Understanding Errors from Multi-Input-Multi-Output (MIMO) Testing of a Cantilever Beam

Fernando Moreu, James Woodall, and Arup Maji

Abstract Multi-Input-Multi-Output (MIMO) testing is challenging due to the need to estimate inputs required to achieve desired responses at multiple locations on the test article of interest. Errors in the estimation of inputs can come from either the forward process (measurement of the Transfer Function (TRF) matrix that captures the input-output relationship) or the inverse process requiring the inversion of a rectangular matrix using the Moore-Penrose Pseudo-Inverse method.

In order to understand how small variation in the location of actuators contribute to errors, a set of SIMO (Single Input Multiple Output) experiments were conducted. A "Smartshaker" actuated a cantilever beam while sensors collected the acceleration response at two locations (close to the input location and at the tip of the cantilever beam). Researchers compared the response from different inputs with that estimated using the Transfer Function (TRF) to determine errors in the forward process. First, an applied Band-Limited-White-Noise (BLWN) signal informed the creation of the TRFs. Subsequently various narrow-band inputs at different frequencies and real-life vibration environments (earthquake and train) excited the structure. The study helps to understand the sources of uncertainty in MIMO vibration testing which can extend to more complex geometries and enables actuator placement that would minimize errors for different real-life inputs.

Keywords MIMO · Multi-input · Vibration · Testing · Error

1 Introduction

Researchers use Multiple Input-Multiple Output (MIMO) testing to take field response data and solve for the inputs to the system [1, 2]. This allows one to use response data which is readily available by the application of sensors to deduce what forces acted on a given structure or dynamic system. These techniques also apply to fields such as active feedback control for dynamic structures [3–5]. Researchers often use a simple beam to conduct MIMO analysis, both in the field and to test aspects of MIMO itself [6–9]. Due to the value offered by MIMO testing in dynamic testing and other engineering fields, there is a wealth of literature and study into the subject. Even so, MIMO still warrants further research and better understanding for many of its facets to better apply and learn from it, and researchers continue to develop new methods and applications for MIMO [10, 11].

One of the fundamental parts of common MIMO testing is the TRF matrix which derives the inputs. This quantity represents all relevant relationships for a test in dynamic analysis, relating all outputs to each of the inputs. Consequently, researchers have investigated the creation of this matrix, developing methods to improve accuracy and reliability when defining the system properties. There are many approaches to this step, depending on factors such as data availability, signal properties, and the conditioning of the TRF matrix [12]. In cases such as with larger structures, the application of a known

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F. Moreu (✉) · J. Woodall
Department of Civil, Construction, and Environmental Engineering, University of New Mexico, Albuquerque, NM, USA
e-mail: fmoreu@unm.edu; jameswoodall@unm.edu

A. Maji
Sandia National Laboratories, Albuquerque, NM, USA
e-mail: amaji@sandia.gov

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force to develop the needed relationships are infeasible, leading to much study in the field of output only modal identification [13–15]. Often, the ambient forces measured for these tests have relatively low magnitude and noise dominates their spectra, as studied by Wang [16], as well as Hazra [17].

MIMO inverts the completed TRF matrix to infer the input signals. In many ways, this is the key step which defines MIMO. Many factors can affect this step, and many researchers continue to investigate problems in the signal processing and the inversion process [18]. Thite [19] investigated how the output sensor locations impact the condition number of the Transfer Function (TRF) matrix, and how the locations may be optimized to minimize the condition number, reducing the error propagation in the inversion process.

Researchers use Finite Element Models (FEMs) in modal analysis and to perform MIMO in virtual experiments. However, when creating a model to represent a system, the presence of sensors on the real member can cause discrepancies between reality and the FEM. When these discrepancies are too severe, researchers must find ways to reconcile the systems [20]. One method of doing this is by mitigating the effect of the sensor on the real system [21–24]. Researchers also reconcile the systems by including the mass of the sensor within the model to match with an experiment. Once the model adequately matches the experiment, researchers may remove the sensor masses from the FEM to better simulate the behavior of the SUT in the field, when no sensors are present. However, many of these methods assume the location of the sensors to be known precisely, and the decoupling procedures can be prone to severe error propagation [25]. The dynamic effect of sensor mass is too pronounced to ignore in many tests, such as those in which the sensor mass is not insignificant compared to that of the structure under test. This paper investigates a case study with imprecise sensor location, and the consequential errors.

2 MISO Test Setup with Small Variation in Actuator Location

Figure 1 describes the SIMO experiments on a steel 34.5 × 1.5 × 0.125 in. cantilever beam with input actuator at 1/3 height and response accelerometers at 1/3 height and the tip. A load cell sensor between the stinger and the beam measured the force applied. Researchers conducted three sets of experiments with the accelerometer located at the 1/3 point and displaced ±7/16″ (1.1 cm) from the 1/3 point. The actuator was a Model K2007E01 “Smartshaker,” manufactured by The Modal Shop Inc., with up to 7 pounds (31.2 N) peak sine force and a maximum stroke length of 1/2″ (1.3 cm). Acceleration sensors were Model 353B03 quartz shear accelerometers, manufactured by PCB Piezotronics. The force transducer was a Model 208C02 ICP® force sensor with a measurement range of 100lbf, also manufactured by PCB Piezotronics.

Researchers conducted the following sequence of experiments; the following section discusses the results:

1. Researchers applied a Band-Limited-White-Noise (BLWN) ranging from 0 to 250 Hz to the three experiment setups with the different sensor locations. The data from these tests informed the creation of the Transfer Functions (TRFs) at the 1/3 point and the tip, with the MatLab™ command *tfestimate*.
2. In the centered case, the stinger applied both narrow-band sinusoidal inputs and various real-life vibration inputs (earthquake, trains) (Fig. 2) to the beam. Researchers collected each of the responses as well as the force applied by the stinger.
3. For each test performed in step 2, researchers estimated the output responses by multiplying the measured inputs with each of the TRFs generated in step 1 (with conversions between time and frequency domain).
4. Researchers compared the measured outputs from step 2 with the estimated outputs from step 3, quantifying the discrepancy with the root mean square error (RMSE, or RMS error) between them. Equation (1) defines the equation for this metric.

$$\mathrm{RMSE} = 100 * \frac{\sqrt{\sum_{i=1}^{n} \frac{(\mathrm{Estimate}_i - \mathrm{Measured}_i)^2}{n}}}{\text{Measured amplitude}} \tag{1}$$

Researchers also performed the same series of steps with the sinusoidal inputs in a virtual simulation to examine the purely theoretical deviations caused by sensor displacement.

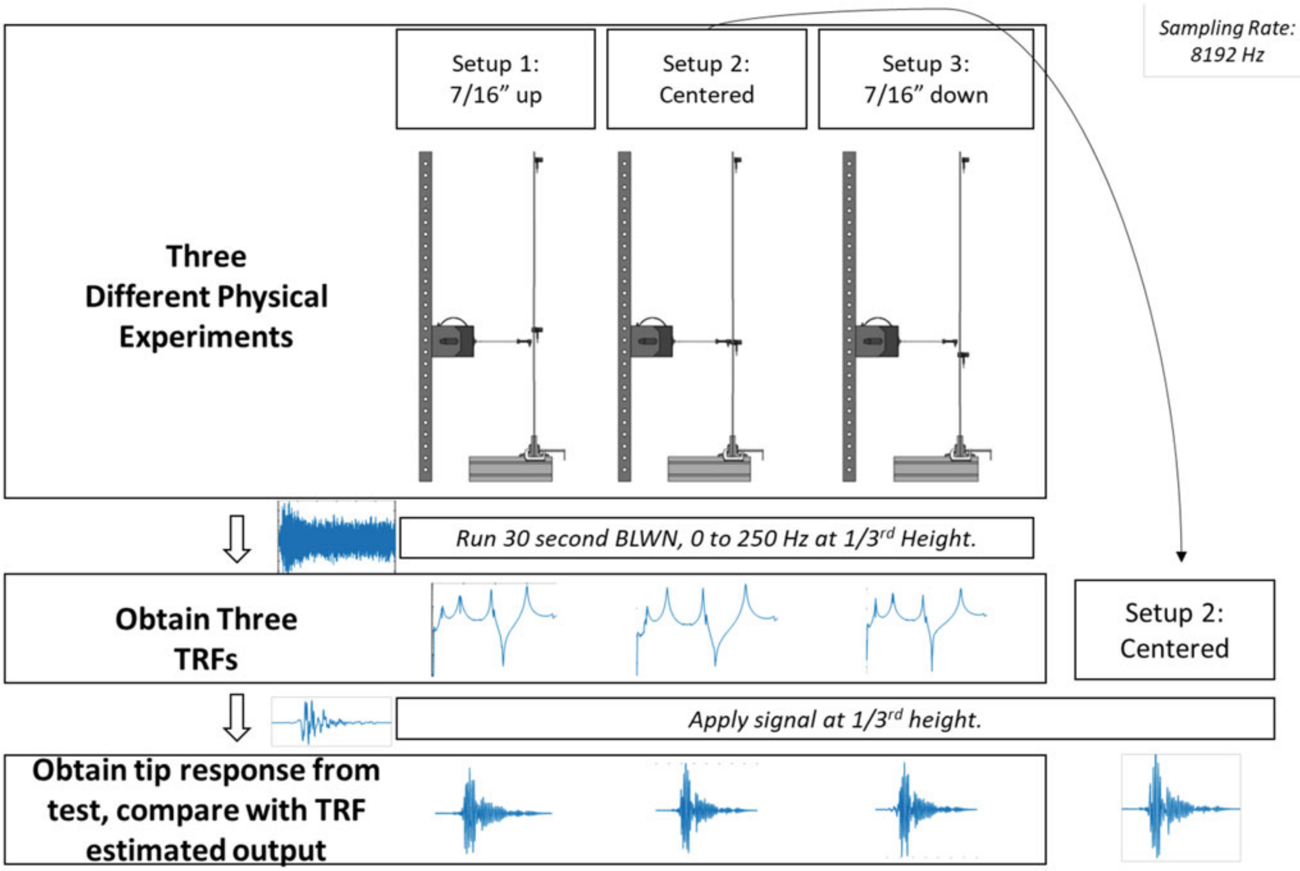

Fig. 1 Schematic of SIMO experiment

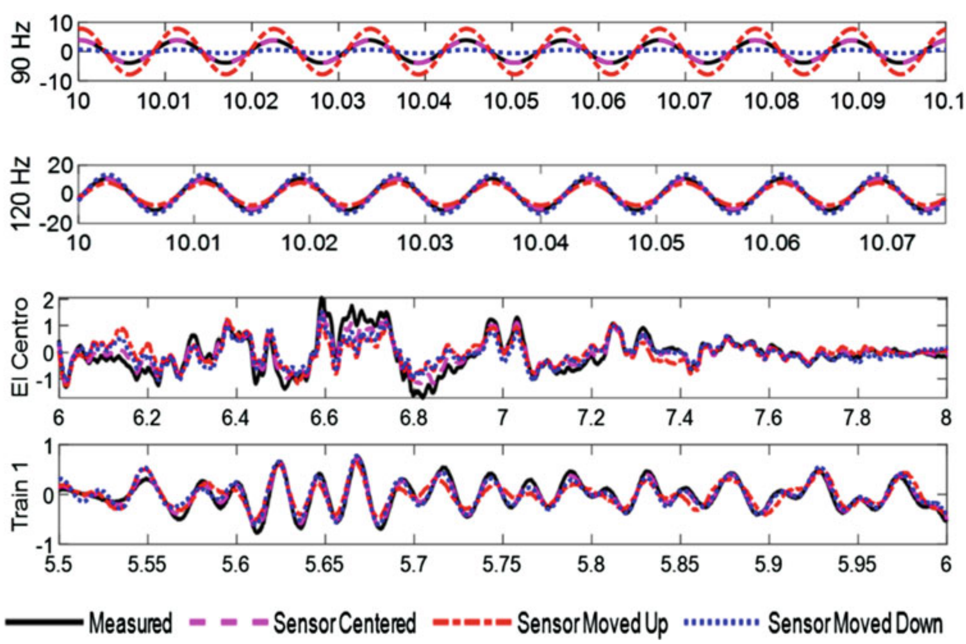

Fig. 2 Examples of time-domain data at 1/3 height for different inputs; measured and estimated

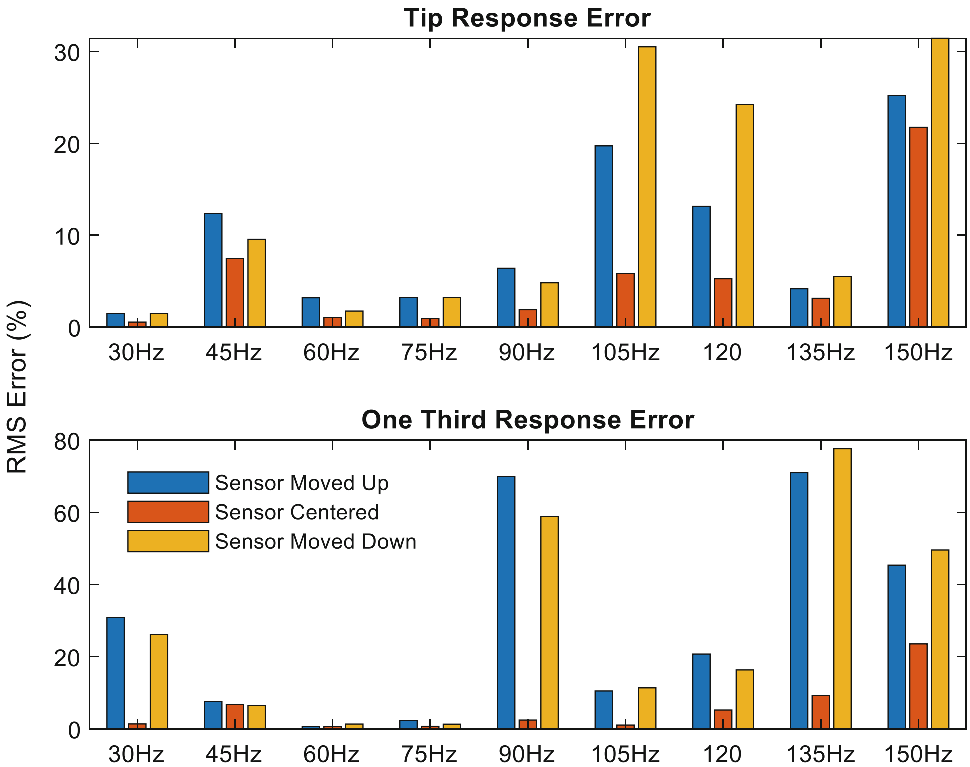

Fig. 3 Experimental RMS error for sinusoidal inputs

3 Test Results and Analyses

Figure 2 shows some example time-domain signals measured at the 1/3 height; the top two are narrow-band inputs at 90 and 120 Hz, followed by the El Centro earthquake and a train crossing a bridge. Figure 3 shows the RMS error between the estimated and measured time-domain signal described earlier for the sinusoidal inputs, while Fig. 4 shows the RMS error for the real-life vibration inputs. In Fig. 4 (right side), the RMS error corresponding to "Train 1" is lower than those corresponding to the BLWN and El Centro earthquake. A closer examination of Fig. 1 shows that the dominant frequency for Train 1 was $\approx$ 45 Hz, which had relatively small error for the narrow-band 45 Hz only input (Fig. 3). However, the BLWN had a higher error since it is an agglomeration of the error across all frequencies. The El-centro error was in-between since it had a dominant low-frequency content and some higher frequency (above 45 Hz) as well. All of the trains have similar frequency content, justifying their similar RMS error levels. Examining the sinusoidal inputs, there are similar trends in error for both the experimental (Fig. 3) and simulation (Fig. 5) cases. Of note is the increase in error for the experimental case at higher frequencies, which is not present in the simulation, while the moved cases remain of similar magnitude. This suggests the presence of some additional factor besides the standard properties used in the model that can dramatically affect results.

4 Conclusions

This paper investigated how small variations in the locations of actuators can affect the accuracy of MIMO testing. A SIMO test and associated analyses determined the accuracy of the forward MIMO process with sensor location uncertainty.

Researchers demonstrate the effects of a small deviation in sensor location on MIMO results. This emphasizes the importance of precise measurements and experimental consistency, as even small changes can result in large increases in error. The sensitivity (variation) of the TRFs (Transfer Functions) due the sensor location can explain errors in response away from the actuator (tip response). Finally, the cumulative effect on real-life loads (earthquake and train-induced vibration) is based on the frequency content of those events and the sensitivity of actuator location at the dominant frequencies. The study

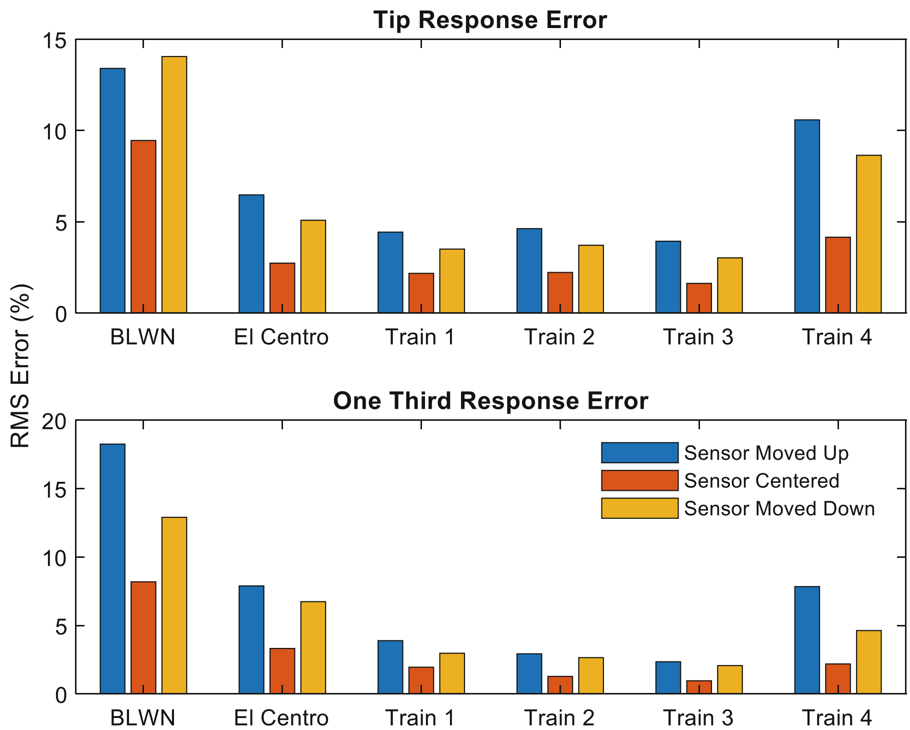

Fig. 4 Experimental RMS error for real-life inputs

Fig. 5 Simulation RMS error for sinusoidal inputs

helps to understand the sources of uncertainty in MIMO vibration testing which can extend to more complex geometries and enables actuator placement minimizing errors for different real-life inputs.

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Fernando Moreu is an assistant professor in the Department of Civil, Construction and Environmental Engineering of the University of New Mexico, with courtesy appointments in the Department of Mechanical Engineering and the Department of Electrical and Computing Engineering. His PhD is from the University of Illinois at Urbana-Champaign in Structural Engineering (2015).

James Woodall is a graduate research assistant in the Department of Civil, Construction and Environmental Engineering of the University of New Mexico, pursuing a MS in structural engineering. His BS is from the University of New Mexico in the Department of Civil, Construction and Environmental Engineering (2018).

Arup Maji is a research scientist in Sandia National Laboratories. His PhD is from the University of Northwestern in Structural Engineering (1988).