# **Evaluation of Partial Safety Factors for the Structural Assessment of Existings Masonry Buildings**



**Pietro Croce, Maria L. Beconcini, Paolo Formichi, Filippo Landi, Benedetta Puccini, and Vincenzo Zotti**

**Abstract** The assessment of existing structures and infrastructures is a primary task in modern engineering, both for its key economic significance and for the extent and the significance of the built environment, nonetheless operational rules and standards for existing structures are often missing or insufficient, especially for masonry constructions. Existing masonry buildings, even in limited geographical regions, are characterized by many masonry types, differing in basic material, mortar, block shape, block texture, workmanship, degree of decay and so on. For these reasons, relevant mechanical parameters of masonry are often very uncertain; their rough estimation thus leads to inaccurate conclusions about the reliability of the investigated structure. In this work, a methodology to derive a refined probabilistic description of masonry parameters is first outlined starting from the analysis of a database of in-situ tests results collected by the authors. In particular, material classes, representing low, medium and high-quality masonry, are identified for a given masonry typology by means of the definition of a Gaussian Mixture Model. The probability density functions so obtained are the fundamental basis for the implementation of probabilistic analysis methods. In particular, the study will focus on the evaluation of masonry classes for compressive strength of stone masonry, considering a relevant database of semi-destructive, double flat jacks, in-situ test results. The statistical properties of the identified masonry classes, which can be used for the direct probabilistic assessment of structural performance of masonry walls under vertical loads, are finally considered for the evaluation of suitable partial safety factors,  $\gamma_M$ , to be used in the engineering practice.

**Keywords** Partial safety factor · Reliability · Existing structures · Masonry · Compressive strength

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### **1 Introduction**

The assessment of existing structures and infrastructures is a primary task in modern engineering, both for its key economic significance and for the extent and the importance of the built environment [\[1\]](#page-11-0), nonetheless operational rules and standards for existing structures are often missing or insufficient, especially for masonry constructions.

In the past centuries, masonry was the main building material; consequently, masonry buildings are a relevant part of existing structures, especially in historical towns [\[2\]](#page-11-1). Mostly, they were built according to empirical rules and architectural canons, far away from the modern design approaches. Despite often they successfully perform their functions over time till today, there is a strong need to "measure" their structural performance, especially in seismic-prone areas, mainly in view of prioritization and planning of maintenance and intervention strategies.

As known, many different types of masonry can be identified in existing masonry buildings in Europe and worldwide [\[3\]](#page-11-2), with significant scatter even in limited geographical regions, differing in basic material, mortar, block shape and texture, workmanship, degree of decay and so on.

Despite existing buildings are commonly declared to be better known than new ones, the relevant mechanical parameters of existing masonry cannot be easily estimated. In fact, mean values and coefficient of variations (COVs) of relevant mechanical properties of a given existing masonry are very scattered, especially in comparison with masonry structures built nowadays. For that reason, the evaluation of masonry parameters cannot overlook the assessment of the related uncertainty that should be properly expressed in probabilistic terms [\[3,](#page-11-2) [4\]](#page-11-3).

In practical cases, masonry's mechanical properties are usually evaluated based on limited semi-destructive or non-destructive tests, taking into account relevant uncertainties.

In the assessment, the first step should be the evaluation of the compressive strength of masonry walls and pillars. Despite a relatively extensive research into masonry structures, the issue of a reliable determination of the load-bearing capacity of existing, mainly historic, stone masonry structures, is still waiting for a satisfactory solution.

A probabilistic description of compressive strength of regular masonry types can be found in [\[3,](#page-11-2) [5\]](#page-11-4), based on the EN1996-1-1 model [\[6\]](#page-11-5), considering tests and the related probabilistic models for masonry units and mortar. But, also considering that the extraction of an appropriate number of samples from the investigated walls is often impossible, this approach cannot be applied to irregular stone masonry. In the following, a procedure to identify masonry classes and their main statistical parameters is proposed, based on masonry compressive strength derived from double flat-jacks tests [\[7\]](#page-11-6). Results are presented for stone masonry, for which a considerable wide database of in situ compression tests on masonry walls was collected by the

authors [\[8,](#page-11-7) [9\]](#page-11-8) in the framework of the in-situ experimental campaign for the assessment of seismic vulnerability of masonry school buildings in the Municipality of Florence.

These probabilistic models for compressive strength are the basis for a reliability analysis devoted to assess, depending on the required target reliability level, partial safety factor  $\gamma_M$  for existing masonry, to be used in the partial factor method implemented in the Eurocodes [\[10\]](#page-11-9).

# *1.1 Experimental Tests for the Evaluation of Masonry Mechanical Parameters*

As anticipated earlier, assessing the relevant mechanical properties of existing masonry walls should need an ad-hoc experimental test campaign, both in the laboratory and in-situ.

Laboratory tests may be, for example, direct compressive tests on masonry samples extracted from the structure, as well as compressive tests on single bricks or blocks associated with tests on mortar samples, such as Darmstadt test [\[11\]](#page-11-10), PNT-G [\[12\]](#page-11-11) or direct compression.

In situ tests are, instead, carried out, for example, by means of single and double flat jacks [\[7\]](#page-11-6). The idea of the test is similar to a standard compressive test, with the difference that it is carried out directly onto the investigated panel, to which the load is applied via two flat jacks, inserted in horizontal cuts, within the panel's thickness. During the tests, four inductive deformation transducers, three vertical and one horizontal (Fig. [1\)](#page-3-0), are used to measure vertical and horizontal displacement in the area between the two cuts, approximately 500 mm from one another.

During the test, it is possible to detect the first cracks occurring in the compressed portion of wall, and to check if they affect the mortar or the stones. The pressure value, which causes the crack formation, is used to estimate the compressive strength of the masonry, while the elastic modulus *E* and the apparent Poisson ratio, *v*, are derived from measurements of vertical and horizontal deformations.

Flat jack tests represent one of the less intrusive method for masonry testing and provide, as shown in [\[8\]](#page-11-7), useful data to obtain a complete mechanical characterizations of masonry walls. Indeed, the compressive strength,  $f_m$ , and the elastic modulus, *E*, are directly estimated by the test, but also shear modulus, *G*, and shear strength,  $\tau_0$ , can be derived by means of appropriate experimental relationships [\[8\]](#page-11-7).

It must be remarked once again that safeguard of the structural integrity calls for a severe limitation of the number of destructive or semi-destructive tests to be carried out on a single structure. As a consequence, even in the most favorable situation, test results only allow to broadly assess the mean value of mechanical parameters and the material's degree of homogeneity throughout the structure, being generally not sufficient to derive the appropriate statistical description of mechanical parameters, which are needed for reliability assessment.



<span id="page-3-0"></span>**Fig. 1** Double flat jacks test on a stone masonry wall

To overcome the lack of information about mechanical properties in terms of probability density functions (*pdfs*) and relevant statistical parameters it is possible to carry out analysis based upon valid databases of tests results carried out on similar masonry panels [\[13\]](#page-11-12).

During the last years, a wide experimental campaign has been carried out by the authors on rather homogenous sets of brick and stone masonry buildings located in the same geographical area (the Municipality of Florence). The results will be discussed in the following.

# *1.2 Database of Test Results*

In the framework of static and seismic vulnerability assessments carried by the authors during the last three years on about 80 masonry school buildings of the Municipality of Florence, a large and consistent database of masonry mechanical parameters has been set up, collecting the results of ad hoc in situ and laboratory tests carried out on several masonry typologies characterizing these buildings. The experimental results, supplemented with literature data made it possible to set up a rational classification for various types of masonry, providing, at the same time, sound information about statistical properties of relevant investigated mechanical parameters.

The buildings differ in size and historical-artistic importance. Most of them date back to the end of 1800 and the beginning of 1900, but more ancient buildings, built before 1700, as well as modern buildings, built after the Second World War, have been also investigated.

The values collected in the database are critically discussed, also referring to the values recommended in the Guidelines for the application of the Italian Building Code [\[14\]](#page-11-13) for the different existing masonry typologies. Moreover, an estimation



<span id="page-4-0"></span>**Fig. 2** In-situ double flat jack test, stress-deformation diagram resulting and test data processing

of the masonry quality index (MQI) [\[15\]](#page-11-14) is provided considering information on masonry quality obtained by visual inspection.

The present study is mainly focused on the analysis of the results of 95 double flat jack tests, performed by three different Laboratories according to ASTM standards [\[7\]](#page-11-6). Among the 95 tests, 67 concern stone masonry walls, 25 solid brick masonry and 3 other masonry types.

For the sake of the consistency and homogeneity of the results, all the information obtained from the in-situ tests have been analysed, processed and evaluated according to a unique procedure. In Fig. [2](#page-4-0) the synthesis report is reported, as an example, for a stone masonry wall.

The main parameters considered are: the normal stress in the masonry panel due to permanent loads,  $\sigma_0$ , the masonry compressive strength,  $f_m$ , the elastic modulus, *E,* and, as ratio between horizontal and longitudinal displacements, the apparent value of the Poisson modulus,  $\nu$ , which is often outside the limits for isotropic and homogenous materials, since determined in a post-cracking state.

The apparent value of the shear modulus, *G*, has been thus estimated adopting the usual relationship for isotropic and homogenous materials, again disregarding cracks.

The elastic modulus *E* and the shear modulus *G* have been evaluated linearizing three different parts of the  $\sigma - \varepsilon$  diagram, to reproduce the masonry behaviour in the mainly elastic and plastic phase. In fact, in the  $\sigma - \varepsilon$  diagram, three interval in terms of normal stresses have been considered, ranging from 10 to 40%, from 40 to 70% and from 70 to 100% of the compressive strength respectively, as summarized, for example, in Fig. [2.](#page-4-0) The first interval corresponds to the quasi-elastic section of the diagram, the intermediate interval refers to the cracked condition, while the higher interval reflects the plastic section.

#### **2 Analysis of Test Results**

Starting from the test results collected in the database, the statistical parameters, i.e. mean and coefficient of variation, of the relevant masonry mechanical parameters have been derived. The results for stone masonry compressive strength,  $f_m$ , the elastic modulus, *E*, and the shear modulus *G* in different conditions, are summarized in Table [1.](#page-5-0)

As expected, data are characterized by high coefficient of variation, especially concerning elastic and shear modulus [\[8\]](#page-11-7), due to the wide variability of masonry properties, even between those belonging to the same typology, depending not only on the quality of the original raw materials, but also on the texture, on the workmanship and on the degradation. In fact, the quality of the mortar, the presence of irregular or dressed stones, as well as the different shape and size of the stones, well justify the existence of different classes within the same masonry typology.

A further analysis is then needed to identify homogenous statistical populations for masonry mechanical parameters. In particular, the general procedure already applied for the identification of concrete classes in  $[16]$  or rebar classes in  $[17]$ , can be used as previously illustrated in [\[13\]](#page-11-12).

The basic idea of the method is to subdivide mechanical tests results by means of a cluster analysis based on Gaussian Mixture Models (GMM), in such a way that homogenous statistical populations for masonry mechanical parameters can be identified. GMM is a cluster algorithm which provides a mixture of Gaussian distributions of vectors processing the subpopulations which are part of the whole Gaussian distribution.

# *2.1 Identification of Masonry Classes*

The cluster analysis has been carried out by means of Gaussian Mixture Model (GMM), which is an algorithm, of unsupervised learning, able to identify subpopulations in of a whole population made up by a mixture of several unknown Gaussian distributions [\[18\]](#page-11-17). Data are analysed with the aim to find a better probabilistic model consisting of different distributions, following the experience-based awareness of a priori existence of different sub-population in the whole dataset. Each identified subpopulation represents a masonry stone class, characterized by its Coefficient of Variation (COV).

<span id="page-5-0"></span>

Let  $X_1, \ldots, X_n$  a random sample of size *n*,  $X_i$  is the p-dimensional random vector with *pdf*  $f(x_i)$  on  $\mathbb{R}^p$ . In a mixture models (MM), with *k* components the distribution  $f(x_i)$  is associated with the following density [\[18\]](#page-11-17):

$$
f(x_i) = \sum_{j=1}^{k} w_i f_j(x_i),
$$
\n(1.)

where  $f_i(x_i)$  are the component densities of the mixture and the quantities  $w_1, \ldots, w_k$  are the mixing proportions (or weights) with

<span id="page-6-1"></span>
$$
\sum_{i=1}^{k} w_i = 1
$$
 (2.)

To speed up the process, an engineering evaluation of a priori value of *k* is needed. In the present study, *k* is set equal to 3, considering low, medium and high-quality stone masonry. Then, the mixture model has been fitted by means of the Expectation Maximization (EM) algorithm, which is a tool able to simplify maximum likelihood problems, starting with an *Expectation* (E) step where a first assignment of each observation to each model is performed, then a *Maximization* (M) step computes the weights, the variance and the mixing probability, and finally the E and M steps are iterated until convergence [\[19\]](#page-11-18).

The results are illustrated in Fig. [3,](#page-6-0) focusing on the compressive strength on masonry. In particular, the frequency histogram is plotted together with the probability density function obtained by fitting the whole dataset with a Normal distribution

<span id="page-6-0"></span>

(in blue) and a Lognormal distribution (in red), while the GMM is shown with blue dashed lines.

As already noted for concrete in [\[16\]](#page-11-15) and rebars in [\[17\]](#page-11-16), the cluster analysis leads to a better evaluation of statistical parameters for masonry compressive strength rather than the analysis of the whole dataset. Indeed, the proper identification of subclasses allows to significantly improve the estimate of the coefficient of variation to be associated with each class: the COVs, which results to be 28% for the lower class, 14% for the intermediate class, and 11% for the upper class, are significantly smaller than that resulting from the analysis of the whole dataset  $(COV = 33\%)$ .

#### **3 Structural Assessment of Existing Masonry Structures**

The assessment of existing structures should be based on the principles of limit states [\[20\]](#page-11-19), selecting the relevant situations (equivalent to those used for design of new structures) and taking into account the updated information on the actual conditions and circumstances under which the structure is required to fulfill its function during its design working life.

In particular, the structural assessment aims to determine the reliability of a structure as a whole or in terms of individual members, with respect to prescribed limit states and for a notional time period. In the assessment of actual reliability, the verifications are mostly based on the partial factor method [\[10,](#page-11-9) [21\]](#page-11-20), but probabilistic methods can also be applied in special cases. In mathematical terms, the following condition should be fulfilled for each relevant limit state:

$$
g(F_d, X_d, a_d, \theta_d) > 0 \tag{3}
$$

where *g* is the limit state function,  $F_d$  is the design value of actions,  $X_d$  is the design value of material properties,  $a_d$  is the design value of geometrical quantities and  $\theta_d$  is the design value for model uncertainty.

#### *3.1 Design Values and Partial Factors*

The suitable knowledge of the parameters of the statistical distribution describing the materials' properties allows to calibrate partial factors to be adopted for the assessment existing structures, in order to achieve a given target reliability. The design or assessment value  $X_d$  is determined from the characteristic value  $X_k$ , by means of the partial factor  $\gamma_m$  for the material resistance, and, in some cases, a conversion factor  $\eta$  [\[10\]](#page-11-9)

<span id="page-7-0"></span>
$$
X_d = \eta \frac{X_k}{\gamma_m}.\tag{4}
$$

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From Eq. [\(4\)](#page-6-1), the partial factor  $\gamma_m$  can be derived, assuming a unit conversion factor  $\eta$ , as follows

$$
\gamma_m = \frac{X_k}{X_d}.\tag{5}
$$

The characteristic values  $X_k$  is generally evaluated, according to EN1990 [\[10\]](#page-11-9), as the 5%-fractile of *X*, while the design value  $X_d$  can be evaluated based on the value of *X* at the FORM design point, i.e. the point on the failure surface  $(g = 0)$  closest to the average point in the space of normalised variables [\[10\]](#page-11-9). In this way, the partial factor on the material strength can be determined in case *X* is described by a normal distribution as

$$
\gamma_m = \frac{X_k}{X_d} = \frac{\mu_X (1 - 1.645 V_X)}{\mu_X (1 - \alpha_R \beta_t V_X)} = \frac{(1 - 1.645 V_X)}{(1 - \alpha_R \beta_t V_X)}
$$
(6)

and in case of Lognormal distribution as

$$
\gamma_m = \frac{X_k}{X_d} = \frac{\mu_X \exp\left(-0.5 \ln(1 + V_X^2) - 1.645\sqrt{\ln(1 + V_X^2)}\right)}{\mu_X \exp\left(-0.5 \ln(1 + V_X^2) - \alpha_R \beta_t \sqrt{\ln(1 + V_X^2)}\right)}
$$
  
= 
$$
\exp\left((\alpha_R \beta_t - 1.645)\sqrt{\ln(1 + V_X^2)}\right)
$$
(7)

where  $V_X$  is the COV of the material properties,  $\alpha_R$  is the sensitivity factor for resistance in the FORM analysis, which can be assumed equal to 0.8 [\[10\]](#page-11-9), and  $\beta_t$  is the target reliability index.

Obviously, the verification consists in checking that the design value of the resistance  $R_d$  is not less of the corresponding design value of the action effects *Ed* :

<span id="page-8-0"></span>
$$
R_d \ge E_d. \tag{8}
$$

The design value of the resistance,  $R_d$ , should be estimated considering the design value of the material properties,  $X_d$ , the geometry  $a_d$  and the model uncertainty  $\theta_d$ . In particular, a model uncertainty factor for the resistance  $\gamma_{Rd}$  is defined, which takes into account the uncertainties in the resisting model and geometrical deviations if these are not modelled explicitly, so that the design value of the resistance,  $R_d$  results.

$$
R_d = R\left(\frac{X_k}{\gamma_M}; a_d\right), \quad \text{where } \gamma_M = \gamma_{Rd}\gamma_m \tag{9}
$$

The partial factor for model uncertainty,  $\gamma_{Rd}$ , can be obtained, in case of normal distribution, as the ratio

$$
\gamma_{Rd} = \frac{1}{\mu_{\theta}(1 - \alpha_R \beta_t V_{\theta})},\tag{10}
$$

and in case of Lognormal distribution as

$$
\gamma_{Rd} = \frac{1}{\mu_{\theta} \exp(1 - \alpha_R \beta_t V_{\theta})},\tag{11}
$$

where  $\mu_{\theta}$  is the mean value,  $V_{\theta}$  is the coefficient of variation, and  $\alpha_R$  is the sensitivity factor in the FORM analysis, assumed equal to 0.32 ("non-dominant resistance variable"). γ*Rd* is generally set equal to 1.1 for new structures [\[2\]](#page-11-1), but higher values are proposed for existing masonry structures, which detailed identification is limited [\[2\]](#page-11-1). In the following calculations a Lognormal distribution with  $\mu_{\theta} = 1$  and  $V_{\theta} = 0.18$  is assumed.

Adopting the distributions previously obtained for the masonry compressive strength,  $f_m$ , the partial factor  $\gamma_M$  can be evaluated depending on the target reliability,  $\beta_t$ , combining Eqs. [\(6\)](#page-7-0) and [\(11\)](#page-8-0). In Fig. [4,](#page-9-0) the results are reported for the whole population of the dataset and for the three identified sub-classes of stone masonry, depending on the adopted  $\beta_t$ .

Target values for the reliability index are given for new structures in the Annex C of EN1990 [\[10\]](#page-11-9) depending on the consequences classes (CC) of the building, i.e. the "*categorization of the consequences of structural failure in terms of loss of human lives or personal injury and of economic, social, or environmental losses*" [\[10\]](#page-11-9). Three consequence classes are defined, CC1 (low) CC2 (medium) and CC3 (high), and the corresponding reliability levels are 4.2, 4.7 and 5.3 for one-year reference period,

<span id="page-9-0"></span>

Masonry	$f_m(N/mm^2)$	$f_{m,k}$ (N/mm <sup>2</sup> )	$\gamma_M$	$f_d(N/mm^2)$
Stone masonry ( $\beta_t = 3.8$ )	1.88	0.86	1.94	0.45
Stone masonry class 1 ( $\beta_t = 3.8$ )	1.14	0.61	1.83	0.33
Stone masonry class 2 ( $\beta$ <sub>t</sub> = 3.8)	1.92	1.47	1.52	0.97
Stone masonry class 3 ( $\beta$ <sub>t</sub> = 3.8)	2.68	2.18	1.46	1.50
Rubble stone $[14]$	$1.00 - 2.00$		$2.00 - 3.00$	$0.33 - 1.00$
Undressed stone $[14]$	2.00		$2.00 - 3.00$	$0.67 - 1.00$

<span id="page-10-0"></span>**Table 2** Characteristic and design values of masonry strength and partial factors

while they are set equal to 3.3, 3.8 and 4.3 for a 50 year reference period. Most existing masonry buildings can be classified to consequences class of failure CC2 or CC3 in terms of loss of human life, but it must be highlighted, in case of ancient building, that the consequence of failure should be considered also for the unrecoverable loss of the historical value of the building. For existing structures, some reduction in the reliability index is often acceptable, a discussion about appropriate target reliability levels can be found, for example, in [\[22\]](#page-11-21) and [\[23\]](#page-11-22), but it must be highlighted that this concept is often not correctly applied.

Assuming a target value  $\beta_t = 3.8$ , the characteristic values, the partial factors and the design values for masonry compressive strength are reported in Table [2.](#page-10-0) These values are finally compared with those provided by the Italian Guidelines [\[14\]](#page-11-13) in terms of mean values, also reported in the table.

#### **4 Conclusions**

The assessment of the structural performance of existing masonry buildings is still a critical issue due to the significant uncertainties characterizing the definition of masonry mechanical parameters. In the paper, a methodology based on Gaussian Mixture Model is outlined to identify masonry classes and their main statistical parameters, mean and coefficient of variation, starting from the analysis of a wide database of in-situ tests results collected by the authors.

In particular, material classes, representing low, medium and high quality stone masonry, are presented focusing on masonry compressive strength, together with the corresponding partial safety factors to be used for the structural verification of masonry walls under vertical loads according to the partial factor method implemented in the Eurocodes.

The obtained probability density functions for the masonry classes provide also a sound basis to perform reliability assessment of existing masonry buildings.

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