Dynamics of a Predator and Prey-Hibernation Model with Genic Mutation and Impulsive Effects

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Abstract. A predator and prey-hibernation system with mutation and impulsive effects is presented in this work. The globally asymptotically stable solution $(\widetilde{x}, \widetilde{y}, 0)$ of system (1) exists. System (1) is proved to be permanent. Finally, simulations are presented to explain the results.

Keywords: Mutation · Predator and prey-hibernation system · Permanence

1 Introduction

Many authors [1,2] indicated that environmental pollutants caused many diseases. Population dynamics are studied by the theories of impulsive differential equations [3–6]. Jiao et al. [7] constructed a predator-prey system with periodic switches and impulses. Jiao and Chen [8] presented a predator-prey system with mutation and impulses. vFor convenience, we make notation $N = n\tau$, and $N + l = (n + l)\tau$.

2 The Model

In this work, we presented a predator and prey-hibernation system with mutation and impulses

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$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= -ax - cx^2 - \beta_1 xz, \\ \frac{dy}{dt} &= -d_1 y - \beta_2 yz, \\ \frac{dz}{dt} &= -d_2 z + k_1\beta_1 xz \\ &\qquad\qquad + k_2\beta_2 yz, \end{aligned}\right\} t \in (N, N+l], \\ \left.\begin{aligned} \triangle x &= 0, \\ \triangle y &= -\mu_1 y, \\ \triangle z &= -\mu_2 z, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} \frac{dx}{dt} &= -d_3 x - \beta_3 xz, \\ \frac{dy}{dt} &= -d_4 y - \beta_4 yz, \\ \frac{dz}{dt} &= -d_5 z + k_3\beta_3 xz \\ &\qquad\qquad + k_4\beta_4 yz, \end{aligned}\right\} t \in (N+l, N+1], \\ \left.\begin{aligned} \triangle x &= (1-\theta_1)b_1 x, \\ \triangle y &= \theta_1 b_1 x, \\ \triangle z &= 0, \end{aligned}\right\} t = N+1, \end{cases} \tag{1}$$

The biological meanings of the varies and the parameters can reference to [7,8].

3 The Lemmas

If $z = 0$, we can easily have the subsystem of system (1)

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &= -ax - cx^2, \\ \frac{dy}{dt} &= -d_1 y, \end{aligned}\right\} t \in (N, N+l], \\ \left.\begin{aligned} \triangle x &= 0, \\ \triangle y &= -\mu_1 y, \end{aligned}\right\} t = N+l, \\ \left.\begin{aligned} \frac{dx}{dt} &= -d_3 x, \\ \frac{dy}{dt} &= -d_4 y, \end{aligned}\right\} t \in (N+l, N+1], \\ \left.\begin{aligned} \triangle x &= (1-\theta_1)b_1 x, \\ \triangle y &= \theta_1 b_1 x, \end{aligned}\right\} t = N+1. \end{cases} \tag{2}$$

Integrating (2), and the stroboscopic map of (2) is

$$\begin{cases} x((N+1^{+}) = [1+(1-\theta_1)b_1] \times \dfrac{ax(N^{+})e^{-[al+d_3(1-l)]\tau}}{a+cx(N^{+})(1-e^{-al\tau})}, \\ y((N+1)^{+}) = \theta_1 b_1 \times \dfrac{ax(N^{+})e^{-[al+d_3(1-l)]\tau}}{a+cx(N^{+})(1-e^{-al\tau})} \\ \qquad\qquad + (1-\mu_1)e^{-[d_1 l+d4(1-l)]\tau} y(N^{+}). \end{cases} \tag{3}$$

Points $G_1(0,0)$ and $G_2(x^*, y^*)$ are gotten as

$$\begin{cases} x^* = Ay^*, \quad \theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1. \\ y^* = \dfrac{a}{cA(1-e^{-al\tau})} \times [\dfrac{\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau}}{1-B} - 1], \\ \qquad\qquad \theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1, \end{cases} \tag{4}$$

here $A = \frac{[1+(1-\theta_1)b_1](1-B)}{\theta_1 b_1} > 0$ and $B = (1-\mu_1)e^{-[d_1 l + d_4(1-l)]\tau} > 0$.

Similarly with reference [7], the following lemmas can easily be obtained.

Lemma 1. *i*) If $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B < 1$, point $G_1(0,0)$ is globally asymptotically stable;

ii) If $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1$, point $G_2(x^*, y^*)$ is globally asymptotically stable.

Lemma 2. *i*) If $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B < 1$, periodic solution $(0,0)$ of system (2) is globally asymptotically stable;

ii) If $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1$, periodic solution $(\widetilde{x}, \widetilde{y})$ of system (2) is globally asymptotically stable, where

$$\begin{cases} \widetilde{x} = \begin{cases} \dfrac{ax^* e^{-a(t-N)}}{a+cx^*(1-e^{-a(t-N)})}, t \in (N, (N+1)], \\ x^{**} e^{-d_3(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \\ \widetilde{y} = \begin{cases} y^* e^{-d_1(t-N)}, t \in (N, (N+l)], \\ y^{**} e^{-d_4(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \end{cases} \tag{5}$$

here x^*, y^* are determined as (5) and x^{**}, y^{**} are determined as

$$\begin{cases} x^{**} = \dfrac{ax^* e^{-al\tau}}{a+cx^*(1-e^{-al\tau})}, \\ y^{**} = (1-\mu_1)y^* e^{-d_1 l\tau}. \end{cases} \tag{6}$$

For (1), we know

$$\begin{cases} \left.\begin{aligned} \frac{dx}{dt} &\le -ax - cx^2, \\ \frac{dy}{dt} &\le -d_1 y, \end{aligned}\right\} t \in (N, (N+l)], \\ \left.\begin{aligned} \triangle x &= 0, \\ \triangle y &= -\mu_1 y, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} \frac{dx}{dt} &\le -d_3 x, \\ \frac{dy}{dt} &\le -d_4 y, \end{aligned}\right\} t \in ((N+l), (N+1)], \\ \left.\begin{aligned} \triangle x &= (1-\theta_1) b_1 x, \\ \triangle y &= \theta_1 b_1 x, \end{aligned}\right\} t = (N+1). \end{cases} \tag{7}$$

Then, we can obtain the following remark.

Remark 3. If $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1$, then

$$x(t) \le x^* + x^{**},$$

and

$$y(t) \le y^* + y^{**},$$

with (x, y) of system (1).

Similar to Ref. [13], we get

Lemma 4. There exists a constant $M > 0$, which makes $x \le \frac{M}{k}, y \le M$ and $z \le M$ for all t large enough.

4 The Dynamics

Theorem 1. Suppose

(H_1):

$$\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1,$$

(H_2):

$$\ln \frac{1}{1+(1-\theta_1)b_1} + al\tau + 2\ln \frac{a + cx^*(1-e^{-al\tau})}{a} + d_3(1-l)\tau > 0,$$

(H_3):

$$\ln \frac{1}{1-\mu_2} + d_2 l\tau + d_5(1-l)\tau$$

$$> \frac{k_1\beta_1}{c} \ln \frac{a + cx^*(1-e^{-al\tau})}{a} + \frac{k_2\beta_2 y^*(1-e^{-d_1 l\tau})}{d_1}$$

$$+\frac{k_3\beta_3 x^{**}e^{-d_3(1-l)\tau}(1-e^{-d_1(1-l)\tau})}{d_3}+\frac{k_4\beta_4 y^{**}e^{-d_5(1-l)\tau}(1-e^{-d_5(1-l)\tau})}{d_5},$$

hold, the solution $(\widetilde{x},\widetilde{y},0)$ of (1) is globally asymptotically stable, and x^*,x^{**} are by (5), y^*,y^{**} are by (7).

Proof. Defining $x_1=x-\widetilde{x}, y_1=y-\widetilde{y}, z_1=z$, for $t\in(N,(N+l)]$, the linear system of (1) are as

$$\begin{pmatrix}\frac{dx_1}{dt}\\ \frac{dy_1}{dt}\\ \frac{dz_1}{dt}\end{pmatrix}=\begin{pmatrix}-a-2c\widetilde{x} & 0 & -\beta_1\widetilde{x}\\ 0 & -d_1 & -\beta_2\widetilde{y}\\ 0 & 0 & -d_2+k_1\beta_1\widetilde{x}+k_2\beta_2\widetilde{y}\end{pmatrix}\begin{pmatrix}x_1\\ y_1\\ z_1\end{pmatrix}.$$

Then, we get the fundamental solution matrix

$$\Phi_1(t)=\begin{pmatrix}\exp(\int_N^t(-a-2c\widetilde{x})ds) & *_1 & *_2\\ 0 & \exp(-d_1(t-N)) & *_3\\ 0 & 0 & M_1\end{pmatrix}$$

where $M_1=\exp[\int_N^t(-d_2+k_1\beta_1\widetilde{x}+k_2\beta_2\widetilde{y})ds]$.

For $t\in((N+l),(N+1)]$, the linear system of (1) are also as

$$\begin{pmatrix}\frac{dx_1}{dt}\\ \frac{dy_1}{dt}\\ \frac{dz_1}{dt}\end{pmatrix}=\begin{pmatrix}-d_3 & 0 & -\beta_3\widetilde{x}\\ 0 & -d_4 & -\beta_4\widetilde{y}\\ 0 & 0 & -d_5+k_3\beta_3\widetilde{x}+k_4\beta_4\widetilde{y}\end{pmatrix}\begin{pmatrix}x_1\\ y_1\\ z_1\end{pmatrix}.$$

Then, we also get the fundamental solution matrix

$$\Phi_2(t)=\begin{pmatrix}\exp(-d_3(t-(N+l)) & *_4 & *_5\\ 0 & \exp(-d_4(t-(N+l)) & *_6\\ 0 & 0 & M_2\end{pmatrix},$$

where $M_2=\exp[\int_{(N+l)}^t(-d_5+k_3\beta_3\widetilde{x}+k_4\beta_4\widetilde{y})ds]$. There is no need to calculate the form of $*_i(i=1,2,3,4,5,6)$.

The linearization of the 4th, to 6th equations of (1) is

$$\begin{pmatrix}x_1((N+l)^+)\\ y_1((N+l)^+)\\ z_1((N+l)^+)\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1-\mu_1 & 0\\ 0 & 0 & 1-\mu_2\end{pmatrix}\begin{pmatrix}x_1((N+l))\\ y_1((N+l))\\ z_1((N+l))\end{pmatrix}.$$

The linearization of the 10th, to 12th equations of (1) is

$$\begin{pmatrix} x_1((N+1)^+) \\ y_1((N+1)^+) \\ z_1((N+1)^+) \end{pmatrix} = \begin{pmatrix} 1+(1-\theta_1)b_1 & 0 & 0 \\ \theta_1 b_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1((N+1)) \\ y_1((N+1)) \\ z_1((N+1)) \end{pmatrix}.$$

The stability of $(\widetilde{x}, \widetilde{y}, 0)$ is by eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\mu_1 & 0 \\ 0 & 0 & 1-\mu_2 \end{pmatrix} \begin{pmatrix} 1+(1-\theta_1)b_1 & 0 & 0 \\ \theta_1 b_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(\tau),$$

which are

$$\lambda_1 = [1+(1-\theta_1)b_1] \exp\Big(\int_0^{l\tau} (-a - 2c\widetilde{x}) ds + d_3(1-l)\tau\Big),$$

$$\lambda_2 = (1-\mu_1) \exp(-[d_1 l + d_4(1-l)]\tau) < 1,$$

and

$$\lambda_3 = (1-\mu_2) \exp\Big[\int_0^{l\tau} (-d_2 + k_1\beta_1\widetilde{x} + k_2\beta_2\widetilde{y}) ds + \int_{l\tau}^{\tau} (-d_5 + k_3\beta_3\widetilde{x} + k_4\beta_4\widetilde{y}) ds\Big].$$

According the conditions of Theorem 1, then $\lambda_1 < 1$, and $\lambda_3 < 1$. Therefore, the $(\widetilde{x}, \widetilde{y}, 0)$ is locally stable.

Choose $\varepsilon > 0$, we get

$$\rho_1 = (1-\mu_2) \exp\Big\{\int_0^{l\tau} [-d_2 + k_1\beta_1(\widetilde{x}+\varepsilon) + k_2\beta_2(\widetilde{y}+\varepsilon)] ds + \int_{l\tau}^{\tau} [-d_5 + k_3\beta_3(\widetilde{x}+\varepsilon) + k_4\beta_4(\widetilde{y}+\varepsilon)] ds\Big\} < 1.$$

From (1), we get $\frac{dx}{dt} \le -ax - cx^2, \frac{dy}{dt} \le -d_1 x$, and $\frac{dx}{dt} \le -d_3 x$, $\frac{dy}{dt} \le -d_4 x$, so

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= -ax_1(t) - cy_1^2, \\ \frac{dy_1}{dt} &= -d_1 y_1, \end{aligned}\right\} t \neq (N, (N+l)], \\ \left.\begin{aligned} \triangle x_1 &= 0, \\ \triangle y_1 &= -\mu_1 y_1, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} \frac{dx_1}{dt} &= -d_3 x_1, \\ \frac{dy_1}{dt} &= -d_4 y_1, \end{aligned}\right\} t \in ((N+l), (N+1), \\ \left.\begin{aligned} \triangle x_1 &= (1-\theta_1) b_1 x_1, \\ \triangle y_1 &= \theta_1 b_1 y_1, \end{aligned}\right\} t = (N+1), \end{cases} \tag{8}$$

From Lemma 3, we have $x \leq x_1, y \leq y_1$, and

$$\begin{cases} x \leq x_1 \leq \widetilde{x} + \varepsilon, \\ y \leq y_1 \leq \widetilde{y} + \varepsilon, \end{cases} \tag{9}$$

From (1) and (14), we also get

$$\begin{cases} \dfrac{dz}{dt} \leq z[-d_2 + k_1\beta_1(\widetilde{x} + \varepsilon) + +k_2\beta_2(\widetilde{y} + \varepsilon)], \\ \qquad\qquad\qquad\qquad t \in (N, (N + l)] \\ \triangle z = -\mu_2 z, t = (N + l), \\ \dfrac{dz}{dt} \leq z[-d_5 + k_3\beta_3(\widetilde{x} + \varepsilon) + k_4\beta_4(\widetilde{y} + \varepsilon)], \\ \qquad\qquad\qquad\qquad t \in ((N + l), (N + 1)], \end{cases} \tag{10}$$

Then,

$$z((N+1)) \leq (1-\mu_2)z(N^+)\exp[\int_N^{(N+l)} (-d_2 + k_1\beta_1(\widetilde{x} + \varepsilon) + k_2\beta_2(\widetilde{y} + \varepsilon)ds$$
$$+ \int_{(N+l)}^{(N+1)} (-d_5 + k_3\beta_3(\widetilde{x} + \varepsilon) + k_4\beta_4(\widetilde{y} + \varepsilon))ds].$$

and $z((N + l)) \leq z(l\tau^+)\rho_1^n$ with $z((N + l)) \to 0$ as $n \to \infty$. For $0 < z \leq z((N + l))e^{(-d_5(1-l)\tau + \frac{k_3\beta_3}{d_3}x^{**} + \frac{k_4\beta_4}{d_4}y^{**})}$ for $N < t \leq (N + l)$. Therefore $z \to 0$ as $t \to \infty$.

For (1) and $t \in (N, (N + l)]$, we have

$$\begin{cases} -(a + \beta_1\varepsilon)x - cx^2 \leq \dfrac{dx}{dt} \leq -ax - cx^2, \\ -(d_1 + \beta_2\varepsilon)y \leq \dfrac{dy}{dt} \leq -d_1 y, \end{cases} \tag{11}$$

and for $t \in ((N + l), (N + 1)]$,

$$\begin{cases} -(d_3 + \beta_3\varepsilon)x \leq \dfrac{dx}{dt} \leq -d_1 x, \\ -(d_4 + \beta_4\varepsilon)y \leq \dfrac{dy}{dt} \leq -d_4 y, \end{cases} \tag{12}$$

then, $w_1 \le x \le m_1, w_2 \le y \le m_2$, with $w_1 \to \widetilde{w_1}, w_2 \to \widetilde{w_2}, m_1 \to \widetilde{x}, m_2 \to \widetilde{y}$, as $t \to \infty$. While (w_1, w_2) and (m_1, m_2) are the solutions of

$$\begin{cases} \left.\begin{aligned} \frac{dw_1}{dt} &= -(a+\beta_1\varepsilon)w_1 - cw_1^2, \\ \frac{dw_2}{dt} &= -(d_1+\beta_2\varepsilon)w_2, \end{aligned}\right\} t \in (N,(N+l)), \\ \left.\begin{aligned} \triangle w_1 &= 0, \\ \triangle w_2 &= -\mu_1 w_2, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} \frac{dw_1}{dt} &= -(d_3+\beta_3\varepsilon)w_1, \\ \frac{dw_2}{dt} &= -(d_4+\beta_4\varepsilon)w_2, \end{aligned}\right\} t \in ((N+l),(N+1)], \\ \left.\begin{aligned} \triangle w_1 &= (1-\theta_1)b_1 w_1), \\ \triangle w_2 &= \theta_1 b_1 w_1), \end{aligned}\right\} t = (N+1), \end{cases} \tag{13}$$

and

$$\begin{cases} \left.\begin{aligned} \frac{dm_1}{dt} &= -am_1 - cm_1^2, \\ \frac{dm_2}{dt} &= -d_1 m_2, \end{aligned}\right\} t \in (N,(N+l)), \\ \left.\begin{aligned} \triangle m_1 &= 0, \\ \triangle m_2 &= -\mu_1 m_2, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} \frac{dm_1}{dt} &= -d_3 m_1, \\ \frac{dm_2}{dt} &= -d_4 m_2, \end{aligned}\right\} t \in ((N+l),(N+1)], \\ \left.\begin{aligned} \triangle m_1 &= (1-\theta_1)b_1 m_1, \\ \triangle m_2 &= \theta_1 b_1 m_1, \end{aligned}\right\} t = (N+1). \end{cases} \tag{14}$$

Here $(\widetilde{w_1}, \widetilde{w_2})$ can be expressed as

$$\begin{cases} \widetilde{w_1} = \begin{cases} \dfrac{(a+\beta_1\varepsilon)w_1^* e^{-(a+\beta_1\varepsilon)(t-N)}}{(a+\beta_1\varepsilon)+cw_1^*(1-e^{-(a+\beta_1\varepsilon)(t-N)})}, t \in (N,(N+1)], \\ w_1^{**} e^{-(d_3+\beta_3\varepsilon)(t-(N+l))}, t \in ((N+l),(N+1)], \end{cases} \\ \widetilde{w_2} = \begin{cases} w_2^* e^{-(d_1+\beta_2\varepsilon)(t-N)}, t \in (N,(N+l)], \\ w_2^{**} e^{-(d_4+\beta_4\varepsilon)(t-(N+l))}, t \in ((N+l),(N+1)], \end{cases} \end{cases} \tag{15}$$

while w_1^*, w_2^* are determined as

$$\begin{cases} w_1^* = A_1 w_2^*, \ \ \theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau} + B_1 > 1, \\ w_2^* = \dfrac{(a+\beta_1\varepsilon)}{cA_1(1-e^{-(a+\beta_1\varepsilon)l\tau})} \times [\dfrac{\theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau}}{1-B_1} - 1], \\ \qquad\qquad\qquad\qquad \theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau} + B_1 > 1, \end{cases} \tag{16}$$

where $A_1 = \frac{[1+(1-\theta_1)b_1](1-B_1)}{\theta_1 b_1} > 0$ and $B_1 = (1-\mu)e^{-[(d_1+\beta_2\varepsilon)l+(d_4+\beta_4\varepsilon)(1-l)]\tau} > 0$, and w_1^{**}, w_2^{**} are determined as

$$\begin{cases} w_1^{**} = \dfrac{(a+\beta_1\varepsilon)w_1^* e^{-(a+\beta_1\varepsilon)l\tau}}{(a+\beta_1\varepsilon)+cw_1^*(1-e^{-(a+\beta_1\varepsilon)l\tau})}, \\ w_2^{**} = (1-\mu_1)w_2^* e^{-(d_1+\beta_2\varepsilon)l\tau}. \end{cases} \tag{17}$$

For any $\varepsilon_1 > 0$, there exists a $t_1, t > t_1$ such that

$$\widetilde{w_1} - \varepsilon_1 < x < \widetilde{x} + \varepsilon,$$

and

$$\widetilde{w_2} - \varepsilon_1 < y < \widetilde{y} + \varepsilon.$$

Let $\varepsilon \to 0$, we have

$$\widetilde{x} - \varepsilon_1 < x < \widetilde{x} + \varepsilon_1,$$

and

$$\widetilde{y} - \varepsilon_1 < y < \widetilde{y} + \varepsilon_1,$$

for t large enough, which indicates $x \to \widetilde{x}, y \to \widetilde{y}$ as $t \to \infty$.

Theorem 2. If (H_1), (H_2) and (H_4):

$$\ln\frac{1}{1-\mu_2} + d_2 l\tau + d_5(1-l)\tau$$

$$< \frac{k_1\beta_1}{c}\ln\frac{a+cx^*(1-e^{-al\tau})}{a} + \frac{k_2\beta_2 y^*(1-e^{-d_1 l\tau})}{d_1}$$

$$+\frac{k_3\beta_3 x^{**}e^{-d_3(1-l)\tau}(1-e^{-d_1(1-l)\tau})}{d_3} + \frac{k_4\beta_4 y^{**}e^{-d_5(1-l)\tau}(1-e^{-d_5(1-l)\tau})}{d_5},$$

hold, (1) is permanent, where x^*, x^{**} are by (5) and y^*, y^{**} are by (7).

Proof. By Lemma 3, we get that $x(t) \leq \frac{M}{k}, y(t) \leq \frac{M}{k}, z(t) \leq M$ for t large enough. From Theorem 1, we know $x(t) \geq \frac{ax^* e^{-al\tau}}{a+cx^*(1-e^{-al\tau})} + x^{**}e^{-d_3(1-l)\tau} - \varepsilon_2 = m_{21}$ and $y(t) \geq y^* e^{-d_1 l\tau} + y^{**}e^{-d_4(1-l)\tau} - \varepsilon_2 = m_{22}$ for t. Thus, we will seek out $m_1 > 0$ making $y(t) \geq m_1$.

By the conditions H_4, we can choose $m_3 > 0, \varepsilon_1 > 0$ small enough to have

$$\sigma = \frac{k_1\beta_1}{c} \ln \frac{(a+\beta_1 m_3) + cz_1^*(1-e^{-(a+\beta_1 m_3)l\tau})}{(a+\beta_1 m_3)} + k_1\beta_1\varepsilon_1 l\tau$$

$$+ \frac{k_2 + \beta_2 z_2^*(1-e^{-(d_1+\beta_1 m_3)l\tau})}{(d_1+\beta_1 m_3)l\tau + k_2\beta_2 m_3 l\tau} + k_2\beta_2\varepsilon_1 l\tau$$

$$+ \frac{k_3\beta_3 z_1^{**} e^{-(d_3+\beta_1 m_3)(1-l)\tau}(1-e^{-(d_1+\beta_1 m_3)(1-l)\tau})}{(d_3+\beta_1 m_3)} + k_3\beta_3\varepsilon_1(1-l)\tau$$

$$+ \frac{k_4\beta_4 z_2^{**} e^{-(d_5+\beta_5 m_3)(1-l)\tau}(1-e^{-(d_5+\beta_5 m_3)(1-l)\tau})}{(d_5+\beta_5 m_3)} + k_4\beta_4\varepsilon_1(1-l)\tau,$$

$$-\ln\frac{1}{1-\mu_2} - d_2 l\tau - d_5(1-l)\tau] > 0,$$

here z_1^*, z_2^* are by

$$\begin{cases} z_1^* = A_2 z_2^*, \quad \theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau} + B_2 > 1, \\ z_2^* = \dfrac{(a+\beta_1 m_3)}{cA_2(1-e^{-(a+\beta_1 m_3)l\tau})} \\ \qquad\qquad \times [\dfrac{\theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau}}{1-B_2} - 1], \\ \qquad\qquad \theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau} + B_2 > 1, \end{cases} \tag{18}$$

and z_1^{**}, z_2^{**} are by

$$\begin{cases} z_1^{**} = \dfrac{(a+\beta_1 m_3) z_1^* e^{-(a+\beta_1 m_3)l\tau}}{(a+\beta_1 m_3) + cz_1^*(1-e^{-(a+\beta_1 m_3)l\tau})}, \\ z_2^{**} = (1-\mu_1) z_2^* e^{-(d_1+\beta_2 m_3)l\tau}. \end{cases} \tag{19}$$

with $A_2 = \frac{[1+(1-\theta_1)b_1](1-B_2)}{\theta_1 b_1} > 0$ and $B_2 = (1-\mu) e^{-[(d_1+\beta_2 m_3)l+(d_4+\beta_4 m_3)(1-l)]\tau} > 0$. $y(t) < m_3$ will be proved that it can not establish. Otherwise,

$$\begin{cases} \left.\begin{aligned} &\frac{dx}{dt} \geq -(a+\beta_1 m_3)x - cx^2, \\ &\frac{dy}{dt} \geq -(d_1+\beta_2 m_3)y, \end{aligned}\right\} t \in (N, (N+l)], \\ \left.\begin{aligned} &\triangle x = 0, \\ &\triangle y = -\mu_1 y, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} &\frac{dx}{dt} \geq -(d_3+\beta_3 m_3)x, \\ &\frac{dy}{dt} \geq -(d_4+\beta_4 m_3)y, \end{aligned}\right\} t \in ((N+l), (N+1)], \\ \left.\begin{aligned} &\triangle x = (1-\theta_1)b_1 x, \\ &\triangle x = \theta_1 b_1 x, \end{aligned}\right\} t = (N+1). \end{cases} \tag{20}$$

By Lemma 4, we have $x \geq z_1$, $y \geq z_2$ with $z_1 \to \widetilde{z}_1, z_2 \to \widetilde{z}_2, t \to \infty$, where (z_1, z_2) is the solution of

$$\begin{cases} \left.\begin{aligned} &\frac{dz_1}{dt} = -(a+\beta_1 m_3)z_1 - cz_1^2, \\ &\frac{dz_2}{dt} = -(d_1+\beta_2 m_3)z_2 \end{aligned}\right\} t \in (N, (N+l)], \\ \left.\begin{aligned} &\triangle z_1 = 0, \\ &\triangle z_2 = -\mu_1 z_2, \end{aligned}\right\} t = (N+l), \\ \left.\begin{aligned} &\frac{dz_1}{dt} = -(d_3+\beta_3 m_3)z_1, \\ &\frac{dz_2}{dt} = -(d_4+\beta_4 m_3)z_2, \end{aligned}\right\} t \in ((N+l), (N+1)], \\ \left.\begin{aligned} &\triangle z_1 = (1-\theta_1)b_1 z_1, \\ &\triangle z_2 = \theta_1 b_1 z_2, \end{aligned}\right\} t = (N+1), \end{cases} \tag{21}$$

and

$$\begin{cases} \widetilde{z}_1 = \begin{cases} \dfrac{(a+\beta_1 m_3)z_1^* e^{-(a+\beta_1 m_3)(t-n\tau)}}{(a+\beta_1 m_3) + cz_1^*(1-e^{-(a+\beta_1 m_3)(t-N)})}, t \in (N, (N+1)], \\ z_1^{**} e^{-(d_3+\beta_3 m_3)(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \\ \widetilde{z}_2 = \begin{cases} z_2^* e^{-(d_1+\beta_2 m_3)(t-N)}, t \in (N, (N+l)], \\ z_2^{**} e^{-(d_4+\beta_4 m_3)(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \end{cases} \tag{22}$$

here z_1^*, z_2^* are by (18) and z_1^{**}, z_2^{**} are by (19) with $A_2 = \frac{[1+(1-\theta_1)b_1](1-B_2)}{\theta_1 b_1} > 0$ and $B_2 = (1-\mu)e^{-[(d_1+\beta_2 m_3)l + (d_4+\beta_4 m_3)(1-l)]\tau} > 0$. Therefore, there exists a $T_1 > 0$ such that

$$x \geq z_1 \geq \widetilde{z}_1 - \varepsilon_1,$$

and

$$y \geq z_2 \geq \widetilde{z_2} - \varepsilon_1.$$

Then

$$\begin{cases} \dfrac{dz}{dt} \geq [-d_1 + k_1\beta_1(\widetilde{z_1} - \varepsilon_1) + k_2\beta_2(\widetilde{z_2} - \varepsilon_1)]z, \\ \qquad\qquad\qquad\qquad t \in (N, (N+l)], \\ \triangle z = -\mu_2 z, t = (N+l), \\ \dfrac{dz}{dt} \geq [-d_5 + k_3\beta_3(\widetilde{z_1} - \varepsilon_1) + k_4\beta_4(\widetilde{z_2} - \varepsilon_1)]z, \\ \qquad\qquad\qquad\qquad t \in ((N+l), (N+1)]. \end{cases} \tag{23}$$

For $t \geq T_1$, Let $N_1 \in N$ and $N_1\tau > T_1$, integrating (28) on $(N, (N+1)], n \geq N_1$, we have

$$y((N+1)) \geq z(N^+)(1-\mu_1)$$
$$\times e^{\int_N^{(N+l)}[-d_1+k_1\beta_1(\widetilde{z_1}-\varepsilon_1)+k_2\beta_2(\widetilde{z_2}-\varepsilon_1)]ds+\int_{(N+l)}^{(N+1)}[-d_5+k_3\beta_3(\widetilde{z_1}-\varepsilon_1)+k_4\beta_4(\widetilde{z_2}-\varepsilon_1)]ds}$$
$$= (1-\mu_1)z(N^+)e^{\sigma},$$

then $z((N_1+k)\tau) \geq (1-\mu_1)^k z(N_1\tau^+)e^{k\sigma} \to \infty$, as $k \to \infty$, which is a contradiction to the boundedness of z. Hence there exists a $t_1 > 0$ such that $z(t) \geq m_1$. The proof is complete.

5 Discussion

In this work, we consider a predator and prey-hibernation model with genic mutation and impulsive effects. If it is supposed that the variables are shown in the table below:

$x(0)$	$y(0)$	$z(0)$	a	c	d_1	d_2	d_3	d_4	d_5	β_1	β_2	β_3	β_4	k_1	k_2	k_3	k_4	θ_1	b_1	μ_1	μ_2	l	τ
1	1	1	0.3	0.1	0.3	0.3	0.3	0.1	0.1	0.4	0.6	0.3	0.4	0.1	0.1	0.1	0.1	0.6	1.5	0.4	0.1	0.5	1

system (1) is permanent (one can see Fig. 1). If it is supposed that another variables are shown in the table below:

$x(0)$	$y(0)$	$z(0)$	a	c	d_1	d_2	d_3	d_4	d_5	β_1	β_2	β_3	β_4	k_1	k_2	k_3	k_4	θ_1	b_1	μ_1	μ_2	l	τ
1	1	1	0.3	0.1	0.3	0.3	0.3	0.1	0.1	0.4	0.6	0.3	0.4	0.1	0.1	0.1	0.1	0.6	1.5	0.4	0.1	0.5	1

there exists a globally asymptotically stable solution $(\widetilde{x}, \widetilde{y}, 0)$ of system (1) (one can see Fig. 2). Our results show that the environmental pollution will reduce biological diversity of the nature world. So we must be in harmony with the environment.

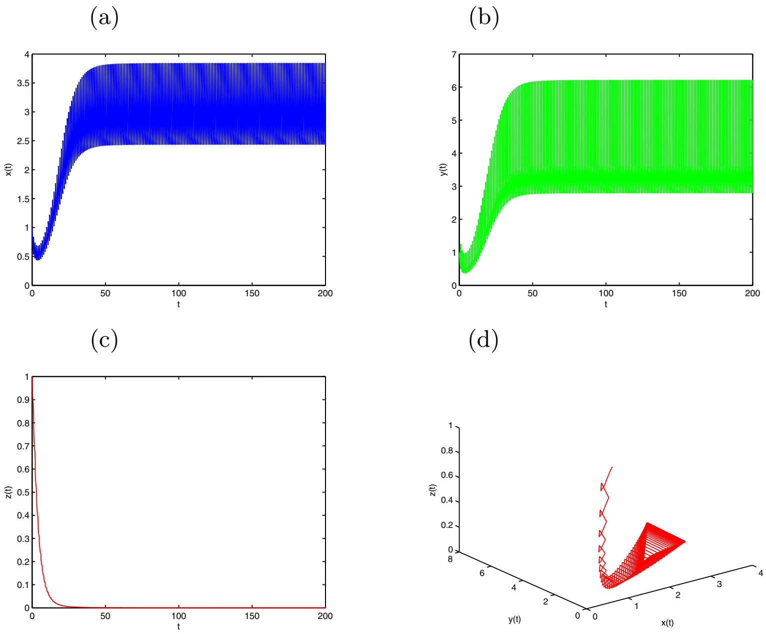

Fig. 1. The permanence of system (2.1) with parameters in the first table.

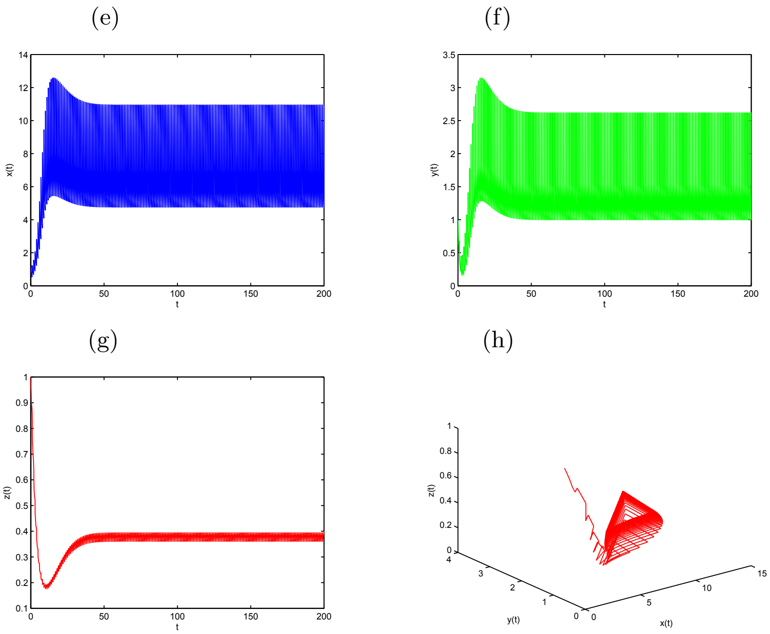

Fig. 2. The dynamics of the globally asymptotically stable microorganism-extinction with parameters in the second table.

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