

# Chapter 17

## Application of Data-Driven Yield Surface to Prediction of Failure Probability for Centrifugal Pump



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**Abstract** The modern world is often faced with the problem of the equipment design lifetime ending. Researches related to determines the residual lifetime, extension of service life, and prediction of failure-free operation is important, especially when it comes to equipment for nuclear and thermal power plants. Such interest is associated with the difficult economic situation, high cost of equipment and its components, requirements for safe working conditions, etc. The main objective of this work is to study the probability of the centrifugal pump failure-free operation. Attention is paying to the water elbow part of a centrifugal pump. Finding the probability of model failures is based on data-driven yield surface application. Takes into account the different behavior of composite materials under tensile and compressive loads, which is analyzed at the micro-level using the finite element method. Going beyond the yield surface indicates the possibility of transition into a plastic state. The proposed method of analysis of a centrifugal pump type WD 16/25 leads to predict the probability of failure during normal operation and in hydro testing mode. Consideration of the influence of corrosion-erosive processes and uniform thinning of the water pump elbow wall, an analysis of the probability of failure-free operation in time is carried out.

**Keywords** Microstructure · Yield surface · Probability · Finite element method · Failure

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## 17.1 Introduction

It's important to predict the failures, especially when it comes to the equipment of nuclear and thermal power plants. The limited financial resources do not allow the renewal of power units, the design lifetime of which is almost exhausted. Therefore, the research deals with predicting the probability of non-failure operation, the assessment of the residual lifetime (Kelin et al. 2019), and extend the design lifetime (Kahlman 2016; Verhuelsdonk 2005), which are in demand and relevant nowadays. During operation, pumping units under the influence of corrosion and erosion inevitably encounter thinning of the housing walls. The negative effects of redistribution of mechanical stresses in the structure are required close attention. It is also necessary to take into account the various modes of operation of the pump, such as normal operating conditions and hydro testing, which is provided by the relevant standards to calculate the strength of the pump equipment (IEC 60041:1991; ISO/TR 17766:2005; ISO 9906:2012).

There are various methods to assess the unit's lifetime. In some works, the reliability and probability of failure are assessed (Cheng et al. 2016; Patel et al. 2005). Other ones are based on deterministic models (Jacobs et al. 2018). The authors use the principle of stress state determination to estimate the operational life. In this work, it is proposed to evaluate the failure-free operation of the pump based on the yield surface data-driven approach. Nowadays, a data-driven approach helps to analysis of the collected data and scientifically make decisions. Widely used in artificial intelligence, engineering (Siddiq 2020; Lvov and Kostromytska 2020), strategy, marketing, policy, medicine, etc.

On the one hand, materials science and modern microscopy have contributed to the development of methods for predicting material properties based on their internal structure. On the other hand, the traditional sciences of continuum mechanics, the theory of elasticity, the theory of vibrations, and reliability provide ample opportunities for multilevel modeling and analysis of materials. During the construction of the model, such a connection leads to the loss of a significant part of the information accumulated during the experimental research. Therefore, the technology of obtaining as complete information on the micro-level as possible and transferring it to the macro model is important, will contribute to the expansion and improvement of existing methods.

As a bridge of communication between the micro and macro levels, approaches based on obtaining large data sets (data mining), and subsequent statistical processing (data science) can serve, which allow obtaining, accumulating, and processing significant amounts of data. The revealed dependences and probabilistic characteristics of the microstructure were used in further modeling of the material—to help improve the quality, accuracy, and completeness of the analysis.

Investigation of the probabilistic characteristics of the anisotropic materials yield surface (Banabic et al. 2010; Shapovalova and Vodka 2019a, b), makes it possible to describe the behavior of a structure under a complex stress state. Computer modeling for predicting material behavior is an alternative to such an approach. The main

principle proposed in this article is to predict possible macro failure of the model based on the study of processes at the micro-level. To evaluate the probability of failure-free operation in the time of the waste dynamic pump (WD 16/25) during normal operation and hydro test mode.

## 17.2 Objectives

The main objective of this study is to use the yield surface data-driven approach to determine the failure-free probability of a centrifugal pump. This objective requires the completion of such tasks:

- construction of a geometric model of the waste dynamic pump (WD 16/25); the simulation of the uniform casing walls thinning process under the influence of erosion effects; meshing the model with finite elements; calculating the stress-strain state at the normal operating conditions and hydro-testing;
- apply the previously obtained material properties (modulus of elasticity) to the most critical pump unit (the water elbow part of the pump); to use a method for assessing the probability of plastic stress occurrence, based on data-driven yield surface application.

## 17.3 Modeling of Centrifugal Pump WD 16/25

The centrifugal pump WD 16/25 is designed for pumping waste, industrial, domestic contaminated, and waste liquids. The analysis takes into account various operating conditions such as normal operating conditions (NOC) and hydro testing (HT) mode. The geometric model of the pump is presented in Fig. 17.1, and consists of: 1—inlet branch, 2—outlet branch, 3—water pump elbow, 4—shaft, 5—motor, 6—stand, 7—bearing, 8—supports, 9—bolts (M8, M12, M14, M16).

According to studies of the pump under similar operating conditions (Kelin et al. 2020), based on the requirements of the standard for strength analyzes (IEC 60041:1991; ISO/TR 17766:2005), a uniform thinning of the walls of the pump elbow is assumed. This thinning corresponds to 18 years of operation with an average running time of 135 h per year (equal to 0.5% thinning per year). A three-dimensional geometric model of complete and partial wall thinning at 0 and 9% is shown in Fig. 17.2.

A finite element (FE) mesh based on linear FE of hexagonal and tetrahedral shapes is applied to the geometric model. The grid used for calculations is shown in Fig. 17.3.

For the calculations, the physical and mechanical characteristics of the material are used for the entire model (steel 20), except for the water pump elbow (orthotropic material, the mechanical properties of which are obtained earlier during the analysis of an artificially generated statistically equivalent material (Shapovalova and Vodka 2020). The corresponding data for the test materials are presented in Table 17.1.

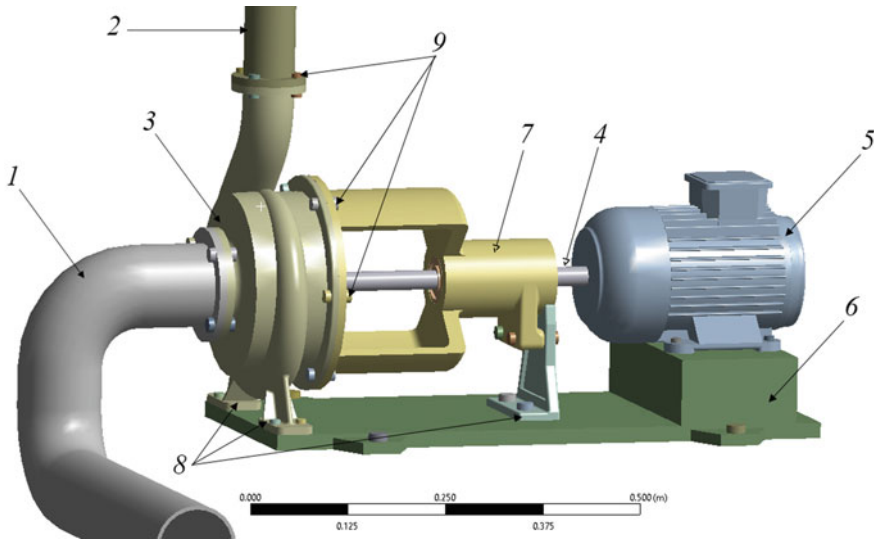


Fig. 17.1 The geometric model of the WD 16/25 pump

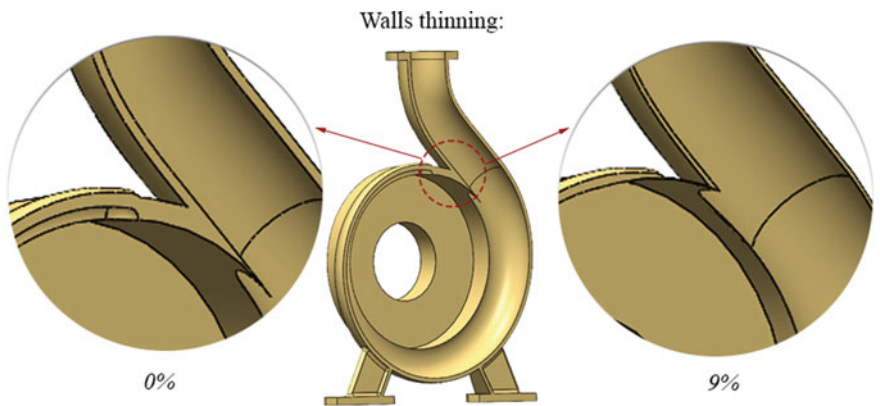


Fig. 17.2 The water elbow part of the pump walls thinning at 0 and 9%

According to strength standards by PNAE G-7-002-86 or similar standards ASME Boiler and Pressure Vessel Code, Vol. III, the nominal allowable stress for elements of equipment and pipelines loaded with internal pressure is selected as the minimum of the following values:

$$[\sigma] = \min\{\sigma_B/2.6; \sigma_{0.2}/1.5\} \tag{17.1}$$

where  $\sigma_B$ —tensile strength;  $\sigma_{0.2}$ —yield strength.

To calculate the stress–strain state, the boundary conditions are set:

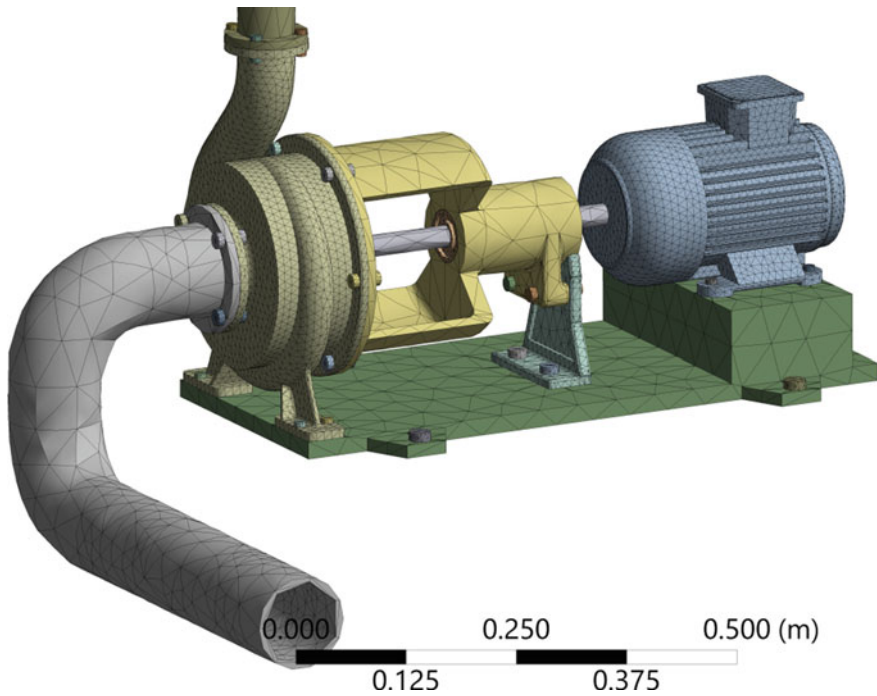


Fig. 17.3 Finite element mesh of the model (general view)

Table 17.1 Mechanical properties of the materials

Elastic modulus, $E$ , GPa	Poisson ratio, $\nu$	Shear module, $G$ , GPa	Ultimate tensile strength, ( $\sigma_B$ ), MPa	Yield strength, ( $\sigma_{0.2}$ ), MPa	Allowable stress, ( $\sigma$ ), MPa	
<i>Steel 20</i>						
200	0.30	79.30	402	216	<b>144</b>	
Artificial microstructure with concentration of inclusions $\psi = 0.100$ (Shapovalova and Vodka 2020)						
$E_x = E_y = E_z$	$\nu_x = \nu_y = \nu_z$	$G_x = G_y = G_z$	Macrolevel <sup>a</sup>		Micro-level	Macrolevel
186.16	0.31	68.93	350	220	<b>3.3</b>	<b>135</b>

<sup>a</sup>Macrolevel data taken from cast iron material properties

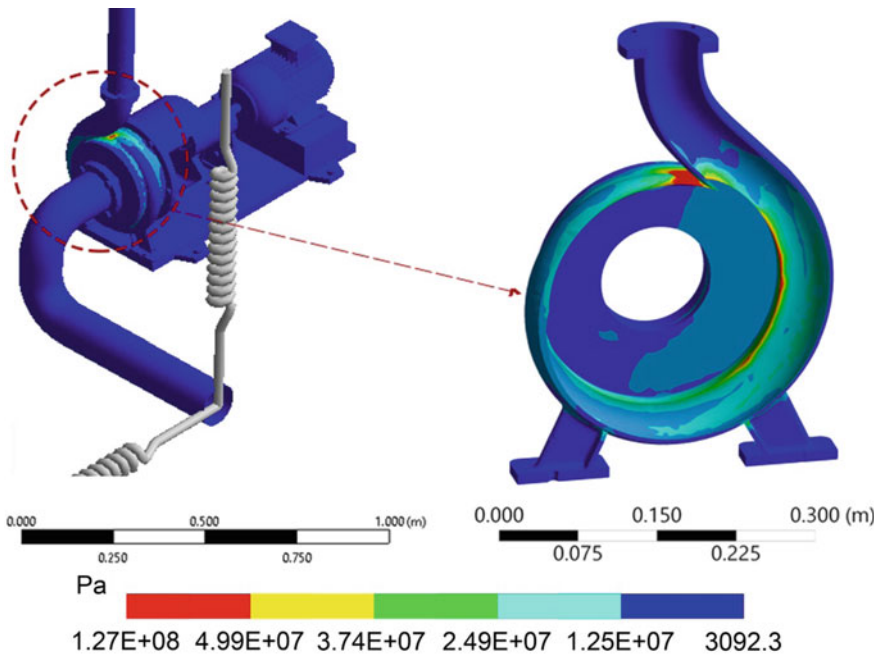
- surface adjacent to the floor—rigid fixation;
- volumetric force—gravity (acceleration of gravity  $g = 9.8 \text{ m/s}^2$ );
- the tightening torque of the bolts in the absence of passport data takes according to Table 17.2;

**Table 17.2** Standard forces of tightening

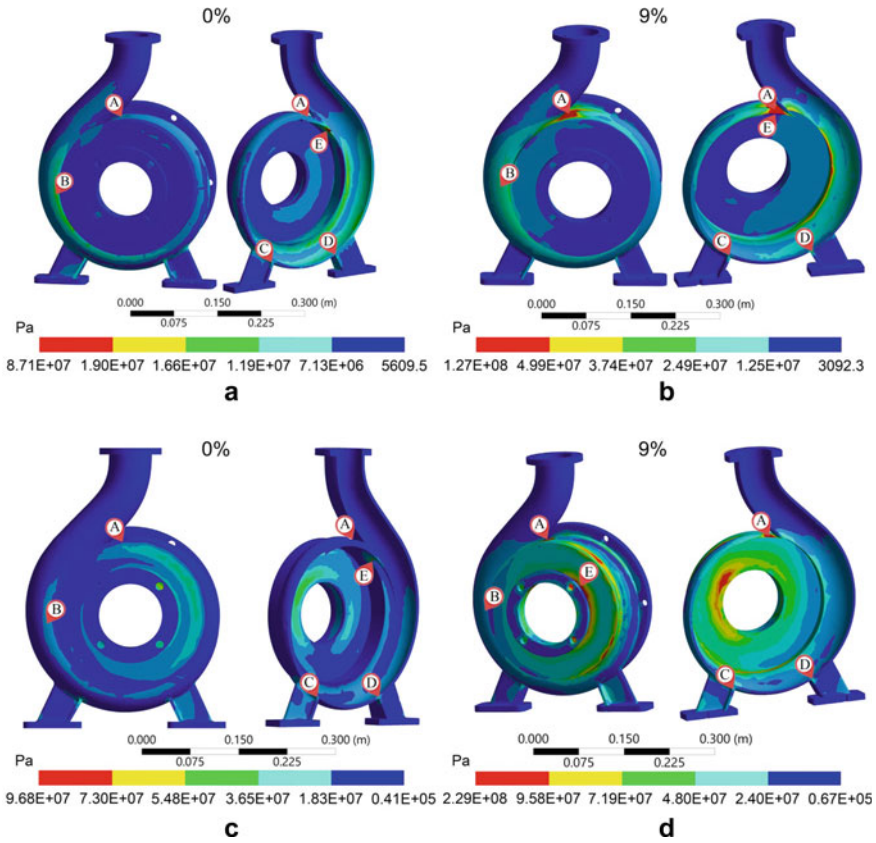
Standard size	Preload force, kN	Standard size	Preload force, kN
M8	3.17	M14	10.10
M12	7.38	M16	20.90

- inlet and outlet nozzles are connected to fragments of pipelines, which are modeled to the nearest support. A rigid fixation is set in the axial direction for the working fluid supply pipe and its outlet pipe, and elastic supports with a rigidity of 0.1 N/m are placed in the plane perpendicular to the axis of these pipes;
- internal pressure during normal operation  $p = 0.2452$  MPa is set in the volume and the outlet pipe; under the condition of hydro testing, the pressure is increasing to 1.5 times ( $p = 0.3678$  MPa) and is set in water pump elbow, in the inlet, and outlet pipes.

According to the result in Fig. 17.4, the maximum stresses occur in the water elbow part of the pump. Therefore, in this part of the model, it is advisable to assess the probability of plastic deformations. The von Mises equivalent stresses are shown in Fig. 17.5 under NOC and during HT mode for the nominal model and at 9% of the elbow walls thinning. According to the results of the calculation (Fig. 17.5a–c), the strength condition is satisfied (the maximum stress value is less than the limit value



**Fig. 17.4** Distribution of equivalent von Mises stresses (Pa), under NOC, at 9% wall thinning



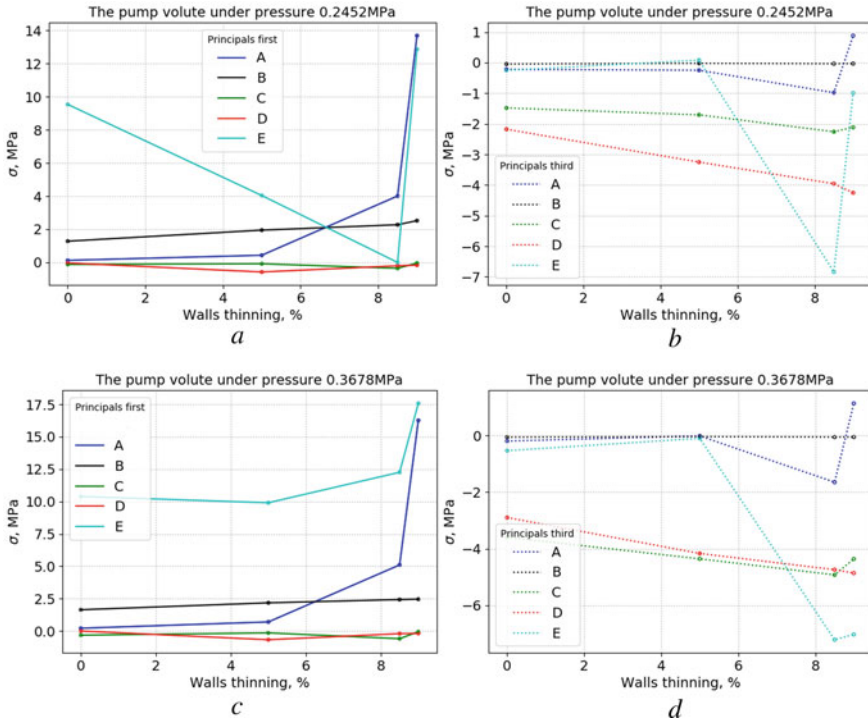
**Fig. 17.5** Von Mises equivalent stress (Pa): **a** under NOC at 0% walls thinning; **b** under NOC at 9% walls thinning; **c** under HT mode at 0% walls thinning; **d** under HT mode at 9% walls thinning

$[\sigma] = 135 \text{ MPa}$ ). During HT mode (Fig. 17.5d) with wall thinning up to 9% in the model arise stresses exceeding the permissible ones (the maximum stress value  $[\sigma] = 229 \text{ MPa}$ ), which indicates the occurrence of plastic deformations of the model at the macrolevel. This is unacceptable according to the standards for this type of equipment (IEC 60041:1991; ISO 9906:2012).

For the analysis of the water elbow part of the pump, five control points are selected, which corresponds: *A*—the upper part of the housing near the outlet; *B*—the outer part of the elbow, which passes into the outlet; *C*, *D*—points of contact water pump elbow with supports; *E*—point with maximum stress in the model.

The dependence of the principal's stress on the water pump elbow walls thinning during the normal operation condition and the hydro test mode in control points are shown in Fig. 17.6.





**Fig. 17.6** The dependence of the principals stress on the elbow walls thinning: **a** the first principals stress under NOC; **b** the third principals stress under NOC; **c** the first principals stress under HT mode; **d** the third principals stress under HT mode

### 17.4 Application of Data-Driven Yield Surface to Prediction of Failure Probability for Centrifugal Pump

The proposed technology for studying the occurrence of plastic deformations involves several stages. At the first step, an artificial microstructure of a statistically equivalent material is created for the analysis of the stress–strain state. Artificial microstructure generation is implemented by establishing the dependence between the size and concentration of inclusions (Shapovalova and Vodka 2019a, b, 2020). The information about the quantity and size of inclusions located on a plane is collecting by using computer vision technology. The mathematical expectation data  $M[R]$  and the variance  $D[R]$  of the radii inclusions dependence on the concentration have been obtained.

The location is followed by a uniform distribution and the size of inclusions is followed to a normal distribution function of concentration. Concentration ( $\psi$ ) is defined as the ratio of the area of the inclusion to the area of the sample. For this study, the concentration of inclusions is equal to  $\psi = 0.100$ .



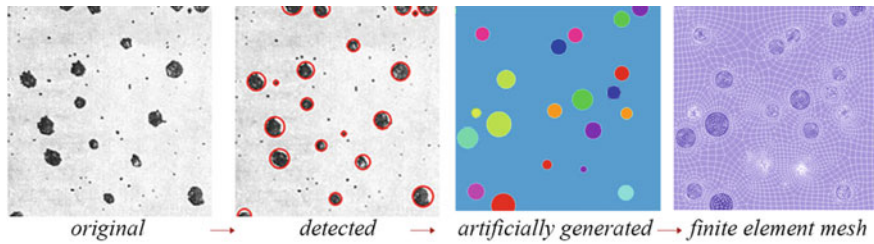


Fig. 17.7 Microstructure research process

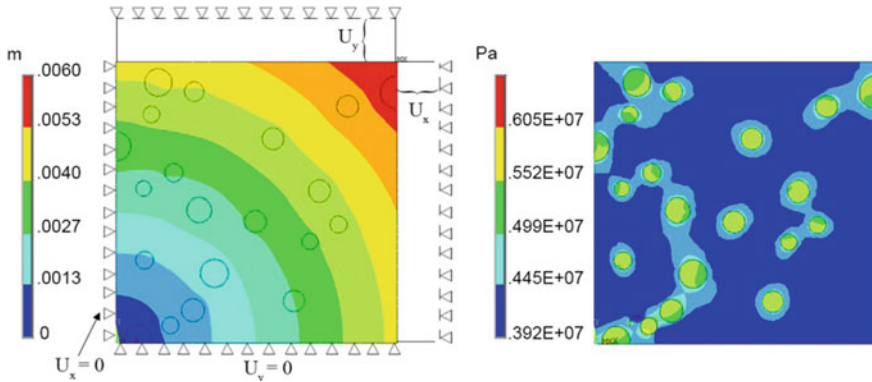
The finite element model construction is based on the artificial generated geometric model of the spheroidal graphite cast iron microstructure. To create the mesh grid, a two-dimensional 8-node finite element with two degrees of freedom in each node is used (Zienkiewicz 1971). For calculation, it is assumed that the main matrix of the investigate sample is isotropic ferrite and the inclusions are an orthotropic graphite material. The corresponding materials properties and elastic constants are given in Table 17.1. Various material properties and their resistance to tension and compression are taken into account. An example of the initial image of the nodular cast iron microstructure, the recognized inclusions, the artificial statistically equivalent generated microstructure, and the mesh of the model are shown in Fig. 17.7.

The next step for the probability of pump failure investigation is the yield surface calculation. One of the tasks of materials engineering is to establish the loading conditions that cause plastic deformation. This is important to determine the load combination which leads to a transition from the elastic to the plastic. In the case of uniaxial loading, this task is not particularly difficult. It is enough to have a relation between stress and strain. Such data can be obtained from experiments on simple tension and compression. However, for materials that are in multi-dimensional stress state conditions, plasticity predicting requires additional information. In the case of a three-dimensional stress state, determining the yield surface is a difficult task. This is due to several technical difficulties caused on the one hand by the complexity of the experimental environment, and on the other hand, by the huge number of samples that need to be tested. This problem is especially acute for composite and heterogeneous materials. To solve this problem, computer simulation methods are used.

In this work to construct the yield surface, the model is considered under different loadings. One of the typical load cases for concentration  $\psi = 0.100$  is shown in Fig. 17.8. The model is represented by a square plate with a side— $l$ . The deformation is set equal to  $\varepsilon_\rho = \Delta/l = 10^{-5}$ , then the displacement is calculated by (2):

$$\begin{aligned} U_x &= \varepsilon_\rho l \cos \Theta \\ U_y &= \varepsilon_\rho l \sin \Theta \end{aligned} \quad (17.2)$$

where  $U_x, U_y$ —displacement along the corresponding axis,  $\Theta = (0 \dots 360)^\circ$  the angle changes in a range, with a step in  $3.6^\circ$ .



**Fig. 17.8** Displacement ( $m$ ) and von Mises equivalent stress (Pa) for the microstructure model,  $\psi = 0.100$

Computer simulation methods are used to calculate the yield surface in a multidimensional stress state. This approach uses the hypothesis of yield strength under difficult loading conditions (Larin et al. 2018; Wu et al. 2020). Finding the yield surface is based on the hypothesis of the maximum distortion energy theory (the Huber-von Mises-Hencky hypothesis), (Ambartsumian 1967). According to it, plastic strains of a sample in a complex stress state occurs when the specific formation energy becomes equal to or exceeds the specific formation energy of the material under the action of a uniaxial stress state.

For statistical equivalent artificial generate microstructure which is consists of two types of materials (ferrite and graphite), the maximum stresses are found. For graphite, the tensile and compressive strengths differ significantly, therefore, separately for each type of stress state, the ratios maximum stresses to the corresponding allowable tensile strength are found. The yield surface is determined by the ratio of the principal stresses to the safety factor (Shapovalova and Vodka 2020). The calculation results of 250 random typical implementations of the yield surface are presented graphically in Fig. 17.9.

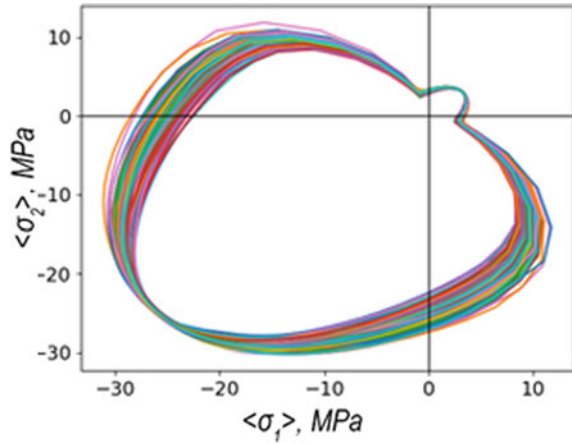
The accumulated statistical information on possible yield surface variants helps to determine the area of stress impact. The construction of a line passing through the origin of the coordinates with control points along it is used (Fig. 17.10). The theta angles that correspond to the loading trajectory are calculated according to (3).

$$\operatorname{tg} \Theta = \frac{\sigma_2}{\sigma_1} \quad (17.3)$$

where  $\sigma_1$  and  $\sigma_2$ —the principal stresses.

Information about the number of the yield surfaces that have fallen into the control points along the line is obtained. This method allowing to define the inverse cumulative distribution function, which in turn determines the parameters of descriptive

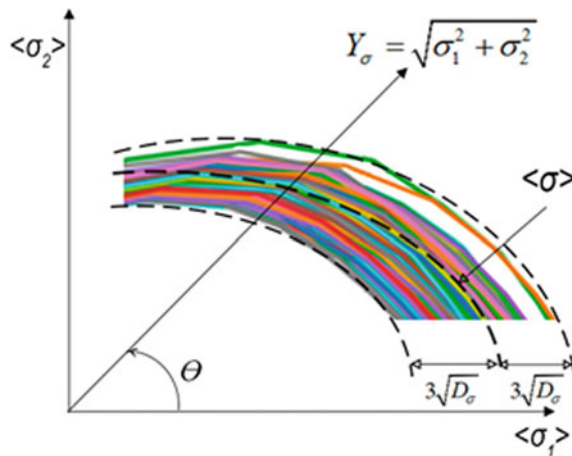
**Fig. 17.9** Typical implementations of the yield surface arrangement for  $\psi = 0.100$

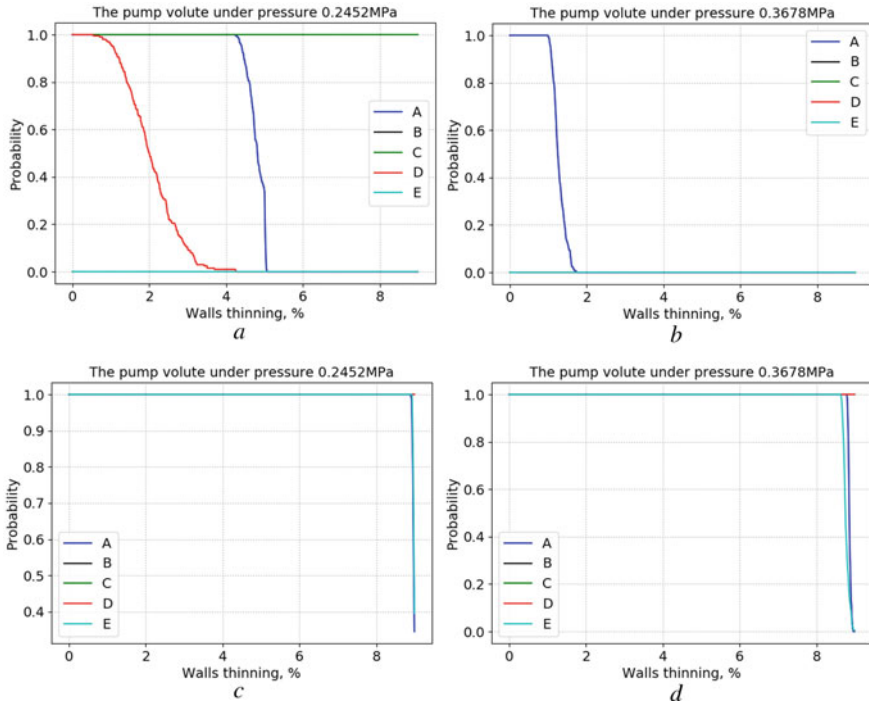


statistics as mathematical expectation (mean), and variance of the random radius function.

In the third step of investigating pump failure-free operation probability, the micro material analysis technology is applied. The material properties for the generated microstructure from Table 17.1 are used, the finite element method is applied for the model with different modes of operation. The probability of failure-free operation in five control points of the water elbow part of the pump depending on the walls thinning under NOC and HT modes are shown in Fig. 17.11. The probability of microplastic deformations at the control points increases even when the walls are thinned by 1%, and tend to 1 already at 4% thinning (Fig. 17.11, a). Which corresponds approximately to 8 years of normal operation. During hydro testing, the probability of microplastic deformations increases to 1 even at thinning close to 2% (Fig. 17.11b), which corresponds to 3–4 years of equipment operation. The

**Fig. 17.10** Bounds on the yield surface stress



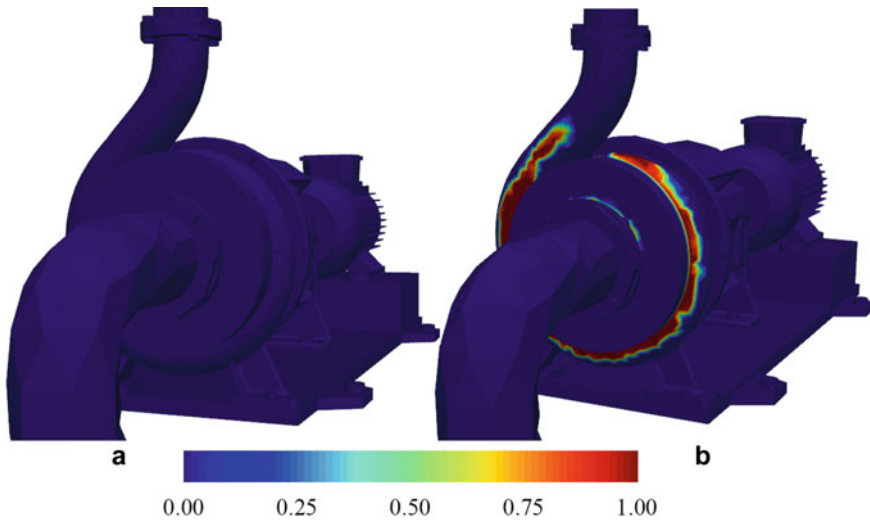


**Fig. 17.11** The probability of failure-free operation of the pump depending on the elbow walls thinning: **a** for NOC at the micro-level; **b** for HT mode at the micro-level; **c** for NOC at the macrolevel; **d** for HT mode at the macrolevel

occurrence of plastic deformations at the micro-level can lead to the development of cracks and structural failure at the macro level. Consequently, such areas require careful study and control over the entire life of the equipment.

The criterion for strength at the macrolevel is the yield stress  $\sigma_{0.2}$ , which already implies 0.2% plastic deformation. The score for an artificially generated structure assumes 0% plastic deformation, and it's much less. But in practice, the  $\sigma_{0.2}$  criterion is used. According to Table 17.1, the passport data to comparing the strength criterion are taken for a similar material by which an equivalent structure is created (spheroidal cast iron 35). Therefore, a transition coefficient is introduced, which corresponds to the expansion of the yield surface curve at the macro level without changing its shape. The results for the control points at NOC and under HT mode are shown in (Fig. 17.11c, d). The results of macroplastic deformation probability in the model occur when the wall thinning is close to 9%, which corresponds to 17–18 years of equipment operation.

Visualization of the entire model plastic deformation probability implemented by using the pyansys (Kaszynski 2020) library (Fig. 17.12). For the case of normal operating conditions with 0% wall thinning, the probability at the macro level and the



**Fig. 17.12** The probability of plastic deformation in the pump under NOC with 0% walls thinning, visualized by pyansys library (Kaszynski 2020). **a** Probability of macroplastic deformation; **b** probability of microplastic deformation

micro-level is different. This corresponds to the onset of microplastic deformations, which do not significantly affect the macrolevel.

## 17.5 Conclusions

The paper discusses steps at studying the probability of the centrifugal pump failure-free operation with data-driven yield surface application. The construction of a geometric model of the WD 16/25 pump was carried out taking into account the uniform thinning of the body walls under the influence of corrosive effects. Distributions of equivalent stresses in the pump construction elements under normal operating conditions and hydro-testing mode are obtained. To the most critical pump unit (the elbow part of the pump) applying previously obtained material properties. The investigation of the elbow part of a centrifugal pump is based on the microstructure estimation of the material for obtaining information about the state and predicting the probabilistic characteristics. A set of yield surfaces is determined from an artificially generated statistically equivalent material structure using the finite element method. Takes into account the different behavior of composite materials under tensile and compressive loads. Going beyond the yield surface indicates the possibility of transition into a plastic state. Understanding the processes in the structure at the micro-level allows one to predict possible macro destruction of the model in advance. The results of the work show the probability of failure-free operation in the time of the pump during normal operation and hydro test mode.

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