OMA: From Research to Engineering Applications

Salvatore Russotto, Alberto Di Matteo, Chiara Masnata, and Antonina Pirrotta

Abstract Ambient vibration modal identification, also known as Operational Modal Analysis (OMA), aims to identify the modal properties of a structure based on vibration data collected when the structure is under its operating conditions, i.e., when there is no initial excitation or known artificial excitation. This method for testing and/or monitoring historical buildings and civil structures, is particularly attractive for civil engineers concerned with the safety of complex historical structures. However, in practice, not only records of external force are missing, but uncertainties are involved to a significant extent. Hence, stochastic mechanics approaches are needed in combination with the identification methods to solve the problem. In this context, this paper's contribution is to introduce an innovative ambient identification method based on the Hilbert Transform to obtain the analytical representation of the system response in terms of the correlation function. This approach opens the pathway for a monitoring system that is user friendly and can be used by people who have little to no knowledge of signal processing and stochastic analysis such as those who are responsible for the maintenance of a city's historical buildings. In particular, this method operates in time domain only. Specifically, firstly the correlation functions matrix $\mathbf{R}_X(\tau)$ is determined based on the recorded time domain data. Next, performing a Singular Value Decomposition (SVD) on $\mathbf{R}_X(\tau)$ for $\tau = 0$ leads to an estimate of the modal matrix $\mathbf{\Phi}$ containing all the modal shapes. In this manner, once $\mathbf{\Phi}$ is known, the entire correlation functions matrix in modal space $\mathbf{R}_Y(\tau)$ is recovered. Further, the analytical signals of the auto-correlation functions in modal space are determined performing the sum of each auto-correlation function with its Hilbert transform. Moreover, since the analytical signal can be expressed in terms of amplitude and phase, then frequencies and damping ratios estimation is possible. Finally, in order to prove the reliability of the method several numerical examples and an experimental test are reported.

Keywords Operational modal analysis · Structural identification · Correlation function · Hilbert transform · Analytical signal

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1 Introduction

A fundamental step for the *Structural Health Monitoring* (SHM) is the structural identification that, from a dynamic point of view, coalesces with the identification of natural frequencies, damping ratios and modal shapes.

In literature there are a lot of dynamic identification methods that can be subdivided into two main classes: *Experimental Modal Analysis* methods (EMA) and *Operational Modal Analysis* methods (OMA) [1]. The first one can identify the nonlinear behavior of a structure but requires the knowledge of the structural input. This aspect is penalizing because the artificial generation of the structural input is complicated and very expensive in the in situ tests. On the other hand, the set-up of the OMA methods is very cheap and simple because these methods don't require the knowledge, and thus the artificial generation of the structural input. Furthermore, since the structural input in the OMA method is an ambient noise due to traffic, wind, ground vibrations, use of the structure and so on, it is possible to identify the dynamic properties in the real operative conditions and the structural input is moldable as a white noise. For these advantages, in the last decades, the researchers focused their attention on the OMA methods.

Another classification of the identification methods can be done considering the domain in which they are developed, in fact OMA methods are subdivided in: time domain methods, frequency domain methods and hybrid domain methods [2]. Frequency domain methods in OMA, like *Peak Picking and Half Power Bandwidth Method* (PP+HP) [3] and *Frequency Domain Decomposition* (FDD) [4, 5], are usually based on the *Power Spectral Density* (PSD) estimation that can be performed by using the Welch's method [6, 7]. The estimation of the PSD with the Welch's method allows to have different advantages due to the decomposition of the structural output in sub-signals to which is possible to apply time windows (Tukey, Hamming, Hanning) and to assign an overlap length. However, the use of this method is difficult for a non-expert user and the choice of the sub-signals length, the kind of window to apply to every sub-signal and the overlap length can influence the results especially in terms of damping ratios estimation. Furthermore, if the modes are very close the identification of the dynamic properties might not be very accurate [3]. There are several time domain methods in literature, such as: *Natural Excitation Technique* (NExT) [8] that, when the excitation is an ambient vibration, it requires that the analytical form of free-vibration and the analytical form of the structural output are the same; *Stochastic Subspace Identification* methods (SSI) [2] divided into *covariance-driven* models (SSI-COV) and *data-driven* models (SSI-DATA); *Auto Regressive Moving Average* models (ARMA) [9] that are articulated into *Auto Regressive* (AR) step and *Moving Average* (MA) step. Hybrid domain methods can be developed in time-frequency domain, like methods based on the Wavelet transform [10], or they can be divided into different steps some in the time domain, others in the frequency domain. An example of the latter is the *Analytical Signal Method* (ASM) [11] that profits by the advantages due to the use of the analytical signal i.e. a high sensitivity to the minimum variations of the natural frequency. Due to the aforementioned advantages the analytical signal is also used for the identification of structural damage [12, 13].

In this paper, a new OMA method founded upon stochastic mechanic's principles is proposed. It uses a *Singular Value Decomposition* (SVD) [14] to reproduce the relationship between the correlation functions matrix in the nodal space and correlation functions matrix in the modal space. The proposed method, called *Time Domain Analytical Signal Method* (TD-ASM) for the similarity with the hybrid method ASM, does not involve the difficulties due to the use of Welch's method, in fact it is developed only in time domain. Furthermore, the analytical signals of the correlation functions are used in order to have a high precision in the frequencies identification.

2 Proposed Method: *Time Domain Analytical Signal Method* (TD-ASM)

2.1 *Identification Algorithm for SDOF Systems*

In this section, a novel OMA method is proposed. This method allows to identify natural frequency and damping coefficient of the SDOF structures enforced by ambient vibration. After the acquisition of the output process $X(t)$, its correlation function $R_X(\tau)$ is calculated. The analytical signal of the correlation function can be estimated by summing the correlation function to its Hilbert transform multiplied by the imaginary unit; in fact, the analytical signal is a complex signal in which the real part is the original function and the imaginary part is its Hilbert transform. Finally, the dynamic properties of the structural system can be estimated from the properties of the analytical signal: amplitude $A(\tau)$ and phase $\theta(\tau)$. The different steps of the proposed method for SDOF structures can be resumed in:

(1) Acquisition of the structural output process $X(t)$;
(2) Estimation of the correlation function $R_X(\tau)$;
(3) Reconstruction of the analytical signal $z_X(\tau)$;
(4) Identification of the dynamic properties.

For SDOF structures the proposed method is very similar to the ASM but it is simpler than the latter because the estimation of the PSD with the Welch's method is removed. In particular, the direct estimation of the correlation function allows to overcome the difficulties introduced by the use of the Welch's method for the evaluation of power spectral density; in fact, the choice of the time windows applied to the sub-signals and the overlap length between two successive sub-signals can influence the results, furthermore the use of the Welch's method requires high specialized skills.

In order to introduce the proposed method in details, a linear SDOF shear-type frame with mass m, stiffness k and damping c is used. The dynamic properties to identify are: the natural frequencies of the structure $f = \sqrt{k/m}/(2\pi)$ and the damping ratio $\zeta = c/(4\pi m f)$.

When the signal of the input force is not acquired and the excitation source is due to ambient vibrations, the key hypothesis of OMA is that the structure can be considered as excited by a white noise process, defined as in [15–18], and, consequently, the stochastic differential equation governing the structural motion is

$$\ddot{X}(t) + 2\zeta\omega_0\dot{X}(t) + \omega_0^2 X(t) = W(t) \tag{1}$$

where ω_0 is the circular frequency that is equal to $2\pi f$. Adding the initial condition to the Eq. (1), the structural response process $X(t)$ can be obtained.

The correlation function $R_X(\tau)$ of the output response process $X(t)$ can be estimated as

$$R_X(\tau) = E[X(t)X(t+\tau)] - \mu_X^2 \tag{2}$$

where μ_X is the mean of the process $X(t)$. Since $X(t)$ is a zero-mean process the Eq. (2) coalesces with

$$R_X(\tau) = E[X(t)X(t+\tau)]. \tag{3}$$

The Hilbert transform $\hat{R}_X(\tau)$ of the correlation function $R_X(\tau)$ is, for definition, the convolution of $R_X(\tau)$ with the signal $1/(\pi\,\tau)$, i.e. it is the response to $R_X(\tau)$ of a linear time-invariant filter having impulse response $1/(\pi\,\tau)$. Therefore $\hat{R}_X(\tau)$ can be calculated as

$$\hat{R}_X(\tau) = \frac{1}{\pi}\wp\int_{-\infty}^{\infty}\frac{R_X(\tilde{\tau})}{\tau-\tilde{\tau}}d\tilde{\tau} \tag{4}$$

where $\wp$ is the principal value.

The analytical signal is calculated as

$$z_X(\tau) = R_X(\tau) + i\hat{R}_X(\tau) \tag{5}$$

and, can be written in polar form as

$$z_X(\tau) = A(\tau)e^{i\,\theta(\tau)} \tag{6}$$

where $A(\tau)$ is the amplitude

$$A(\tau) = \sqrt{R_X^2(\tau) + \hat{R}_X^2(\tau)} \tag{7}$$

and $\theta(\tau)$ is the phase

$$\theta(\tau) = \arctan\left[\frac{\mathrm{Im}[z_X(\tau)]}{\mathrm{Re}[z_X(\tau)]}\right] = \arctan\left[\frac{\hat{R}_X(\tau)}{R_X(\tau)}\right]. \tag{8}$$

Taking into account the Euler's formula, the analytical signal in Eq. (6) can be expressed in the form

$$z_X(\tau) = A(\tau)\cos(\theta(\tau)) + iA(\tau)\sin(\theta(\tau)). \tag{9}$$

The correlation function of a SDOF structure enforced by a white noise can be approximated, for $\tau > 0$, as

$$R_X(\tau) = Qe^{-2\pi f\zeta\tau}\cos\left(2\pi\bar{f}\tau\right) \tag{10}$$

and thus its Hilbert transform can be expressed as

$$\hat{R}_X(\tau) = Qe^{-2\pi f\zeta\tau}\sin\left(2\pi\bar{f}\tau\right) \tag{11}$$

where $\bar{f} = f\sqrt{1-\zeta^2}$ is the damped frequency and Q is a constant equal to the variance of the structural response.

Replacing Eqs. (10) and (11) in the Eq. (5) it leads to

$$z_X(\tau) = Qe^{-2\pi f\zeta\tau}\cos\left(2\pi\bar{f}\tau\right) + iQe^{-2\pi f\zeta\tau}\sin\left(2\pi\bar{f}\tau\right) \tag{12}$$

and thus, using Eqs. (12) and (9) is clear that

$$A(\tau) = Qe^{-2\pi f\zeta\tau} \tag{13}$$

and

$$\theta(\tau) = 2\pi\bar{f}\tau. \tag{14}$$

The instantaneous damped frequency can be calculated performing the first derivative of the phase and dividing by 2π, i.e.

$$\bar{f}(\tau) = \frac{1}{2\pi}\frac{d}{d\tau}[\theta(\tau)]. \tag{15}$$

By performing the average of the instantaneous damped frequency it is possible to obtain the damped frequency of the structure like

$$\bar{f} = E\left[\bar{f}(\tau)\right]. \tag{16}$$

The damping ratio can be obtained from the logarithm of the amplitude that has a linear form as it can be seen from the following equation

$$\ln[A(\tau)] = \ln[Q] - 2\pi f \zeta \tau = c_2 + c_1 \tau. \tag{17}$$

From the linear form of the Eq. (17) it is clear that the angular coefficient of the amplitude's logarithm is related to the structural damping ratio; in particular, the damping ratio is obtained as

$$\zeta = -\frac{c_1}{2\pi f}. \tag{18}$$

Since only the damped frequency is identified, we need to take into account that $f = \bar{f} \Big/ \sqrt{1 - \zeta^2}$, and thus the Eq. (18) reverts to

$$\zeta = \sqrt{\frac{\bar{c}_1^2}{1 + \bar{c}_1^2}} \tag{19}$$

with $\bar{c}_1 = c_1 \big/ \left(2\pi \bar{f}\right)$.

2.2 *Identification Algorithm for MDOF Systems*

The proposed method, introduced in the previous section, requires another step if applied to the MDOF systems. As a matter of fact, the components of the correlation functions matrix are "multi-component" functions and then they have a not well-behaved Hilbert transform. Therefore, it needs a "mono-component" correlation function, such as the modal correlation function, to restore the efficiency of the Hilbert transform. In light of the above, the identification method for MDOF systems can be resumed as:

(1) Output processes $\mathbf{X}(t)$ acquisition;
(2) Correlation functions matrix $\mathbf{R}_{\mathbf{X}}(\tau)$ estimation;
(3) Singular Value Decomposition of $\mathbf{R}_{\mathbf{X}}(0)$ and modal shapes identification;
(4) Calculation of the correlation functions matrix in the modal space $\mathbf{R}_{\mathbf{Y}}(\tau)$;
(5) Reconstruction of the analytical signals of the auto-correlation functions in the modal space;
(6) Frequencies and damping ratios estimation.

In order to extend the proposed method to a MDOF system, a shear-type MDOF frame with mass matrix $\mathbf{M}$, stiffness matrix $\mathbf{K}$ and damping matrix $\mathbf{C}$ is used. The dynamic properties to be identified are: the modal matrix $\mathbf{\Phi}$, the natural frequencies f_i and the damping ratios ζ_i where $i = 1, 2, \ldots, N$ and N is the number of degree of freedom of the system. In this case the differential equations system that governs the structural motion is

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\mathbf{V}W(t) \tag{20}$$

where $\mathbf{V}$ is the influence vector. The correlation functions contained into the correlation functions matrix are estimated as

$$R_{X_iX_j}(\tau) = E\big[X_i(t)X_j(t+\tau)\big] - \mu_{X_i}\mu_{X_j} \tag{21}$$

where μ_{X_i} and μ_{X_j} are respectively the averages of the i-th and j-th response process.

As it is well known, the differential equation system in Eq. (20) can be expressed in the modal space pre-multiplying for the modal matrix $\mathbf{\Phi}$ and taking into account the modal transformation

$$\mathbf{X}(t) = \mathbf{\Phi}\mathbf{Y}(t) \tag{22}$$

where $\mathbf{X}(t)$ is the response process in the nodal space and $\mathbf{Y}(t)$ is the response process in the modal space. The Eq. (20) expressed in the modal space thus becomes

$$\ddot{\mathbf{Y}}(t) + \mathbf{\Lambda}\dot{\mathbf{Y}}(t) + \mathbf{\Omega}\mathbf{Y}(t) = -\mathbf{\Phi}^T\mathbf{M}\mathbf{V}W(t) \tag{23}$$

in which $\mathbf{\Omega} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$ is a diagonal matrix containing the squares of the circular frequencies of the system and $\mathbf{\Lambda} = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}$ is, for classically damped system, a diagonal matrix that have the i-th term on the diagonal equal to $2\zeta_i\omega_i$.

The auto-correlation functions contained into the diagonal of the correlation functions matrix expressed in the modal space are

$$R_{Y_iY_i}(\tau) = E[Y_i(t)Y_i(t+\tau)] - \mu_{Y_i}^2 \tag{24}$$

where μ_{Y_i} is the average of the i-th response process in the modal space.

Since the modal matrix $\mathbf{\Phi}$ is unknown, the proposed method takes into account the relationship between the correlation functions matrix expressed in the nodal space $\mathbf{R_X}(\tau)$ and the correlation function matrix expressed in the modal space $\mathbf{R_Y}(\tau)$ [14], i.e.

$$\mathbf{R_X}(\tau) = \mathbf{\Phi}\mathbf{R_Y}(\tau)\mathbf{\Phi}^T. \tag{25}$$

In order to decompose $\mathbf{R_X}(0)$ in the product of three matrices is possible to use a SVD as it is reported in the following equation

$$\mathbf{R_X}(0) = \mathbf{U}\mathbf{S}\mathbf{V}^H. \tag{26}$$

In Eq. (26) $\mathbf{S}$ is a diagonal matrix that contains the singular values of $\mathbf{R_X}(0)$, $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices that contain respectively the left singular vectors and the right singular vectors of $\mathbf{R_X}(0)$; and the apex H denote the conjugated transpose. If $\mathbf{R_X}(0)$ is a normal matrix, i.e. if it is square and $(\mathbf{R_X}(0))^H \mathbf{R_X}(0) = \mathbf{R_X}(0)(\mathbf{R_X}(0))^H$, then $\mathbf{U} = \mathbf{V}$. Since $\mathbf{R_X}(0)$ is a normal and real matrix, Eq. (26) becomes

$$\mathbf{R_X}(0) = \mathbf{U}\mathbf{S}\mathbf{U}^T. \tag{27}$$

If the frequencies are well separated, then $\mathbf{R_Y}(0)$ is almost a diagonal matrix and thus $\mathbf{S} \approx \mathbf{R_Y}(0)$ and $\mathbf{U} \approx \mathbf{\Phi}$. In light of the above, the modal matrix $\mathbf{\Phi}$ is estimated performing a SVD of $\mathbf{R_X}(0)$ and the correlation functions matrix in the nodal space can be calculated with the inverse formula of Eq. (25), i.e.

$$\mathbf{R_Y}(\tau) = \mathbf{\Phi}^T \mathbf{R_X}(\tau)\mathbf{\Phi} \tag{28}$$

The analytical signals $z_i(\tau)$ of the auto-correlation functions in the modal space are calculated as

$$z_i(\tau) = R_{Y_iY_i}(\tau) + i\hat{R}_{Y_iY_i}(\tau) \tag{29}$$

where $\hat{R}_{Y_iY_i}(\tau)$ is the Hilbert transform of $R_{Y_iY_i}(\tau)$.

For each degree of freedom of the system, the frequencies and the damping ratios can be estimated with the same procedure proposed for SDOF system. In particular, applying Eqs. (7) and (8) is possible to calculate respectively the amplitudes $A_i(\tau)$ and the phases $\theta_i(\tau)$ of the analytical signals, by using Eqs. (15) and (16) the damped frequencies $\bar{f}_i$ can be estimated and, finally, Eqs. (17–19) can be used to estimate the damping ratios ζ_i of the system.

3 OMA: From Research to Engineering Applications

As regards, the paper's contribution is to provide a user friendly method, that can be used by people who have little to no knowledge of signal processing and stochastic analysis such as those who are responsible for the maintenance of a city's historical buildings. To aim at this, all the aforementioned steps have been implemented into an algorithm in MatLab environment reported in the Appendix.

Specifically, this algorithm only requires as input the time vector and the recorded structural outputs, then automatically returns all steps necessary to provide the dynamic properties of the structure. In sequential order the aforementioned algorithm estimate: the correlation functions matrix in the nodal space, the modal shapes, the correlation functions matrix in the modal space, the analytical signals of the auto-correlation functions in the modal space, the amplitudes, the phases, the damped frequencies and the damping ratios. To assess the reliability of the method and the algorithm, several numerical simulations and an experimental test are reported as it follows.

3.1 Validation of the Proposed Algorithm Through a SDOF System

In order to prove the reliability of the identification algorithm for SDOF system, a numerical simulation on a linear SDOF shear-type frame was performed for different values of the damping coefficient ζ. In particular, the SDOF structure has a natural frequency $f = 30$ Hz and the damping ratio is variable between 0.01 and 0.10 with a step equal to 0.01. The structural input $W(t)$ has been generated with the Shinozuka's formula [19] and the number of the generated samples is equal to 1000. Every sample has a duration of 100 s with sampling frequency 1000 Hz. The instantaneous damped frequency $\bar{f}(\tau)$ and the logarithm of the amplitude $A(\tau)$ are depicted respectively in Figs. 1 and 2 for $\zeta = 0.02$.

The results obtained by using the proposed method and PP + HP are reported, with the relative discrepancies $\varepsilon\%$, in Tables 1 and 2. These results suggest that both methods are reliable for the estimation of the natural frequency and damping ratio in a SDOF system. However, the proposed method has less discrepancy than PP + HP, both in terms of damped frequency estimation and in terms of damping ratio estimation. In this section the proposed method is compared only with PP + HP because other automated algorithm in MatLab environment, like FDD.m [20] and SSICOV.m [21], can be used only for MDOF systems.

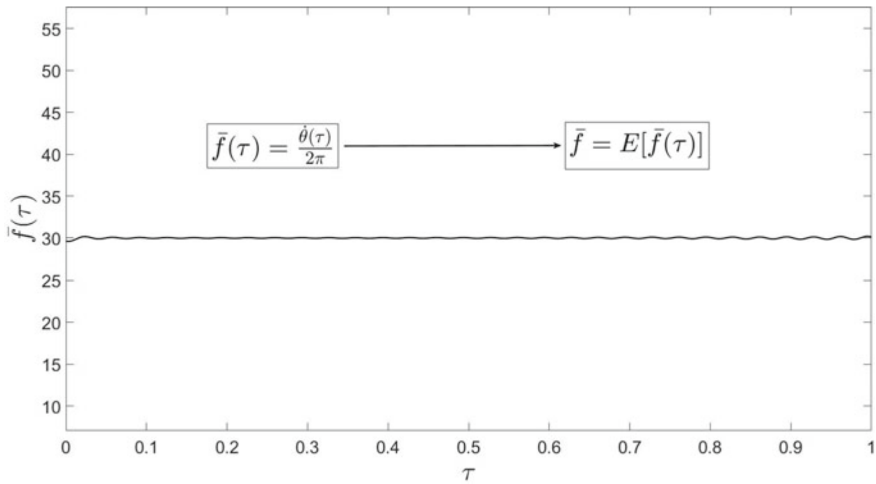

Fig. 1 Instantaneous damped frequency for $f = 30$ Hz and $\zeta = 0.02$

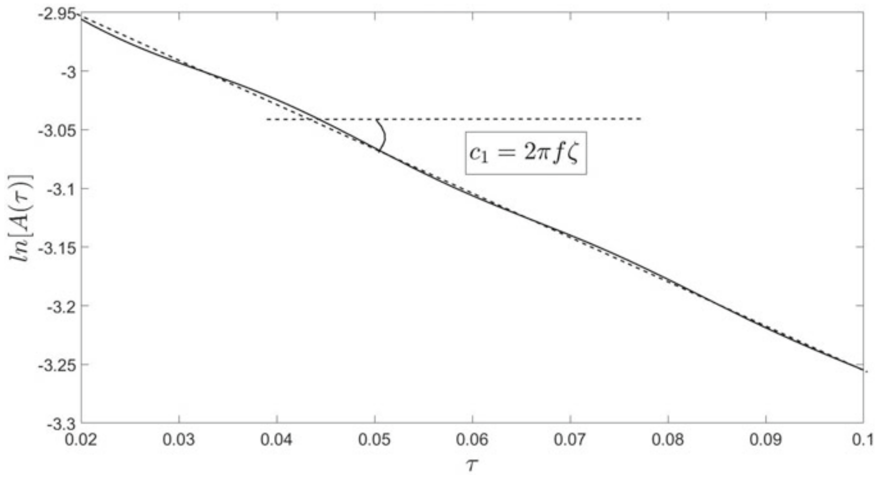

Fig. 2 Amplitude's logarithm for $f = 30$ Hz and $\zeta = 0.02$

Table 1 Comparison among the exact damped frequency and the estimated damped frequency

Damping ratio	Exact frequency	TD-ASM frequency	Discrepancy ε%	PP + HP frequency	Discrepancy ε%
0.0100	29.9985	29.9999	0.0046	29.9900	0.0283
0.0200	29.9940	29.9967	0.0090	29.9700	0.0800
0.0300	29.9865	29.9901	0.0121	29.9500	0.1217
0.0400	29.9760	29.9798	0.0125	29.9500	0.0867
0.0500	29.9625	29.9647	0.0074	29.9500	0.0416
0.0600	29.9460	29.9434	0.0087	29.9500	0.0135
0.0700	29.9264	29.9130	0.0447	29.9400	0.0454
0.0800	29.9038	29.8694	0.1152	29.9400	0.1209
0.0900	29.8783	29.8064	0.2405	29.9400	0.2067
0.1000	29.8496	29.7173	0.4433	29.5600	0.9703

3.2 Validation of the Proposed Algorithm Through a MDOF System

In order to prove the reliability of the identification algorithm for MDOF systems, a numerical simulation on a linear 3DOF shear-type system was performed at various values of the damping coefficient ζ_1. In particular, the range of variation of the first damping ratio is $\zeta_1 = 0.05 \div 0.10$ with a step equal to 0.01. ζ_2 and ζ_3 are calculated considering a Rayleigh damping. The shear-type 3DOF frame used for the numerical simulations has mass $m_j = 794$ kg for $j = 1, 2, 3$ and

Table 2 Comparison between the exact damping ratio and the estimated ramping ratio

Exact damping ratio	TD-ASM damping ratio	Discrepancy ε%	PP + HP damping ratio	Discrepancy ε%
0.0100	0.0100	0.3102	0.0104	3.6540
0.0200	0.0200	0.0063	0.0202	1.1639
0.0300	0.0300	0.0623	0.0299	0.4492
0.0400	0.0400	0.0485	0.0394	1.4300
0.0500	0.0500	0.0157	0.0492	1.5617
0.0600	0.0599	0.1252	0.0595	0.7978
0.0700	0.0698	0.2866	0.0703	0.4566
0.0800	0.0796	0.5158	0.0795	0.6002
0.0900	0.0908	0.8370	0.0886	1.5493
0.1000	0.1013	1.2787	0.1022	2.2490

stiffness $k_j = 6.18 \times 10^6 \mathrm{N}/\mathrm{m}$ for $j = 1, 2, 3$ and thus the natural frequencies are $f_1 = 6.2489$ Hz, $f_2 = 17.5091$ Hz, $f_3 = 25.3014$ Hz, and the modal shapes are $\phi_1 = \left[0.328\ 0.591\ 0.737\right]^T$, $\phi_2 = \left[0.737\ 0.328\ -0.591\right]^T$, $\phi_3 = \left[0.591\ -0.737\ 0.328\right]^T$. The results in terms of damped frequencies (for each value of $\zeta_1 = 0.05 \div 0.01$) evaluated by using the proposed method are compared with those obtained by SSI-COV [21] and FDD [20], as reported in Table 3.

From these results it is apparent that all methods are performing very well, in particular the SSI-COV results have the least discrepancy, but SSI-COV is not direct as the proposed method TD-ASM, since it requires the knowledge of a parameter related to the first frequency, that is a priori unknown. This means that it needs at least a Fourier Transform of the signal to get this value, while the proposed method does not require any preliminary information of the unknown characteristics. Further, results in terms of damping ratios are compared with SSI-COV only, since the algorithm FDD.m does not allow the damping ratios' evaluation. Also in this case the proposed method gets satisfactory results as shown in Table 4 (for each value of $\zeta_1 = 0.05 \div 0.01$).

As regards the modal shapes, Fig. 3 reports the discrepancies of results obtained with the proposed method, FDD and SSI-COV with respect the exact ones at different values of damping ratios. Also in this case, the proposed method and SSICOV (both developed in the time domain and based on the correlation function) are more precise than FDD (developed in the frequency domain and based on the PSD) especially for the first modal shape that gives the major contribution to the total structural motion.

Table 3 Comparison between the exact damped frequencies and the estimated damped frequencies

ζ_1	Mode	Exact	TD-ASM	ε%	FDD	ε%	SSI	ε%
0.05	1	6.2411	6.2363	0.0770	6.1646	1.2261	6.2433	0.0356
	2	17.4872	17.4529	0.1961	17.8833	2.2650	17.4595	0.1584
	3	25.2522	25.0554	0.7794	25.3296	0.3065	25.2753	0.0916
0.06	1	6.2376	6.2262	0.1823	6.1646	1.1713	6.2426	0.0802
	2	17.4776	17.4344	0.2472	17.8833	2.3213	17.4334	0.2529
	3	25.2305	24.9680	1.0405	25.3296	0.3926	25.2255	0.0201
0.07	1	6.2335	6.2255	0.1290	6.1646	1.1065	6.2381	0.0734
	2	17.4662	17.4128	0.3060	17.8833	2.3880	17.4147	0.2950
	3	25.2049	24.7680	1.7334	25.3296	0.4947	25.1682	0.1455
0.08	1	6.2288	6.2223	0.1042	6.1646	1.0315	6.2353	0.1036
	2	17.4531	17.3857	0.3858	17.8833	2.4652	17.4209	0.1844
	3	25.1753	24.3554	3.2568	25.3296	0.6129	25.0907	0.3362
0.09	1	6.2234	6.2133	0.1625	6.1035	1.9271	6.2361	0.2033
	2	17.4381	17.3575	0.4626	17.8833	2.5528	17.3813	0.3257
	3	25.1417	24.2051	3.7254	25.3296	0.7473	25.1235	0.0723
0.10	1	6.2175	6.2222	0.0761	6.1035	1.8326	6.2377	0.3251
	2	17.4215	17.3323	0.5117	17.8833	2.6510	17.3420	0.4562
	3	25.1041	24.1537	3.7860	25.3296	0.8982	25.1216	0.0698

3.3 Validation of the Proposed Algorithm Through Experimental Test

In order to prove the reliability of the proposed method on real structures, an experimental test was performed on a three-story frame. The set-up of the experimental test is reported in Fig. 4. In particular, the structure was excited by a broad-band noise from 0.01 to 80 Hz through an electro-magnetic shaker APS-ELECTRO-SAIS. The input and the output signals were recorded using piezo-electric accelerometers Brüel&Kjaer 4507 002 connected to the acquisition unit NI PXIe 1082. Some tests of 240 s with sampling frequency equal to 1000 Hz were performed and the proposed algorithm was used to obtain the modal shapes, the frequencies and the damping ratios of the structure. Since the tests are performed on a real system that has unknown dynamic properties, is impossible to report a discrepancy between the exact properties and the identified properties. However, the results obtained by the used methods are reported in Tables 5 and 6.

From these results, it is clear that all the used algorithms well identify the same frequencies and that the differences between the damping ratios identified with the proposed algorithm and SSICOV.m are very low.

Table 4 Comparison between the exact damping ratios and the estimated damping ratios

ζ_1	Mode	Exact	TD-ASM	ε%	SSI	ε%
0.05	1	0.0501	0.0519	3.6703	0.0481	3.9291
	2	0.0500	0.0493	1.2739	0.0521	4.2302
	3	0.0623	0.0623	0.0697	0.0646	3.6640
0.06	1	0.0601	0.0607	0.9509	0.0582	3.2513
	2	0.0600	0.0597	0.3966	0.0629	4.8558
	3	0.0748	0.0746	0.2821	0.0739	1.1238
0.07	1	0.0702	0.0727	3.6792	0.0666	5.0789
	2	0.0699	0.0699	0.1102	0.0726	3.8543
	3	0.0872	0.0824	5.5115	0.0858	1.6690
0.08	1	0.0802	0.0838	4.5188	0.0757	5.5309
	2	0.0799	0.0793	0.7807	0.0850	6.3065
	3	0.0997	0.0915	8.2092	0.1034	3.6762
0.09	1	0.0902	0.0953	5.6554	0.0848	6.0208
	2	0.0899	0.0894	0.5921	0.0926	2.9736
	3	0.1122	0.1038	7.4214	0.1106	1.4040
0.10	1	0.1002	0.1046	4.3784	0.0935	6.7502
	2	0.0999	0.1004	0.4290	0.1015	1.6203
	3	0.1246	0.1081	13.2609	0.1215	2.5302

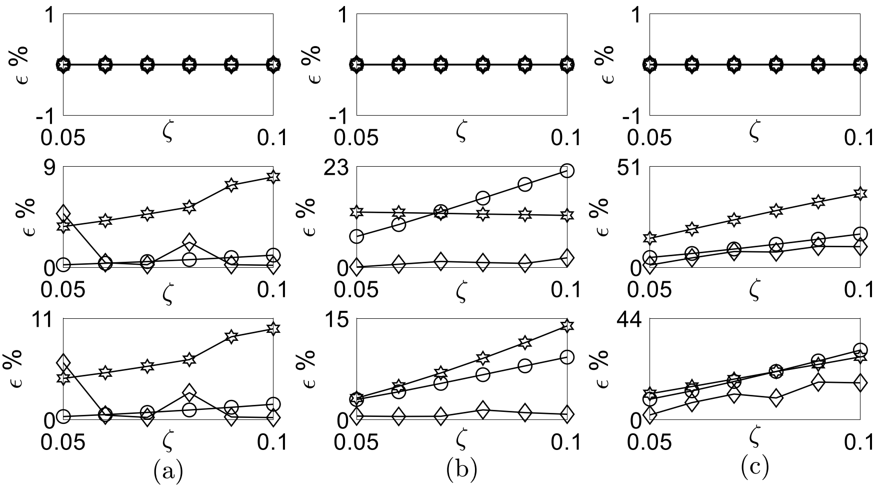

Fig. 3 Discrepancy of results obtained with the proposed method TD-ASM (circular marker), FDD (hexagram marker) and SSICOV (diamond marker) with respect the exact ones at different values of damping ratios: first mode **a**, second mode **b**, third mode **c**

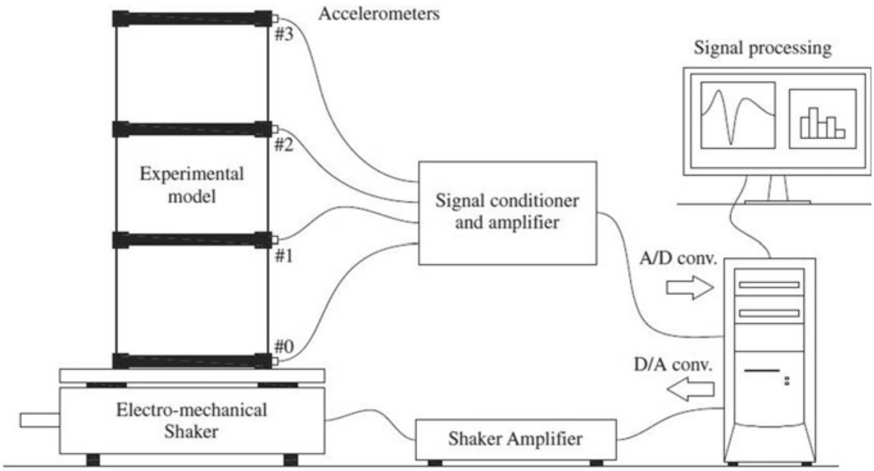

Fig. 4 Experimental set-up

Table 5 Comparison between TD_ASM.m, SSICOV.m and FDD.m in terms of damped frequencies and damping ratios

Mode	Frequencies			Damping ratios		
	TD-ASM	FDD	SSICOV	TD-ASM	FDD	SSICOV
1	2.1438	2.2125	2.1511	0.0853	–	0.0717
2	6.0719	6.0730	6.0868	0.0075	–	0.0080
3	8.7321	8.7280	8.7285	0.0013	–	0.0016

Table 6 Comparison between TD_ASM.m, SSICOV.m and FDD.m in terms of modal shapes

TD-ASM			FDD			SSICOV		
ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.7306	0.3794	−1.2784	2.0377	0.3328	−1.4188	1.9186	0.7549	−1.4351
1.9751	−0.8388	0.6139	2.5393	-0.7784	0.7477	2.3581	−0.8516	0.7672

4 Conclusions

This paper introduces an innovative ambient identification method based on the Hilbert Transform to obtain the analytical representation of the system response in terms of the correlation function. It leads to identify the modal shapes performing a SVD of the correlation function matrix in $\tau = 0$, the frequencies by the phase of each analytical signal of the auto-correlation functions in the modal space and the damping ratios by the amplitude of each analytical signal of the auto-correlation functions in the modal space. The numerical simulations prove that for SDOF structures the proposed method can identify the dynamic properties better than the PP+HP and for MDOF structures the proposed method TD-ASM can identify very well the dynamic properties of the structural systems, especially in terms of the damping ratios. The performed experimental tests prove that the dynamic properties identified with the proposed method are similar to the properties identified by other automated algorithm and that the proposed algorithm have some advantages compared to the others. However, the greatest advantage of TD-ASM is that it is user friendly, in fact the developed MatLab function requires only the time vector and the recorded outputs and can be used also by a not-expert user. As a concluding remark, the authors wish that this approach could open the pathway for a monitoring system that is user friendly and can be used by people who have little to no knowledge of signal processing and stochastic analysis such as those who are responsible for the maintenance of a city's historical buildings.

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Appendix

The proposed algorithm, entirely reported in this appendix, requires as input only the output signals (X) and the time vector (time). It calculates automatically the frequencies (fid), the damping ratios (Z_ID_LOG) and the modal shapes both normalized with respect to the first component of each mode (PHI_IDNN) and not-normalized (PHI_ID). The entire developed MatLab function, called TD_ASM.m, is shown below.

```
function [PHI_ID,PHI_IDNN,fid,Z_ID_LOG] = TD_ASM(X,time)
% number of performed tests
n_cam=size(X,2);
% number of degree of freedom
ndof=size(X,3);
% tau vector
tau=[-fliplr(time(2:end)) time];
% length of time vector
N=length(time);
% time sampling
dt=time(2)-time(1);
% Correlation functions matrix estimation (nodal space)
for j=1:n_cam
for jj=1:ndof
for ii=1:ndof
Rx(j,ii,jj,:)=xcorr(squeeze(X(:,j,ii)),squeeze(X(:,j,jj)),'biased');
end
end
end
Rx_m=squeeze(mean(Rx,1));
% SVD and modal shapes estimation
[PHI_ID RY0 U]=svd(Rx_m(:,:,N));
for ii=1:ndof
PHI_ID_N(:,ii)= PHI_ID(:,ii)/PHI_ID(1,ii);
End
% Correlation functions matrix estimation (modal space)
for tt=1:length(tau)
Ry(:,:,tt)= PHI_ID'*Rx_m(:,:,tt)*PHI_ID;
End
% Calculus of: analytical signals, instantaneous frequencies and
% amplitudes
for ii=1:ndof
Zy(ii,:)=hilbert(squeeze(Ry(ii,ii,:)));
fist(ii,:)=gradient(unwrap(angle(squeeze(Zy(ii,:)))),dt)/(2*pi);
A(ii,:)=abs(squeeze(Zy(ii,:)));
End
% Damped frequencies estimation
for ii=1:ndof
[NPi NPf]=TimeStopSelection(fist(ii,:));
f_id(ii)=mean(fist(ii,N+NPi:N+NPf));
end
% Damping ratios estimation
for ii=1:ndof
[NPi NPf]=TimeStopSelection(A(ii,:));
y=log(A(ii,N+NPi:N+NPf));
x=tau(N+NPi:N+NPf);
C_coeff(1:2)=polyfit(x,y,1);
c1_bar=-C_coeff(1)/(2*pi*f_id(ii));
zid_log(ii)=sqrt(c1_bar^2/(1+c1_bar^2));
end
% Dynamic properties sorting
[fid ind]=sort(f_id);
for ii=1:ndof
PHI_ID(:,ii)=PHI_ID(:,ind(ii));
PHI_IDNN(:,ii)=PHI_ID_N(:,ind(ii));
Z_ID_LOG(ii)=zid_log(ind(ii));
end
end
```

In this code only two kind of interactions with the user are requested. The first one is the organization of the input data i.e. the time vector that is a row vector and

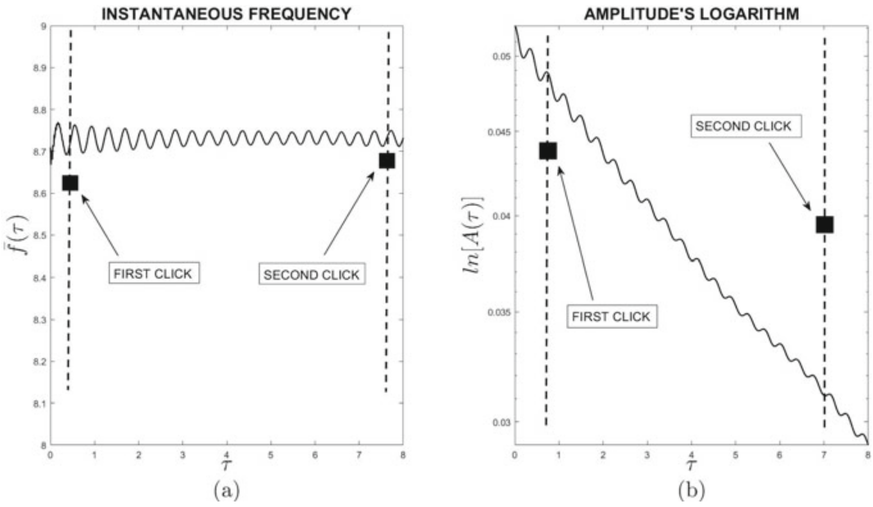

Fig. 5 Interactive graphic interface of TimeStopSelection.m: for frequency estimation **a**, for damping ratio estimation **b**

the structural output process that is a three-dimensional array. The second one is a step of the function TimeStopSelection.m that is contained into TD_ASM.m and that requires the choice of the time interval to be used to perform the average in Eq. (16) and to identify the coefficient c_1 in Eq. (17). In order to simplify this step, TimeStopSelection.m has an interactive graphic interface that allows to choose, with few clicks, the aforementioned time intervals as reported in Fig. 5a, b respectively for frequency identification and damping ratios identification.

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