Footbridge Vibrations and Modelling of Pedestrian Loads

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Abstract Vibrations in footbridges can be annoying and hence it is useful already at the design stage to be able to predict levels of footbridge vibrations in order to ensure that serviceability-limit-state requirements will be fulfilled. For the studies of the paper footbridge vibrations are assumed brought about by pedestrians. Walking parameters such as load amplification factors, pacing frequency, pacing speed, and pedestrian weight determine the characteristics of the loading. By nature, these parameters are stochastic and hence the studies of this paper will handle some of the walking parameters as random variables. This has the effect that predictions of footbridge vibration levels end up being random variables. The paper will consider and examine how selected decisions related to setting up the calculation framework can influence the outcome of design stage predictions of footbridge vibration levels.

Keywords Footbridge vibrations · Walking loads · Walking parameters · Stochastic load models · Serviceability limit state

Nomenclature

a	Bridge acceleration
i	Integer
v	Pacing speed
L	Bridge length
α	Dynamic load factor
σ	Standard deviation
f_1	Bridge fundamental frequency
m_1	Bridge modal mass
t	Time
Q	Modal load
ζ_1	Bridge damping ratio
Θ	Phase
f_s	Step frequency
l_s	Step length
F	Walking load
W	Weight of pedestrian
μ	Mean value
Φ	Mode shape

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1 Introduction

A footbridge may be flexible and is potentially prone to vibrate due to human excitation. The dynamic forces caused by pedestrians might not cause ultimate-limit-state issues, but the serviceability limit state may be of concern.

A well-known and critical scenario was that occurring on the Millennium Bridge in London [1]. Here, both vertical and horizontal motion caused by pedestrians showed to be a problem. The problematic conditions often relate to coincidence between bridge natural frequencies and frequencies of human in motion. In this case excessive structural vibrations may occur and if not properly designed, the bridge may be unfit for its intended use.

In this paper focus is on the vertical action of pedestrians. The action has for decades been modelled as a deterministic load [2–4]. More recently, in for instance [5–8], the stochastic nature of the action has been considered and modelled.

Hence, this paper will adapt the stochastic line of thinking, hereby modelling central walking parameters as random variables. Doing so recognises that parameters such as pedestrian step frequency, step length and dynamic load factors are in fact stochastic properties by nature.

A comprehensive line-up of the probabilistic scenario is introduced in [9]. The present paper takes off-set in the general approach, but also considers, adapts and evaluates simplifications along the way.

For all investigations off-set is taken in artificial footbridges. They will be assumed pin-supported, and focus will be on the vertical bridge response at midspan in the form of accelerations. As a result of the fact that the loading is stochastic, the response will be stochastic too, and the output of calculations will be the acceleration quantiles. These are derived combining Newmark time integration with Monte Carlo simulations.

The paper aims at exploring how sensitive the stochastic nature of footbridge vibration is to decisions related to defining dynamic load factors and how sensitive it is in regard to assumptions made for the stochastic nature of step frequencies of the pedestrians.

Section 2 describes basic model assumptions.

Section 3 describes the different study angles of the paper, and Sect. 4 summarises conclusions.

2 Modelling of Walking Loads

The walking load models considered in this paper rely on a modal load assumption in which the modal load $Q(t)$ acting on the footbridge and generated by the pedestrian is derived using Eq. (1).

$$Q(t) = \Phi(t)F(t) \tag{1}$$

$F(t)$ represents the vertical force imparted at the position of the pedestrian while crossing the bridge, and $\Phi(t)$ is the mode shape function. Only the first bending mode is considered to dominate the response, and hence this will be taken as a half-sine sinusoidal. It will depend on the pacing speed v of the pedestrian, and in consequence hereof on the step frequency f_s and step length l_s of the pedestrian as a result of the relationship shown in Eq. (2).

$$v = f_s l_s \tag{2}$$

The mode shape function is calculated using Eq. (3).

$$\Phi(t) = \sin(\pi v t/L) \tag{3}$$

This load model, $F(t)$, is the model introduced in [9]. The mathematical expression for $F(t)$ is seen in Eqs. (4–6):

$$F(t) = \sum_{i=1}^{5} F_i(t) + \sum_{i=1}^{5} F_i^S(t) \tag{4}$$

$$F_i(t) = W\alpha_i \sum_{\overline{f}_j = i-0.25}^{i+0.25} \overline{\alpha}_i\left(\overline{f}_j\right)\cos\left(2\pi \overline{f}_j f_s t + \theta\left(\overline{f}_j\right)\right) \tag{5}$$

$$F_i^S(t) = W\alpha_i^S \sum_{\overline{f}_j^S = i-0.75}^{i-0.25} \overline{\alpha}_i^S\left(\overline{f}_j^S\right)\cos\left(2\pi \overline{f}_j^S f_s t + \theta\left(\overline{f}_j^S\right)\right) \tag{6}$$

Reference is made to [9], for a detailed description.

Here it suffices to mention that W represents the static weight of the pedestrian. Furthermore, that f_s represents the step frequency.

The model consists of main load harmonics (Eq. 5) and subharmonics (Eq. 6). The latter due to the fact that "the fundamental period of the force time history is equal to the time required to make two successive steps, rather than one" [9].

A governing parameter for the loading is the dynamic load factors, α_i, $(i = 1, 2, \ldots, 5)$.

For the main harmonic, α_1 (the first load harmonic), the following mean value, μ, and a standard variation, σ, is assumed.

$$\mu = -0.2649 f_S^3 + 1.3206 f_S^2 - 1.7597 f_S + 0.7613; \quad \sigma = 0.16\mu \tag{7}$$

As would appear, the dynamic load factor is modelled as a random variable, and the distribution is assumed Gaussian.

Table 1 defines the assumptions made for the dynamic load factors, for $\alpha_i (i = 2, 3, 4, 5)$, and the corresponding mean values (μ) and standard deviations (σ).

The subharmonic load factors α_i^S are derived from the main harmonic load factor, α_1, in the way described in [9].

Having set up the load, Newmark time-integration allows for computing bridge response and Monte Carlo simulations for establishing a statistical basic for the response. From this, acceleration quantiles can be derived, such as the acceleration quantile a_{95}, which will be the parameter in focus in this paper for describing the acceleration level of the considered footbridges.

For each bridge 100.000 simulations were conducted.

3 Studies of This Paper

3.1 Impact of Decisions Related to Modelling the Load Amplification Factor

The purpose of this study is to explore how different decisions related to modelling the dynamic load factor influence estimates of the stochastic bridge response in the form of the acceleration quantile a_{95}.

A simplified load model is assumed, namely one that only considers the first harmonic, α_1. Leakage of energy around this load harmonic is also disregarded.

The following assumptions were made as regards the walking parameters, see Table 2.

Hence, it is assumed that step length and step frequency are random variables, and Gaussian distributions are assumed to apply.

By this approach, the load amplification factor (Eq. 7) would vary depending on the outcome of f_s in simulations from one pedestrian crossing to the next.

Table 1 Mean values and standard deviations

–	α_2	α_3	α_4	α_5
μ	0.07	0.05	0.05	0.03
σ	0.030	0.020	0.020	0.015

Table 2 Mean values and standard deviations

–	μ	σ	Reference
W	750 N	0 N	[9]
l_s	0.71 m	0.071 m	[9]
f_s	1.87 Hz	0.186 Hz	[9]

An alternative assumption for the calculations could be to assume that the load amplification factor assumes a constant value, namely the value that can be calculated by assuming the mean value of the step frequency f_s for the calculations of the dynamic load factor (in Eq. 7).

This was done for the SDOF footbridges listed in Table 3.

The combination of values of f_1, m_1, and L is believed to be fairly realistic for SDOF pin-supported footbridges, as m_1 and L drop with increase in f_1.

In terms of the deviations between outcomes of calculations of a_{95}, obtained by the two approaches, Table 4 shows the results. The difference between results is normalised to the result obtained assuming that the dynamic load factor would vary from one bridge crossing to the next.

The results signify that it may not be totally off to employ a simplified approach for computations, when it comes to settling on the dynamic load factor. Even though the simplified approach somewhat violates the stochastic nature of the problem, fairly reasonable results are obtained.

It is underlined that the investigations presented here do not reflect the uncertainty of the stochastic nature of the dynamic load factor. For such information see for instance [8].

3.2 *Impact of Decisions Related to Modelling the Step Frequency*

To this end of investigation different sets of assumptions related to modelling the stochastic nature of step frequency are considered.

The step frequency will be modelled as a random variable, but assumptions related to mean value and standard deviation need to be made.

For the investigations of this paper, the assumptions listed in Table 5 are considered.

The models represent different proposals that can be found in literature [8].

Here, they are considered as input data for computing a_{95} for different footbridges.

For the study, the bridges tabulated in Table 6 are considered.

The dynamic characteristics for the bridges do not exactly correspond to those in the previous study, but this is not of primary importance.

Table 3 Modal properties of bridges and bridge lengths (L)

Property	Unit	Bridge		
		A	B	C
f_1	Hz	1.6	1.9	2.2
ζ_1	%	0.5	0.5	0.5
m_1	10^3 kg	61.5	44.0	32.5
L	m	54.0	45.0	39.0

Table 4 Deviations in terms of estimates of a_{95}

Deviation	Unit	Bridge		
		A	B	C
a_{95}	%	+10	1	−20

Table 5 Mean values and standard deviations

Model	Unit	μ	σ
I	Hz	1.87	0.186
II	Hz	1.99	0.173
III	Hz	2.20	0.300

Table 6 Modal properties of bridges and bridge lengths (L)

Property	Unit	Bridge		
		A	B	C
f_1	Hz	1.85	2.00	2.20
ζ_1	%	0.5	0.5	0.5
m_1	10^3 kg	46.2	39.5	32.6
L	m	46.5	43.0	39.1

Table 7 Acceleration quantile a_{95}

	Model for step frequency		
Bridge	I (m/s^2)	II (m/s^2)	III (m/s^2)
A	0.3093	0.2879	0.1750
B	0.3904	0.4256	0.3442
C	0.2743	0.4715	0.5033

In terms of bridge response, and focusing on the response characteristic a_{95}, Table 7 summarises the results computed for footbridge A, B, and C. Values of a_{95} are provided for the three different step frequency models.

For bridge A there is a maximum difference of (0.3093–0.1750=) 0.1343 m/s^2 between results obtained for the three stochastic models for step frequency. If this value is normalised by the minimum number 0.1750 m/s^2, one obtains a 77% difference in results for bridge A.

Doing the same calculations for bridge B and C results in 24% and 83% differences, respectively.

Hence, the choice of step frequency model assumed for computing a_{95} has a relative high impact on the result.

4 Conclusion and Discussion

In the paper the influence of decisions as regards settling on a framework for pedestrian load models for evaluating footbridge response at the design stage was examined. Focus was on estimation of the footbridge acceleration response occurring at midspan of footbridges. The acceleration quantile a_{95} (the acceleration level exceeded in 5% of the pedestrian crossings) was chosen for investigation.

For the investigations, different artificial SDOF and pin-supported bridges were considered so as to widen the perspective of conclusions.

One issue addressed was on ways for choosing the dynamic load amplification factor for a computational prediction of footbridge response. Another issue was on choosing parameters of a stochastic model for the step frequency of pedestrians for entering into the calculations.

Both choices might potentially affect the outcome of the predicted stochastic nature of bridge response and hence serviceability-limit-state evaluations for footbridges.

As for the dynamic load factor, different methods for extracting the main governing load amplification factor were examined. It turned out that a simplified technique not fully in accordance with the stochastic nature of the pedestrian traffic provided fairly reasonable results (errors in predictions of a_{95} of maximum 20% for the investigated bridges).

Whereas the investigations in terms of the dynamic load factor focused on a technique for simpler processing of data, the investigations in terms of choosing parameters for a stochastic model for step frequency directly relate to actual uncertainties.

Solutions to this challenge are not provided here but it is interesting to notice that the calculations of this paper suggest up to 83% deviations in estimates of a_{95} depending on which bridge is considered and which input parameters are chosen for modelling the stochastic nature of step frequencies.

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