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Alice Wonders *Ed.*

Math in the Time of Corona

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Editor

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Introduction

Alice Wonders

It may be obvious to many readers that the title of this book, *Math in the Time of Corona,* has been drawn from the highly acclaimed novel by Gabriel García Márquez, *Love in the Time of Cholera*. The volume editor, Alice Wonders, holds a fictitious name that represents the mathematics publishing group at Springer Nature. Be sure to catch the accompanying video to this Introduction showing quick clips of each member of the Mathematics group, expressing "I am Alice." The video is meant to add a dimension of fun and familiarity before diving into the short essays that follow. Three of the essays included in the book also include accompanying videos. Be sure to check for supplementary material listed on the HTML version of each chapter or the link to Electronic Supplementary Material in the PDF files. [Please note that this book is published in electronic form only.]

Covid-19, like Cholera, has affected the entire world. At the time that this book is published, the pandemic is ongoing and implementation of vaccinations have been uneven throughout the globe since the beginning of 2021. However, there has been a renewed hope for a return to normalcy though the timing will no doubt vary worldwide.

The pandemic has certainly changed the way we have been living day to day since early 2020. Teachers, instructors, academics, have been challenged to adopt new online teaching methods, methods which require more time in both preparation and delivery. Essays in this volume vary in topic and are written by members of the greater mathematics community, hence the use of "Math" in the book title. They recount or describe significant or noteworthy discoveries, musings, award winnings, eureka moments, challenges, solutions, inspirations, etc. that have resulted from, or

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have occurred during, an unprecedented global pandemic. Several of the authors have been involved in starting new research and devising new methodologies related to society's response to the outbreak and its ability to self-organize during a dramatic and complex situation. Some contributions describe how mathematical models and the management of big data have proved to be fundamental tools for the interpretation of epidemic activity and development of coping mechanisms.

Wishing contributors and readers good health and prosperity moving forward.

April, 2021 Alice Wonders

Things Lost and Won During the Plague

Karim Adiprasito

1 Blissfully Unaware in the Mountains

In early March of this year, I was attending a conference in Austria, giving a minicourse about my work on the g-theorem. The virus had made its way from its origin in China to Italy, and a few of the Italian participants were worried about their families. We heard that countries were closing their borders and, as people tend to do when they are getting nervous, many were making jokes, while others were getting scared about getting back to their country (indeed, Trump announced that the US would be closing its borders that week).

Nevertheless, I was hardly concerned. It was a very nice week, and I spent it hiking and thinking about mathematics. I had recently discovered a relation between l^2-cohomology of certain contractible manifolds (which is a useful way to think about the geometry of unbounded spaces, measuring how much like a valley a space might look) and some things I had been working on, and was pretty absentmindedly stepping through the melting snow as I was trying to get my mind around the way that symmetry might enter into a calculation I had in mind, trying different things. I tend to match my walking to my thoughts, so before I knew it, my feet were wet and I was in the middle of the mountains.

At other times, I was explaining some details of my talk to students, but I slowly noticed that people were vanishing because their home countries were stopping flights.

My wife contacted me when I would get home, starting to get worried about Denmark (where we live) closing its borders, and I myself was getting kind of nervous. But to be honest, I was still getting sidetracked in my thoughts, often

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returning to mathematics instead of focusing on getting home. In the mountains, I tend to forget all emails and worries, so I kept stepping into the snow, thinking about this and that. In this situation, I hardly registered that I also got an email, from Volker Mehrmann, president of the EMS, when I was about to leave for a workshop in Sweden before returning home. I was so deep in thoughts, I do not remember getting that email in that week, and was only able to reconstruct it from the dates on the email. I must have been entirely unaware it said something about a prize at this point.

2 The Trip to Stockholm

To be honest, I should have cancelled that trip. But there was a close friend, Igor Pak, in Stockholm that told me he was bored, lonely at Institut Mittag Leffler and wanted to talk mathematics, and with him, that was always terribly fun. Any discussion with him is marvellous, devolving into a mix of anecdotes, jokes and scientific discourse. These are often fruitful (we had recently resolved an old conjecture of Vladimir Berkovich concerning of singularities with two other colleagues) but most of all, they are enjoyable.

It was only there that it hit me that I won the EMS prize for the work I had been presenting in Austria. I told him that, we got a delicious marzipan cake, tea and beer, and got to celebrating, discussing mathematical problems and whatever was new in our lives. And I was feeling incredibly proud of having won, being honored in such a way.

It was a very serene atmosphere; the Institute Mittag Leffler is already very isolated, outside of the busy Stockholm center. And equipped with a spacious library in a labyrinthine mansion that filled me with joyful curiosity every time I visited. It was one of those buildings you like to go looking for the secret levers to open hidden pathways.

But these days, it was particularly empty. Still, I had a friend, and a few other mathematicians were still around. The complimentary fruit bowls and cookies were left unguarded, ready for the looting by Igor and me.

It is this what I would miss most during the next months. Sure, we now have video-conferences and online seminars, but they always have a specific goal, a beginning and end. And drinking, celebrating and discussing mathematics without a particular goal is something that is hard to replace.

We were discussing particularly how we could simplify a particular construction we had come up with years prior, triangulating 4-dimensional euclidean space with acute polyhedra. This is the beauty of discussing math with Igor, it always went into wild directions, going over algebraic to topological ideas and back. And then, musing over other things again. We were wondering why the restaurants in that area of Stockholm, a particularly wealthy one, were so full, we were wondering (they may not know how to cook, we theorized). We discussed whose stock might rise in the coming days (finding out why we were not living in the wealthy neighborhood).

Overall, it was remarkable how calm the Swedes stayed, while many of the mathematicians we knew, often not locals, were panicking, leaving the country.

3 Returning Home, into the Lockdown

Still, the virus caught up with us. Igor was worried about his mother, and wanted to see her in Moscow, and I was getting nervous about Denmark's closed borders. But most of all, I wanted to celebrate with my wife too. I was going to miss this atmosphere during the coming months, and the ability to talk freely about whatever topic came to our minds. And while the time lockdown had its ups (being free to think about new things, I mostly stayed away from online seminars, for they miss the most important part about what makes mathematical discourse so interesting) I am looking forward to meeting friends again, discussing math and joking again.

Karim Adiprasito

Has the coronavirus affected the Ferran Sunyer i Balaguer Prize?

Manuel Castellet

The Ferran Sunyer i Balaguer Foundation, created in Barcelona in memory of the Catalan mathematician Ferran Sunyer, yearly awards an international prize for a mathematical monograph of an expository nature, presenting the latest developments in an active area of research in Mathematics in which the applicant has made important contributions. The prize, amounting to 15,000 euros, is provided by the Foundation and the winning monograph is published in Birkhäuser's series *Progress in Mathematics*.

Since its first edition in 1993, we have had years with many applications and others with few, but we have almost always had high-quality submissions; only three times did the prize's scientific committee (made up of a local mathematician, two editors of *Progress in Mathematics* and two mathematicians from other countries, all of recognized prestige) proposed to the Foundation's Board of Trustees not to award the prize.

2020 has been unique for two reasons: the extraordinary quality of two of the submitted monographs and the COVID-19 pandemic.

On the one hand, the committee recommended exceptionally awarding two monographs, one on algebraic topology and the other on differential geometry, both of the same value. The authors of the monographs work in four different countries: Belgium, France, Italy and Spain, the four European countries most affected by the coronavirus.

The Board of Trustees met to approve the proposal of the committee on March 12 at 3:30 PM. This was the last academic activity that took place in person in Catalonia; that afternoon at 5:00 PM all events were cancelled and the next day the universities closed.

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We managed to save the award, but could not celebrate the solemn award ceremony normally held in the noble hall of a 17th century building on the day of the feast of Saint George, April the 23rd. This is the day of the patron saint of Catalonia and also the day of books and roses, when the streets are filled with people buying and giving books and roses to their loved ones; truly a unique day.

These facts may be more or less anecdotal, but the most serious and saddest part of this relationship between the Ferran Sunyer Prize and the coronavirus is that, unfortunately, on March 24, Josep Vaquer, one of the first members of the Foundation's Board of Trustees, died at the age of 91 from COVID-19 disease. Josep Vaquer, Professor Emeritus of the University of Barcelona, was the first professor of mathematics that I had at the University, an excellent teacher and a man involved, jointly with his wife, in various social causes, especially teaching the most in need, visiting prisoners and advising and helping women in different very difficult life situations. A great man.

Josep Vaquer

Manuel Castellet

Intensive Collaborative Work on COVID-19 Modeling

Arni S. R. Srinivasa Rao and Steven G. Krantz

1 Introduction

This is a story of a COVID-19 modeling collaboration between two scientists who live at a considerable distance and who have been collaborating since 2017. They are developing several novel ideas to understand the spread of the epidemic. They work closely at developing, planning, and policy-making decisions.

The broader subject of mathematics and the subject of quantification are centuries old and these have been of great value not only for theoretical developments but appreciated for their positive roles in society. This includes understanding planetary motions as well as other useful ideas in ecology and environmental studies. However, mathematical ideas have been also in use for understanding the propagation of the number of people infected and the spread of epidemics since the 18th-century cholera epidemic in Europe, the 20th-century plague epidemic in England and India, the Spanish flu epidemic in the U.S. and other countries, and for several infectious diseases like HIV/AIDS, avian influenza, etc.

Modeling the novel coronavirus (COVID-19 or SARC-nCov2) since it has been reported from mid-December 2019 in China has very particular characteristics. Several modeling experts across the world and other people working on quantification of epidemics are studying the matter from several different points of view. The ensuing results could range from

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(i) the uncertainty of the virus genomics, and its etiological properties,
(ii) the uncertainty of what will be best treatment options if infected,
(iii) how to contain the virus within an infected area,
(iv) level of infectivity,
(v) list of all clinical signs and symptoms,

and so forth. Even in the initial days (late December of 2019, early January of 2020) it was not clear whether the virus would be spreading to other countries outside China. There were even suppositions that, if the virus reached the U.S., then it could be easily contained.

The authors began by asking the following question: how do we construct wavelets (a kind of improved version of the Laplace transform from harmonic analysis) based on an epidemic from only partial information on that epidemic? The goal was to be able to construct a complete picture of an epidemic from partial information. We developed rigorous mathematical methods while addressing these questions and published our first paper in the *Journal of Theoretical Biology* [1]. We accelerated our preparation of this article from January–February, 2020 as we started to realize the importance of the methods developed due to the emergence of COVID-19, and we focused as well on the novel coronavirus during the revision of our manuscript when it was accepted in March, 2020. Our further collaborations on corona started very quickly as we have engaged in intensive study of the development and applications of various mathematical and probabilistic techniques.

In this article, we will describe a collection of ideas we are developing for COVID-19 in the U.S.A., Europe, and Asia. Note that several new mathematical ideas have been explored and developed, which range from simple differential models to detailed age-structured models to wavelets. We also look at set-theoretic and Venn diagram expressions, conformal mapping principles, Fisher-Rao metrics, etc.. We will describe how our quantifications assisted in practical planning and also describe how we collaborated so intensively within the span of a few months. Our activities have included articles published/prepared, grant proposals written, invitations to give talks, mini-symposia, peer-review requests, special issues of journals, etc..

2 Under-Reporting of COVID-19

First, we have published studies on the pandemic covering several countries. Just before our first paper got accepted, we decided to study the under-reporting of corona in various countries until the first week of March. Such an idea was a natural consequence of our first paper. In this study, we have constructed a simple differential equation model with two variables—those who are particularly susceptible to the virus and those who have a virus. Our model tries to obtain numerically the number of newly infected cases in each country from the day that a country has reported more than ten cases until the first peak that occurred [2, 3]. We had to calibrate the parameters of transmission based on the reported trend and from growth in reported cases.

In addition, other factors such as population density, people living in urban areas, population age-structure, etc., were also considered. This study received attention worldwide as this was the first model-based under-reporting that considered a detailed parameter. Once the projected new cases were obtained through the model, we have compared them with the reported cases with the same data to obtain the rate of reported cases to the total number of cases. Such a ratio was plotted through Meyer wavelets, which are very flexible structurally and robust too. The notable feature of wavelets is that they are localizable both in the space variable and the phase variable.

These wavelets helped us to visually see the difference between actual cases of corona as per the model and the reported cases of the corona. What we predicted until the end of March (either predicted to be severe or not) turned out to be true for several countries, for example, Italy, Germany, Spain, South Korea etc.

Other countries that we considered were China, India, France, Iran, the UK, and the U.S.A. We have also analyzed the data on general health preparedness of rich countries and the status of COVID-19 response during the first two months of the pandemic [4].

Apart from the papers mentioned above which published our under-reporting-related research, our work has attracted the attention of 40–60 worldwide newspapers (both print and OnLine) and TV channels in several languages. Details of the matter can be found through Internet searches. It was a great experience for us to communicate the various questions from the media and devise our mathematics—either the wavelets or the differential equations or detailed modeling assumptions—to simplify to a language that can be understood by non-experts. After all, science is meant to be implemented and communicated to help society!

3 ICU and Non-ICU Admissions in the U.S.A.

We have visualized a collection of sets of populations to understand the spread of COVID-19 with and across these sets. Some of the sets within the collection are mutually exclusive and some have overlapping populations. We have also divided these sets further based on whether the populations within each set have any medical underlying conditions like hypertension, cardiovascular illness, lung diseases, and combinations of these diseases. The idea was to use these sets to predict what fractions of these sets of people with underlying conditions in the U.S. population who are aged 65+ will likely have COVID-19 during April–June, 2020, and to find how many of them within the collection will likely be hospitalized [6]. We further provided the number of hospitalizations that required ICU (Intensive Care Unit) admissions and those which do not. We developed age-structured models for prediction and considered wavelets for understanding the difference in magnitudes of various sets of predicted populations with COVID-19.

4 Social Distancing

We have considered the degree of lockdowns and social distancing in our age-structured type of models, combined with wavelets to predict the number of new cases that could emerge during the months of May–June, 2020 [5]. One of the novel features of this work is that we have considered new cases of COVID-19 that were identified and those that were not identified. Among those that were not identified, we have varied the degree of adherence to the social distancing norms to predict new transmissions within the U.S.

5 Virtual Tourism Technology and Information Geometry

Because of the pandemic, the airlines, hospitality, and tourism industries have been financially affected. Considering the need of the hour, we have proposed to develop a new technology called LAPO (live-stream with actual proportionality of objects). This technology, if developed and implemented, proposes to use mathematical and statistical principles such as Fisher-Rao metrics and Rao's distances and conformal mappings for maintaining consistency between visual distances and angles of objects in actual tourist locations and that of 2D images and 3D videos [5]. Such applications of information geometry principles and angle preserving techniques and the related technologies are novel to the literature. A related concept is referred to as the Fisher-Rao metric. Our LAPO concept will make travel accessible and safer for more people, and will be fun for young and old.

6 Timely Assistance Provided in Planning and Response

Through our collaborations, we have had an opportunity to assist in timely planning and preparing COVID-19 responses at various levels. For example, we received an invitation from the Governor's office through the university to provide Georgia state-level predictions in March 2020. Again, in April 2020, we received a request to provide dates at which the model-based peaks of COVID-19 cases in Georgia and for a couple of counties would occur. Local media and TV reported these results, see for example (WRDW, April 14, 2020, Augusta Chronicle, April 4, 2020). We have also assisted the Bulgarian Academy of Sciences when their President of the Academy approached us for providing model-based predictions of national COVID-19 numbers. Our results were used by their national preparedness team. We have also received several requests to give Webinars, mini-symposia, invitations for peer-review from leading journals, invitations to write articles, and of course received lots of appreciation and support from our respective universities. We both are also co-editing a special issue on the theme Wavelets, Dynamical Models, and Algorithms in Understanding and Preparedness for the COVID-19 Pandemic for a reputed journal in mathematics, namely, the *Journal of Mathematical Analysis and its Applications*.

7 Artificial Intelligence Frameworks

One of us (Rao) has been involved in developing artificial intelligence frameworks for identifying the coronavirus cases and implementing the technologies through the hospital systems and mobile-based surveys [7]. Such ideas received wide media attention of 150–170 news outlets across the globe as that was the first idea proposed for identification of the COVID-19. One can search on the Internet for all media reporting of this work. Through the probabilistic approach proposed in their work, it is possible to construct probabilities of identifying a person with certain disease symptoms.

8 Rewards of Distance Collaboration in the Time of COVID-19

Rao's background is in mathematical and stochastic modeling analysis and Krantz's background is in theoretical mathematics (mostly complex analysis). We bring very different perspectives to this work. The result has been a rewarding collaboration and a strong new personal friendship.

As described in Figure 1, we have made use of the mathematical, statistical, and probabilistic techniques in a timely fashion and worked rigorously with a passion to be able to assist in the COVID-19 response.

Of course, neither of us is a stranger to distance collaboration. Krantz has more than 60 co-authors all over the world, and Rao has about a dozen. But it is new to both of us to have a meeting of minds between a medical school and a theoretical mathematics department. The COVID-19 virus has played an elemental role in making this happen. The shared concern and the shared passion for making a difference have been a driving force for both of us. Practicing applied mathematics that can save and improve people's lives is certainly a worthwhile endeavor.

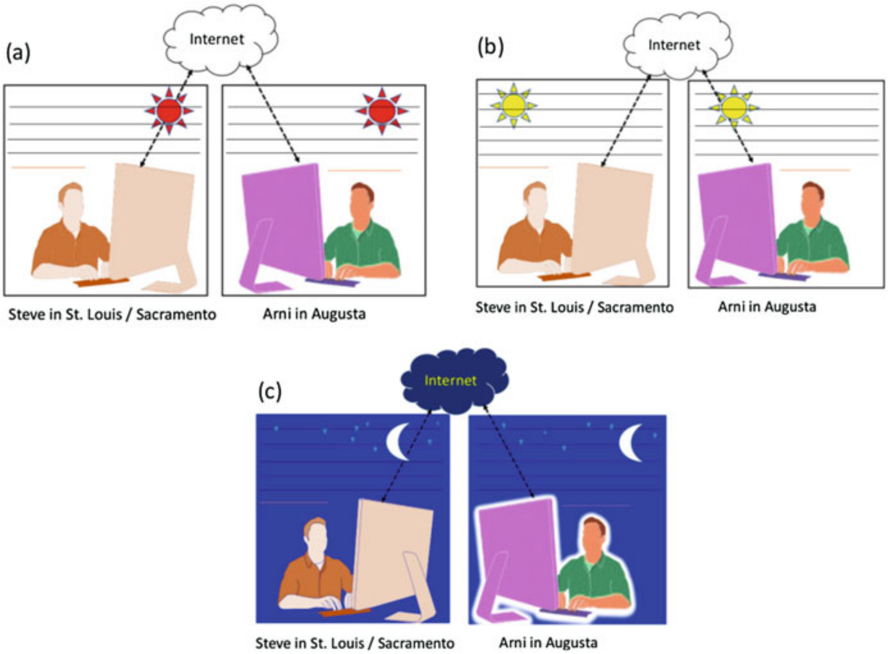

Fig. 1 Krantz and Rao collaborating long distance and long hours.

References

1. Krantz, S. G., Polyakov, P., Rao A. S. R. S. (2020). True Epidemic Growth Construction Through Harmonic Analysis *Journal of Theoretical Biology*, Volume 494, 2020, 110243
2. Krantz, S. G. and Rao, A. S. R. S. (2020a). Level of under-reporting including under-diagnosis before the first peak of COVID-19 in various countries: Preliminary Retrospective Results Based on Wavelets and Deterministic Models *Infection Control & Hospital Epidemiology*, 41(7), 857–859. doi:10.1017/ice.2020.116
3. Rao, A. S. R. S., Krantz, S. G., Kurien, T., Bhat, R., Kurapati, S. (2020). Model-based retrospective estimates for COVID-19 or coronavirus in India: continued efforts required to contain the virus spread *Current Science* 118 (7), 1023–1025
4. Krantz, S. G., Rao, A. S. R. S. (2020b). Preparedness and Response within Individual G7 Countries for COVID-19: Was there a Failure, in Spite of US Economic Strength, of Health Intelligence in the US to Assess the prior situation? (NIH/WIKI IMAGE preprint)
5. Rao, A. S. R. S., Krantz, S. G. (2020) Data science for virtual tourism using cutting edge visualizations: information geometry and conformal mapping, *Cell Patterns*, 10.0106/j. patter.2020.100067

6. Rao, A. S. R. S., Miller, D. D., Berman, A. E., Hess, D. C., Krantz, S. G. (2020). Immediate and Near Future Prediction of COVID-19 Patients in the US Population Aged 65+ With the Prior Medical Conditions of Hypertension, Cardiovascular and Lung Diseases: Methods, Models and Acute Care Estimates (medRxiv)
7. Rao, A. S. R. S. and Vazquez, J. A. (2020). Identification of COVID-19 Can be Quicker through Artificial Intelligence framework using a Mobile Phone-Based Survey in the Populations when Cities/Towns Are Under Quarantine *Infection Control & Hospital Epidemiology* (Cambridge University Press), 41(7), 826–830. doi:10.1017/ice.2020.61

Arni S. R. Srinivasa Rao
(Photo taken by Kim Ratliff)

Steven G. Krantz
(Photo taken by Randi Diane Ruden)

The Home-Field Paradox

Peter Winkler

In *Mind-Benders for the Quarantined*, a challenging mathematical puzzle was—and still is, as of this writing—being offered each week by the National Museum of Mathematics ("MoMath") to a list of subscribers that currently number about 10,000, located in 50 states and more than 80 countries. Every Sunday a new puzzle is sent out, followed by a hint on Tuesday, a bigger hint on Thursday, and a solution on Saturday. This (free) service was begun in March 2020, with the idea of supplying brain fodder for home-bound puzzle mavens during the COVID-19 pandemic.

The puzzles were collected or composed by the author; the paradox discussed below arises from a Mind-Bender puzzle that he devised earlier, while the COVID-19 outbreak was raging in Wuhan, for use in a MoMath course for executives called "Probability and Intuition."

The paradox concerns two baseball teams and a statistician. The teams are the Appleton Aardvarks and the Brockville Bandicoots; they are traditional rivals in their league, and meet every year to play a series of games. The first team to win four games gets to take home the coveted Cassowary Cup and display it proudly until, perhaps, the next year's series.

The teams are evenly matched except that playing on home turf confers a small advantage; in any given game between the Aardvarks and the Bandicoots, the home

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team has a 54% chance of winning.[1] There are no ties. By tradition, every year the first three games of the series are played in Appleton and all remaining games in Brockville.

The league statistician has been looking over the records of this very long-running series and has made a curious discovery. Intrigued, she proceeds to do some math.

1 Where Are Most of the Games Played?

Every year the Cassowary Cup contest takes 4, 5, 6 or 7 games to be decided. The statistician's intuition tells her that on average, the number of games played ought to be somewhere between 5 and 6. If so, it would follow that over the long haul, fewer than 3 games per year would be played in Brockville. Since three games are always played in Appleton, the statistician expects most series games to have been played in Appleton.

But just because a number n can be 4, 5, 6 or 7 doesn't mean these values are equally likely; the "expected value" of n, i.e., the long-run average, could be anything between 4 and 7.

So the first thing our statistician does is to compute the average number of games played in the series. To keep things simple, she begins with the assumption that the outcome of every game is a 50–50 proposition.

The probability that $n = 4$ is easy to calculate; for the series to be over after only four games, the second, third, and fourth games must be won by the same team that won the first game. The probability of that happening, under the 50–50 assumption, is 1/8.

For the series to last five games, one team must win four of the first five games played and the other team one of the first four. There are eight ways this can happen: ABBBB, BABBB, BBABB, BBBAB and the same sequences with A and B swapped. Each of these outcomes has probability $(1/2)^5 = 1/32$, so altogether the probability that $n = 5$ is $8 \times 1/32 = 1/4$.

Our statistician, being both lazy and clever, now realizes that she may be able to save herself some trouble by jumping to the $n = 7$ case. For the series to go seven games the teams must each win three of the first six, and remembering her elementary combinatorics, she calculates that the number of ways that could happen is $\binom{6}{3} = (6 \times 5 \times 4)/(3 \times 2 \times 1) = 20$. Each of these outcomes for the first six games has probability $1/2^6 = 1/64$, so the probability that the series goes to seven games is $20 \times 1/64 = 5/16$.

[1] Statistics show that 54% is, approximately, the fraction of major-league baseball games won by the home team—notwithstanding the outcome of the 2019 World Series in which every game was won by the visitors.

Since the only other possibility is that the series lasts exactly six games, and the probabilities must sum to 1, she deduces that the probability that $n = 6$ is $1 - (1/8 + 1/4 + 5/16) = 5/16$.

Finally, the expected value of n is the sum of its values times their probabilities; thus, $4 \times 1/8 + 5 \times 1/4 + 6 \times 5/16 + 7 \times 5/16$ which works out to 5 13/16. So, indeed: in the long run, an average of 5 13/16 = 5.8125 games are played in each series, 3 in Appleton and the remaining 2.8125 in Brockville.

2 Where Are Most of the Games *Really* Played?

Our statistician's intuition tells her that the average number of games played per series won't change much if she takes into account the slight advantage enjoyed by the home team. Still, as a scientist, she knows she should make the calculation, and she knows from the statistics of the whole league that playing at home boosts a team's probability of winning to 54% over an equally talented adversary.

This makes her calculation tougher but it's not fundamentally different. For example, there are still two ways that the series can end in only four games, but one of these—namely, AAAA—entails just one "upset" (game won by the visitors), while the other requires three upsets. Thus the probability that $n = 4$ changes to $.54^3 \times .46 + .46^3 \times .54 = 0.12499488$, quite close to the $1/8 = 0.125$ that she computed before.

When she's done with her recalculation, she finds that the expected number of games played in a series, allowing for home-field advantage, is 5.81531800192, just a bit higher than before and still between 5 and 6.

3 So Which Team Has the Advantage?

It seems to our statistician that her own calculations show that the Appleton Aardvarks are (slight) favorites in any Cassowary Cup series, since on average more games are played in Appleton. But something bothers her.

She knows that the series can be thought of as "best of seven," that is, the cup goes to the team that wins the most of seven games. She asks herself: "would it make any difference if all seven games were always played?". Of course, it *shouldn't* make any difference, because the outcome is determined once one of the teams has won four games; that's why they terminate the series at that point. But she imagines that, perhaps because tickets have been sold and the players love to play even when the fate of the cup is sealed, they go ahead and play the rest of the games with equal ferocity.

All the games count equally in the end, and four are played in Brockville, so it seems that it is the Brockville Bandicoots who should be the favorites.

The statistician is now seriously confused. She's convinced herself that indeed, the Bandicoots are more likely to win any series. But how can it be that home-field advantage in the seventh game, which probably won't be played, is just as valuable as home-field advantage in the first game, which is always played?

Suddenly, one day while she is waiting for a taxi, it hits her: the home-field advantage in late games *is there when the Bandicoots need it*. The statistician lives in the city and doesn't own a car, because there's a reliable taxi service and she figures that a car that's there when she needs it is just as good as a car that's there all the time. The same applies to home-field advantage in the seventh game. If the Bandicoots have already won the series by then, they didn't need that advantage; if they've already lost the series by then, it wouldn't have helped.

(In fact the probability that the Bandicoots win the series is 51.254831047%. This is not hard to calculate because the series is a toss-up if it lasts at most six games. Thus the Bandicoots' gain over 50% is 4% times the probability that a seventh game is needed, the latter being 0.31370776192.)

4 What Had the Statistician Discovered?

Finally, our statistician is at peace with herself—because her conclusions explain what she observed in her compilation of data from the past *many* years of the Cassowary Cup series. Yes, most games were played in Appleton; and yes, the scheme actually favors Brockville. So what did she find?

That most of the games were won by the Aardvarks, but most of the series by the Bandicoots!

Peter Winkler

Data Analysis and Predictive Mathematical Modeling for COVID-19 Epidemic Studies

Alfio Quarteroni, Luca Dede', and Nicola Parolini

1 Introduction

The health crisis that affected the entire planet from early 2020 has raised huge questions about our society's response and ability to self-organize for facing a dramatic and complex problem. In this scenario, the mathematical models and the management of big data prove to be fundamental tools for the interpretation of the epidemic, its understanding and for supporting digital health.

World public opinion, especially the one accustomed to decades of well-being and extraordinary achievements of scientific and technological research, reacted with surprise, if not astonishingly, in the face of the substantial unpreparedness in tackling a health problem, which however was not too dissimilar from others already sadly known in human history, even very recent. In fact, just in the last two decades we witnessed at least three similar viral epidemics (Ebola, SARS and MERS) which, despite not having proved to have the same geographic spread as COVID-19, are not completely resolved yet.

From the outset, Mathematics was indicated as an essential discipline for providing, for example, forecasts of the course of the infection for time intervals of days,

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months, and even years, in terms of the reproduction number (R0 at the beginning, Rt in the following stages after adopting containment measures), the date of occurrence of the peak and the extent of the same, and, more sadly, the number of victims. These are numbers to which the public has quickly become accustomed thanks to fast but necessarily approximate information, as well as often not very reliable.

Very likely, these expectations were cooled by the observation that the peak concept, mantra of the month of March during the outbreak in Italy, did not seem so conclusive, and the curves of the new daily infected did not have that regular and symmetrical trend (in the growth and decreasing phases) of a Gaussian.

Most of the data sets provided daily by Authorities were affected by bias and errors: some more acceptable, due to the lack of coherence and the delay with which they were collected at a territorial level, others more substantial, almost all underestimating figures (just to name a couple, the number of new infections, due to an insufficient swabs, and the number of deaths attributed to the COVID-19). Furthermore, for several weeks many deaths - occurring at home or in elderly residential care homes - were not initially attributed to COVID-19.

The problem of bias, inconsistency and incompleteness of data has plagued all Countries (not only Italy), either because of unreadiness and unpreparedness, or (sometimes) because of a distorted political use of the crisis. Whatever the cause, this has made understanding the epidemiological process even more difficult. A lesson that we would certainly do well to learn, in view of the inevitable long tail of the pandemic.

Moreover, knowledge of the data (even of all the data, as correct and complete as possible) can, on the one hand, allow us to represent a process, but not necessarily to interpret and manage it. This is done by epidemiological models, based on mathematical equations that use data but provide forecasts, that is, the evolution of variables (the solutions) over time, which serve to fully characterize the epidemiological process.

2 Mathematical Models

Mathematical models for the study of epidemics have existed for over a century. The most famous was developed way back in 1927 by two Scotts, William Kermack and Anderson McKendrick, formulated to explain the rapid growth and subsequent decrease in the number of infected people observed in some epidemics, in particular plague and cholera.

In these models, it is essential to identify the variables that can describe the process, for example the number of infected, susceptible, exposed to contagion, hospitalized, healed or, unfortunately, deceased people (see, for example, [1], for a recently proposed multi-compartimental model for COVID-19). But there are also models with many more variables, and therefore many more equations. These variables will constitute the solutions of the model; their dependence on time will allow us to understand how they will vary in the future according to the data collected in the past, and therefore to have a complete quantitative description of the dynamics of the epidemiological process.

Multi-compartimental models can be further improved by including a spatial dependence though the so-called multi-city approach, where a multi-compartimental ODE problem is solved for each local geographic unit with additional coupling terms accounting for the mobility of people [2].

The model also allows describing different scenarios, depending on, for example, the application of different containment strategies (lockdown, social distancing, closing of schools, factories, shops, theaters and museums, for example), or possible availability of drug treatments or vaccine administration to increasing percentages of the population, infected or susceptible. It is also possible to introduce control variables in order to allow epidemic governance within certain limits. It is the task of the builders of mathematical models to identify the variables necessary to build the mathematical tool with effective predictive and control capacities. On the other hand, it is the task of govern and health authorities to prepare the legal and technical structures for data collection.

3 The COVID-19 Case

In the case of the spread of an infection that occurs by proximity, such as COVID-19, it is essential to identify the infected people as soon as possible and the contacts they had before being confined. Of the two tasks, the first is more problematic than the second, because the infection, in the case of COVID-19, has no manifest effects before a few days - a variable number (presymptomatic) - or even may have no effect at all (the case of the so-called asymptomatics).

The time frame in which a person can transmit the virus to other people is therefore very wide and, given the human density of the most common living conditions (most of the World's population now lives in cities), the number of infections increases, as is now commonly said, exponentially. Doctors have identified some signs of possible infection (increase in temperature, cough, respiratory problems, reduced sense of taste . . .) that can be used to assess personal conditions; it is quite evident now that this type of anamnesis must be accompanied and strengthened by a system of health monitoring and control on the territory that receives the first reports and investigates them, confirming or denying them.

4 The Use of Apps

Several apps are now available for monitoring (or tracking) the contacts among individuals and, sometimes, for collecting biometric data. The simpler, in theory, is the control of contacts, with technological solutions that we can all already have in our pocket today, literally, thanks to our smartphones.

In this context, the Italian government has recommended the *Immuni App*, an exposure notification solution based on Bluetooth technology, in line with that proposed by Apple and Google, revolving around user privacy, entrusting everyone with the task of isolating themselves and spontaneously reporting to the health service in case a notification of a contact with a person who has proved positive.

However, the management of the evolution of the epidemic could more effectively take advantage of the geolocation of individuals, i.e. the tracking of movements (always via smartphone); in this way, a punctual control action on the individual person can be implemented, through the analysis of the people mobility. This type of action can be enhanced through alternative recognition systems, such as cameras with facial recognition or other identification methods (a strategy that was pursued in several countries in far east Asia).

Each of these methods necessarily takes for granted an active and voluntary participation of individuals. This moves in a context of transparency that allows everyone to understand what is the spatial and temporal scope of the collection of information, what its purposes are and, of course, who is the Data Protection Authority that should ensure adequate security measures and confidentiality policies in the data treatment.

If in fact the implementation of tracking people through Bluetooth technology (with decentralized data management) does not have particular repercussions on people's privacy (no movement is detected, nor any declaration of positivity is associated with individuals), geolocation and donation voluntary biometric data assume that the citizen delivers information on his health and habits.

These data are used to feed the models that describe and predict the progress of the epidemic.

5 The Data Lake

Thanks to mathematical models and machine learning algorithms it is possible to extract indications to simulate, predict and optimize. Big data and machine learning are words that have become commonplace in order to improve and make various industrial and social processes more effective and efficient. Today, these same tools can be used in emergency or post-emergency healthcare settings.

There are two other fundamental aspects, in addition to tracking: testing (with swabs and serological tests), and treatment (the famous 3Ts that are so much talked about). Tracking and testing are extraordinary data generators (indeed, Big Data means gigantic quantities of heterogeneous data). A crucial factor for their effective use is that of their *reconciliation*, which is preliminary to the construction of a data lake that feeds the epidemiological models described above. This is an activity in which the Veneto Region appeared to stood out par excellence from the early outbreak in Italy, thanks to the georeferencing of infected cases, the monitoring of infected micro-clusters, both in homes and in workplaces, to the monitoring of the state of infection of care homes operators and to that of the guests of the elderly residential care homes.

The availability of a sufficiently rich and reliable data lake also becomes essential in order to exploit the extraordinary predictive power of epidemiological models as effectively as possible: identifying potential new outbreaks, anticipating the early detection of the infected, optimizing the health management and therapeutic treatment, also through the exploration of different scenarios of strengthening or relaxing containment measures.

As already noted, the availability of reliable and geographically distributed data, to be used to initialize these models and for model calibration purposes, is fundamental for the reliability of the predictive power of space-time models. By model calibration we mean the mathematical procedure that allows an optimal choice of the several parameters that characterize the mathematical model. One instance is provided by the mobility maps, that is the availability of data that represent the amount of individuals travelling between cities in a given time frame (e.g. daily). An example is provided in Fig. 1 where color intensity is increasing with the number of travelers.

In Fig. 2 we report a comparison between the results predicted by a numerical model that generalizes the one given above by introducing a multi-city scenario and accounting for the inter-city mobility of people (see [2]). Parameter calibration is made by accounting for the first 110 days of the epidemic.

Often, however, these data are incomplete, for example the number of deaths for which information for many municipalities has often been lacking (for an example, see Fig. 3), or the data that describe the flow of social or mobility relationships on networks.

Fig. 1 Intercity mobility map in Italy.

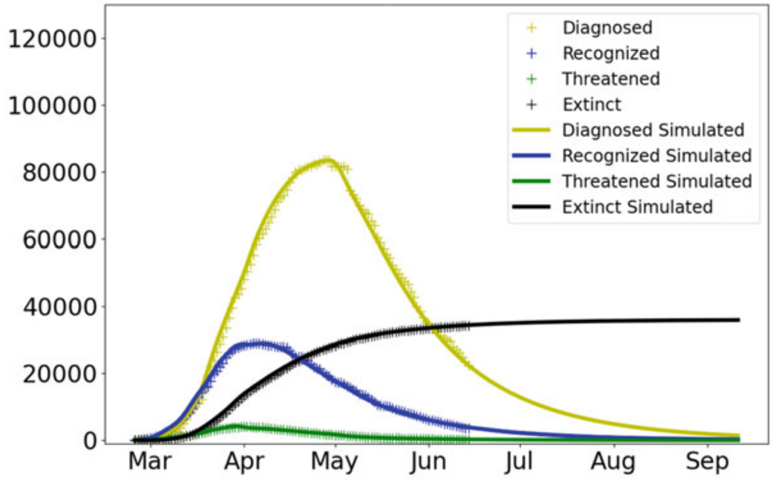

Fig. 2 Comparison between real figures and those predicted by the mathematical model described in [2].

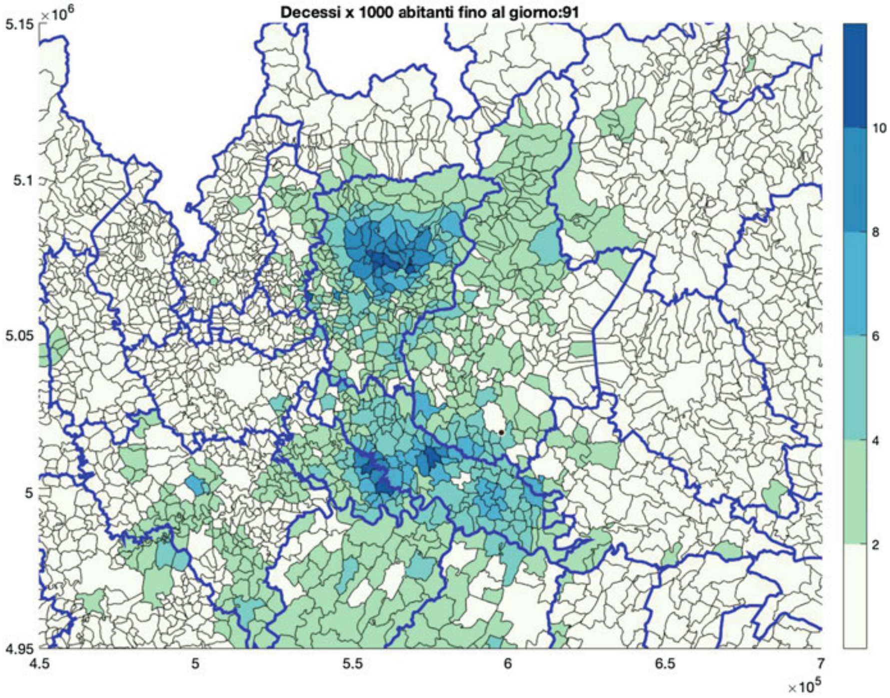

Fig. 3 Estimated number of deaths per 1000 inhabitants for municipality until March 31th in Northern Italy, including Lombardy region, elaborated from data released by ISTAT on May 4th 2020 [3].

Mathematical and statistical models can allow to remedy, at least partially, even this incompleteness.

References

1. Giulia Giordano, Franco Blanchini, Raffaele Bruno, Patrizio Colaneri, Alessandro Di Filippo, Angela Di Matteo, and Marta Colaneri. Modelling the COVID-19 epidemic and implementation of population-wide interventions in italy. *Nature Medicine*, pages 1–6, 2020.
2. Edie Miglio, Nicola Parolini, Alfio Quarteroni, Marco Verani, and Stefano Zonca A spatio-temporal model with multi-city mobility for COVID-19 epidemic, MOX Report 42/2020.
3. Impatto dell'epidemia COVID-19 sulla mortalità totale della popolazione residente - Primo Trimestre 2020, https://www.istat.it/it/files//2020/05/Rapporto_Istat_ISS.pdf.

A View from Lockdown: Mathematics Discovered, Invented, and Inherited

Alexandre Borovik

The classical platonist/formalist dilemma in philosophy of mathematics can be expressed in lay terms as a deceptively naive question:

Is new mathematics discovered or invented?

Using an example from my own mathematical work during the Coronavirus lockdown, I argue that there is also a third way: new mathematics can also be inherited. And entering into possession, making it your own, could be great fun.

1 Your Best Friend, the Subconscious

I confess, with some embarrassment, that my life in lockdown is comfortable and happy. I wake up at sunrise and take an hour long walk in the local park (conveniently, a wilderisation project), meeting on my way only foxes and birds – among them the resident grey heron, *Ardea cinerea*, an elegant and dignified bird.[1] After a light breakfast and coffee, I start doing mathematics, that is, I sit at my desk and look out of the window. Thinking is a hard job, and I soon become tired, move to a sofa

[1]Today I have seen a relatively rare atmospheric phenomenon: a full arch double rainbow, of very intense colour, at the very moment when the rising sun was crossing the horizon. Some would perhaps see that as a good omen and symbol of hope, or a tribute to the National Health Service (badly painted rainbows are everywhere all over the country), but I instead started to recover, in my head, a geometric explanation of the old conundrum: why, in the two arches of a double rainbow, colours change in opposite orders: from blue to red in the inner, and from red to blue in the outer arch? I leave this problem to the readers as an exercise.

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and take a nap. On waking up, I am refreshed, and return to mathematics – and more often than not I have some new ideas for my work; they came to me during my sleep. This cycle is repeated, with breaks for meals and tea. My wife uses meals for briefing me about Covid and other news, I myself do not follow current affairs.

Perhaps at this point I have to touch on one of the best kept secrets of mathematics:

mathematics is done in the subconscious.

A mathematician has to maintain good relations with his or her subconscious. The subconscious is not a properly domesticated beast, but it responds well to attention and kindness. It is like our rabbit, Cadbury the Netherland Dwarf (one of the wilder breeds of pet rabbits). When he is in good spirits, Cadbury grooms me, combing with his incisors the skin on my arm, apparently trying to relieve me of my (non-existent, I hope) fleas – this is a natural social behaviour of rabbits. While I doze, my subconscious combs the deepest recesses of my memory for morsels of mathematics which could be relevant to, or just somehow associated with the mathematics that I am trying to do in my conscious state. The subconscious is a wordless creature and brings its catch to the surface as a kind of uncertain, instantly disappearing visual image akin to a single frame inserted in a film reel (the "inverse vision" as described by William Thurston [10]). Then another miracle happens: someone or something else in my mind looks at the catch and says: "well, this is . . ." – and gives the name, usually an already well-known term of mathematical language.[2]

A few days ago, a crucial ingredient of a proof on which I was working was brought that way to the surface after hibernating in the depths four decades – completely forgotten and never touched by me. Now, when I write this story, I am able to recall the particular paper where I first encountered this concept, and the monograph which I consulted to learn more about it.[3] All that had happened in about 1976, when I was an undergraduate student. I never touched the stuff since then.

I reached this state of nirvana only at the end of May, when the heroic attempts to teach online had been paused for summer. In lockdown before that, I lived

[2] You may find more on that in my paper [4, Section 6.1].

[3] For the mathematician reader: the paper and the monograph were the seminal paper by Hall and Higman [9] and the classical book by Curtis and Reiner [7]. And the concept was the *enveloping algebra* of a representation. Hall and Higman revolutionised the finite group theory by observing that if L/K and M/L are sections in a finite group G for $1 \leqslant K \triangleleft L \triangleleft M \leqslant G$, with $K \triangleleft M$ and L/K being an elementary abelian group of order p^l for prime p, then the action of M/L on L/K by conjugation is a representation $M/L \to \mathrm{GL}_l(\mathbf{F}_p)$ and can be usefully studied by methods of representation theory. The images of elements from M/L generate a subring (called the enveloping algebra) in the matrix algebra $\mathrm{M}_{l\times l}(\mathbf{F}_p)$. If the representation is irreducible, this subring is the matrix algebra $\mathrm{M}_{m\times m}(\mathbf{F}_{p^n})$ for $mn = l$ [7, Lemma 70.5]. I made the following basic observation only now, in lockdown: the group G is much more complex than it looks at the first glance because the general linear groups $\mathrm{GL}_{m\times m}(\mathbf{F}_{p^n}) \subset \mathrm{M}_{m\times m}(\mathbf{F}_{p^n})$ naturally lives and *acts* inside G (in terminology of model theory, it is *interpretable* in G). Moreover, the groups $\mathrm{GL}_{m\times m}(\mathbf{F}_{p^n})$ are some of the best understood finite groups. This brought dramatic simplifications into the model theoretic problem I was working on.

comfortably, taught online quite productively, wrote papers, but did not achieve the wholeness of being that I experience now.

2 Mathematics Inherited

And now I turn to the issue indicated in the title of my notes:

mathematics discovered, invented,[4] *and inherited.*

My story is about mathematics which is neither discovered no invented, but inherited. Indeed I argue that

new mathematics can also be inherited.

Some years ago I made my paper [3] public where I described an example from my own work, a convoluted pre-history of a few simple but powerful mathematical ideas and the way they were inherited, or ignored, or re-discovered by new generations of mathematicians. That old paper was too technical, but I borrow from it a couple of softer passages.

My lockdown episode fits into the same pattern of inheritance: I recovered from my memory a few elementary mathematical concepts which I learned decades ago and have not used for 40+ years; they belonged to canonical stuff, occasionally taught in senior undergraduate courses, found in textbooks and therefore securely fossilised. This happened because I worked on sufficiently hard problems in model-theoretic algebra (one of them could be traced back to 1994) [6, Problem B.38, p. 365]. I realised that I needed to get some good understanding of how the traditional old stuff could be used in the new non-traditional environment.

And I suddenly got a rather frivolous idea. I decided to experiment with the principle that I formulated years ago, perhaps in my undergraduate years:

mathematics can be done with a matchstick on a moldy wall of a prison cell.

For many years, I maintained a list of rain day problems, something that I could do without access to the literature – I thought the list could be useful if I was confined to a bed in a hospital with just paper and a pencil to save me from boredom. I included one of these problems in my book [2, Section 11.2]: it is about development of Euclidean geometry from an alternative system of axioms – something that, I had a good reason to believe, could indeed be done with a matchstick on a prison cell wall. Now, in lockdown, I decided to turn my research project into a matchstick exercise – work on it from the first principles, without looking into any books or any external sources of information, and even not making notes on paper.

[4]This philosophical dilemma – discovered vs invented – has interesting practical implications: a mathematical formula can be patented in the USA, but not in the UK. For American lawmakers and lawyers, the formula is invented, for British – discovered.

To my joy, I almost instantly started to feel that I was developing a much deeper insight into the problem than I would otherwise have had, and that the emerging proof was, as mathematicians would say, the 'right' proof. It is now a cute little paper [5] (its wording is more formal than my original sketch). It was incredible fun.

3 Ernst Haeckel: Ontogeny Recapitulates Phylogeny

My lockdown experience was an illustration of Haeckel'principle as expressed in the title of this section.

In the past I was privileged to work with Israel Gelfand, one of the great mathematicians of the 20th century. He made a clear distinction between the two modes of work in mathematics expressed by Russian words '*prIdumyvanie*' and '*prOdumyvani*e', very similar and almost homophonic. The former means 'inventing,' the latter 'properly thinking through' and was used by Gelfand with the meaning

> *'thinking through starting from the origins, fundamentals, first principles.'*

Gelfand valued *prOdumyvanie* more than *prIdumyvanie*, he was convinced that *prOdumyvanie* yielded deeper results.

I was lucky that I followed Gelfand's advice and restricted myself to *prOdumyvanie* – this gave me a few days of happiness and an immense intellectual joy.

4 Inoculation Against Lockdown Blues

Finally, I have to explain what allows me to be happy under lockdown.

Mathematics is a proselytising cult, and by the age of 16 I swallowed its dogmata hook, line, and sinker, and became an unwavering convert. Myself and friends at FMSh, the Physics and Mathematics Boarding School at Novosibirsk University, knew that we were to become professional researchers in physics or mathematics, and, moreover, we knew that we had no other choice because this was the only way available to us to maintain some degree of intellectual freedom. At that time, in Soviet specialist schools like FMSh this was a commonplace sentiment – see [8].

During my first year at university another colour was added to this vision of the world: the explicit understanding (shared by my friends) that

> *mathematics was the best escape route from reality.*

At the next stage of my professional development, when I was a kind of a postdoc, I got hold of the Russian translation of the novel *Theophilus North* by Thornton Wilder [11]. I already knew earlier works by Wilder, and open the book with some anticipation.

Theophilus, the narrator and protagonist of the novel, tells about himself at the very beginning of the novel:

> At various times I had been afire with NINE LIFE AMBITIONS – not necessarily successive, sometimes concurrent, sometimes dropped and later revived, sometimes very lively but under a different form and only recognized, with astonishment, after the events which had invoked them from the submerged depths of consciousness.

He gives a curious list:

> a saint, an anthropologist among primitive peoples, the archaeologist, the detective, the actor, the magician, the lover, the rascal, – and a free man.

I immediately put the book aside and started my own list. The translator did not use the word ‘ambitions’ (perhaps because Soviet people were not supposed to have ambitions, this word had negative connotations); the one used could be translated back as ‘role’ or ‘field of activity’. This made my spontaneously produced list a bit more precise. Most entries are irrelevant now, but the last one matters: political exile. Still a young man, I suddenly discovered that I was prepared to face this fate.

Should I add anything else to claim that growing up in certain political environments is the best inoculation against lockdown blues later in life?

Acknowledgements This paper would not be written without help and support from my wife Anna who shares with me life and lockdown.

My work described here is a small fragment of a joint project with Ayşe Berkman.[5] Previously, we did most of our joint work in the Nesin Mathematics Village in Şirince, Turkey,[6] and we were planning to meet there during the Easter break and then again in summer. . .I also badly missed the planned in advance intensive work sessions with two my other co-authors, Adrien Deloro and Şükrü Yalçınkaya, also supposed to take place in the village. This is the price I have to pay for the joys of lockdown.

References

1. Berkman, A., Borovik, A.: Groups of finite Morley rank with a generically sharply multiply transitive action. J. Algebra **513**, 113–132 (2018). arXiv:1802.05222[math.GR].
2. Borovik, A.V.: Mathematics under the Microscope. Notes on Cognitive aspects of Mathematical Practice. Amer. Math. Soc., Providence, RI (2010).
3. Borovik, A.: Mathematics discovered, invented, and inherited. Selected Passages from Correspondence with Friends **1**, 13–28 (2013). ISSN 2054–7145. arXiv:1309.3073v2[math.HO].

[5] We are removing ‘sharpness’ from the assumptions of the main result of our paper [1].

[6] https://nesinkoyleri.org/en/nesin-villages/.

4. Borovik, A.: Mathematics for makers and mathematics for users. In: Humanizing Mathematics and its Philosophy: Essays Celebrating the 90th Birthday of Reuben Hersh (B. Sriraman ed.), Birkhauser, 309–327 (2017). https://doi.org/10.1007/978-3-319-61231-7$_$22.
5. Borovik, A.: Finite group actions on abelian groups of finite Morley rank. arXiv:2008.00604v1 [math.GR] (2020).
6. Borovik, A., Nesin, A.: Groups of Finite Morley Rank. Oxford University Press (1994).
7. Curtis, C.W., Irving R.: Representation Theory of Finite Groups and Associative Algebras. AMS Interscience Publishing, Providence, RI (1962).
8. Gerovitch, S.: "We teach them to be free". Specialized math schools and the cultivation of the Soviet technical intelligentsia. Kritika: Explorations in Russian and Eurasian History **20** no. 4, 717–754 (2019).
9. Hall, P., Higman, G.: On the p-length of p-soluble groups and reduction theorems for Burnside's problem, Proc. London Math. Soc. **6**, 1–42 (1956). https://doi.org/10.1112/plms/s3-6.1.1.
10. Thurston, B. P.: On proof and progress in mathematics, Bull. Amer. Math. Soc. **30**, 161–177 (1994).
11. Wilder, T.: Theophilus North. Harper and Row (1973).

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Exponentials and the Virus

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When we heard the reports of a novel virus emerging from China, it had become clear to many of us that it was growing exponentially and would soon spread across the globe. Initially called nCov-19 or novel coronavirus 2019, the virus was renamed SARS-CoV-2 when analysis showed that it was closely related to the coronavirus that led to the SARS epidemic of 2003.

"We" in this case refers to a group of doctors, scientists, and business leaders. Several members are epidemic experts having dedicated a significant portion of their lives to the tracking, prediction, and study of contagions. Others in the group have been analyzing exponential phenomena of many types including biological, digital and other trends that quickly increase in orders of magnitude.

All of us recognized that it has historically been very difficult to effectively communicate the magnitude of exponentially-driven trends. Let's consider a few such trends and then return to the virus.

Moore's law is an exponential trend that has now permeated our culture. In 1965, Gordon Moore noted that the number of transistors on a chip typically doubles every two years while the cost of the chip halves in the same period. He later revised the time period to every 18 months. It turns out that not only has Moore's law been fairly reliable since 1965, we can trace an exponential trend in computing from as early at 1910 (see Fig. 1).

Yet, despite the grand success of Moore's law, it has still been very challenging in the last 50 years to communicate the changes that would come about due to exponential phenomenon. There was still skepticism in the 1980s that computers would become household staples. Experts failed to appreciate that plunging costs

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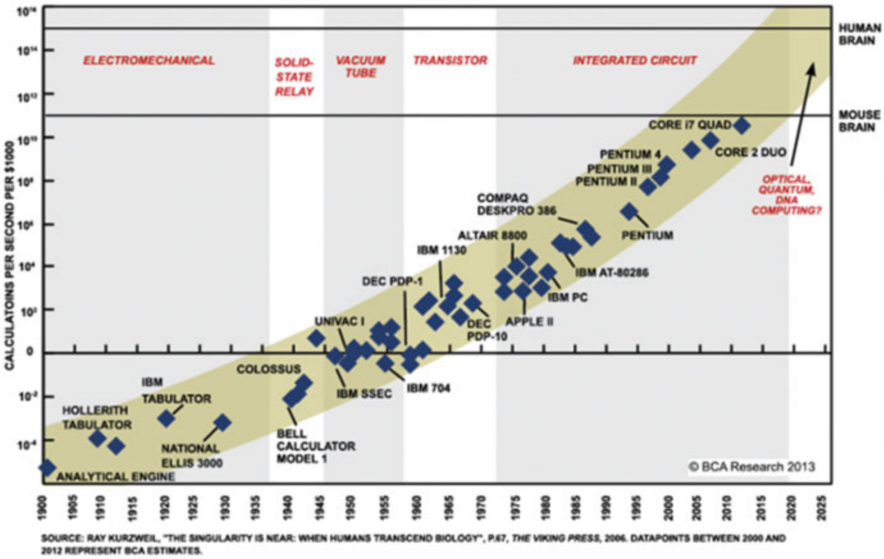

Fig. 1 Exponential trends in computing power. (*Source:* Kurzweil.net *with permission)*

and rising capabilities would make computing ubiquitous not only in the home but in mobile phones and in wristwatches.

Even today, many experts do not fully appreciate that inexpensive and powerful computing will continue to transform many industries. For example, a recently developed blood pressure cuff has its own chip and SIM card on board and has no need to be paired with a mobile phone. This biomarker device, and others like it with such computing capabilities, is enabling a new paradigm of healthcare.

The human mind has evolved to be more attuned to short-term perturbations such as the rustle in the leaves that could signal a tiger or other danger. Even as our species turned to agriculture, we understood the seasons that drive the planting and harvesting of our crops and which repeat each year; they may fluctuate a bit year to year, but that generally stay within a comprehensible range.

There is a wonderful fable from the 13th century about a king who promises a reward to a valiant subject. The subject asks for nothing more than a grain of rice on the first day and then a mere doubling of this each day for 64 days to correspond to the 64 squares on a chessboard. The king, upon hearing such a “modest” request is pleased and grants it with no objection. Naturally, such a request could never be fulfilled, since after just 32 doublings, the king would have to pay out 7+ billion grains of rice.

At 64 doublings the total comes to a staggering 18,446,744,073,709,551,615 grains of rice which is many times the total world production of all grains with modern technology, let alone the much smaller output of rice in the 13th century. This led Ray Kurzweil to refer to the “second half of the chessboard” phenomenon which is that once we get to about 32 doublings it becomes very apparent that the

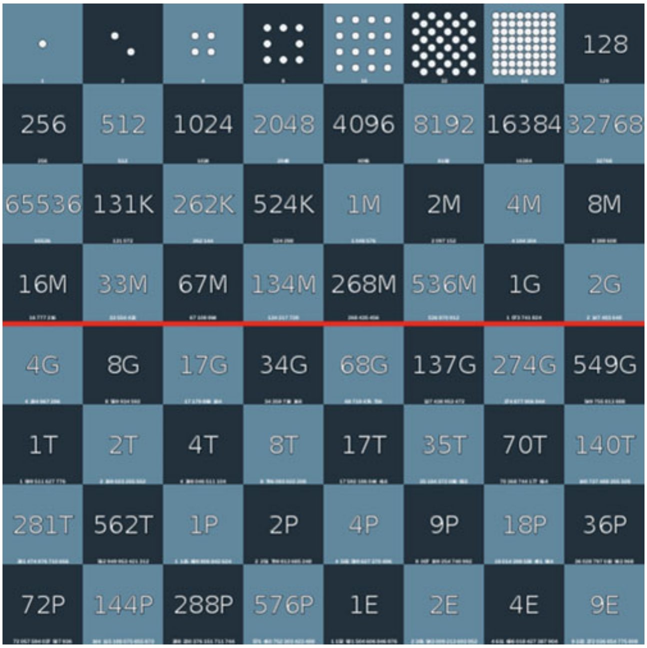

Fig. 2 Doubling a grain of rice each square of a chessboard. After 32 doublings we get to 4+ billion grains on the new square and a total of more than 7+ billion grains to that point. G = giga or billion (10^9); T = tera or trillion (10^{12}; P = peta or quadrillion (10^{15}) and E = exa or quintillion (10^{18}). (*Source:* Kurzweil.net *with permission)*

trend moves beyond any typical sense of human-centered trend prediction (see Fig. 2).

Storytellers have long appreciated that humans cannot wrap their heads around exponentials. Yet, despite knowing this, we continue to avoid training students in this way of thinking in the standard curricula, preferring instead to teach a form of mathematics devoid of excitement, application, and relevance. Mention is sometimes made of compound interest or rabbits breeding, but the implications are not followed through with substantive engagement. We have failed to equip students with the tools necessary to analyze and manage exponential trends.

Turning now to the pandemic, the mathematics of viral spread is straightforward. The index case, sometimes called patient zero, infects a certain number of people; these individuals in turn infect others around them, who in turn infect still others and so on. The average number of individuals that an infected person infects is called R-naught or R_0.

If R_0 is below 1 then the virus will die out and not spread in an epidemic-like fashion. If R_0 is equal to 1 then the virus will spread in a steady way, but not get out of control. If, however, R_0 is greater than 1 then the virus will spread exponentially if left unchecked. R_0, combined with the incubation period of the pathogen, leads to the doubling rate which, as we saw in the fable of the king, can quickly get out of hand.

We have known for centuries that physical distancing and related practices can significantly reduce the spread of a virus. Since the 1300s, Italy kept sailors on board ships for 40 days—*quaranta giorni*—before permitting them to disembark; this, of course, is the origin of the term quarantine.

During the Influenza pandemic of 1918, cities such as NY, Philadelphia, and St. Louis all eventually shut down schools, closed bars and churches, and banned large gatherings. Today, we call these practices non-pharmaceutical interventions (NPIs) as they are one of our only toolsets in the face of novel pathogens, particularly in the absence of treatments and vaccines. In 1918, each city implemented NPIs at different times. What we see in the historical data is that even small timing differences of weeks or even days made a big difference in the per capita mortality rate in these cities.

When our group began to engage community leaders with the projections of impending mortality of SARS-CoV-2 we were initially met with high degrees of skepticism. Even though we shared what had happened in 1918 and in other pandemics, there was a high degree of resistance to take "drastic" measures to close schools. Humans tend to look for compromises and half-measures so that they can continue life as usual even in the face of clearly changing circumstances. We now know that keeping the schools open would have been the drastic measure. To their great credit, the community leaders quickly understood the implications and promptly implemented many of the NPIs. There is no doubt that all this work together prevented many hospitalizations and deaths.

Now when students ask us how mathematics impacts the real world, we have a clear example of how embracing the mathematics of the exponential saved many, many lives. No impact can be greater.

Jack D. Hidary

The Consolation of Math in Plague Time

Barry Mazur

In reflecting about what I might do for the community, for my students, my family, myself, during these pandemic times, I found myself wrestling with preliminary thoughts, working them up into some notes that eventually took the form of an article.[1] When I submitted it to the Mathematical Intelligencer, it was suggested that I also submit an excerpt of it, or a short similar essay, to this volume. I'm very happy to do this!

I'm a mathematician devoted to rather theoretical issues. If I were an applied mathematician I am sure that I'd be delighted to be pressed into service: collecting, sorting, and classifying data. And formulating and calibrating models that help in interpreting what the data wants to tell us about what has happened in the past and what we can expect for the future.

But how can pure mathematicians be of help? Besides, of course, teaching Multivariable Calculus and Probability Theory to the future generation of epidemiologists and practitioners, and just homeschooling children or grandchildren and keeping contact with students; usually necessarily Zoom contact.

Well, we can just try to be avid students of the work of our applied colleagues—close listeners, and appreciaters. In a broader arena, we can look out for what we can do for the good of others. . . . but also: we could be looking in, for some mode of consolation.

As for "looking out," our government, our communities, our common humanity, our families—all need the closest of attention—and there is even welcome energy now—*now*— for the righting of long-lingering wrongs. The world faces a palimpsest of hundreds of thousands of personal misfortunes, tragedies some surely are; but

[1]**Math in the Time of Plague**.

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we also see the emergence, perhaps, of vibrant energy[2]. to meet challenges, and effect some kind of long-range change.

As for "looking in," we have those gems of constancy: mathematical thought, and mathematics per se to understand and appreciate. Mathematics seems to be blessed with eternal youth—the freshness of new conjectures, methods, results, overviews, analogies—and, most curiously, the freshness of the old ones too! Every advance seems to offer us a higher perch, from which we see more, and then have more new questions to excite our imagination. And yet the old questions never lose their allure. A mathematician friend of mine once said that every time he thinks about the Pythagorean Theorem he is enchanted anew.

And mathematical ideas don't necessarily decrescendo during adverse times.

In one of those—from today's perspective, ridiculously minor—personal bad times, I managed to prove some (equally minor) lemma. Made happy by realizing that I actually could work in such times, I cheerily called my lemma *my consolation prize*. Happily, in good times, there is such exhilaration in doing mathematics—and in bad times: consolation.

But to move from the ridiculous to the sublime, there is the *Annus Mirabilis:* the bubonic Great Plague year 1665-6 some months of which Isaac Newton spent in a countryside retreat, to escape the plague in Cambridge, and... to invent Calculus. He wrote:

> ... in those days I was in the prime of my age for invention and minded Mathematics and Philosophy more than at any time since.

I'm sure we all can cite grand examples of intense focus of intellectual energy during times of stress or hardship—from the composing of *Buchenwaldlied* by Hermann Leopoldi to Shakespeare's grand creations through a filigree of plagues.[3]

I was once very moved by an instance of *focus of mathematical energy* during a time of stress that I witnessed close up. I attribute the type of arresting *focus* I'm about to allude to—to the striking transcendental quality of mathematical ideas: transcendental, at the very least, in the naive sense that they endure; they transcend any aspect of our material surround.

I was relatively (in fact, actually) young, talking math with—and trying to understand the mathematics of—an older mathematician, when he suddenly seized up with an attack—an attack that later was seen to be caused by some internal bleeding.

I drove him to the emergency room of the hospital nearby (in Orsay, France) but we had to wait for quite a while—since that was the day when everyone drives home from summer vacation and the nearby highways were full of accidents.

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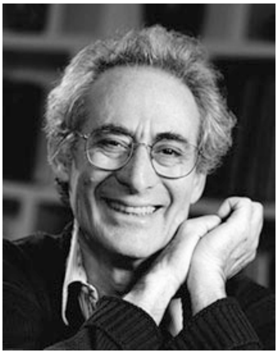
[2]E.g.: Black Lives Matter

[3]E.g., see Andrew Dickson's *Shakespeare in lockdown: did he write King Lear in plague quarantine?* (The Guardian; March 22 2020: https://www.theguardian.com/stage/2020/mar/22/shakespeare-in-lockdown-did-he-write-king-lear-in-plague-quarantine).

No matter how severe my older friend's illness was, and how much he needed immediate care, he ranked low in the triage of that emergency room because of those accidents. Consequently, we waited for an interminable—it seemed to me—time till he too could be wheeled in as a patient. During all that time he was—relaxedly it seemed to me—thoroughly engaged in our discussion, absorbed by some topological issue (I won't say what, exactly, it was, since (a) that's irrelevant and (b) you would think I rigged it if I said).

He was fully involved in these ideas, unfazed by where we were, and why—rather: he was transported by contemplating a certain mathematical analogy. But I—nervously—was quite otherwise, counting every new gurney that entered.

I said, some paragraphs ago, that "mathematical ideas endure; they transcend any aspect of our material surround." But that is too passive a claim. Mathematical ideas do better: they urge **us** to endure, to transcend. This is the real gift of Math in the time of Corona.



Barry Mazur (Photo taken by Jim Harrison)

Singularities at Home, Unofficial Observations

D.-C. Chang and B.-W. Schulze

A story about Albert Einstein deals with his desire for a working place in a remote lighthouse for himself and his ideas. The following remarks are devoted to aspects of communication in mathematical sciences with the outside world, sometimes under unusual circumstances. Various considerations are developed from the viewpoint "singularities alone at home". Singularities belong to the challenges in geometry and mathematical analysis and they may be rather "small" like the tip of a pencil and fit into a home office though applications to the real world are not necessarily situated in the hermitage of a researcher. We do not solve this contradiction here but recall remarkable episodes around personalities in singular situations, not only from the present exceptional crisis but also from the past.

1 Introduction

The "home office" as a hideaway, evoked by an aggressive wave of infection, has certainly a different intention than the above-mentioned idea of being undisturbed

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somewhere. There is hope that the results of systematic research in the medical sciences, often induced by new ideas and combined with hard practical work, will be easing the situation in the end. Just as optimism and positive thinking have vitalizing effects on one's general health, the scientific progress is being stimulated by a relaxed and humorous approach throughout all spaces of time, see also the setting of the novel [16]. In order to explain some effects it is helpful to remember that Euler's exponential function should be taken seriously—not only in economy—when some quantity is growing or decaying. Clearly, optimism is more justified in the present situation when we may observe exponential decay of infection. However, if we are consuming too much from a resource, for instance, the earth (mixed up with a piece of cheese), we disappear after a while (not the cheese), because we do not experience the inevitable end. In other words, exponential effects can be pleasant or unpleasant and we should not ignore mathematical rules in favor of wishful thinking.

Here we present a few impressions or opinions in connection with activities around mathematics and other arts.

2 Remarkable Personalities

The practical consequences of systematic scientific research may be so serious that we are inclined to believe the most visible personalities are the most unapproachable. A representative of the Max-Planck Society once remarked that "usual" people sometimes realize scientists as "higher beings," though they also have their unspectacular issues of daily live. When the second author of the present thoughts, labeled by "A2", was relatively young, during a visit in Göttingen he watched a scene at the door of the city hall where protestors threw tomatoes and eggs at Karl Friedrich von Weizsäcker, the Federal President of Germany at that time. Fortunately, the President was not hit, but A2 came to the improper conclusion that an indication of political success seems to be a welcome target for attacks of that kind. Such greetings are certainly not a common reaction among researchers, although the scientific judgment may find other ways, e.g., in terms of writing negative reports or of backbiting letters against competing specialists. Nevertheless, in very exceptional cases real crimes have happened, while a home office is relatively safe. Let us ignore here really dangerous incidents since those are absolutely out of the question, but soft conflicts may be amusing for those who are not involved. Moreover, the evolution of differences in scientific opinions, beginning with ignoring a new idea or claiming a priority, may affect long periods of history in a scientific discipline, in particular, in mathematics. Information in this direction is filling libraries. Not only classical mathematicians, including Georg Cantor, cf. also [13], became skeptical when they tried to find out whether a proposition is true. A standard insight in this connection is that people can never detect a lie if they always believe the liar. Let us cut out luxuries like "truth" in mathematics; the truth about the "truth" in this context could be disappointing. A simple exercise is asking the innocent researcher: "Can you decide, whether the assertion 'no rule without exception' is a true claim or whether it is just the conclusion of its opposite." Principles like that are too well-known as suitable inmates for being trapped in a home office since a majority of people would accept such truths anyway. Otherwise, a mysterious secret, hidden somewhere, may fascinate and give a home office worker the feeling of protecting something of value, including the satisfying hope that it is non-trivial. This can be a compensation for the suffered pain of not immediately sharing a result with friends. In any case, we may expect an excellent playground for schooling mathematically gifted students in "right opinions" belonging to the subject of "fake truth"; the winner is the one with the right answer.

Other fields of human culture prefer other notions of correct thinking. For instance, in the beginning of the past century, relatively close to other catastrophes, quantum theory was first dominated by the matrix mechanics of W. Heisenberg and his uncertainty relation. Valid interpretations remained uncertain in many respects during this period. The explanation by E. Schrödinger of discrete energy levels of electrons in atoms via discrete eigenvalues of a differential operator came later, and a colleague asked Schrödinger why he preferred a description of a discrete phenomenon by a "continuous" model of analysis, a differential operator, over the main stream dominated by the matrix formalism of W. Heisenberg. His answer (in rough translation) in a letter was: "It is new for me that the truth of a scientific opinion is determined by a majority of votes." The freedom of research and teaching in academic life has been discussed by scientists at many occasions, see, e.g., [14]. Schrödinger explained in one of his original works that his motivation has been operator-valued analogies of classical rules of Hamiltonian mechanics. A2 found this letter in the State Library in Leipzig when he was student in mathematics there. A few years later (1967), W. Heisenberg gave a talk at the University of Leipzig; the Schrödinger operator was not explicitly mentioned in his lecture; he focused on discrete phenomena and compared appearances in 4-dimensional space-time with a mush of discrete quantities, similar to a cream of wheat, including a hypothetically quantized "existence" of the time, so far often understood as a continuum; also the Planck-time played a role. The story around this visit has been mentioned also in [15]. A nilpotent Lie group in Harmonic Analysis became known under the name Heisenberg group, see results and cooperation [1, 2] of the first author A1.

3 Episodes During Scientific Meetings

Singularities of a geometric configuration may be locations where smoothness of the underlying geometry is violated. For instance, cones or wedges contain such locations, where conical singularities are single points of dimension zero while edges may be of higher dimension. Conical as well as other types of singularities easily fit into any home office. Although these may be so tiny, they are far from being boring or ridiculous. Information is also given in [7, 12]. The above mentioned quantization of classical Hamiltonians gave rise to Fourier integral operators, introduced and widely promoted and applied by L. Hörmander with spectacular success, cf. [6]. Special cases are pseudo-differential operators, including their coordinate invariance under suitable smooth bijective substitutions of variables in phase space, cf. also properties around Egorov's theorem. Symbols in this context are analogies of Hamiltonian functions in phase space or on a conic Lagrangian manifold, while the operators themselves are obtained by some quantization. For instance, propagation and reflection of singularities belonged to the consequences. Such a concept has been developed in several variants, e.g., by Maslov's school with his canonical operator and the study of asymptotic solutions in parameter-dependent situations. However, when the underlying configuration is not smooth and has geometric singularities, say, of conical, edge-, or higher corner type, coordinate invariance in the former sense is violated from the very beginning, compared to the "smooth building" of structures. Traditional mathematical expectations of dealing with phenomena in neighborhoods of such singularities become much more complex, cf. [9]. Impressions about what can happen with symbols and quantizations close to singularities associated with (non-complete) Riemannian metrics, describing, e.g., polyhedral configurations, are described in [4], see also [10, 11] for the case of conical and edge singularities and references therein. The strata of the respective singular space generate hierarchies of symbols. The "strata conditions" generalize the concept of boundary conditions when we talk about manifolds with (smooth) boundaries, see the index theorem of Boutet de Monvel, [3], and we observe hierarchies of quantizations. More details in corner situations are given in [5]. The outcry of justified excitement from the world of all those who contributed ideas and efforts over years to make the research program around singular analysis successful and popular is absolutely understandable. But the home office is too small, in any respect, to house all the basic achievements together with the representatives of this particular world of remarkable phenomena and special cases of singular geometries. Therefore, the authors resisted the temptation to write more than a few pages and instead create an online outline and illustrate a more specific approach.

Cones may be regarded as the first step in generating higher corner spaces by repeatedly forming cones and wedges of links produced in the preceding step. However, there remains the task of formulating corresponding pseudo-differential techniques closer to those from the smooth case. Geometric singularities give rise to new analytic challenges, e.g., to characterizing solutions to partial differential equations with asymptotes close to singularities. In this framework it is adequate to study quantizations in terms of the Mellin transform and to apply meromorphic symbols. Other phenomena belonging to the recent development are connected with analysis on infinite straight singular cones, see also [5, 8] and the references therein.

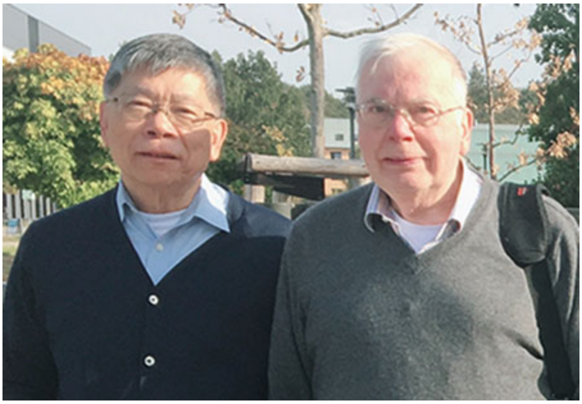
Let us close this part of the consideration with an episode from the seminar activities at the University of Potsdam during the years between 1989 and 2000. During that time V. Kondratiev visited Potsdam and gave several talks on analysis near conical singularities in the seminar, originally published in [12], using handwritten slides, projected to a screen. He left us a real conical point as a souvenir on the screen itself, immortalized on the tissue for several years.

Individual experiences among colleagues in mathematics and difficulties for the scientific life itself are stated in an authentic report by G. Wildenhain [17]. His reflections of mathematics in isolation, caused by ideological localization, has been an absurd horror-generalization of the present home office situation, except for those who find it cozy and acceptable. Events of similar kind are also the topic of [15].

4 Other Remarks and Observations

The “home office” existence of creative activities of sciences and arts in past centuries has not always been seen as an exotic realization of a condition of creativity and deep feeling of spiritual content of our life. In computer-dominated communication a different style of working is becoming more and more normal. Complaining about the permanent flood of information in increasing speed and complexity may sound old-fashioned and dowdy. However, in the past century scientists like Max Planck developed a series of philosophical thin booklets containing some advises for getting particularly broad scientific progress. During his studies in Leipzig, A2 read such scripts in the years before 1989. Those were illegal during that time, but a religious fellow student managed to get access to them. It was dangerous to have such material at home, since A2’s “home office” was a room rented in a private apartment. This was one of the reasons that this treasure got lost after some years. But A2 never forgot the really important viewpoints for research, applied in this case to a kind of home office situation behind the iron curtain. The idea was that substantial scientific progress on a broad front in a new and deep field may be successful by the efforts of different individuals where every participant produces a thin but very deep hole. Such a strategy may be compared with an approach for removing a huge massive rock on a street of some unaccessible material by penetrating it at first and at the end succeeding. This story is not included in [15], but among other propositions of Max Planck in these booklets. In a sense we may enjoy some home office feeling like a partition of unity where every single part is locally doing necessary jobs with global results, modulo some smoothing remainders, and cooperating at the right moment. This is especially true in the modern day, where online teaching and virtual conferences make collaboration efforts more clear. In any case, localized entities seem to be like an unusual “aggregation state” of research aspects. By studying events or intentions of other representatives of the community we may find more fascinating geometric and algebraic structures and motivations from index theory, see, in particular, [9].

Additionally, musical live of the past presents remarkable episodes. However, they are not always authentic. Sometimes an episode is “more true” than the real truth. In any case, let us remember an answer of the baroque composer Ch. W. Gluck from the times of the Sun King Louis XIV of France. About his beautiful and impressive works of music he has been asked by an admirer: “What kind of musics do you like most”? The answer was quite direct:“Of course, my own.” Questions in a similar direction may refer to a feeling of taste or smell of some specific approach or a discipline in mathematics. Individual expectations or accidents help to navigate a future unknown field, or to observe and count differences in parallel work. For instance, it is known that G. Leibniz spent about 15 years trying to find a manageable formalism of infinitesimal calculus. Later on this was systematically promoted and employed by L. Euler. The word “infinitesimal” sounds rather local, predestinated for a home office situation. More recent achievements in [6] with wave front sets are even micro-local. So the trajectory through the history has many glorious stages. It is not possible here to appreciate the influence of all researchers even during shorter periods, but I. Newton with his calculus and his pioneering works brought the insight on the nature of gravity to a new level. The far reaching consequences have been really global, though the corresponding apple itself from the famous legend was certainly small enough for a home office. A. Einstein on the other hand came to a well-known geometric interpretation of gravity, and discoveries in cosmology are still going on. In the meantime, uncomfortable locations like black holes enjoy the status of fascinating singularities, and it is the merit of enthusiastic researchers to encode their secrets. Let us finally note, that a certain concept of the philosophy of G. Leibniz was the so-called “Monadenlehre” (“Lehre” in the sense of teaching a knowledge field). At the end he came to the conclusion to be himself a “Monade”, an entity of some complex meaning, but probably not a trivial conical singularity in geometric terms. When the mathematician B. Fedosov [9] from Moscow spent some years as a guest at the Max-Planck research group in Potsdam, he made the remark: “Analysis with singularities is teaching people conservativeness”.



D.-C. Chang and B.-W. Schulze

References

1. C. Berenstein, D.-C. Chang and J. Tie, Laguerre Calculus on the Heisenberg Group, AMS/IP series in Advanced Mathematics, **22**, International Press, Cambridge, Massachusetts, 2001.
2. O. Calin, D.-C. Chang and P. Greiner, Geometric analysis on the Heisenberg group and its generalizations, AMS/IP series in Advanced Mathematics, **40**, International Press, Cambridge, Massachusetts, 2007.
3. L. Boutet de Monvel, *Boundary problems for pseudo-differential operators*, Acta Math. **126** (1971), 11–51.

4. D.-C. Chang, B.-W. Schulze, *Ellipticity on spaces with higher singularities*, CHINA SCIENCE Math. **60**, 11 (2017), 2053–2067.
5. D.-C. Chang, S. Khalil and B.-W. Schulze, *Analysis on regular corner spaces*, to appear in Journal Geometric Analysis, (2020).
6. L. Hörmander, *Fourier integral operators I*, Acta Mathematica. **127** (1971), 79–183.
7. Yu. Egorov, V. Kondratiev, and B.-W. Schulze. On the completeness of root functions of elliptic boundary problems in a domain with conical points at the boundary. Comptes Rendus Acad.Sc. Paris, Ser. I 334 (2002), 649–654.
8. Ju. V. Egorov and B.-W. Schulze, *Pseudo-differential operators, singularities, applications*, Oper. Theory: Adv. Appl. **93**, Birkhäuser Verlag, Basel, 1997.
9. B. Fedosov, *Deformation Quantization and Index Theory*, Mathematical Topics, Vol 9, Akademie-Verlag, Berlin, 1996.
10. J.B. Gil, B.-W. Schulze, and J. Seiler, *Holomorphic operator-valued symbols for edge-degenerate pseudo-differential operators*, Differential Equations, Asymptotic Analysis and Mathematical Physics, eds. M. Demuth et al. Mathematical Research, Vol. **100**, Akademie Verlag (1997), pp. 113–137.
11. J.B. Gil, B.-W. Schulze, and J. Seiler, *Cone pseudodifferential operators in the edge symbolic calculus*, Osaka J. Math. **37** (2000), 221–260.
12. V.A. Kondratyev, *Boundary value problems for elliptic equations in domains with conical points*, Trudy Mosk. Mat. Obshch. **16**, (1967), 209–292.
13. H. Meschkowski, Probleme des Unendlichen, Vieweg und Sohn GmbH, Braunschweig 1967.
14. J. Osel, Vorsingen für's Forschergeld. Wie Platzhirsche der Wissenschaft schaden. Spiegel Online, 20. 12. 2011.
15. B.-W. Schulze, Erlebnisse an Grenzen - Grenzerlebnisse mit der Mathematik. Birkhäuser, Springer Basel 2013.
16. B.-W. Schulze, Die Perle des Universums. Munich 2020, www.thalia.de ebook. Amazon publ. ISBN 9798676128128.
17. G. Wildenhain, With disgust and outrage. Comment on "Life in and after GDR. Interviews with Horst Sachs and Gustav Burosch. Mitt.Dtsch.Math. Ver. 3, 23–33, 1997.

An Essay in the Time of Corona

Carlos Kubrusly

1 Is There?

One hundred and twenty two days ago, I was dining with a friend in a empty restaurant: a piece of grilled rump and chips with cream cheese on pita bread. So simple a meal, and yet so far way now. The world was already at the dawn of a new age of perplexity. After taking my friend to her place, I headed to mine into lockdown. This was on a Sunday, March 15 of the virulent year of 2020; it was just four months before but it seems four long decades ago. This is a COVID 19 pandemic scenario.

At that time I was thinking on revisiting tensor products, a subject on which I had written some papers a dozen years before in connection with transferring properties from a pair of Hilbert-space operators to their tensor product. It was quite a fashionable subject at the time. So perhaps this Corona social distance enforcement and consequent home imprisonment might be a chance to give it a try. As an aftermath came the next few lines addressed to a wide audience.

There is a conventional protocol to build up a tensor product of Hilbert spaces where a reasonable crossnorm comes nicely and naturally from the factors' inner products. There is, however, an intriguing point: why is it, and where does it come from? As George Pólya [3] taught us: *is there an easier question to ask*? So, are there other ways to construct those tensor product spaces? Are they somehow equivalent, thus boiling down to the same thing? Yes, indeed. Here is the yellow brick road.

Mathematics Subjects Classification (2010): 46M05, 47A80, 15A63; Keywords Algebraic tensor product

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2 Is This a Possible Start?

$$\begin{array}{ccc} \mathcal{X}\times\mathcal{Y} & \xrightarrow{\ \phi\ } & \mathcal{Z} \\ & {\scriptstyle\theta}\searrow & \uparrow{\scriptstyle\Phi} \\ & & \mathcal{T} \end{array}$$

These are all rooted on firm grounds, no abstract nonsense required: $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ and $\mathcal{T}$ are linear spaces (all over the same field), and a pair $(\mathcal{T}, \theta)$ is a tensor product of $\mathcal{X}$ and $\mathcal{Y}$ if (a) θ is a bilinear map whose range spans $\mathcal{T}$, and (b) for every bilinear map ϕ into any $\mathcal{Z}$ there is a linear transformation Φ for which the above diagram commutes. These are the axioms of tensor product whose definition can be rewritten as "a tensor product space $\mathcal{T}$ of $\mathcal{X}$ and $\mathcal{Y}$ has the universal property with respect to a bilinear map θ on the Cartesian product $\mathcal{X} \times \mathcal{Y}$" , and so θ factors every bilinear map ϕ through $\mathcal{T}$ thus "linearising ϕ by Φ". Enough is enough.

It is a rather clean start and it seems impressive to me how much follows from these axioms. Basically all properties of concrete tensor products (yes, they do exist) follow from such an abstract formulation. First of all if $(\mathcal{T}, \theta)$ and $(\mathcal{T}', \theta')$ are tensor products of $\mathcal{X}$ and $\mathcal{Y}$, then they are essentially (i.e., up to isomorphism) the same, and so it is usual to write $\mathcal{X} \otimes \mathcal{Y}$ for "the" tensor product space $\mathcal{T}$ of $\mathcal{X}$ and $\mathcal{Y}$. Also, the linear space $b[\mathcal{X} \times \mathcal{Y}, \mathcal{Z}]$ of all bilinear maps ϕ from the Cartesian product $\mathcal{X} \times \mathcal{Y}$ to an arbitrary linear space $\mathcal{Z}$ is essentially equal (i.e., isomorphic) to the linear space $\mathcal{L}[\mathcal{X} \otimes \mathcal{Y}, \mathcal{Z}]$ of all linear transformations Φ from the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ to $\mathcal{Z}$, thus showing how and why tensor products linearise bilinear maps; and many more properties follow from the axioms including, of course, the dimension identity: $\dim \mathcal{X} \otimes \mathcal{Y} = \dim \mathcal{X} \cdot \dim \mathcal{Y}$.

3 Plus ça Change, Plus c'est la même Chose

It was said above that concrete tensor products do exist. In fact, the most common interpretations of tensor products are the so-called quotient space and linear maps of bilinear maps realisations. The former states that a quotient space $\mathcal{S}/\mathcal{M}$ can be made into a tensor product space of $\mathcal{X}$ and $\mathcal{Y}$, where $\mathcal{S}$ is the free linear space generated by the Cartesian product $\mathcal{X} \times \mathcal{Y}$ of linear spaces $\mathcal{X}$ and $\mathcal{Y}$ (which is not a linear space itself) and $\mathcal{M}$ is an appropriate linear manifold of $\mathcal{S}$. The latter in its simplest form goes as follows.

Suppose all linear spaces are complex (i.e., over the field $\mathbb{C}$) and let ψ: $\mathcal{X} \times \mathcal{Y} \to \mathbb{C}$ be an arbitrary bilinear map (in this case it is called a bilinear form). Associated to each pair $(x, y) \in \mathcal{X} \times \mathcal{Y}$ consider a functional $x \otimes y$ defined by $(x \otimes y)(\psi) = \psi(x, y)$ for every bilinear form ψ. This is called a single tensor which is itself a linear functional (i.e., a linear form) on the linear space of bilinear forms, that is, $x \otimes y$: $b[\mathcal{X} \times \mathcal{Y}, \mathbb{C}] \to \mathbb{C}$ is linear; a linear map of bilinear maps. Consider the linear space $\mathcal{T}$ spanned by the collection of all single tensors, and define a bilinear map $\theta\colon \mathcal{X} \times \mathcal{Y} \to \mathcal{T}$ by setting $\theta(x,y) = x \otimes y$ for every $(x,y) \in \mathcal{X} \times \mathcal{Y}$. This supplies a tensor product $(\mathcal{T}, \theta)$ of $\mathcal{X}$ and $\mathcal{Y}$. Now constrain the bilinear forms ψ to products of linear forms, say, $\psi(x,\, y) = \mu(x) \cdot \nu(y)$. Then to each $(x,y) \in \mathcal{X} \times \mathcal{Y}$ associate a single tensor given by $(x \otimes y)(\mu,\, \nu) = (x \otimes y)(\psi) = \psi(x,\, y) = \mu(x) \cdot \nu(y)$ for every pair of linear forms $\mu\colon \mathcal{X} \to \mathbb{C}$ and $\nu\colon \mathcal{Y} \to \mathbb{C}$. A tensor product is still obtained in this particular case. Thus write $\mathcal{X} \otimes \mathcal{Y}$ for the tensor product space $\mathcal{T}$ of $\mathcal{X}$ and $\mathcal{Y}$ spanned by the collection of all new single tensors as above.

As for the good old Hilbert-space case equip $\mathcal{X}$ and $\mathcal{Y}$ with inner products $\langle\, \cdot\, ;\, \cdot\, \rangle_{\mathcal{X}}$ and $\langle\, \cdot\, ;\, \cdot\, \rangle_{\mathcal{Y}}$. In this context, linear forms μ on $\mathcal{X}$ and ν on $\mathcal{Y}$ are identified with vectors u in $\mathcal{X}$ and v in $\mathcal{Y}$ by $\mu(\cdot) = \langle\, \cdot\, ; u \rangle_{\mathcal{X}}$ for some $u \in \mathcal{X}$ and $\nu(\cdot) = \langle\, \cdot\, ; v \rangle_{\mathcal{Y}}$ for some $v \in \mathcal{Y}$ according to the Riesz Representation Theorem in Hilbert space. Hence

$$(x \otimes y)(u,v) = \langle x\, ; u \rangle_{\mathcal{X}} \langle y\, ; v \rangle_{\mathcal{Y}} \quad \text{for every} \quad u \in \mathcal{X} \;\text{ and }\; v \in \mathcal{Y},$$

which is the conventional protocol to build a single tensor $x \otimes y$ in a Hilbert-space setting, and therefore is the initial step to build up a tensor product space $\mathcal{X} \otimes \mathcal{Y}$ of Hilbert spaces $\mathcal{X}$ and $\mathcal{Y}$. The more you change it the more it is the same thing.

4 There Is

The previous section title was originally coined by the French writer Jean-Baptiste Alphonse Karr [2]. But here it was stolen from Somerset Maugham's novel *Then and Now* [4]. As a matter of fact, the title is his entire Chapter 1. The plot is about a short period in the life of Niccolò Machiavelli in sixteen century Italy which had been written towards the end of Maugham's career. However, most of Maugham's novels have been written around the 1920s by the time of the American Pandemic [1] (or have they called it "little flu" as well? — then and now). If Machiavelli himself was alive and kicking during this time of Corona, then perhaps we would be safer than we are now on this side of the pond. Wanderings in the time of Corona cannot be confined to mathematics and virtual lectures (otherwise one most certainly would not be writing this but tied up in a Renaissance lunatic asylum). So rereading old books has become part of a new normal. Since a huge and sunny beach is just around the corner, perhaps rereading old books can share space with a contemplated new revival. I do not know that I will avoid a stroll by seaside before long.

References

1. N. Bristow, *American Pandemic*, Oxford University Press, Oxford, 2017.
2. A. Karr, *Plus ça Change, Plus c'est la Mème Chose*: *Notes de Voyage*. (Date de l'edition originale: 1871), Hachette Livre-BNF, Paris, 2013.
3. G. Polya, *How to Solve it*, 2nd ed, Doubleday Anchor Books, New York, 1957.
4. W. Somerset Maugham, *Then and Now*, William Heinemann, London, 1946.

Carlos Kubrusly

Truth by Close Proximity

Christian Constanda

At the end of the spring 2020 semester, when teaching was concluded online, I had an unusual conversation with a student via Microsoft Teams. I will try to reproduce it as accurately as I can, using the direct dialog style to eliminate redundant verbosity. Since we, mathematicians, aim to make things as simple as possible, I will abbreviate myself to P (for professor) and my interlocutor to S (for student). Here is the gist of the discussion.

S: There has been some controversy lately about mathematics, its nature, origin, and interpretation, and I would be interested to have your views on the subject.

P: What controversy? I see no controversy where mathematics is concerned. Can you be more specific?

S: For example, I read that $2 + 2$ should not always be taken to be equal to 4. It may, perhaps, be equal to 5?

P: It may, if we ban the use of the number 4, counting the positive integers as 1, 2, 3, 5, 6, ... They do that with the thirteenth floor in some hotels. It's not as if there is no thirteenth floor—they just call it the fourteenth. It's stupid, and no educated person should condone this practice.

S: But can we not find a different designation for the answer of $2 + 2$ without having to delete the number 4?

P: We certainly can. If we counted in base 4, then we would have $2 + 2 = 10$, pronounced 'one-zero' and not 'ten'.

S: In class, you told us that mathematics is the combination of three fundamental ingredients...

P: Yes. Numbers, logic, and the power of abstraction. I did say that, and I stand by it.

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S: You gave me a glimpse of the flexibility of mathematics when it comes to labelling numbers. Can we not also have some degree of flexibility where logic is concerned?

P: Logic is defined as a system of principles governing correct and reliable inference, a particular method of reasoning or argumentation that can be applied to any branch of knowledge. Flexibility? In mathematics? You mean, applying a different set of rules than those we have always accepted to prove or disprove conjectures?

S: Yes, in a certain sense. You just said that logic is a "particular method of reasoning". If this 'particular' method is replaced by another 'particular' one that is, well, complete and free of self-contradiction, shouldn't we accept its conclusions, even though they may appear to differ from what we hold to be established truth?

P: I cannot say unless I am given the opportunity to examine such a different system. Do you happen to have one in mind?

S: I do, and it's very simple to explain.

P: Let's hear it, then.

S: In the current grading scheme, a final average of 90 is awarded an A, is it not?

P: It is.

S: But what would you do if someone had an average of 89.5? Would you not be thinking that perhaps this was due to a possible slight inaccuracy in the allocation of points to various parts of the solution, or to your mood on the day as affected by environmental or other subjective factors? Would it not seem to you a bit inequitable to give an A to someone who scored 90 and a B to someone who scored 89.5 over six quizzes, three class tests, and a final exam? Would your conscience not feel a pang of guilt if you were to base your decision on such a minute difference? Would you not be inclined, in the interest of fairness and justice, to extend the A also to the 89.5 candidate?

P: Depending on the overall circumstances, yes, I might.

S: Very well. We have now established that 89.5 is A-standard. Then, applying the same logic again, would you not conclude that 89 is also A-standard?

P: I see where you are going. You have just indirectly enunciated a theorem that I would call 'Truth by Close Proximity', which, for the non-mathematical mind, could be summarized as "If a is close enough to b, then a is equal to b."

S: Sounds about right.

P: Unfortunately, there are three essential things that are wrong with your alternative logic. First, it is based on feelings and emotions, not on cold and indisputable factual reality. Second, what exactly is meant by 'close enough'? What precise, specific number would quantify this detail? And third—and most important and dangerous aspect of it—your theorem is open to symmetric application.

S: I don't understand...

P: What is your final score in my course?

S: 75.

P: So I gave you a C, but you are cleverly trying to angle for an A. Let me ask you this: what grade would an average of 59 attract?

S: An F, I guess.

P: Correct. Using your alternative logic, would you not then agree that 59.5 should also carry an F?

S: But...

P: And then, would not a 60 have the same fate? You see, the same theorem you used to plead for an A for your 75 average, can be applied to give you an F for that same score.

S: In that case, perhaps a B...

P: Now, another one of those factors that go into deciding your grade is that I am the one who applies the existing rules here. My advice to you is that you should quit while you are ahead. Take the well-deserved C and concentrate on your next project. Logic diversity has no place in mathematics. Anyone who thinks otherwise is either uneducated, or has ulterior motives, not always honorable.

I wonder how this student's career will evolve. He is certainly intelligent, has originality of thought, and is focused on his objective, whether worthy of the effort or not, which makes me think that some day he might end up in a government position.

If you would like to read more stories like this, check out my Springer book *Dude, Can You Count?*

Christian Constanda

A Teacher and an Administrator in the Time of Corona

D. C. Struppa

On March 16th, we got the stay-at-home order from the Emperor of California. As President of Chapman University this threw my entire life into a spin: all of a sudden I had the responsibility to convert the university from, well, the usual way we run a university, to a remote operation. Classes and administrative systems went remote, students had to leave their residence halls, and finally parents, students, faculty started to call frantically. On top of that we had to plan for a financial shortfall that ended up being \$13.5M just for the spring (and we are now facing a loss of \$120M for this incoming academic year). And so, on that fateful day, I sent a message to the entire community (faculty, parents, students, trustees, news organizations, busybodies everywhere) and declared that Chapman was going remote.

Not good. Not good at all. Until a few days before closing down, I had received thousands of emails from students and parents claiming that I was risking innocent lives (*you have blood on your hands* being the kindest of the comments I was reading). As soon as we went remote, those same people who had compared me to Rasputin switched into a different gear and we were now asking to give tuition back (*if I wanted an online education I would have gone to the University of Phoenix* the most common remark). Great! And then the immediate accusation of having waited too long, as well as the dual accusation of having rushed into the decision.

To top it all off, I had my own two classes to deal with. I was teaching an honors class entitled *Three Infinities* in which I was working with my friend and colleague Marco Panza (professor of philosophy in Paris, but currently a Presidential Fellow at Chapman) to show our non-mathematical students how mathematicians deal with the idea of infinity, and to illustrate this process with three main examples: projective

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geometry, calculus, and set theory. I was also teaching another, less philosophical, class on my own, on mathematical methods for the physical sciences: the usual, fairly traditional class, where juniors and seniors are exposed to a wealth of mathematical methods ranging from the Laplace Transform, to the basics of Calculus of Variations.

The infinity class was the easiest one... we just went to zoom, and we managed to continue the conversation with our students. It wasn't as much fun because my friend Marco and I love to fight in our joint classes. I am a mathematician, and he is a philosopher. I know math, he thinks he knows math (he actually does, but I would never acknowledge this in his presence). He knows philosophy, I think I know philosophy. We have been friends for 40 years, and we both have a temper, and being both Italians, we are loud, and we fight on absolutely everything. He is one of those guys that disagree with you even when you agree with them. And the students love that. They love to see me angry at his obstinacy, and to see him angry at my lack of subtlety. Through these completely unscripted battles, students learn several things that I believe are very important. They learn that people can disagree, even harshly, without stopping respecting and loving each other. They learn that mathematics is not foreign to intellectual disputes about its meaning, and that such disputes add richness and texture to our oh-so-beautiful discipline. This conflict, this expression of love through animosity, was difficult to replicate via zoom. And the students were struggling to follow our complicated arguments on the small screen of their computer. I was zooming from my home office, and Marco from his home office... not the same thing as being in the same room. So, we resolved to write a few notes for the class, with the immediate and modest goal of offering an additional support to our students. The outcome... we now have 250 pages of notes, that are becoming a book. And if you happen to read the book, you will immediately see why I am irritated at Marco (whose second chapter is awfully complicated) and why he is irritated at me (since I apparently trivialize important concepts... or so he says).

Fortunately I was alone in my other class, and I had nobody to argue with. I could teach what I wanted, and how I wanted. But I soon realized that teaching Laplace transforms and the Euler-Lagrange equation via zoom was truly a complex challenge. And so, after discussing it with my students, I went to a hybrid format. I decided to set up a small whiteboard in my home (remember, we could not even go to our offices, because the Emperor, I mean the Governor, had forbidden us to go anywhere, in order to supposedly tame the virus). Armed with the whiteboard and my iPhone, I decided I was going to tape short videos for my students, and then use them to give them additional support through this strange period. To add complexity to the situation, I should confess I am a luddite. Without my two young daughters Arianna and Athena, I am completely unable to even use my iPhone correctly, and one of my fondest memories is to look at my beautiful girls smiling at me with an air of desperation when I ask them how to do something with the settings of the damn phone: I absolutely hate this contraption. So the challenge was of course the fact that I had no idea of how to shoot a video with my iPhone, how to post it on Canvas (or whatever is the name of our system), or to use another very mysterious thing that my daughter calls Dropbox. So, I had to solicit the help of Athena, my 12 year old, to

The author with his daughters, Athena (left) and Arianna (right)

edit my videos. The price to pay was $10 per video, plus she imposed her request to have her introduce some of the videos herself... what a disaster!

I waddled through this experience though, and I ended up enjoying it quite a bit. I tried to make my videos short (no more than twenty minutes each) and to convey through them my enthusiasm for the topic. I also learned new things. As I was trying to find a good series of examples for the basic ideas of calculus of variations, I bumped into the absolutely beautiful proof that Johann Bernoulli gave in 1696–97 to show that the brachistochrone was the cycloid. As I learned, Bernoulli had offered different proofs of this fact, but the one that I found absolutely stunning was the one in which he uses Fermat's principle that states that the path between two points taken by a beam of light is the one that takes the least time. Bernoulli then imagined that the medium between the two points has variable density, and uses this, together with the law of refraction, to derive, in a few very simple steps, the equation for the brachistochrone. Wow! What a magnificent piece of work. I was so excited about this, that I shared with my friends on campus, and the (somewhat long) video that I made (accessible as an Extra Source Material to this essay) gave the students both the background (Fermat and the law of refraction) as well as the solution to the problem. Not only is the proof incredibly elegant, but it is one of the finest examples of a proof of a mathematical statement achieved through physics, something that has now stimulated me and another friend of mine, to look in a different way to some problems we have been working on for a while (if this concept fascinates you, I would recommend reading Mark Levi's book *The Mathematical Mechanic*).

Despite my initial concerns, this new way of teaching has met with great enthusiasm from my students (if the course evaluations are any proof, I got remarkably high marks for these two courses, with only one student expressing discomfort at the use of videos), and I have become such a convert that last week, in a marathon session lasting a day and a half, I have already recorded videos for the entirety of my forthcoming class in ordinary differential equations. This time, however, the taping has taken place in our newly renovated high technology classes (here go another few millions. . .) with much higher quality video and audio. Corona has not been (and continues not to be) a great thing. It has killed people, it has devastated the economy, and it has made our jobs harder and different. But there is, like in all things, a silver lining. Out of corona I now have a new book (which I would have never written otherwise), and a new appreciation for the use of technology. Who knows, maybe next time I need to adjust the brightness on my iPhone I won't even need to call my daughters.

Ups and Downs

Robert C. Penner

I am a mathematician by training, whose early work in topology, geometry and dynamics has found applications in high energy physics and theoretical biology. I have held the René Thom Chair in Mathematical Biology at the IHÉS since 2014, after having been a frequent visiting professor there for decades. Since June, 2019, I have been studying viral glycoproteins, leading to the two papers *Backbone free energy estimator applied to viral glycoproteins* and *Conserved high free energy sites in human coronavirus spike glycoprotein backbones*, both in the *Journal of Computational Biology*. In the first, I propose a geometric method to predict promising targets for antiviral drugs or vaccines across all viruses, and the sequel applies these methods specifically to human coronaviruses, thus pushing forward the current efforts to fight SARS CoV-2, the virus causing COVID-19. These works are surveyed in a *Scientific American* article from May 19, 2020. The IHÉS asked me to recount how a pure mathematician has found his way to work on virology, and here is that story.

My first paper on RNA was published in 1992, co-authored with my close friend and onetime colleague Mike Waterman, sometimes called the "father of computational biology." We would celebrate (or bemoan) the beginning of each academic year at USC with a deep-sea fishing trip, for it is late summer when the yellowtail amberjack run in the warm waters off southern California. Waiting for something to bite, he mentioned his recent work, which I immediately recognized as a bastardized version of Poincaré duality. This led to our first paper on spaces of RNA secondary structures, which was well received but had no major impact until much later. But this set the ball in motion, and he invited me, over the next decades, to any seminar he thought might be accessible and of interest to me. Some years later, we ran a private meeting at USC on macromolecules funded by the bio-philanthropist Peter

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Preuss, and among the star-studded attendees was Alexeii Finkelstein, a leading world authority on protein, who plays crucial subsequent roles. We instantly became friends. His book entitled “Protein Physics: a course of lectures,” written with his teacher Oleg Ptitsyn, is a masterpiece, and I devoured it.

Macromolecules—specifically, RNA and protein—were my gateway to biology, a separate and comprehensible piece of a dauntingly enormous puzzle. Macromolecules, after all, are essentially one-dimensional objects that interact along sites, just as are the strings of high-energy physics. And I immediately saw ways to extrapolate up 25 or so orders of magnitude, from the Planck scale to the Angstrom scale, the basic combinatorics of my earlier work in string theory. Once in a seminar at Caltech, the eminent physicist John Schwarz laughed out loud at my remark, because one of his great insights decades earlier was the same but different: strings were originally a model for protons whose combinatorics he had scaled down twenty-odd orders of magnitude to strings with exactly the same remark about invariance of combinatorics under rescaling. Down and up, up and down.

After nearly 25 years at USC in the early 2000s, I undertook a move to Aarhus, Denmark. Once my friend and colleague there, Joergen Andersen, and I were making dinner, and he asked if I had any crazy ideas for applied Teichmüller theory. I offered up two: one on color quantization, and the other on the topology of proteins, the latter of which I had already hatched after the Preuss seminar. This evolved into our first paper on protein topology and later on protein geometry, basically for us the natural transition from the Z/2-graph connections of fatgraphs to the SO(3)-graph connections we finally studied with a large team in Aarhus, including multiple academic departments, from molecular biology to biophysics to physics to nanotechnology and mathematics. It was actually during a visit of Alexeii Finkelstein to Aarhus, his last moments there sitting in the coffee lounge with us, that the passage to SO(3)-graph connections came to light, extending ideas developed earlier by Joergen and me. Alexeii and I apparently have a habit of making progress in the last seconds of our visits together. . .as will happen again.

This turned into a multi-year project leading to a kind of spectacular result. Proteins are basically—and over-simplistically—a concatenation of peptide groups, small units each comprised of 6 atoms, forced to lie in a plane owing to quantum effects. Each such plane admits a canonical orientation that derives from the chemistry and contains a specified vector in the direction of the peptide bond it contains. *Voilá:* a peptide group gives a positively oriented orthonormal three-frame, so any ordered pair of such gives a well-defined rotation of three-dimensional space, or in other words, an element of the Lie group SO(3) (Figure 1). We took an unbiased and high-quality subset of the Protein Data Bank (PDB), the repository of all known three-dimensional protein structures, and computed the rotations of all the hydrogen bonds between peptide groups within it and found, quite remarkably, that Nature employs only a third of the volume of SO(3). Moreover, within that third, the data clusters into thirty well-defined regions, which reproduced, refined and extended the known classification of such hydrogen bonds. The results were sufficiently striking that the paper appeared in the prestigious journal Nature-Communications, a non-trivial feat coming, as it did, from outsiders to the field.

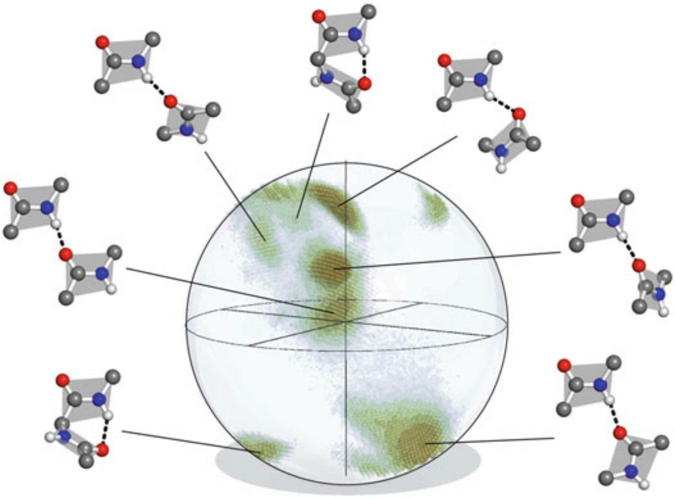

Fig. 1 Depicted is the density of backbone hydrogen bonds plotted in SO(3), and selected peptide group rotations are indicated.

Things sat there for about five years. I continued working in math/physics and on RNA, as this database of protein geometry just sat quietly in repose. I wanted to move from Aarhus because, as it turned out, I was not so good at socialism and grew tired of paying 108 percent world marginal tax on my Danish income. No kidding!

Having visited the IHÉS on and off for decades, I jumped at the chance to call it my part-time and now full-time academic home, not the least of which would be the chance to interact with Misha Gromov, who had been a critical sounding board for me by email for years. We have both spent decades studying biology and attending seminars, and Paris is a treasure trove of biological talent, just as it is for mathematics or physics. Discovering on arrival that I held a chair in *mathematical biology* and with my understanding only of macromolecules, the first several quarterly visits to the IHÉS were spent reading and reading, thousands of pages of biology texts and then research papers more targeted to my evolving interests. I would often go to Paris for evening walks following biology seminars at the Institut Curie.

Five or so years later, again enters Alexeii Finkelstein in mid-2019, since Misha and I had invited our common friend to spend a few weeks with us at the IHÉS. My own intentions were the selfish pursuit of trying to figure out what to do next with the protein clusters, and we spent several weeks without conclusion speaking of this among other things.

First thing in the mornings in France, I always start with a small regimen of exercises and calisthenics while watching the American PBS news from the previous evening. On one such morning during Alexeii's visit, there happened to be a science segment with Anthony Fauci from NIH talking about the freshly-stated goal of finding a universal vaccine target for influenza, something about sexy new visualization methods and a remark about some protein or other, which a little online homework identified as hemagglutinin. I had one tool at my disposal, one stick with which to poke this protein, namely, I could run the method from Denmark and see which clusters occurred among its hydrogen bonds between peptide groups. Here I can only say that there was a lucky accident, for one of the hydrogen bonds was incredibly rare: among the 1166165 bonds in the database, influenza hemagglutinin exhibited one bond from the cluster called B5e, the second-least populous with only 295 examples. This jumped off the page and showed how incredibly rare was this hydrogen bond in the universe of all hydrogen bonds between peptide groups in the whole PDB.

I showed Alexeii and Misha, and we discussed other aspects of this fascinating protein hemagglutinin. But it was not until the very last seconds of Alexeii's visit, when he came to say goodbye—just like in Aarhus six or seven years before—that we at once said: the bond is so rare that if we can target it with a drug or vaccine, then such a drug or vaccine is unlikely to have side-effects! It was a shared eureka moment—less momentous than it seemed at the time I suppose—but nevertheless a good insight that brought to the forefront using the protein database of clusters to find vaccines. The train had left the station, as Alexeii left for Italy.

The first months of exploration were confused. I had only the clusters, so membership in a small one like B5e was obviously remarkable. I knew from the outset that there could be outliers in the bigger clusters which were equally so, but I had no sensible way to compare them. I nevertheless undertook awkwardly studying whole collections of viral glycoproteins with the same result: that B5e and a couple of the other small clusters typically occurred. A pattern was already emerging. Also, my first impression of remarkable hydrogen bonds, or exotic as I came to call them, was that they pinpointed places on the viral glycoprotein of extreme geometry, places that stuck out a lot and most especially stuck in a lot. This was not unreasonable since after all it had been geometry that pinpointed them. It was a fun if misguided enterprise, virus after virus, finding an exotic site and feeling a rush of *gotcha!* each time, like when you finally swat an annoying fly.

I was compiling a list of these exotic sites and planned a paper with a detailed analysis of influenza and a supplementary table of viral targets. Alexeii and I were back and forth online daily with him now back home in Puschino and a fellow named Sergiy Garbuzinskiy from his lab helping me with the analysis. Misha and I were in extended discussions on this every day. A joint paper by Alexeii and me was envisioned and even written entitled "Universal Influenza and Dengue Fever Targets."

In the course of compiling the table to squash all viruses I could find on the PDB—though I was still learning which were the correct proteins and knew little, as one more example, I studied Rift Valley Fever Virus, RVFV, and found a signal stronger than ever. It was B5e again all right, but there was another measure—something we had called "stress" in an abandoned paper with the Danish group—which measured how rare was the given bond in its cluster. There was a hydrogen bond in RVFV more exotic by this measure than any I had seen before. A quick look online uncovered that there was an expert on RVFV right there in Paris, a fellow Pablo Guardado-Calvo at the Institut Pasteur, and I boldly wrote to him explaining my feeble understanding of things at the time and describing the exotic site I had discovered for RVFV. I was thrilled that he answered immediately even though he was at that moment on summer holiday, I suspect surprised that a mathematician had somehow targeted the RVFV fusion peptide with geometry. He made several excellent suggestions in response to my emails, as I worried about pestering him, ruining his holiday and poisoning our relationship. We made plans to meet upon his return.

Pablo came and spent the day at the IHÉS with us. It was fantastic for Misha and me, learning so much so quickly. And for Pablo, I think there was the curiosity about seeing what was this fabled place, the IHÉS. When Pablo left, Misha and I were positively struck with how great was this young man, how much he knew and we could learn from him. This was first of several visits of Pablo to the IHÉS and mine to Pasteur. We have become friends, and I owe him huge gratitude for all he has taught me.

Likewise with Alexeii and Sergiy, my learning curve was steep and fun. By now, I understood that the abandoned Danish notion of stress gave a measure of the free energy using the Pohl–Finkelstein formalism that Alexeii and co-workers had first explained. I was so committed to the idea of clusters, however, and there still was lacking any sensible way to compare across clusters. Misha and I worked hard on this, how to sensibly combine Boltzmann distributions.

It was Sergiy who discovered that the site I had found for influenza was well known, called the fusion pocket. There was even a sticky antibody described in the literature by a fellow Jimmy Kwang and company out of Singapore, and the antibody gave 100 percent protection against infection in mice. I wrote to Kwang and his collaborators to ask why there had been no follow up, but they never responded. Pablo later explained that mice are not a good model for humans, and moreover the gurus of influenza in the states probably felt that other sites were more promising. This more or less killed the first paper since my universal site was the known fusion pocket, but it also gave a proof-of-concept for whatever it was I was finding with my still primitive methods.

I understood the basics of the Boltzmann distribution but had never really computed with it. So I turned to my colleague Thibault Damour, who works on gravitational waves and who indulged me to listen and explain things. He had me probe the clusters, only to find that the distribution of hydrogen bonds within them failed quite spectacularly to resemble a normal distribution. He taught me further details of Boltzmann distributions as I still struggled to figure out how to combine or compare them. It was a frustrating period.

With a eureka one morning, I awoke and saw that after all these years of living with the data in clusters, having learned which were large or small, other properties too, and some of their cartography in SO(3), that they were entirely immaterial to the current circumstance. Indeed with the Danish group, we had computed a density on SO(3) itself, one big and beautiful density, no need to combine anything, just apply the Pohl–Finkelstein quasi Boltzmann Ansatz to the whole density! Thibault surely helped me come this understanding, and it was revolutionary enough that it took some convincing before Misha bought into it.

So now I was in business to compute and compute. It was great! I finally could look at the distribution of free energy across the entire database from PDB and plotted it. In fact, another dear friend for many decades, Greg McShane, a geometer who now enjoyed computer and stat studies of all sorts, had come to Paris from Grenoble to visit so that we could see The Cure at the Rock en Seine concert. He dove in and wrote codes for me that were crucial for the ongoing further analysis.

Having read and studied many texts and papers on viral glycoproteins, including supervision from Pablo on which PDB files to study, I was off and running. By now, I understood that high free energy targeted unstable sites, not geometrically significant ones, though the unstable fusion peptides are typically hidden from the immune system in caverns or troughs, consistent with the initial findings. I also had several examples of the different fusion mechanisms clear in my mind, and Pablo and I had a number of great meetings that cemented my understanding.

So the chemistry and math were perfect, and the biology absolutely clear. I had come to anthropomorphize viruses and could empathize with their search for the love of their lives. It became clear through this understanding why they would capitalize on hydrogen bonds in this pursuit just as I would do. But the physics was still messed up: I could not resolve the overall energy distribution with the known energies of various motifs such as alpha helices. This was terribly troubling. If this was right, then everything must be perfect, and the physics just did not make sense. As Misha said at one point: if the physics is wrong, it is like having a beautiful meal before you but useless silverware.

There was still another conceptual hurdle to overcome, and Alexeii was frustrated with my inability to understand: the free energy is NOT that of the hydrogen bond itself, but rather that of the protein detail which it stabilizes. It is a subtle distinction and took me forever to comprehend. With this final realization, all fell into place. The artificial manipulations I was trying in order to resolve the physics fell away, and all was perfect even giving an internal consistency check to the whole theory: the extreme energies in the distribution were exactly where they should be, just below the bounds of protein stability.

This led to the first paper in the *Journal of Computational Biology*. The second paper applies these tools to the known coronaviruses diseases which afflict human beings, and in particular provides several sites of interest for vaccine/drug/test targets for the SARS-CoV-2 virus that causes COVID-19 (Figure 2). Because of the lockdown in France and the consequent lack of interruptions, it was 2 months of 12–15 hours per day of work that led quickly to the sequel on human coronaviruses.

It is exciting to feel involved. I also am fortunate to be so passionate about a project that I have been able to pursue while in lockdown and distracted from the evident fears we all share. I obviously hope that my sites will be useful for taming COVID-19, but only experiments can finally measure their utility. Quite rightly, a biologist should only care if they do so.

The method has presumptive further applications throughout biology, of course to other viruses, but also in principle for example, to neurodegenerative diseases like Alzheimers, which involve inappropriate protein folding, and to cancer metastasis, which relies on cell motility—really in any context where proteins change their backbone geometry using hydrogen bonds.

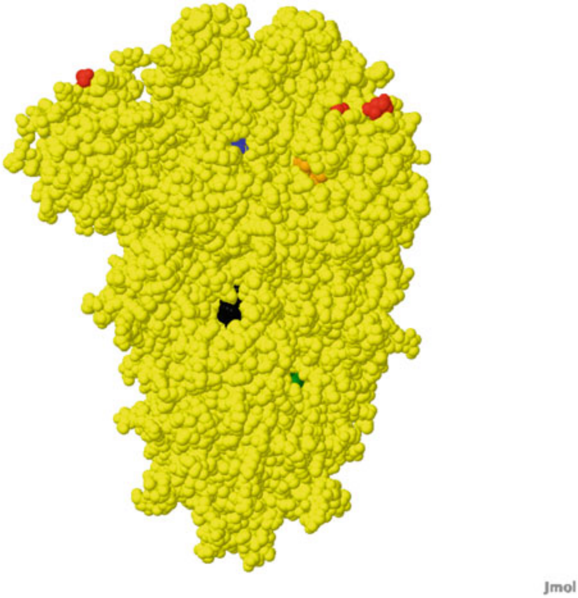

Fig. 2 The spike glycoprotein of SARS CoV-2, where the conserved sites of high free energy providing promising antiviral targets are highlighted in color.

With these many other potential applications of my methods across biology, I hope to recruit others to employ this new tool. Most good ideas do not work, but this seems to be one that may.

Robert C. Penner

Off Line from No Line to On Line

Yves Nievergelt

1 No Line

By sheer luck, my institution's quarterly academic schedule, coupled with adjustments by its administration, provides a whole month to move instruction out of classrooms and into the aether, but is with a late surprise from the State's Governor.

Near the end of winter quarter, on Thursday 5 March 2020, the University cancels instruction and final examinations in classrooms and offices for the last week of the quarter, which would be 16–20 March. Again by sheer luck, that evening comes an unrelated invitation to drop in at the Instructional Technology unit. So the next day I drop in and get a quick but very useful introduction to a system called *Canvas*.

On Wednesday 11 March, the University mandates instruction to occur fully on line in the spring quarter, with its start delayed by one week, to 6 April. I have never taught anything on line. Fortunately, a guardian angel comes to the rescue. My colleague Becky Sommers is the director of a lab that routinely holds meetings on line through something called *Zoom*, and invites us who have not taught on line to participate in such meetings as an introduction to *Zoom*. There we discover how to go to break-out rooms, mute participants, and share screens.

Now being confident that I can be a remote talking head, on Friday 20 March, after turning in my grades, I take the opportunity to get some exercise over the weekend, camping and cross-country skiing on the last skiable snows up in the North Cascades.

Back at the office, on Monday 23 March, a message from the University relays that the Governor's Stay Home Stay Healthy orders take effect in 48 hours. That means teaching *on line but not from the office*. I don't have access to the Internet of anything and spotty cellular coverage at home. I am now asking friends about their

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experience with various carriers and frantically searching for things called *modem* and *hot spot*. I'm investigating everything from land lines and underwater cables to satellites and smoke signals, eventually settling for and ordering a connection from one carrier. From another carrier, I order a cell phone service with a hot spot and no limits on data, because I'm not sure yet from where I'll be broadcasting.

On the last day of freedom, Wednesday 25 March, from the office I take home the desktop computer and as many books and notes that will fit in a car. The next day, a technician comes in and connects the house to the Internet of everything.

At home a week later, I email my classes to announce a test run for the next day. On Thursday 2 April at 10 a.m. the connection proves sufficiently fast and reliable!

2 Off Line

Thanks to the U.S. Postal Service, there is no need to leave the house for six weeks, after which time the Governor's orders ease up a bit. Splitting wood to heat the house and cook on the wood stove helps fight the cold and rainy weather. Fortunately, my neighbors get me into gardening to get fresh air. Yet, as put starkly by Camilla van der Waerden, "There we discovered how difficult it is to produce food" [3, p. 317].

3 On Line

At 8 a.m. on Monday 6 April, I start the first live Zoominar with a class in basic multivariable calculus, followed at 11 a.m. by a class in basic complex analysis. As the academic quarter progresses, the University offers optional Webinars about teaching on line, which repeatedly emphasize making courses relevant to students. Hence, first and second derivative tests for minima find applications to fitting curves to data, beginning with linear regression, and culminating with non-linear regression, fitting parameters to sizes of populations consisting of individuals who are susceptible, ill, or removed by the coronavirus. Figure 1 shows results computed with data available on 19 April. Meanwhile, in complex analysis, since the right-hand side of the system in Figure 1 consists of polynomials, power series in several variables enter a proof that the solution is an analytic function of time and all the parameters [1, Ch. VII]. This conclusion, though perhaps not its proof, may have guided Kermack & McKendrick in the derivation of their approximation of the solution [5, p. 713]. More sophisticated models may also use splines as additive terms to model seasonal variability, specifically, ten cubic basis splines with seven internal knots and no intercepts [6, Fig. 1, p. 861, and Supplementary Materials, p. 33]. The terminology of splines and their knots comes from flexible rulers tied to weights at a few places on drafting tables in engineering [4]. Mathematical splines also help in the computation of optimal orbits and flight trajectories. "Splines

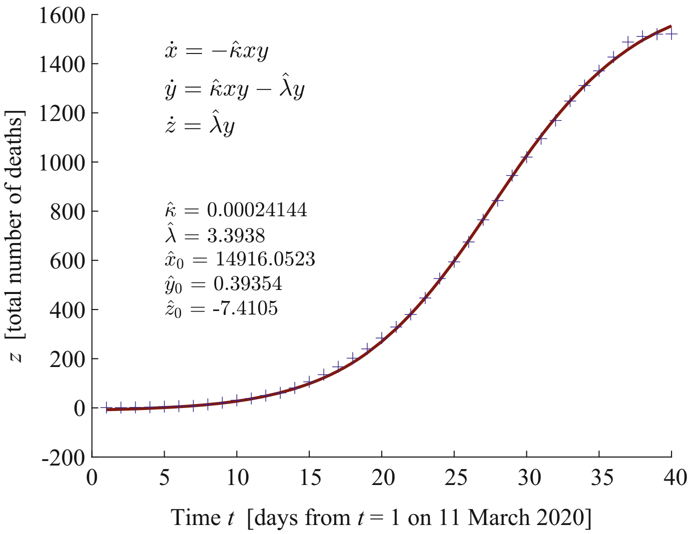

Fig. 1 Curve fitted to data from *Public Health Agency of Sweden* [7].

demonstrate some of the good things that happen when you get the math right!" (Tom Grandine, cited in [2]).

The moral of the story teaches us and students to pay attention to mathematics from fields outside our interests. As a case in point, the authors of reference [6] incorporated in their model of the spread of the coronavirus a mathematical tool, splines, that was perfected at Citroën, Renault, General Motors (GM) and The Boeing Company to design automobiles, aircraft, helicopters, rockets, and spacecraft [2].

Yves Nievergelt (Photo taken by Larry Conboy, Eastern Washington University's photographer.)

References

1. Cartan, H.: Elementary Theory of Analytic Functions of One or Several Complex Variables. Dover, New York, NY (1995).
2. Davis, P.: B-splines and geometric design. SIAM News **29**(5) (1996).
3. Dold-Samplonius, Y.: Interview with Bartel Leendert van der Waerden. Notices of the American Mathematical Society **44**(3), 313–320 (1997).
4. Grandine, T.A.: The extensive use of splines at Boeing. SIAM News **38**(4) (2005).
5. Kermack, W.O., McKendrick, A.G.: A contribution to the mathematical theory of epidemics. Proceedings of the Royal Society A **115**(772), 700–721 (1927).
6. Kissler, S.M., Tedijanto, C., Goldstein, E., Grad, Y.H., Lipsitch, M.: Projecting the transmission dynamics of SARS-CoV-2 through the postpandemic period. Science **368**(6493), 860–868 (2020).
7. Public Health Agency of Sweden–Official statistics at arcgis: https://www.arcgis.com/sharing/rest/content/items/b5e7488e117749c19881cce45db13f7e/data (2020).

Dodging a Bullet

Loring W. Tu

This article is an expanded version of a letter to the Editor of the *Notices of the American Mathematical Society* (AMS). In it, I give an inside look at a cancelled meeting of the AMS. Someday, when one looks back at mathematics in this unusual period in history, it may be useful to have a few contemporaneous first-hand accounts.

I was the local organizer of the Sectional Meeting of the AMS at Tufts University scheduled for March 21–22, 2020. The impetus for this meeting, at least for my department, was to showcase the university and the Department of Mathematics, for although we have made great strides in the last thirty years, Tufts is still not widely known. Even with the help of my colleagues and the competent staff at Tufts Conference and Event Services, I must have put in at least one hundred hours over the past two years into organizing this meeting, from personally inspecting every room and its audiovisual equipment to raising funds, to lining up presenters for invited addresses and moderators for contributed paper sessions, to finding alumni speakers for a non-academic career panel, to inviting the provost to give welcoming remarks, to arranging for food and coffee, and to planning a reception for 250, as well as acting as the liaison to the AMS, and a host of other responsibilities.

Meanwhile, in early 2020, the coronavirus was raging in China and Italy, but there were only a few cases in the United States, mostly concentrated in Washington and California. In late February and early March, the federal authorities were assuring the country that the situation was under control. No one was wearing masks. In fact, it was recommended that other than medical personnel, people should not wear masks because they would not offer protection if improperly worn.

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On March 2, there were only two confirmed cases in Massachusetts, both infected from traveling abroad, and no deaths. Later it emerged that a number of people who had attended a Biogen leadership conference in Boston on February 27 had tested positive, but the extent of the infection was still not known. Back then, officials and scientists in Massachusetts were also minimizing the danger, reiterating for several days that the risk posed by the virus was low and claiming that there was no evidence of community spread in the state. All indications were that the AMS meeting would go on as scheduled.

A total of 415 participants had preregistered. Counting those who would register on site, we expected five hundred participants. It would be the biggest event the Tufts Mathematics Department had ever hosted. On March 6, I placed an order for the reception on the Saturday of the meeting. In the experience of the AMS, around half of the participants—250 people—would attend it. Of course I hoped the participants would remember Tufts for the mathematics learned and professional connections made, but I was also aiming for what makes any conference truly memorable: an unforgettable reception! To this end, I had raised enough money from deans, the mathematics department, and other faculty to make it lavish. Instead of pizzas, cheese and crackers, and chips and dips, there would be various kinds of fresh sushi, shrimp, and other hot and cold hors-d'oeuvres served on platters by roving waiters.

Around that time, the AMS was about to mail seventy boxes of books to Tufts for its book display during the meeting and we at Tufts made arrangements to receive and store them. By Monday, March 9, all preparations for the meeting were in place and my biggest worry then was whether the totes of coffee we ordered would cool off too fast and whether there was a way to have continuous service of hot coffee at the meeting. We were ready for the conference. However, since the coronavirus situation was fast developing, there was always a hint of uncertainty.

On March 10, when everyday life still felt normal, the Tufts administration, ever cautious, cancelled the meeting. This was even before the AMS had cancelled anything, including the Virginia meeting scheduled for the coming weekend, and way before the lockdown ordered by the state. Initially, I felt a rush of disappointment—two years of work down the drain. The participants, most of whom had by then bought plane tickets and booked hotels, would have to scramble. The cancellation caused massive inconvenience for hundreds of participants. Later that day, because the number of cases in Massachusetts had spiked from forty-one a day before to ninety-two, Gov. Baker declared a state of emergency. Of the ninety-two, a whopping seventy-seven were from the Biogen conference. On March 13, President Trump declared a national emergency.

As time went on, the number of cases in Massachusetts exploded, from 2 on March 2 to 10,000 a month later and 100,000 two months later with 6,700 deaths. It was then I realized that Tufts had dodged a bullet. If the meeting had been scheduled for two weeks earlier, on March 7–8, it would certainly have taken place. With five hundred people from the four corners of the earth converging at Tufts (except participants from China and Italy, whom Tufts had already barred based on CDC travel warnings), the meeting would certainly have become a superspreader event, worse than the Biogen meeting, which had only 175 participants. Many people would have become ill, and instead of being showcased, Tufts would live in infamy. We were lucky that the meeting was scheduled for when it was, with just enough warning of a disaster to come for it to get cancelled.

Loring W. Tu

The Co-Video Mathematician

Steven J. Miller

Many of my experiences during the pandemic and the response are probably similar to most. Some days I can't see the differences from how it was before unless I look closely. I go for a hike with my family or watch my son play ball or my daughter do gymnastics. It feels normal till I look more closely and see how spread out we are, how little conversation there is in the stands. Other times what we have lost is clear, especially on days when my kids should be in school and are instead home with me. I fight the same depression that we all face.

My family is fortunate. My wife and I are both professors (she in marketing at UMass Amherst, I in mathematics at Williams), so we could shift our jobs online. For me it was particularly easy, since for the past six years I've been recording all my lectures and posting them on YouTube (see https://web.williams.edu/Mathematics/sjmiller/public_html/). I was teaching two sections of multivariable calculus when we moved online; it took a minute to write my students and say that 2020 is now 2018, and for the rest of the semester we'll watch the videos from the last time I taught the class; my students even sent out a meme of me enjoying drinks on a beach in the new normal! While this "freed" up time, there was a long line of demands on it too. The three biggest were teaching, my kids, and the regional school committee where I serve as vice-chair and chair of the now very busy education sub-committee. Though there are connections between them, I'll break into three sections.

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1 Teaching

As I had been online for years, it was natural to reach out and help colleagues to transition. In addition to posting my videos, I had also opened up my classes to students at other schools, and taught online classes with colleagues over the summer (Introduction to Data Science, with Ella Foster-Molina, Jingchen (Monika) Hu, Moataz Khalifa and Natalia Toporikova, with tremendous administrative support by Liz Evans). Thus I could share first hand experiences of things that worked and things that most assuredly did not! In addition to helping friends at universities, I also did a lot with elementary and secondary school teachers. I had been giving continuing education lectures for years with the Teachers as Scholars program (https://www.teachersasscholars.org/), as well as designing modules and running on-line classes with the Value of Computational Thinking across Grade Levels team (with Midge Cozzens and many, many others, supported by DIMACS and the NSF: https://www.comap.com/Free/VCTAL/index.html). I moved this year's course online, and spent many weeks helping others do the same for theirs. There were two parts to this. Obviously there are conversations with the instructors; these were pretty straightforward and fast. What took time is that many of the participants had little-to-no experience using online technology, so for several weeks I had drop in sessions where people across Massachusetts would pop in and we would practice using zoom and other programs.

Not surprisingly, it was natural to use these technologies to move conferences online. I was an organizer for Mel Nathanson's Combinatorial and Additive Number Theory in June (http://www.theoryofnumbers.com/cant/), and the 19th International Fibonacci Conference in July (http://www.pmf.unsa.ba/fibonacci19/index.php). Costs were way down (on the order of a few hundred dollars at most for certain zoom licenses), as were barriers to attending. We had a lot more participants than before as people could just zoom in, and many people who would not have been able to travel to speak could do so now. Of course, creating a schedule is no longer an easy task as we juggle the different time zones. For one conference we did everything on local time; for the other we broke the world into three time zones (China Standard, European Central, US Pacific), and each day chose times that worked for two of the three; while daylight issues make one world-wide time zone impractical, as an organizer it is nice to ponder! Further, it was trivial to record the talks, and both conferences will have webpages with talks and slides. While it was a solution, we still have work to do. The important social aspect was greatly diminished; I found it worked better when I was with people I already knew rather than people I was meeting for the first time. I miss going out for lunch with friends I haven't seen for awhile, and young mathematician I've just met. I've made a lot of great connections, on both sides of the table, through conferences over the years. I have, however, elected to cancel a special session I was organizing for an AMS Sectional; I am zoomed out. I feel a bit bad, as it does provide an opportunity for early career mathematicians, but seeing the strong programs from the other sessions I've rationalized my decision.

As I serve on a regional school committee, I had been thinking about the effects the coronavirus might have on our kids' education for months before we entered lockdown. My early call for a discussion of contingency plans was (somewhat but not entirely understandably) not met with any sense of urgency or need, as it just seemed inconceivable that it would be more than a temporary inconvenience. The response was very different in the mathematics community. I've been the director of the Williams College SMALL REU (Research Experiences for Undergraduates) program for almost a decade now; each year several professors guide 25–45 students in original research. We are one of many programs across the country, and when I raised my fears on the REU directors list we immediately started discussions about how the summer might unfold. Yunus Zeytuncu took point and organized multiple zoom meetings for directors where we brainstormed possibilities. We started with some hope that perhaps we could have some programs in-person, maybe multiple programs would re-locate to the same part of the country, but well before the summer began we knew we would be entirely remote. I found these conversations exceptionally helpful as we started planning for various contingencies. Further, many of us had different start dates, so the programs which began first were able to give advice on how to build a community and what challenges the remote transition caused. In the end I believe our program worked. While I do not know my students as well as I did in previous years, I do know them, and they showed that they can do good, original, collaborative work in the most challenging of situations.

It wasn't just conferences that moved, of course. Overall, we in math were fortunate that we could mostly shift our programs online (I was looking forward to my colleague Eyvi Palsson and his graduate student Sean Sovine joining me in person in Williamstown, and had to settle with them being remote like our students); many of my colleagues in the lab sciences were not so lucky, and several of their students asked me if I had anything for them. While I found some work for a few, I knew there were many more and that I wasn't the only one fielding such questions. Eventually four REU directors (Ben Brubaker, Adam Sheffer (who really started these conversations), Yunus Zeytuncu and I) decided to try to run a Polymath-REU (see https://geometrynyc.wixsite.com/polymathreu). The goal was to propose a few problems, have some post-docs, grad students or, in some cases, advanced undergraduates serve as TAs/mentors to help students do research in large groups. We tried to find problems that could be pushed in many different directions, and in addition to the four of us we also had groups run by Kira Adaricheva, Patrick Devlin, Vic Reiner, Alexandra Seceleanu and Tingting Tang. I cannot sing enough praise for the assistants who helped run these groups, as many of us were already overcommitted with tasks (including our standard REUs, and we needed to make sure those students still had the expected experience). We decided on a bit of a "Lord of the Flies" approach; we essentially accepted anyone from anywhere who had taken any proof based class and had a letter of recommendation from a professor. This has led to the highest acceptance rate in REU history (I think it was around 85%), with over 300 students participating from around the world. We knew some would be more active than others, but as several projects had different directions one could go, the hope was that students could find problems for themselves to study and people to work with. To date several groups have already presented at conferences, and results are being written up for publication.

Like everyone else, there is a lot more I could say about how the pandemic's response has affected me professionally, but there is one item in particular that should be mentioned. It led me to change what I was teaching for Fall '20. At Williams we have a wonderful program; the following is a brief description (see https://gaudino.williams.edu/ for more):

> *The Gaudino Fund was established by Professor Gaudino's students and friends after his death to perpetuate his legacy of reflection on uncomfortable and experiential learning. This learning process, nurtured both inside and outside the classroom, involves an ideology-free reflection on one's own assumptions and experience, deep self-questioning, and empathy for distinct human realities quite different from one's own.*

Robert Jackall, a former Gaudino scholar and sociology professor emeritus, and two alumni (Chris Alberti and Paul Lieberman) contacted me about the need to have rational, data based discussions on Covid.

> Paul: *The basic backstory is that we had had great success two decades ago with a course that quickly responded to the crisis of 9/11, sponsored by an endowed college fund named for a long-deceased political philosophy professor who believed there was no higher calling than challenging long-held beliefs and any group's conventional wisdom. We thus reached out to you, as the faculty head of the PBK chapter who had shown the willingness (and courage) to do just that. In addition, whereas a brilliant sociologist had been ideal to lead the initiative on terrorism after 9/11, here the fact that you were a mathematician was a profound plus because the public debate amid the pandemic over "what to do?" (the political philosopher's favorite question) was "infected" – pardon the pun – by our society's rampant misunderstanding and misuse of statistics.*

> Chris: *The country needs to have a frank conversation about acceptable risk, based on a rigorous examination of the data, as it makes judgments that are essentially political and not just matters of public health or economic policy. Your offering the course at both 100 and 300 levels is an example of your entrepreneurship and should facilitate broad participation. It would be a master stroke if the College allowed alumni to audit the course in real time.*

This challenge resonated with me, as for weeks I had been in misery seeing the Covid discussions centered on the wrong facts and the wrong statistics. I agreed to change my teaching plans and teach Math 119 / 312: The Mathematics of Pandemics and Cost-Benefit Analyses of Responses (the homepage https://web.williams.edu/Mathematics/sjmiller/public_html/119/ is under construction, but will be done before students return!). I've reached out to a diverse group of people (government, industry and academia, Republicans and Democrats, ...), and have been busily reading and preparing for the course. I have yet to reach out to anyone who hasn't been eager to be involved and share their perspective. The course will be freely available for anyone to audit, possibly in real-time; it is a lot of work, but it is work that is worth doing, and work that hopefully can have a real impact.

2 My Children

The above paragraphs describe the professional impact; the impact on home was quite different and more intense. Initially we were told our kids would be going home for two weeks, and then the time home kept lengthening and lengthening until we were told they would not come back until next year at the earliest. It didn't help that we constantly had changing guidelines from the state. For the first month we were essentially told to treat it as a holding pattern, with teachers instructed not to cover new material but rather to spend their time reinforcing what was done before. Unfortunately, as the time at home lengthened such a strategy becomes harder to support and more dangerous to implement. Thankfully one of my kid's teachers was working to give them new content from the start, correctly saying that if this were not done then they would not be ready for next year. But this was the exception rather than the rule in our schools. Numerous national articles talk about the educational costs of the choices we made, with a net *loss* in math and reading from the months at home.

These costs do not affect all equally; my kids actually ended up more advanced in math than they would have otherwise, as we did frequent math lectures. Like most things in life, the initial iterations were terrific examples of mistakes that could be made. Probably the worst thing I did was to do a long 30 to 40 minute lecture, thinking I needed to do the time equivalent of the class they were missing. Quickly, though, these become 10 to 15 minute windows on good, interesting mathematics. I recorded all of these talks, with my 5th and 7th grade kids interacting, and posted on my homepage https://web.williams.edu/Mathematics/sjmiller/public_html/math/talks/talks.html and Facebook page, and was heartened to see many friends and colleagues use these with their kids. Lectures included From Pythagoras to π, What do you MEAN (how to choose the right statistic), numerous mathematical games, induction and sums, from Pascal's triangle to Calculus, and many probability problems (from calculating the odds of different splits in card games, to estimating how long an Imperial officer would survive serving under Darth Vader). It was fun preparing these and other outreach activities with my kids, and I'm thankful to the Teachers as Scholars program and the Journal of Number Theory / Elsevier (see https://www.journals.elsevier.com/journal-of-number-theory/, where I serve as a Managing Editor) for support and co-sponsorship.

It was time consuming to make these lectures. As I wanted them to be of use for others, I didn't want to just write on a piece of paper with my kids watching. I took the time to create good slides with fun images, as I would link those with the talks online. Also, much of the material that I wanted to do is not normally done in a pre-algebra setting, so it was an interesting experiment to think what do I truly need. With a bit of work, though it is faster to know calculus, one can bypass it and be in a situation where simple algebra suffices (but boy do you now have a lecture on the importance of being able to manipulate expressions to see relationships!). We talked a lot about trying to estimate answers, trying to find upper and lower bounds, trying to see if what we found is reasonable. I've since used these slides for mentoring high school and college students, and hope to write some up for publication (if anyone is interested, I'd love a paper with "Darth Vader" and "James Bond" in the title; email me at sjm1@williams.edu).

3 School Committee

The last area I want to mention is my service on the school committee. I've enjoyed serving on it for years; I was able to use my math background several times to help the towns in our region to make good decisions. These ranged from fixing the capital projects allocation formula to correctly assigning Chapter 70 revenues from the state to each school in the district (it's a long story, worthy of a separate note). The greatest challenge we have had, of course, is dealing with the school closures last spring and trying to decide what to do right now. These have been the most painful discussions I have had in my life; it's still a bit too close to home to write about. There is no good decision; there are only decisions that are not as bad as others. One has to balance physical health with mental health, weighing the costs of various return-to-school plans against their gains.

I chair the education sub-committee, and I have run these meetings under the *All Hands On Deck!* model. We are in it together; anyone from the community is welcome to speak at any time. What are the concerns? What are the problems? What resources do we have? For example, to get a sense of what it would be like for our students if they return, school committee members rode the bus to the last meeting. Busing is going to be a huge challenge, as the capacities will be tremendously reduced. However, the experience was quite useful as several inefficiencies stood out. The first was the original guidance did not have siblings sitting next to each other. The second is we fill in the bus from back to front. While I do not think this is an issue because there will still be separation, masks, and open windows, it seems sub-optimal to have the back fill up and then the front. This means the first few groups will be near each other for the entire ride. Why not fill every other row when we start to have larger separation, and then once we get to the front then we return to filling up the back. It does mean you have some kids walking down the center for a short time to get to the back, but I think that is better. . . .

This is just one of many examples. Everything has to be rethought; my days are spent reading articles and considering various cases. For much of this, being a mathematician is quite helpful. Interestingly I'm teaching a class on sphere packing in the fall, and when I saw the guidelines from the state on socially distant classroom seating, my first thought was that we shouldn't use the square but rather the hexagonal lattice! Unfortunately, most problems are not so easy. There are real health concerns to consider for students, staff, teachers and families. There are the technology issues on what kind of remote learning can we do (synchronous or asynchronous). There are the consequences of our decisions on families, especially those whose jobs are not as flexible. As you would expect, for every option we have

The Miller Family courtesy of Cecilia Hirsch and the Front Porch Project

people on both sides saying that the other choice would be a terrible hardship on them.

The state has required us to create three plans: in-person, hybrid, remote. The preliminary plan was due on July 31st; it was a wonderful example of a starting point. It couldn't be anything else, given not just the challenges of the problem but also some local issues (our superintendent left on medical leave, our assistant superintendent had already accepted another job and thus was available for only a week); we were fortunate in being able to hire a great former superintendent to step in, but in a ten day window we had three supers! We are fortunate that the population of our county has been declining for years, and two of our three schools were built for significantly larger populations. Thus we can get all kids back, if we wish, with 6 feet separation in the elementary schools; in the new middle/high school this is not as easy. The preliminary plan had 10th, 11th and 12th grades entirely remote, which concerned many members of the community. Through many good discussions we identified the issues and were able to get them in one day a week, and now it's up to two (the same as grades 7 thru 9). Or at least this is the proposed plan; there is a lot of fear (some justified, some not) about the true risks and dangers to students, teachers and families in returning. Based on the research I have read, I have strong opinions about what benefits outweigh what costs, and I hope I am able to convince my colleagues. We were supposed to vote on a final plan on Thursday August 6th, and then the superintendent was to submit to the state on Monday the 10th. I was able to convince my colleagues that the 6th was too soon as things are changing so rapidly, both guidelines from the state and responding to community concerns. Fortunately less than an hour after convincing my chair that we needed a meeting on the 10th to give our staff as much time as possible, the state announced that we have an extra week to decide.

Today is Monday, August 10th. The superintendent has a forum on Tuesday the 11th, and the school committee votes on the 13th. I've chosen to finish writing this article now, before the decisions are made, as I think this illustrates the new normal we are living in. The constant uncertainty, trying to prepare for what might happen, needing plans to adapt on the fly.

Or perhaps a slightly more positive note. At our last meeting the superintendent described the process as building (or perhaps it was repairing) a plane in mid-air; I countered with the story of the USS Yorktown and the battle of Midway. Through cracking the Japanese code and some ingenious deceptions, we knew they were going to attack Midway in force, and when and how. It was essential to have as many of our aircraft carriers there as possible. The Yorktown was damaged and it was estimated it needed two weeks in drydock for repairs; 48 hours later it was moving westwards, with repair crews onboard working around the clock, using every moment of time. Though it was sunk, it played a key role in the success in the battle that turned the tide in the war in the Pacific. Perhaps this is a better ending, remembering the heroic deeds that we have accomplished. And the price that must be paid.

What you always wanted to know about . . . exponential growth

Christiane Tretter

Never would I have imagined that mathematical curves like the graph of the exponential function would gain such importance for the wider public. I am less surprised that people find it difficult to interpret them and that they underestimate the danger so clearly signaled by their rapid growth. On my long way from the first day in school in a small town in Bavaria to a chair for mathematics at the University of Bern in Switzerland I was accompanied by stereotypes about a subject without which no modern society could live in health, security and prosperity and which, at the same time, is disregarded so much. Like most scientists I have also been accompanied by a lack of understanding for a profession that puts pure gain of knowledge and basic research above economical gain of profit and social prestige.

On March 3, 2020, when most people in Switzerland and in many other countries were still living their normal lives, for the first time ever in my university career, I stood in front of a ghostly empty big lecture hall in the building of Exact Sciences of the University of Bern. Usually, this hall is filled with more than 200 students of mathematics, physics and computer science whom I introduce into the abstract world of mathematical analysis. The transition of such large size lectures to online teaching had to be implemented over the weekend before, from Friday night to Monday morning. The transition of all other courses took place two weeks later, equally quickly, after the Swiss government's decision on March 13 to shut down schools, universities and large parts of public life to prevent the spread of the novel corona virus. Nobody would have ever expected that centuries-old traditional institutions such as universities react so rapidly and efficiently to an unexpected crisis of this dimension.

Neither in the empty benches of the lecture hall of my 'Analysis 2' course nor in the screen of my laptop in my other course 'Mathematics for Dara Science' I could

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see interested faces or doubtful eyes from which I could guess where an additional explanation was needed. On the other hand, it was never easier to find timely examples and convincing applications for the topics of these lectures. In the analysis course I had just started a new topic, differential equations – which describe not only physical processes, but also the spread of epidemics. The solution of the simplest of all differential equations y'=cy with positive constant c exhibit exponential growth which also dominates the dramatic start of epidemics. The entire development of epidemiological events in time can be described, forecast and simulated by sophisticated mathematical models, e.g. to test the efficacy of political decisions to stop the spread before implementing them. All these tools have been developed over decades, driven by human curiosity and the ambition to unveil the secrets and laws of the universe, nature and live.

To study, model and control the spread of infectious diseases in our modern complex networks of worldwide mobile individuals many more tools are needed beyond the understanding of exponential growth. Mathematics offers a lot of them, including graph theory, spectral theory, probability theory and statistics. Will it still be socially acceptable to freely admit or even suggest that one was always bad in math after this crisis? Or should one, instead, rather dare to understand the seemingly incomprehensible? Why not give it a try with exponential growth, e.g. on https://www.youtube.com/watch?v=Kas0tIxDvrg&feature=emb_title by 3blue1brown, a YouTube channel for `Animated Math'' – not only for adults . . . .

While teaching changed rapidly and radically for all, the effects of the crisis on research differ substantially. Subjects where experiments and excursions are central have been badly hit by the lock-downs of universities. Scientists directly working in COVID19 related fields have been exposed to huge pressure from all sides to produce results and, at the same time, sometimes to unfair attacks which reveal the complete ignorance of how science and scientists work. Researchers in mathematics like me were luckier since we were not cut off from the basic infrastructures for our research: a powerful brain, a laptop or computer with fast internet connection and online access to international scientific journals and library resources. Whatever the circumstances or subject, and with the same worries about our loved ones at home, we have all continued to educate students and future scientists and to do our research quietly, almost unnoticed by the public.

My working days during the shutdown started early and ended very late, even later than usual. During the week, as director of the Mathematical Institute, I could spend them in the empty building of Exact Sciences where, in normal times, it is hard to concentrate on inventing difficult mathematical proofs, writing grant proposals or advancing joint research projects with international collaborators. In mathematics the almost unlimited means of communication around the globe already started to revolutionize research some decades ago. Hardly anyone these days works alone because it is simply more fun to work together – smashing yet another cliché. In the present crisis communication with colleagues all over the world has seen a further peak.

Yet, some essential things are missing, not only personal encounters such as meeting my research group for a coffee in my office, lively discussions and joint experiences at international conferences and workshops. In my work as Editor-in-Chief of a scientific journal, the dimension of this pandemic both in space and time became obvious to me very early when reading between the lines of the emails of authors or referees from China, and later from Italy, Spain or the UK. I have always been prepared to wait at least a year until anything else than discipline can come to our rescue against the new virus. Some things that we will have to miss for longer are not necessary for life, but still needed, especially in times of extreme workloads. These non-essential human needs are very personal. On February 11, 2020, after my last 'real' talk in the mathematical colloquium at University College London and its preparation the days before, though being tired, I went to see an opera in Covent Garden in the evening – something I did not have a chance to do when I was younger. Sometimes, when I listen to music now, it makes me very sad not to know when this will be possible again and, immediately, I dismiss this thought remembering the heart-breaking pictures from hospitals flooded with patients fighting for their lives . . . but after a while hope returns.

Having the stamina to wait long for results and success, to pursue a chosen path consequently, to sacrifice a lot for a distant goal, to endure psychologically difficult situations, and to share knowledge free of purpose and happily with others,: maybe it is these qualities of researchers and people in academia - which cannot be measured by any evaluation - that have been carrying universities through this crisis so successfully and that will eventually bring us the desired vaccine or cure for COVID19?

This text is an updated version of a text published in German on April 28, 2020, at https://www.unibe.ch/universitaet_bern_in_zeiten_coronas/carte_blanche/christiane_tretter/index_ger.html by the University of Bern, Switzerland.

Christiane Tretter

Glass and Lace

Marjorie Senechal

Two unrelated events in January, 2020, on opposite coasts of North America, precipitated these inconclusive musings.

- January 12: a concert in Northampton, Massachusetts, where I live. A mindboggling and mindbending afternoon: cellist Matt Haimovitz and pianist Simone Dinnerstein playing Beethoven sonatas and solo works by Philip Glass.
- January 20: the first corona virus case in the United States was confirmed in faraway Seattle. It spread eastward with the speed of sound. In what seemed an instant, my calendar was erased for the foreseeable future, except for my weekly piano lessons, which moved to Zoom.

Unusual juxtapositions, to say the least! The concert and the coronavirus, and also Beethoven and Glass. Beethoven is the predominant musical figure in the transitional period between the Classical and Romantic eras; orchestras around the world had planned 250th birthday galas, but the virus cut them short. By contrast, Glass's work has been called "minimalist;" he himself describes it as "music with repetitive structures." Either way, at 83 he is one of the most influential composers of the late 20th century.

So I had expected dissonances in style, sound, and centuries. Instead, I heard consonances that seemed, somehow, familiar. The two duos, the composers and the performers, pointed me to a rabbit hole. The lockdown of normal life set me free to explore it. I grabbed my piano and jumped in.

To explore, in a time of corona virus, is to remember, reread, listen, probe, and search online. Here are some of the things I found.

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An interview with the young Icelandic pianist, Vikingur Olafsson, who had recently released an album of Philip Glass's piano works. Said Olafsson of Glass,

> To me, he's like the Mondrian of music. He's taking primary colors and exploring what that means. At his best he's getting to the essence of music. . . . I came to the conclusion that it's not a repetition. It's a rebirth. It's not treading the same path, but traveling in a spiral.

An aside in Milan Kundera's *The Book of Laughter and Forgetting*.[1]

> Variation form was Beethoven's favorite toward the end of his life. At first glance, it seems the most superficial of forms, a simple showcase of musical technique, work better suited to a lacemaker than to a Beethoven. But Beethoven made it a sovereign form (for the first time in the history of music), inscribing in it his most beautiful meditations.

A memory. It's 1998. Geometers from around the world have gathered in Rome to celebrate the work of the Dutch graphic artist M. C. Escher and the centennial of his birth.[2] The program includes a guided tour of the Palace of the Vatican. There they are! Plato and Aristotle, talking as they stroll through the School of Athens! Michaelangelo's Pieta breaks my heart. But soon the guide stops, exasperated:

> What's the matter with you people? These are the world's greatest works of art, and you're just looking at the floors!

A floor mosaic in the Vatican.

A passage in *The Assayer*, by the great astronomer Galileo Galilei (1623), that I'd first read in college:

[1]Milan Kundera, *The Book of Laughter and Forgetting* (*Kniha Smichu i Zapomneni*), 1978; translated from the French by Aaron Asher, Harper Perennial 1996

[2]*M. C. Escher's Legacy: a centennial celebration*, D. Schattschneider and M. Emmer, eds, Springer 2003. A collection of articles coming from the M. C. Escher Centennial Conference, Rome 1998.

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[3]

Amazon delivers to rabbit holes, so I ordered Glass's "The Piano Collection" and tried it out. I also ordered his memoir, *Words without Music*. [4]

Like me, but four years earlier, Philip Glass left high school without graduating and entered the University of Chicago. "Whenever possible, which turned out to be all the time," he wrote, "the books we studied would be first-hand, primary sources. We were never given summaries to read or even commentaries, unless they themselves rose to the level of a primary source. So, for example, . . in physics we reenacted the experiments of Galieo with rolling balls and inclined planes. . . .This early exposure would be reflected in *Galileo Galilei*, which I composed forty-five years later, in which his experiments become a dance piece -- the balls and inclined planes are there."

Another memory. I'm seventeen, sitting at an oval table with a dozen or so fellow UC students and a professor whose name I've forgotten. We've spent hours in the physics lab rolling balls down inclined planes; now we're discussing Galileo's explanation in his masterpiece, *Two New Sciences*.

"How would *you* explain this to Simplicio, Miss Wikler?" the professor asks me.

Who, me?

Yes, you!

But Ah'm jest a teenager from Kentucky! What do Ah know?

He may have smiled, but he didn't laugh. And so, accented and unschooled, I joined the eternal conversation that the University of Chicago, the School of Athens, and Galileo too, called education.

"The effect on me was to cultivate and understand in a firsthand way the lineage of culture," Glass continued. "The men and women who created the stepping-stones from earliest time became familiar to us -- not something `handed down' but actually known in a most immediate and personal way."

In the rabbit hole, the conversation continues in my head (and on my piano).

Stepping stones:

from the inlaid designs in the Vatican's parquet floors to Escher's interlocking lizards,

from the "Moonlight Sonata" to "Einstein on the Beach" and back again.[5]

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[3]Galileo Galilei, *The Assayer*, 1623, translated by Stillman Drake, *DIscoveries and Opinions of Galileo*, Anchor Books, 1957.

[4]*Philip Glass: The Piano Collection*, Wise Publications, 2009; *Words Without Music: a memoir*, W.W. Norton, 2015.

[5]Here in the rabbit hole my piano stays untuned. But Glass is interesting even so. An Mp3 of my attempts to merge fragments of his "Einstein on the Beach" and Beethoven's Moonlight Sonata is posted as extra.

from the lacemaker's threads and needles to the deepest meditations;

from triangles and circles to the *maria* on the moon.

Wholes and parts; themes and variations; causes and effects; big pictures and brush strokes. The mysteries at the heart of all great art and science.

The more we know, the more we don't. So we keep on talking.

"The accepted idea when I was growing up," Glass wrote later in his book, " was that the late Beethoven quartets or the Art of Fugue or any of the great masterpieces had a platonic identity -- that they had an actual, independent existence. . . . I had really thought of the interpreter as a secondary creative person. I never thought he was on the same level with Beethoven or Bach. But after I had spent some time thinking about all that and began playing myself, I saw that the activity of playing was itself a creative activity . . ."

Here in the rabbit hole, I'm trying to translate that into math.

Marjorie Senechal

Covid-19 Analytics: Proposed Projects for Undergraduate Research

Reza O. Abbasian and John T. Sieben

For many in academia Spring Break 2020 will be remembered as a turning point. At least for the foreseeable future we needed to find a new way to operate. We realized that our time would be used differently since national meetings and recreational travel would not be possible for a period of time. Was there any way we could eek something positive from this pandemic? Upon taking inventory it appeared that our new assets would be time to research, availability of students with whom to research, some new, interesting, and important topics to which we could apply math techniques, and an abundance of data. Looking back, we see the COVID-19 pandemic has certainly lived up to our expectations. It has created opportunities to showcase mathematical and statistical software as tools to utilize real life data for modeling and analysis. With Covid-19 as a constant threat, the wise course of action appeared to be isolation. But our isolation need not be unproductive. Through Zoom we were able to initiate undergraduate research with our students and colleagues to prepare several projects that will be useful in the undergraduate classroom.

In this chapter, we will develop a series of projects that demonstrate the importance of quantitative modeling and analysis. The projects are all suitable for undergraduate research, in fact, some were developed with our summer researchers, others came from discussions with our colleagues. The projects will pose questions, that will challenge students to separate myth from facts. A unifying feature of our projects is that they ask students to investigate the questions using readily available software and data on the Covid-19 pandemic. Both authors are firm believers in the use of real data and available technology for modeling and analysis of real-life problems. Some of the projects are the results of the authors work with student researchers during the summer of 2020 and several others are created as part of the NSF funded grant # 1905246 titled " Mathematics and Statistics Across the

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Curriculum: Empowering Non-STEM Students to Appreciate and Use Quantitative Modeling” .

The goal of “ Mathematics and Statistics Across the Curriculum” project is to increase the use of mathematical and statistical modeling in mostly non-STEM fields. The desired impact is to transform the educational landscape so that students and faculty in the social sciences, applied studies, and selected STEM disciplines, build appreciation of quantitative methods and their utility in non-STEM fields. It is hoped that after a brief exposure to mathematical and statistical modeling using technology , the student will see the importance of the quantitative justification for decision making. Of interest to us is the use of technology in creating mathematical models for those problems which are relevant to current events and affect society. The Covid-19 pandemic, in many ways, has affected us adversely but it has provided us with massive amount of data which can act as a laboratory for teaching modeling and analysis of real-life problems.

In the following descriptions of projects, we have intentionally avoided the inclusion of detailed step-by-step quantitative analysis, mathematical jargon and cumbersome calculations. The goal is to make these projects available to an audience who may or may not have extensive math-stat training. In preparing these projects, we utilized statistical software such as R and Minitab and math software such as Maple or WolframAlpha. However, all these projects can be completed by utilizing many other software packages, most of which are free.

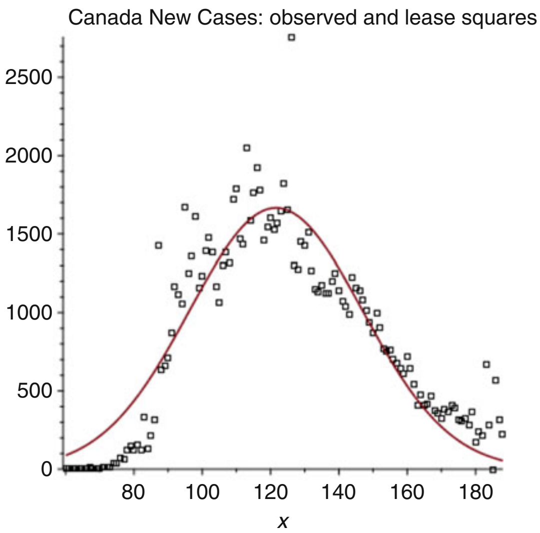
Project 1: What is a suitable mathematical function to model the number of daily new infections? Clearly the data and hence the appropriate model, to some extent, depends on the region/country affected by the pandemic, the approach which was taken to control the pandemic, the available resources, the geography of the region, and other factors. We also wanted to use data which are reported by health authorities, free of political pressure.

Data including daily number of new infections, recovered patients, number of deaths, etc. is readily available online. In addition to https://ourworldindata.org which is a widely used source, there are many other reliable sources such corona-virus. JHU.edu. We have listed other sites for capturing relevant data (ex: see The World Health organization site: COVID-19.who.int) . We encourage the use of data from sources that have been historically reliable such as northern and western European countries. Within countries known for reliable data, we find groups of countries that have employed markedly different approaches to controlling the pandemic. For example, Germany, Italy and France all resorted to a shelter-in-place approach with gradual loosening of the restriction. Sweden used a “hands off” approach hoping for the recovered population to quickly become large enough to bestow a “herd immunity” on the country. The approach of New Zealand was to enforce the strictest set of restriction and complete closing of the borders. (By August 10 New Zealand had been 100 days without a new Covid-19 case. On August 11 when this was written, one family had three members test positive. They have been quarantined and the source of the virus is being investigated.)

We note that an opportunity to discuss the importance of smoothing data arises here in a natural way. The Covid-19 data is very often reported daily. When daily numbers are examined they seem to fluctuate wildly. These fluctuations are at best a distraction and at worst may hide patterns within the data. Data smoothing, even something as simple as using a running average, is a welcome addition to the discussion. See Project 6 below for a specific example.

When attempting to model the New Cases for countries with a shelter-in-place approach (followed by gradual opening), often it will be observed that the graph of new cases vs time has an approximate bell-shaped (Gaussian) structure. We asked our research student to take a close look at the data and to supply reasons why the approximating curve is in fact not fully normal. Students with a more advanced knowledge of regression may attempt to fit a more suitable model to the data. The with just a bit of guidance was able to utilize Minitab's nonlinear regression command to create the best fits regression equation. One can also use other statistical software such as R, or a Computer Algebra System (CAS) such as Maple. As an example, below is a scatter plot of new cases in Canada with the Gaussian function $y = a * e^{-\frac{(x-b)^2}{c}}$ superimposed. The values of the parameters a, b, and c are determined using least squares method and the CAS software Maple. Our student created similar graphs for several other countries. The results confirmed that the containment approach had a marked effect on the outcome. For students with a background in statistics, one could include different measures of regression fit such as coefficient of correlation and for more advanced students a discussion about the size of the mean square of the residual.

Canada, New Cases, April, May, and June, early July 2020: graph and regression by Maple



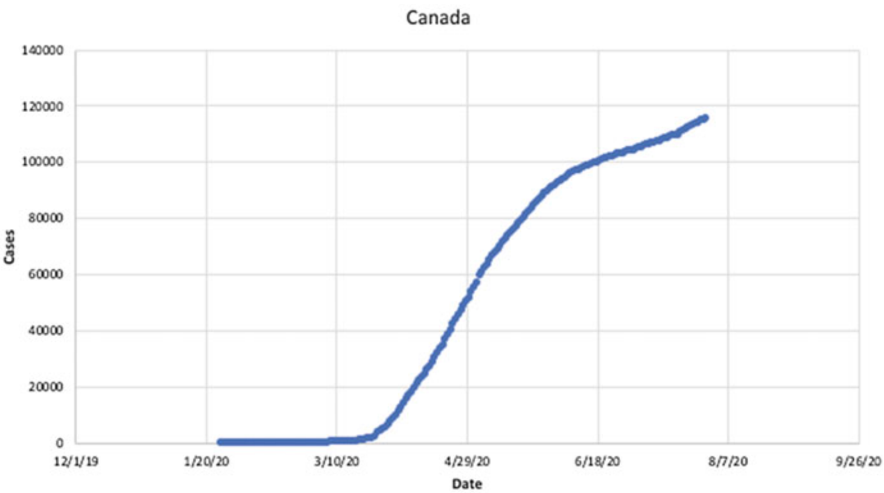
Using cumulative graph to explain inflection point

Using cumulative graph to explain inflection point

1 Project 2: Practical significance of inflection point.

Students in calculus classes learn how to find an inflection point but often do not understand its practical significance. Using the same source of data from project 1, one can form a cumulative graph showing the total number of new cases (or deaths) for each country. On the cumulative graph there should be at least one inflection point. This will be a teaching moment to explain the meaning and importance of inflection points to students. It is very likely that the first inflection point results from the graph going from positive concavity to negative concavity (slowing the rate of new infections.) Most likely it will be observed that the first inflection point occurs shortly after strict shelter in place rules were enforced. In some cases, such as US states like Texas or Florida, one might see two inflection points referring to an initial success, and then a return to increasing rates of new cases shortly after the reopening of businesses, beaches and bars.

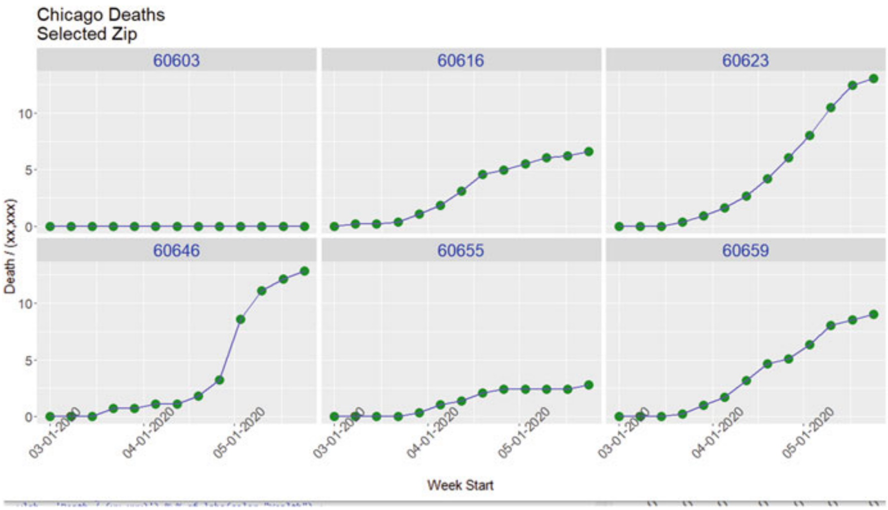
For students with a background in calculus, one can fit a logistic equation to a cumulative graph and then show that the derivative (marginal change) would amount to the graph produced in projects 1. For countries with a "herd immunity" approach, for obvious reasons, there won't be a pronounced inflection point. Following is an example of the cumulative graphs for the total number of infections in Canada which was create using Minitab. The approximate location of the inflection point is late-April. Shelter in place started on March 28. And as expected, within a few weeks the sign of concavity changed from positive (increasing rate of infection) to negative (decreasing rate of infection). A suitable addition to this project would be creating and discussing the details of a table showing the dates of the shelter in place for various countries (or states within US) and the date when inflection points occurs.



Cumulative cases from mid-March through early August

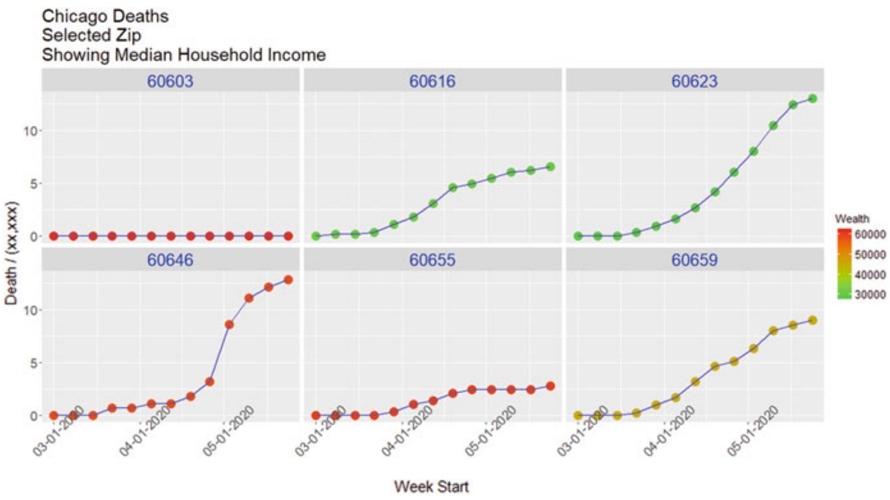
2 Project 3: Does Covid- 19 disproportionally affect the minorities and low-income areas of a city?

For major cities, data showing the median income, percentage of minorities, number of daily infections and number of death due to Covid-19 are available, though one may need to consolidate data from several sources. Data is available in several online sites, for example, see https://data.cityofchicago.org/Health-Human-Services/COVID-19-Cases-Tests-and-Deaths-by-ZIP-Code/yhhz-zm2v As a first step in this project, students should graph the various dependent variables (number of daily infections, median income, etc.) for different zip codes in a large city such as New York, Chicago or Houston and visually inspect the graph. Does it appear that the number of cases or number of deaths are independent of the zip code? There are several statistical tools available to test the disproportional number of Covid-19 infections versus, say, income or % minority in a zip code. Following is an example created for the city of Chicago. In these examples we drew graphs of the number of deaths per ten-thousand residents for six selected zip codes. The zip codes were selected by geographic location, north, center and south side. A glance at the six resulting graphs suggest that the deaths resulting from covid-19 are not independent of zip code.



Having observed that the pattern of death is unique to each zip code, it is natural to ask if we can attribute the difference to a characteristic of the zip code. To illustrate this concept, we created two additional graphs in which we color coded the graphs, first according to the population density of their zip codes and then to the wealth of the zip code. In the interest of space, we reproduce only one of these graphs.

In the graph below we see that the zip codes are colored by wealth of the zip code. A reasonable supposition might be that wealthier regions will have less deaths due to an infectious disease. But the graph for zip = 60603 (top left) shows very high wealth and very low death numbers but the graph for zip = 60646 (bottom row, left) also shows much wealth but high death numbers. Wealth alone is not the answer.



In the second graph (not reproduced here) the population density of the zip code is revealed. The graph suggests that population density will not be a powerful predictor of the number of death due to Covid-19. There are other similar characteristics of a region that might contribute to the patterns of death we observe due to Covid-19. Discovering in the Chicago example that wealth and population density are not strong predictors of Covid deaths leads one to speculate on what characteristics might be strong predictors of the number of deaths in a region. This leads in a naturel way to project four.

3 Project 4: What is (are) the best predictor(s) for the number of new cases or death in a country?

Let your dependent variable be the number of cases say per million population and Independent variable to consider are population density, wealth (as measured by Gross National Income (GNI)), system of government (democratic or authoritarian), geography of a country (island or not), etc. One can easily create a regression equation and measures the strength of these predictors. Following is an example of a simple linear regression with y = total cases as per million as a dependent variable and several independent variables: population density, GNI, average age as continuous predictors and shelter in place, type of government, and island nation or not as categorical variables. As a simple assignment, students should be encouraged to observe the signs (+ or -) of each coefficient and interpret. For example, the positive sign of the coefficients for population density and GNI would indicate that as expected countries with high population density will have more infections. However, somewhat unexpectedly, higher GNI also contributes to an increase in the number of infections. So, to protect yourself from covid-19, it is best to be young, live in a less-than-prosperous and sparsely populated island country governed by a non-democratic system! The regression equation and p-value tables are given. For students with a background in statistics, p-values indicates the significance of each independent variable. For students without a sufficient background in statistics, one can explain the significance of a variable in terms of the p-value without a lengthy discussion.

4 Regression equation

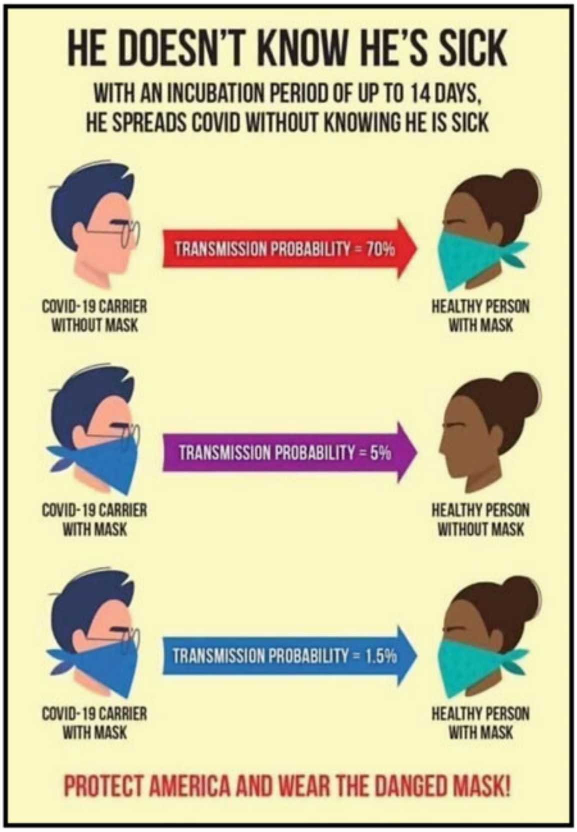
totalCase	=	12103 + 0.248 Popdnsty + 0.0886 GNI - 271 Av. Age - 3561 Shelter + 2792 Dem - 2644 Island

5 Coefficients

Term	Coef	SE Coef	T-value	P-value	VIF
Constant	12103	5446	2.22	0.040	
Popdnsty	0.248	0.199	1.25	0.228	2.36
GNI	0.0886	0.0405	2.19	0.043	1.66
Av. Age	-271	165	-1.64	0.118	1.56
Shelter	-3561	1765	-2.02	0.060	1.31
Dem	2792	2640	1.06	0.305	2.57
Island	-2644	1591	-1.66	0.115	1.38

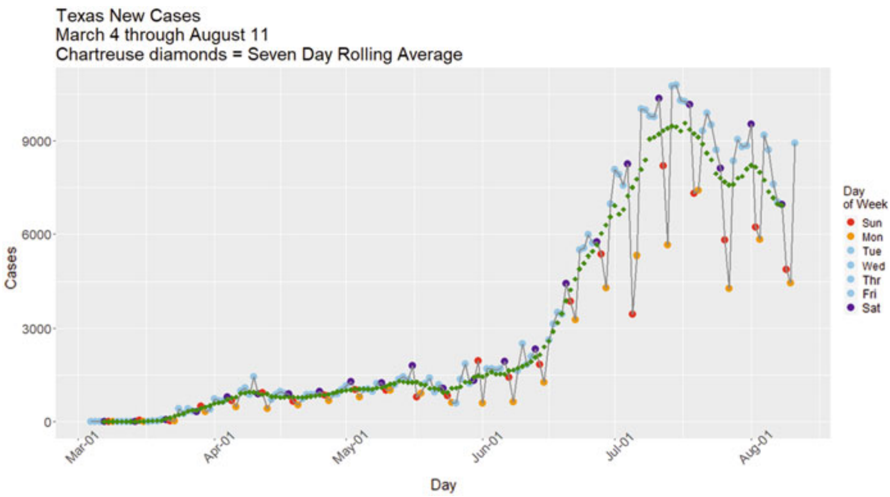
6 Project 5: Investigating a claim about use of masks in preventing Covid-19

A diagram widely distributed in social media shows the probability of infecting another person (assuming no social distancing is observed) for three different scenarios: A) both wearing masks (1.5%) , B) only the infected person wearing a mask (5%) and C) only the non-infected wearing a mask (70%). There is no evidence or study given. The cases illustrated seem plausible, since studies do show that the risk of infection is the lowest for case C and highest for case A. However, the specific numbers are not substantiated. The diagram does give us an excellent chance to engage students in defining metrics that will measure the claims made in the diagram, gathering relevant and reliable data, cleaning the data, and using the data to verify (or refute) the claims made it the diagram. For example, the percentage of health care workers infected with Covid-19 could be used as an estimate of the rate of infection of a person (in this case health care worker) as she/he an is caring for an infected person. Various studies suggest (see https://www.cidrap.umn.edu/news-perspective/2020/05/studies-1-healthcare-workers-had-covid-19) this number to be around 1% which is consistent with the numbers given on this diagram(1.5%)



Project 6: At the outset of the outbreak, there were several articles emphasizing the dangerous nature of Covid-19, compared to other viruses such as flu. These articles mentioned 2.6 as the rate of transmission. Namely each infected person, on

the average, infects 2.6 other people. Is that correct? In fact, the rate of transmission is not constant, it is rather a function of many variables, such as a country's approach to containment, age of the population, geography, etc., but most importantly it is a function of time. To study this phenomenon, one can pick a few countries or US state/cities, plot the number of infections starting on say March 15 and try to fit a piecewise (mostly) exponential function to the data. The ratio of two consecutive days, $f(x_2)/f(x_1)$ is a rough estimate of the transmission rate. Then one can plot various values of the transmission rate as a function of time to create a new function. It is entirely possible that the average value of this function is approximately 2.6. However, it is more likely that the maximum value is 2.6. This is a useful exercise in demonstrating the process of building and understanding the behavior of an exponential function, dealing with piecewise functions and the idea of a variable rate of increase. Following is an example showing a scatter diagram of the number of daily infections in Texas from the outset of the pandemic to August 1, 2020. As one can see from the graphs good fit can be obtained with a linear function followed by a Gaussian (bell-shaped) curve) The second graph is the daily number of infections smoothed by applying a seven-day rolling average.



Daily new cases fluctuate, more so in recent weeks than in the early days of Covid-19. A pattern that is easy to identify from this graph is the number of cases reported on Saturday, Sunday, and Monday. Week after week the cases decrease Saturday to Sunday, show a further decrease on Monday, but rebound on Tuesday. It does not seem reasonable that the number of cases is behaving in this way. More likely the pattern is due to the reporting cycle among the counties in Texas. For this reason, among others, we have also plotted the seven-day rolling average vs Julian Date. (Graph not reproduced but it is the same graph as the chartreuse diamond overlay above.)

The seven-day rolling average smooths the data considerably. It appears that to model this data one could do piecewise model with the first 90 days (approximately March 3 through June 1) following a linear model and the days between June 1 and July 27 being a reasonable approximation to a Gaussian curve. The data after July 27 shows another increase and breaks the bell-shaped pattern, perhaps to start another Gaussian curve with different parameters.

Using the seven-day rolling average to calculate $\frac{f(t_{n+1})}{f(t_n)} \approx R_n$ gives 146 daily estimates of the transmission rate. Descriptive statistics of the 146 values gives as a mean of 1.072 with a minimum of 0.778 and maximum of 2.77 which supports our view that the reported rate of infection (2.6) refers to the maximum not the average. This is an excellent opportunity to demonstrate to students 1) the dependence of regression on the data. For example, if the amount of mask wearing and social distancing changes our regression equation will no longer describe the physical situation and 2) that the rate of growth or decay is generally not constant, rather it is a function of time.

7 Concluding thoughts

These are challenging times in the USA and in the world. The authors acknowledge that while Covid-19 has forced an adjustment of their usual activities compared to the privations that other's have suffered we have been exceptionally fortunate. We are socially isolated but remain healthy and have used the time in isolation to think of ways to engage our students as they return to university life. This experience has reinforced our long-held belief that life is full of opportunities to practice mathematics, statistics, and quantitative analysis in general. We thank the people at Springer for this opportunity to share some of our reflections.

Reza O. Abbasian

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The Optimal Lockdown Strategy Against Virus Propagation and Economic Loss

Filippo Gazzola

1 Introduction

The present paper is my personal contribution to the volume *Math in the Time of Corona* whose title voluntarily reminds the novel **El amor en los tiempos del cólera** [12, Love in the time of cholera] by Gabriel García Márquez (Colombia, 1927–Mexico, 2014), worldwide known because of his Nobel Prize and of his originality. His novel [11] starts as follows. *El día que lo iban a matar, Santiago Nasar se levantó a las 5:30 de la mañana para esperar el buque en que llegaba el obispo*. Another of his novels [9] ends as follows. *La mujer se desesperó. "Y mientras tanto qué comemos?", preguntó, y agarró al coronel por el cuello de franela. Lo sacudió con energía. "Dime, qué comemos". El coronel necesitó setenta y cinco años - los setenta y cinco años de su vida, minuto a minuto - para llegar a ese instante. Se sintió puro, explícito, invencible, en el momento de responder: "mierda"*. In honor of Márquez, this paper could have started and ended with the very same sentences: *El día que lo iban a matar... mierda*. But, at the very last minute, I decided to organize it as I did below, by recalling, from time to time, the novels of Márquez..

This paper gives some hope and some suggestions on how to behave in case of future pandemic. It also shows that mathematics can explain everything, including virus propagation and lockdown strategies. This fact is well-known since Leonardo da Vinci (Italy, 1452–France, 1519) who said *Nessuna certezza delle scienze è dove non si pò applicare una delle scienze matematiche, ovver che non sono unite con esse matematiche*. Incidentally, we observe that Leonardo failed to receive the Nobel Prize only because Alfred Bernhard Nobel (Sweden, 1833–Italy, 1896) was born almost four centuries later. Have a nice time while reading this paper!

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2 The Logistic Equation

In 1798, in his study of the “future improvements of Society”, the English economist Thomas Robert Malthus [14] suggested a model for the dynamics of populations. His monograph was published under the pseudonym of J. Johnson, only much later discovered to be Malthus himself. The Malthus model assumes the existence of infinite quantities of both space and food, and that the variation of a population of individuals (e.g. viruses) merely depends on the natality rate $n>0$ and on the mortality rate $m>0$, taken as fixed constants. The relevant parameter is their difference $\rho=n-m$. If the population is initially (at time $t=0$) made by $y_0>0$ individuals and if we denote by $y(t)$ the population at time t, the model states that, in average, in any interval of time Δt there is a quantity of individuals born which is proportional to the population and to the interval of time, that is, equal to $ny(t)\,\Delta t$. Similarly, it states that the number of deaths in the same interval of time is $my(t)\,\Delta t$. The population at time $t+\Delta t$ is the given by the population at time t plus the born population and minus the dead population. By taking the limit as $\Delta t\to 0$, we obtain a *linear* differential equation and, assuming that there is just one affected human at time $t=0$, namely $y(0)=1$, its unique solution is given by the exponential $y(t)=e^{\rho t}$, which is increasing if $\rho>0$ (natality larger than mortality) and decreasing if $\rho<0$; if $n=m$, then $y(t)$ remains constant. In case of pandemic, $y=y(t)$ is also proportional to the number of humans affected by the virus. Clearly, this model is fairly simplified and gives inaccurate responses. The weakest point is the initial assumption of infinite quantities of space and food, an assumption allowing for an unbounded increment of the population, while nobody expects an invasion of viruses. In fact, the lack of food and space decreases the natality rate and increases the mortality rate: then $\rho=\rho(y)$ is decreasing with respect to y, which leads to a nonlinear equation.

It was the Belgian mathematician Pierre François Verhulst [15, 16] who introduced the so-called *logistic equation* in 1838, an equation able to take into account the decrement of food and space as the population increases. More precisely, the probability to have an increment of affected people is proportional to the remaining population with risk of being affected. This leads to the logistic equation, which is a *nonlinear* differential equation. Since this equation belongs to the class of Bernoulli equations (taking their name from the work of the Swiss mathematician Jacob Bernoulli [3] in 1695), it may also be solved explicitly. Denoting by p the overall human population and assuming again that $y(0)=1$, the resulting solution is

$$y(t)=\frac{p}{1+(p-1)e^{-\rho t}} \tag{2.1}$$

whose graph is displayed in the top picture in Figure 1 for $\rho=0.1$, $p=10^5$. This graph shows a good agreement with the curves of the Coronavirus affections during year 2020, all over the world [17], see the reproduction of the graphs in the bottom pictures. Therefore, the logistic equation is nowadays considered a good model to describe the dynamics of populations.

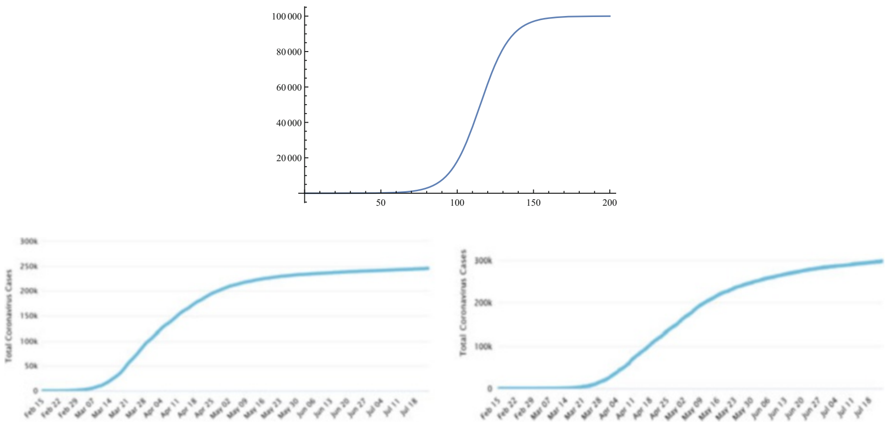

Fig. 1 Top picture: graph of the solution (2.1) of the logistic equation for $\rho = 0.1$, $p = 10^5$. Bottom pictures: graphs of the affected population in Italy (right) and in the UK (left); source [17]

3 The Model and Its Responses

In order to build a model able to take into account the problems that arose during the Coronavirus propagation, we make a couple of preliminary remarks. First, one may wonder whether a continuous function is well-suited to measure the number of affected humans which is an integer number. This may be explained by taking into account the *percentage of affection* of each human. More precisely, the affection itself cannot be considered just on or off, different degrees of affection are present in humans, starting with weak symptoms until a full infection is reached. Therefore, the continuity assumption is fully justified.

The second remark concerns the *lockdown*, namely a control imposed by the Governments during the Coronavirus diffusion in order to reduce the speed of propagation of the virus. The lockdown consists in prohibiting to a certain amount of the population to circulate freely. As shown by a statistic published in the Financial Times [2], each Country/Region decided its own lockdown strategy in order to minimize the number of affected people and the economic impact, see also [6]. Clearly, these two targets are competing with each other: a strong lockdown decreases the number of affected people but increases the economic losses and viceversa. Since also the economic losses themselves represent an affection of the population in terms of welfare, these two numbers may somehow be put on the same level. In particular, economic losses induce both decrements of salaries and of the quality of food and, hence, increment of diseases due to the bad quality of food and decrement of the possibility of treating health problems due to lower incomes. Moreover, an iterated quarantine prevents primary medical treatments and has physical and psychological consequences such as post-traumatic stress symptoms, confusion, and anger [5]. In the sequel, we call

directly affected the humans attained by the virus,
indirectly affected the humans losing health as a consequence of the lockdown.

Roughly (and sadly) speaking, the indirectly affected humans represent **la otra costilla de la muerte** [8, the other rib of death].

The data from [2] show the results, in terms of the (directly and indirectly) affected humans, depending on the adopted lockdown strategies. But, among so many data, it appears impossible to derive an *optimal lockdown strategy* able to minimize the overall affected humans. The purpose of our model is to suggest a new optimal control problem aiming to determine the best lockdown strategy.

Assume that a pandemic virus has affected one human, among a given population formed by $p \gg 1$ humans, and denote by $\alpha = \alpha(t)$ the lockdown control parameter, namely the measure of the restrictions imposed by the Government at time t in order to reduce the directly affected humans. Consider (for instance) the case where $p \geq 5000$ and $0 \leq \alpha(t) \leq p-1$, although much larger p are expected to better describe a population. The control α may also be discontinuous: $\alpha(t) = 0$ means a null lockdown strategy (everybody is free to circulate) while $\alpha(t) = p-1$ means a total lockdown (no free movement is allowed).

Denoting by $y_1(t)$ and by $y_2(t)$, respectively, the directly affected and indirectly affected humans at time t, in the control problem introduced in [13], the unknowns y_1 and y_2 are governed by two different logistic equations and, therefore, are independent quantities. The precise model is described by the following system:

$$\begin{cases} \dot{y}_1(t) = \rho\left(1 - \dfrac{y_1(t)}{p - \alpha(t)}\right) y_1(t) \ , \\ \dot{y}_2(t) = k\alpha(t)\big(\alpha(t) + 1 - y_2(t)\big) y_2(t) \ , \\ y_1(0) = 1 \ , \quad y_2(0) = 1 \ , \end{cases} \qquad \alpha(t) \in [0, p-1] \quad \forall t \geq 0 \ . \tag{3.1}$$

Some individual may be affected *both* directly and indirectly so that the constraints are that $y_1 \leq p$ and $y_2 \leq p$, but there is no such constraint on their sum. The two equations in (3.1) emphasize the opposite impact that α has on y_1 and y_2: *larger* α *yield both smaller* y_1 *and larger* y_2 *and viceversa*. We assume that $\rho > 0$ (natality larger than mortality among viruses), that $k > 0$ (positive speed for indirect affections), these parameters having obvious meanings: ρ and k represent, respectively, the speed of propagation of direct and indirect affections. If a Region has good quality of health assistance then ρ is small, while a bad health assistance means that ρ is large. On the other hand, if a Region has wealth (in any sense) then k is large, while if it is poor then k is small (e.g., low wealth means low risk for economic losses).

The derivation and a full interpretation of (3.1) is given in the original paper [13]. The purpose of the Government is to choose the optimal lockdown control α minimizing the amount of overall affected humans $y_1(T) + y_2(T)$ at some future time $T > 0$. Clearly, the optimal choice of the Government strongly depends on the values of the parameters ρ and k: large ρ and small k suggest a strategy with less direct affections and viceversa. The optimal lockdown strategy should follow the already mentioned principle that *larger* α *yield both smaller* y_1 *and larger* y_2, and viceversa. If $\alpha(t) = 0$ (null lockdown) for all t, then $y_2(t) = 1$ for all t. If $\alpha(t) = p - 1$ (full lockdown) for all t, then $y_2(t) = 1$ for all t. This means that in the two extremal cases, either the unique initially directly affected individual or the unique initially indirectly affected individual remain with **cien años de soledad** ([10], in solitude for one hundred years if we take $T = 100$).

If we consider a "lazy Government" that aims to take a decision once forever and never change strategy, then the admissible lockdown strategy are constants and the results in [13] show that

> *For lazy Governments, the optimal strategy is never to have null lockdown while it can be a total lockdown if the propagation rate k of indirectly affected humans is sufficiently small.*

As expected, the total lockdown strategy is more convenient if the propagation rate ρ of directly affected humans is large, compared to the propagation rate k of indirectly affected humans. These results give strength to our model. We recall that a total lockdown was imposed by some African Countries [1].

But the optimal lockdown strategy needs not to be constant and, in this case, the detection of the optimal strategy is more delicate. In 1956, the Russian mathematician Lev Pontryagin, together with his students [4], formulated what is nowadays called the Pontryagin Minimum Principle; we refer to [7] for a bibliography list of 54 items which appeared before year 1961. Thanks to this principle we could show that the optimal lockdown strategy is always bounded away from zero and, in some cases, it can be a total lockdown. More precisely, if a Government is ready to modify its strategy during the different phases of the pandemic, the results in [13] show that

> *a null lockdown is never the best strategy;*
> *a total lockdown is the best strategy if the indirect diffusion coefficient k is small;*
> *a total lockdown is not the best strategy if the indirect diffusion coefficient k is large;*
> *the possible transition from a total lockdown to a weaker strategy is not continuous.*

We believe that these results validate further our model and we hope that the suggested model might be considered as a good starting point towards a "perfect model" able to take into account also other factors, such as memory effects or different payoff functionals. For the interested reader, the paper [13] is complemented with several remarks and some possible future developments.

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References

1. Africanews, March 27, 2020, https://www.africanews.com/2020/03/27/coronavirus-hub-impact-of-outbreak-across-africa/
2. S. Bernard, C. Tilford, J. Burn-Murdoch, K. Fray, *Coronavirus tracked: the latest figures as the pandemic spreads*, https://www.ft.com/coronavirus-latest , Financial Times, March 24, 2020
3. J. Bernoulli, *Explicationes, annotationes et additiones ad ea, quae in actis sup. de curva elastica, isochrona paracentrica, et Velaria, hinc inde memorata, et paratim controversa legundur; ubi de linea mediarum directionum, alliisque novis*, Acta Eruditorum, 1695
4. L.S. Pontryagin, V.G. Boltayanskii, R.V. Gamkrelidze, E.F. Mishchenko, *The mathematical theory of optimal processes*, Wiley, 1962
5. S.K. Brooks, R.K. Webster, L.E. Smith, L. Woodland, S. Wessely, N. Greenberg, G.J. Rubin, *The psychological impact of quarantine and how to reduce it: rapid review of the evidence*, Lancet, Vol.395, March 14, 2020 (available on line at www.thelancet.com)
6. S. Flaxman et al, *Estimating the number of infections and the impact of non-pharmaceutical interventions on COVID-19 in 11 European countries*, Imperial College COVID-19 Response Team, March 30, 2020
7. A.T. Fuller, *Bibliography of Pontryagm's Maximum Principle*, Journal of Electronics and Control 15, 513-517, 1963
8. Gabriel García Márquez, *La otra costilla de la muerte*, 1948
9. Gabriel García Márquez, *El coronel no tiene quien le escriba*, 1958

10. Gabriel García Márquez, *Cien años de soledad*, 1967
11. Gabriel García Márquez, *Crñica de una muerte anunciada*, 1981
12. Gabriel García Márquez, *El amor en los tiempos del cólera*, 1985
13. F. Gazzola, *An optimal control problem for virus propagation and economic loss*, DOI: 10.13140/RG.2.2.11271.83364
14. T.R. Malthus, *An essay on the principle of population*, J. Johnson, London, 1798
15. P.F. Verhulst, *Notice sur la loi que la population poursuit dans son accroissement*, Correspondance Mathématique et Physique 10, 113–121, 1838
16. P.F. Verhulst, *Recherches Mathématiques sur la loi d'accroissement de la population*, Nouveaux Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Bruxelles, 18, Art. 1, 1–45, 1845
17. Worldometer, *COVID-19 Coronavirus pandemic*, https://www.worldometers.info/coronavirus/, 2020

Research Seminars: A New Hope

Edgar Costa, Bjorn Poonen, David Roe, and Andrew Sutherland

A long time ago, in a world that now seems far, far away, the Boston area hosted seven weekly research seminars in number theory. Frustrated by the task of scanning the seminar homepages every weekend to find talks of interest, one of us (Edgar Costa) created a website to collect them automatically into one page. That site, Bean Theory, proved to be a valuable resource for local number theorists, especially those new to the community.

When the pandemic reached the U.S. and we moved our seminars online, we were pleasantly surprised to see some mathematical friends from faraway places participating. We realized that audiences for seminars like ours might be even larger if there were an effective way to advertise them. Conversely, there might be online seminars that we might like to attend, if only we knew about them. This inspired two of us (Edgar Costa and David Roe) to begin creating a website that mathematicians worldwide could use to find seminars, listed in their own time zone, with direct videoconference links. Because of our experience developing Bean Theory and the L-functions and Modular Forms Database (a database of objects in arithmetic geometry), we were able to get a running start. The rest of us soon joined in designing and developing the site, which we called mathseminars.org.

Even though we all put aside other work to complete our project quickly, we were not the first to produce a website listing online mathematics seminars. By the time our site was ready, a few other websites existed, but our site had the advantage that its content was crowdsourced. We realized from our experience with Bean Theory

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that this would make it easy for our list to grow: any seminar organizer with a web browser could add new talks without our direct involvement, after being endorsed by any other content creator on the site (a measure intended to reduce spam).

And indeed it grew, even faster than we had anticipated! During the first 28 days after the April 10, 2020 launch, the site received 411,015 page views from 83,618 visitors.

> mathseminars.org is like the departure board at O'Hare if you could just get on any flight you wanted and they were all free!—Jordan Ellenberg

Mathematicians thanked us for enabling them not only to continue learning about new research but also to reconnect with their colleagues at a time when many were feeling isolated. Conference organizers scrambling to host events online found that they could run them entirely on our site. The American Mathematical Society decided to partner with our site. Researchers in physics asked if their seminars could be included; in response, we developed functionality to include fields beyond mathematics and renamed the site researchseminars.org to reflect its broader scope. As of September 2020, the site covers biology, chemistry, computer science, earth sciences, economics, and physics in addition to mathematics. We continue to add new fields as soon as there is sufficient demand and expert consensus on what the subtopics should be.

Do we expect researchseminars.org to continue growing after in-person talks become common again? Yes! Many features we incorporated were designed to be useful for in-person seminars as well:

- the site relieves organizers of the task of designing their own seminar website, while still letting them embed the seminar schedule on an external website;
- it lets speakers enter and update their talk information without burdening organizers;
- it gives participants one place to find all talks in their vicinity;
- it allows participants to filter conferences and talks according to their interests;
- it allows participants to subscribe to a single personalized calendar that updates automatically as new talks are added;
- it provides a central repository with links to slides and video recordings; and
- it provides a historical record.

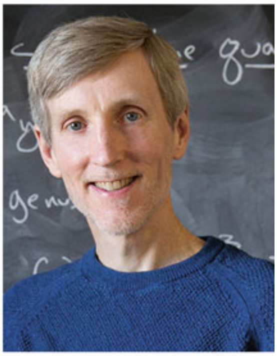
Despite the challenges posed by the sudden transition to online research, hosting talks virtually also has benefits. Attending a talk no longer requires proximity to the host institution, only a reliable internet connection. Researchers with health issues or family obligations can participate more easily. Finally, hosting events online reduces travel, saving researchers' time and reducing their carbon footprint. Now that organizers have experienced some of these benefits, we hope that many in-person seminars will include an online presence going forward, and we designed our site to facilitate this.

We invite all researchers to explore researchseminars.org, contribute conferences and talks, and help spread the word! We encourage researchers in fields not yet covered to assemble a group of colleagues willing to help plan and promote a new section of the site. Many improvements to the website are still possible, and the code is open source, so people enthusiastic about building a better site are welcome to join the project; many others have already contributed.

Edgar Costa

Bjorn Poonen

David Roe



Andrew Sutherland

Mathematics Indicates That an HIV-Style Strategy Could Be Applied to Manage the Coronavirus

Julie Rowlett

1 Can We Adapt Strategies Used to Fight HIV to Create Strategies to Fight the Coronavirus?

We live with many viruses that have no vaccine and no cure. One notable example is HIV. Although many effective treatments have been developed, there is still neither a cure nor a vaccine for HIV. Nonetheless, most people do not live in constant fear of HIV, in spite of the fact that it is a deadly and incurable virus. How do we manage this? How do you protect yourself from HIV?

You might answer that you abstain from sex with partners that have not been tested for HIV, or that you use condoms with new sexual partners. These are examples of effective methods that when used correctly prevent or reduce transmission between people. With the coronavirus and HIV, we highlight in Figure 1 mitigation strategies for these two viruses that are somewhat—albeit not perfectly—analogous. On the one hand, both HIV and the coronavirus can be transmitted by people who do not have any symptoms [3, 7, 8, 14], so that both viruses can be invisible threats. On the other hand, the transmission routes for HIV are much more specific and intimate compared with the transmission routes for the coronavirus. Nonetheless, we may be able to use what we have learned in the past forty years fighting HIV and apply it to fight the novel coronavirus.

The good news is: we have very recently obtained a mathematical proof that an HIV-style strategy could work [9]. As with all theoretical mathematics, there are certain caveats that should be mentioned. First, the mathematical model in [9] is rooted in evolutionary game dynamics, that assumes individuals are rational and act in their best self-interest. The model makes no predictions for individuals who do not

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HIV mitigation measure	coronavirus mitigation measure
testing	testing
abstinence	isolation
if you have had sex with someone who may have HIV, abstain from sex until you have a negative test result	stay home if you feel sick or if you have had contact with someone exposed to coronavirus
condom use	face mask use
informing previous sexual partners if you test positive for HIV	contact tracing if you test positive for coronavirus

Fig. 1 There are many similarities between HIV and the coronavirus, like the fact that both viruses can be transmitted by people who show no symptoms. To fight these invisible enemies, effective mitigation measures are also somewhat analogous. Of course, the analogy is far from perfect because these viruses are also quite different. For example, the level of intimacy required to contract HIV compared to the coronavirus is much greater. Nonetheless, we may be able to apply lessons from fighting HIV to battle our new enemy. These mitigation measure analogies are only a few; there may be further analogous measures that have escaped our attention

fit that description. Second, it is currently unknown whether or not infection from the coronavirus and subsequent recovery grants long-term immunity [12, 13, 17]. Our model errs on the side of caution by making no assumption regarding long-term immunity; that is, we assume that immunity is either not conferred or is short-lived.

2 The Disease Dilemma: To Mitigate or Not to Mitigate, That Is the Question

Our mathematical model combines the epidemiological model for the spread of diseases that do not grant lasting immunity together with the game theoretic model for predicting the evolution of human behaviors according to the replicator equation [9]. The epidemiological compartmental model is known as SIS (also known as SI) and has two compartments into which the population is categorized: susceptible and infectious. The classical SIS model does not incorporate human behavioral choices and changes, but human behavioral choices affect the spread of disease. People can choose to change their usual behavior to include mitigation measures to reduce the transmission rate [25]. Moreover, people are not stuck with their choice; they are free to change their behaviors based on their perception of cost versus benefit. The World Health Organisation [5] and numerous other references including [6, 10, 18] argue that it is reasonable to describe this situation with the Prisoner's Dilemma (PD) as depicted in Figure 2.

In the context of coronavirus mitigation, research indicates that the most common transmission route is airborne [11, 24, 26, 27], so that wearing a mask to prevent

 What should I do?	 Wear a mask.	 Don't wear a mask.
 Wear a mask.	If two people both wear masks, they both receive payoffs = R.	If one person wears a mask, their payoff is S, and if the other person doesn't, their payoff is T.
 Don't wear a mask.	If one person wears a mask, their payoff is S, and if the other person doesn't, their payoff is T.	If two people both do not wear masks, they both receive payoffs = P.

Fig. 2 In the 'disease dilemma' people have the choice to cooperate, mitigating the spread of the disease, or defect, making no change to their regular behaviour. This is described by the non-cooperative game shown here in normal form. Image sources and license: openclipart.org, CC0 1.0

coronavirus transmission may be compared to wearing a condom to prevent the transmission of sexually transmitted infections. We therefore focus on the mask mitigation measure. Alice and Bob choose whether to cooperate, by wearing a mask, or defect, by not wearing a mask. If Alice cooperates while Bob defects, then Alice pays the cost of buying the mask, which is represented by $-B<0$. Alice receives some protection from her mask, represented by $\varepsilon>0$, but the main benefit is to everyone around Alice, similar to the reason surgeons wear masks not to protect themselves but to protect the person on the operating table [20]. Consequently, if Bob does not wear a mask, he pays no cost but receives a benefit of $T>\varepsilon>0$. Alice's total payoff is therefore $-B+\varepsilon$. Since the benefit to Alice is relatively small, we assume that $\varepsilon<B$. If both Alice and Bob cooperate, then they both pay the cost $-B$, but they also receive the maximal protective benefit of $T+\varepsilon$, and their total payoffs are thus $T-B+\varepsilon$. Consequently, defining

$$C := B - \varepsilon,$$

the payoffs satisfy

$$S = -C < P = 0 < R < T = R + C. \tag{2.1}$$

This particular representation of the Prisoner's Dilemma is known as the Donor-Recipient game and is given in normal form in Figure 2. To generalize the two player game to model interactions within the entire population, the payoffs (2.1) are modified by a quantity $N(k)$, where k is the average number of social contacts each individual in the society has. Making a standard set of simplifying assumptions as in [15, 16, 21–23], the payoffs R and P remain as in (2.1), whereas the social network structure, that can be interpreted as peer pressure, now modifies the payoffs S and T

$$S = -C + N(k), \quad T = R + C - N(k), \quad N(k) := \frac{Rk - 2C}{(k+1)(k-2)}, \quad k \in \mathbb{N}\backslash\{2\}, \quad N(2) := R. \tag{2.2}$$

2.1 A Dynamical System That Combines Disease Spread with Human Behavior Choices

The hybrid SIS-PD dynamical system we obtained in [9] is

$$\dot{I}(t) = \big([1 - x(t)]\beta_D + x(t)\beta_C\big)I(t)(1 - I(t)) - \gamma I(t), \tag{2.3}$$

$$\dot{x}(t) = x(t)(1 - x(t))[\alpha_1(\beta_D - \beta_C)I(t) - \alpha_2(C - N(k))]. \tag{2.4}$$

Above, the quantities on the left are differentiated with respect to time, t. The frequency of cooperators in the population at time t is $x(t)$, while the frequency of defectors is $1 - x(t)$. The positive quantities $\beta_C < \beta_D$ are the rates of transmission for cooperators and defectors, respectively. The rate at which infected individuals become susceptible again is γ; if D is the average duration of the infection, then $\gamma = 1/D$. The rate of infectious individuals in the population is $I(t)$, and $1 - I(t)$ is the rate of susceptible individuals in the population.

There are three timescales in the model: disease transmission, PD-payoff transmission, and information transmission. The timescale of disease transmission is t, while the PD-payoff timescale is $\alpha_2^{-1}t$ with $\alpha_2 > 0$. The timescale at which individuals receive disease-related information is $|\alpha_1|^{-1}t$. The parameter α_1 may be positive, negative, or zero and describes the frequency and accuracy of information disseminated about the disease. Large values of $|\alpha_1|$ correspond to frequent exposure to information regarding the disease. When $\alpha_1 > 0$, this corresponds to accurate information recommending disease avoidance, whereas when $\alpha_1 < 0$, this corresponds to (mis)-information which may suggest either the disease is harmless or that it is beneficial to contract the disease. For the sake of brevity, we refer to [9] and related work by Poletti et al. [25] for the derivation and justification of the system (2.3) and its ability to accurately represent the spread of disease combined with the influence of human behavior choices.

It is a straightforward exercise to calculate all equilibrium points of the hybrid dynamical SIS-PD system (2.3). The asymptotically stable equilibrium points are summarized in Table 1, with the key values of the information timescale parameter α_1 below

Table 1 These are all of the asymptotically stable equilibrium points of the SIS-PD model (2.3). They depend on the value of α_1 in relation to $\check{\alpha}_1$ and $\hat{\alpha}_1$ that are defined in (2.5). Below, $x^* = \frac{\beta_D}{\beta_D - \beta_C} - \frac{\gamma}{(\beta_D - \beta_C)(1 - I^*)}$ and $I^* = \frac{\alpha_2(C - N(k))}{\alpha_1(\beta_D - \beta_C)}$.

Range of α_1	Unique stable equilibrium point for (2.3) with α_1 in this range
$\alpha_1 < \check{\alpha}_1$	$x = 0, I = 1 - \gamma/\beta_D$
$\check{\alpha}_1 \le \alpha_1 \le \hat{\alpha}_1$	$x = x^*, I = I^*$
$\hat{\alpha}_1 < \alpha_1$	$x = 1, I = 1 - \gamma/\beta_C$

$$\check{\alpha}_1 := \frac{\beta_D}{\beta_D - \gamma} \frac{\alpha_2(C - N(k))}{\beta_D - \beta_C} \quad \text{and} \quad \hat{\alpha}_1 := \frac{\beta_C}{\beta_C - \gamma} \frac{\alpha_2(C - N(k))}{\beta_D - \beta_C}. \tag{2.5}$$

3 An HIV-Style Strategy to Combat Both the Current and Future Pandemics

These results suggest a strategy to fight both diseases that do not to confer immunity as well as new diseases, since it is unknowable whether contracting and recovering from a new disease grants immunity [4]. Mathematically, the strategy is to aim for the equilibrium point with $x = 1$, and $I = 1 - \gamma/\beta_C$, that is obtained if α_1 is sufficiently large, corresponding to frequent reminders of effective mitigation measures. In the limit, $x(t) \nearrow 1$, so the entire population of rational individuals acting in their best self interest cooperates. As mitigation measures become increasingly effective, $\beta_C \searrow \gamma$, so that rate of infectious individuals $I(t)$ tends to zero. Of course, as we learned in analysis, a limit may never actually be reached, but it can be approached. The strategy we suggest, aiming for this limit, consists of two key steps.

Step 1: Study the new virus to understand its particular features and thereby identify effective mitigation measures for this particular new virus. Mathematically, the goal is to determine measures that minimize β_C, the rate of transmission among cooperators.

Step 2: Raise public awareness of the dangers of the disease caused by the virus, similar to the 'Grim Reaper Ad Campaign' used in Australia to fight HIV [1, 2]. In the case of covid-19, the disease caused by the coronavirus, healthy individuals are at risk of sustaining lung damage [28], brain and neural system damage [29–31], organ damage [32], sterility [33], or the worst outcome, death. Conclude the advertisement with a positive and empowering message: a clear explanation of the effective mitigation measures. Mathematically, the ad campaign is used to increase the information transmission parameter α_1, so that the unique stable equilibrium point in the dynamical system (2.3) is that with $x = 1$, and $I = 1 - \gamma/\beta_C$.

3.1 Concluding Remarks: Outsmarting Viruses and Evolving to a New Normal

We have learned to live with HIV and reduce the harm it causes by outsmarting the virus. If the predictions of evolutionary game dynamics hold true, we can learn to live with coronavirus and reduce the harm it causes by outsmarting it with a strategy in the same spirit as the strategy we use to manage HIV.

Julie Rowlett

References

1. *Building on success 1 – a review of gay and other homosexually active men's HIV/AIDS education in Australia*, Commonwealth of Australia, Commonwealth Department of Health and Family Services, 1998.
2. *20 years after Grim Reaper ad, AIDS fight continues*, ABC News Australia (April 4, 2007).
3. R. M Anderson, H. Heesterbeek, D. Klinkenberg, and T D. Hollingsworth, *How will country-based mitigation measures influence the course of the covid-19 epidemic?*, The Lancet **395** (2020/03/21), no. 10228, 931–934.
4. A. L. Barthold S.W. & Smith, *Duration of challenge immunity to coronavirus JHM in mice*, Archives of Virology **107** (1989), 171–177.
5. C. Blouin, N. Drager, and R. Smith, *International trade in health services and the GATS: current issues and debates*, The World Bank, 2006.
6. E. Fukuda and J. Tanimoto, *Impact of stubborn individuals on a spread of infectious disease under voluntary vaccination policy*, Proceedings of the 18th Asia Pacific Symposium on Intelligent and Evolutionary Systems, Volume 1. Proceedings in Adaptation, Learning and Optimization **1** (2015).
7. M. Gandhi, D. S. Yokoe, and D. V. Havlir, *Asymptomatic transmission, the achilles' heel of current strategies to control covid-19*, New England Journal of Medicine **382** (2020), no. 22, 2158–2160, available at https://doi.org/10.1056/NEJMe2009758.
8. X. He, E. H. Y. Lau, P. Wu, X. Deng, J. Wang, X. Hao, Y. C. Lau, J. Y. Wong, Y. Guan, X. Tan, X. Mo, Y. Chen, B. Liao, W. Chen, F. Hu, Q. Zhang, M. Zhong, Y. Wu, L. Zhao, F. Zhang, B. J. Cowling, F. Li, and G. M. Leung, *Temporal dynamics in viral shedding and transmissibility of covid-19*, Nature Medicine **26** (2020), no. 5, 672–675.

9. C.-J. Karlsson and J. Rowlett, *Decisions and disease: a mechanism for the evolution of cooperation*, Scientific Reports **10** (2020), no. 1, 13113.
10. P. Kaushik, *Covid-19 and the Prisoner's Dilemma*, 2020.
11. S. L, P. F, D. G. G, B. P, P. MG, B. M, P. J, D. G. A, P. P, and M. A, *Airborne transmission route of covid-19: Why 2 meters/6 feet of inter-personal distance could not be enough*, Int J Environ Res Public Health **17** (2020), no. 8.
12. T. Liu, S. Wu, H. Tao, G. Zeng, F. Zhou, G. Fangjian, and X. Wang, *Prevalence of IgG antibodies to SARSCoV-2 in Wuhan - implications for the ability to produce long-lasting protective antibodies against SARS-CoV-2*, medRxiv (2020).
13. Q.-X. Long, X.-J. Tang, Q.-L. Shi, Q. Li, D. Hai-Jun, J. Yuan, J.-L. Hu, X. Wei, Y. Zhang, L. Fa-Jin, K. Su, F. Zhang, J. Gong, B. Wu, X.-M. Lui, J.-J. Li, J.-F. Qui, J. Chen, and A.-L. Huang, *Clinical and immunological assessment of asymptomatic SARS-CoV-2 infections*, Nature Medicine (2020).
14. H. Nishiura, T. Kobayashi, T. Miyama, A. Suzuki, S.-m. Jung, K. Hayashi, R. Kinoshita, Y. Yang, B. Yuan, A. R. Akhmetzhanov, and N. M. Linton, *Estimation of the asymptomatic ratio of novel coronavirus infections (covid-19)*, International Journal of Infectious Diseases **94** (2020/05/01), 154–155.
15. H. Ohtsuki and M. A. Nowak, *Evolutionary games on cycles*, Proc. Royal Soc. B **273** (2006), no. 1598, 2249–2256.
16. ——, *The replicator equation on graphs*, J. Theor. Biol. **243** (2006), no. 1, 86–97.
17. M. Pollán, B. Pérez-Gómez, R. Pastor-Barriuso, J. Oteo, M. A. Hernán, M. Pérez-Olmeda, J. L. Sanmartín, A. Fernández-García, I. Cruz, N. Fernández de Larrea, M. Molina, F. Rodriguez-Cabrera, M. Martin, P. Merino-Amador, J. L. Paniagua, J. F. Muñoz Montalvo, F. Blanco, and R. Yotti, *Prevalence of SARS-CoV-2 in spain (ENE-COVID): a nationwide, population-based seroepidemiological study*, The Lancet (2020).
18. E. T Richardson, M. B. Barrie, C. T Nutt, J D. Kelly, R. Frankfurter, M. P Fallah, and P. E Farmer, *The Ebola suspect's dilemma*, The Lancet Global Health **5** (2017), no. 3, e254–e256.
19. D. Romer, S. Sznitman, L. F. Salazar, P. A. Vanable, M. P. Carey, M. Hennessy, L. K. Brown, R. F. Valois, B. F. Stanton, T. Fortune, and I. Juzang, *Mass media as an hiv-prevention strategy: Using culturally sensitive messages to reduce hiv-associated sexual behavior of at-risk african american youth*, American Journal of Public Health **12** (2009), no. 99, 2150–2159.
20. R. O. J. H. Stutt, R. Retkute, M. Bradley, C. A. Gilligan, and J. Colvin, *A modelling framework to assess the likely effectiveness of facemasks in combination with 'lock-down' in managing the covid-19 pandemic*, Proceedings of the Royal Society A **476** (2020).
21. J. Tanimoto, *A simple scaling in the effectiveness of supporting mutual cooperation in donor-recipient games by various reciprocity mechanisms*, BioSystems **96** (2009), 29–34.
22. ——, *Fundamentals of evolutionary game theory and its applications*, Evolutionary Economics and Social Complexity Science, vol. **6**, Springer, 2015.
23. M. A. Taylor C. & Nowak, *Transforming the dilemma*, Evolution **61** (2007), no. 10, 2281–2292.
24. R. Zhang, Y. Li, A. L. Zhang, Y. Wang, and M. J. Molina, *Identifying airborne transmission as the dominant route for the spread of covid-19*, Proceedings of the National Academy of Sciences **117** (2020), no. 26, 14857–14863, available at https://www.pnas.org/content/117/26/14857.full.pdf.
25. P. Poletti, B. Caprile, M. Ajelli, A. Pugliese, and S. Merler, *Spontaneous behavioural changes in response to epidemics*, Journal of Theoretical Biology, **260** (2009), no. 1, 31–40.
26. N. R. Jones, Z. U. Qureshi, R. J. Temple, J. P. J. Larwood, T. Greenhalgh, and L. Bourouiba, *Two metres or one: what is the evidence for physical distancing in covid-19?* BMJ **370** (2020), available at https://doi.org/10.1136/bmj.m3223.
27. L. Bourouiba, *Turbulent gas clouds and respiratory pathogen emissions: potential implications for reducing transmission of COVID-19*, JAMA **323** (2020), no. 18, 1837–1838.

28. Y. Wang, C. Dong, Y. Hu, C. Li, Q. Ren, X. Zhang, H. Shi, and M. Zhou, *Temporal changes of CT findings in 90 patients with COVID-19 pneumonia: a longitudinal study*, Radiology, **296** (2020), no. 2, E55–E64.
29. N. Kanberg, N. J. Ashton, L.-M. Andersson, A. Yilmaz, M. Lindh, S. Nilsson, R. W. Price, K. Blennow, H. Zetterberg, and M. Gisslén, *Neurochemical evidence of astrocytic and neuronal injury commonly found in COVID-19*, Neurology (2020), available at https://doi.org/10.1212/WNL.0000000000010111.
30. R. W. Paterson, R. L. Brown, L. Benjamin, R. Nortley, S. Wiethoff, T. Bharucha, D. L. Jayaseelan, G. Kumar, R. E. Raftopoulos, L. Zambreanu, V. Vivekanandam, A. Khoo, R. Geraldes, K. Chinthapalli, E. Boyd, H. Tuzlali, G. Price, G. Christofi, J. Morrow, P. McNamara, B. McLoughlin, S. T. Lim, P. R. Mehta, V. Levee, S. Keddie, W. Yong, S. A. Trip, A. J. M. Foulkes, G. Hotton, T. D. Miller, A. D. Everitt, C. Carswell, N. W. S. Davies, M. Yoong, D. Attwell, J. Sreedharan, E. Silber, J. M. Schott, A. Chandratheva, R. J. Perry, R. Simister, A. Checkley, N. Longley, S. F. Farmer, F. Carletti, C. Houlihan, M. Thom, M. P. Lunn, J. Spillane, R. Howard, A. Vincent, D. J. Werring, C. Hoskote, H. R. Jäger, H. Manji, M. S. Zandi, and the UCL Queen Square National Hospital for Neurology and Neurosurgery COVID-19 Study Group, *The emerging spectrum of COVID-19 neurology: clinical, radiological and laboratory findings*, Brain (2020), available at https://doi.org/10.1093/brain/awaa240.
31. J. Helms, S. Kremer, H. Merdji, R. Clere-Jehl, M. Schenck, C. Kummerlen, O. Collange, C. Boulay, S. Fafi-Kremer, M. Ohana, M. Anheim, and F. Meziani, *Neurologic features in severe SARS-CoV-2 infection*, New England Journal of Medicine **382** (2020), no. 23, 2268–2270.
32. D. McGonagle, J. S. O'Donnell, K. Sharif, P. Emery, and C. Bridgewood, *Immune mechanisms of pulmonary intravascular coagulopathy in COVID-19 pneumonia*, The Lancet Rheumatology **2** (2020), no. 7, e437–e445.
33. M. Yang, S. Chen, B. Huang, J.-M. Zhong, H. Su, Y.-J. Chen, Q. Cao, L. Ma, J. He, X.-F. Li, X. Li, J.-J. Zhou, J. Fan, D.-J. Luo, X.-N. Chang, K. Arkun, M. Zhou, and X. Nie, *Pathological findings in the testes of COVID-19 patients: clinical implications*, European Urology Focus **6** (2020), no. 5, 1124–1129.

Orchestral Turbulence

Raymond O. Wells Jr.

I have spent most of my career as an active mathematician, and now, in my retirement years, I find myself in a perplexing situation. Not too long ago (in June of 2020), I was elected president of the Board of Directors of the Boulder Philharmonic Orchestra in Boulder, Colorado, where we have lived for more than a decade. To be elected to such an office in the middle of the coronavirus pandemic is quite unsettling, to say the least. When we planned the new slate of officers for the orchestra some four months earlier, no one had any idea that we were going to be confronted with this bizarre situation.

The orchestra has canceled its last two spring concerts, has had to raise emergency funds to help pay the musicians for the canceled performances, and is scrambling to plan for a 2020–2021 season, when the usual venues are closed due to the pandemic. Moreover, two staff members have been laid off, and the cash flow has changed dramatically. This is indeed a turbulent time for the orchestra.

Furthermore, not long ago, George Floyd was murdered in Minneapolis. This has caused additional national turmoil as well as the rise of the Black Lives Matter movement. This too is affecting most of American society. Finally, the question of opening schools in the fall, or how that might happen, is another part of the current national debate. All three of these national issues: the pandemic itself, the racism discussion our society is having, and the effect of the pandemic on our schools, have made an impact on our orchestra in different ways, as I describe in this essay.

The pandemic effect on the orchestra is a very complex problem, and our board and dedicated staff are working hard on coming up with virtual concerts using various digital media which are available these days. It is quite difficult, and it is certainly not clear whether the decisions we are making now will be successful for the future of this sixty-two-year-old professional orchestra. The situation is

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Fig. 1 The Boulder Philharmonic Orchestra

incredibly difficult for the musicians of our orchestra, all of whom are part-time employees (with union contracts), most of whom perform gigs with other musical organizations in the area, all of which have the same problems we do. Some such performing arts groups have simply canceled the 2020–2021 season completely.

I have been involved in other non-profit civic and arts organizations before, but this situation is certainly the most challenging I have ever faced personally. As they say, we shall see what we will see.

What is interesting is to see the creativity of our artists and administrators. Our music director, Michael Butterman, created a YouTube video of the last movement of the Firebird Suite by Igor Stravinsky, which was supposed to have been performed at our concert in March. It was quite amazing and well done. It portrayed all of the musicians performing in their own homes, with the music being synchronized for the broadcast. The video showed closeups of individual musicians (e.g., several trumpet players being highlighted in the video) and then zoomed out to show all the performers at the same time and many variations of this idea. For much of the video, the music director was shown conducting, at times highlighted but mostly as one of several images on the the screen. He was also conducting from his own home! It had a magical feel and represented so well the times we are in.

I talked to Michael recently and asked how he did it, and what was it like to be conducting to an empty room. He explained the process, and I found it fascinating. Here’s what he told me. First, the sound and video engineer who works with the orchestra and helped create the video found a recording the orchestra and Michael had made some years ago of the Firebird Suite. Then Michael was filmed conducting to the sound track of the recording he had made (he knew what was going to happen, moment by moment). Then the video of his conducting and the sound track were sent to the musicians so that each of them could hear (via earphones) the performance and see the conductor (via a monitor) as they performed their individual parts, which were recorded. The engineer and one of the members of the orchestra compiled the audio and video recordings to create a composite performance.[1] Yes, Michael said, it was weird to be conducting to an empty room!

We are planning what looks like an all digital season (for the fall for sure), and a sticky part has been the union contracts for our professional musicians. How do you compensate for digitally streamed performances versus live auditorium performances? Our executive director and the union representatives have finally come to an agreement after more than a month of negotiations at both the local and the national level. We are now able to offer a digital season for our subscribers, and we have had to make some substantive compromises to be able to make this happen.

Another problem concerns our subscribers and our patrons. We had season ticket subscribers who didn't get to see the last two performances of the season, and we gave them several options. The vast majority donated the value of their tickets back to the orchestra, which increased our overall donations at the time. Moreover, we also had fewer expenses (e.g., rental of the 2000-seat auditorium on the university campus). Both of these facts did help us on cash flow at the time. So far our major donors are sticking with us. Now we are announcing our digital season, and we will soon see how many of our subscribers will stay loyal to us through these trying times.

The Black Lives Matter movement affected us as well. At a meeting where we were revising our by-laws and updating our mission statement, one of our board members became very passionate and he questioned, in so many words, whether we can just revise our mission statement and say we are going to perform music when the turmoil around us was asking us to do more. He argued that we should have a value statement as well as a mission statement. His passion was well received, and our board adopted a new value statement, which I am sharing here in the form that was sent to the Boulder Philharmonic Orchestra (Boulder Phil) community in an e-broadcast.

To the Boulder Phil community:

The Boulder Phil joined our colleagues across the music world in pausing our music yesterday to hear the important black voices speaking now, the voices calling for justice in America. Pausing one day was not enough; we must keep listening – and elevating and acting – today, tomorrow, and for every tomorrow after, until all voices can rise up together in a just land.

The path to justice begins with empathy, with our ability, however imperfectly, to imagine ourselves in the places of others, to fundamentally recognize our common humanity. Music is empathy in sound. To put our lives into tones and share them with another is a profound way to know and be known. Music seeks a shared vision to be better.

Music is aspirational, and we shall be too.

Boulder Philharmonic Orchestra

[1] You can see the video here: https://www.youtube.com/watch?v=nQbkkVzDfOo.

Fig. 2 The Boulder Phil brass ensemble in a school auditorium

It was an interesting and important moment for all of us. Since that time our orchestra is taking additional steps to address the issues of diversity and equity in our orchestra and our community. This is not easy, but it is very important for all of us to strive for improvement in the long standing inequities in our society.

The Boulder Phil has prided itself for at least a decade on its interaction with schools and students in the Boulder area. Approximately 10,000 students each year (primarily elementary and middle school students) get to experience either a special concert with Maestro Michael in the large symphony hall (about 4,000 students in four special concerts over two days), or they meet small ensembles (brass, winds, strings, etc.) in their classrooms. It's a powerful program that we are very proud of, and major funding has been provided on a regular basis by several major benefactors in Boulder.

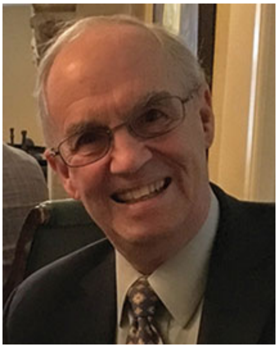
With the question of opening of schools and the nonavailability of our large symphony hall, we are faced for our education and outreach program with the same sort of problems that our regular orchestral programs are facing. Our education and community engagement director, Sara Parkinson, and her collaborators in the schools are coming up with creative solutions to this problem. I have seen some samples of this, and they are doing a great job. Some of these innovations will be new ways to interact with the students even after the schools have opened and we have some sense of normality in our lives.

What have we learned from this experience? First and foremost is that in interacting digitally with artists and audiences, it became clear that there can be powerful interactions between performers and patrons not possible in a performance or even a recital setting. One example that has evolved has been our Events of Note, a recital series with a guest artist, usually in a private home. We have had several such events post pandemic onset via Zoom and YouTube where the artist is a part of a Zoom discussion before, during and after watching a YouTube playlist with presentations of music by the artist, performed for the occasion.[2] The discussions are vivid and all can participate, whereas at the wine and cheese part of the event in the private home only a few get to talk to the artist at a time. One important point for the orchestra, financially, is that our patrons are paying for these events. just as they did for recitals at a private home (not quite as much, of course).

For instance, in May, we had a well-known violinist in a Zoom chat from her apartment in New York. She was one of the soloists whose concert was canceled. After the introductions and after listening to her perform several pieces of music, she was asked how she was doing in New York with the pandemic. She replied that she knew 10 people personally who had died from the virus and she was having to wash her clothes in her bathtub, due to her fear of using the laundry room or a laundromat, it was difficult to get salt or butter, etc. It was a tough time in New York! Her short personal monologue was very vivid for all of us in Boulder. The more recent recitals have not been quite as dramatic! But they have been equally moving and personal.

I think the digital version of sharing music will be a strong supplement to our normal concerts and recitals, and we are only now learning how to do this.

My hope is that, after sixty-two years of being an important, and beloved by many, part of Boulder, the Boulder Philharmonic Orchestra will continue to fulfill its mission of providing superb orchestral music to its community. I am personally very pleased to be a part of trying to make this happen.



Ronny Wells

[2]The Zoom quality has not been sufficient for broadcasting the music as part of the Zoom discussion, but we are sure this will be coming.

Applied Mathematics in the Time of Corona: A Survival Guide

Alain Goriely

"I think that a mathematician is well suited to be in isolation"
Attributed to Sophus Lie, after his prison release
following a wrongful arrest in Paris (1870).

1 A Looming Storm

When and where did you first hear about COVID-19? This may not be a vivid memory like the time you heard about 9/11. Yet, we can all remember when the importance of this new disease first entered our consciousness for good. For me, it was early January. I was visiting my friend Yibin Fu in Tianjin and in the middle of my visit I was told not to worry. "Not to worry about what?" I asked, very worried. "There is a new disease, it is like SARS, people die, but it is confined to Wuhan, more than a 1,000km away. Nothing to worry about". Shortly after my return, the academic drums started to beat. First, a distant hum, the noise would soon be deafening. Robin Thompson, a bright young researcher in mathematical epidemiology in our department wrote one of the first papers [5] on the disease and gave, by the end of the month, an overcrowded talk about it. The picture was clear, with a reproduction number between 2 and 3, mortality rates hovering around 1%, and no natural immunity, the virus had all the potential to become a massive epidemic. But, if confined to China, there is really nothing to worry about, I reasoned naively. The following month would see the dominos falling one after the other. Travel bans were established, new individual cases popped up around the world, the first serious outbreaks appeared in Northern Italy, and panic took over populations and governments alike. The rest, as they say, is history. By mid February, we could clearly see the storm looming over the British shores. An extensive lockdown was inevitable. We would soon pack our books and papers and stay home for an indefinite period of time. Filled with a sense of dread, frustration, and impotence, my thoughts kept

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returning to the same questions: How can we, as a modeling community, help? How can I, as an individual, help?

2 The Shortcomings of Academia

There is no doubt that modeling has a clear and important role to play in such circumstances. Microscopes and telescopes allow us to probe the smallest and largest scales of the universe. Mathematical modeling is the only scientific crystal ball that we have to look at possible futures. It turns out that epidemiological models are the bread and butter of applied mathematics with a very large literature and striking successes to pandemic control, as I had already showcased in my little book [3]. Large specialized groups around the world worked on the problem and, at first, it was not clear that unsolicited help would actually improve the situation. I know many of them and admire their work. The problem was in good hands, I thought. Yet, soon enough, three main problems emerged that exposed clear shortcomings of our regular academic system in a time of crisis.

First, basic epidemiological models, such as the Suscetible-Infected-Recovered model, are deceptively simple to understand and code. Anyone who ever took an undergraduate course in differential equations can set up the system within minutes on a home computer and produce nice-looking curves showing the rise and fall of an infected population without realizing that the real problem is to get the correct parameters from the data and to properly extend such models for the current crisis. This apparent simplicity created a tsunami of low-quality preprints that soon clogged the peer-reviewing system.

Second, there was no discussion, hence no clear consensus, across the multiple models developed by the leading groups. Typically, such consensus arises through the long multi-year academic cycle of publications, exchanges, testing, validation, and scientific meetings. The usual time frame for proper scientific debate, like the one for the development of a vaccine, was not suitable for the crisis.

Third, new reports were produced constantly by university research groups, by companies, by government departments. At the government level, how could the proper advice be given when so many reports flooded the system? Which ones were good enough to inform policy decisions?

In the UK, a group of modelers attached to the venerable institution of the Royal Society decided to address this problem by creating a new national initiative for the Rapid Assistance in Modelling the Pandemic headed by Mike Cates in Cambridge. RAMP was born and the call for volunteers was soon answered by thousands of scientists around the country and around the world. In Oxford, Philip Mani and I were soon volunteered to lead the effort. Of particular interest to RAMP was an ongoing effort into a systematic literature review initiated by Robin Thomson and Eamonn Gaffney. Soon, Philip and I were tasked by RAMP to set up a new mechanism to quickly review models, softwares, and reports of possible interest to the various scientific committees and departments advising the government. The

Rapid Review Group that we created is divided into 6 different subject panels and staffed by an all-star team of 120 dedicated expert volunteers. It stands ready to assess critical scientific work with a typical turnaround of 24 to 48 hours, bypassing the typical monthlong process offered by scientific journals. As the pandemic tides rose, every sector of the economy was in need of some form of modeling and the demand surged. Our group of volunteers, fueled by the collective ideal of scientific quality has been busy churning out new reviews to meet the demand and plans to continue its work through 2021. This new structure provides a shortcut to the academic cycle by both filtering out irrelevant work and giving further support to quality work. The way I see it, our small contribution is to maintain the highest level of academic standards in the midst of a crisis. Personally, the process was initially exhausting, but it gave me a small window of observation into the shifting interests of the government and allowed me to develop a better understanding of the entire field. It also made me realize that, now more than ever, there is a need for thorough review, critical assessment, and quality scientific advice. Whether or not scientific advice is heard and acted upon by governments is another story, one that still needs to be written.

3 My Body for Science

Living in Oxford provided me with another unexpected way to contribute. Oxford has always been a centre of excellence for the development of new vaccines and it was no surprise that the Jenner Institute was the first to come up with a new vaccine candidate, ChAdOx1 nCov-19, based on their earlier work on the MERS and SARS vaccines [6]. By April, researchers from the Jenner Institute announced a phase 2 safety trial and asked for volunteers, soon followed by a phase 3 efficacy trial. Like everyone else, I was in lockdown at the time. There are only so many papers one can write and breads one can bake every day and I soon realized that apart from organizing rapid reviews my intellectual skills were not really needed for the crisis. But maybe, I thought, science could make some use of my body and I jumped on the opportunity. I enrolled in the trials and was eventually selected. Was my goal purely selfish or purely altruistic? Probably neither (or both). Clearly, the vaccine could offer some protection, but this assumes that you actually get the real stuff rather than the common meningitis vaccine in this randomized trial (about a fifty-fifty percent chance). It also assumes that the vaccine will actually work. Early indications are positive but the trial in the UK, with 12,330 participants, will run until August 2021. We will probably not know its outcome before January 2021, at the earliest. No, my main motivation was mostly intellectual. I found the process and the stakes fascinating and I wanted to be part of it so that I could experience it first hand, in the same way that I spent a day in neurosurgery when I first became interested in the brain. The abstract world that we build through modeling needs to be anchored in reality and the trial was a chance to ground myself. I also enjoy the poetic justice of self-metamorphising into a datum after having spent so many years abusing data.

The Phase 2 (safety) trial shows promising result [1]. Yes, the vaccine is safe and does trigger an immune response. In their paper, the authors list the standard side effects following the injection of ChAdOx1 compared to Meningitis. They report the fraction of people experimenting a given symptom for either vaccine. Now, here is an interesting Bayesian problem. If I have experienced a given symptom but not others, what is the probability that I have received the COVID vaccine? Many family members and friends are also part of the trial. Some have experienced some of these symptoms, some haven't. As you can imagine, this possibility of assigning probability leads to interesting (socially-distanced) conversations around the dinner table. Yet, I will refrain myself from actually assigning probability as it would partially unblind the experiment and risk skewing the results. A good datum knows its place and the exercise is left to the reader.

4 The Newfoundland Story

Through the reviewing process and a nearly morbid fascination for data and graphs related to the evolution of the crisis, I became increasingly acquainted with many of the scientific challenges related to COVID. Yet, I was still reluctant to jump into the fray, assuming that better researchers are on it. Too many cooks spoil the broth. However, my long-time friend and collaborator, Ellen Kuhl from Stanford, did not share my restraints. As an extreme athlete, a marathoner, a triathlete and an iron-(wo)man, Ellen is fearless. Early on, she realized that the methods we had developed to model the propagation of toxic proteins on the brain connectome [2] could be readily adapted to the evolving crisis. Ellen combines a unique ability for modeling, amazing technical skills, and a great intuition for good problems. She quickly built elegant data-driven models for the spread of the diseases around the world. Eventually, she convinced me to collaborate on a couple of COVID projects, one of which would become the central evidence of a case in front of the Supreme Court of Newfoundland and Labrador [4].

The island of Newfoundland is part of the Canadian province of Newfoundland and Labrador. Following a travel ban on May 5, 2020, this Atlantic province enjoyed the rather exceptional and enviable position of having the coronavirus pandemic under control. By July 3, 2020, it had a cumulative number of 261 cases, with 258 recovered, 3 deaths, and no new cases for 36 days. The same day, the Atlantic Bubble opened to allow air travel between the four Atlantic Provinces, Newfoundland and Labrador, Nova Scotia, New Brunswick, and Prince Edward Island, with no quarantine requirements for travelers. With respect to COVID, the inhabitants of the province are in a dangerous position as they have the highest rates of obesity, metabolic disease, and cancer nationally, and an unhealthy lifestyle with the highest rate of cigarette smoking among all provinces. Despite its success in eliminating the virus, the government found itself in a precarious position. Its travel ban, Bill 38, was being challenged by a Halifax resident who was denied entry for her mother's funeral in the Spring and the lawsuit was further supported by the Canadian Civil

Liberties Association. They are seeking a declaration from the provincial Supreme Court in St John's that the travel ban is unconstitutional, a decision that could apply to the entire country. Determined to keep control of its borders, the Office of the Attorney-General reached out to Ellen. Would her models be applicable to this situation? What would happen during gradual or full reopening under perfect or imperfect quarantine conditions?

Ellen and I had been talking about a hypothetical problem like this one. If the virus is eliminated from a region, can it come back, like a boomerang, when restrictions are eased? Newfoundland seemed to be the perfect case study for us, and with the help of her outstanding Post-doc, Kevin Linka and Dr Proton Rahman, a clinical epidemiologist and professor of medicine at Memorial University of Newfoundland, we jumped at the opportunity to test some of our ideas. Soon, we converged on a network model where each node represents a US state or a Canadian province. On each node, we run a local Suscetible-Exposed-Infected-Recovered epidemiological model and model air traffic by a graph Laplacian-type transport process as commonly done for network transport. Parameters are estimated by Bayesian inference with Markov-chain Monte Carlo sampling using a Student's t-distribution for the likelihood between the reported cumulative case numbers and the simulated cumulative case numbers (Fig. 1).

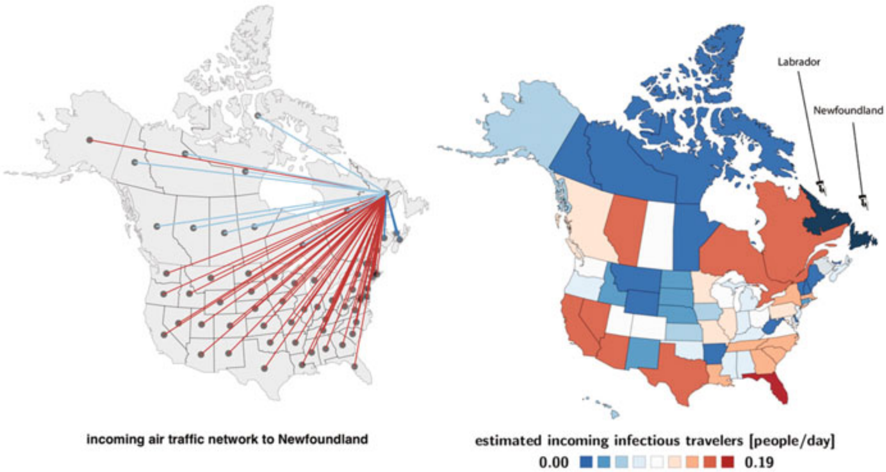


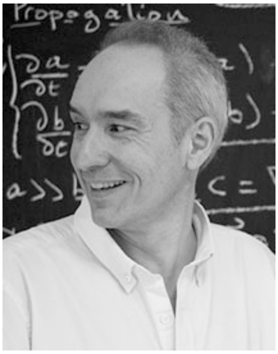
Fig. 1 Left: Mobility modeling. Discrete graphs of the Atlantic Provinces, of Canada, and of North America with 4, 13, and 64 nodes that represent the main travel routes to Newfoundland and Labrador. Dark blue edges represent the connections from the Atlantic Provinces, light blue edges from the other Canadian provinces and territories, and red edges from the United States. Right: Estimated COVID-19 infectious travelers to Newfoundland and Labrador. Number of daily incoming air passengers from the Canadian provinces and territories and the United States that are infectious with COVID-19. Figures adapted from [4]

Conceptually, the model is quite simple. I have a natural preference for parsimony when it comes to modeling complex phenomena as the assumptions are completely known and in full display. This is a personal choice and the outcomes of such models should be seen as estimates rather than hardcore forecast. What we found is quite interesting. Using air traffic information from the previous 15 months, we showed that opening Newfoundland to the Atlantic provinces or the rest of Canada would have negligible effects on the evolution of the disease as prevalence dropped considerably in Canada. Yet, opening the airports to the USA would lead to 2-5 infected passengers entering the island a week, with as many as 1-2 asymptomatic travelers. Without an air-tight quarantine system, the disease would infect 0.1% of the Newfoundland population within 1 to 2 months.

In the first week of August, evidence were presented to the court. The Chief Medical Officer of Health Dr. Janice Fitzgerald opened with the following quote: "*In 1775 the American revolutionary Patrick Henry declared, 'Give me liberty or give me death.' In this case, if the applicants' remedy is granted, it will result in both.*" The same week Proton testified in court about our model, its assumptions, and our findings. To my surprise, the scientists were heard and on 17 September, the judge rendered his verdict. In his ruling, Justice Burrage declared that *"The upshot of the modelling ... is that the travel restriction is an effective measure at reducing the spread of COVID-19 in Newfoundland and Labrador.*" He concluded that yes, the ban was legal and justified. Having an impact on the lives of Newfoundlanders, however small, is a strange but rather pleasant feeling.

5 Final Proof

The lockdown has been difficult for most people. Despite creating a feeling of helplessness, it has been also a time of personal reflection. Like many, I took the opportunity to build my bread-baking skills and ended producing a couple of loaves every other day for the last few months. The process is both soothing and fascinating. As days go by, it started to dawn on me that bread making is very much like mathematical modeling. It is a process that deeply relies on science yet is so complicated that craft, techniques, and tricks are necessary ingredients. Like modeling, baking cannot be taught but has to be practiced. You can only learn by putting your hands in the dough, literally and figuratively. The beauty of the living dough and the evolving model is that you'll never quite know what you'll end up with. At the end of the last proof, your creation can be a thing of beauty and pride or a miserable failure. Yet, the drive and curiosity that get me out of bed in the early morning are the same, and the pleasure of breaking the crust with my family or sharing mathematics with friends and colleagues, a source of constant pleasure.



Alain Goriely

References

1. Pedro M Folegatti, Katie J Ewer, Parvinder K Aley, Brian Angus, Stephan Becker, Sandra Belij-Rammerstorfer, Duncan Bellamy, Sagida Bibi, Mustapha Bittaye, Elizabeth A Clutterbuck, et al. Safety and immunogenicity of the chadox1 ncov-19 vaccine against sars-cov-2: a preliminary report of a phase 1/2, single-blind, randomised controlled trial. *The Lancet*, 2020.
2. Sveva Fornari, Amelie Schäfer, Ellen Kuhl, and Alain Goriely. Spatially-extended nucleation-aggregation-fragmentation models for the dynamics of prion-like neurodegenerative protein-spreading in the brain and its connectome. *Journal of Theoretical Biology*, 486:110102, 2020.
3. Alain Goriely. *Applied mathematics: a very short introduction*. Oxford University Press, 2018.
4. Kevin Linka, Proton Rahman, Alain Goriely, and Ellen Kuhl. Is it safe to lift covid-19 travel bans? the newfoundland story. *Comp. Mech.*, 2020.
5. Robin N Thompson. Novel coronavirus outbreak in wuhan, china, 2020: intense surveillance is vital for preventing sustained transmission in new locations. *Journal of clinical medicine*, 9 (2):498, 2020.
6. Neeltje van Doremalen, Teresa Lambe, Alexandra Spencer, Sandra Belij-Rammerstorfer, Jyothi N Purushotham, Julia R Port, Victoria A Avanzato, Trenton Bushmaker, Amy Flaxman, Marta Ulaszewska, et al. Chadox1 ncov-19 vaccine prevents sars-cov-2 pneumonia in rhesus macaques. *Nature*, pages 1–8, 2020.

Glimpses at Corona: La Boqueria and Notre Dame

Alexander J. Hahn

The setting is the crowded Mercat de la Boqueria in Barcelona, Spain. It is by reputation one of the world's best fresh food markets (Figure 1). The history of the market goes back to the 13th century when street vendors, peasants, and nearby farmers would come to sell their wares from makeshift, open-air stalls in the city's center. The name is believed to derive from the language of the region. In Catalan, *boc* means "goat", so that a *boqueria* would be a place where goat meat is sold. The many shoppers and tourists who walk into La Boqueria are immersed in a world of delicate scents, vibrant colors, and rich flavors. In the central part of the market, they'll find the rich seafood displays, and toward the back many of the butcher and delicatessen shops, offering aromatic meats, hams and sausages. Completing this mosaic are the displays of the scores of fruit vendors—each in just a few available square meters—with varieties of apples, pears, peaches, plums, apricots, cherries, grapes, and oranges. In competition with each other, they vie for the attention of locals and tourists who press ahead from stall to stall. One of several merchants sells Valencia oranges. His fruit is large, round, polished, and beautiful. Arranged in a large pyramid, they glow rich in color and light. A sign proclaims them to be "extra dulce" and a larger sign announces that they cost 0.99 euros per kilogram. See Figure 2. An excited boy reaches to pull an orange from the bottom of the stack . . . his mother, recognizing an impending disaster, stops him just in time. The boy's interest in the oranges now turns to the question: "how many oranges are there in this pile, mama?" Mama shrugs, but tells him that this is a question for his older sister. The older sister, a mathematics student at the Universitat de Barcelona, reflects: "hmm, how many oranges might there be? Can their number be estimated?" After some back and forth with her brother, she assumes that the pyramid consists of oranges through and through and that there is no inner structure of wood or

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Fig. 1 Tourists and natives busily shopping on September 13, 2009 at the La Boqueria market in Barcelona. Image credit and permission from *FeaturePics*

Fig. 2 A carefully arranged display of oranges at the La Boqueria market. Many thanks to the enthusiastic, fun-loving world traveler Alexandra Kovacova for posting this image on her informative and richly illustrated website https://www.crazysexyfuntraveler.com/la-boqueria-market-in-barcelona/ and for permitting its use in this article

cardboard that supports the display. She regards the oranges at the very top of the stack to be rearranged in two horizontal layers, counts the oranges in the horizontal rows of each of the two rising sides of the pile, and lists the resulting numbers as

$$13, 13, 12, 12, 11, 11, 10, 10, 9, 9, 8, 8 \text{ and } 13, 13, 12, 12, 11, 11, 10, 10, 9, 9, 8, 8$$

on a sheet of paper. Her brother counted also and objects that his sister's count is not accurate. His sister reassures him, "we're only looking for a good estimate, and you'll see the method behind my choice of numbers in a moment." In her approach to the question, she has replaced the stack in front of her by a streamlined pyramid that has a flat top, and consists of 12 horizontal layers of oranges, all arranged in squares. Starting at the bottom of the pyramid and going up, the sizes of these squares, in terms of the oranges that they contain, are

$$13 \times 13,\ 13 \times 13,\ 12 \times 12,\ 12 \times 12, \ldots,\ 9 \times 9,\ 9 \times 9,\ 8 \times 8,\ 8 \times 8.$$

Her brother agrees that the number of oranges in the merchant's stack is approximated by the sum,

$$\begin{aligned} 8^2 + 8^2 + 9^2 + 9^2 + 10^2 + 10^2 + 11^2 + 11^2 + 12^2 + 12^2 + 13^2 + 13^2 \\ = 2(8^2 + 9^2 + 10^2 + 11^2 + 12^2 + 13^2). \end{aligned}$$

Our college student knows the sum of squares formula

$$1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6},$$

and explains it to her brother. Using it with $n = 13$ and then once more with $n = 7$, she concludes that the streamlined pyramid contains

$$2\left(\frac{(13)(14)(27)}{6} - \frac{(7)(8)(15)}{6}\right) = 2(13 \cdot 7 \cdot 9 - 7 \cdot 4 \cdot 5) = 2(819 - 140) = 1,358$$

oranges. This is the older sister's approximation of the number of oranges in the stack. Since it is hard to imagine that anyone would actually stack so many oranges, the two agree that the merchant's pile must have an internal scaffolding structure that supports the oranges on display, and that with the exception of the top of the pile, the oranges that are visible are more or less all of them. Using the streamlined version of the stack once more, brother and sister compute the number of oranges in the first ten rows of each of the two slanting sides as

$$\begin{aligned} 13 + 13 + 12 + 12 + 11 + 11 + 10 + 10 + 9 + 9 \\ = 22 + 22 + 22 + 22 + 22 = 5(22) = 110. \end{aligned}$$

Since the oranges of the rising edge that the two slanting sides share are counted twice, the first ten rows of the two sides contain $110 + 110 - 10 = 210$ oranges.

Adding the 100 or so oranges at the top of the stack, they conclude that there are about 300 oranges in the display.

Our story about Barcelona's Mercat de la Boqueria has illustrated some key features of a successful mathematics classroom. After all, at its best, a classroom is a marketplace of ideas. In the same way that the merchant emphasizes the quality of his oranges, the instructor of a mathematics course needs to make a strong case for the relevance of the subject. In the same way that the merchant displays his Valencias in an attractively organized way, the instructor needs to present the subject articulately and coherently. Just as a shopper examines the oranges, questions their quality, and asks for samples, the student needs to engage the relevant concepts, concentrate on the arguments that are made, examine difficult details, and study examples that offer tangible illustrations. Just as shoppers learn about the products being offered and their prices in conversations with each other, students should profit from carefully designed collaborative opportunities to explore concepts and facts, and to engage relevant problems.

By March and April of the year 2020, the coronavirus—having already wreaked havoc in China—had burst into Europe and the Americas, quickly infecting hundreds of thousands of people, putting medical facilities under siege, and killing thousands. In order to contain the spread of the virus, many governments put travel restrictions in place, ordered its citizens to wear masks, to keep several arm-lengths from each other when in public spaces, and to remain sequestered at home in non-emergency situations. The negative impact on the social, commercial, and economic life in the countries involved was predictable. In Spain, Barcelona and the surrounding province of Catalonia were hit especially hard.

How did the spreading virus affect the goings-on at Mercat de la Boqueria? A comparison of Figures 1 and 3 confirms that most of the market's vibrant life had been snuffed out. A majority of its 250 food stalls were closed. Entire sections, normally a riot of colors and mouthwatering aromas, were locked behind gray metallic shutters. A cleaning team disinfected the spaces every day. In place of the 40,000 to 50,000 visitors that used to pass through the market each day, only a few dozen customers were allowed to enter at a time. Once inside, they needed to maintain the required "social distance" from one another. It was forbidden to touch any of the products on display. Payments with credit card or cell phone were allowed, but cash payments were not accepted. Not surprisingly, customers were no longer inclined to linger. Figure 3 shows a lone customer leaving an almost deserted market.

Barcelona struggled with the silence that the pandemic spread over the city. Commercial venues faced the difficult question of how to reopen and to restart their economic lives. This includes La Boqueria. The tourists who had been overrunning the market in recent years were now absent, and La Boqueria was beginning to reclaim its role as a neighborhood market. The proprietor of the fruiteria *Vidal Pons* comments, "we have to go back to our essence, that of being a neighborhood market," and adds that "now with no tourism, we are focusing on online selling." Accompanying the sharp decrease of the spread of the virus in Spain during May and June were calls for the easing of the restrictions that had been put into place. Not surprisingly, this included La Boqueria. A vendor at the fish stall *Palmira* made this explicit, when she urged a small group of customers to "spread the word, the markets are open, and the fish is fresh!"

Fig. 3 A lone shopper leaving La Boqueria. Image courtesy of *Culinary Backstreets*, a well-documented and extensive global guide to the local food scene in cities around the world. See https://culinarybackstreets.com/. Many thanks to *Culinary Backstreets* for the use of this image

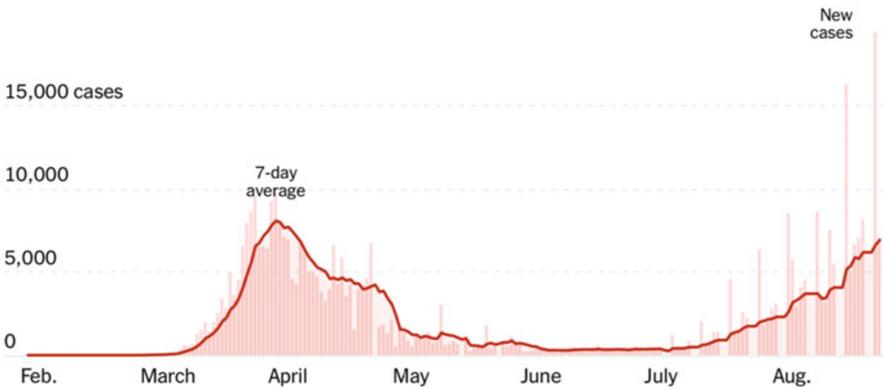

Fig. 4 The reported number of infections in Spain in rolling 7 day averages from February until late August, 2020. Spain has a population of close to 47 million. New York Times, Aug. 26, 2020

However in July, 2020, a new reality began to set in. The numbers of infections in Spain—as the graph of Figure 4 indicates—began to rise again. This resurgence in coronavirus cases had a two-fold explanation. On the one hand, large numbers of people took advantage of the warmer weather to venture into the streets and to the beaches and ignored the mask-wearing and distance-keeping advisements. And on the other, the testing of the population for the virus, initially confined mostly to older patients arriving at hospitals, was expanded. Even though they displayed no symptoms or only mild symptoms, young people were also being infected. The age group from 20 to 40 years began to account for 40% of all cases in Spain. Around four million people in the Barcelona metropolitan area were instructed to remain indoors and to leave only for essential reasons. As a consequence, cinemas, theaters and nightclubs were closed, and restaurants and bars were limited to half capacity. Non-essential businesses had to interact on 'appointment only basis' with their clients. The health ministry of Catalonia announced that "we must go backwards so that we do not have to return to a total lockdown of the population in the coming weeks."

In the meantime, in early March 2020 on the other side of the Atlantic in New York City, college students at Columbia University were huddled in libraries studying for their midterms. But on the evening before many of the first exams, an email from the university's president confirmed what had previously been just a rumor: The entire university would be moving to online instruction as a result of the growing threat of the coronavirus in New York City. Figure 5 gives a sense of the rapid flow of infections through the populations of United States and Europe during the month of March. Once the World Health Organization declared the coronavirus a global pandemic, a follow-up recommendation was sent to Columbia's students

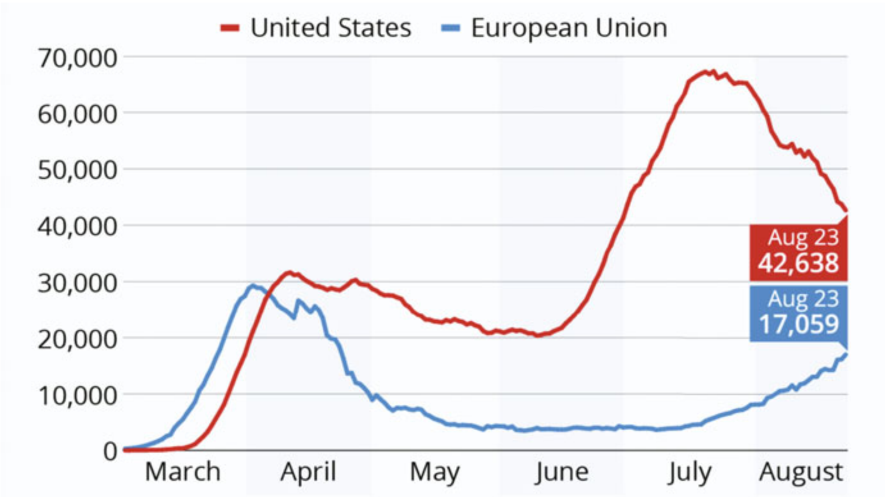

Fig. 5 The graphs of the rolling 7 day average of the total number of infections in the European Union countries and those of the United States from March until late August of 2020. The uptick in the number of cases in the EU is due primarily to infection surges in Spain and France. Note that the EU's population of close to 450 million is about 35% greater than the 330 million population of the US. Note also that there is an unknown variable: the different testing strategies for the virus. Graph by the Coronavirus Resource Center, Johns Hopkins University

informing them not to come back to the university after spring break and that online instruction would extend to the remainder of the semester. Harvard, Princeton, as well as Notre Dame made similar announcements to its students, and thousands of colleges and universities across the country followed suit.

Online instruction meant that the interaction between instructor and students, traditionally in the classroom, person to person, and face to face, gave way to a computer screen to computer screen format conducted via video conferencing platforms such as Zoom. These tools can be used in simple and also sophisticated ways. A television-like transmission of a lecture is easy. However, to turn such a platform into an effective, multi-aspect teaching tool and learning environment represents a challenge. A combination of a high-functioning laptop or desktop with built-in camera and microphone, headphones, and an additional monitor allows the instructor to transmit the visual content of the lecture and to observe the students' faces and the frowns, confusion, and distraction that they express. Basic functions of the instructor's online setup, such as discussion boards, breakout rooms, and polling features, allow an instructor to pause for questions, respond with clarifying comments, quickly gauge students' confidence with the material, divide students into small discussion groups, launch collaborative problem solving sessions, and facilitate one-on-one instructor-student check-ins and peer-to-peer mentoring. To succeed in doing all this is difficult enough in the real, face-to-face classroom. But in the online classroom, it requires a skillfully organized effort not dissimilar to that of a conductor presiding over a symphonic musical performance. Even then, it cannot replicate the three dimensional, live fabric of a dynamic, real classroom.

In this way, American universities were able to complete the spring semester of the academic year 2020. The fall semester would start differently. While many universities continued with the online approach, many others returned to traditional, in person instruction, even though—as Figure 5 indicates—there was a sharp rise in the number of corona infections in the month of July nationally. The University of Notre Dame was one of them.

A mathematics professor and an economics professor[1] pursued the question as to whether a mid-size university could successfully proceed with in-person instruction during the pandemic. They developed a mathematical simulation that involved a highly transmissible hypothetical virus at a hypothetical university with 20,000 students and 2,500 instructors interacting daily for 100 days. It studied the spread of this hypothetical virus and evaluated the efficacy of various interventions. The study concluded that without serious restrictions, more than 2,000 people would be infected within 30 days of the first infection, and that in time over 20,000 people, or about 90% of the total campus population, would become infected. The model also studied the infection outcomes under a combination of strategies that included mask-wearing, daily randomized testing of 3% of the university community,

[1]Refer to https://www.insidehighered.com/news/2020/06/22/working-paper-models-covid-spread-university.

quarantine and contact tracing, as well as the teaching of all classes with 30 or more students in an online-only mode. Under these assumptions, the cumulative infections were kept below 66 cases (out of 22,500) in more than 95% of the simulations. Conducting all classes with more than 30 students in an online format was determined to be the most effective measure for keeping infection rates within acceptable limits.

Only time will tell how well universities will be able to respond to the unforeseen and unimagined challenges that the coronavirus presents. While a number of universities have already been forced to return to all online instruction, Notre Dame held to its decision to bring all of its students to campus for an abbreviated fall semester of in-person classes. In the Fall of 2019, the university community consisted of about 12,600 students (of which 6,900 resided on campus, 5,100 off campus, and 600 off site), 1,350 faculty, and 6,100 secretaries, maintenance personnel, and food workers.

We'll start our discussion of Notre Dame's experience with a short digression. Unlike the earlier piece of fiction about the number of oranges in the display at the La Boqueria market, the following story is true. In early August, a philosophy professor at Notre Dame, took a break from the pandemically challenged preparations of his fall classes and turned his thoughts to an ongoing interest of his: the study of numbers. He noticed that small odd positive integers could be expressed as differences between two squares. He observed that

$$1 = 1^2 - 0^2, 3 = 2^2 - 1^2, 5 = 3^2 - 2^2, 7 = 4^2 - 3^2,$$
$$9 = 5^2 - 4^2, 11 = 6^2 - 5^2, 13 = 7^2 - 6^2, \ldots$$

and thought that this pattern might continue. Could it be, he wondered, that any odd positive integer can be expressed as the difference between consecutive squares? When he did not see the path to a proof, our philosopher turned to a colleague in the mathematics department for assistance. After a brief consultation, he knew what to do. He let an arbitrary odd positive integer be given and expressed it in the form $2n+1$ for some integer $n \geq 0$. His computation

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1$$

verified that $2n+1$ is indeed equal to the difference between two consecutive squares. He had rediscovered a beautiful fact that Pythagoras had observed over 2500 years ago.

Our philosopher's successful excursion into the field of elementary number theory has since found a parallel in Notre Dame's return to in-person instruction on August 10th. Early on there was an alarming spike in the numbers of infected students. The suspicion was that off-campus partying had infected groups of students who then carried the infection to the campus. The university responded to this flareup with a precautionary two-week pause and a return to online classes. Administrators observed that students living off campus had a much higher coronavirus-infection rate than students living on campus. The pause kept those two groups of students apart. The contact tracing that followed, revealed that while off-campus students where the cause of the spiking infections, the source of the transmissions were not large parties, but small, indoor gatherings of students not wearing masks. Students thought erroneously that they were safe, as long as they didn't gather in massive numbers. What was learned, moved Notre Dame to tighten its procedures for students, faculty, and staff. The two-week pause squashed the spike and was followed by a resumption of in-person instruction within a strictly enforced adherence to a thorough infection-controlling protocol. This included the following provisions:

Masks must be worn at all times and in all places—both outside and inside—except by students in their assigned residence hall rooms and by faculty and staff when alone in a private office. Physical distancing is required and any gathering is limited to 10 people or fewer (with some specified exceptions). This applies to on-campus as well as off-campus situations. Within the dorms up to two additional residents from the same section of the dorm may gather in another student's room as long as the door is left open and all present wear masks and keep appropriate distances. Students need to eat their meals exclusively either outdoors or in their residence hall rooms. Students, faculty, and staff are required to complete an online Daily Health Check. Individuals who report corona related symptoms are issued a 'red pass' and directed to receive rapid tests, to self-isolate, and to get medical attention. Those who contract the virus are subject to quarantine. People that an infected person had been in contact with are traced and tested for the virus.

Faculty are expected to deliver their courses in "dual-mode" instruction that is simultaneously in-class and Zoom-assisted online. Faculty whose age or health situation falls within a high risk category or involves other special circumstances are permitted to teach exclusively online. Students scheduled for an "alternating attendance" course join the class in person or online on alternating class meetings. Those who receive a red pass on their Daily Health Check or test positively are moved into quarantine or isolation and are able to continue to participate in their classes online. All classes must be be recorded to allow students to review previously presented material and to facilitate a return to an exclusively online approach should this become necessary.

A snapshot or "dashboard" of the campus infection picture is published daily. Figure 6 depicts the dashboard for the 10th of September, 2020. The bar graph—the number of tests are shown in gray bars, the number of positive outcomes in brown, and the graph of the running 7 day average in green—confirms that the earlier flareup has ended and that the virus has been brought under control. In recent weeks, the number of students moving out of quarantine has been greater than the number moving into quarantine. All indications are that the measures and strategies that Notre Dame has adopted are dealing with the pandemic successfully. It seems probable—so far so good—that the university's effort is sustainable.

The coronavirus stories for the University of Notre Dame and Barcelona's Mercat de la Boqueria contrast sharply. Notre Dame is open and carrying out its educational mission. On the other hand—in response to the rising infection

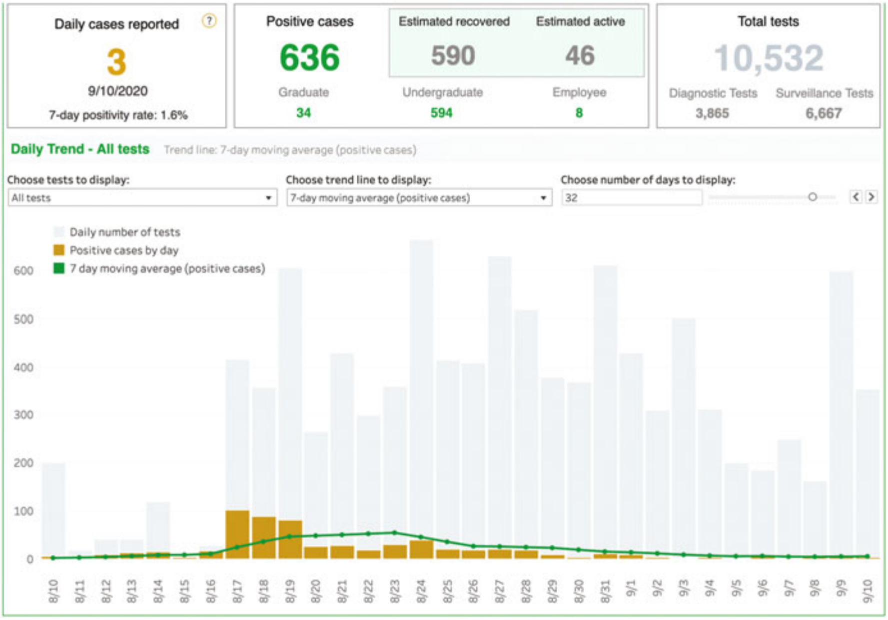

Fig. 6 This bar graph of Notre Dame's Covid-19 Dashboard provides a snapshot of the campus infection situation from the start of the fall semester on Aug. 10 to Sept. 10, 2020. Credit, University of Notre Dame

curve of Figure 4 and as Figure 3 illustrates—La Boqueria has only a small number of shoppers and its stalls are largely closed. The difference is easily explained by considering the two environments. Notre Dame is a community that is largely self-contained. It's close to 20,000 individuals live and work in an environment that can be subjected to stringent protocols and careful monitoring. The market La Boqueria, on the other hand, is subject only to the already discussed weaker protocols and precautions of the city of Barcelona and the province of Catalonia. Notre Dame is a microcosm of a restricted number of people, La Boqueria on the other hand, is a market that is open to the millions of residents of the city.

While Notre Dame opted for a "primarily in person" instructional model, most colleges and universities have been more cautious. The pie chart of Figure 7 shows how the close to 3000 colleges and universities in the U.S. have approached the instruction of their students in response to the threat of the virus. In the same way that states, counties, and municipalities in the U.S. have needed to react to increases in the number of daily infections, U.S. colleges and universities have needed to adopt a fluid approach to control the infections on their campuses. Some, like Notre Dame, were able to respond to spikes successfully with temporary pauses in in-person instruction, but for others such pauses were not successful, and they were forced to turn to online instructional delivery.

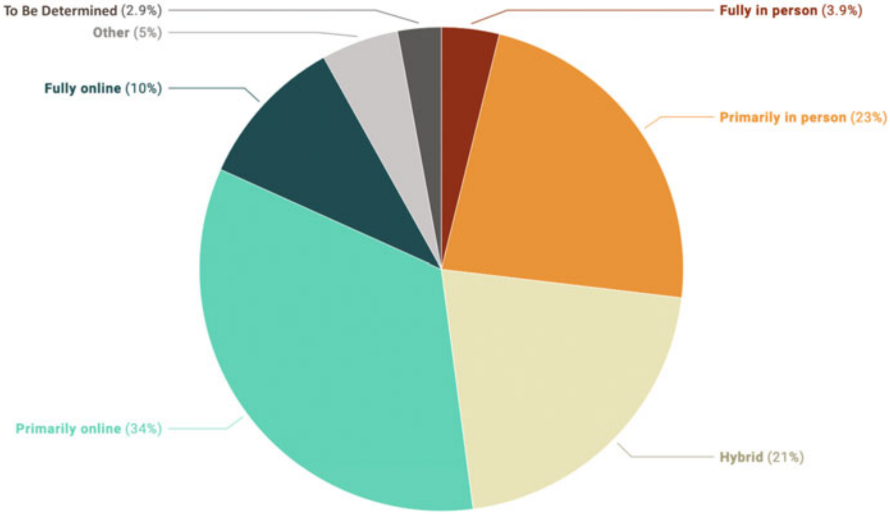

Fig. 7 Breakdown of the reopening models of nearly 3000 two-year and four-year public and private colleges and universities in the U.S. Data from the Chronicle of Higher Education, August 28, 2020. Source: College Crisis Initiative at Davidson College

We have had a glimpse at the worldwide turmoil that the coronavirus has caused. The challenge has been to keep the disruptions to the functioning of the social, commercial, and educational channels to a minimum while at the same time to safeguard peoples' health and lives. This has brought about new patterns of behavior and new modi operandi. Time will tell what kind of "new normal" stable states will emerge.

Alexander Hahn is Mathematics Professor Emeritus at the University of Notre Dame. His most recent work, the book *Basic Calculus of Planetary Orbits and Interplanetary Flight: The Missions of the Voyagers, Cassini, and Juno*, was published during the corona pandemic by Springer Publishing in April, 2020.

Alexander J. Hahn

Soap Bubbles Vanitas Venice

Michele Emmer

I was a professor at the *Ca' Foscari University* in Venice for 7 years. I had my studio in the *Ca' Dolfin* palace, famous for the 10 canvases by Gianbattista Tiepolo which are currently at the *Metropolitan Museum* in New York, the *Hermitage* in St. Petersburg and the *Kunsthistorisches Museum* in Vienna. I know the city of Venice very well and in 1991 I wrote a book on the geometries of the city *La Venezia perfetta* (The Perfect Venice) [1] and a revised and reduced version of the book was published in English with the title *Venetian Geometry, or the Perfect Venice* in one of the three volumes dedicated to the city by Alain Vircondelet *Venice Art and Architecture* published in 2006 by Flammarion [2] and partially in the magazine on architecture and Math *Nexus* [3]. A second edition of my book with a new cover and introduction was published in 2019 [4]. In 2013 I was elected a member of the *Istituto Veneto di Scienze, Lettere e Arti* (IVSLA) founded in Venice by Napoleon. Since then I have organized my yearly conferences on mathematics and culture at the Institute which is based in Palazzo Franchetti on the Canal Grande, one of the most famous Venetian palaces, unique for its history and its architectural features (Fig. 1).

In my book on Venice I talked about the labyrinthine structure of the city, its topology (a few years later at the 2008 *Venice Biennale of Art* there was the exhibition *Topological Gardens* [5] by the US artist Bruce Naumann, who studied mathematics for three years), of the forms found in churches, palaces, squares: helices, symmetries, geometric decorations, glasses called *reticello* (crossing of spirals). The famous starshaped solid, officially invented by Kepler in 1619 in the volume *Harmonices Mundi*, made in mosaic many years earlier on the floor of the Basilica of San Marco based on drawings by the artist Paolo Uccello. A geometric and scenographic city, a perennial open-air stage.

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Fig. 1 Palazzo Franchetti, IVSLA, Venice. Photo C. Morucchio © by permission

In March 2019 I organized the last (for the moment I hope) *Imagine Math, Mathematics and Culture* conference (the first was in 1997) at IVSLA and at the same time in the other building of the Institute, Palazzo Loredan, in Campo Santo Stefano two hundred meters from Palazzo Franchetti, I organized an exhibition of the famous Italian artist Mimmo Paladino, with the ten posters he created just for the conference [6].

One of the posters became the cover of the last Springer Series volume *Imagine Math 7* published in October 2020 [7] (Fig. 2).

Simultaneously with the conference and the exhibition in Venice, a major exhibition on soap bubbles in art and beyond, in architecture, cinema, biology, and of course mathematics, opened in Perugia, curated by Marco Pierini and myself. Also included in the exhibition were the two volumes by Joseph Plateau published in 1873 *Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires* [Experimental and theoretical statics of liquids subject to only molecular forces] Paris, Gauthier-Villars [8].

His experiments and observations have given rise to some of the most interesting mathematical problems, fascinating many researchers. The problems of minimal surfaces, surfaces with curvature, Plateau's problem, Bernstein's problem, singularities. In short, the *Calculus of Variations*. Fields medals (Jesse Douglas, Enrico Bombieri, Alessio Figalli) were also won working in these sectors while Karen Uhlenbeck won the Abel Prize in April 2019 and *The New York Times* dedicated two full pages to her by the title *A Mathematical Universe in a Bubble* [9]. Two medals for Italy, the only ones.

Fig. 2 M. Paladino, *Without title*, sketch for the conference *Mathematics and culture*, 2018, work on paper, mixed media, mm. 1000x 800. Private collection by permission

The exhibition took place in Perugia in the Palazzo dei Notari, one of the most prestigious venues for Italian painting from the fourteenth century to the Renaissance, with works by Piero della Francesca, Beato Angelico and obviously Perugino. The subject of soap bubbles has interested me since I graduated in mathematics in 1970 at the University of Rome presenting a thesis on the works of Renato Caccioppoli. Some of his ideas were later used by the famous Italian mathematician Ennio De Giorgi to introduce the *Theory of Perimeters*, one of the ways to solve problems of calculus of variations related to minimal surfaces. I started my activity at the University of Ferrara as an assistant to Mario Miranda, then one of De Giorgi's main collaborators.

Since soap films and soap bubbles are not only models for 3D minimal surfaces and surfaces with assigned mean curvature but are also very beautiful and colorful objects, I immediately became interested in images of soap bubbles in art, starting in the meantime my personal small collection of paintings and objects. Very important was the influence of my father who since 1938 started and invented a new way of making art documentaries on art, among the most famous ones the one on *Giotto* [10], *Leonardo da Vinci* [11], which won a Golden Lion at the *Venice International Film Festival* in 1952, *Goya*, best art film festival in Berlin, 1951 [12] and *Picasso* [13], a film made with the Catalan artist in 1953.

I had a large collection of art books at home and I was lucky enough to meet many artists personally. At the beginning of the eighties I started the series of my films on *Mathematics and art*, they would be 20 at the end, and one of the themes was precisely soap bubbles. The film was made in part at Princeton University's Math Department with Fred Almgren and Jean Taylor [14]. Jean had just proved the

Fig. 3 A. Romako, (copy from) *Zwei mit Seifenblasen spielende Kinder*, oil on canvas, end XIX Century, private collection by permission

correctness of Plateau's observations on the singularities of soap films [15]. Jean and Fred had published an article about their research in the *Scientific American* in 1976 [16], which included beautiful color images of soapy structures.

The exhibition in Perugia opened on March 9, 2019, the conference in Venice and the Paladino exhibition on March 30, 2019. The first images of soap bubbles in art appear in the 16th century, especially in the countries of Northern Europe, Holland, Belgium, Germany, Denmark, (Fig. 3) side effect of the spread of the new soap probably invented in today's Turkey. Bubbles games became very popular probably, many artists made paintings on the theme. For this reason, even scientists started being interested in phenomena concerning both the formation of color on soapy surfaces and their forms. Among the first Isaac Newton. In a 19th-century painting, Newton discovers color on soap bubbles looking at a child playing with them.

Many famous painters made works with bubbles from Chardin (at the exhibition the painting at the *Hermitage* Museum in St. Petersburg) to Nestcher, up to John Millais, Eduard Manet, to get to contemporary art Man Ray, Paolini and Max Beckman. Soapy structures play a key role in modern architecture, after their first use by the German architect Frei Otto in the sixties of the twentieth century. One of the projects based on Plateau's models was the *Olympic Swimming Pool* in Beijing for the 2008 Olympic Games. The original *maquette* of the project was part at the exhibition.

In Art history, the theme of soap bubbles becomes a sub-genre of the more general theme of the Vanitas, *Vanitas Vanitatum et Omnia Vanitas* as written in *Ecclesiastes* 1: 2. In Goltzius' most famous and widespread work of 1594 *Quis evadet?* (who escapes) appear as symbols of Vanitas bubbles, smoke, dried flowers, a skull. In preparing the exhibition a work by Agostino Carracci was identified (Fig. 4). With the same title and practically identical to that of Goltzius, the exact date is not known (Fig. 4).

Fig. 4 A. Carracci, *Quis evadet*, bulino, around 1590, Reggio Emilia, Italy, Biblioteca Panizzi, Gabinetto delle stampe "A. Davoli". By permission

At the exhibition in Perugia I did not want the Vanitas theme to be the prevalent one, and therefore there was only one painting explicitly dedicated to Vanitas, a still life with bubbles and skull. I wanted the playful, fun aspect to be prevalent, present since the beginning of the spread of soap bubbles in art. And the other scientific and artistic aspects. The exhibition was titled *Soap Bubbles. Forms of Utopia between Vanitas, Art and Science.* The word Vanitas could not be missing in the title to give a correct location in the history of art [17–21].

A year and a few months have passed since then. I worked on the publication of the volume *Imagine Math 7*, with the proceedings of the 2019 Venice conference, with the ten Paladino posters inserted. I am also editing another volume of the series *Imagine Math 8* not connected to any conference in Venice, since nothing could be organized due to the Covid-19 virus. And streaming was unthinkable. The venue, Venice, is an essential part of the conference. Streaming meant destroying the idea.

In Italy we were in lockdown for three months. It was not possible to move, to see anyone, not even children and grandchildren who live far away. Writing, even more than using the computer as a video phone, was an essential way to survive and try to maintain a delicate mental balance. And writing, loneliness led to reflect. To reflect on life, on work, on loved ones, on those who died in this period.

So the Vanitas I wanted to remove from the soap bubbles exhibition resumed its role. Soap bubbles, the fragility of life, and therefore of art, of everyone's role, everyone's work and research. Is it all Vanitas, do we just try to forget, trying to build our own space like a soap bubble destined to burst? Research on minimal surfaces, on the problem of Plateau, on those fragile forms which become the symbol of the uselessness of life? And does research in this field, and in that of the history of art, have a meaning? Are not our efforts useless, irrelevant when only health research, biology, vaccinations seem to have a real importance and all the rest are vain chimeras as Mathilde Wesendonck wrote for the lieder of Richard Wagner? [22] (Fig. 5)

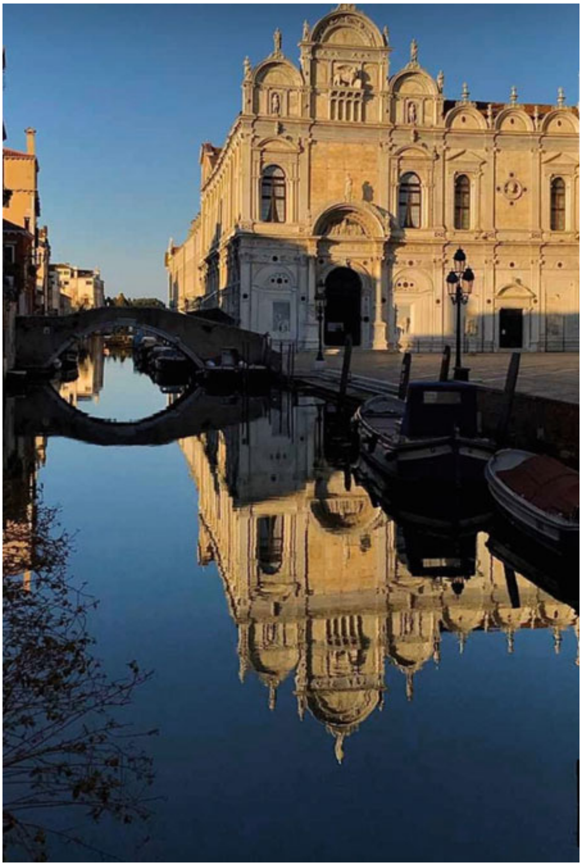

Fig. 5 Scuola Grande of San Marco, Venice. March 2020. Photo P. Freguia © by permission

Sag, welch wunderbare Träume
Halten meinen Sinn umfangen,
Daß sie nicht wie leere Schäume
Sind in ödes Nichts vergangen?

(What wondrous dreams are these/
Holding my mind in thrall,/That they,
like insubstantial foam,/Don't barren
emptiness recall?)

And the town, that town that was for hundreds of years the ideal scenario for many events of human civilization, that Venice celebrated, told, saw, revised, reworked, decadent, (and leaving apart the *Death in Venice* by Thomas Mann), geometric, fragile, she too was a looming symbol of Vanitas. As demonstrated once again by the tragic high water that flooded the city on November 12, 2019, lower only than the devastating one of 1966 (Figs. 6, 7, and 8).

In Venice where you could not go due to the virus, Venice where the quarantine was invented in response to the plague. The term originates from the isolation for an indefinite number of days which was imposed on the crews of ships as a preventive measure against the diseases that raged in the fourteenth century, including the plague. A document of 1377 states that before entering Ragusa, today's Dubrovnik in Croatia, it was necessary to spend 30 days (about thirty) in an isolated place, usually the nearby islands off the coast, waiting for any symptoms of plague to

Fig. 6 Piazza San Marco, Venice, March 2020. Photo P. Freguia © by permission

Fig. 7 *Acqua alta* (High Tide) in Venice, November 12, 2019. It was impossible to trace the author even though many newspapers on line have used it.

Fig. 8 P. Greenaway, 92 *Drawings of Water*, catalogue, Centro internazionale della Grafica Venezia, drawing n. 10 [23]. Private collection by permission

appear. In 1448 the Venetian Senate extended the period of isolation up to 40 days giving rise to the term quarantine (originally, Venetian form for forty). Venice was the first to issue provisions to stem the spread of the plague, appointing three guardians of public health in the early years of the Black Plague (1347). The first hospital was founded by Venice in 1403, on a small island adjacent to the town.

Those scenographic geometries have become empty, fascinating and tragic. The empty, deserted city. And that exhibition of bubbles, in which the reference to Vanitas was just an expedient to stay in the wake of the great exhibitions of the year, seems to have taken place years ago, far in time, in another era.

And did it make sense to keep trying to write, to bring to terms books that will only be virtual, as virtual as the town of Venice observed only by drones? I don't have an answer to these questions, everyone will have their own. But I continued to write and will continue to write, as I am doing even now. Because writing testifies, tells, remains, the memory, the memory. And the very title of the Springer series that I invented is obviously taken from John Lennon's *Imagine* which imagines the possible future. True, everything is Vanitas, but fortunately we never think about it for too long.

Imagine all the people
Living life in peace
You, you may say I'm a dreamer
But I'm not the only one
I hope someday you will join us
And the world will be as one.
. . . ..

John Lennon, *Imagine,* 1964.

PS:
To have a look at the exhibition: https://youtu.be/fFHh9hi5fwM
An excerpt from the movie *Soap Bubbles*: https://youtu.be/iys6zVOMiqc

References

1. M. Emmer, *La Venezia perfetta*, Centro Internazionale della Grafica, Venezia, 1993.
2. A. Vircondelet, *Venice Art and Architecture*, Flammarion, Paris, 2006, pp. 34-65.
3. M. Emmer, *Minimal Surfaces and Architecture: New Forms*, Nexus, **15** pp. 227-239, 2013.
4. M. Emmer, *La Venezia perfetta*, Centro Internazionale della Grafica, Venezia, 2nd ed., 2019.

5. C. Basualdo, M. R. Taylors, eds, *Topological Gardens Bruce Naumann*, Yale University Press, 2009.
6. M. Emmer, ed., *Mimmo Paladino, Sulla Mathematica*, catalogue, Palazzo Loredan, Venezia, 2019.
7. M. Emmer, M. Abate, eds, *Imagine Math 7*, Springer, 2020.
8. J. Plateau, *Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires* Paris, Gauthier-Villars., 1873.
9. S. Roberts, *A Mathematical Universe in a Bubble*, The New York Times, 10 April 2019.
10. L. Emmer, film maker, *Giotto racconto da un affresco*, Dolomiti Film prod, 1938.
11. L. Emmer, film maker, *Leonardo Da Vinci*, Documento Film prod., 1952
12. L. Emmer, film maker, *Goya, I disastri della guerra, La festa di San Isidro*, Colonna Film, 1950.
13. L. Emmer, film maker, *Picasso*, 1953. New version, RAI2, 2000.
14. M. Emmer, film maker, *Soap Bubbles*, series Art and Math, 25', Film 7 prod. Rome, 1984.
15. J. Taylor, The Structure of Singularities in Soap-Bubbles-Like and Soap-Films-Like Minimal Surfaces, Ann. Math., 103, pp. 489-539. 1976.
16. F. Almgren, J. Taylor, *The Geometry of Soap Bubbles and Soap Films*, Scientific American, pp. 82-93, 1976.
17. M. Emmer, M. Pierini, curators, *Soap Bubbles. Vanitas and Utopia between Art and Science*, catalogue, ebook, Silvana editors, 2019.
18. M. Emmer, F. Greco, *Bubbles, Bubbles, Bubbles, for children*, Aguaplano, 2019.
19. M. Emmer *Bolle di sapone tra arte e matematica*, Bollati Boringhieri, Torino, 2009, Viareggio Award, Best Italian Essay 2010.
20. M. Emmer, Soap Bubbles in Art and Science: from the Past to the Future of Math Art, in The Visual Mind, MIT Press, pp. 135-142 1993.
21. M. Emmer, From Soap Bubbles to Fields Medals: an Exhibition, in M. Emmer, M. Abate, eds, Imagine Math 7, Springer, 2020.
22. R. Wagner, music, M. Wesendonck, text, *Traume - Studie zu Tristan und Isolde*, lieder, dicember 1857.
23. P. Greenaway, *Senza titolo*, in M. Emmer, *92 Drawings of Water: Peter Greenaway,* catalogue, Centro Internazionale della Grafica, Venezia, march 2006, in M. Emmer, ed *Matematica e cultura 2007*, Springer Italia, Milano, 2006, pp. 311-314. All 92 drawings made for the Art book *Tulse Luper in Venice*, Volumina, Turin, 2004.

Thoughts at the Time of an Epidemic

Alexander Soifer

We have nothing to lose except everything
—Albert Camus, October, 1957

March 7, 2020. I arrive in Deerfield Beach, Florida, for an International Conference on Combinatorics, Graph Theory and Computing. Wyndham Hotel's general manager, Claude Dubois, a handsome Frenchman, and I share a hug. I am a regular here.

March 10, 2020. At the conference, I do not see a single mask. All seems normal as in the years past, although Elizabeth Loew of Springer does not arrive due to the declared state of emergency in Florida and general concerns about the virus. My university Chancellor Venkat Reddy sends faculty a message ordering all courses to move online, not right away but in early April, in three weeks, after the Spring Break.

March 14, 2020. As I am leaving the hotel for the airport, a concern over the virus begins to register in our life – Claude Dubois and I bid each other farewell with touching elbows. Still no masks anywhere, even at the airport and on board of the plane. One of my students informs me that he is moving his family away from the city of Colorado Springs to a remote little town due to his fear of a panic in Colorado Springs. I find it hard to believe that my city will panic – until the day after.

March 15, 2020. Colorado Springs, I am in a supermarket. Empty shelves everywhere, customers' carts are full of whatever is still available, hours-long lines to cashiers. People look tense, ready to explode. My university Chancellor orders all courses to move online immediately, overnight. A student asks me: "How are you going to teach online if you've never done that?" – "Carefully," I reply. It is true that I have never tried online instruction, but I am promised a 30-min intro to Microsoft Teams by our IT specialist, Jackie.

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March 18, 2020. I dive into online teaching for mathematics and European cinema. It is not as hard as I feared because during the first two months of the semester my students and I developed a bond, a unity of purpose. Our sessions in Microsoft Teams are reminiscent of visiting friends over Skype.

April 11, 2020. The first personal and professional tragedy – I lose a coauthor and friend of my Princeton years, the most ingenious mathematician, John Horton Conway. A few lines of my remembrance have just been published [1].

July 6, 2020. Another tragedy, Ron Graham succumbs to a fatal hereditary disease. I feel like an orphan in Ramsey Theory, as our Captain leaves the ship. My tribute is now out [2].

July 12–19, 2020. China is to host the 14th International Congress on Mathematical Education (ICME-14) in Shanghai, a quadrennial forum that brings together ca. 4000 scholars – a la Olympics in our sport of mathematics. I am invited to give a keynote address in the Topic Study Group "Mathematical competitions and other challenging activities." Professional duty outweighs fears of going to the birthplace of COVID-19, but a U.S. announcement of a 14-day mandatory quarantine for everyone coming back from China causes me to pause. Then the U.S. Department of State issues a shocking China – Level 4 [highest] warning, "Do not Travel." More precisely (emphasis in bold is in the posted warning):

> Do not travel to the People's Republic of China (PRC), including the Hong Kong Special Administrative Region (SAR), due to **COVID-19 and arbitrary enforcement of local laws.**
>
> **Country Summary:**
>
> The PRC government arbitrarily enforces local laws, including by carrying out arbitrary and wrongful detentions and through the use of exit bans on U.S. citizens and citizens of other countries without due process of law. The PRC government uses arbitrary detention and exit bans:
>
> - to compel individuals to participate in PRC government investigations,
> - to pressure family members to return to the PRC from abroad,
> - to influence PRC authorities to resolve civil disputes in favor of PRC citizens, and
> - to gain bargaining leverage over foreign governments.
>
> In most cases, U.S. citizens only become aware of an exit ban when they attempt to depart the PRC, and there is no reliable mechanism or legal process to find out how long the ban might continue or to contest it in a court of law.
>
> U.S. citizens traveling or residing in the PRC or Hong Kong, may be detained without access to U.S. consular services or information about their alleged crime. U.S. citizens may be subjected to prolonged interrogations and extended detention without due process of law.
>
> Security personnel may detain and/or deport U.S. citizens for sending private electronic messages critical of the PRC government.

No keynote address is worth free food and lodging in a Chinese prison. I cancel my fully arranged trip to China. Soon after, the Congress is postponed by one year. ICME-14 Chair Professor Wang proudly declares that China has conquered COVID-

19 while America and others (Europe) failed to stop the spread. Let me quote his August 13, 2020 e-mail, explicitly promoting Chinese government's deception:

> China's opening-up policy remains unchanged. It is entirely due to COVID-19 that China is now taking the necessary restrictions or quarantines on its citizens and international travelers from pandemic countries and regions, and the restrictions on the routes and flight frequency of international passenger air travel. Once the global pandemic alert is lifted, the Chinese government will not restrict the normal entry and exit of foreigners for any reason or in any form. As we all know, the US State Department's political accusation in travel advisory for China is groundless and ridiculous. It can be interpreted as a political manipulation by a small group of people for a certain purpose, and it should be completely ignored.
>
> I hope my explanation will clear up your confusion.
>
> With best regards,
> Jianpan Wang, ICME-14 chair

Our "confusion"? Detention of innocent American, Canadian, Australian, and other nationals when they attempt to *depart* [!] from China "should be completely ignored"? After that statement, I do not think I will go to China in July, 2021.

Some amusing invitations follow:

- "It is our great pleasure and privilege to welcome you to join the World Gene Convention, which will take place in Macao. On behalf of the Organizing Committee, we would be honored to invite you to be a chair/speaker in Module 1: Breakthroughs in Gene while presenting about *E15: From Squares in a Square to Clones in Convex Figures*."
- "It is our great pleasure and privilege to welcome you to join the Annual World Congress of Food and Nutrition, which will be held in Singapore. On behalf of the Organizing Committee, we would be honored to invite you to be a chair/speaker at Session 405: Foodborne Diseases, Carcinogenic Food while presenting about *E23: More about Love and Death* at the upcoming WCFN."

In both cases the organizers somehow dug out the titles of my "Further Explorations" from *The Colorado Mathematical Olympiad* books (Springer, New York, 2011 and 2017) and interpreted the Olympiad problem titles literally!

The COVID-19 epidemic forced me to postpone the ready-to-run 37th Soifer (formerly Colorado) Mathematical Olympiad from April, 2020 to April, 2021. Nevertheless, thinking about the Olympiad proved productive. One Olympiad train of thought took me to a research space. In 2010, I created – and in 2011, used the following problem in the Colorado Mathematical Olympiad (CMO).

Won't you be my neighbor? (*A. Soifer*, 2010, problem 5 of the 28th CMO in 2011)

Each unit square of a 2011×2011 square grid is colored in one of 2011 colors so that each color is used. A pair of distinct colors is called a *neighbor pair* if they appear as colors of a pair of unit squares sharing a side.

1. Find the maximum M of the number of neighbor pairs.
2. Find the minimum m of the number of neighbor pairs.

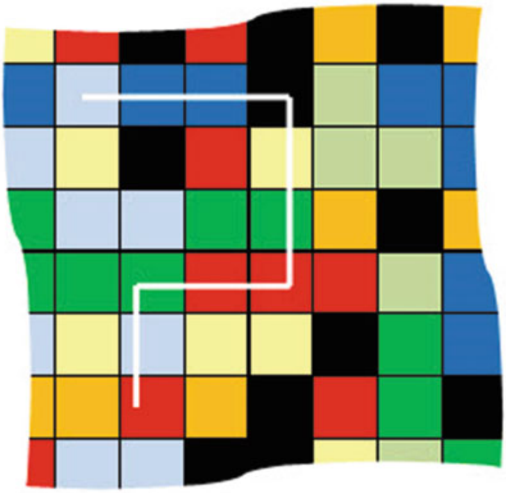

Fig. 1 A rook's path connecting any pair of squares

Solution. Form a graph G with 2011 vertices, one per color, and two vertices adjacent if and only if the corresponding colors form a neighbor pair somewhere on the colored grid. The graph G is connected, for there is always a rook's path on the grid connecting unit squares of any pair of colors (Fig. 1). Where the color along the path changes, we get an edge in G. Thus, every two vertices of G are connected by a path along its edges, and hence, G is connected.

Any tree on 2011 vertices features the minimum number 2010 of edges in a connected graph (an easy proof by mathematical induction); the complete graph K_{2011} (i.e., the graph where every pair of vertices is adjacent) sports the maximum number $\binom{2011}{2}$ of edges. All there is left to demonstrate is a coloring of the grid that induces a tree graph T, and a coloring of the grid that induces the complete graph K_{2011}.

In order to achieve a tree, we first color the 2011×2011 square grid in a chessboard fashion in color 1 and a temporary color 2012. We then replace color 2012 with colors 2, 3, . . ., 2011 with each of these colors used on at least one unit square. We get a tree with the root at the vertex corresponding to color 1, and 2010 edges connecting this vertex to all other 2010 vertices of the graph (Fig. 2).

Let us now induce the complete graph K_{2011}.

The row 1 we color in two alternating colors, 1 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 2, 3, . . ., 1006.

The row 2 we color in two alternating colors, 2 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 3, 4, . . ., 1007; and so on.

The row 2011 we color in two alternating colors, 2011 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 1, 2, . . ., 1005.

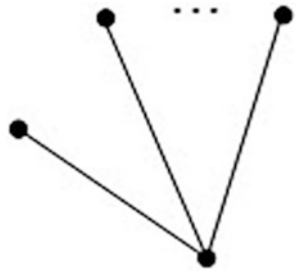

Fig. 2 A tree induced by an appropriate grid coloring

Due to a cyclic nature of coloring, it suffices to verify that color 1 neighbors on each of the other colors. It neighbors on colors 2, 3, . . ., 1006 in the first row. For $1007 \le i \le 2011$, color i neighbors on color 1 in the row i, where color i neighbors on colors i+1, i+2, . . . , i+1006 (we calculate these sums modulo 2011, i.e., subtract 2011 from the color number as soon as it exceeds 2011). Since $i + 1 \le 2012$ and $2013 \le i+1006$, color 1 will appear as a neighbor of color i in the row i.

This problem must have entered my subconscious universe. Consequently I created the following related problem:

Problem of Neighboring Colors (A. Soifer). Each unit square of an $n \times n$ square grid is colored in one of m colors, so that every two colors appear on unit squares that are neighbors somewhere on the grid, i.e., share a side. Find maximum of m, which we will denote as max(m).

I immediately came up with optimal colorings for small n, and thus arrived at a conjecture:

Neighboring Colors Conjecture (A. Soifer). max(m)= 2n–1.

It was not hard to show that for any n, max(m) $\le$ 2n–1, but a proof of the conjecture in general case eluded me. And so I shared the problem with the 1990 and 1991 Colorado Mathematical Olympiad winner Matthew Kahle, now a Professor of Mathematics at Ohio State University, who in turn shared it with his Ph.D. student Francisco Martinez-Figueroa. The three of us achieved good progress, but still did not prove the conjecture completely. First, we learned that in 2010 Keith Edwards showed that max(m) = 2n–1 for all sufficiently large n, but it was not clear how large n must be, and the proof was not constructive. We proved the following results constructively and published them in *Geombinatorics* at the time of pandemic, in April-2020:

Small *n* Theorem [3]. For any $n \le 8$, max(m)=2n–1.

Lower Bounds Theorem [3]. For any positive integer n, max(m) $\ge$ 2n–9. Moreover:

- if $n \equiv 0$ or 1 (mod4), then max(m) $\ge$ 2n–6;
- if $n \equiv 2$ (mod4), then max(m) $\ge$ 2n–7.

As you can see, a beautiful Olympiad problem led me to create an even more exciting problem, which proved to be too hard for the Olympiad. However, it gave three generations of mathematicians an opportunity to enjoy working together. It could give you a pleasure of advancing the proof (or disproof) of the conjecture further!

This has been a most depressing time in my 42 years on these shores. Yet as much as I could compose myself, I have been writing a new, much expanded edition of *The Mathematical Coloring Book*. Its ca. 600 pages promise to double. My white collie poppy Bellissimo takes me daily for a stroll around Quail Lake, and sometimes to high country for hunting. No, Bellissimo and I will never hurt beautiful Colorado wild animals – we hunt for mushrooms. In the fall-2020 semester, I am teaching – in person – mathematics and my new course "World War II History through Films of Individual Tragedy": https://www.youtube.com/watch?v=MZ_U9WYjwqQ&feature=youtu.be. Join my course or just be my guests to view these incredible cinematic works of Polish, Russian, French, Lithuanian, and American film directors.

Summing up, I made two fundamental observations while living with epidemic:

Man has been trying to conquer Nature, and Nature responded with hurricanes, tsunamis, and epidemics.

Man created virtual reality as a reflection of his life, and virtual reality returned the favor.

Alexander Soifer. Teaching at the time of the COVID-19 pandemic, September 4, 2020

References

1. Soifer, A., Remembering John Horton Conway, Geombinatorics XXX(2), October 2020, 53–55.
2. Soifer, A., Remembering Ronalrd Lewis Graham, Geombinatorics XXX(2), October 2020, 56–62.
3. Kahle M., Martinez Figueroa, Francisco, and Soifer, A., A Square Grid Coloring Problem, Geombinatorics XXIX(4), April 2020, 167–184.

Dancing on Stilts

Mark Saul

I thought I knew how to teach. After spending 35 years in classrooms from grades 3 through 12, I could go into a classroom with 15 min notice and deliver a meaningful, if not polished, lesson. I could do it for advanced students, for average learners, and for struggling students. And I could keep inattentive students down to one or two out of thirty.

Then came COVID, and I had to learn to teach all over again.

I had the intellectual part down, pretty much. I knew where the students were and what they should be guided to next. But I didn't have eye contact. I didn't have body language. Worst of all, I didn't have an expansive chalk board.

The heart of the teaching process is motivation—getting the student to want to learn. The heart of motivation, in turn, is emotional—the bond between teacher and student that makes each party eager to please and communicate with the other. How do I bond with students when I only see their heads—and sometimes not even those? When I cannot tell what they are doing with their hands or their bodies?

It's as if someone made ballet dancers do the Dance of the Cygnets on six-foot high stilts.

What follows is a description of some of the best experience I've had in online teaching, some unresolved questions, and some analysis of lessons I've learned.

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1 A Child's Insight

"I know how to tell how many divisors a number has. You factor it into primes. . . ." Alejandro was with a virtual group of four enthusiastic ten year olds, in the midst of exploring a problem. He gave the usual result, using his own somewhat makeshift words. But not too distant really from what I would have said:

"If N factors as $p^a q^b r^c$: : :, then the number of divisors is (a+1)(b+1)(c+1):::". His description was less economical, but still accurate.

His virtual friend Xue said: "That's great. Let's look it up on Wikipedia."

I was thinking, at this point: how nice! They can just conjure up Wikipedia and look something up. This would be clumsy in the usual classroom. I'm already tapping into new resources.

But my hasty self-congratulation was quickly blown away by Xue's next comment.

"No. Let's not look it up. Let's pretend we don't know it and see if we can prove it."

Dear Reader: I swear to you, on Galois' grave, that I am not making this up. Nor the rest of the vignette I will be recounting here.

2 The Venue

During the COVID Spring (2020), I was part of a team developing an online 'webinar' for the Julia Robinson Mathematics Festival (JRMF) program. At the time, I was its Executive Director. In normal times, we ran non-competitive after-school mathematics events ("Festivals") in which students are offered interesting games, puzzles and problems, assisted by a facilitator. Since face-to-face work with students became impossible, we sought to continue the work virtually.

The program met with success. We worked on the presentation of a problem each week, polishing it for a group of about 200 students who 'tune in' to the event internationally. The students are split into groups of fewer than ten, and put in breakout rooms to discuss the problem. An adult facilitator guides the discussion, not to achieve a particular goal, but as a moderator, letting the students' insights emerge naturally. Facilitators meet for half an hour after the webinar, to pool their experiences and offer ideas for refining the program.

We have found that one teacher working with fewer than ten students is about right. But even for this small group, it's important to have a second leader present. I always had a "technical assistant" who would play several roles. First, she would take care of software technicalities. There is always something about the software that I don't know or that I have yet to explore. My assistant would jump in and tell me how to perform a particular action—or tell me that the platform would not support what I want.

A second role for the "technical assistant" is not really technical at all. She can keep track of the students. Which ones are responding? Which have their faces hidden? What have asked a question in the chat that I haven't noticed? In front of a classroom, I get this feedback directly. In a virtual environment, the reactions of students must often be inferred. If I'm sharing the screen, I don't see the group as a whole. And I don't have time to make the inferences. My assistant would tell me whom to call on if I have a leading question.

The problems we use are "low threshold, high ceiling". That is, very young students can work on them, have fun, and achieve insights that will eventually take them farther. More advanced students can use them to engage in thorny issues or deep mathematical concepts. For examples of such problems (and an open invitation to participate in these webinars) see www.jrmf.org.

Attendance has grown steadily. We find that students who come to the webinar tend to return. Thus we have created a virtual community, all around the globe, of students who enjoy mathematics. The vignette below showcases some of what we have learned from this experience.

There are two points that are not illustrated here. The first is that online teaching takes much more time than classroom teaching, and usually about twice as much. The teacher cannot be as nimble in reacting to student needs. The students may not be able to understand as quickly what the teacher is saying. Switching between screens, writing on screens, taking notes—these are all common classroom tasks that take longer in a virtual setting than they do in a real time classroom. Dancing on stilts…

On the other hand, the existence of a 'chat stream,' a private and asynchronous communication channel, is an asset that the real time classroom lacks. The shy student can enter a question in chat without having to insert herself among more lively peers. An assistant instructor can tend to the chat, doing triage on the questions. Some questions should come to the attention of the teacher. Others can be answered in private chat. In a formal classroom, such private communication is usually seen as disruptive. In a virtual classroom, this happens smoothly and naturally.

3 The Zoom Room

One week, I was assigned a room with four energetic and highly motivated young students, each about 10 years old. The facilitators were familiar with these four. When we first started the program, we found them difficult to work with. They had often gone far into the problem: the amount of time they spent on it could not tell us that. They often had bits of mathematical background that other students lacked. And their youthful and overflowing exuberance made it hard to integrate them into a group. They were always a challenge to any facilitator.

So we decided to create a special breakout room for them, the "Zoom Room" where they could race ahead. The success of this effort varied with the mood of the children. At best, they urged each other forward and vied with each other for insight. At worst, they would try to show off to each other what they already knew, without contributing to either the group effort or their own knowledge.

This week the group clicked. I was delighted to find that the four boys (they were all boys) worked beautifully together as a team, and got further than any one or two of them could have in the short time available. I led them with but a light touch of the reins.

They did not solve the given problem. They didn't even work on it. They created their own, and the last thing I wanted to do was confine them to what I thought they should be learning.

Here (briefly) was the problem we had set them:

> *Given a large square with integer sides, how can you tile it with smaller (possibly unequal) squares, also with integer sides?*

The actual problem was presented in a more structured way, to offer `on ramps' to the mathematics. It's an interesting problem, combining elements of combinatorics and geometry. And, as is typical of JRMF problems, it can be worked on many different levels. I was eager to find out where the discussion would go with my four young students. It took a turn that I could not have predicted--or prepared for.

They looked at the first problem and immediately answered that with 1×1 squares, you can tile any $N\times N$ square. Seeing this as a special case is a sophisticated insight for children that young.

They then went on to consider the question of tilings with 2×2 squares. I asked if you could tile a 7×7 square with 2×2 squares. They again saw that they couldn't, and articulated the reason: 2 does not divide 7.

So I asked, "If A does not divide B, then clearly an $A\times A$ square cannot tile a $B\times B$ square. But maybe the tiling might fail, even if A does divide B. Maybe there's some other reason you can't tile the BxB square." My point, the difference between a necessary and a sufficient condition, was new to them. Very generally, I find that the core difficulty in learning mathematics—for anyone, at any level—is the logical structure behind the assertions or computations. Even these sophisticated students had to take a minute to understand what I was saying.

In fact this condition is sufficient as well as necessary. They seemed to understand this particular example, but I am not so sure that they will understand the distinction between a necessary and a sufficient condition in another context. No matter. They are ten years old.

To guide the discussion a bit, and to get what I could out of their intense engagement, I asked how many ways they can tile a 7×7 square with identical squares. Dan (I am not using the students' real names here) immediately said, "Only with 1×1, because 7 is prime."

"No," countered Alejandro. "You can tile it with one big fat 7×7 square. Does that count?"

"Well," said Titus, "A prime number has only two divisors: one and itself. So we can use the same idea to count these tilings, if we count 7×7 as a tiling."

Titus may have wanted simply to show what he already knew. But this seemingly innocent and perhaps boastful remark turned out to be a fertile one. Dan generalized immediately: “For an 8×8 square, there are four tilings.” (He meant tilings with identical squares, and so will we, from hereon in.) “That’s because 8 has four divisors: 1, 2, 4, and 8.”

And this is where we came in. Alejandro took up Xue’s challenge, and his ten-year old explanation was wonderfully simple. “Say there are two primes, p and q. Say the number is p^2q^3 You just make a picture.” He shared his screen and drew this:

1	p	p^2
q	pq	p^2q
q^2	pq^2	p^2q^2
q^3	pq^3	p^2q^3

In another group, Alejandro’s explanation would have been a mystery. But these four looked at it and understood.

Sharing the screen took a moment, but this group was all ears. In a real classroom, there would be no transition necessary. Here there was, and with a less motivated group this would be a weak juncture. But for these students it was not an issue.

“You need a 1 to count the 1,” said Dan, “and also the singles: q; q^2; q^3.”

“Right,” said Xue. “So if p is squared, you have three columns, not two. That’s why we add one to the number on top.” He meant the exponent.

“But what if it’s like $p^2q^3r^4$?” asked Alejandro... and then answered his own question.

“Oh. It’s the same thing. You can just list the twelve divisors we have already down the side, and list r; r^2; r^3; r^4 on the top.” As facilitator, I squirmed a bit at the error. But in this virtual environment, no one saw it. And knowing these kids, I remained silent.

“No,” said Titus quickly. “You need five columns: 1; r; r^2; r^3; r^4.”

(Here again, with a less sophisticated group, I would have had to draw an example or two—which would have occasioned another disruption. But these students did not need more.)

“That’s right,” said Alejandro. The silence had paid off: it made the point better and faster than I could have. The interaction at once exploited the benefits of kids working together and increased the bond between them. Boastfulness and ego were quickly put on the back burner.

Silence is an important classroom element (the technical term is ‘wait time’). The reader will find several places in this narrative where it was important for the teacher to hold his tongue. In a real classroom, the silence can be filled with non-verbal cues, both from the teacher and from the students. In a virtual classroom, silence is harder to manage. But with this group, there was no problem. It was always productive. Always golden.

I didn’t want to rest there. They could recite the formula. They could prove it. I wanted to make sure they could use it. So I asked them a question that they were unlikely to have seen before: What two-digit number has the most divisors?

Their thought was swift, and collective. They quickly saw that they had to look at prime divisors and balance the number of divisors with the exponent in the formula. All this without writing anything down.

Titus led off: "It probably should have lots of 2's and 3's. Because we don't want the number to get too big."

Xue: "Well, it can't have more than six 2's, because 2^7 is already 128. And 2^6 is 64 and has seven divisors." He had intuited the formula for the case of a single prime. I did not need to call his attention to this special case.

Titus again: "What if we put in a 3? Three times 32 is 96. It has. . . " He thought a minute. "It has 6×2 : : : twelve divisors."

I didn't have to ask him to explain. Indeed, I didn't have time. Alejandro jumped right in: "It depends on the exponents. The primes don't matter. They just can't be too big."

Xue: "Can we have a 5 as a prime factor? Well, we can't have two 5s. We can, but that will give us 25, 50, 75, and they don't have enough divisors." He was imagining what applying the formula would do, and his intuition told him (correctly) that these numbers would have fewer divisors than the 12 that they already saw for the number 96.

Dan: "And if we have one five, the rest of the number is 20 or less. We would need 6 or 7 divisors for that kind of number. Can we do it?"

Silence.

Then Dan again: "Seven divisors can't work. It's prime. Six divisors? It's 2×3, so we need pq^2. That's 2×3^2 or $2^2 \times 3$. Eighteen or twelve. Five times these give 90 or 60. Each of these also has 12 divisors."

Alejandro: "I don't think we can beat 12. We just have to look at 2's and 3's. No. We can't get 13 or 14 divisors. We would need too high a power." (I didn't stop him. everyone seemed to understand.) "Can we get 15 divisors? We'd need $2^2 \times 3^4$. That's too big. Or $2^4 \times 3^2$. What is that? 16×9. No, still too big."

Titus: "So only 12 divisors."

I asked, "Which two-digit numbers have 12 divisors?" The list came tumbling out of them, and they all contributed.

This last episode was sort of a wormhole in the timing of the class. Less sophisticated students would need to work more examples. In a real environment, I would have divided the students in groups. I would have assigned each group a different set of computations, then pooled the results. I may even have had the whole class work at blackboards around the room, if the venue supported that move.

None of this was necessary here. But how would I have duplicated the orchestration of a group effort virtually? I'm not sure.

4 Generalization

Unbidden, the group asked the next question: “What three-digit number has the most divisors?” They started working on this, and the ideas flowed. Ramsey Makan, my technical assistant, himself quite young, had been listening. The number 720 came up, and someone remarked that this was 6!.

Ramsey asked them, “How many divisors does 6! have?” They worked it out. Then of course they started thinking about factorials in general. Titus was out of the discussion for a few minutes, then came back.

“I wrote a Python program to list the divisors of n!.” They all wanted to see, so Titus ran it, for n = 1 through 6.

Titus’ productive absence would have been difficult to arrange in a real classroom.

“Can it do 10!?” someone asked. Titus ran it for n = 10. The screen went blank.

“The numbers are pretty big,” he finally said. “So it’s going slow.”

And indeed it was. The program was using brute force. I wanted to keep the momentum of the group up, so I said: “Can you figure it out yourselves? Maybe you can beat the machine.”

And they did. When the number finally popped up on the screen, it matched their result.

This “John Henry” episode is unlikely to have occurred in a real classroom environment. In the virtual environment, it was completely natural.

With time running out, I wanted to leave them with something to work on. So I said: “Suppose you know the number of divisors of 12!. Suppose some wizard told you how many there were. Would there be more divisors of 13!? Or fewer?”

The group responded easily: “More.” And then Dan said, “Twice as many. Because 13 is prime.” This was met by a chorus of “Oh, yeah.”

“But it wouldn’t work for 14! if you knew 13!,” said Xue. Then, a moment later, “What would work?”

They started thinking. Titus said: “Four times as many...”

Titus’s idea was not quite right. But the time was up. The breakout room was closing. I said goodbye and the webinar came to a close.

5 Conclusion

My experience with these four students may not generalize easily. After all, I had the wind behind me: the kids came with their own motivation. But it does give us a picture of what can happen when students encounter each other virtually.

I like to take the Long View on the development of technology. Very early automobiles were ‘horseless carriages’. Headlights were lanterns. Trunks were strapped to the rear to hold luggage. But in time the design adapted to the technology.

Distance learning is still in its infancy. We are still constantly comparing the virtual classroom with the real classroom. But we can look forward to a time when the techniques of distance learning are valued for their own sake, and not as an imitation of the classroom.

Mark Saul

Public Engagement 280 Characters at a Time

Oliver Johnson

1 Research and Group Testing

I am Professor of Information Theory at the University of Bristol in the UK, where I am Director of the Institute of Statistical Science in the School of Mathematics. If I expected a turbulent year, I would have guessed it would have been due to the effects of Brexit. Despite that, as for many contributors to this volume, in early 2020 mathematical life for me was proceeding as normal – with research visits to Cambridge and Frankfurt in January, along with hosting a French co-author and marking 300 first year exam scripts. Obviously, I was aware of the beginnings of the COVID story, particularly since I had planned a research visit to Hong Kong in mid-February. As the COVID news story started to grow, and with regret, I cancelled this trip as a precaution, it still felt like a distant event taking place overseas.

Had I been asked to predict what relevance my mathematical background might have to a pandemic, I might have assumed it would come through my research in group testing, sometimes known as pooled testing. This is a search strategy that can be used to detect infectious disease when tests are scarce and the disease is rare. We combine samples together in a pool and test them together: if no person in the pool is infected then the test will be negative, so each negative test allows us to remove some people from consideration. It is an interesting combinatorial problem to design an optimal test strategy, and to develop practical (polynomial-time) algorithms to identify the infected individuals from the test results, particularly when the tests are performed all at once (non-adaptively). Following several years of research on group testing, we published a 200 page survey monograph [1] in November 2019.

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Since group testing is a natural strategy for detecting coronavirus infections, and since it has been demonstrated in practice that PCR tests are powerful enough to detect one infected person from a pool of thirty or more, there has been a considerable amount of interest in our monograph, and in creating new group testing testing strategies and detection algorithms. However, I have not personally been particularly involved in this, pleased though I am to see group testing reach wider prominence.

2 Public Engagement via Twitter

Instead, I have found myself using mathematics in a more unexpected way. In 2012, I had signed up to Twitter, as a way of communicating with my undergraduate students. Originally, my plan was to try to summarise each lecture in a single tweet, to focus my attention on what the essential messages were. Since then, I had found more use for the site, as a way of networking formally and informally, and keeping in touch with mathematical developments. At the start of 2020, I had roughly 1,000 followers at my account @BristOliver.

As cases started growing in the UK in early March, as my Chinese students started wearing masks on campus, and thoughts started turning to possible contingency plans, like many of my mathematical colleagues I was keen to understand exactly what was going on. One obvious way to do this was to track case numbers – plotted on a logarithmic scale, with a regression line fitted, as a simple way to spot trends. Immediately it was clear that there was a problem. The conventional wisdom of 5 day doubling did not seem to be justified by the data, which was showing 25–30% daily increases. I started to tweet the kind of crude plots I was generating, with some kind of brief commentary as to what I observed.

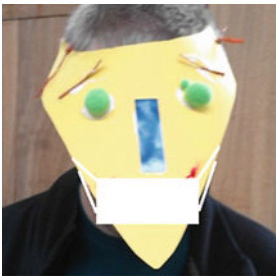
Gradually, these plots began to gain interest. People who followed me retweeted them, other people found them interesting enough to follow me, and so on. Ironically in the context, I found my follower numbers starting to grow at what felt like an exponential pace. At some stage, I started to be followed by a number of journalists and commentators, and found myself being invited to write for the website of the *Spectator* magazine, and to appear on BBC Radio and television, attempting to summarise the coronavirus numbers for a wider audience. I have unexpectedly found myself in the position of acting as a mathematical point of contact for journalists, and have continued to try to explain the latest developments. As of late November, I have just over 7,000 Twitter followers, and receive around a million views per week, although I have little doubt that these numbers will plunge once vaccinations reduce the danger of the virus!

3 Lessons About Public Engagement

While the coronavirus pandemic has been a unique experience for all of us, I have learned a lot about public engagement at this time. For example:

1. There is a genuine interest from the public to find out more about the mathematical principles underlying the situation. Professional mathematicians might regard many of these, such as logarithms, exponentials and percentages, as obvious. However, you can't assume too much knowledge – it is important to be able to explain these things clearly, and people are always happy to have a reminder of what it all means. It may be second nature to mathematicians that unchecked exponential growth at any rate is a serious problem, but you can't take it for granted that everyone knows that.
2. One of the most effective explanatory tools for the pandemic has been data visualisation. However, it is worth remembering that many people will see your graph on a small phone screen in bad lighting conditions. I am not a graphics expert, and usually produce very crude graphs. However, simple plots can be very effective, and less can be more in this sense. For example, in September 2020 I started plotting the trend for hospital beds occupied by COVID patients in the North West of England. This was simply a track of the latest data plotted on a logarithmic scale, and a best fit line pointing to the level of the first wave peak. Watching this graph evolve made it clear what direction things were heading in, and emphasised the need for action.
3. It is possible to get people to take on new ideas using Twitter. For example, I have always made the point that daily data is uncertain, and that the point estimates in the headlines actually come with confidence intervals. It is important to remind everyone that we are observing a complicated situation through noisy proxy measurements. I believe it is good for scientists to admit that they are not omniscient and infallible, and confidence intervals are a small way to do this.
4. In the same way, I have tried not to extrapolate trends to an unreasonable extent. The natural state of the epidemic is exponential growth or decay, so it is often enough to simply state the trend, and remark 'if this continued for two weeks then . . .'. Personally, I have tried to not take a too dogmatic position on either side of the debate about lockdowns and other interventions, and to take a centrist viewpoint.
5. I have learned that, despite what you might hear, most people on the Internet are nice. The journalists I have talked to have all been working incredibly hard to convey a complicated story in difficult circumstances, and have always been happy to take on board the nuances of the numbers. There remain a small percentage of Twitter users who are not so lovely, and I have gradually come

to embrace the block button to deal with people who I feel are acting in bad faith. However, the potential for negative experiences should not deter you from joining the Twitter conversation, and I believe more mathematicians should be using it as a medium to explain what we do.



Oliver Johnson

References

1. M. P. Aldridge, O. T. Johnson, and J. M. Scarlett. Group testing: an information theory perspective. *Foundations and Trends in Information Theory*, 15(3–4):196–392, 2019.

Three Stories on the Importance of Putting It on the Back Burner

Serge Tabachnikov

In the time of COVID, it is hard not to think about Boccaccio's *Decameron*, a collection of tales told by a group of young men and women who escaped the Black Death that was ravaging Florence by sheltering themselves at a secluded villa. In fact, in July of 2020, The New York Times Magazine published "The Decameron Project", a collection of 30 short stories of contemporary authors [14].

And so I was thinking about some mathematical tales from my experience unified by a single theme.

I believe the following scenario is familiar to many mathematicians: trying to solve a problem, one hits the wall and has to put the problem on hold. And later, sometimes much later, and for no apparent reason, an idea for a solution appears, previously hidden in plain sight.

I shall present three such stories from my practice. I do not mean to imply that one continues to think about the problem subconsciously; it is rather that a break is needed for viewing the problem from a different perspective. In my case, the problems involved were not particularly deep or hard, but in each case the "waiting period" was 15 years or more.

1 Foucault Pendulum

My first story started when I was in high school. This was in the Soviet Union in the early 1970s. We had a substantial physics course, and one of the topics in dynamics was inertial forces.

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Fig. 1 Saint Isaac's Cathedral in Saint Petersburg

These are fictitious forces that appear to act on mass-points when the motion is described in non-inertial systems, such as rotating ones. One of these forces is the Coriolis force, see, e.g., chapter 27 of [1]. The Coriolis force, due to the rotation of the Earth, explains why the moving bodies in the Northern hemisphere are deflected to the right (and to the left in the Southern hemisphere).

In particular, our high school physics textbook explained that it is due to the Coriolis force that the plane of oscillation of the Foucault pendulum is turning, and this phenomenon is the more prominent the closer one gets to the Pole.

Foucault pendulums could be seen in many places: churches often were transformed into science museums, and a Foucault pendulum that proved the rotation of the Earth was a prominent exhibit. The first Foucault pendulum that I saw as a child was very impressive: it was in Saint Isaac's Cathedral in Leningrad, and with its 98 m, it was the longest in the world (this Foucault pendulum was exhibited from 1931 until 1986) (Fig. 1).

I had problems with understanding inertial forces sufficiently well, and for many years I felt that I did not really understand why the Foucault pendulum does what it does (apparently, I was not unique: see [8] for the history of controversies related to the Coriolis effect). Until I realized, many years later, that the Foucault pendulum was a purely differential geometrical phenomenon.

Imagine that the Earth is a (not rotating) unit ball, and let γ be a closed path on its surface. What happens if one traverses the path γ holding a pendulum in one's hand? *The plane of oscillation of the pendulum will turn through the angle equal to the total geodesic curvature of* γ.

To see this, approximate the path by a spherical polygon. While one walks along a side of this polygon, that is, an arc of a great circle, the plane of oscillation does not

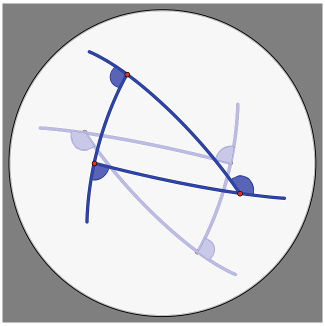

Fig. 2 Sum of exterior angles of a spherical polygon

turn relative to this side. At a corner, one turns by the exterior angle of the polygon, but the plane of oscillation remains the same. It follows that the total turn of the plane of oscillation is the sum of the exterior angles of the polygon which, in the continuous limit, becomes the total geodesic curvature of the path γ. See Fig. 2.

One can calculate this total curvature using the Gauss-Bonnet theorem: if γ is a simple curve, then its total curvature equals 2π minus the area inside γ. In particular, if γ is a circle of spherical radius r, then its total curvature equals $2\pi \cos r$.

The rotation of the Earth forces the suspension point of the Foucault pendulum to move along a circle of latitude, explaining the phenomenon in purely differential geometrical terms. See section 20.9 of [3]. A similar explanation can be given if the pendulum is replaced by a well lubed bicycle wheel kept horizontally, see [5].

The geometrical approach makes it possible to replace the surface of the Earth by other Riemannian surfaces, assuming that the only force acting on the masses is a constant normal "gravitational" force.

For example, consider the case of the hyperbolic plane. If a pendulum traverses a circle of radius r, then its plane of oscillation will turn through $2\pi \cosh r$. In particular, for a countably many values of the radius, the turning angle is a multiple of 2π, so the plane of oscillation will return to its initial position.

2 Cone Eversion

The second story started when I was an undergraduate student, in the late 1970s. My advisor, Dmitry Fuchs, suggested to me the following research problem.

In the punctured plane two unit vector fields are given, the gradient of the distance to the origin and minus this gradient. These fields are, of course, homotopic as non-vanishing vector fields: just turn the vectors of the first field through 180°. The problem is to construct a homotopy in the class of non-vanishing *gradient* vector fields.

The problem was posed by M. Kransnoselsky in the 1970s in the talk titled "Mathematical divertissement". The existence of a homotopy follows from the Gromov-Phillips theorem, see [4] or the book [2] where this problem appears as

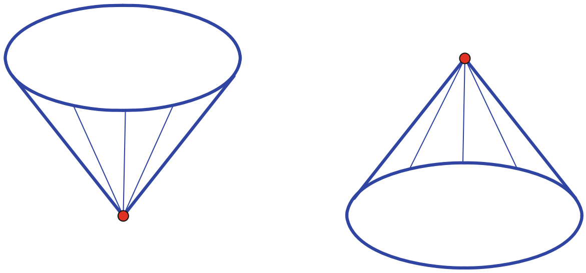

Fig. 3 Two punctured cones

an exercise. The Gromov-Phillips theorem belongs to the theory of h-principle, and the proofs in this theory are not constructive. This makes the problem interesting and non-trivial.

A famous example of a surprising result of this theory is the existence of a sphere eversion that follows from the Smale-Hirsch theorem. There are several constructions of such an eversion; one of them, due to W. Thurston, is presented in the film "Outside In" [13].

Our problem can be presented in a more geometric way as a cone eversion problem: one wants to deform the punctured cone $z = \sqrt{x^2 + y^2}$ to the cone $z = -\sqrt{x^2 + y^2}$, Fig. 3, in such a way that the tangent plane to the intermediate surfaces is never horizontal.

At the time, I did not produce a convincing construction of the required homotopy, and for many years I was not thinking about this problem at all. And then, out of the blue, I recalled the problem, and a solution presented itself quite clearly.

Here are heuristics that lead to a solution. First of all, instead of the whole punctured plane, one may consider a narrow open annulus whose center line is the unit circle (which is topologically the same), and the surfaces involved are the graphs of functions defined in this annulus.

The initial and the terminal cones are made of a horizontal unit circle with radial segments attached to its every point and having constant positive, respectively, negative, slope. One can assume that each intermediate surface is made of a core closed space curve that sits over the plane unit circle and the radial segments attached to its every point.

The condition that the tangent planes are never horizontal translates to the following restriction: if the core curve has a horizontal tangent at some point, then the slope of the radial segment at this point must differ from zero.

Thus the whole process can be described by two time-dependent functions of one variable (parameterizing the unit circle): the height of the core curve over the unit circle and the slope of the radial segment at the respective point.

The reader interested in the formulas and some pictures can find them in [11] or chapter 27 of [3]. But perhaps it is better to watch a recent movie made by S. Li and featuring this cone eversion [6].

3 Equitangent Problem

The third story goes back to the late 1980s. Influenced by V. Arnold, I was interested in various variants of the four vertex theorem and its ramifications (this theorem asserts that a simple closed curve has at least four curvature extrema). I asked myself the following question: *given a plane oval, is it true that every closed path around it contains a point such that the tangent segments from it to the oval are of equal length?*

If the path is very close to the oval, then these "equitangent" points are located near the oval's vertices, and there are indeed at least four of them. I believed that there are necessarily four such points on every curve that goes around the oval. I published this conjecture in [10], but I could neither prove nor disprove it.

A somewhat stronger evidence for the conjecture is provided by the following result, motivated by the flotation theory: the locus of points from which an oval is seen under a fixed angle contains at least four equitangent points [9]. These curves foliate the exterior of the oval, so the equitangent points abound.

For many years I did not think about this conjecture. And then, one day, a counterexample dawned on me (see [12]).

First, one considers an equivalent formulation of the problem: can a chord (of variable length) make a circuit inside an oval so that the angles that it makes with the oval are never equal?

Second, the problem can be discretized: the oval is replaced by a convex polygon, the chord is described by its endpoints and the directions of the support lines to the polygon at its endpoints. Furthermore, the motion of the chord is also piecewise linear: at each step, only one of the four elements (an endpoint or a support direction) varies. After such a discrete example is constructed, its smoothing yields an oval with the desired property.

See Fig. 4, borrowed from section 2.17 of [7], for the construction.

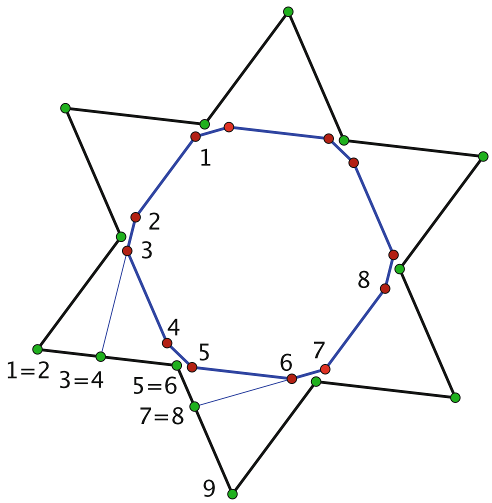

Fig. 4 A discretized construction

The dodecagon has a sixfold rotational symmetry, and the star of David is a path around it that is free from equitangent points. Due to the sixfold symmetry, only 1⁄6 of the whole process is described.

The starting chord is 15 with the support directions at its endpoints 12 and 56. The terminal chord is 37 with the support directions 34 and 78. The terminal chord differs from the starting one by rotation through $\pi/3$.

The whole process is as follows (the first item in each triple is the chord, the second and the third are the support directions at its endpoints):

$$(15, 12, 56) \mapsto (25, 12, 56) \mapsto (25, 23, 56) \mapsto (35, 23, 56) \mapsto$$
$$(35, 34, 56) \mapsto (36, 34, 56) \mapsto (36, 34, 67) \mapsto (37, 34, 67) \mapsto (37, 34, 78).$$

The author

References

1. V. Arnold. *Mathematical methods of classical mechanics.* Second edition. Springer-Verlag, New York, 1989.
2. Ya. Eliashberg, N. Mishachev. *Introduction to the h-principle*. Amer. Math. Soc., Providence, RI, 2002.
3. D. Fuchs, S. Tabachnikov. *Mathematical omnibus. Thirty lectures on classic mathematics*. Amer. Math. Soc., Providence, RI, 2007.
4. A. Haefliger. *Lectures on the theorem of Gromov.* Proceedings of Liverpool Singularities Symposium, II (1969/1970), pp. 128–141. Lecture Notes in Math., Vol. 209, Springer, Berlin, 1971.
5. M. Levi. *A "bicycle wheel" proof of the Gauss-Bonnet theorem.* Exposition. Math. **12** (1994), 145–164.
6. S. Li. https://www.youtube.com/watch?v=DMO9Qg3fGVg&feature=youtu.be
7. B. Khesin, S. Tabachnikov. *Fun problems in geometry and beyond*. SIGMA Symmetry Integrability Geom. Methods Appl. **15** (2019), Paper No. 097, 21 pp.
8. A. Persson. *The Coriolis Effect: 400 years of conflict between common sense and mathematics*. Hist. Meteorol. **2** (2005), 1–24.

9. E. Raphaël E., J.-M. di Meglio, M. Berger, E. Calabi. *Convex particles at interfaces*. J. Phys. I France **2** (1992), 571–579.
10. S. Tabachnikov. *Around four vertices*. Russian Math. Surveys **45** (1990), 229–230.
11. S. Tabachnikov. *A cone eversion*. Amer. Math. Monthly **102** (1995), 52–56.
12. S. Tabachnikov. *The (un)equal tangents problem*. Amer. Math. Monthly **119** (2012), 398–405.
13. https://www.youtube.com/watch?v=wO61D9x6lNY
14. https://www.nytimes.com/interactive/2020/07/07/magazine/decameron-project-short-story-collection.html

Solitude Is Conducive to Work and Contemplation

Krishnaswami Alladi

Since mid-February 2020, the world has been tormented by a new Coronavirus (dubbed Covid 19). It attacks the lungs, injures the immune system, and spreads through contact and through the air we breathe, and is more lethal than the viruses we have endured in the past. The world did not know how to stem the tide of this virus except through self isolation and lockdowns. This coronavirus brought everything to a standstill. It has had a devastating effect on education, travel, business, and our day to day lives. In the sphere of education, it forced all classes from elementary school level to graduate courses to be conducted virtually. This is definitely not as effective as live in class instruction, but we need to manage with the situation at hand. We should be thankful that we live in the era of internet communication and so it is possible for most of us to conduct our work from home. As I pen these lines in the last quarter of 2020, we are told that a new vaccine has been found which will be made available in the first half of 2021.

Human beings have overcome the most formidable circumstances. Examples abound of persons who did outstanding work undeterred by the impediments during the most troubling times. I provide two very different examples of monumental work done during times of isolation and disaster.

When the bubonic plague hit England in 1664–1665, and Cambridge University was closed, Isaac Newton isolated himself at his home in Woolsthorpe. He thought deeply about limits, and during these two years, invented (among other things) "the method of fluxions" (calculus). During World War II, when the city of Los Angeles and its surroundings were under blackout, the astronomer Walter Baade utilized the absence of background light pollution to map the heavens from the Mt. Wilson Observatory, and to even view and photograph the Andromeda Galaxy which is 2.5 million light years away from the earth. The works of Newton and Baade done

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during periods of isolation and disaster, have revolutionized their fields in major ways.

Long hours of contemplation is key for creativity in mathematics and music, more so than in other fields of study. And solitude often provides the time for contemplation. Thus I am sure that research mathematicians to whom the pursuit of their subject is their first priority, often to the neglect of various duties(!), would have continued to make significant progress in their investigations. Book writing, or even writing up research papers on work completed, takes time. The real thrill in research is in the discovery, and so many top mathematicians in the forefront of research, lecture on their work, but take time to write it up. But periods of isolation can be, and are, used to make progress on various writing projects.

The great mathematician G. H. Hardy said that the most creative years of a mathematician are the years of youth, and so he felt that the years past 40 are best spent by a mathematician in writing books based on the knowledge and experience gained over time. And he did write a number of influential textbooks. In my case, I continued to do research after the age of 40, but decided to spend a significant part of my time in the last decade (I am 65 now) writing books. I utilized the isolation in the last six months to complete two book projects.

My book "Ramanujan's place in the world of mathematics" was published by Springer in 2012 for Ramanujan's 125th birth anniversary. It is a collection of essays that I had written on Ramanujan related themes since the Ramanujan Centennial in 1987. I say Ramanujan related themes because much of the book is about mathematical luminaries in history like Euler, Jacobi, L. J. Rogers, MacMahon, Hardy, and Littlewood, whose lives and works have things in common with Ramanujan. The idea is that we can understand and appreciate Ramanujan and his mathematics better if we study him in comparison with the lives and works of other mathematical giants who have links with Ramanujan. There are also articles in the book about current developments in the enchanting world of Ramanujan. This book was written to be of appeal to both experts and lay persons. Much has happened since 2012, and so I decided to publish an enhanced Second Edition which has six additional articles. These include my detailed review of "The Man Who Knew Infinity" movie on Ramanujan, a report of the work of the SASTRA Ramanujan Prize Winners, and a report of a conference held at The Royal Society, London, in 2018 to commemorate Ramanujan on the Centenary of his election as Fellow of the Royal Society. I completed work on Edition 2 during these months of isolation and sent it to Springer for publication. Edition 2 is expected to appear in the first half of 2021.

The second book is my academic autobiography entitled "My Mathematical Universe – People, Personalities and the Profession" to be published by World Scientific. The book includes a discussion of the work and personalities of the many remarkable and eminent mathematicians with whom I have had the pleasure of interacting since my days of youth, and about some issues facing the profession, many of which I have dealt with as Chair of mathematics at the University of Florida for a decade (1998–2008). I completed the First Draft of this book in early November 2020 and will spend the next six months polishing the narrative to get it ready for publication.

I have also used the time to help a PhD student on his thesis problem in analytic number theory. The problem I gave him stems from my own PhD thesis of 1978 in which I introduced a duality principle that links the largest and smallest prime factors of an integer using the Moebius function. I exploited this duality to obtain asymptotic estimates for sums of the Moebius function over numbers with restricted prime factors. In the last few years, this duality principle has been generalized in significant ways to algebraic number fields. The problem I gave my PhD student was to study certain interesting series involving the Moebius function (not investigated previously) in the context of this duality principle. We continued to make progress on these questions during these six months of isolation.

During the isolation brought forth by the pandemic, mathematicians worldwide have presented their work in seminars and conferences that have been conducted virtually. One major advantage of holding a university weekly seminar virtually is that it can be viewed by audiences all over the world, whereas these weekly seminars previously had only a local audience, unless the talks were recorded and made available later.

The pandemic has had varied effects on conferences. Some conferences have been cancelled or postponed, whereas others have been converted to a virtual format. I will mention two conferences where I was invited to be a speaker.

The year 2020 is the centenary of passing of the Indian mathematical genius Srinivasa Ramanujan. In January 1920, three months before his death in April that year in Madras, India, Ramanujan wrote his last letter to Hardy outlining his latest discovery – the mock theta functions. Thus 2020 is also the centenary of the discovery of the mock theta functions, considered to be among Ramanujan's deepest contributions. So a conference was scheduled to take place in May 2020 at Vanderbilt University to mark the centenary of the birth of mock theta functions. But the organizers decided to postpone this conference to 2021 instead of holding it virtually.

To mark the centenary of Ramanujan's passing, the Ramanujan Mathematical Society scheduled a conference in Cochin, India, in December 2020. In view of the pandemic, the organizers converted the conference to a virtual format instead of postponing or cancelling it. So I will give my talk at the Cochin conference virtually as all other speakers will do.

With regard to the SASTRA Ramanujan Prize which is given annually at a conference at SASTRA University in Kumbakonam (Ramanujan's hometown) in India around December 22 (Ramanujan's birthday), we made the selection of the winner of the 2020 SASTRA Prize in October this year, but decided not to hold the SASTRA Conference; the prize will be given in December 2021 along with the 2021 Prize. We decided not to have a virtual conference this year, because one of the purposes of inviting the winner to Kumbakonam is to show him/her Ramanujan's home from where a thousand theorems emerged, the temple where he worshipped, and the high school he attended.

There is one major advantage to holding conferences virtually: they can be conducted with almost no budget! Thus there is no need to apply to funding agencies for conference support, or ask the university administration for seed money. I hope that owing to this advantage, virtual conferences do not become the norm in the future. The main importance of having regular conferences with a gathering of participants is that outside of the sessions where the lectures are presented, participants can get together informally for discussions about their work. Such informal discussions contribute significantly to the progress of research.

Covid 19 would not have been a barrier for mathematicians to pursue the problems that intrigued them. In fact I would suspect that if some data is collected by professional societies like the AMS regarding the work done by mathematicians during this period of isolation, it would actually show an increase and not a decrease. This is not an argument in favor of lockdowns! It only shows that mathematicians will work undeterred by the most formidable circumstances and that the isolation helped in the contemplative nature of their work. Of course we all want a vaccine to be produced soon to effectively counter the virus so that life would return to normalcy for the benefit of all.

Krishnaswami Alladi

Counting Syllables, Shaping Poems: Reflections

JoAnne Growney

Girls who change
lightbulbs change
everything!

One of my mental habits is to try to find connections between disparate facts, events, and preferences and a consequence of this habit is that I find more and more links between two of my favorite subjects, poetry and mathematics. My career was in mathematics (a professor at Bloomsburg University in Pennsylvania) but I have also been able, after my children were grown, to find time for reading and writing and sharing poetry. In retirement, I've investigated a variety of math-poetry linkages, including the French group ***OULIPO***, a group of mathematicians and poets who engaged in and invented some new math-and-literature connections. My growing desire to share what I'd been learning led me to begin, in 2010, a blog, "Intersections – Poetry with Mathematics," which continues to this day (with more than a thousand postings).

Some mathematical patterns in poetry are pleasing to the ear and match with body rhythms. The ten-syllable lines of the sonnet each occupy one breath—with each line composed of five iambs (da-DUMs), matching the beats of the human heart. The fourteen-line sonnet thus draws the reader not only into an excursion for the mind but also for the physical self. Occasionally I attempt the difficult task of writing a sonnet or a villanelle (which has the same ten-syllable-per-line pattern as the sonnet and has also some repetition of lines) and feel proud when I achieve that complex blend of sound with meaning.

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But poetic patterns simpler than the sonnet suit me better when I have an idea that I want to shape and share quickly AND in well-chosen words. I have observed that by following a pattern of syllable counts my word-choices are constrained in a way that helps them to be imaginative and effective. As a sample of this, I offer below a couple of syllable-square poems from a collection published about a year ago by *Math Horizons* under the title, "Give Her Your Support." Here are two samples from that article:

Little Women	Smart Girl Speculating
In school, many Gifted math girls. Later, so few Famed math women!	Last Sunday's paper had an essay by a clown who said "as long as I play dumb people let me do what I want." And I cannot stop wondering.

To publicize my math-poetry blog, I have often posted links on Twitter; during the pandemic Twitter became even more special—particularly in April which is National Poetry Month. National Public Radio (to which I often listen) regularly celebrates the arts and in April, NPR issued a call for poetry postings on Twitter (requiring poems with less than 280 characters). And I began to explore, including some thoughts about the coronavirus AND some notions focused on April as National Mathematics and Statistics Awareness Month.

My Twitter handle is @MathyPoems and here are several of my tweets.

On April 2: **A Coronavirus Fib**

Don't
touch
me with
your fingers—
use your heart—we must
keep bodies distant and stay safe!

A "Fib" is a 6-line poem whose syllable-counts follow the Fibonacci numbers.

On April 3: **A Fib for the Season**

Birds
sing.
Trees bud.
Daffodils
and tulips open.
My eyes and nose and heart love spring!

On April 4: **Smart Girl Speculating** (shown above, offered on Twitter untitled)

On April 13: **Pandemic** (a Haiku)

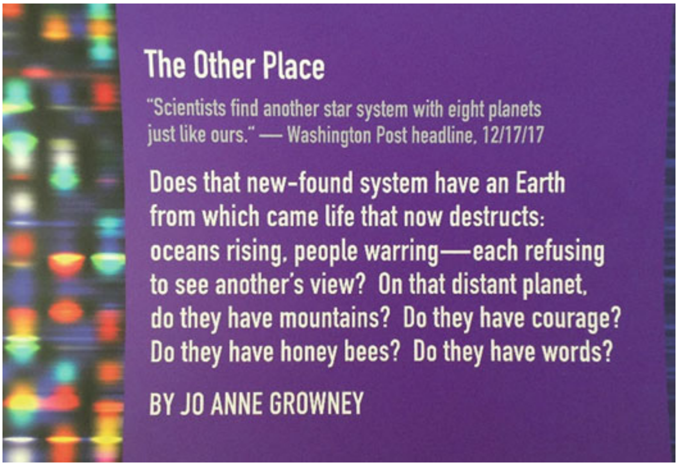
Exponential growth:
small numbers doubling quickly—
a world upended!

On April 17: The 3 × 3 syllable square with which this article began appeared as a Tweet

On April 24: Quarantine—
that other world
in which no moments
notice where I am.

And so on . . .

One of my ongoing concerns is climate change and I can't resist voicing that concern here and sharing a poem, "The Other Place," that appears on a poster in my study—a poem that was a contest winner in 2018 and appeared on busses in Arlington, Virginia.



For Twitter explorers I invite visits to my postings found using my Twitter handle @MathyPoems, and I encourage all with a bit of interest in the many connections that exist between mathematics and poetry to visit my blog, "Intersections – Poetry with Mathematics."

Additional Notes Information about the establishment of **National Poetry Month** (which happened in 1996) is available at the website for the Academy for American poets at this link.

President Reagan in April, 1986 established **National Mathematics Awareness Week** and this celebration in 1999 evolved into **National Mathematics Awareness Month** and in 2017 grew into **Mathematics and Statistics Awareness Month**. Details for the 2020 Celebration are available here.

JoAnne Growney

Experimental Math Outreach and Popularization During COVID-19

Naomi Simone Borwein

Since the pandemic began "Love in the Time of Corona" has become a popular meme in mass culture that superficially resonates with a large cohort. While cultural acquisition attests to the transmission of the iconography of Gabriel Garcia Marquez's 1985 *Love in the Time of Cholera*—only a small percentage of this cohort have actually read Marquez's text. It is a novel based aesthetically in its own critical moment, at the threshold of experimental modernist and postmodernist imperative, and imbued with ideas about mathematical metaphor, mechanism, formal logic, pattern, sequence, and popularizations about chaotic and dynamic systems. In the same way, mathematics that is deeply rooted in cultural imagination can be loosely understood through the visual, symbolic, metaphoric, or even apocryphal uses in popular culture. Marquez's work suggests a colourful experimental tension between logic and imagination, during plague, as a process of abstraction set against a backdrop of popularized 20th century fiction and society. In keeping with the theme of this Springer volume on "math in the time of corona," and in concert with the underlying math of Marquez's novel, this paper explores the idea of visualization in popular and digital uses during COVID-19, to examine how experimental approaches may or may not differ from pre-COVID conditions, as examples of the way experimental math exists through visualization for outreach and popularizing as it moves from one set of platforms to another. COVID is impacting outreach and learning pathways, through experimentation and innovative translation of visual math between platforms.

Modern experimental mathematics has flourished in a digital age, of computation supported by connectivity and distance, but media forms and approaches—nature of collaboration, digital resources, technological literacy, inquiry-based models, and experimentation—have undergone considerable change. Thus this paper attempts to

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answer the following questions by inspecting if the same hold true for outreach. What part does COVID-19 have to play in that transition? The stimulation of ideas in real time? In the digital environment? In experimental and *virtual analysis*?

1 Multi-sensory Experiments Before COVID-19

Consider experimental math as a research practice that is e-collaborative and is both computational and digitally assisted. Experimental math in popularization can be done through a broad spectrum of visual approaches, for example, mathematical pictures, the arts, music and immersive spaces. In these spaces, multi-sensory elements incorporate a toolbox of experimental methods. Take the example of a pre-COVID environment, a geometric outreach activity from the priority research centre for Experimental and Constructive Mathematics (CECM) at Simon Fraser University in the 1990s. This geometric and spatial concept progression in an outreach session is grounded in ideas about learning pathways that follow ‘concept progression’, or the gradual progression of concepts from early learning, through elementary and secondary learning into university learning (see studies by Daro et al. [8] and Shapiro [24]). It assumes both experimentation as a tool for conceptual knowledge, and experimental math as a digitally or concretely assisted mechanism underlying these progressions.

The outreach at CECM was not constrained by grade specific-curriculum learning expectations. What is interesting in this outreach is that the Grade 11–12 students were given the task of building 4–12-sided shapes using paper and balsa, with calculation of the internal angles. This progressed to origami folding and associated measurements, then on to viewing solid geometric objects that aided student visualization of internal angles, area, and volume. Visual proofs were given by rotating geometric images on touch screens capable of building 2D and 3D, then 4D images that furthered the understanding of concepts of space.

The experimental context of these outreach activities allowed for a transition in representations of various objects—for instance, from hollow to solid to virtual or immersive. The intention was to build conceptual thinking from the primary math of lower elementary shapes, through areas and volumes of secondary math in the 3D plane to academic research math in 4D.

Clearly some of outreach explorations translate more or less well into the digital resources that continue to be available and of use, even as platforms for visualization change.

Models that are underlying these forms of experimental outreach, including fun, innovation, math-in-action models, or multi-sensory learning forms, are often targeting an audience that spans multiple grades and includes an interested general public. An excellent example would be Helaman Ferguson who produces sculptural math in nature.

1.1 On-Line in the Digital Pre-COVID Era

A wealth of digital and/or interactive material for popularization and outreach assuredly existed before COVID. The Topic Study Group No. 07 explored "Popularization of Mathematics" at the International Congress of Mathematical Education (ICME), later published in the *Proceedings of the 13th International Congress on Mathematical Education* 2017, where the editors emphasize virtual forums, visual arts, and new technology alongside inquiry/research based projects. The 2012 volume *Raising Public Awareness of Mathematics* showcases an impressive range of popularization projects that happened in various countries. Globally, forms of math popularization employ a variety of experimental, embodied, or sensory approaches; these include topics and platforms such as

- *Mathmagical circus*, a text by Martin Gardner
- MoMath Exhibitions, National Museum of Mathematics, New York
- Random Walk on math interface
- Imaginary open mathematics
- Mathcitymap
- Mathematical Selfies, Western Carolina University [410]
- Snow sculpture math with an online presence by H. Ferguson et al.
- The International Mathematical Knowledge Trust (IMKT), a worldwide digital library for people and software systems.
- Braiding (crocheting) as a math task in Ester Dalvit's *Using Braids to Introduce Groups: from an Informal to a Formal Approach*
- Origami outreach

This diverse and playful subset of resources creates part of the foundation for extensions in COVID-era outreach.

2 Preliminary Examples During COVID-19

There is a move afoot by individual mathematicians and scholars to teach, intrigue, or enliven the constrained environment we find ourselves within today. This impulse is nothing new.

But, as if writing "The story of COVID-19" [23] as an experimental transition of highly visual elements across platforms in popular culture, consider the shift from visualizations of π to E_8 in the media *circa* 2020. In *American Mathematical Society* (AMS) blogs, a reshuffling of earlier material, means students can access visualizations of a variety of mathematical ideas, not just from π and E_8, but cooking Gaussian curvature, and "Journeys to the Distant Fields of Prime," all drawing on popular interest in mathematical constants.

Under 'Math in the Media' the AMS offers visual math. Highlighting how preexisting outreach platforms are being negotiated during COVID, listed on the AMS site under *Visual Math* an interactive virtual exhibition tour by the sculptor

Anton Bakker can be accessed at the National Museum of Mathematics in New York, written about by Denise M. Watson for The Virginian-Pilot, November 10, 2020.

Here the AMS, like the *Canadian Mathematical Society* (CMS), engages with an active and evolving conversation on "Mathematics and COVID-19," such as in the Discussion Series: Mac Hyman of Tulane University talks Mathematical Modeling and COVID-19.

Internet math is a virtual space of mainstream and academic data and collaboration in the shadows of social media—on Facebook, Twitter, instagram, and more. There exists a large repository of visual math exploration of fractals, mandalas, and various popular engagements with π and e, as much as simply visualizing the pandemic, from "COVID Math" to "COVID Visualizers" that depict global morbidity and infection rates.

Experimentation resonates across cultural spaces. For instance, Eric Andrew-Gee writing "What Quebec's COVID-19 experiment can teach us about the second wave" (Globe and Mail, November 20, 2020) underscores the perhaps unanticipated functionality of active, action-based research and experiment in mathematical modeling of disease: "That's our country for you," said economics professor Pierre-Carl Michaud, "We have a packet of experiments, and we can try to find the best recipe" [1]. His description of a recipe in this media piece suggests a pattern rule, or a toolbox through experiment, merging the modelling pandemics and disease contagions with other popular ideas such as chaos or apocalypse, like Mathematical Modelling of Zombies [25]. Conversely, in the journal *Chaos, Solutions & Fractals* Volume 140, November 2020, "Mathematical models of Ebola and COVID-19" "virus pathogen in the environment" by Zizhen Zhang and Sonal Jain describes randomness of disease vectors while citing 'random walks' [28].

What we see then showcases the broad uses and abuses of visualization and experimentation in popularization and research during COVID.

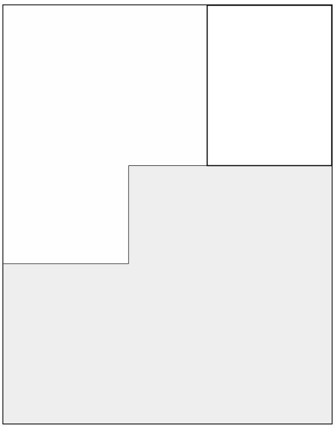
In 2020, Barry Mazur's "Math in the Time of Plague," published in *The Mathematical Intelligencer*, draws on Euclid's construction of the perpendicular bisector of a line segment. He explores the idea of basic conceptual understanding, by exploring the idea of "axiomatic setup"—that there is no hint that the lines intersect. Contrasted to these "primitive bedrocks of thought," what Mazur calls "common mathematical sensibility" recalls popular math knowledge. Mazur describes the process of "design[ing] fundamental online techniques to accommodate this moment" [18] and drawing on an experimental digital tool box that intersects with popular culture and in the connection has injected some of this sense of chaos into experimental space and practice.

Kevin Hartnett describes "Math After COVID-19" at the beginning of the Western pandemic in February, in *Quanta Magazine*; and, the byline reads "Modern mathematics relies on collaboration and travel. COVID-19 is making it increasingly difficult" [12]. With growing interest and the need to function in a virtual space, it is necessary to incorporate digital resources and form experimental research communities through new technologies, or new uses of pre-existent technologies—from ZOOM to Microsoft TEAMS to Cinderella. Digital spaces for popularization and interactive sites provide excellent interactive spaces for the dissemination of math games and support systems that are popping up to help the mathematically inclined person suffering from ennui. All of these games and sites become part of a new COVID-era learning pathway.

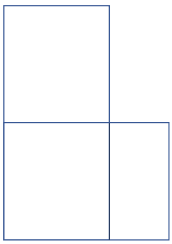
3 One Learning Pathway

Mathematics sometimes called the quest for patterns? [22, p. 15]

Basic visualizations are a staple of popular math—pre-COVID, during the pandemic, and certainly after. In virtual outreach patterning, tiling, and fractal geometry are often used to educate students while attracting a math literate but non-specialized audience. One pathway for fractal patterning and visualization, that I have developed, which is useful for looking at a transmission between digital platforms, moves from the math behind the honeycomb formation and hexagons, to pinwheel fractal tiling and pinwheel fractal stages, to The L Shape Problem (where we ask what the area of a given shape is?) to the more complex tessellations of The Golden Bee Problem (see Michael Barnsley's work) as a discussion of rotation and tiling, pattern/non-pattern, and fractal geometry. Clearly it enables a popular exploration of Euclidian geometry (e.g., hexagons) with fractal geometry.



Golden Bee Problem



L-Shape Problem

To describe the common use of outreach in more depth, take the example of the hexagon honeycomb problem, as popular math widespread in mass media. It can be used to teach patterning and the writing of mathematical expressions, which in turn moves quite naturally to the pinwheel fractal; the student is asked to write an expression to show the fractal formation of the pinwheel structure, visualized in stages from the first level represented by a single square to the third level, a simple fractal, where clusters of squares form fractal nodes—with parallels in the formation of the snowflake.

The next level of this basic visualization is the L-Shape Problem, an elementary problem where the L shape is used to learn, for example about area. Through emphasizing area, geometry, and patterning, it creates a stepping stone, conceptually, to further visualize the Golden Bee Problem, as an example of Penrose tiling and fractal geometry, here, multiple tiles in the shape of a 'b' (in two sizes) make the pattern.

Photo by Naomi Borwein

For a hexagonal tessellation like in a honeycomb—imagine how ceramic tiles repeat on a kitchen floor forming a pattern. Consider one visualization that is a learning sequence or pathway for exploring such patterning. This outreach task instructs a student or interested general audience to use 22 tiles to begin a pattern. They must then explain how that pattern is continued; and by having them create a non-pattern with those 22 tiles, as a comparison, they are engaging with inquiry-based learning. In any number of interactive or immersive digital spaces, this outreach can be achieved. Much like the CECM exercise, they can reproduce the original pattern with hexagonal cubes or produce a quadrilateral honeycomb, as original or solid objects or virtual manipulatives (concrete learning aids), and muse about how it becomes more complicated.

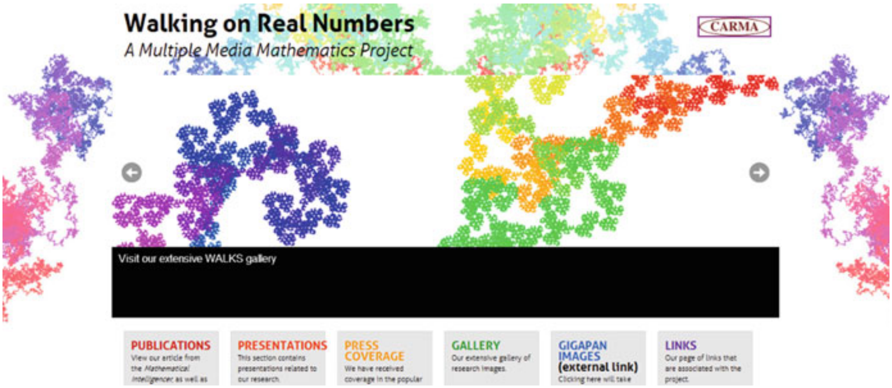
There is ironically a lot of rhetoric in the mainstream about these ideas, and they even tie in, tangentially, to the butterfly effect and colony collapse. Equally, fractals often connect to popular chaos theories (e.g., J. W. Bloom) as much as being rooted in the appeal of visualizations.

4 An Extension of a Learning Path

Interactive implementation of random walks are an example of pre-pandemic outreach that continues to endure. Representing the idea of randomness, a color-coded visualization of π to 100,000 digits, entitled "walking on real numbers" was published as a random walk in *Wired Magazine* in an article by Samuel Arbesma:

Randomness as a mathematical feature is often misunderstood in mass media. But the pictures are not only mathematical, they are visual candy for an interested general audience.

The following is a screenshot of "Walking on Real Numbers A Multiple Media Mathematics Project." It is an Interactive Random Walk, as a digital tool and website:



There are plenty of existing digital interfaces that help build public awareness of mathematics, such as IMAGINARY an Open Source Math Exhibition Platform [20], as well as numerous other resources that are actively evolving under the new constraints in which we find ourselves; such constraints encourage acts of inquiry, experiment, and discovery.

5 Modern Experimental Math

Far removed from the realm of popular visualization, the impact of modern experimental math as a theory and method is paradigmatically felt in the *Philosophy of Mathematics*. Here "Mathematical knowledge" once "regarded as certain since antiquity, because of formal proofs and deductive reasoning with respect to valid rules and axioms" is inherent in the very structure of knowledge systems that governed life. However, "this belief was shaken several times during the history of mathematics" and "most recently through computational experimental mathematics" itself emblematic of, or even eliciting, this shift in mathematics and mathematics education [21, p. 232].

Perhaps the most succinct definition of what exactly modern experimental mathematics is, is given in *The Computer as Crucible*.

> Experimental mathematics is the use of a computer to run computations that are sometimes no more than trial-and-error tests to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search [5, p. 9].

During COVID-19 this transformative approach pushes the practitioner into further uncharted waters where the very dichotomies between beautiful proof and the ugly underbelly of mathematical research have to be renegotiated as much as virtualizing the element of research and outreach practice.

While in "About This Journal" in the first issue of *Experimental Mathematics* published in 1992, David Epstein, Silvio Levy, and Rafael de la Llave wrote

> Experiment has always been, and increasingly is, an important method of mathematical discovery. (Gauss declared that his way of arriving at mathematical truths was "through systematic experimentation".) Yet this tends to be concealed by the tradition of presenting only elegant, well-rounded and rigorous results.... we consider it anomalous that an important component of the process of mathematical creation is hidden from public discussion [9, p. 1].

Almost thirty years later, much has changed, and much has remained poignantly static.[1]

While experimental math is no longer as contentious in the academic terrain of applied mathematics, in the journal *Experimental Mathematics* (2020) it is still noted that

> many mathematicians have been reluctant to publish experimental results. Those who have tried it have sometimes found the best-known mathematical journals unwilling to accept such material, regardless of merit. Experimental Mathematics is an effort to change this situation. We envision it as something akin to a journal of experimental science: a forum where experiments can be described, conjectures posed, techniques debated, and standards set. We strongly believe that such a forum will further the healthy development of mathematics [2].

This "systematic" experimentation and "the process of mathematical creation" [2] can be rethought in light of new impediments caused by the pandemic, where mathematical discovery can still be framed within the constraints of scientific experimentation, through platforms, and digital and virtual analysis.

6 Conclusion

Will COVID become a paradigm shifter?

[1]Portentously, Cristian Calude (2016) postulates in "Postface" that "in the not too distant future mathematicians will use them [proof assistants] as they today use LaTeX" [7, p. 432]. In 2020, such projections are more realistic, and the constraints of pandemic health regulations have afforded a space to push the limits of experimental computational research and pedagogy. And likewise, the longevity of attacks, or suppression of methods (exploratory, discovery, experiment) that describe "The End of Proof?" through the disavowal of modern experimental math [26], have been mitigated by scholarship published during covid-19 that bolsters these methods [6]. Invoking the phrasing of Francis Fukuyama's "The end of history?" [10] about the horrors of WWII being so extreme and destructive that confronting apocalyptic life damaged the very nature of thought and in doing so it signified the end of history "as we know it". Covid is doing the same, extreme pandemic catalyzes change and highlights the fact that experimental math has been paradigm shifting, even if H. K. Sorenson and others view this as a pejorative.

Clearly pre-COVID outreach is being extended, and with these extensions learning pathways are being altered—much like, for example, spatial and geometric visualizations. It is far too soon to know if there is or will be a dramatic difference in how experimental math exists in popular culture, or outreach after COVID, or which digital resources and hybrid methods will last. As visualization models move from one set of platforms to another, what does a pandemic driven transition in modern experimental math look like so far? The embodied, tactile, and multi-sensory nature of experimental outreach is replaced by many interactive hybrid models, various degrees of guided inquiry, and aspects of play. And the online experience offers a shield, from social pressure and peer-benchmarking that often exacerbates math fear. There is a redistribution of the student experience of outreach math in embodied, immersive, and digital formats. In popularization activities such as tiling and fractals visualization, or random walks, there is a suggestion both of increasing transmission and transition of visual math and its platforms today, where real-time and asynchronous forms of interaction, distance collaboration, lockdowns, virtual symposiums, interactive resources and outreach models, draw on a toolbox of experimental approaches as Bazar and others note. In the 1990s, the World Wide Web and computational power were foundational to modern experimental mathematics. Today, visible are the virtual extensions of multi-sensory learning pathways used in CECM outreach, as described at the beginning of this paper, and resources adapted through experimental translation in various new platforms. These interfaces and pathways will continue to co-evolve under current conditions.[2]

[2]Like the unimaginable mathematics of Borges's "The Library of Babel", Marquez's works, textually explore revolutions of myth, convey infinity through the vanishing point (as in "A very old man with enormous wings" [17] and *One Hundred Years of Solitude* [15]) or disembodied consciousness (in *Cholera* [16]) as a fugue state—oddly reminiscent of the cotton wool of deep proof and thinking. Marquez's cholera epidemic is explored through popularized myths of scientific method, a reinterpretation of the scientific practice of epidemiology research and the exploration of discourses of reason and logic: through a humanist lens that underlies the development and practice of modern experimental math: "From youthful enthusiasm he had moved to a position that he himself defined as fatalistic humanism" where "[e]ach man is master of his own death, and all that we can do when the time comes is to help him die without fear of pain. But, despite these extreme ideas, which were already part of local medical folklore," [16] and perhaps that resonates with a discourse on sublime beauty, truth, mystery, and surcease as a facet of 19th century engagement within research mathematics [11, 13], at the end of the novel, figures are left floating on a steamboat, sempiternally navigating up and down along the riverbed; it is an existential statement about the space of abstraction and a modernist symbol for consciousness, distilled thought: "They spoke of themselves, of their divergent lives, of the incredible coincidence of their lying naked in a dark cabin on a stranded boat when reason told them they had time only for death" [16]. There is something current but tacit in this imperative. While some researchers in the absence of concrete materials atavistically return to the space of the mind, cloistered in a mathematical hermeticism, this COVID-19 pandemic equally affords a moment of experientialism and experimentalism. Yet consider for a moment, *Love in the Time of Cholera* [16] alongside *One Hundred Years of Solitude* [15], with its epidemic of sleeping sickness and a colourful experimental tension between logic and imagination as a process of abstraction; this as a binary of reverse negatives mirrors plague and reason as part of proof against a backdrop of popularized 20th century fiction and society. Experimental math practice in the age of COVID-19 auspiciously relies on transformative digital engagement, which is germane to exploring the metacognitive dimensions of collaborative and connective approaches and platforms for research, necessitated by distance.

Naomi Simone Borwein

Bibliography

1. Andrew-Gee, Eric. "What Quebec's COVID-19 experiment can teach us about the second wave". (2020, 11, 20). *The Globe and Mail*.
2. Anonymous, Aims and Scope, *Experimental Mathematics*. (2020).
3. Behrends, Ehrhard, Crato, Nuno, Rodrigues, Jose Francisco (Eds.), Raising Public Awareness of Mathematics, Springer, Preface, 2012, vi.
4. Borwein, J. M., and D. H. Bailey. *Mathematics by experiment: Plausible reasoning in the 21st century.* Natick, MA: AK Peters, 2008.
5. Borwein, J., & Devlin, K., "The computer as crucible: An introduction to experimental mathematics," *The Australian Mathematical Society* 36, no. 3 (2009): 208–213.
6. Calkin, N. "What Newton Might Have Known Experimental Mathematics in the Classroom." *American Mathematical Monthly* 121, no.1 (June 13, 2020).
7. Calude, C S. "Postface." In *The Human Face of Computing, Advances in Computer Science and Engineering: Texts 9*, 429–432. Imperial College Press, 2016.
8. Daro, P., Mosher, F. A., and Corcoran, T. B. "Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction." In *Commons Consortium for Policy Research in Education (CPRE)*. 2011.
9. Epstein, D., Levy, S. & de la Llave, R. "About This Journal." *Experimental Mathematics* 1, no.1 (1992): 1.
10. Fukuyama, Francis. "The End of History?." *The National Interest* 16 (1989): 3–18.
11. Hardy, G. H. *A mathematician's apology.* Cambridge, UK: Cambridge University Press, 1967.
12. Hartnett, K. "Math After COVID-19". *Quanta Magazine*, April 28, 2020.
13. Lockhart, P. *A mathematician's lament, with a foreword by Keith Devlin*, New York: Bellevue Literary Press, 2009.
14. Lykken, D. "The genetics of genius." In Andrew Steptow (ed.), *Genius of the Mind*. Oxford: Oxford University Press, 1998.
15. Márquez, G. G. *One hundred years of solitude.* UK: Penguin, 2014.
16. Márquez, G. G. *Love in the time of cholera*, Translated from the Spanish by Edith Grossman. Kindle edition: Penguin, 2014.
17. Márquez, G. G. "A very old man with enormous wings: A tale for children." In *Leaf storm: And other stories*. NY: Harper Collins, 2005.
18. Mazur, B. "Math in the Time of Plague." *The Mathematical Intelligencer*, 42, 1–6 (2020).

19. Mercat C., Vennebush P., Budd C., et al. “Popularization of Mathematics.” In *Proceedings of the 13th International Congress on Mathematical Education. ICME-13 Monographs.* edited by Gabriele Kaiser, pp. 405–408. Cham: Springer, 2017.
20. Matt, A. D. “IMAGINARY and the Open Source Math Exhibition Platform.” In Behrends E., Crato N., Rodrigues J. (eds) *Raising Public Awareness of Mathematics*, pp. 167–185. Springer, Berlin, Heidelberg, 2012.
21. Rott, B., Leuders, T. & Shahl, E. “Epistemological judgements in mathematics.” In *Views and beliefs in mathematics education: Results of the 19th MAVI Conference*, edited by Carola Bernack-Schler, Ralf Erens, Timo Leuders, Andreas Eichler, pp. 227–238. Berlin: Springer, 2015.
22. du Sautoy, M. *Symmetry: A journey into the patterns of nature*. New York, NY: Harper Collins Publishers, 2008.
23. Sengupta, S. “The story of COVID-19: A comparative analysis.” *Science Translational Medicine* 12, no. 547, 2020.
24. Shapiro, B. “Developing understanding: Research on science learning and teaching over time.” In *Canadian Journal Science, Mathematics, and Technology Education*, 4(1), 2004, 1–6.
25. Smith, R., ed. *Mathematical modelling of zombies.* Ottawa, ON, Canada: University of Ottawa Press, 2014.
26. Sorenson, H. K., “The End of Proof?” In *Mathematical Cultures: The London Meetings 2012–2014* edited by Brendan Larvor, pp. 39–160. Basil, Switzerland: Birkhauser, 2016.
27. Sullivan, John M., Mathematical Pictures: Visualization, Art and Outreach in raising public awareness ebook, 279–293.
28. Zhang, Zizhen, and Jain, Sonal, “Mathematical models of Ebola and COVID-19 with fractional differential operators: Non-Markovian process and class for virus pathogen in the environment”. In *Chaos, Solutions & Fractals* Volume 140, November 2020, Accessed 12 November 2020.

Counting Heads: Individual-Based Models of Disease Dynamics

Are Magnus Bruaset

During the Covid-19 pandemic, there has been considerable use of mathematical models to monitor and predict the disease transmission, subject to measures taken in terms of social distancing, quarantine regulations, advice to use home office, etc. In particular, such models have provided valuable input to important decisions taken by health authorities and governments around the world. One particular class of such models is based on creating a fictitious population that closely replicates the behavioural patterns of the real population of interest. These individual-based models are a useful complement to the more established compartment models based on differential equations.

1 Contact Graphs

Think back to a typical day of your life pre-Corona:

> You spent the night in your bed at your regular address, or you slept over at a friend's place, or maybe at the hotel next to the convention center where you have attended a business conference over the past days. You woke up, got dressed and went for breakfast alone at the kitchen counter, at the family table, or in the hotel restaurant. Then you went to school or work, either by public transportation, or by driving or biking. At your destination, you attended a meeting or lecture in a big room with tens of people, and you had a discussion with your two closest colleagues. In between these events, you had lunch with a group of people and you engaged in a couple of conversations around the coffee maker. After finishing your daytime chores, you went to the gym for an hour, or you picked up your two-year old daughter from kindergarten. Back at your place, you prepared dinner, which you enjoyed in the company of your family or friends. After some evening work or watching a TV show, you gave your significant other a warm hug before you rolled in for an early

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night. Tomorrow is a new day, with lots of things to do, lots of places to be, and lots of people to meet.

Each normal day, most people would move around and engage with other people, more or less as described above. Each time you get in close contact with another person, say within the distance of two meters for a period of some minutes, there is a significant potential for transfer of disease from one person to the other. This could be influenza or just an old-fashioned cold, but nowadays we are mostly concerned with Covid-19 — the infection caused by the SARS-CoV-2 virus. The transmission happens typically by respiratory droplets from the other person reaching your face or by physical contact with the other person, directly or via shared objects like a door handle. Depending on the situation and the characteristics of the disease, contagion might be left in the air for some time, resulting in airborne disease transmission.

Drafting a mental picture of all the contacts you make with other people, you can imagine a graph where each node and its corresponding edge represent one specific contact — who and where you met, and for how long. Obviously, this *contact graph* grows quickly over time. For anyone that moves a lot around and meets many people, this graph easily gets large. Lifting the view from the individual to a group of people or the local community, all these individual graphs connect via intersecting nodes due to physical encounters between people, and become a huge and tangled graph. Projecting this further to a large population, the accumulated graph becomes extremely complicated — a social version of the Gordian knot.

For highly contagious diseases, the frequency, and duration of close contacts, as well as the physical distance and environment involved, are key parameters in understanding disease transmission. Quite naturally, taking measures that influence these parameters in a favourable way can effectively reduce or even strangle the outbreak of disease. In the Covid-19 pandemic, transmission by droplets and physical contact seem to be the dominating causes of disease spreading. Therefore, from a very early stage, the health authorities have strongly advised people to keep distance to minimise the number of contact points and the number of different people they meet. In many communities, during periods of high disease transmission, the public has been asked to work from home as much as possible, periodically stay home from school, and avoid public transportation if possible. All these measures are ways of breaking down the contact graphs of as many people as possible, and thereby reducing the probability of an infected person transmitting the virus to other persons.

2 Modelling Disease Spreading

Epidemiological models are routinely used for understanding and assessing outbreaks of contagious diseases with widely different spreading mechanisms, such as influenza, measles, malaria, dengue fever, and HIV. Today, with the Covid-19 pandemic affecting all countries world-wide, the notion of such models has moved out of the realm of the scientists and into media. The modelling efforts to understand and predict the pandemic has been plentiful [4], and several of these models are used as input to important decisions made by national health authorities and governments around the globe [3, 11, 25].

Using mathematics to model the dynamics of infectious diseases is, however, not a new idea. Paraphrasing the comprehensive overview of mathematical models addressing disease dynamics by Siettos and Russo [23], it dates at least back to 1776 when Bernoulli published his analysis of mortality caused by the smallpox outbreak in England. His work was later refined and extended by several outstanding mathematicians, but it was not until 1911 that modern mathematical epidemiology was established by Ross [22], who introduced a mechanistic approach in terms of discrete equations approximating the temporal dynamics of malaria. From 1927 to 1933, Kermack and McKendrick established the approach of deterministic compartmental epidemic modelling through a series of papers [17–19].

The papers by Kermack and McKendrick formed the basis for *state-space* (or *continuum*) models in epidemiology, typically consisting of a system of differential equations that can be used to predict how an epidemic will evolve over time. In its simplest form, such models divides the population into three compartments: susceptible (S), infected (I), and removed (R), see for instance Siettos and Russo [23] or Brauer et al. [8]. People in the S group are healthy and might get sick, the ones in the I group are sick and can infect others, while those belonging to the R group do no longer take part in the disease dynamics since they are immune after having been sick or have died. Over time, individuals move between the three compartments depending on their state of health, according to this system of ordinary differential equations (ODEs):

$$\frac{dS}{dt} = -\alpha SI, \;\; \frac{dI}{dt} = \alpha SI - \beta I, \;\; \frac{dR}{dt} = \beta I.$$

Here, α is the average probability for transmitting the disease and $1/\beta$ is the average length of time that that an infected person is contagious and can transmit the disease. This continuum-style model describes disease evolution on a coarse scale by averaging over the total population. The model can be refined in several ways, for instance by introducing more compartments and additional interactions between the compartments to better catch the characteristics of the disease, such as including presymptomatic transmission or time-limited immunity [8, 20]. It is also possible to establish several models, each for one subpopulation, and link these based on data describing mobility, such as traffic data sourced from mobile network operators [25].

The continuum approach to modelling also explains the relevance of the critical parameter $R_0 = \beta S/\alpha$ evaluated at time $t = 0$. This *basic reproduction number* essentially tells how many persons will get the disease from an already infected individual if no precautions are taken nor any effective treatment is available. When using dynamic models to monitor and predict the continuous evolution of an ongoing epidemic, one usually estimates the reproduction number at different stages of the epidemic subject to the effect of all measures taken at that time. The model above prescribes that the disease will die out if the reproduction number is less than 1, and it will escalate into an epidemic state if the number is larger than 1. Therefore, the reproduction number is an indicator of whether measures are working and which situation society will be facing over the next weeks, which explains why this entity is often referred to by the press during the Covid-19 pandemic.

But what if we think back to the contact graphs described above — can these be of any help in the modelling? The state-space approach has in its way averaged the information in all contact graphs, and distilled it into an average probability of disease transmission regardless of who, where, and when the transmission takes place. The resulting model cannot distinguish between different situations with different risks for disease transmission, nor easily adapt to measures implemented in society, such as a requirement to wear face masks or restrictions on how many people can travel on the same bus. For this to enter the SIR-type model, one needs to re-estimate the probability α, which is a highly nontrivial task. By doing so, you lose information about the type of each contact, which could be useful for more detailed modelling. After all, it makes a lot of difference whether you kiss your friend on the cheek or wave at him from a distance.

3 Models Based on the Contact Graph

As shown in Figure 1, there is a jungle of methods available for constructing mathematical models of how an infectious disease spreads. For now, let us focus on what the figure denotes as agent-based simulations, also often referred to as *individual-based models* (IBMs). Also these methods come in different flavours, and a comprehensive overview of IBMs in epidemiology is given by Willem et al. [26].

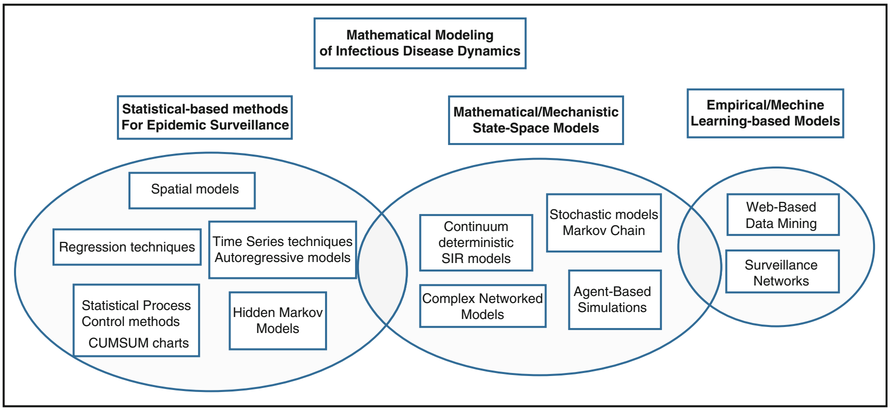

Fig. 1 A classification of mathematical models for spreading of infectious diseases. The figure appeared originally in [23].

Directly or indirectly, IBMs use the contacts graphs of individuals as the script for "playing out" the lives of the people in the population. However, this is not done for real individuals in the real population, but for the virtual people (or agents) in a fictitious population. That is, creating a fictitious population with the same statistical profile as the real population is the first step in establishing a representative model. Most countries are naturally divided into regions, counties and municipalities, and even large cities can be sectioned into city quarters or administrative units. Usually, there are publicly available demographic data such as the number of inhabitants, composition of households, and distributions of gender, age, salaries, and categories of work mapped to these different spatial units. Thus, we can fold the demographic information into the fictitious population, respecting the spatial distribution of the different features. Once this is done, we can add available information about houses and apartments, schools, kindergartens, and workplaces like offices, shops and public services to the model. Put simply, think of it as a blend of the demographic data from your national statistical bureau with geographical context data listed on Google Maps. In this way, we can equip the individuals in our fictitious population with a home address, a typical daytime location at work or school, etc. If we also apply available information about local traffic patterns, in terms of use of private cars and public transportation, we can create a dynamic population where virtual people move around. Using traffic data from mobile phone networks, we can calibrate this dynamic model such that it becomes a reasonable representation of the real community that we are modelling. We can then also create contact events, based on knowledge of how people generally meet at the workplace, in schools, in the shopping mall, etc. Exactly who meets who, when and where would be the result of drawing random numbers from appropriate statistical distributions. If this starts to sound as the inner workings of a computer game like The Sims, it is not a coincidence. This type of discrete modelling is indeed used in such games, as well as in quantitative research on different aspects of social dynamics [9]. You can also find similar mindsets in science, for instance in molecular dynamics and other particle-based simulations [5].

So how does this model include the mechanisms for disease spreading? In contrast to the "coarse-grained" and averaged probability of disease transmission in the continuum-based models, we can now use several different probabilities that adapt to the characteristics of the actual people meeting (for instance age and health profile) and the environment they meet in (such as inside a bus or at an office). There exists several IBMs for disease spreading that are in use, also for the Covid-19 pandemic [10, 16, 24]. Most of these are research codes that are not open to the public, but there exist also a couple of open source codes like FRED [15] and EMOD [7] that provide a deeper insight in the implementation of a typical IBM for this kind of purpose. As in the continuum models, it is natural to adopt the compartments of S, I, R, and possibly others, to describe the infection status of each individual. However, the mechanisms for moving people between the compartments as time passes is different from the continuum case, as in the IBM approach each transition is based on the simulation of the discrete events in the contact graphs.

One immediate advantage with the IBMs is the more or less one-to-one relation between each of the involved probabilities affecting disease transmission and the observations from clinical studies of how the disease spreads among different groups of people in different situations. Second, with this apparatus up and running, we have a very flexible way of doing experimental studies. For instance, what happens if there is a cluster of 10 people in this local community that gets infected and start moving around? This is exactly the type of events that accelerated the Covid-19 pandemic in Norway when groups of skiing enthusiasts returned to their home communities from vacation in the Alps during a school vacation late February. Another very relevant use of these models is to simulate the possible effects of measures taken to limit contagion. For instance, from a conceptual viewpoint you could easily model the effect of allowing only 20 or less passengers on each bus, changing the opening hours of shops, or closing bars and restaurants. You could impose social distancing in the model by allowing only a fraction of all contacts to take place closer than a given distance, or add the effect of infected people being isolated and exposed contacts being quarantined. A compelling visual presentation of disease dynamics and how measures like social distancing and quarantining affects the spreading of disease can be found on YouTube [2].

Also in the IBMs, there is need for assumptions on how changes to the conditions governing our virtual community affects different parameters in the model, but at least this relationship would be transparent and interpretable. These parameters, which can become plentiful for an advanced model, constitute both the curse and the blessing of IBMs in epidemiology. Siettos and Russo [23] embrace these methods, saying "In contemporary mathematical epidemiology, agent-based modelling represents the state-of-the-art for reasoning about and simulating complex epidemic systems." On the other hand, Railsback and Grimm [21] see the flexibility in these models as a potential threat to validity, and states "they can be calibrated to say anything", which is a quite frequent criticism of IBMs. Willem et al. [26] claims, however, that "This is partly a result of not capturing the difference between the calibration of IBMs and equation-based models."

Comparing state-space models and IBMs, we see that there are pros and cons with both approaches. For instance, while it is conceptually simple to implement the actions of societal measures in IBMs, it is difficult to assess population-wide key parameters such as the reproduction number. Also, IBMs tend to be computationally more expensive than solving the ODE systems, at least when you are dealing with very large populations and a correspondingly high number of events to handle. On the other hand, with the computational speed and large memories of today's supercomputers it is still very feasible. Regardless of the chosen model, it will need calibration based on ground facts, such as updated parameters distilled from hard data such as the number of confirmed positive cases in the real population, the number of people admitted to hospitals, mortality numbers, and changes to societal measures. These are all facts that change over time, so re-calibration must be done frequently.

One interesting possibility for calibrating IBMs could be to connect the model to aggregated data sourced by a contact tracing app. This holds the potential of an almost continuous calibration of the epidemic models based on live data streams, since the contact tracing app has the purpose of determining when an app user is in the proximity of other app users [13]. The idea is that if a user tests Covid-19 positive, all other users that have been close to the infected person within a certain period of time are advised to quarantine and get themselves tested. Generally, such apps rely on Bluetooth to record the contacts and do the processing in a distributed way [1, 12, 14]. In theory, the app could also collect statistical information about when, how, and where close contacts appear and communicate that back to the data storage used for model calibration. However, the whole idea of contact tracing challenges the privacy of individuals. While there has been put significant effort in finding ways to perform this task in a privacy-friendly way, it naturally gets even more challenging if the tracing app should also compile input to aggregated statistics of how people move around and connect. One possible approach, based on a strategy for distributed data aggregation, has recently been published [6].

Summarising pros and cons, the inevitable conclusion is that there should be room for both modelling approaches, as well advanced statistical analysis, since these modelling strategies can strengthen and complete each other when used in a balanced way.

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References

1. ROBust and privacy-presERving proximity Tracing protocol (2020). [https://github.com/ROBERT-proximity-tracing/documents]
2. Simulating an Epidemic (2020). [https://www.youtube.com/watch?v=gxAaO2rsdIs]
3. Adam, D.: Modeling the pandemic - The simulations driving the world's response to COVID-19. Nature **580**, 316–318 (2020)
4. Adiga, A., Dubhashi, D., Lewis, B., Marathe, M., Venkatramanan, S., Vullikanti, A.: Mathematical Models for COVID-19 Pandemic: A Comparative Analysis. Journal of the Indian Institute of Science **100**(4), 793–807 (2020). doi: 10.1007/s41745-020-00200-6. URL https://doi.org/10.1007/s41745-020-00200-6
5. Al-Khayat, O., Bruaset, A.M., Langtangen, H.P.: A Lumped Particle Modeling Framework for Simulating Particle Transport in Fluids. Communications in Computational Physics **8**(1),

115–142 (2010). doi: 10.4208/cicp.030509.141009a. URL http://www.global-sci.com/issue/abstract/readabs.php?vol=8&page=115&issue=1&ppage=142&year=2010

6. Bruaset, A.M., Lines, G.T., Sundnes, J.: Data aggregation and anonymization for mathematical modeling and epidemiological studies. In: Elmokashfi, A., Lysne, O., Naumova, V. (eds.) The Smittestopp App for Digital Contact Tracing: Technology, Validation, Privacy Aspects, and Lessons Learned. Springer Nature (2021)
7. Bershteyn, A., Gerardin, J., Bridenbecker, D., Lorton, C.W., Bloedow, J., Baker, R.S., Chabot-Couture, G., Chen, Y., Fischle, T., Frey, K., Gauld, J.S., Hu, H., Izzo, A.S., Klein, D.J., Lukacevic, D., McCarthy, K.A., Miller, J.C., Ouedraogo, A.L., Perkins, T.A., Steinkraus, J., Ten Bosch, Q.A., Ting, H.F., Titova, S., Wagner, B.G., Welkhoff, P.A., Wenger, E.A., Wiswell, C.N.: Implementation and applications of EMOD, an individual-based multi-disease modeling platform. Pathogens and disease **76**(5), 1–10 (2018). doi: 10.1093/femspd/fty059
8. Brauer, F., Castillo-Chavez, C., Feng, Z.: Mathematical models in epidemiology and immunology, vol. 28. Springer (2000). doi: 10.14708/ma.v28i42/01.1879
9. Cornell, S.J., Suprunenko, Y.F., Finkelshtein, D., Somervuo, P., Ovaskainen, O.: A unified framework for analysis of individual-based models in ecology and beyond. Nature Communications **10**(1), 1–14 (2019). doi: 10.1038/s41467-019-12172-y. URL http://dx.doi.org/10.1038/s41467-019-12172-y
10. Cuevas, E.: An agent-based model to evaluate the COVID-19 transmission risks in facilities. Computers in Biology and Medicine **121**(April) (2020). doi: 10.1016/j.compbiomed.2020.103827
11. Eker, S.: Validity and usefulness of COVID-19 models. Humanities and Social Sciences Communications **7**(1), 1–5 (2020). doi: 10.1057/s41599-020-00553-4. URL http://dx.doi.org/10.1057/s41599-020-00553-4
12. Elmokashfi, A., Lysne, O., Naumova, V. (eds.): The Smittestopp App for Digital Contact Tracing: Technology, Validation, Privacy Aspects, and Lessons Learned. Springer Nature (2021)
13. Ferretti, L., Wymant, C., Kendall, M., Zhao, L., Nurtay, A., Abeler-Dörner, L., Parker, M., Bonsall, D., Fraser, C.: Quantifying SARS-CoV-2 transmission suggests epidemic control with digital contact tracing. Science **368**(6491), 0–8 (2020). doi: 10.1126/science.abb6936
14. Google and Apple: Exposure Notification (2020). [https://www.google.com/covid19/exposurenotifications/]
15. Grefenstette, J.J., Brown, S.T., Rosenfeld, R., Depasse, J., Stone, N.T., Cooley, P.C., Wheaton, W.D., Fyshe, A., Galloway, D.D., Sriram, A., Guclu, H., Abraham, T., Burke, D.S.: FRED (A Framework for Reconstructing Epidemic Dynamics): An open-source software system for modeling infectious diseases and control strategies using census-based populations. BMC Public Health **13**(1) (2013). doi: 10.1186/1471-2458-13-940
16. Hoertel, N., Blachier, M., Blanco, C., Olfson, M., Massetti, M., Rico, M.S., Limosin, F., Leleu, H.: A stochastic agent-based model of the SARS-CoV-2 epidemic in France. Nature Medicine **26**(9), 1417–1421 (2020). doi: 10.1038/s41591-020-1001-6
17. Kermack, W.O., McKendrick, A.G.: A contribution to the mathematical theory of epidemics. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **115**(772), 700–721 (1927). doi: 10.1098/rspa.1927.0118. URL https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1927.0118
18. Kermack, W.O., McKendrick, A.G.: Contributions to the mathematical theory of epidemics. II. The problem of endemicity. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **138**(834), 55–83 (1932). doi: 10.1098/rspa.1932.0171. URL https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1932.0171
19. Kermack, W.O., McKendrick, A.G.: Contributions to the mathematical theory of epidemics. III. Further studies of the problem of endemicity. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **141**(843), 94–122 (1933). doi: 10.1098/rspa.1933.0106. URL https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1933.0106

20. Kristiansen, I.S., Burger, E.A., Blasio, B.F.D.: Covid-19: Simulation models for epidemics. Tidsskrift for Den norske lægeforening (2020). URL https://tidsskriftet.no/en/2020/03/kronikk/covid-19-simulation-models-epidemics
21. Railsback S. F. ; Grimm, V.: Agent-based and Individual-based Modeling: A Practical Introduction. Princeton University Press (2011)
22. Ross, R.: The prevention of Malaria. John Murray (1911)
23. Siettos, C.I., Russo, L.: Mathematical modeling of infectious disease dynamics. Virulence **4**(4), 295–306 (2013). doi: 10.4161/viru.24041
24. Silva, P.C., Batista, P.V., Lima, H.S., Alves, M.A., Guimares, F.G., Silva, R.C.: COVID-ABS: An agent-based model of COVID-19 epidemic to simulate health and economic effects of social distancing interventions. arXiv **139** (2020)
25. The Norwegian Institute of Public Health: Coronavirus modelling at the NIPH (2020). [https://www.fhi.no/en/id/infectious-diseases/coronavirus/coronavirus-modelling-at-the-niph-fhi/]
26. Willem, L., Verelst, F., Bilcke, J., Hens, N., Beutels, P.: Lessons from a decade of individual-based models for infectious disease transmission: A systematic review (2006–2015). BMC Infectious Diseases **17**(1), 1–16 (2017). doi: 10.1186/s12879-017-2699-8