Dynamics of a Single Population Model with Non-transient/Transient Impulsive Harvesting and Birth Pulse in a Polluted Environment

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Abstract. In this paper, we present a single population model with non-transient/transient impulsive harvesting and birth pulse in a polluted environment. The sufficient conditions for system permanence is presented.

Keywords: Single population system · Non-transient/transient impulses · Birth pulse · Permanence

1 Introduction

Many investigations [1–4] devoted into impulsive equations. Clack [5] has studied the logistic equation with optimal harvesting. The environmental toxicant decreases the carrying capacity in polluted environments [6,7]. They are assumed that the inputting toxicant was continuous. Liu et al. [8] considered that the environmental toxicant is often emitted with regular pulse. In this paper, we do notation as $N = n\tau$ and $L = l\tau$.

2 The Model

In this work, we present

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$$
\begin{cases}
\dfrac{dx}{dt} = -(d_1 + \beta_1 c_o)x + c_1 x^2,\ t \in (N, (N+L)], \\
\left.\begin{aligned}
&\triangle x = -\mu x, \\
&\triangle c_0 = 0, \\
&\triangle c_{e1} = 0, \\
&\triangle c_{e2} = 0,
\end{aligned}\right\} t = (N+L), \\
\dfrac{dx}{dt} = -(d_2 + E + \beta_2 c_o(t))x + c_2 x^2,\ t \in ((N+L), (N+1)], \\
\left.\begin{aligned}
&\triangle x = bx, \\
&\triangle c_0 = 0, \\
&\triangle c_{e1} = D(c_{e2} - c_{e1}) + \mu_1, \\
&\triangle c_{e2} = D(c_{e1} - c_{e2}) + \mu_2,
\end{aligned}\right\} t = (N+1), \\
\left.\begin{aligned}
&\dfrac{dc_o}{dt} = fc_{e1} - (g+m)c_o, \\
&\dfrac{dc_{e1}}{dt} = -h_1 c_{e1}), \\
&\dfrac{dc_{e2}}{dt} = -h_2 c_{e2},
\end{aligned}\right\} t \in (N, (N+1)].
\end{cases}
\tag{1}
$$

The biological meanings of the varies and parameters can reference [8–10].

3 The Dynamics

For (1), one subsystem of (1) is

$$
\begin{cases}
\dfrac{dx}{dt} = -d_1 x + c_1 x^2,\ t \in (N, (N+L)], \\
\triangle x = -\mu x, t = (N+L), \\
\dfrac{dx}{dt} = -(d_2 + E)x + c_2 x^2,\ t \in ((N+L), (N+1)], \\
\triangle x = bx, t = (N+1).
\end{cases}
\tag{2}
$$

Integrating (2), we get

$$
x(t) = \begin{cases}
\dfrac{d_1 x(N^+) e^{-d_1(t-N)}}{d_1 + c_1 x(N^+)(1 - e^{-d_1(t-N)})}, t \in (N, (N+L)], \\
\dfrac{(d_2+E) x((N+l)^+) e^{-(d_2+E)(t-N)}}{(d_2+E) + c_2 x((N+l)^+)(1 - e^{-(d_2+E)(t-N)})}, \\
\qquad t \in ((N+L), (N+1)].
\end{cases}
\tag{3}
$$

Stroboscopic map for (2) is

$$x((n+1)\tau^+) = \frac{M_1}{M_2}, \tag{4}$$

where $M_1 = (1+b)(1-\mu)d_1(d_2+E)e^{-[d_1 l+(d_2+E)(1-l)]\tau}x(N^+)$, $M_2 = d_1(d_2+E) + [(d_2+E)c_1(1-e^{-d_1 l}) + (1-\mu)d_1 c_2 e^{-d_1 l\tau}(1-e^{-(d_2+E)(1-l)\tau})]x(N^+)$.
We rewrite (4) as

$$x((N+1)^+) = \frac{A_0 x(N^+)}{d_1(d_2+E) + B_0 x(N^+)}. \tag{5}$$

where

$$A_0 = (1+b)(1-\mu)d_1(d_2+E)e^{-[d_1 l+(d_2+E)(1-l)]\tau},$$
$$B_o = (d_2+E)c_1(1-e^{-d_1 l}) + (1-\mu)d_1 c_2 e^{-d_1 l\tau}(1-e^{-(d_2+E)(1-l)\tau}).$$

We get points $G_1(0)$ and $G_2(x^*)$ of (4), and

$$x^* = \frac{d_1(d_2+E)[(1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} - 1]}{(d_2+E)c_1(1-e^{-d_1 l}) + (1-\mu)d_1 c_2 e^{-d_1 l\tau}(1-e^{-(d_2+E)(1-l)\tau})}, \quad (1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} > 1. \tag{6}$$

Condition $(1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} < 1$ is made as (C_1), and condition $(1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} > 1$ is made as (C_2).

Theorem 1. *i*) Suppose (C_1) holds $G_1(0)$ of (4) is globally asymptotically stable;

ii) Suppose (C_2) holds, $G_2(x^*)$ is globally asymptotically stable.

Proof. Marking x^n as $x(n\tau^+)$, we rewrite (4) as

$$x_{n+1} = F(x_n) = \frac{Ax_n}{d_1(d_2+E) + Bx_n}. \tag{7}$$

i) Suppose (C_1) holds, then,

$$\frac{dF(x)}{dx}\Big|_{x=0} = (1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} < 1. \tag{8}$$

Then, locally stable $G_1(0)$ exists, Furthermore, it is globally asymptotically stable.

ii) Suppose (C_2) holds,concerning to $G_1(0)$, we get

$$\begin{aligned}\frac{dF(x)}{dx}\Big|_{x=0} = \frac{Ad_1(d_2+E)}{[d_1(d_2+E)+Bx]^2}\Big|_{x=0} \\ = \frac{A}{d_1(d_2+E)}. \\ = (1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau} > 1,\end{aligned} \tag{9}$$

then, $G_1(0)$ is unstable.

Concerning $G_2(x^*)$, we also get

$$\begin{aligned}\frac{dF(x)}{dx}\mid_{x=x^*} &= \frac{Ad_1(d_2+E)}{[d_1(d_2+E)+Bx]^2}\mid_{x=x^*} \\ &= \frac{1}{(1+b)(1-\mu)e^{-(d_1 l+(d_2+E)(1-l))\tau}} < 1.\end{aligned} \tag{10}$$

Then, locally stable $G_2(x^*)$ exists. Furthermore, it is globally asymptotically stable.

Similarly with Reference [6], the following lemma can easily be proved.

Theorem 2. *i*) Suppose (C_1) holds, 0 of (1) is globally asymptotically stable; *ii*) Suppose (C_2) holds, $\widetilde{x(t)}$ of (1) is globally asymptotically stable, and

$$\widetilde{x(t)} = \begin{cases} \dfrac{d_1 x^* e^{-d_1(t-N)}}{d_1 + c_1 x^*(1-e^{-d_1(t-N)})}, t \in (N, (N+L)], \\ \dfrac{(d_2+E)x^{**}e^{-(d_2+E)(t-N)}}{(d_2+E)+c_2x^{**}(1-e^{-(d_2+E)(t-N)})}, t \in ((N+L),(N+1)]. \end{cases} \tag{11}$$

here x^* is by (6), and x^{**} is by $x^{**} = \frac{(1-\mu_1)d_1x^*e^{-d_1 l\tau}}{d_1+c_1x^*(1-e^{-d_1 l\tau})}$.

For (1), another subsystem of system (1) is obtained as

$$\begin{cases} \left.\begin{aligned} \frac{dc_o}{dt} &= fc_{e_1} - (g+m)c_o, \\ \frac{dc_{e1}}{dt} &= -h_1 c_{e1}, \\ \frac{dc_{e2}}{dt} &= -h_2 c_{e2}, \end{aligned}\right\} t \neq N, \\ \left.\begin{aligned} &\triangle c_o = 0, \\ &\triangle c_{e_1} = D(c_{e_2} - c_{e_1}) + \mu_1, \\ &\triangle c_{e_2} = D(c_{e_1} - c_{e_2}) + \mu_2, \end{aligned}\right\} t = N. \end{cases} \tag{12}$$

Integrating (12), we get

$$\begin{cases} \widetilde{c_o} = c_o(N^+)e^{-(g+m)(t-N)} \\ \qquad\qquad + \dfrac{fc_{e1}(N^+)(1-e^{-(h-g-m)(t-N)})}{h-g-m}, t \in (N,(N+1)], \\ \widetilde{c_{e1}} = c_{e1}(N^+)e^{-h_1(t-N)}, t \in (N,(N+1)], \\ \widetilde{c_{e2}} = c_{e2}(N^+)e^{-h_2(t-N)}, t \in (N,(N+1)]. \end{cases} \tag{13}$$

The stroboscopic map of (12) is

$$\begin{cases} c_o((N+1)^+) = c_o(N^+)e^{-(g+m)\tau} \\ \qquad\qquad + \dfrac{fc_{e1}(N^+)(1-e^{-(h-g-m)\tau})}{h-g-m}, t \in (N,(N+1)], \\ c_{e1}((N+1)^+) = (1-D)c_{e1}(N^+)e^{-h_1\tau} + Dc_{e2}(N^+)e^{-h_2\tau} + \mu_1, \\ \qquad\qquad t \in (N,(N+1)], \\ c_{e2}((N+1)N^+) = Dc_{e1}(N^+)e^{-h_1\tau} + (1-D)c_{e2}(N^+)e^{-h_2\tau} + \mu_2 \\ \qquad\qquad t \in (N,(N+1)]. \end{cases} \tag{14}$$

A unique fixed point of (14) is

$$\begin{cases} c_o^* = \dfrac{f[\mu_1(1-(1-D)e^{-h_2\tau}) + \mu_2 De^{-h_2\tau}](1-e^{-(h-g-m)\tau})}{M_3}, \\ c_{e1}^* = \dfrac{\mu_1[1-(1-D)e^{-h_2\tau}] + \mu_2 De^{-h_2\tau}}{M_4}, \\ c_{e2}^* = \dfrac{\mu_2[1-(1-D)e^{-h_1\tau}] + \mu_1 De^{-h_1\tau}}{M_4}, \end{cases} \tag{15}$$

where $M_3 = (h-g-m)(1-e^{-(h-g-m)\tau})[(1-(1-D)e^{-h_1\tau})(1-(1-D)e^{-h_2\tau}) - D^2e^{-(h_1+h_2)\tau}]$ and $M_4 = [1-(1-D)e^{-h_1\tau}][1-(1-D)e^{-h_2\tau}] - D^2e^{-(h_1+h_2)\tau}$.

Writing (14) as a map, and defining it as $F : R_+^3 \to R_+^3$

$$\begin{cases} F_1(c) = c_o(N^+)e^{-(g+m)\tau} \\ \qquad\qquad + \dfrac{fc_{e1}(N^+)(1-e^{-(h-g-m)\tau})}{h-g-m}, t \in (N,(N+1)], \\ F_2(c) = (1-D)c_{e1}(N^+)e^{-h_1\tau} + Dc_{e2}(N^+)e^{-h_2\tau} + \mu_1, \\ F_3(c) = Dc_{e1}(N^+)e^{-h_1\tau} + (1-D)c_{e2}(N^+)e^{-h_2\tau} + \mu_2. \end{cases} \tag{16}$$

Lemma 3. Suppose $D > \frac{1}{2}$ holds, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ of (16) is globally asymptotically stable.

Proof. We mark $(c_o^n, c_{e1}^n, c_{e2}^n)$ as $(c_o(n\tau^+), c_{e1}(n\tau^+), c_{e2}(n\tau^+))$. Rewriting linearity of (16) as

$$\begin{pmatrix} c_o^{n+1} \\ c_{e1}^{n+1} \\ c_{e2}^{n+1} \end{pmatrix} = M \begin{pmatrix} c_o^n \\ c_{e1}^n \\ c_{e2}^n \end{pmatrix}. \tag{17}$$

Obviously, the stabilities of $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is by eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of M, which are less than 1.

Suppose $D > \frac{1}{2}$ holds, $0 < e^{-h_i} < 1 (i = 1, 2)$, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ exists, and

$$M = \begin{pmatrix} e^{-(g+m)\tau} & \frac{f(1-e^{-(h-g-m)\tau})}{h-g-m} & 0 \\ 0 & (1-D)e^{-h_1\tau} & De^{-h_2\tau} \\ 0 & De^{-h_1\tau} & (1-D)e^{-h_2\tau} \end{pmatrix}. \tag{18}$$

For

$$\lambda_1 = e^{-(g+m)\tau} < 1,$$

$$\lambda_2 = \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) + \sqrt{[(1-D)(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2D)e^{-(h_1+h_1)\tau}}}{2}$$

$$= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) + \sqrt{[D(e^{-h_1\tau} + e^{-h_2\tau})]^2 + (1-2D)(e^{-(h_1-h_1)\tau})^2}}{2}$$

$$< \frac{e^{-h_1\tau} + e^{-h_2\tau}}{2} < 1,$$

$$\lambda_3 = \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) - \sqrt{[(1-D)(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2D)e^{-(h_1+h_1)\tau}}}{2}$$

$$= \frac{(1-D)(e^{-h_1\tau} + e^{-h_2\tau}) - \sqrt{[(1-D)(e^{-h_1\tau} - e^{-h_2\tau})]^2 + 4D^2e^{-(h_1+h_1)\tau}}}{2}$$

$$< \frac{(1-D)\min\{e^{-h_1\tau}, e^{-h_2\tau}\}}{2} < 1.$$

That is, $\lambda_i < 1 (i = 1, 2, 3)$, then, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is locally stable. Then, it is globally asymptotically stable.

Similar to Reference [9], we get

Lemma 4. (12) has a unique globally asymptotically stable periodic solution $(\widetilde{c_o}, \widetilde{c_{e1}}, \widetilde{c_{e2}})$ of (12), and

$$\begin{cases} \widetilde{c_o} = c_o^* e^{-(g+m)(t-N)} \\ \qquad + \dfrac{f c_{e1}^*(1 - e^{-(h-g-m)(t-N)})}{h-g-m}, t \in (N, (N+1)], \\ \widetilde{c_{e1}} = c_{e1}^* e^{-h_1(t-N)}, t \in (N, (N+1)], \\ \widetilde{c_{e2}} = c_{e2}^* e^{-h_2(t-N)}, t \in (N, (N+1)]. \end{cases} \tag{19}$$

where $c_o^*, c_{e1}^*, c_{e2}^*$ are by (15).

Remark 5. $m_o \le c_o(t) \le M_o$, $m_{e1} \le c_{e1}(t) \le M_{e1}$ and $m_{e2} \le c_{e2}(t) \le M_{e2}$ hold for t large enough, and $m_o = c_o^* e^{-(g+m)\tau} + \frac{fc_{e1}^*(1-e^{-(h-g-m)\tau})}{h-g-m} - \varepsilon > 0$, $M_o = c_o^* + \frac{fc_{e1}^*}{(h-g-m)} + \varepsilon$, $m_{e1} = c_{e1}^* e^{-h_1\tau} - \varepsilon > 0$, $M_{e1} = c_{e1}^* + \varepsilon > 0$ $m_{e2} = c_{e2}^* e^{-h_2\tau} - \varepsilon > 0$ and $M_{e2} = c_{e2}^* + \varepsilon > 0$.

Thinking (1), we get

$$\begin{cases} \dfrac{dx}{dt} < -d_1 x + c_1 x^2, \\ \qquad\qquad t \in (N, (N+L)], \\ \triangle x = -\mu x, \\ \qquad\qquad t = (N+L), \\ \dfrac{dx}{dt} < -(d_2+E)x + c_2 x^2, \\ \qquad\qquad t \in ((N+L), (N+1)], \\ \triangle x = bx, \\ \qquad\qquad t = (N+1), \end{cases} \tag{20}$$

and

$$\begin{cases} \dfrac{dx}{dt} > -(d_1+\beta_1 M_o)x + c_1 x^2, \\ \qquad\qquad t \in (N, (N+L)], \\ \triangle x = -\mu x, \\ \qquad\qquad t = (N+L), \\ \dfrac{dx}{dt} > -(d_2+E+\beta_2 M_o)x + c_2 x^2, \\ \qquad\qquad t \in ((N+L), (N+1)], \\ \triangle x = bx, \\ \qquad\qquad t = (N+L), \end{cases} \tag{21}$$

with their comparative impulsive systems

$$\begin{cases} \dfrac{dx_1}{dt} = -d_1 x_1 + c_1 x_1^2, \\ \qquad\qquad t \in (N, (N+L)], \\ \triangle x_1 = -\mu x_1, \\ \qquad\qquad t = (N+L), \\ \dfrac{dx_1}{dt} = -(d_2+E)x_1 + c_2 x_1^2, \\ \qquad\qquad t \in ((N+L), (N+L)], \\ \triangle x_1 = bx_1, \\ \qquad\qquad t = (N+1), \end{cases} \tag{22}$$

and

$$\begin{cases} \dfrac{dx_2}{dt} = -(d_1 + \beta_1 M_o)x_2 + c_1 x_2^2, \\ \qquad\qquad t \in (N, (N+L)], \\ \triangle x_2 = -\mu x_2, \\ \qquad\qquad t = (N+L), \\ \dfrac{dx_2}{dt} = -(d_2 + E + \beta_2 M_o)x_2 + c_2 x_2^2, \\ \qquad\qquad t \in ((N+L), (N+1)], \\ \triangle x_2 = b x_2, \\ \qquad\qquad t = (N+1)\tau. \end{cases} \tag{23}$$

Marking $(1+b)(1-\mu)e^{-[(d_1+\beta_1 M_o)l+(d_2+E+\beta_2 M_o)(1-l)]\tau} < 1$ as (C_3) and $(1+b)(1-\mu)e^{-[(d_1+\beta_1 M_o)l+(d_2+E+\beta_2 M_o)(1-l)]\tau} > 1$ as (C_4).

Similar to Theorem 2, we can get

Theorem 6. *i*) Suppose (C_3) holds, 0 of (23) is globally asymptotically stable;

ii) Suppose (C_4) holds, $\widetilde{x_2(t)}$ of (23) is globally asymptotically stable, and

$$\widetilde{x_2} = \begin{cases} \dfrac{d_1 x^* e^{-d_1(t-N)}}{(d_1+\beta_1 M_o)) + c_1 x^*(1 - e^{-(d_1+\beta_1 M_o))(t-N)})}, \\ \qquad\qquad t \in (N, (N+L)], \\ \dfrac{(d_2+E+\beta_2 M_o))x^{**} e^{-(d_2+E+\beta_2 M_o))(t-N)}}{(d_2+E+\beta_2 M_o)) + c_2 x^{**}(1 - e^{-(d_2+E+\beta_2 M_o))(t-N)})}, \\ \qquad\qquad t \in ((N+l), (N+1)]. \end{cases} \tag{24}$$

here x_2^* is by

$$x_2^* = \frac{M_4}{M_5}, (1+b)(1-\mu)e^{-[(d_1+\beta_1 M_o)l+(d_2+E+\beta_2 M_o)(1-l))\tau} > 1, \tag{25}$$

where $M_4 = (d_1 + \beta_1 M_o)(d_2 + E + \beta_2 M_o)[(1+b)(1-\mu) e^{-[d_1+\beta_1 M_o)l+(d_2+E+\beta_2 M_o)(1-l)]\tau} - 1]$, $M_5 = (d_2+E+\beta_2 M_o)c_1(1-e^{-(d_1+\beta_1 M_o)l}) + (1-\mu)(d_1+\beta_1 M_o)c_2 e^{-(d_1+\beta_1 M_o)l\tau}(1-e^{-(d_2+E+\beta_2 M_o)(1-l)\tau})$ and $x_2^{**} = \frac{(1-\mu_1)(d_1+\beta_1 M_o))x_2^* e^{-(d_1+\beta_1 M_o)l\tau}}{(d_1+\beta_1 M_o))+c_1 x_2^*(1-e^{-(d_1+\beta_1 M_o)l\tau})}$.

Theorem 7. *i*) Suppose (C_1) and (C_3) hold, $(0, \widetilde{c_o}, \widetilde{c_{e1}}, \widetilde{c_{e2}})$ of (1) is globally asymptotically stable;

ii) Suppose (C_2) and (C_4) hold, (1) is permanent.

Proof. *i*) From condition (C_1), (22), Theorem 2, and the theorem of the impulsive equation, we can have $x(t) \leq x_1(t) = x_1(t) \to 0$ as $t \to +\infty$. From condition (C_3) (23), Theorem 6, and the theorem of the impulsive differential equation, we can have $x(t) \geq \widetilde{x_2(t)} \to 0$ as $t \to +\infty$. So, the globally asymptotically stable periodic solution $(0, \widetilde{c_o(t)}, \widetilde{c_{e1}(t)}, \widetilde{c_{e2}(t)})$ exists.

$ii)$ From condition (C_2), (22), Theorem 2, we can have

$$x(t) \le x_1(t) \le \widetilde{x_1(t)} - \varepsilon \le \frac{d_1 x^* e^{-d_1 l\tau}}{d_1) + c_1 x^*(1 - e^{-d_1 l\tau})}$$

$$+\frac{(d_2+E)x^{**}e^{-(d_2+E)(1-l)\tau}}{(d_2+E) + c_2 x^{**}(1 - e^{-(d_2+E))(1-l)\tau}} - \varepsilon \triangleq m_1.$$

From condition (C_4), (23), Theorem 6, we can easily obtain

$$x(t) \ge x_2(t) \ge \widetilde{x_2(t)} - \varepsilon \ge \frac{d_1 x^* e^{-d_1 l\tau}}{(d_1+\beta_1 M_o)) + c_1 x^*(1 - e^{-(d_1+\beta_1 M_o)l\tau}}$$

$$+\frac{(d_2+E+\beta_2 M_o))x^{**}e^{-(d_2+E+\beta_2 M_o))(1-l)\tau}}{(d_2+E+\beta_2 M_o)) + c_2 x^{**}(1 - e^{-(d_2+E+\beta_2 M_o))(1-l)\tau}} - \varepsilon \triangleq m_2.$$

From above discussion and Remark 5, we can have $m_1 \le x(t) \le m_2$, $m_o \le c_o(t) \le M_o$, $m_{e1} \le c_{e1}(t) \le M_{e1}$, $m_{e2} \le c_{e2}(t) \le M_{e2}$. This completes the proof.

4 Discussion

In this work, we consider we construct a single population model with non-transient/transient impulsive harvesting and birth pulse in a polluted environment. From the conditions of Theorem 7, we can deduce that the transient impulsive harvesting amount μ has a threshold μ^*. If $\mu > \mu^*$, the globally asymptotically stable $(0, \widetilde{c_o(t)}, \widetilde{c_{e1}(t)}, \widetilde{c_{e2}(t)})$ exists. If $\mu < \mu^*$, system permanence. We can also obtain the threshold l^* for non-transient impulsive harvesting intervals. Our results present a biological management researching basis in a polluted environment.

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