On Health-State Transition Models for Risk-Based Structural Health Monitoring

A. J. Hughes, R. J. Barthorpe, and K. Worden

Abstract A desire for informed decision-making regarding the operation and maintenance of structures provides motivation for the development and implementation of structural health monitoring (SHM) systems. One approach to decision-making in SHM is to adopt a risk-based framework in which failure events and decidable actions are attributed costs/utilities. Optimal maintenance strategies may be pursued by considering the probability of occurrence of future failure events in conjunction with associated costs. In order to forecast future failure events, a probabilistic model that describes the degradation of the structure over time is required; in the state-space formulation of risk-based SHM, this model is equivalent to the transition probabilities from possible current health-states of the structure to future health-states.

The current paper aims to demonstrate how such models may be determined using information gathered during the operational evaluation stage of the structural health monitoring paradigm. This information may include knowledge of the operational and environmental conditions under which the structure will operate, in addition to initial physics-based modelling of the structure. A probabilistic transition model describing the degradation of a four-bay truss is developed here, with finite element simulation used to yield knowledge of the load paths within the structure when it is in differing health-states. The paper concludes with a discussion of the importance of probabilistic degradation models within SHM decision-making. The discussion highlights the challenges that arise due to the lack of data available prior to the implementation of an SHM system and suggests for how these may be overcome.

Keywords Risk · Decision-making · Probabilistic graphical models · Degradation modelling

1 Introduction

Structural health monitoring (SHM) is a field of engineering that is concerned with damage detection in structures and infrastructure via the development and implementation of data acquisition and processing systems [1]. A key motivation for the development and implementation of SHM systems is to facilitate the decision-making processes associated with the operation and management of high-value or safety-critical assets. One approach to decision-making in the context of SHM is by the use of a probabilistic risk-based framework based upon probabilistic graphical models (PGMs) [2], in which actions on, and failure modes of the structure are assigned costs and optimal decisions are made through the maximisation of expected utility gain, or the minimisation of expected utility loss.

An agent tasked with making decisions regarding the operation and management of a structure may utilise health-state information inferred via an SHM system to make better informed and more optimal decisions. However, given solely information regarding the structural health-state at the current instance in time, the agent may only make well-informed decisions ad hoc. In order to make well-informed decisions on policies that include preventative actions, the agent requires information about the future health-states of the structure. This information can be gained by developing transition models that forecast future health-states given the current health-state and each decidable action. For the case that the decided action is 'do nothing' the health-state transition model will forecast the degradation of a structure.

Degradation models of differing complexities have been used within the field of engineering for reliability assessment, maintenance planning and prognosis [3]. In general, the models can be categorised in terms of a combination of the following

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criteria; physics-based or data-based, deterministic or probabilistic and continuous state or discrete state. A commonly used degradation model is Paris' law for crack growth given by the following equation,

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where a is the crack length, N is the load cycle, ΔK is the stress intensity range and C and m are constants. After a little thought one can reason that Eq. (1) is a deterministic, physics-based model of a continuous state. Different categories of degradation model are applicable in different scenarios depending on the context. For example, in a situation where little is known of the underlying physics governing the degradation, but data are readily available, one may opt for a data-based model. Conversely, if the physics are known but data availability is low, a physics-based model may be more suitable. Whether continuous or discrete states are modelled also depends on the nature of the application; considerations for this include the required model fidelity and the computational cost/time. Without delving too far into metaphysics, it is reasonable to assert that, in general, the future is inherently uncertain. For this reason, with regard to the use of deterministic versus probabilistic models, the latter have a distinct advantage as they are capable of representing uncertainty. Fortunately, many deterministic degradation models can be used to obtain probabilistic outputs via methods such as sequential Monte Carlo sampling [4].

In the context of SHM and decision-making, a variety of health-state transition models have been employed. In [5], a probabilistic interpretation of Paris' law is used to develop a degradation model in a maintenance decision process for a simulated wind turbine tower. In [6], a continuous health-state variable is given nonlinear Gaussian transition models in a partially observable Markov decision process (POMDP) based on a normalised unscented Kalman filter; this approach has the property that there is a non-zero probability that the health-state transitions to a less-damaged state, meaning that the structural degradation is not strictly monotonic. In [7], qualitative data obtained from the inspection of mitre gate components is used to derive a health-state transition matrix for a Markovian decision process for optimal maintenance decisions.

The current paper aims to present a general methodology for determining a health-state transition matrix for use in a probabilistic risk-based decision paradigm for the operation and maintenance of structures as developed in [2]. The methodology will be demonstrated using a case study of a four-bay truss. Finally, the importance of health-state transition models within the risk-based decision framework will be discussed, and the challenges associated with their development will be highlighted.

2 Probabilistic Risk-Based SHM

The approach proposed in [2] facilitates decision-making in the context of SHM by incorporating aspects of probabilistic risk assessment into a probabilistic graphical model framework. For brevity, here, a short introduction to probabilistic graphical models is provided, followed by a summary of the risk-based decision framework; for a more comprehensive explanation, the reader is directed to the original paper.

2.1 Probabilistic Graphical Models

Probabilistic graphical models are graphical representations of factorisations of joint probability distributions and are a powerful tool for reasoning and decision-making under uncertainty. For this reason, they are apt for representing and solving decision problems in the context of SHM, where there is uncertainty in the health-states of structures. While there exist multiple forms of probabilistic graphical model, the key types utilised for the risk-based decision frameworks are Bayesian networks (BNs) and influence diagrams (IDs) [8].

Bayesian networks are directed acyclic graphs (DAGs) comprised of nodes and edges. Nodes represent random variables and edges connecting nodes represent conditional dependencies between variables. In the case where the random variables in a BN are discrete, the model is defined by a set of conditional probability tables (CPTs). For continuous random variables, the model is defined by a set of conditional probability density functions (CPDFs).

Figure 1 shows a simple Bayesian network comprised of three random variables X, Y and Z. Y is conditionally dependent on X and is said to be a child of X while X is said to be a parent of Y. Z is conditionally dependent on Y and can be said to be a child of Y and a descendant of X while X is said to be an ancestor of Z. The factorisation described by the Bayesian network shown in Fig. 1 is given by $P(X, Y, Z) = P(X) \cdot P(Y|X) \cdot P(Z|Y)$. Given observations on a subset of nodes

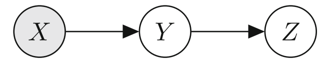

Fig. 1 An example Bayesian network

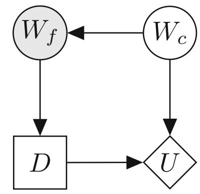

Fig. 2 An example influence diagram representing the decision of whether to go outside or stay in under uncertainty in the future weather condition given an observed forecast

in a BN, inference algorithms can be applied to compute posterior distributions over the remaining unobserved variables. Observations of random variables are denoted in a BN via grey shading of the corresponding node, as is demonstrated for X in Fig. 1.

Bayesian networks may be adapted into influence diagrams to model decision problems. This augmentation involves the introduction of two additional types of node that are shown in Fig. 2: decision nodes, denoted as squares, and utility nodes, denoted as rhombi. For influence diagrams, edges connecting random variables to utility nodes denote that the utility function is dependent on the states of the random variables. Similarly, edges connecting decisions nodes to utility nodes denote that the utility function is dependent on the decided actions. Edges from decision nodes to random variable nodes indicate that the random variables are conditionally dependent on the decided actions. Edges from random variable or decision nodes to other decision nodes do not imply a functional dependence but rather order, i.e. that the observations/decisions must be made prior to the next decision being made.

To gain further understanding of IDs, one can consider Fig. 2. Figure 2 shows the ID for a simple binary decision; stay home and watch TV or go out for a walk, i.e. $domain(D) = \{\text{TV, walk}\}$. Here, the agent tasked with making the decision has access to the weather forecast W_f which is conditionally dependent on the future weather condition W_c. The weather forecast and future condition share the same possible states $domain(W_f) = domain(W_c) = \{\text{bad, good}\}$. The utility achieved, U, is then dependent on both the future weather condition and the decided action. For example, one might expect high utility gain if the agent decides to go for a walk and the weather condition is good.

In general, a policy δ is a mapping from all possible observations to possible actions. The problem of inference in influence diagrams is to determine an optimal strategy $\mathbf{\Delta}^* = \{\delta_1^*, \dots, \delta_n^*\}$ given a set of observations on random variables where δ_i^* is the ith decision to be made in a strategy $\mathbf{\Delta}^*$ that yields the *maximum expected utility* (MEU). Defined as a product of probability and utility, the expected utility can be considered as a quantity correspondent to risk.

2.2 Decision Framework

A probabilistic graphical model for a general SHM decision problem across a single time-slice is shown in Fig. 3. Here, a maintenance decision d is shown for a simple fictitious structure $\boldsymbol{S}$, comprised of two substructures $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, each of which is comprised of two components; c_{1-2} and c_{3-4}, respectively.

The overall decision process model shown in Fig. 3 is based upon a combination of three sub-models; a statistical classifier, a failure-mode model and a transition model.

Within the decision framework, a random variable denoted $\boldsymbol{H}_t$ is used to represent the latent global health-state of the structure at time t. For this decision process, a posterior probability distribution over the latent health-state $\boldsymbol{H}_t$ is inferred via observations on a set of discriminative features $\boldsymbol{\nu}_t$. It is assumed that the generative conditional distribution $P(\boldsymbol{\nu}|\boldsymbol{H})$ is learned implicitly or explicitly, depending on the choice of statistical classifier.

The failure condition of the structure $F_{\boldsymbol{S}}$ is represented as a random variable within the PGM and is conditionally dependent on the health-states of the substructures denoted by the nodes hs_1 and hs_2. The health-states of the substructures are dependent on the local health-states of the constituent components denoted by the nodes hc_{1-4}. The local health-states of the components are summarised in the global health-state vector $\boldsymbol{H} = \{hc_1, hc_2, hc_3, hc_4\}$. The conditional probability tables defining the relationship between random variables correspond to the Boolean truth tables for each of the logic gates in the fault tree defining the failure mode $F_{\boldsymbol{S}}$ [9, 10]. This failure-mode model is repeated in each time-step. The failure states associated with the variable $F_{\boldsymbol{S}}$ are given utilities via the function represented by the node U_F. As it is necessary to consider the future risk of failure in the decision process, these utility functions are also repeated for each time-step.

Figure 4 shows the influence diagram of the transition sub-model. By interpretation of the graphical model shown in Fig. 4, one can realise that the transition sub-model is solely formed of the conditional probability distribution $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d_t)$

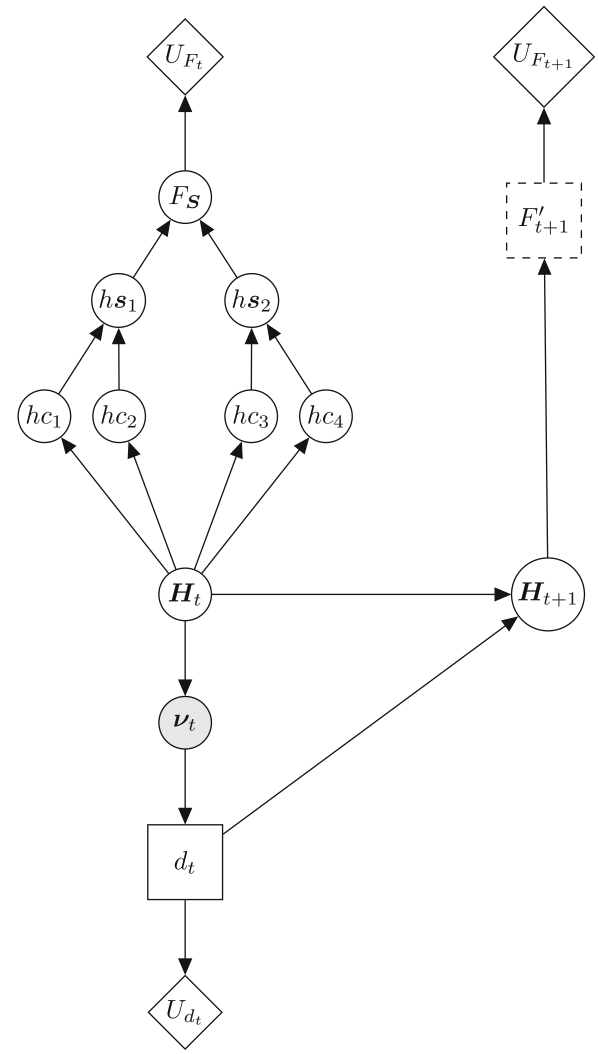

Fig. 3 An influence diagram representing the partially observable Markov decision process over one time-slice for determining the utility-optimal maintenance strategy for a simple structure comprised of four components. The fault tree failure-mode model for time $t+1$ has been represented as the node F'_{t+1} for compactness

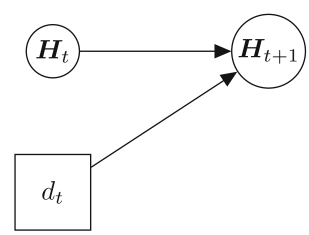

Fig. 4 An influence diagram representing the transition sub-model of the overall SHM decision process

and that the future health-state $\boldsymbol{H}_{t+1}$ is dependent only on the current health-state and the action decided in the current time-slice. An underlying assumption of the decision framework presented in [2], that facilitates the modelling process, is that structures can be represented as a hierarchical combination of discrete substructures/regions. A consequence of this assumption is that the health-states of interest are all represented as discrete random variables, hence, the transition models required are matrices. For a given decided action a, and assuming a finite number N of possible discrete global health-states, the conditional probability table $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d_t = a)$ is given by an $N \times N$ square matrix whose i, jth entry is the probability of transitioning from the ith to the jth health-state and $i, j \in \mathbb{Z} : 1 \leq i, j \leq N$. Additionally, it is assumed that the Markov decision process is stationary, i.e. $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d_t = a)$ is invariant with respect to t. Because of this stationarity, assuming no intervention is made ($d_t =$ 'donothing$'$ $\forall\, t$), the future global structural health-state is forecast as,

$$P(\boldsymbol{H}_{t+n}) = P(\boldsymbol{H}_t) \cdot P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d_t = \text{'do nothing'})^n \qquad (2)$$

where n is the number of discrete time-slices forecast over, and $P(\boldsymbol{H}_t)$ and $P(\boldsymbol{H}_{t+n})$ are $1 \times N$ multinomial probability distributions over the global health-states at times t and $t + n$, respectively.

3 Developing Transition Models for Risk-Based SHM

As with the established paradigm for conducting an SHM campaign (detailed in [1]), the risk-based approach is formed of several distinct stages. The risk-based approach consists of: operational evaluation, failure-mode modelling, decision modelling, data acquisition, feature selection and statistical modelling. Most crucial to the development of transition models is the operational evaluation stage. The current section outlines the information that must be obtained for the development of transition models, provides discussion around the quantification of the uncertainty in operational conditions and offers an explanation of how the quantified uncertainty may be used in conjunction with a physics-based model to develop transition models.

3.1 Operational Evaluation

The operational evaluation stage, for both the traditional and probabilistic risk-based structural health monitoring paradigms, seeks to assess the context in which a structural health monitoring campaign is to be conducted. It is during this stage that the operational and environmental conditions for the structure of interest are considered. Furthermore, failure modes of interest are determined and key health-states of the structure identified.

For the development of transition models in the probabilistic risk-based approach, during the operation evaluation stage, it is necessary to identify factors that will influence the way in which the structure will degrade. Many of these factors may be specific to the type of structure on which SHM is being conducted. Information regarding the operational conditions that must be obtained includes the anticipated forcing amplitudes, locations and temporal variations. These operational conditions will influence the fatigue life of the structure. Environmental conditions are also important to consider. Examples of important environmental factors include operating temperatures and the presence/absence of water. The anticipated operational temperature ranges are important to consider as these potentially introduce thermally induced stresses in addition to other temperature effects on material properties such as fracture toughness. Furthermore, whether the structure will be in the presence of water is a key factor as this may introduce structural degradation mechanisms such as corrosion and erosion. An important consideration to make when considering operational and environmental conditions is that degradation mechanisms may interact with one another. A notable example of this effect occurring is within the core of light-water nuclear reactors where stainless steel structural components experience accelerated brittle fracturing as a result of interplay between multi-physical phenomena in a process known as irradiation-assisted stress corrosion cracking (IASCC) [11].

With the operational and environmental conditions of the structure considered and potential degradation mechanisms determined, the failure modes of interest for the structure and critical substructures, components and joints can be identified. Subsequently, it is important to define damage for each critical substructure, component and joint, i.e. the possible local health-states. Depending on factors such as materials and local operational and environmental conditions, different components/joints may be susceptible to different types of damage; for example, composite components may experience delamination, whereas metallic components may experience fatigue cracking. For each component, criteria for each of the relevant failure mechanisms should be specified.

Irrespective of the type of damage associated with each component/joint, it is reasonable to assert that the discrete random variables corresponding to the local health-states will have a cardinality of at least 2. In the most simple case, each local health-state variable could possess states corresponding to ‘undamaged’ and ‘failed’, where the ‘failed’ state represents the component being unfit-for-purpose. In some scenarios, it may also be desirable to consider extents of damage and the functionality of the component/joint at varying damage extents. Some components/joints may possess health-states associated with the presence of damage whilst continuing to function at their full, or partial capacity. Although these states are not necessarily associated with any immediate risk with regard to the failure of the global structure, they may still be important to consider as they may increase the propensity for transitioning to other more advanced damage states that do have high risk associated. An example of a component that may require this consideration is a load-bearing structural member in which partial thickness cracks may form.

3.2 Handling Uncertainty

For most applications of structural health monitoring, perfect knowledge of the operational and environmental conditions will not be available prior to the implementation of the system. It is for this reason, that uncertainties should be considered and quantified where possible. While there exists a number of methodologies for the quantification of uncertainty, including interval analysis and Dempster-Shafer theory [12, 13], here, it is considered reasonable to continue using probability theory for consistency with the probabilistic risk-based decision framework.

For each of the key environmental and operation conditions, statistical distributions quantifying the ranges, likely values and/or variance in the conditions should be elicited from an expert judgement, and where possible, observed data. In a Bayesian setting, these distributions may be updated as measurements are collected, and the transition models re-estimated.

3.3 Generating Transition Models

To generate the degradation transition models, a physics-based model is required. The function of the model is to simulate the structure and specifically its critical components in each of the global health-states and under specified operational and environmental conditions. The simulated structure can then be evaluated with respect to the failure criteria identified in the operational evaluation stage to determine whether state transitions occur.

With respect to modelling the degradation of a structure, the purpose of the physics-based model is to determine a distribution over the quantities of interest in which the failure criteria are specified, conditioned on the uncertain operational and environmental conditions. In the case that the physics-based model employed is inherently stochastic (such as a probabilistic fracture mechanics model), this conditional distribution may be determined analytically. In the case that the physics-based model employed is deterministic (such as a finite element model), this distribution may be determined by applying sampling methods to the probability distributions for the operational and environmental conditions, and querying the physics-based model accordingly.

Once a distribution over the quantities of interest has been determined, a distribution over local failure events can be produced by executing the logical operations defining the failure criteria. Again, this distribution is conditioned on the operational and environmental conditions. This conditional distribution over local failure events can then be mapped into transitions in the global health-state by utilising the definition of $\boldsymbol{H}$ as a vector containing the local health-states of the critical components, joints and substructures.

At this stage, it is necessary to marginalise out the variable operational and environmental conditions to obtain the distribution $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 0)$. Additionally, to ensure a valid probability distribution is produced, normalisation should be carried out.

Developing transition models for specific actions (such as repairs) is typically a problem that is highly dependent on the context.

4 Case Study: Four-Bay Truss

To demonstrate how probability distributions quantifying uncertainty in operational conditions may be used in conjunction with a physics-based model to generate a transition model for a risk-based SHM decision process, the methodology was applied to a case study of a physical four-bay truss structure identical to that used in [14], and shown in Fig. 5. The truss was composed of 20 aluminium members, each with a cross-sectional area of 177 mm^2. The horizontal and vertical members of the truss possessed lengths of 250 mm, resulting in the overall structure having a length of 1 m and a height of 0.25 m. The members were pinned together using steel bolts in lubricated holes. For illustrative purposes, fictitious operational conditions were assumed.

To avoid obfuscating the development of the transition model, it was elected to ignore the failure of joints and the horizontal and vertical members and instead focus on the failures of the cross-members. Denoting the local health-states of the eight cross-members as hm_9 to hm_{16}, the global health-state of the structure can be expressed as the vector $\boldsymbol{H} = \{hm_9, \ldots, hm_{16}\}$. Additionally, for the purposes of demonstration, binary health-states for each of the 8 cross-members were considered resulting in 256 possible global health-states. From hereon in, a convenient referencing scheme for the global health-states is adopted where $\boldsymbol{H}$ is given a superscript corresponding to the decimal representation of the 8-bit binary number (with ascending powers of two from left to right) specified by the vector $\boldsymbol{H}$, i.e. the undamaged health state

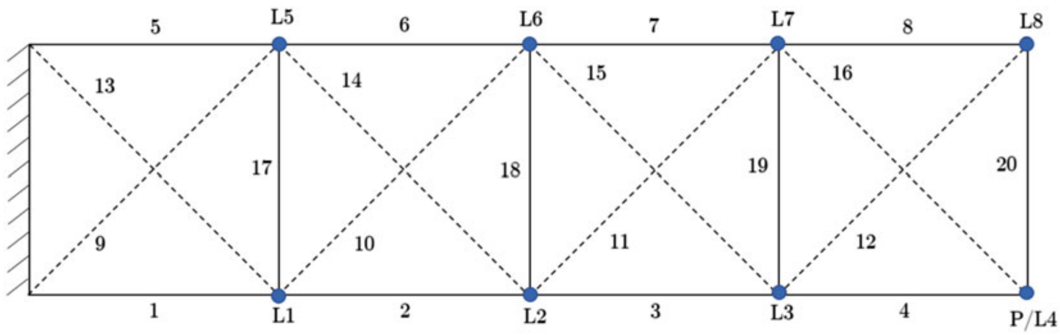

Fig. 5 A two-dimensional four-bay truss comprised of 20 members, eight of which are removable and denoted by a dashed line. Loads are applied at points L, and a preload is applied at point P. Load positions are shown as blue dots. The bays are numbered left to right from 1 to 4

$\boldsymbol{H} = \{0, 0, 0, 0, 0, 0, 0, 0\}$ is denoted as $\boldsymbol{H}^0$, and the health-state corresponding to the failure of the cross-members in the first bay $\boldsymbol{H} = \{1, 0, 0, 0, 1, 0, 0, 0\}$ is denoted as $\boldsymbol{H}^{17}$.

Finally, a binary decision d was considered for the structure, with possible courses of action 'do nothing' and 'perform maintenance'; for conciseness, these actions will be denoted with $d = 0$ and $d = 1$, respectively. In this case study, it is assumed that the 'perform maintenance' action is equivalent to the replacement of all cross-members with the structure consequently returned to its undamaged state.

4.1 Operational Conditions

Operational conditions were assumed for the structure such that the stress experienced in cross-members has a degree of stochasticity. Specifically, it was assumed that there would be uncertainty in both the load and the location that the load is applied to the structure at each time-step. In addition to the variable load, a constant preload of 5 kg was applied to the structure at point P.

The magnitude of the load w was assumed to vary in accordance with the discrete uniform distribution,

$$w \sim \mathcal{DU}(0, w_{max}; n) \tag{3}$$

where w_{max} was determined such that $P(\boldsymbol{H}^0_{t+1}|\boldsymbol{H}^0_t, d_t = 0) = 0.8$ and each load magnitude had probability of $P(w) = \frac{1}{n}$ with $n = 100$.

The position of the load was also assumed to vary according to a discrete uniform distribution over 8 candidate locations labelled L1 to L8 in Fig. 5. This distribution may be formalised as:

$$L \sim \mathcal{DU}(1, 8) \tag{4}$$

Hence, the operational conditions can be summarised as a vector $\boldsymbol{c}_o = \{w, L\}$. In total, 800 possible operational conditions were considered.

4.2 Failure Criteria

For each cross-member, three modes of failure were considered; yielding under tension, buckling under compression, and supercritical crack growth.

A cross-member was considered to have failed by yielding, if the tensile stress in the member exceeded the ultimate tensile stress of aluminium, where $\sigma_{UTS} = 300$ MPa. The event of a cross-member m_i failing via yielding is denoted as Y_i.

A cross-member was considered to have failed by buckling when the compressive stress within a member exceeded the buckling stress σ_b. The critical buckling stress for a slender beam is given by the following equation [15],

$$\sigma_b = \frac{\pi^2 EI}{A(KL)^2} \tag{5}$$

where E is the Young's modulus, I is the cross-sectional second moment of area, A is the cross-sectional area, K is the effective length factor and is dependent on the boundary conditions, and L is the length of the member. As the truss was constructed in a way that allows in-plane rotation at the ends of each member, a pinned-pinned boundary condition was assumed, resulting in an effective length factor of $K = 1$. Taking the Young's modulus of aluminium to be $E = 70\,\text{GPa}$, the critical buckling stress was found to be a compressive stress of $\sigma_b = 270\,\text{MPa}$. The event of a cross-member m_i failing via buckling is denoted as B_i.

The final failure method considered for the cross-members was supercritical crack growth. For this failure mechanism, it was assumed that each member possessed a crack in the centre across the entire width of the member and at the midpoint along the length with probability 0.1. The size of the crack in meters was assumed to be continuous uniformly distributed according to,

$$2a \sim \mathcal{U}(0, b) \tag{6}$$

where $2a$ is the crack size and $b = 0.0125$ and is the half width of the cross-members.

Assuming the cross-members can be modelled as a finite plate and with plane strain conditions, the mode I stress intensity factor K_I for a cracked member can be given by the following equation [16],

$$K_I = G\sigma\sqrt{\pi a} \tag{7}$$

where σ is the applied stress, and G is a geometric factor given by,

$$G = \frac{1 - \frac{a}{2b} + 0.326(\frac{a}{b})^2}{\sqrt{1 - \frac{a}{b}}} \tag{8}$$

A cracked cross-member was considered to have failed when the stress intensity factor exceeded the critical stress intensity factor K_c. For the aluminium members, it was taken that $K_c = 24\,\text{MPa} \cdot \text{m}^{\frac{1}{2}}$. The event of a cross-member m_i failing via supercritical cracking is denoted as C_i.

The initial variable structural conditions can be summarised in a vector $\boldsymbol{c}_s = \{2a_9, \ldots, 2a_{16}\}$, where $2a_i$ is the crack length present in cross-member m_i. Here, it should be noted that the $\boldsymbol{c}_s$ is considered independently of $\boldsymbol{H}$.

4.3 Transition Modelling

To determine the stresses within the structure under the variable operational and structural conditions, a finite element model of the truss was developed. The finite element model was validated with a set of strain measurements taken from the physical truss in its undamaged condition.

A wrapper function was produced to iterate over the global health-states $\boldsymbol{H_t}$. Additionally, the function was used to generate random samples $\boldsymbol{c}^*$ from the probability distributions specifying the uncertain operational and structural conditions $\boldsymbol{c} = \{\boldsymbol{c}_o, \boldsymbol{c}_s\}$. Afterwards, the function queried the finite element model to obtain the stresses in the cross-members for the given global health-state and a random sample of operational and structural conditions.

Asserting $d = 0$, for an initial global health-state $\boldsymbol{H}_t$ and a randomly sampled set of conditions $\boldsymbol{c}^*$, a health-state transition was defined as $\boldsymbol{H}_{t+1} = \boldsymbol{H}_t + \delta\boldsymbol{H}$ where $\delta\boldsymbol{H} = \{\delta hm_9, \ldots, \delta hm_{16}\}$ is an 8-bit binary vector and,

$$\delta hm_i = \mathbb{1}[(Y_i \vee B_i \vee C_i)|\boldsymbol{H}_t, d = 0, \boldsymbol{c}^*] \tag{9}$$

where $\mathbb{1}$ denotes the indicator function and $\vee$ denotes the inclusive-or logical operator. Here, Eq. (9) corresponds to evaluating cross-member failures with respect to the previously discussed criteria for yielding, buckling and cracking. Subsequently, the conditional probability of transitioning from $\boldsymbol{H}_t^i$ to $\boldsymbol{H}_{t+1}^j$ given $\boldsymbol{c}^*$ was specified such that,

$$P(\boldsymbol{H}_{t+1}^{j}|\boldsymbol{H}_{t}^{i}, d=0, \boldsymbol{c}^{*}) = \begin{cases} 1 & \text{if } \delta\boldsymbol{H} = \boldsymbol{H}_{t+1}^{j} - \boldsymbol{H}_{t}^{i} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

To populate the transition matrix $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 0)$, the variability in the conditions $\boldsymbol{c}$ must be marginalised out and the distribution normalised. This was achieved by calculating the i, jth entry of the transition matrix as,

$$P(\boldsymbol{H}_{t+1}^{j}|\boldsymbol{H}_{t}^{i}, d=0) = \frac{\sum_{1}^{N_s} P(\boldsymbol{H}_{t+1}^{j}|\boldsymbol{H}_{t}^{i}, d=0, \boldsymbol{c}^{*})}{N_s} \tag{11}$$

where N_s is the number of queries of the finite element model per $\boldsymbol{H}_t$.

The transition model for the action corresponding to ‘do nothing’ was estimated with the described procedure using $N_s = 10^4$. The heatmap of the resulting transition matrix $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 0)$ is shown in Fig. 6. A dominant lighter colour line can be seen along the diagonal in Fig. 6; this indicates that the structure has a tendency to remain in the same health-state over a single time-step. Furthermore, it can be seen that the elements in the lower-right triangle of the graph (which corresponds to the lower-left triangle of the transition matrix) consists entirely of zero elements; a result of the implicit constraint imposed through Eqs. (9) and (10) that the structure monotonically degrades. Taking the $\log_{10}$ of the conditional probability distribution (with an offset of +0.01 so that zero elements may be plotted with finite values) reveals further structure in the transition matrix as lower probability transitions are made more visible, as can be seen in Fig. 7. Figure 7 shows that the transition matrix has fractal pattern akin to the Sierpiński triangle. Due to the fact that the global health-state is represented as an 8-bit binary vector, the set of all allowable transitions assuming only monotonic degradation (i.e. once a bit is ‘turned on’, it cannot be ‘turned off’), form a Sierpiński triangle [17]. The possible transitions shown in Fig. 7 are a subset of the Sierpiński triangle with some elements missing due to physical effects disallowing some transitions; for example, if the truss were to collapse due to the failure of the first bay, then the members in the other bays would no longer be able to fail as the structure would cease to support the load.

For completeness, the transition matrix for the ‘perform maintenance’ action $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 1)$, was specified by making the assumption that the replacement of all cross-members returns the structure to the undamaged health-states, as shown by the following function:

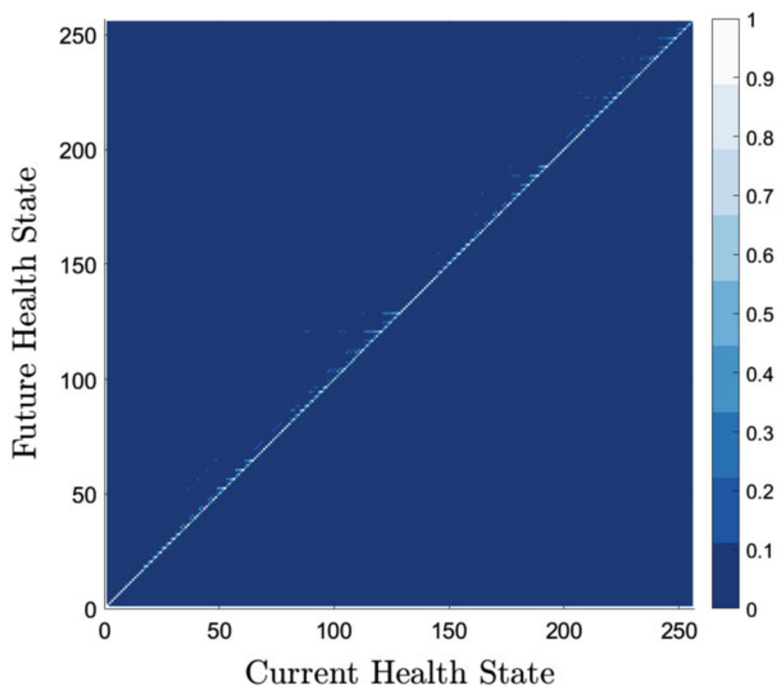

Fig. 6 A heatmap showing the transition matrix $P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 0)$

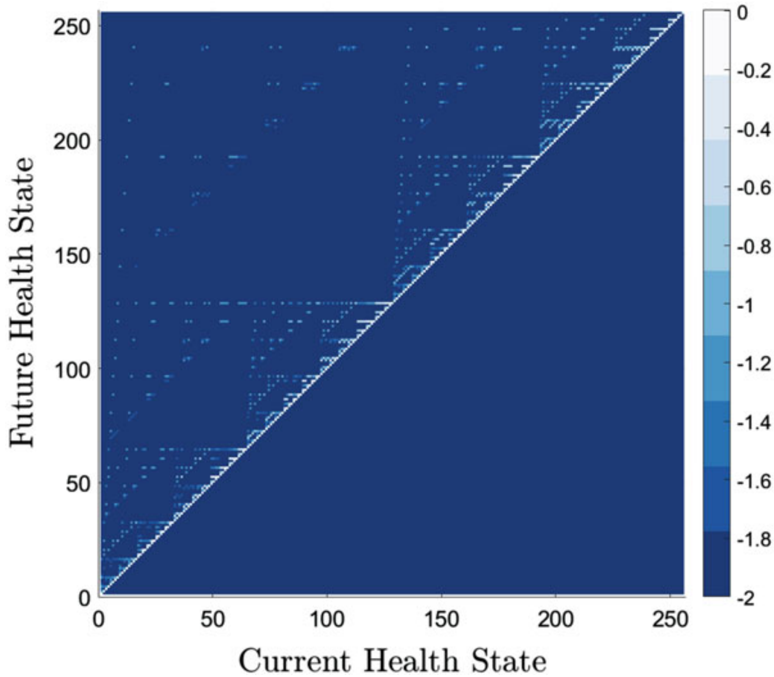

Fig. 7 A heatmap showing the log probability of the transition matrix with an offset, $\log_{10}(P(\boldsymbol{H}_{t+1}|\boldsymbol{H}_t, d = 0) + 0.01)$

$$P(\boldsymbol{H}^j_{t+1}|\boldsymbol{H}^i_t, d = 1) = \begin{cases} 1 & \text{for } j = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

The current section has demonstrated a sensible methodology for developing a health-state transition model for a structure by means of a case study. The next steps would be to evaluate and test the transition model, though this is omitted here as it is outside the scope of the current paper.

5 Discussion

The current sections aim to highlight and discuss the importance of health-state transition models in the context of risk-based decision-making for SHM and for the specific problem of prognosis. Additionally, discussion will be made around the challenges associated with the development of the transition models.

5.1 *Importance of Transition Models*

In general, when it comes to decision-making, possessing information or beliefs regarding future events/states is crucial. This statement becomes most apparent when taking this notion *ad absurdum*. At one extreme, if one possesses no information or belief regarding future events/states, then there is no reason for one to expect that any single course of action is better than any other. At the other end, if one somehow becomes clairvoyant and possesses perfect information regarding future events/states, then it follows that one would be able to make perfect decisions such that maximum rewards may be reaped.

As it happens, almost all decision problems, including those pertaining to SHM, fall somewhere between these two extremes, where belief and partial knowledge regarding future events/states is possessed. Nonetheless, in the context of SHM, increased expected utility gain provides a strong argument for striving towards improved knowledge regarding future health-states by the development of transition models.

In addition to allowing closer to optimal decisions to be made within the risk-based framework, a good transition model allows for a pseudo-prognosis for the structure to be made by utilising Eq. (2). By propagating the belief in the current health-state forward in time according to Eq. (2), and by evaluating the risk of failure associated with the predicted distribution over future health-states, at each time-step until the risk exceeds the cost of one of the candidate courses of action, one can obtain an estimate for the anticipated number of time-steps until an action should be taken. Whilst this result is not as powerful as a true-prognosis that yields remaining useful life, this information is still beneficial as it provides the expected time available to execute a course of action.

5.2 Challenges

There are numerous challenges associated with the development of transition models.

A primary challenge pertains to the validation of transition models. For many applications of SHM, the monitoring campaign will be for a newly built structure from which data are yet to be acquired at the time that the transition model must be developed. Without any observed state transitions to validate the model, one must rely solely on prior knowledge of the underlying physics that govern the degradation. One possible option is to independently validate the physics-based models used to develop the transition model via hybrid testing, or performing experiments on individual components or substructures. Alternatively, in situations where an SHM system is being retrofitted to an existing structure there may be historical data detailing health-state transitions that may be used to validate the degradation model.

The issue of validation is further complicated if the structure of interest is unique. For such a structure, even in a scenario where one is able to update the transition model with observed state transitions, it is possible, and in many cases likely, that only a small subset of the total possible state transitions will be observed throughout the operational lifetime; thereby leaving potentially large portions of the transition model without validation. In the context of population-based SHM [18–20], a single transition model may be applied to all members of a fleet of homogeneous structures and also updated with state transitions observed from each instance of the structure. The process of continually validating transition models online may be achieved through active learning [21].

Another challenge is the cost, both in terms of money and time, associated with the development of transition models. The development cost of a transition model will depend highly on the complexity of the structure for which a model is being developed, and the range of operational and environmental conditions that must be considered. For complex structures, the high-fidelity models capable of the multi-physics that may be required to simulate all the necessary failure mechanisms to develop a transition model are expensive and time-consuming to develop, often requiring teams of highly skilled engineers. The financial argument for the development of such models should be constructed and evaluated during the operational evaluation stage of the SHM process, taking into account whether the structure is of high-value, or safety-critical.

The computational cost of the development and implementation of the transitional model should also be considered. During the development of the transition model, it is possible that a physics-based model is queried numerous times. For complex structures, and high-fidelity models these simulations required large computing times. As the number of influential operating and environmental conditions increases, the number of samples required to adequately cover the input space will also increase. Taking this factor into account with the possibility that high-fidelity physics-based simulations may need to be queried many times, the calculation of the transition models may have prohibitively long computation times. A possible solution to this issue would be to use a surrogate model, where an interpolation function that is relatively cheap to query is trained on a subset of the outputs of the physics-based model.

Finally, a challenge pertaining to maintenance action transition models is left as an open topic for research and discussion. In a few limited cases, such as when repair corresponds to replacement of all failed components (as is assumed for the case study in the current paper), it may be reasonable to assume that the structure returns to its original undamaged case. However, in general, for less extreme and more realistic approaches to structural repair, this does not hold and, in fact, it is possible that the state to which the structure transitions was not considered during the original development of the transition model [22]. Here, the challenge lies with determining reasonable assumptions that allow one to avoid redeveloping the transition model after every intervention, or to conceive of methods for adapting the health-states considered within the risk-based decision framework.

6 Summary

The aim of the current paper has been to present a general methodology for developing structural health-state transition models for use in a probabilistic risk-based decision framework for SHM. Using a four-bay truss for a case study, a degradation model in the form of a probabilistic transition matrix was developed by considering uncertain operational conditions in conjunction with a physics-based model. Finally, discussions were made focussing on the challenges with developing health-state transition models but also on the importance of the models for both the risk-based decision framework, and their application to the problem of prognosis in SHM.

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