Optimal Linear Quadratic Stabilization of a Magnetic Bearing System

Abdelileh Mabrouk[1], Olfa Ksentini[1](✉), Nabih Feki[1,2], Mohamed Slim Abbes[1], and Mohamed Haddar[1]

[1] Laboratory of Mechanics, Modeling and Manufacturing (LA2MP), National Engineering School of Sfax, Ministry of Higher Education and Research of Tunisia, BP 1173, 3038 Sfax, Tunisia
slim.abbes@enis.tn, Mohamed.haddar@enis.rnu.tn
[2] Higher Institute of Applied Sciences and Technology of Sousse, University of Sousse, 4003 Sousse, Tunisia
nabih.feki@issatso.u-sousse.tn

Abstract. Magnetic bearings' modelling is performed using different methodologies. Generally, the linear modelling of these actuators, which is required for the use of linear controllers, gives an approximation of the nonlinear relation between the bearing load and the control current. However, this approach may have some disadvantages because the model is linearized around an equilibrium position. Thus, the performance of the linear design can decreases if there is a perturbation of the system. Although, when the variation of the rotor displacement is too small, a linearized control law can offer valid results. The main objective of this work is to study the stabilization of an electromechanical system based on the linear quadratic (LQ) control method. The stabilization of a high sped rotating shaft supported by four α-degree oriented magnetic bearings will be studied. The linearization around an equilibrium position is performed to adopt the linear control law. Some simulations results will be illustrated to evaluate the performances of the proposed controller.

Keywords: Magnetic bearings · Dynamic behaviour · LQ control · High speed · Weight

1 Introduction

Magnetic suspensions are efficient solutions for very different field. They can support small electric machines up to huge mechanics such as some compressors. The suspended parts can be in a stationary position (telescopes) and can be subjected to high speeds (centrifuges, turbines…). Mechanical suspensions have a limited rotational speed due to mechanical problems and overheating which causes unbalancing phenomenon at high speed and lead to a significant vibration. The balancing issues can be avoided by the use of magnetic bearings because the axis of inertia can be adjusted with the axis of rotation. A controlled magnetic bearing allows the rotor positioning with a great

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precision. In addition, a high speed could be reached due to the absence of contact in a magnetic bearing. The reduction of mechanical wear leads to lower maintenance costs and a longer system life. In magnetic bearings, adaptive stiffness could be used in vibration isolation, for avoiding critical speeds and external disturbances. For that, it's necessary to understand the behaviour of this system. Many techniques was been developed in literature for the magnetic bearing modelling. Rigid modelling method was used by some researchers (Toumi and Reddy 1992; Fan et al. 1992; Chen et al. 2007) to supervise the magnetic bearing system behaviour. Others (Hentati et al. 2013; Ding et al. 2015; Yanliang et al. 2006) presented a finite element model to study the dynamic behaviour of a spindle. Results showed that there is a difference between the rigid and the flexible model. The control of the magnetic bearing system was the objective of other researchers (Schweitzer and Lange 1976). Different methods was been adopted for the shaft stabilization. Zhuravlyov et al. (2000) had tested the validity of an LQ regulator to stabilize a MB system in a high speed motion and the ability of this controller to minimize copper losses in coils. Also, Barbaraci and Mariotti (Barbaraci and Mariotti 2012) kept a contactless motion during the increase of the velocity by varying the rotor angular speed. Moreover, a Linear Quadratic Gaussian (LQG) controller (Hutterer et al. 2014; Gu et al. 2004) was used to compensate the gyroscopic effect of a high angular velocity rotor. Results were compared to those obtained from a decentralized PID controller.

The aim of this paper is to control a high-speed rotating shaft supported by four magnetic bearings with an LQ method. It will be organized as follows: a shaft supported by α-degree oriented symmetric magnetic bearings elements is modelled using rigid method, and then an LQ regulator is adopted for the stabilization of the system, finally different gain values are tested to identify the adequate one for the system control.

2 Model Presentation

A four degrees of freedom model corresponding to the translational and the rotational components is studied. It's composed from a shaft actioned by two magnetic bearings (see Fig. 1).

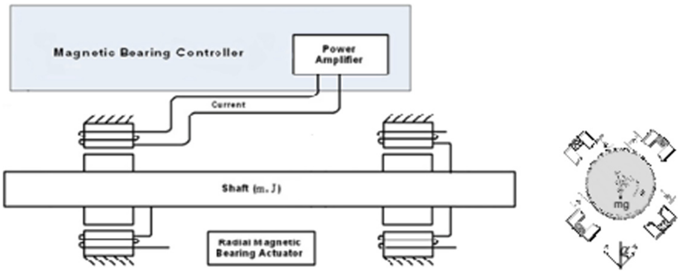

Fig. 1. The linear control of a shaft supported by two radial bearings

Using Euler-Lagrange equation, and based on the linearized expression that binds the force and the current of the electromagnetic bearings (Schweitzer and Lange 1976; Toumi and Reddy 1992), the motion equation without (1) and with (2) control are obtained respectively as follows:

$$M\ddot{q} + G\dot{q} + K_X q = f_g \tag{1}$$

$$M\ddot{q} + G\dot{q} + K_X q = f_g + K_I U \tag{2}$$

Where,

- M = diag (m, m, J, J) is the mass matrix with m is the rotor mass and J is the moment of inertia of the shaft and $f_g = \frac{mg}{\cos\alpha}$, g is the gravitational acceleration, $\alpha = 45°$.

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Omega J_z \\ 0 & 0 & \Omega J_z & 0 \end{bmatrix} \tag{3}$$

- K_X: is the Displacement stiffness matrix. It depends on the bearing displacement stiffness k_x and the distance d between the centre of the rotor and the bearing position as it's shown in 4.

$$K_X = \begin{bmatrix} -2k_x & 0 & 0 & 0 \\ 0 & -2k_x & 0 & 0 \\ 0 & 0 & -2k_x d^2 & 0 \\ 0 & 0 & 0 & -2k_x d^2 \end{bmatrix} \tag{4}$$

- K_I: is the Current stiffness matrix. It depends on the bearing current stiffness k_i and the distance d. It's defined in 5.

$$K_I = \begin{bmatrix} k_i & 0 & k_i & 0 \\ 0 & k_i & 0 & k_i \\ k_i d & 0 & -k_i d & 0 \\ 0 & k_i d & 0 & -k_i d \end{bmatrix} \tag{5}$$

- X = $[x\beta]$ is the DOF's vector
- I = $[I_{xa} I_{ya} I_{xb} I_{yb}]$ is the current vector with I_a and I_b are respectively currents in bearings A and B in X and Y directions.

3 LQ Control

Electromagnets exerted attractive forces in order to maintain the rotor in an adequate position. The magnetic bearing rotor requires greater values of control parameters to be positioned accuracy . The aim from some control strategy is to minimize the control values to be in optimized conditions. Therefore, Linear-Quadratic (LQ) controller is suggested as a solution for an optimal AMB control (Lurie 1951; Kwakernaak and Sivan 1972) (Fig. 2).

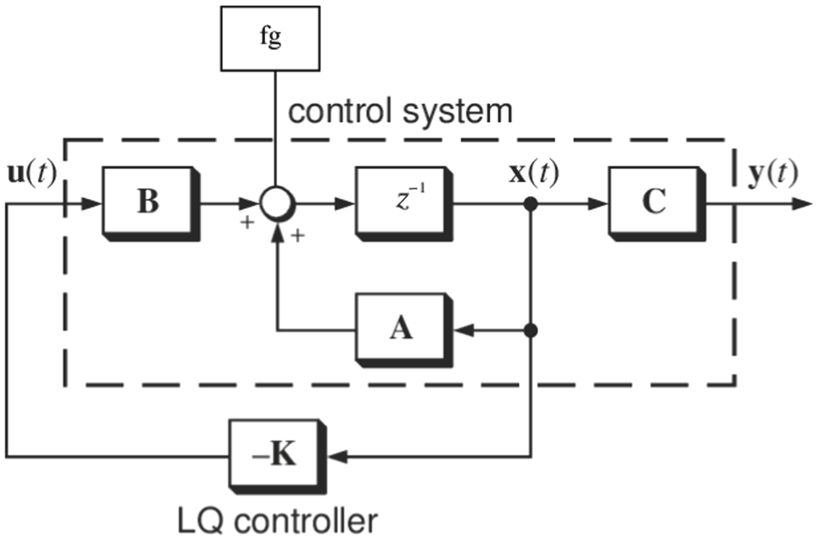

Fig. 2. Block diagram for LQ control method

To adopt this control method a state space representation is required. So the equation of motion can be written as follows.

$$\left\{ \dot{X}(t) \right\} = [A]\{X\} + [B]\{u\} \tag{6}$$

$$\{Y\} = [C]\{X\} \tag{7}$$

Where X(t) is a state variables vector, Y(t) is the vector of output variables, u(t) is the vector of input variables and A, B, C are constant matrices. These matrices and vectors are detailed in Eqs. (8) and (9).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\beta}_x \\ \dot{\beta}_y \\ \ddot{x} \\ \ddot{y} \\ \ddot{\beta}_x \\ \ddot{\beta}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\ 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\ 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0\ 0 & 0 & 1 \\ -\frac{2k_x}{m} & 0 & 0 & 0 & 0\ 0 & 0 & 0 \\ 0 & -\frac{2k_x}{m} & 0 & 0 & 0\ 0 & 0 & 0 \\ 0 & 0 & -2k_x.\frac{d^2}{J} & 0 & 0\ 0 & 0 & -\Omega J_z \\ 0 & 0 & 0 & -2k_x.\frac{d^2}{J} & 0\ 0 & \Omega J_z & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \beta_x \\ \beta_y \\ \dot{x} \\ \dot{y} \\ \dot{\beta}_x \\ \dot{\beta}_y \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{k_i}{m} & 0 & \frac{k_i}{m} & 0 \\ 0 & \frac{k_i}{m} & 0 & \frac{k_i}{m} \\ k_i.\frac{d}{J} & 0 & -k_i.\frac{d}{J} & 0 \\ 0 & k_i.\frac{d}{J} & 0 & -k_i.\frac{d}{J} \end{bmatrix} \begin{bmatrix} I_{xa} \\ I_{ya} \\ I_{xb} \\ I_{yb} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ fg \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} x_a \\ y_a \\ x_a \\ y_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & d & 0 \\ 1 & 0 & 0 & -d \\ 0 & 1 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \beta \\ \dot{\beta} \end{bmatrix} \tag{9}$$

The adequate control command gives a minimized quadratic integral performance index J (see Eq. (10)) when it brings the system from an arbitrary to a zero-state position.

An optimal motion control is also based on the selection of the optimum states Q (symmetric and positive matrix) and R (symmetric and positive matrix) which represent respectively the weighting and the control-weighting matrix.

By this way, the quadratic cost function J which involves these two matrices is minimized.

$$J = \int \left(q^t\, Q\, q + P\, u^t R\, u\right) dt \tag{10}$$

where:

$Q = 0.1 \cdot [I_n]$
$R = 10 \cdot [I_n]$

with, I_n is the identity matrix.

The command is optimized when an optimal gain K is chosen (see Eq. 10).

$$u = -K\, X(t) = -P^{-1} B^t P\, X(t) \tag{11}$$

where, P is the solution of the steady-state matrix Riccati equation (Zhuravlyov et al. 2000) and K is the system gain.

4 Simulation Results

The simulation is carried out into two steps. In the first one, the command current is not introduced. The objective from this step is to follow the system behaviour without control and to see if it's necessary to control the system by the LQ approach. If the system is not stabilized, it is crucial to adopt the LQ method in a second step to maintain the shaft at an equilibrium position.

Figure 3 and 4 show the displacement in the X and Y directions and the rotation around X and Y respectively. They are obtained after the simulation of Eq. 1 where the command current is not taken into account (without control).

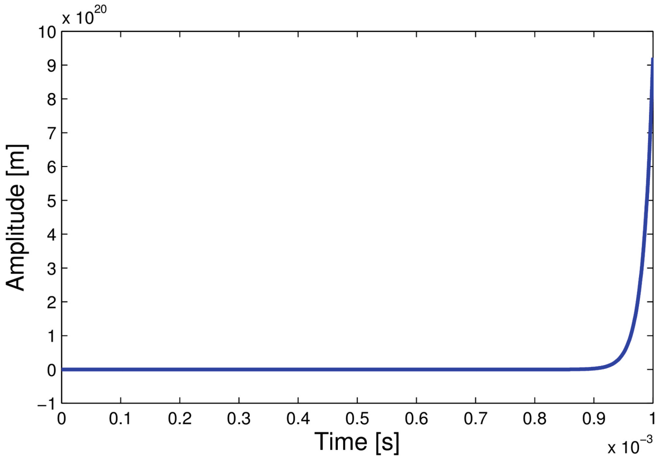

Fig. 3. Rotor displacement in the X and Y directions without control

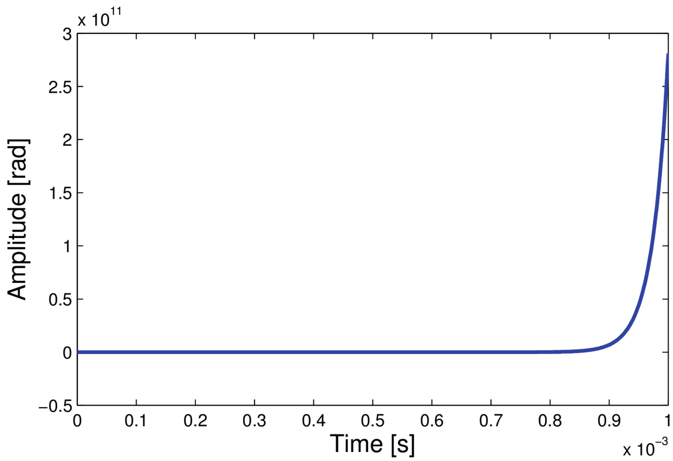

Fig. 4. Rotor rotation around the X and Y directions without control

It can be noticed that the command current has a great influence on the displacement and the rotation. In fact, if it is not taken into account the system responses increase rapidly. In fact, the response grows exponentially thus the rotor may fall down or touch the magnet. It is a critical phenomenon, which can affect the behaviour of the system. Therefore, it is necessary to introduce a control law to stabilize this system.

As it has mentioned previously a linear method LQ is adopted for the system control. The Eq. 2 is solved and results of the displacement in x direction and the rotation around Y direction are presented respectively in Fig. 5 and 6.

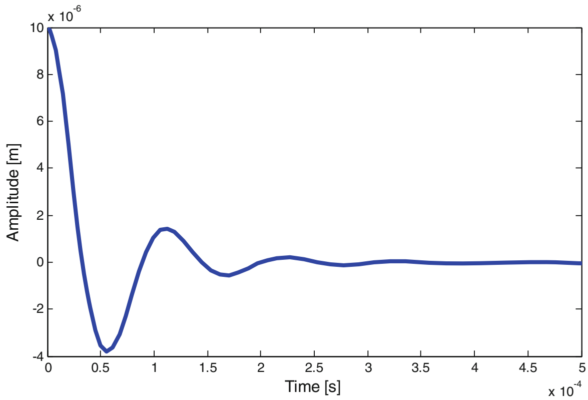

Fig. 5. Displacement of the rotor in the X and Y directions with LQ control.

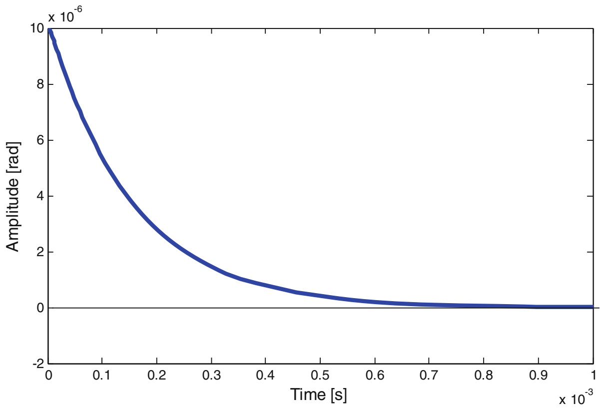

Fig. 6. Rotation of the rotor around the X and Y directions with LQ control.

Within 0.8 ms, the shaft is stabilized at the equilibrium position. This result proves the LQ regulator effectiveness. By examining Fig. 5, it is observed that the response fluctuates around the axes of rotation, which correspond to the transient step. This fluctuation is not noticeable in the rotation response. In fact, the weak displacement does not give an important rotation. So in order to ameliorate the rotor accuracy, it is indispensable to supervise the displacement behaviour by adjusting the gain of the system to eliminate vibration.

To provide an efficient evaluation of LQ controller and to find the best conditions of stabilization, different gain values are tested. Results obtained after the simulation are shown in Fig. 7 and 8. From this parametric study, the optimal gain will be extracted.

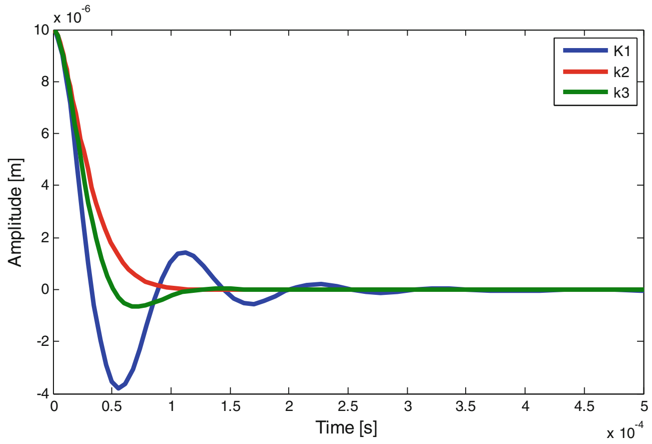

Fig. 7. Control of the rotor displacement in the X and Y directions with different gain values

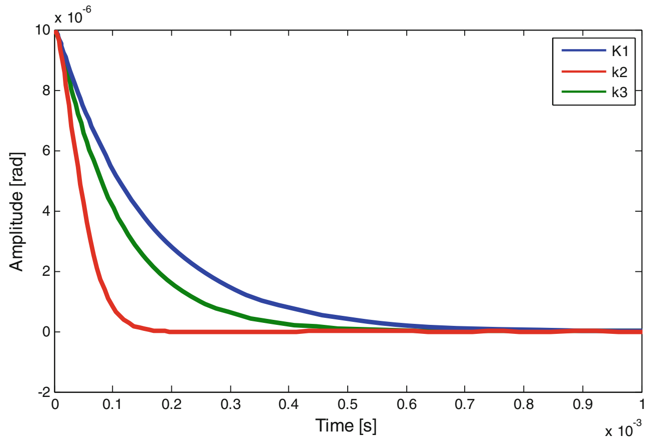

Fig. 8. Control of the rotor rotation around X and Y directions with different gain values

It is observed that the first matrix gain K1 shows a damped behaviour with an undershoot of more than 3 μm and a reduced time of control. The behaviour is ameliorated in the case of gain K3 where the undershoot and the stabilization time are minimized. The system behaviour under the gain K2 shows the best result in term of absence of damped behaviour compared to the first and the second case with a short rise time (down to about 10 times compared to the gain K1). That's why, it's better to choose the adequate gain for the system stabilization to minimize vibration. In fact matrices Q and R, where Q defines the weights of the states and R defines the control signal weight in the cost function J are used to control the signal behaviour. The more their values are increased the more the gain values increased and the more the signal behaviour is penalized.

A larger value of Q allows stabilizing the system with little changes in the states. So, because there is a trade-off between the two parameters, we kept usually Q and we altered R. In the case of a limited control current (saturation zone) we perform a less weighted energy strategy with a large R values (expensive control strategy) otherwise we choose a small R without penalizing the states behaviour (cheap control strategy).

Finally, an LQ controller proves it ability to stabilize a rotor suspended by oriented symmetric MBs in high speed movement without touching magnets.

5 Conclusion

In this paper, an electro-mechanical model of a high-speed rotating shaft actioned by an oriented magnetic bearing system is studied. The system diverges when the command current is not taken into account. The stabilization of this model is obtained using an LQ control method. A parametric study is elaborated to choose the adequate gain for the system stabilization with the minimum vibration. For all tested cases, the LQ controller proves its ability to stabilize a rotor supported by four magnetic bearings without touching magnets.

Further works are in progress in order to introduce other methods of control and to compare them with this method. Also, asymmetrical bearings positions will be tested to prove the influence of bearings positions.

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