

Evaluation of the Acoustic Performance of Perforated Multilayer Absorber Materials

Amine Makni^{1,2}, Marwa Kani², Mohamed Taktak^{1,2} (🖾), and Mohamed Haddar¹

¹ Laboratory of Mechanics, Modeling and Production (LA2MP), National School of Engineers of Sfax, University of Sfax, BP N° 1173, 3038 Sfax, Tunisia mohamed.haddar@enis.rnu.tn
² Faculty of Sciences of Sfax, BP N° 1171, 3000 Sfax, Tunisia mohamed.taktak@fss.rnu.tn

Abstract. Acoustic absorber based on perforated plate and air cavity presents a fundamental liner used in many acoustic applications as a liner applied at the wall of duct systems to reduce noise. The use of this kind of absorber in form of multi-layer is generally used to improve their acoustic performance and the attenuation capacity is increased. Such absorber has an acoustic performance varying with their intrinsic and geometrical proprieties (perforation rate and the thickness of the plate, cavity length, etc...). To quantify and qualify the acoustic performance of these absorbers an acoustic indicator can be used which is the acoustic absorption coefficient. In this paper, the transfer matrix method is used to compute this coefficient in the case of the presence of plane wave. For this, several cases of multilayer are studied and investigated. After that, and for each studied configuration, a parametric study of the influence of the multilayer materials parameters on this coefficient for various configurations is carried on to deduce the more influent parameters on the acoustic performance of this kind of absorber.

Keywords: Plane waves \cdot Acoustic response \cdot Perforated material \cdot Multilayer absorber \cdot Porous material \cdot Helmholtz resonators \cdot Transfer matrix \cdot Absorber coefficient

1 Introduction

Helmholtz resonators are generally used in many industrial fields (transport, building, etc...) for sound attenuation or impact energy dissipation. Thus, an accurate and thorough understanding of the acoustic behavior of these materials and the effect of their parameters is necessary. So a parametric study on the variation of these parameters was carried out by several researchers. In fact, (Nakai and Yoshida 2013) had made a simulation of the normal incidence absorption coefficient of perforated panels with and without glass wool. (Dengke et al. 2014) have studied the influence of the position of porous material on absorption of perforated plates. Also (Hai and Yadong 2014) studied the acoustic absorption coefficient for different perforated plate's configurations. The transfer matrix method is also used to the study of this kind of absorbers as presented in the work of Gerges and Jordan (2005), Campa and Camporeale (2012) and Campa and Camporeale (2010).

To improve the acoustic performance of this kind of absorber, it can be used in form of multilayer, each layer is a resonator. For this case, an electric – acoustic analogy can be used as presented in Mina et al. (2013) and Congyun and Qibai (2005). Also, the air cavity can be replaced by a porous material to enlarge the frequency domain of the absorption. In this article, a parametric study of multilayer absorber materials composed by air, porous materials and perforated plates is presented based on the transfer matrix method. In the second section, we are interested by determining of the absorption coefficient expressed as a function of the impedance of the resonator surface. This is based on the Elnady model (Elnady and Boden 2003) to determine the impedance of the perforated plate, and the Johnson - Allard model (Sellen 2003; Henry 1997; Allard and Champoux 1992) to determine the surface impedance of the cavity. In the third section, the studied configurations are presented and in the fourth section, a parametric study of different studied configurations of multilayer materials is made to determine the influential parameters on this coefficient.

2 Theoretical Basis

2.1 The Transfer Matrix

The transfer matrix method is used to model electric quadrupoles. This method can be adapted to model one-dimension acoustic system as presented in Lee and Kwon (2004). The acoustic transfer matrix links the pressures (p_i) and the acoustic velocities (u_i) in two different positions r and r + 1 in the same axe as presented in Eq. (1).

$$\begin{bmatrix} p_r \\ u_r \end{bmatrix} = [T] \begin{bmatrix} p_{r+1} \\ u_{r+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{r+1} \\ u_{r+1} \end{bmatrix}$$
(1)

With [T] is the transfer matrix.

2.2 Computation of Multilayer Absorber Transfer Matrix

The studied multilayer materials are a succession of layers composed even by perforated plate and an air cavity (Helmholtz resonator) or by perforated plate and a porous material (absorber). So, to model multilayer absorbers, two types of matrices must be computed: the transfer matrix of the plate P and the transfer matrix of the cavity S. Then the total transfer matrix T of the layer is given by the Eq. (2):

$$\boldsymbol{T} = \boldsymbol{P} \cdot \boldsymbol{S} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$
(2)

2.2.1 Transfer Matrix of the Plate

The perforated plate (Fig. 1) is characterized by:

- The plate thickness *t*,
- The of perforation rate σ_p ,
- The whole diameter d.



Fig. 1. Perforated plate

The transfer matrix of the perforated plate is given by Lee and Kwon (2004) and Narayana and Munjal (1986):

$$\begin{pmatrix} p_1 \\ u_1 \end{pmatrix} = \begin{bmatrix} 1 \ \rho_0 c_0 \zeta \\ 0 \ 1 \end{bmatrix} \begin{pmatrix} p_2 \\ u_2 \end{pmatrix} = \boldsymbol{P} \begin{pmatrix} p_2 \\ u_2 \end{pmatrix}$$
(3)

 $\rho_0 c_0$ are respectively the air density and the sound speed.

 ζ is the plate impedance expressed as follows (Elnady and Boden 2003):

$$\zeta = \operatorname{Re}\left\{\frac{ik}{\sigma_p C_D} \left[\frac{t}{F(k'_s d_p/2)} + \frac{\delta_{re}}{F(k_s d/2)}\right]\right\} + i \operatorname{Im}\left\{\frac{ik}{\sigma_p C_D} \left[\frac{t}{F(k'_s d/2)} + \frac{\delta_{im}}{F(k'_s d/2)}\right]\right\}$$
(4)

with C_D is the discharge coefficient, $k = \frac{2\pi}{c_0} f$ is the constant of propagation of air and f is the frequency, δ_{re} and δ_{im} are correction coefficients:

$$\delta_{re} = (0, 2).d + (200).d^2 + (16000).d^3 \text{ and } \delta_{im} = (0, 2856).d$$
 (5)

$$F(k_s d/2) = 1 - \frac{J_1(k_s d/2)}{k_s \frac{d}{2} J_0(k_s d/2)}$$

and $F(k'_s d/2) = 1 - \frac{J_1(k'_s d/2)}{k'_s \frac{d}{2} J_0(k'_s d/2)}$ (6)

$$k'_{s} = \sqrt{-i\omega/\nu'}$$
 and $k_{s} = \sqrt{-i\omega/\nu}$ (7)

With ω is an angular frequency, k_s and k'_s are respectively the Stokes number of insulating and conductive walls,

 $v = \frac{\mu}{\rho}$ is the kinematic viscosity and

$$\nu' = \frac{\mu'}{\rho} = \frac{(2, 179)\mu}{\rho}$$
(8)

 μ is the dynamic viscosity and ρ is the material density.

2.2.2 Transfer Matrix of an Air Cavity

This matrix is analytically obtained by writing pressure p_1 at x = 0 and pressure p_2 at x = L out the form of plane wave as detailed in the work of (Beranek and Vér 1992) (Fig. 2). The matrix S of an air cavity is expressed as follows (with Z_0 is the air impedance):



Fig. 2. Construction of the transfer matrix of the cavity

2.2.3 Transfer Matrix of a Cavity Containing Porous Material

The porous material is considered like an equivalent fluid. For this study, we use the Lafarge – Allard model (Lafarge et al. 1997) for computing the acoustic impedance (Z_C) in a porous material. This impedance is given in the following equation:

$$Z_C = \sqrt{\rho(\omega).K_{LA}(\omega)} \tag{10}$$

$$\rho(\omega) = \alpha_{\infty}.\rho_0.\left[1 - j\frac{\sigma.\phi}{\rho_0.\alpha_{\infty}.\omega}.\sqrt{1 + \frac{4.j.\rho_0.\alpha_{\infty}^2.\omega.\eta}{\sigma^2.\phi^2.\Lambda^2}}\right]$$
(11)

$$K_{LA}(\omega) = \gamma . P_0. \left[\gamma - \frac{\gamma - 1}{1 + \frac{\eta . \phi}{j . \omega . \rho_0 . N_{pr} . k_0'} . \sqrt{1 + \frac{4 j . \omega . \rho_0 . N_{pr} . k_0'^2}{\eta . \phi^2 . \Lambda'^2}} \right]$$
(12)

In those equations, ω is an angular frequency, $\rho(\omega)$ is the effective density, $K_{LA}(\omega)$ is the dynamic compressibility, α_{∞} is the material tortuosity, ϕ is the material provide provide

Finally, and when the pores of the material are saturated with air, the transfer matrix is given by:

$$\begin{bmatrix} S_p \end{bmatrix} = \begin{bmatrix} \cos(k(\omega).L) & j.Z_C.\sin(k(\omega).L) \\ (j/Z_C)\sin(k(\omega).L) & \cos(k(\omega).L) \end{bmatrix}$$
(13)

With
$$k(\omega) = \omega \cdot \sqrt{\frac{\rho(\omega)}{K_{LA}(\omega)}}$$
 (14)

 $k(\omega)$ is the wave number.

2.3 Computation of the Acoustic Absorption Coefficient

The reflection coefficient is computed from the transfer matrix coefficients as follows:

$$R = \frac{T_{11} - \rho_0 c_0 T_{12}}{T_{11} + \rho_0 c_0 T_{21}} \tag{15}$$

The absorption coefficient α is given by Eq. (16) (Scarpato et al. 2013):

$$\alpha = \frac{4\text{Re}(Z/\rho_0 c_0)}{\left[1 + \text{Re}(Z/\rho_0 c_0)\right]^2 + \left[\text{Im}(Z/\rho_0 c_0)\right]^2}$$
(16)

Z is the system impedance.

3 Studied Configurations

Firstly, the multilayer Helmholtz resonators are obtained with a series of perforated plates separated by air cavity (Fig. 3). Table 1 defines the geometric parameters of the different used layers.

Secondly, the air cavity is replaced by a porous material. Three kinds of porous material were used. Table 2 gives the acoustic intrinsic parameters of these materials.

This study was performed to determine the effect, on the absorber coefficient versus frequency, of the layers order, the plate and the cavity geometric parameters, the porous material kind and its intrinsic parameters. The parametric study was realized for the two and three layers materials.

	Thickness of the plate t (mm)	Diameter of the holes d (mm)	Rate of perforation σ_p	Length of the cavity Lc (mm)
Layer 1 (M1)	1	1	0.025	21
Layer 2 (M2)	0.8	0.3	0.05	20
Layer 3 (M3)	1	1	0.025	8

Table 1. Geometrical parameters of studied layers

	Rockwool	Glass wool	Acousticel
α_{∞} : tortuosity	2.1	1	1.38
σ : resistivity (<i>N</i> . <i>m</i> ⁻⁴ . <i>s</i>)	135000	9000	22000
Ø: porosity	0.94	0.99	0.95
η : viscosity (<i>Pa.s</i>)	0.1	0.1	0.1
Λ : viscous characteristic length (<i>m</i>)	$0.49 e^{-4}$	$1.92 e^{-4}$	$1.7 e^{-4}$
Λ' : thermal characteristic length (<i>m</i>)	$166 e^{-6}$	$576 e^{-6}$	5.1 e ⁻⁴
k'_0 : thermal permeability (m^2)	$k_0' = \eta/\sigma$	$k_0' = \eta/\sigma$	$0.83 e^{-8}$
γ : report of the heat capacity and mass	1.4	1.4	1.4

Table 2. Intrinsic parameters of different porous materials



Fig. 3. (a) Two layers Helmholtz resonator, (b) Three layers Helmholtz resonator

4 Results and Discussion

4.1 Multilayer Resonators

The curves of absorption coefficient versus frequency show a number of peaks of absorption equal to the number of the layers.

The parametric study was performed firstly on the plate's parameters (the plates order, the plate thickness, the holes diameter and the plate rate of perforation) and secondly on the cavity parameters (the cavity length).

4.1.1 Effect of the Plate's Order

According to the Fig. 4, the better absorption is obtained when the layer 2 is placed in the first position with values reaching 1.

In terms of peaks frequencies, we note a slight modification between all the configurations, except in the second peak of the three layers' system (Fig. 4b) when we place the third layer in the first place.

This can be explained by the fact that the layer 2 has the perforated plate with the greatest perforation rate that increases the acoustic resistivity of the layer and therefore the increases the absorption phenomenon.



Fig. 4. Effect of the plate's order in a: (a) two layers resonator, (b) three layers resonator

4.1.2 Effect of the Plate's Thickness

Figure 5 shows that this parameter has a low effect on the first peak value and its frequency. In all configurations, when the thickness of the first plate increases (Figs. 5a and 5c), the absorption coefficient decreases and the peak's appearance frequency remains constant (in the second and the third peak). But when the thickness of the second or third plate increases, the absorption coefficient increases (except in Fig. 5d) and the peak's appearance frequency decreases (in the second and the third peak).

4.1.3 Effect of the Plate's Holes Diameter

According to Fig. 6a and Fig. 6c, we note that the evolution of absorption coefficient value is inversely proportional to that of the holes diameter of the plate 1 in all the absorption peaks.

When the hole's diameter of the plate 2 or those of the plate 3 increases, the absorption coefficient value increases, this is observed on the second and third peaks of Fig. 6. But on the first peaks, the effect of the plate's 2 holes' diameters and the plate's 3 holes' diameter are different: The increasing of the plate's 2 holes' diameters causes a decreasing of the absorption coefficient value (Fig. 6b and Fig. 6d), and the increasing of the plate's 3 holes' diameters have no effect on the absorption coefficient value (Fig. 6e). Also, the hole's diameter variation has no effect on the peaks frequencies.

4.1.4 Effect of the Plate's Perforation Rate

In Fig. 7, the absorption coefficient values in the first peaks are modified only when the plate 1 perforation rate change (Fig. 7a and Fig. 7c).

So when the plate 1 perforation rate increases, the peak value decreases and the peak appearance frequency increases.

In the second and third peaks, the evolution of the absorption coefficient value is proportional to that of the plate 1 perforation rate, and is inversely proportional to that of the plate 2 and the plate 3 perforation rates. In those peaks, the plate's 1 perforation rate has no effect on the peak's occurrence frequency. The latter presents a proportional evolution with that of the plate's 2 and plate's 3 perforation rate.



Fig. 5. Absorption coefficient versus frequency resulting from the thickness variation of: (a) plate 1 in two layers resonator, (b) plate 2 in two layers resonator, (c) plate 1 in three layers resonator, (d) plate 2 in three layers resonator, (e) plate 3 in three layers resonator

4.1.5 Effect of the Cavity Length

Figure 8 shows that the cavity length evolution has a low effect on the absorption coefficient value and the peak's frequencies. In the second and third peaks, the peak's frequencies decrease when the layer's 1, layer's 2 or layer's 3 cavity length increase. The absorption coefficient values increases when the layer's 2 or layer's 3 cavities lengths increases, and its decreases when the slayer's 1 cavity length increases.

4.2 Multilayer System with Cavities Containing Porous Material

When the cavity contains a porous material, the absorption peaks become wider and the absorption covers a larger frequency domain. The parametric study was performed firstly on the plate's parameters with Rockwool porous material, and secondly on the



Fig. 6. Absorption coefficient evolution versus frequency resulting from the holes diameter variation of: (a) plate 1 in two layers resonator., (b) plate 2 in two layers resonator, (c) plate 1 in three layers resonator, (d) plate 2 in three layers resonator, (e) plate 3 in three layers resonator

porous material's parameters. The porous material's parameters are material's order, the tortuosity and the viscous characteristic length.

4.2.1 Effect of the Plate's Order

According to Fig. 9, we obtained the same curves shape between the two and three layers systems. The curves, in this figure, show two types of evolutions. The first type is when the system 1 or system 3 is placed in the first case; the absorption reaches the value of 30% of absorption, and then decrease. The second type, when the system 2 is placed in the first position, present a continuous increasing of the absorption, the maximum measured is 33% of absorption. As showed in the first study, when the layer 2 (having the greatest perforation ratio) is in the first place, the multilayer system is more absorbent.



Fig. 7. Absorption coefficient evolution versus frequency resulting from the perforation rate variation of: (a) plate 1 in two layers resonator, (b) plate 2 in two layers resonator, (c) plate 1 in three layers resonator, (d) plate 2 in three layers resonator, (e) plate 3 in three layers resonator

4.2.2 Effect of the Plate's Thickness

Using Rockwool porous material and according to Fig. 10, only the variation of the plate's thickness n°1 has an effect on the absorption coefficient. So in the two or three layer systems, the increasing of this thickness causes the decreasing of the maximum value of the absorption and his occurrence frequency.

4.2.3 Effect of the Plate's Hole's Diameter

In Fig. 11, only the effect of the hole diameter variation of plate 1 was presented because there are the only parameters that affect the absorption coefficient. So when this parameter increases, the absorption coefficient value and his occurrence frequency decreases slightly.



Fig. 8. Absorption coefficient evolution versus frequency resulting from the cavity length variation of: (a) system 1 in two layers resonator, (b) system 2 in two layers resonator, (c) system 1 in three layers resonator, (d) system 2 in three layers resonator, (e) system 3 in three layers resonator



Fig. 9. Influence of the plate's order using the Rockwool porous material in: (a) two layers absorber, (b) three layers absorber



Fig. 10. Absorption coefficient evolution versus frequency resulting from the thickness variation of: (a) plate 1 in two layers absorber, (b) plate 2 in two layers absorber, (c) plate 1 in three layers absorber, (d) plate 2 in three layers absorber, (e) plate 3 in three layers absorber



Fig. 11. Absorption coefficient evolution versus frequency resulting from the holes diameter variation of: (a) plate 1 in two layers absorbent. (b) plate 1 in three layers absorber

4.2.4 Effect of the Plate's Perforation Rate

The same effect of the perforation rate variation of plate 1 is observed on the absorption coefficient evolution in the two and three layers absorbers. We obtained a proportional evolution of the absorption value and the absorption maximum occurrence frequency compared with the plate's perforation rate evolution (Fig. 12).



Fig. 12. Absorption coefficient evolution versus frequency resulting from the perforation rate variation of: (a) plate 1 in two layers absorber, (b) plate 1 in three layers absorber

4.2.5 Effect of the Porous Material's Order

Using plate 3, the better absorption coefficient, in two and three layers absorber, is detected when the "rigid glass wool" is placed in the first position. This is observed at frequencies less than 3000 Hz. At higher frequencies (frequencies greater than 4000 Hz), the system with Rockwool porous material, placed in the first position, gives the maximum of absorption (Fig. 13).



Fig. 13. Influence of the porous material's order in: (a) two layers absorber, (b) three layers absorber

4.2.6 Effect of the Porous Material's Tortuosity

In the two layers absorber, the order of layers is: layer 1 backed by the layer 2. In the three layers absorber, the order of the systems is: layer 1 backed by layer 2 backed by the layer 3. The principal effects are observed when the variation of the tortuosity is made on the first porous material in front of the incident wave. This is the same using an absorber with two and three layers absorbers. In those absorbers and according to Fig. 14, the absorption coefficient evolution is proportional to that of the porous material's tortuosity. This behavior is reversed when the frequency reaches the value of 5000 Hz in absorber with two layers absorber, and 2700 Hz in absorber with three layers absorber.



Fig. 14. Influence of the porous material's tortuosity in: (a) two layers absorber, (b) three layers absorber

4.2.7 Effect of the Viscous Characteristic Length

In the two layers absorber, the order of the layers is: layer 1 backed by the layer 2. In the three layers absorber, the order of layers is: layer 1 backed by layer 2 backed by the layer 3. The major effects are observed when the variation of the tortuosity is made on the first porous material in front of the incident wave. This is the same using an absorber with two (Fig. 15a) and three degrees of freedom (Fig. 15c). According to Fig. 15, the absorption coefficient evolution is proportional to that of the porous material's viscous characteristic length. This behavior is reversed when the frequency reaches the value of 6000 Hz in absorber with two layers absorber, and 2800 Hz in absorber with three layers absorber.



Fig. 15. Absorption coefficient evolution versus frequency resulting from the viscous characteristic length variation of: (a) material 1 in two layers absorber, (b) material 2 in two layers absorber (c) material 1 in three layers absorber, (d) material 2 in three layers absorber, (e) material 3 in three layers absorber

5 Conclusion

In this paper, the transfer matrix method is used to determine the absorption coefficient of multilayer absorbers composed by perforated plates, air and porous materials. Then, a parametric study of the geometric and intrinsic parameters is made to determine the influence of geometrical and intrinsic parameters of these absorbers on the absorption coefficient. For multilayer resonators, it is observed that the parameters of the first plate have no influence on the first peak of the absorption. These parameters have an effect on the frequency and the values of the second and third peaks of absorption in the case of two or three layers' resonators. For multi layers containing porous materials, the results showed that the variations in the tortuosity and the viscous characteristic length have an influence on the absorption coefficient in the opposite of other parameters like the flow resistivity and porosity.

References

- Nakai, T., Yoshida, K.: Simulation of normal incidence sound absorption coefficients of perforated panels with/without glass wool. In: Proceedings of Meetings on Acoustics, Montreal, Canada, vol. 19, p. 015043 (2013)
- Dengke, L., Daoqing, C., Bilong, L., Jing, T.: Improving sound absorption band width of microperforated panel by adding porous materials. In: Proceedings of Inter-Noise Congress (2014)
- Hai, X., Yadong, L.: Sound absorption characteristics of the perforated panel resonator with tube bundles. In: Proceedings of the 21st International Congress on Sound and Vibration, ICSV 21 (2014)
- Gerges, S.N.Y., Jordan, R.: Muffler modeling by transfer matrix method and experiment entail verification . J. Braz. Soc. Mech. Sci. Eng. **17**(2), 132–140 (2005)
- Campa, G., Camporeale, S.M.: Eigenmode analysis of the thermo acoustic combustion instabilities using a hybrid technique based on the finite element method and transfer matrix method. Adv. Appl. Acoust. **1**, 1 (2012)
- Campa, G., Camporeale, S.M.: Application of transfer matrix method in acoustics. In: Proceedings of the COMSOL Conference (2010)
- Elnady, T., Boden, H.: On semi-empirical liner impedance modeling with grazing flow. In: Proceedings of 9th AIAA/CEAS (2003)
- Sellen, N.: Changing the surface impedance of an active control material: application to the characterization and optimization of an acoustic absorber,. Ph.D. thesis, Lyon central school, France (2003) (in French)
- Henry, M.: Measurements of the parameters characterizing a porous medium. Experimental study of acoustic behavior of foams at low frequencies, Ph.D. thesis, Maine University, France (1997) (in French)
- Allard, J.F., Champoux, Y.: New empirical equations for sound propagation in rigid frame fibrous materials. J. Acoust. Soc. Am. 91(6), 3346–3353 (1992)
- Lee, D.H., Kwon, Y.P.: Estimation of the absorption performance of multiple layer perforated panel systems by transfer matrix method. J. Sound Vib. **278**, 847–860 (2004)
- Narayana, R.K., Munjal, M.L.: Experimental evaluation of impedance of perforates with grazing flow. J. Sound Vib. 108(2), 283–295 (1986)
- Beranek, L., Vér, I.L.: Noise and Vibration Control Engineering: Principles and Applications. Wiley-Interscience, New York (1992)
- Lafarge, D., Lemarinier, P., Allard, J.F., Tarnow, V.: Dynamic compressibility of air in porous structures at audibles frequencies. J. Acoust. Soc. Am. **102**(4), 1995–2006 (1997)
- Scarpato, A., Ducruix, S., Schuller, T.: Optimal and off-design operations of acoustic dampers using perforated plates backed by a cavity. J. Sound Vib. 332, 4856–4875 (2013)
- Mina, S., Nagamura, K., Nakagawa, N., Okamura, M.: Design of compact micro-perforated membrane absorbers for polycarbonate pane in automobile. Appl. Acoust. **74**(4), 622–627 (2013)
- Congyun, Z., Qibai, H.: A method for calculating the absorption coefficient of a multi-layer absorbent using the electro-acoustic analogy. Appl. Acoust. **66**, 879–887 (2005)