Optimal Resource Allocation for Coverage Control of City Crimes

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1 Introduction

As the complexity and challenge of the world grow, drug abuse, large population, shifting demographics, and advanced technologies bring more complicated criminal situations. One of the core responsibilities of governments is to protect citizens from crimes. Current law enforcement strategies can deal with a large scale of crimes. The decline of crimes in the United States is a positive sign for the safety of all citizens and this decline is credited to effective law enforcement strategies (Morris, 1997). However, due to the changing scale and nature of criminal activities, there would be insufficient people and support to scale up previous efforts achieved by traditional approaches (Hipp & Kane, 2017). As every second counts in the crisis, limited law-enforcement resources pose significant challenges to effectively and efficiently protect the city. Therefore, current situations of crimes call for new strategies of the law enforcement allocation to achieve faster responses, reduced costs, and highly efficient operations.

In the United States, crimes can be classified into two main categories, violent crimes and property crimes. Violent crimes consist of assault, robbery, rape, and murder (Freilich & Pridemore, 2007). Property crimes consist of burglary and theft. The crime rates corresponding to various types of crimes from 1981 to 2018 in

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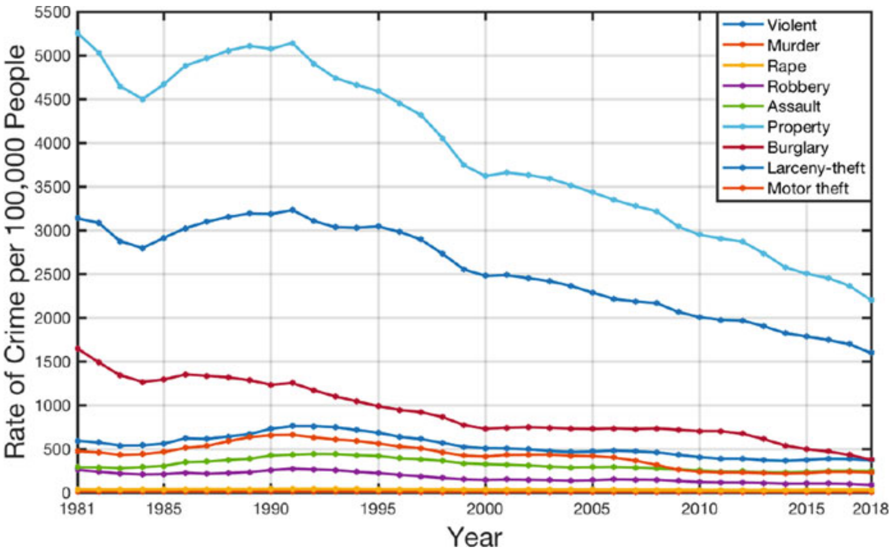

Fig. 1 Crimes in the United States from 1981 to 2018

the United States are shown in Fig. 1. The estimated number of violent crimes was 1,206,836 in 2018. Aggravated assault accounted for roughly 67% of the total violence. The estimated number of robberies was 282,061 and firearms were used in 38.5% of all robberies. The rough number of murders was 16,214 and firearms were used in 72.68% of murders. The estimated number of property crimes was 7,196,045 and resulted in losses of around $16.4 billion. The larceny-theft accounted for 72.5% of the property crimes (U. S. Department of Justice Federal Bureau of Investigation, 2018). Law enforcement officers call for better strategies that can help control and reduce crimes in the United States.

As the fourth largest city of Pennsylvania, Erie has an area of 19.37 square miles and a population of 101,786 people. The crime rate of Erie is relatively low compared to other large cities in New York, Ohio, and Pennsylvania. However, as shown in Fig. 2, Erie's crime rate is higher when compared to the national average, which is mainly because of economic factors that plague the city. Another reason for the high crime rate is the limited law-enforcement resources. In 2003, the police force had 214 officers. That number decreased to 161 officers in 2016. This resulted in Erie reaching its highest crime rate in 2008. Even in 2018, the crime rate in Erie was still higher than rates in 70.1% of U.S. cities (Crime rate in Erie, Pennsylvania (PA), 2020).

In this paper, we develop a new optimal learning algorithm to characterize multi-scale distributions of crimes and then determine an optimal policy for coverage control of city crimes. First, we categorize crimes into low, medium and high severity levels. Then, we characterize and model crime distributions for various severity levels. Second, we develop an optimal policy for coverage control to allocate limited resources of the law enforcement in areas of interest. Third, the model performance is measured based on average response time of an agent to

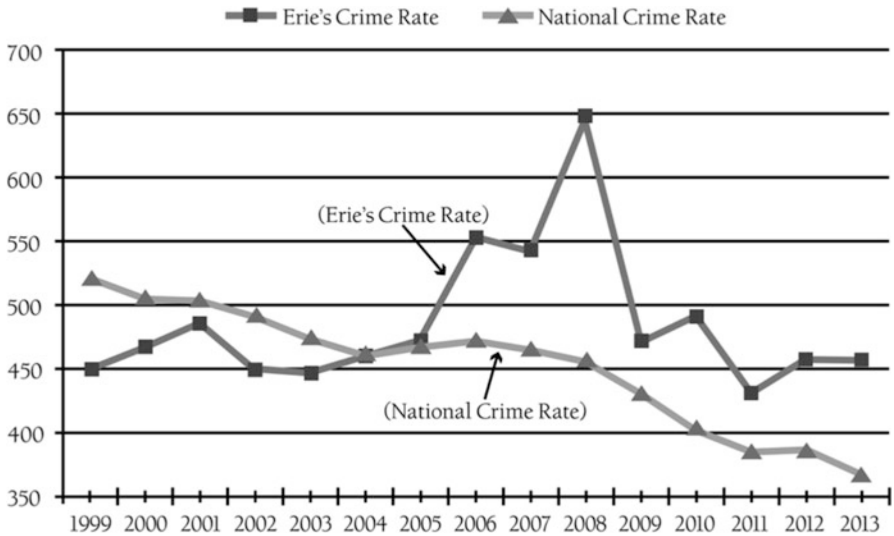

Fig. 2 Erie crimes vs. national average (Jefferson Educational Society, 2015)

reach crime scenes. Experimental results demonstrate that the proposed algorithm is effective and efficient in optimizing resource allocation for the coverage control of city crimes and shows a better performance in terms of average response time to crime scenes than zone-random patrol and uniform allocation. The rest of this paper is organized as follows: Section 2 introduces the state of the art in optimal resource allocation for city crimes. Section 3 presents the proposed methodology of optimal resource allocation for coverage control. Section 4 shows the experimental design and results, and Sect. 5 includes the conclusions arising from this investigation.

2 Research Background

Data analytics are widely studied by law enforcement officials and criminal investigators to analyze crime data. It is a challenging task to discover and understand the complex patterns of crimes from multiple perspectives. Risk Terrain Modeling (RTM) is one of the techniques to localize places where there is a high probability that a crime will occur. Also, RTM is able to identify risk factors of criminal events (Kocher & Leitner, 2015). A fuzzy association rule mining is developed by Buczak (Buczak & Uhrig, 1996) to investigate the underlying community crime pattern. As the crime data get big and complex, manual and visual analytics are shown to be limited in the ability to handle a large volume of data. A space-time and multivariate visualization system (VIS-STAMP) is then developed to identify hidden patterns of aggravated assault, robbery, burglary, etc. This approach is implemented to analyze the crime data of Philadelphia, Pennsylvania by Guo and Wu (Guo, 2009). Clustering techniques are commonly utilized to extract the crime pattern. Malleson et al. identify the area with significant crime rates by exploring crime clusters (Malleson & Andresen, 2016). Different features are given various weights in order to improve the accuracy of the model and remove outliers. The clustering model is leveraged to investigate how frequently crimes occur at different times. A weighting scheme is also developed to handle limitations of clustering techniques. Phillip et al. integrated crime data analytics along with socio-economic and socio-demographic factors to discover patterns that may contribute to the development of future crime occurrences (Phillips & Lee, 2012).

Crimes are distributed over a spatial region of interests (e.g., Erie, PA) and evolve over time. In order to predict the trend of crimes, it is necessary to investigate the spatiotemporal variations of crime patterns. In other words, how crime pattern changes in the region of interests? and what are the variations of crime data over different periods of time? Spatiotemporal modeling is conducive to better predict the occurrence of crimes. Although very little has been done to investigate spatiotemporal variations of crimes, spatiotemporal modeling has been developed and applied in many other disciplines, e.g., spatiotemporal modeling of electrical potentials on the body surface (Yao et al., 2018) and in the heart (Yang et al., 2013), and spatiotemporal modeling of service accessibility over a large geographic area (i.e., Georgia State, USA) for 13 years (Serban, 2011). The cluster confidence rate boosting has been utilized to identify the hierarchical structure of spatiotemporal patterns (Yu et al., 2016). Kotevska et al. have developed a model for temporal trends with multivariate spatiotemporal data streams to improve the performance of prediction (Kotevska et al., 2017).

In spite of rapid advancements in crime data analytics, very little has been done to utilize crime data for optimal allocation of law enforcement resources. As the complexity of crimes grows, current practice of law enforcement, which is simply increasing the number of resources to cover and control crimes, may not be effective. There is an urgent need to effectively and efficiently control crimes through optimal allocation of limited resources.

3 Research Methodology

This paper presents a new strategy for optimal allocation of law enforcement resources towards coverage control of city crimes. We develop an optimal learning algorithm to characterize multi-scale distributions of crimes and determine optimal coverage control of city crimes. First, we characterize and model distributions of city crimes within the area of interest. Second, the area of interest is partitioned into regions which optimally cover nonuniformly distributed crimes. Third, an optimal learning algorithm is developed to allow agents to asymptotically converge to centroids of corresponding regions.

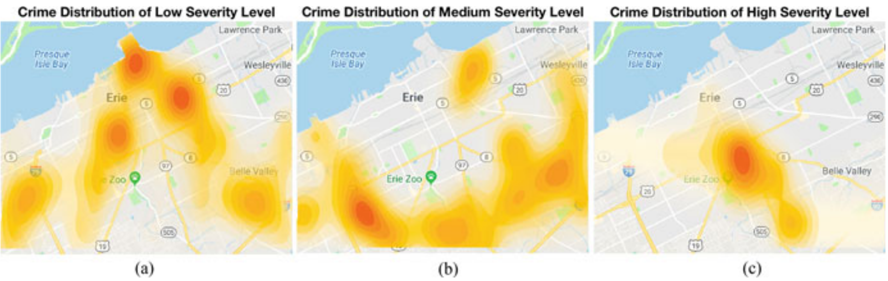

Fig. 3 Crime distribution: (**a**) Low severity; (**b**) Medium severity; (**c**) High severity

3.1 *Crime Distribution Modeling*

Crimes are categorized into three severity levels, namely low, medium, and high. The low severity level includes crimes such as theft, abandoned vehicle and burglary without force. Crimes of the medium severity level include drunkenness, disorderly conduct, burglary with force, etc. The high severity level involves missing person, assault, aggravated assault by prisoner, etc. Distributions of crimes at three severity levels are modeled to demonstrate the coverage of crimes in Erie city. Figure 3a shows the distribution of crimes at low severity level. The distribution of medium severity level is shown in sub-figure (b). Figure 3c demonstrates the distribution of crimes with high severity. The density of crimes varies from low to high while the color changes from light to dark. The proposed algorithm is implemented to cover and control crimes based on crime distributions with constrained resources.

3.2 *Optimal Coverage Control*

Let Ω be a convex with information density following Gaussian distribution as shown in Fig. 4a. If we only have one law enforcement agent to control the area Ω, intuitively, we will place the agent at the location with the highest density of crimes to minimize the information loss. The information loss for infinite points on the space Ω is formulated as

$$C(\theta_i) = \int_{\Omega} d\left(\|s - \theta_i\|\right) \sigma(s) ds \tag{1}$$

where $d(\|s - \theta_i\|)$ is a distance function between the agent θ_i and the location s on space Ω, $\sigma(s)$ is the information density at the location s. In this paper, the information density is referred to as the density of crimes at location s. The information loss increases along with the increment of function $d(\|s - \theta_i\|)$ and the information density $\sigma(s)$. Further, if we possess n agents and a complex distribution

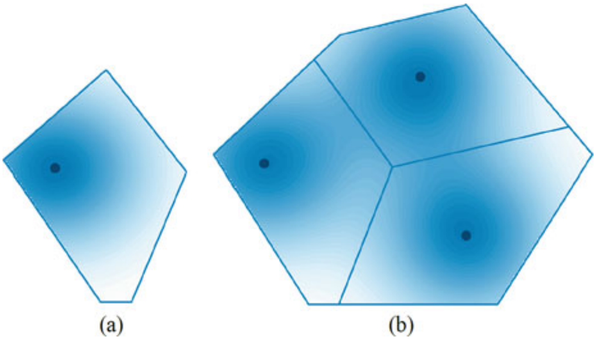

Fig. 4 Polygons with information densities following: (**a**) Gaussian distribution; (**b**) Complex distribution

on space Ω as shown in Fig. 4b, then we need to divide the space Ω into n regions and place one agent in each region. The objective is to minimize the total information loss through determining optimal tessellation and locations of agents, which is formulated as

$$C(\boldsymbol{\Theta}, \boldsymbol{\omega}) = \sum_{i=1}^{n} \int_{\omega_i} d(\|s - \theta_i\|)\, \sigma(s) ds \tag{2}$$

where $\boldsymbol{\Theta}$ is the agent set and $\boldsymbol{\Theta} = \{\theta_1, \theta_2, \ldots, \theta_n\}$, ω_i represents the region of space Ω, and $\boldsymbol{\omega} = \{\omega_1, \omega_2, \ldots, \omega_n\}$ is the tessellation, and n is the total number of agents.

Given a space Ω, Voronoi region V_i of agent θ_i is defined as

$$V_i = \left\{ s \in \Omega : \|s - \theta_i\| \le \left\| s - \theta_j \right\|, \forall j \ne i \right\} \tag{3}$$

where the set $\boldsymbol{V} = \{V_1, V_2, \ldots, V_n\}$ is the Voronoi tessellation of space Ω if $V_i \cap V_j = \varnothing$ for $i \ne j$. Each V_i in the set is referred to as Voronoi region corresponding to agent θ_i. Voronoi tessellation has different definitions using different distance functions. In this paper, we use the Euclidean norm to define Voronoi tessellation. The definition of Voronoi region ensures that all crime locations s within region V_i are the closest to agent θ_i. Therefore, the optimal tessellation for space Ω is the Voronoi tessellation.

Tessellation $\boldsymbol{\omega}$ in the objective function is then replaced with the Voronoi tessellation $\boldsymbol{V}$, and formulated as

$$C(\boldsymbol{\Theta}, \boldsymbol{V}) = \sum_{i=1}^{n} \int_{V_i} d(\|s - \theta_i\|)\, \sigma(s) ds \tag{4}$$

We compute the polar moment of inertia about the agent location as

$$J_{\theta_i} = \int_{V_i} \|s - \theta_i\|^2 \sigma(s) ds \tag{5}$$

If we set the function $d(\|s-\theta_i\|)=\|s-\theta_i\|^2$, then the objective function can be reformulated as

$$C\left(\boldsymbol{\Theta}, \boldsymbol{V}\right)=\sum_{i=1}^{n}\int_{V_i}\|s-\theta_i\|^2\sigma(s)ds=\sum_{i=1}^{n}J_{\theta_i} \tag{6}$$

The mass and centroid of mass of each Voronoi region are formulated as

$$m\left(V_i\right)=\int_{V_i}\sigma(s)ds \tag{7}$$

$$c\left(V_i\right)=\frac{1}{m\left(V_i\right)}\int_{V_i}s\cdot\sigma(s)ds \tag{8}$$

We substitute J_{θ_i} in the objective function based on the parallel axis theorem and obtain

$$C\left(\boldsymbol{\Theta}, \boldsymbol{V}\right)=\sum_{i=1}^{n}J_{c(V_i)}+\sum_{i=1}^{n}m\left(V_i\right)\|\theta_i-c\left(V_i\right)\|^2 \tag{9}$$

From Eq. (9), it may be noted that the objective function is minimized when agent θ_i is located at $c(V_i)$. Therefore, optimal locations of law enforcement agents are determined as centroids of Voronoi regions.

3.3 Asymptotic Convergence of Optimal Allocation

We propose an optimal learning algorithm for agents to search for optimal locations. There are other existing methods, such as sequential sampling and Lloyd algorithm. However, sequential sampling algorithm is computationally expensive for a large space Ω (Du et al., 1999). Lloyd algorithm is a special case of the gradient flow algorithm with a step size equals to 1 (Du et al., 2006). The large step size poses the drawback that agent locations may never reach a steady state. The agent's movement follows

$$\theta_i\left(t+1\right)=\theta_i(t)-\alpha\cdot\frac{\partial C}{\partial\theta_i} \tag{10}$$

where $\frac{\partial C}{\partial\theta_i}=2m\left(V_i\right)\left(\theta_i-c\left(V_i\right)\right)$. Here, we assume the agent's movement obeys the first order dynamical behavior

$$\dot{\theta}_i=-\alpha_{step}\left(\theta_i-c\left(V_i\right)\right) \tag{11}$$

where α_{step} is the step size. Thus, agents converge asymptotically to their optimal locations by

$$\theta_i\,(t+1) = \theta_i\,(t) - \alpha_{step}\,(\theta_i\,(t) - c\,(V_i)) \tag{12}$$

4 Experimental Design and Results

We design a three-way layout experiment to test the performance of proposed algorithm as shown in Fig. 5. First, crimes are categorized into low, medium, and high severity levels and their crime distributions are modeled. Second, limited resources of the law enforcement are allocated with the proposed algorithm of optimal coverage control which includes settings of 8, 12, and 20 agents. Third, to evaluate the effectiveness and efficiency of the proposed algorithm, we measure the performance based on the response time of an agent to reach crime scenes and benchmark with both zone-random patrol and uniform allocation of law enforcement.

4.1 Optimal Allocation of Law Enforcement Agents

The proposed algorithm for optimal coverage control of city crimes is implemented for the three severity levels to optimally allocate law enforcement resources. We consider various settings of law enforcement agents, including 8-agent, 12-agent, and 20-agent allocations. Here, 12-agent allocation is utilized to demonstrate performance of the proposed algorithm.

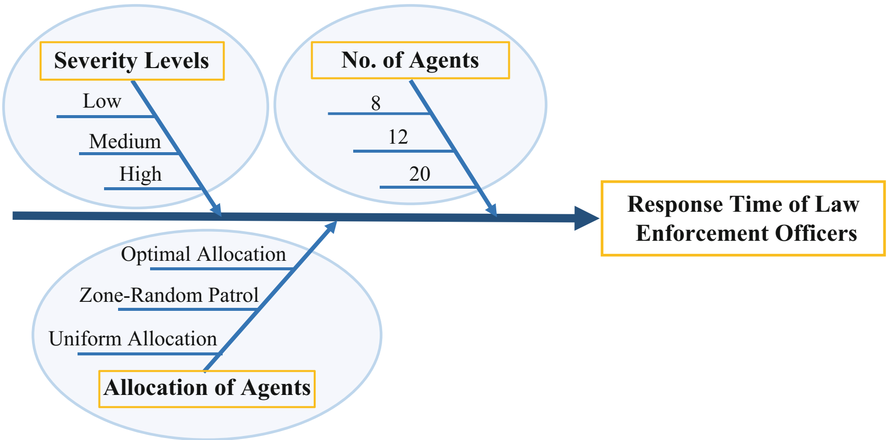

Fig. 5 The design of experiment for performance evaluation

Fig. 6 Optimal allocation of 12 agents for the low severity level. (**a**) Optimal locations; (**b**) Routes from random starting locations to the optimal locations; (**c**) Crime distribution of low severity level

Fig. 7 Optimal allocation of 12 agents for the medium severity level. (**a**) Optimal locations; (**b**) Routes from random starting locations to the optimal locations; (**c**) Crime distribution of medium severity level

Fig. 8 Optimal allocation of 12 agents for the high severity level. (**a**) Optimal locations; (**b**) Routes from random starting locations to the optimal locations; (**c**) Crime distribution of high severity level

For crimes of the low severity level, Fig. 6a shows optimal locations of 12 agents. In Fig. 6b, routes of 12 agents from random starting locations (black dots) to optimal locations (larger red dots) are shown with black lines. Figure 7a b demonstrate the optimal allocation and routes of 12 agents for crimes at the medium severity level, respectively. The optimal allocation of 12 agents to cover and control high-severity crimes is demonstrated in Fig. 8a. Routes of 12 law enforcement agents to optimal locations are shown in Fig. 8b. Notably, optimal allocation of law enforcement agents varies depending on the distribution of crimes. Under a certain crime distribution as shown in sub-figure (c) of Figs. 6, 7 and 8, agents are likely to move towards the area with high crime density.

4.2 Benchmark with Zone-Random Patrol and Uniform Allocation

With the proposed algorithm for optimal coverage control of city crimes, law enforcement agents can provide an effective and efficient service to citizens. By positioning agents at optimal locations, the proposed algorithm can shorten response times from agents to crime scenes. Response time of an agent to reach crimes within its corresponding region is a key metric to quantify the performance of the proposed algorithm, which is formulated as

$$T_R = \frac{\sum_{i=1}^{n} \sum_{\gamma}^{m_i} \left\| c_\gamma - \theta_i \right\| \cdot t_R}{n} \tag{13}$$

where c_γ represents the crime, m_i is the total number of crimes in i th region, t_R is the time it takes for a law enforcement agent to travel a unit distance, T_R is the response time of one agent to reach all crimes within its corresponding region.

The performance of optimal resource allocation is benchmarked with zone-random patrol and uniform allocation of limited resources. We use zone-random patrol to benchmark because it is very common in existing patrols. In the zone-random patrol, Erie area is divided into 2 zones as shown in Fig. 9b and half of the agents are randomly allocated in each zone. All agents are uniformly allocated in the uniform allocation. We implement three strategies for various settings of law enforcement resources at all three severity levels and summarize response-time performances in Table 1. As shown in the table, response times of agents from optimal locations to crime scenes are much shorter than from zone-random patrol and uniform allocation. Specifically, law enforcement agents will give faster

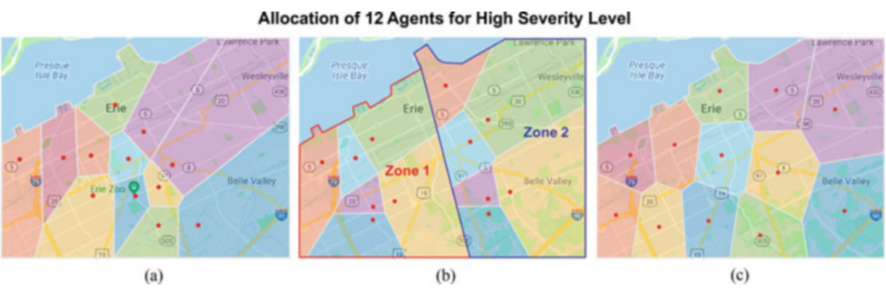

Fig. 9 Allocation of 12 agents for high severity level: (**a**) Optimal allocation; (**b**) Zone-random patrol; (**c**) Uniform allocation

Table 1 Response-time performances of optimal allocation, zone-random patrol and uniform allocation

	8-agent Setting			12-agent Setting			20-agent Setting		
Severity Level	Optimal	Zone-random Patrol	Uniform	Optimal	Zone-random Patrol	Uniform	Optimal	Zone-random Patrol	Uniform
Low	30.45	36.64	43.98	26.73	31.64	34.92	19.06	22.76	24.43
Medium	16.03	22.78	27.14	10.95	18.36	21.21	9.53	10.68	10.98
High	3.53	4.37	4.95	2.62	3.33	4.39	1.94	3.16	3.31

responses to crimes with the proposed algorithm. Further, the response time of an individual agent is shorter if we increase the number of law-enforcement agents.

5 Conclusions

As the scale and nature of crimes become more and more complex, resources of law enforcement become insufficient. Protecting citizens through effective and efficient strategies therefore becomes a challenging task, especially when the law enforcement resource is constrained. In this paper, we develop a new optimal learning algorithm to characterize multi-scale distributions of crimes and then determine an optimal policy for coverage control of city crimes. First, we categorize crimes into low, medium, and high severity levels. Then we characterize and model crime distributions for various severity levels. Second, we develop an optimal policy for coverage control to allocate limited resources of the law enforcement in the areas of interest. Third, the model performance is measured based on the response time of an agent to reach crime scenes. We evaluate and benchmark the performance of the optimal policy of coverage control with zone-random patrol and uniform allocation. Experimental results show that the proposed algorithm is effective and efficient in optimizing the allocation of limited law enforcement resources and demonstrate a better performance than zone-random patrol and uniform allocation of limited resources.

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