eq)
        then have "P t a" by (metis d4 d5 eqrel_def tfl_some)
        moreover have "P a x" using d3 d4 by (simp only: ‘a ∈ U ∧ T =
eqrel P U a‘ eqrel_def)
        ultimately show ?thesis by (meson ‘a ∈ U ∧ T = eqrel P U a‘ a1
d3 d5 qo_on_imp_transp_on transp_on_def)
      qed
      ultimately show ?thesis
        by (metis (full_types) a1 b1 d3 d5 eqreldom qo_on_imp_reflp_on
qo_on_imp_transp_on transp_on_def)
```

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    qed
    moreover have "P x y"
    proof -
      have "P x t"
      proof -
        obtain a where "a ∈ U ∧ T = eqrel P U a" by (smt d1 eqcl_def
mem_Collect_eq)
        then have "P x a" using d3 d4 by (simp only: eqrel_def)
        moreover have "P a t" by (metis ‘a ∈ U ∧ T = eqrel P U a‘ d4
d5 eqrel_def tfl_some)
        ultimately show ?thesis by (meson ‘a ∈ U ∧ T = eqrel P U a‘ a1
d3 d5 qo_on_imp_transp_on transp_on_def)
      qed
      moreover have "P t (Eps Y)" using d2 d5 by blast
      moreover have "P (Eps Y) y" by (metis b4 c3 eqrel_def tfl_some)
      ultimately show "P x y"
        by (metis a1 b3 b4 d3 d5 eqreldom qo_on_imp_reflp_on
qo_on_imp_transp_on transp_on_def)
    qed
    ultimately show "Q x" using b3 d3 by blast
  qed
  ultimately show "∃Y∈eqcl P U. strict P (Eps Z) (Eps Y) ∧ P (Eps Y)
(Eps X) ∧
         (∀T∈eqcl P U. strict P (Eps Z) (Eps T) ∧ P (Eps T) (Eps Y) ⟶
(∀x∈U. T x ⟶ Q x))"
  by blast
qed


lemma dfinductrel:
  assumes  qocond: "qo_on P U"
       and pocond: "antisymp_on P U"
       and dccond: "dc_onrel P U"
       and dfcond: "df_on P U"
     and indcond1: "qinduct P U Q"
     and indcond2: "qinduct2 P U Q"
  shows "∀ x ∈ U . Q x"
proof -
    obtain H where a1: "H = (λ x y . x = y ∨ x ∈ U ∧ y ∈ U ∧ P x y)"
by blast
    interpret h: dfdcorderrel "H" "strict H"
    proof
      fix x y
      show "strict H x y = strict H x y" by blast
    next
      fix x
      show "H x x" by (meson a1 qocond qo_on_def reflp_on_def)
    next
      fix x y z
      assume b1: "H x y" and b2: "H y z"
```

```
      have "transp_on P U" using qo_on_def qocond by blast
      then show "H x z" using b1 b2 by (simp only: transp_on_def a1,
blast)
    next
      fix C
      assume b1: "chain_on H C UNIV" and b2: "C ≠ {}"
      have b3: "(∃ x ∈ C . ∃ y ∈ C . x ≠ y) ⟹ chain_on P C U"
      proof -
        assume "∃ x ∈ C . ∃ y ∈ C . x ≠ y"
        then obtain x y where c1: "x ∈ C ∧ y ∈ C ∧ x ≠ y" by blast
        have "C ⊆ U"
        proof -
          have "¬ (∃ c ∈ C . ¬ c ∈ U)"
          proof
            assume "∃ c ∈ C . c ∉ U"
            then obtain c where "c ∈ C ∧ c ∉ U" by blast
            thus "False" using c1 b1 by (metis a1 chain_on_def)
          qed
          thus ?thesis by blast
        qed
        moreover have "∀ x ∈ C . P x x" by (meson calculation
qo_on_imp_reflp_on qocond reflp_on_def subset_iff)
        ultimately show ?thesis using b1 by (unfold chain_on_def, simp
add: a1, fastforce)
      qed
      thus "∃x. glbrel H UNIV C x"
      proof (cases "chain_on P C U")
        case True
        then have d1: "chain_on P C U" by blast
        then obtain x where d2: "x ∈ U ∧ glbrel P U C x" using dccond
dc_onrel_def b2 by blast
        have d3: "C ⊆ U" using d1 by (simp only: chain_on_def)
        have "glbrel H UNIV C x"
        proof (unfold glbrel_def lb_def, rule conjI)
          show "∀y∈C. H x y"
          proof
            fix y
            assume "y ∈ C"
            thus "H x y" using d2 d3 by (auto simp add: a1 glbrel_def
lb_def)
          qed
        next
          show "∀y∈UNIV. (∀ya∈C. H y ya) ⟶ H y x"
          proof(intro ballI impI)
            fix y
            assume d4: "∀ z ∈ C. H y z"
            then have "∀ z ∈ C . P y z" by (metis a1 chain_on_def d1)
            moreover have "y ∈ U" using d4 a1 b2 d3 by auto
            ultimately show "H y x" using d2 d3 by (auto simp add: a1
glbrel_def lb_def)
```

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          qed
        qed
        thus ?thesis by blast
      next
        case False
        then have c1: "∀ x ∈ C . ∀ y ∈ C . x = y" using b3 by blast
        obtain c where "c ∈ C" using b2 by blast
        then have "C = { c }" using c1 by blast
        then have "glbrel H UNIV C c" using a1 by (unfold glbrel_def
lb_def, simp)
        thus ?thesis by blast
      qed
    next
      fix x y
      assume "H x y" and "H y x"
      thus "x = y" using pocond by (simp only: antisymp_on_def a1,
blast)
    next
      fix a b c d
      assume "(H a b ∧ H b d) ∧ H a c ∧ H c d"
      thus "H b c ∨ H c b" using dfcond by (simp only: df_on_def a1,
blast)
    qed
    show "∀ x ∈ U . Q x"
    proof
      fix x
      assume c1: "x ∈ U"
      obtain Q' where c2: "Q' = (λ x . x ∈ U ⟶ Q x)" by blast
      have "qinduct H UNIV Q'" using indcond1 by (unfold qinduct_def,
simp add: a1 c2, blast)
      moreover have "qinduct2 H UNIV Q'"
      proof(unfold qinduct2_def, simp, intro allI impI)
        fix z x
        assume d1: " Q' z ∧ H z x ∧ ¬ H x z ∧
              (∀e. H z e ∧ ¬ H e z ∧ H e x ⟶ (∃e'. H z e' ∧ ¬ H e'
z ∧ strict H e' e))"
        show "∃y. H z y ∧ ¬ H y z ∧ H y x ∧ (∀t. H z t ∧ ¬ H t z ∧ H
t y ⟶ Q' t)"
        proof (cases "z ∈ U ∧ x ∈ U")
          case True
          then have d2: "z ∈ U ∧ x ∈ U" by blast
          then have "Q z ∧ strict P z x" using a1 d1 c2 by blast
          moreover have "∀e ∈ U. strict P z e ∧ P e x ⟶ (∃e' ∈ U.
strict P z e' ∧ strict P e' e)" using a1 d1 by blast
          ultimately obtain y where d3: "y ∈ U ∧ strict P z y ∧ P y x
∧ (∀t∈U. strict P z t ∧ P t y ⟶ Q t)"
            using indcond2 d2 by (unfold qinduct2_def, blast)
          then have "H z y ∧ ¬ H y z ∧ H y x ∧ (∀t. H z t ∧ ¬ H t z
∧ H t y ⟶ Q' t)" using d2 d3 a1
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          by (case_tac "y = z", simp, case_tac "x = z", simp, simp,
metis c2 qo_on_def qocond reflp_on_def)
         thus ?thesis by blast
       next
         case False
         thus ?thesis using a1 d1 indcond2 by (unfold qinduct2_def,
blast)
       qed
     qed
     ultimately have "Q' x" using h.df_induction[of "Q'" "x"] by (simp
add: qinduct_def qinduct2_def)
     thus "Q x" using c1 c2 by blast
   qed
qed

lemma dfinductrel2:
  assumes  qocond: "qo_on P U"
       and dccond: "dc_onrel P U"
       and dfcond: "df_on P U"
     and indcond1: "qinduct P U Q"
     and indcond2: "qinduct2 P U Q"
  shows "∀ x ∈ U . Q x"
proof
  fix x
  assume a1: "x ∈ U"
  have "∀ X ∈ (eqcl P U) . (unpr U Q) X"
  using qocond dccond dfcond indcond1 indcond2
        eqcrelqo eqcrelantisymp eqcreldc eqcreldf eqcrelqind eqcrelqind2
        dfinductrel by (metis (mono_tags, lifting) qo_on_imp_reflp_on)
  then have "(unpr U Q) (eqrel P U x)" using a1 using eqcl_def by blast
  thus "Q x" using a1 by (metis eqrel_def qo_on_imp_reflp_on qocond
reflp_on_def unpr_def)
qed

lemma (in preorder) dfinduct2:
  assumes dccond: "dc_onrel (op ≤) UNIV"
    and dfcond: "df_on (op ≤) UNIV"
    and ind1: "⋀x. (⋀y. y < x ⟹ Q y) ⟹ Q x"
    and ind2: "⋀z x. (Q z ∧ z<x ∧ (∀ e . (z<e ∧ e≤x ⟶ (∃ e' .
z<e'∧e'<e)))) ⟹
                              (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶
Q t))"
  shows "Q x"
proof -
  have "qo_on (op ≤) UNIV" by (metis local.eq_refl local.order_trans
qo_on_def reflp_on_def transp_on_def)
  moreover have "qinduct (op ≤) UNIV Q" using ind1 by (metis
iso_tuple_UNIV_I local.less_le_not_le qinduct_def)
  moreover have "qinduct2 (op ≤) UNIV Q" using ind2 by (unfold
qinduct2_def, simp add: local.less_le_not_le)
```

```
  ultimately show ?thesis using dccond dfcond dfinductrel2 by blast
qed

theorem (in dcorder) df_induction:
assumes
diamond_free: "∀ a b c d . (a ≤ b ∧ b ≤ d) ∧ (a ≤ c ∧ c ≤ d) ⟶ (b
≤ c) ∨ (c ≤ b)"
    and ind1: "∀ x.   (∀ y. y < x ⟶ P y) ⟶ P x"
    and ind2: "∀ z x. (P z ∧ z < x ∧ (∀ e . (z<e ∧ e≤x ⟶ (∃ e' .
z<e'∧e'<e))) ) ⟶
                      (∃ y . z<y ∧ y≤x ∧ (∀ t . z<t ∧ t≤y ⟶ P t))"
shows "∀ x . P x"
proof -
  have a1: "dc_on (op ≤) UNIV" using dc_on_UNIV by blast
  have "dc_onrel (op ≤) UNIV"
  proof (unfold dc_onrel_def, intro allI impI)
    fix C
    assume b1: "chain_on (op ≤) C UNIV ∧ C ≠ {}"
    then obtain x where "glb (op ≤) C x" using a1 using local.dc by
blast
    then have "glbrel (op ≤) UNIV C x" by (simp add: glb_def
glbrel_def)
    thus "∃x∈UNIV. glbrel (op ≤) UNIV C x" by blast
  qed
  moreover have "df_on (op ≤) UNIV" using diamond_free df_on_def by
blast
  ultimately show ?thesis using dfinduct2 ind1 ind2 by blast
qed

end
```

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