On a Single Server Queueing Inventory System with Common Life Time for Inventoried Items

Khamis Abdullah K. AL Maqbali(✉), Varghese C. Joshua, and Achyutha Krishnamoorthy

Department of Mathematics, CMS College Kottayam, Kottayam, Kerala, India
{khamis,vcjoshua,krishnamoorthy}@cmscollege.ac.in

Abstract. We consider a single server queueing inventory model. The customers arrive according to Markovian arrival Process (MAP). The service is assumed to follow Phase type(PH) distribution. An inventory of commodities is attached to the service station. The common life time (CLT) for inventoried items is assumed to follow Phase type(PH) distribution. The inventoried items perish all together. In this case, the supply of items is immediately in local purchase to bring the inventory level to maximum inventory level S. The inventory is not allowed to go down to zero because of local purchase. Each service requires a unit of commodity for service. This unit is instantaneously taken at the beginning of the service. The replenishment of inventory follows (s, S) policy with lead time positive. The lead time follows exponential distribution. In the case of local purchase, the outstanding order of the normal purchase (wait until replenishment) is cancelled. Service of a customer begins only when the server is free. Otherwise, the arriving customer joins the buffer. Steady state analysis of the queueing inventory model is performed. Some performance measures are computed under steady state. A numerical example is presented.

Keywords: Queueing inventory · Lead time · Common life time · Local purchase · Phase type distribution · Markovian arrival process · Matrix analytic method

1 Introduction

In many real life situations, customer, who needs inventoried items to complete his service, may arrive to service station according to Markovian arrival process. After that, he may go through different phases to complete his service in order to get the inventory. Moreover, common life time (CLT) for inventoried items may go through different phases until perishing, before they are taken by

Supported by the Indian Council for Cultural Relations (ICCR) and Ministry of Higher Education in Sultanate of Oman.

A. Dudin et al. (Eds.): ITMM 2020, CCIS 1391, pp. 186–197, 2021.
https://doi.org/10.1007/978-3-030-72247-0_14

customers. The Markovian Arrival Process (MAP) is more general than Poisson process. MAP keeps the memoryless property of the Poisson process(partial memoryless) [3].

Many papers studied queueing inventory models. For example, Krishnamoorthy et al. [4] studied a PH/PH/1 queueing inventory system under (s, S) policy when the lead time is zero. AL Maqbali, Joshua and Krishnamoorthy [1] studied M/PH/1 queueing inventory system under (s, S) policy with lead time positive. Also, Krishnamoorthy and Shajin [5] studied the MAP/PH/1 queueing inventory system under (s, S) policy with lead time positive. In addition, Divya et al. [2] studied MAP/PH/1 queueing inventory system with processing of service items under vacation and N-policy with impatient customers. In their study, customers arrive according to MAP and service time follows two different phase type distribution. The inventory processing time follows phase type distribution. Moreover. Nair and Jose [8] studied the MAP/PH/1 production inventory model with varying service rates under (s, S) policy with lead time positive.

Some papers studied queueing inventory systems with common life time. For instance, Shajin et al. [10] studied a MAP/PH/1 queueing inventory system with Markovian lead time to bring the inventory level to its maximum. In their study, the common life time of inventoried items follows independent exponential distribution. Besides this, their study provided an interesting application of queueing inventory model with common life time as medicines with the same expiry date. Moreover, Shajin et al. [9] studied a MAP/M/1 and M/M/1 queueing inventory system with advanced reservation and cancellation for the next K time frames a head in the case of overbooking. In their study, the common life time (CLT) of inventoried items follows Phase type distribution.

Some papers studied queueing inventory systems with local purchase. Local purchase was introduced by Krishnamoorthy and Raju [6]. Krishnamoorthy, Varghese and Lakshmy [7] studied an (s, S) production inventory model with positive service time under local purchase.

As mentioned above, Krishnamoorthy and Shajin [5] and Divya et al. [2] studied MAP/PH/1 queueing inventory system. Then Shajin et al. [10] studied a MAP/PH/1 queueing inventory system with common life time. In this paper, we consider an MAP/PH/1 queueing inventory model under (s, S) policy with lead time positive. Besides this, we consider PH distributed common life time (CLT) for inventoried items. According to the common life time (CLT), the inventoried items perish all together. The supply of items is immediately in local purchase to bring the inventory level to the maximum inventory level S.

This model can be described as follows: customers arrive according to Markovian Arrival Process (MAP) with representation (D_0, D_1) of order y. The service is assumed to follow PH-distribution with representation (β, T) of order m. An inventory of commodities is attached to the service station. The inventoried items have common life time (CLT) which follows PH-distribution with representation (α, W) of order l. We assume that the inventoried items perish all together. In this model, the inventory is not allowed to go down to zero because of local purchase. In order to keep customer goodwill during stock out, the supply of items is immediately in local purchase to bring inventory level to S. Local purchases are purchased at a higher cost than the regular order (wait until replenishment) procedure.

Each service requires a unit of commodity for service. This unit is instantaneously taken at the beginning of the service. The replenishment of inventory follows (s, S) policy with lead time positive. The lead time follows exponential distribution with rate θ. When $1 \leq i \leq s$, the replenishment occurs to bring the inventory level i to S according to the rate of lead time. In the case of local purchase, the outstanding order of the normal purchase (wait until replenishment) is cancelled. According to (MAP), the first arriving customer instantaneously takes one item of the inventory at the beginning of his service. Then, the service of this customer immediately follows Phase type (PH) distribution. When service station is available, the next arriving customer takes one time at the beginning of his service and the service of this customer instantaneously follows Phase type (PH) distribution. Otherwise, this customer must wait in the buffer until the availability of service station. This process goes on.

According to types of blood group, blood bank has store for each blood group. The motivation for the model comes from the inventory management of one store in bank blood. For example, patients deal with one type of blood group in this store. They arrive according to Markovian Arrival Process (MAP). When the service station is available, the service of this patient follows Phase type (PH) distribution in the hospital and one blood bag is immediately taken from store to the patient at the beginning of his service. The common life time (CLT) for blood bags may go through different phases until perishing, before patients take the blood bags. In this case, the supply of blood bag is immediately in local purchase.

2 Mathematical Description of the Model

The model discussed above can be studied as a level Independent Qusi-Birth-Death (LIQBD) process. We introduce the following notations.

At time t:

$N(t)$: the number of customers in the system.
$I(t)$: the number of items in the inventory and these items are the same type.
$L(t)$: the phase of common life time.
$M(t)$: the phase of service.
$Y(t)$: the phase of the arrival process.

$X(t) = \{(N(t), I(t), L(t), M(t), Y(t)); t \geq 0\}$ is a continuous time Markov Chain (CTMC) with state space

$\Omega = \{(0, i, l_1, y_1); 1 \leq i \leq S; 1 \leq l_1 \leq l; 1 \leq y_1 \leq y\} \cup \{(n, i, l_1, m_1, y_1); n \geq 1; 1 \leq i \leq S; 1 \leq l_1 \leq l; 1 \leq m_1 \leq m; 1 \leq y_1 \leq y\}$.

The terms of transitions of the states are shown in the Table 1.

The infinitesimal generator Q of the continuous time Markov Chain (CTMC) is given by

Table 1. Intensities of transitions

From	To		Transition rate
$(0,1,l_1,y_1)$	$(1,S,l_1,m_1,y_1^{'})$	$1\le l_1\le l; 1\le m_1\le m$	$d_{y_1y_1^{'}}(1)\ \beta_{m_1}$
$(0,i,l_1,y_1)$	$(1,i-1,l_1,m_1,y_1^{'})$	$2\le i\le S$	$d_{y_1y_1^{'}}(1)\ \beta_{m_1}$
(n,i,l_1,m_1,y_1)	$(n+1,i,l_1,m_1,y_1^{'})$	$1\le n; 1\le i\le S$	$d_{y_1y_1^{'}}(1)$
$(0,i,l_1,y_1)$	$(0,i,l_1,y_1^{'})$	$1\le i\le S; y_1\ne y_1^{'}$	$d_{y_1y_1^{'}}(0)$
(n,i,l_1,m_1,y_1)	$(n,i,l_1,m_1,y_1^{'})$	$1\le n; 1\le i\le S; y_1\ne y_1^{'}$	$d_{y_1y_1^{'}}(0)$
$(0,i,l_1,y_1)$	$(0,S,l_1,y_1)$	$1\le i\le s$	θ
(n,i,l_1,m_1,y_1)	(n,S,l_1,m_1,y_1)	$1\le n; 1\le i\le s$	θ
$(1,i,l_1,m_1,y_1)$	$(0,i,l_1,y_1)$	$1\le i\le S$	$\tau^0_{m_1}$
(n,i,l_1,m_1,y_1)	$(n-1,i-1,l_1,m_1^{'},y_1)$	$2\le i\le S\ ;\ 2\le n$	$\tau^0_{m_1}\beta_{m_1^{'}}$
$(n,1,l_1,m_1,y_1)$	$(n-1,S,l_1,m_1^{'},y_1)$	$2\le n$	$\tau^0_{m_1}\beta_{m_1^{'}}$
(n,i,l_1,m_1,y_1)	$(n,i,l_1,m_1^{'},y_1)$	$1\le n; m_1\ne m_1^{'}; 1\le i\le S$	$\tau_{m_1m_1^{'}}$
$(0,i,l_1,y_1)$	$(0,S,l_1^{'},y_1)$	$1\le i\le S$	$w^0_{l_1}\alpha_{l_1^{'}}$
(n,i,l_1,m_1,y_1)	$(n,S,l_1^{'},m_1,y_1)$	$1\le n; 1\le i\le S$	$w^0_{l_1}\alpha_{l_1^{'}}$
$(0,i,l_1,y_1)$	$(0,i,l_1^{'},y_1)$	$1\le i\le S; l_1\ne l_1^{'}$	$w_{l_1l_1^{'}}$
(n,i,l_1,m_1,y_1)	$(n,i,l_1^{'},m_1,y_1)$	$1\le n; 1\le i\le S; l_1\ne l_1^{'}$	$w_{l_1l_1^{'}}$

$$Q=\begin{pmatrix} B_{00} & B_{01} & & & \\ B_{10} & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & & \ddots & \ddots & \ddots \end{pmatrix};$$

where

$$B_{00}=\begin{pmatrix} \Upsilon_1 & O_{(yls)\times[(S-s-1)yl]} & \Upsilon_2 \\ O_{((S-1-s)yl)\times(syl)} & \Upsilon_3 & \Upsilon_4 \\ O_{(yl)\times(syl)} & O_{(yl)\times((S-1-s)yl)} & \Upsilon_5 \end{pmatrix};$$

B_{00} is a square matrix of order (Syl);

where

$$\begin{aligned}
\Upsilon_1 &= I_{(s)}\otimes((I_l\otimes D_0)+(W_l\otimes I_l)-[\theta I_{(yl)}]);\\
\Upsilon_2 &= e_s\otimes[[(\alpha\otimes W_l^0)\otimes I_y]+\theta I_{ly}];\\
\Upsilon_3 &= I_{(S-1-s)}\otimes((I_l\otimes D_0)+(W_l\otimes I_l));\\
\Upsilon_4 &= e_{(S-1-s)}\otimes[(\alpha\otimes W_l^0)\otimes I_y] \text{ and}\\
\Upsilon_5 &= (I_l\otimes D_0)+(W_l\otimes I_l)+[(\alpha\otimes W_l^0)\otimes I_y].
\end{aligned}$$

$$B_{01}=\begin{pmatrix} O_{(yl)\times((Syml)-(yml))} & I_l\otimes(\beta\otimes D_1) \\ I_{(S-1)}\otimes(I_l\otimes(\beta\otimes D_1)) & O_{([S-1]yl)\times(ylm)} \end{pmatrix};$$

B_{01} is a matrix of order $(Syl) \times (Syml)$.

$$B_{10} = \left(I_S \otimes [I_l \otimes (T_m^0 \otimes I_y)] \right);$$

B_{10} is a matrix of order $(Syml) \times (Syl)$.

$$A_2 = \begin{pmatrix} O_{(yml)\times[(S-1)yml]} & [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]] \\ \Gamma & O_{[(S-1)yml]\times(yml)} \end{pmatrix};$$

A_2 is a square matrix of order $(Syml)$;
where $\Gamma = (I_{(S-1)} \otimes [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]])_{[(S-1)yml]\times[(S-1)yml]}$.

$$A_0 = \left(I_{(mlS)} \otimes D_1 \right);$$

A_0 is a square matrix of order $(Syml)$.

$$A_1 = \begin{pmatrix} \varphi_1 & \varphi_2 \\ (O_{[(S-1-s)yml]\times(ylms)}, \varphi_3) & \varphi_4 \\ O_{[yml]\times([S-1]ylm)} & \varphi_5 \end{pmatrix};$$

A_1 is a square matrix of order $(Syml)$. where

$\varphi_1 = \left(I_s \otimes ([\{I_l \otimes [(I_m \otimes D_0) + (T_m \otimes I_y)]\} + (W_l \otimes I_{(ym)})] - \theta I_{(yml)}) \right);$
φ_1 is a matrix of order $(syml) \times (ylms)$;
$\varphi_2 = \left(O_{(syml)\times[(S-s-1)ylm]} \ (e_s \otimes [[(\alpha \otimes W_l^0) \otimes I_{(ym)}] + \theta I_{(yml)}])_{(syml)\times(yml)} \right);$
φ_2 is a matrix of order $(syml) \times (yml)$;
$\varphi_3 = \left([I_{(S-1-s)} \otimes [\{I_l \otimes [(I_m \otimes D_0) + (T_m \otimes I_y)]\} + (W_l \otimes I_{(ym)})]] \right).$
φ_3 is a matrix of order $((S-1-s)yml) \times ((S-1-s)yml)$;
$\varphi_4 = \left([e_{(S-1-s)} \otimes [(\alpha \otimes W_l^0) \otimes I_{(ym)}]] \right);$
φ_4 is a matrix of order $((S-1-s)yml) \times (yml)$;
$\varphi_5 = \left([\{I_l \otimes [(I_m \otimes D_0) + (T_m \otimes I_y)]\} + (W_l \otimes I_{(ym)})] + [(\alpha \otimes W_l^0) \otimes I_{(ym)}] \right);$
φ_5 is a matrix of order $(yml) \times (yml)$.

3 Steady-State Analysis

3.1 Stability Condition

Theorem 1. *The stability condition of the queueing inventory model with common life time for inventoried items under study is given by*

$$\lambda < \mu$$

Where $\lambda = (\sum_{i=0}^{(mlS)} \boldsymbol{\pi_i}) D_1 e_y$ *;* $\boldsymbol{\pi_i}$ *is a row vector of order* (y)
and $\mu = (\sum_{i=0}^{S+1} \boldsymbol{\pi_i}) \wedge e_{(yml)}$ *;*
where $\wedge = [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]]$ *and* $\boldsymbol{\pi_i}$ *are row vectors of order* (yml).

Proof. Let $A = A_2 + A_1 + A_0$. We can realize that A is an irreducible matrix. Thus, there exists the stationary vector $\boldsymbol{\pi}$ of A such that

$$\boldsymbol{\pi} A = 0$$

$$\boldsymbol{\pi} e = 1.$$

The Markov chain with generator Q is stable if and only if

$$\boldsymbol{\pi} A_0 e < \boldsymbol{\pi} A_2 e.$$

Recall, $A_0 = \left(I_{(mlS)} \otimes D_1 \right)$; A_0 is a square matrix of order $(Syml)$.

$$\begin{aligned}
\boldsymbol{\pi} A_0 e &= (\boldsymbol{\pi_0}, \boldsymbol{\pi_1}, \boldsymbol{\pi_2}, \ldots, \boldsymbol{\pi_{(mlS)}}) \begin{pmatrix} D_1 & & & \\ & D_1 & & \\ & & \ddots & \\ & & & D_1 \end{pmatrix} \begin{pmatrix} e_y \\ e_y \\ \vdots \\ e_y \end{pmatrix}_{(mlS)} ; \\
&= (\boldsymbol{\pi_0} D_1, \boldsymbol{\pi_1} D_1, \boldsymbol{\pi_2} D_1, \ldots, \boldsymbol{\pi_{(mlS)}} D_1) \begin{pmatrix} e_y \\ e_y \\ \vdots \\ e_y \end{pmatrix}_{(mlS)} ; \\
&= (\boldsymbol{\pi_0} D_1, \boldsymbol{\pi_1} D_1, \boldsymbol{\pi_2} D_1, \ldots, \boldsymbol{\pi_{(mlS)}} D_1) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}_{(mlS)} e_y; \\
&= (\boldsymbol{\pi_0} D_1 + \boldsymbol{\pi_1} D_1 + \boldsymbol{\pi_2} D_1 + \cdots + \boldsymbol{\pi_{(mlS)}} D_1) e_y; \\
&= (\boldsymbol{\pi_0} + \boldsymbol{\pi_1} + \boldsymbol{\pi_2} + \cdots + \boldsymbol{(mlS)}) D_1 e_y; \\
&= (\sum_{i=0}^{(mlS)} \boldsymbol{\pi_1}) D_1 e_y \\
&= \lambda.
\end{aligned}$$

Recall,

$$A_2 = \begin{pmatrix} O_{(yml)\times[(S-1)yml]} & [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]] \\ \Gamma & O_{[(S-1)yml]\times(yml)} \end{pmatrix};$$

A_2 is a square matrix of order $(Syml)$.
where $\Gamma = (I_{(S-1)} \otimes [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]])_{[(S-1)yml]\times[(S-1)yml]}$.

To be more clear, we rewrite matrix A_2 as following:

$$A_2 = \begin{pmatrix} O & \cdots & & & \wedge \\ \wedge & O & \cdots & & O \\ O & \wedge & O & \dots & \vdots \\ \vdots & O & \wedge & & \\ \vdots & \vdots & & \ddots & \vdots \\ & & & \wedge & O \end{pmatrix};$$

where $\wedge = [I_l \otimes [T_m^0 \otimes (\beta \otimes I_y)]]$.

$$\boldsymbol{\pi} A_2 e = (\boldsymbol{\pi_0}, \boldsymbol{\pi_1}, \boldsymbol{\pi_2}, \dots, \boldsymbol{\pi_{(S+1)}}) \begin{pmatrix} O & \cdots & & & \wedge \\ \wedge & O & \cdots & & O \\ O & \wedge & O & \dots & \vdots \\ \vdots & O & \wedge & & \\ \vdots & \vdots & & \ddots & \vdots \\ & & & \wedge & O \end{pmatrix} \begin{pmatrix} e_{yml} \\ e_{yml} \\ e_{yml} \\ \vdots \\ e_{yml} \\ e_{yml} \\ e_{yml} \end{pmatrix}_{(S)};$$

$$= (\boldsymbol{\pi_0}\wedge, \boldsymbol{\pi_1}\wedge, \boldsymbol{\pi_2}\wedge, \dots, \boldsymbol{\pi_{(S+1)}}\wedge) \begin{pmatrix} e_{yml} \\ e_{yml} \\ e_{yml} \\ \vdots \\ e_{yml} \\ e_{yml} \\ e_{yml} \end{pmatrix}_{(S)};$$

$$= (\boldsymbol{\pi_0}\wedge, \boldsymbol{\pi_1}\wedge, \boldsymbol{\pi_2}\wedge, \dots, \boldsymbol{\pi_{(S+1)}}\wedge) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}_{(S)} e_{(yml)};$$

$$= (\boldsymbol{\pi_0} \wedge + \boldsymbol{\pi_1} \wedge + \boldsymbol{\pi_2} \wedge + \cdots + \boldsymbol{\pi_{(S+1)}}\wedge) e_{(yml)};$$
$$= (\boldsymbol{\pi_0} + \boldsymbol{\pi_1} + \boldsymbol{\pi_2} + \cdots + \boldsymbol{\pi_{(S+1)}}) \wedge e_{(yml)};$$
$$= (\sum_{i=0}^{S+1} \boldsymbol{\pi_1}) \wedge e_{(yml)};$$
$$= \mu.$$

Then, $\boldsymbol{\pi} A_0 e = (\sum_{i=0}^{(mlS)} \boldsymbol{\pi_1}) D_1 e_y = \lambda$ *and*

$$\boldsymbol{\pi} A_2 e = (\sum_{i=0}^{S+1} \boldsymbol{\pi_1}) \wedge e_{(yml)} = \mu.$$

Since $\pi A_0 e = \lambda$ and $\pi A_2 e = \mu$, then the queueing inventory model under study is stable if and only if

$$\lambda < \mu$$

3.2 Stationary Distribution

According to Stewart [11], we can obtain the stationary distribution of the Markov chain under study by solving the set of Eqs. 1 and 2.

$$\mathbf{X}Q = 0 \tag{1}$$

$$\mathbf{X}e = 1. \tag{2}$$

Let $\mathbf{X}$ be decomposed with Q as following :

$$\begin{aligned}
&\mathbf{X} = (\mathbf{X}_0, \mathbf{X}_1, \dots) \text{ where } \mathbf{X}_0 = (\mathbf{X}_{01}, \mathbf{X}_{02}, \dots, \mathbf{X}_{0S});\\
&\mathbf{X}_{0k} = (\mathbf{X}_{0k1}, \mathbf{X}_{0k2}, \mathbf{X}_{0k3}, \dots, \mathbf{X}_{0kl}) \text{ for } k = 1, 2, 3, \cdots, S;\\
&\mathbf{X}_{0kr} = (x_{0kr1}, x_{0kr2}, x_{0kr3}, \dots, x_{0kry}) \text{ for } r = 1, 2, 3, \cdots, l;\\
&\mathbf{X}_i = (\mathbf{X}_{i1}, \mathbf{X}_{i2}, \dots, \mathbf{X}_{iS}) \text{ for } i = 1, 2, 3, \cdots;\\
&\mathbf{X}_{ik} = (\mathbf{X}_{ik1}, \mathbf{X}_{ik2}, \mathbf{X}_{ik3}, \dots, \mathbf{X}_{ikl});\\
&\mathbf{X}_{ikr} = (\mathbf{X}_{ikr1}, \mathbf{X}_{ikr2}, \mathbf{X}_{ikr3}, \dots, \mathbf{X}_{ikrm});\\
&\mathbf{X}_{ikrj} = (x_{ikrj1}, x_{ikrj2}, x_{ikrj3}, \dots, x_{ikrjy}) \text{ for } j = 1, 2, 3, \cdots, m.
\end{aligned}$$

From Eq. 1, we get set of equations as following.

$$\mathbf{X}_0 B_{00} + \mathbf{X}_1 B_{10} = 0; \tag{3}$$

$$\mathbf{X}_0 B_{01} + \mathbf{X}_1 A_1 + \mathbf{X}_2 A_2 = 0; \tag{4}$$

$$\vdots$$

$$\mathbf{X}_{i-1} A_0 + \mathbf{X}_i A_1 + \mathbf{X}_{i+1} A_2 = 0 \text{ for } i \geq 2.$$

where i is a positive integer number.

There exists a constant matrix R such that

$$\mathbf{X}_i = \mathbf{X}_{i-1} R \text{ for } i \geq 2 \; . \tag{5}$$

We can rewrite the Eq. 5 as following

$$\mathbf{X}_i = \mathbf{X}_1 R^{i-1} \text{ for } \; i \geq 2.$$

We can use the matrix quadratic Eq. 6 to obtain the matrix R.

$$R^2 A_2 + R A_1 + A_0 = 0. \tag{6}$$

The matrix R can be obtained from $R_{k+1} = -V - R_k^2 W$ and $R_0 = 0$; where $V = A_0 A_0^{-1}$ and $W = A_2 A_1^{-1}$. Then, we can find X_0 and X_1 by solving Eqs. 3 and 4. After that, we must normalize X_0 and X_1 by using the normalizing condition $\mathbf{X_0} + \mathbf{X_1}(I - R)^{-1} e = 1$. Then, we use $\mathbf{X}_i = \mathbf{X}_1 R^{i-1}$ for $\; i = 2, 3, \dots$.

4 Performance Measures

Under steady state, some performance measures of this queueing inventory model can be obtained as following:

1. Expected number of customers in the system
$$E[N] = \sum_{i=0}^{\infty} i\mathbf{X}_i e.$$
2. Expected number of items in inventory.
$$E[I] = \sum_{i=0}^{\infty} \sum_{k=1}^{S} k\mathbf{X}_{ik} e.$$
3. Probability that the server is idle
$$b_0 = \sum_{k=1}^{S} \mathbf{X}_{0k} e.$$

5 Numerical Example

For the arrival process, we consider Markovian arrival process (MAP) with representation (D_0, D_1) of order $y = 3$, where

$$D_0 = \begin{pmatrix} -8 & 1.5 & 1 \\ 1.5 & -6 & 1.5 \\ 1 & 1 & -7 \end{pmatrix} \text{ and } D_1 = \begin{pmatrix} 1.5 & 1.5 & 2.5 \\ 0.5 & 1 & 1.5 \\ 2.5 & 1.5 & 1 \end{pmatrix}.$$

For the service process, we consider PH-representation (β, T) of order $m = 3$, where

$$\beta = \big(0.2, 0.5, 0.3\big),$$
$$T = \begin{pmatrix} -12 & 3 & 4 \\ 6 & -13 & 3 \\ 5 & 3 & -14 \end{pmatrix} \text{ and } T^0 = -Te = \begin{pmatrix} 5 \\ 4 \\ 6 \end{pmatrix}.$$

For the common life time (CLT) for inventoried items, we consider PH-representation (α, W) of order $l = 3$, where

$$\alpha = \big(0.3, 0.4, 0.3\big),$$
$$W = \begin{pmatrix} -0.35 & 0.1 & 0.2 \\ 0.3 & -0.41 & 0.1 \\ 0.3 & 0.2 & -0.52 \end{pmatrix} \text{ and } W^0 = -We = \begin{pmatrix} 0.05 \\ 0.01 \\ 0.02 \end{pmatrix}.$$

We fix the rate of lead time $\theta = 0.6$ and $s = 3$.

Now, we analyze the effect of S on the performance measures of the system in the Table 2.

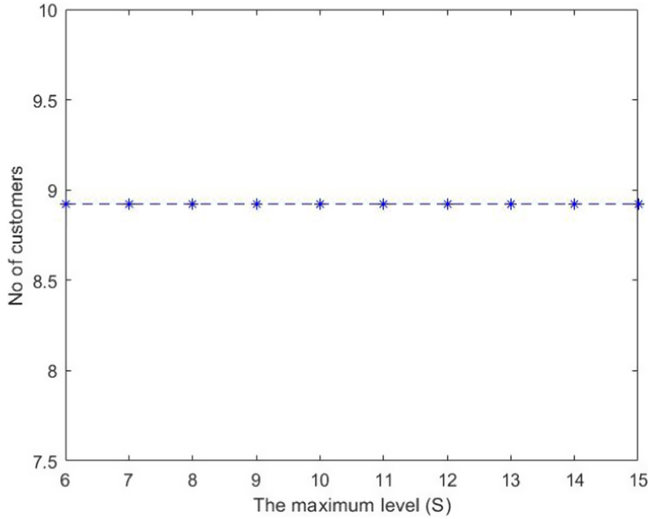

Fig. 1. Effect of S on expected number of customers

Table 2. Effect of S on various performance measures

S	E[N]	E[I]	b_0
6	8.9228	3.7312	0.1014
7	8.9228	4.2546	0.1014
8	8.9228	4.7748	0.1014
9	8.9228	5.2932	0.1014
10	8.9228	5.8109	0.1014
11	8.9228	6.3283	0.1014
12	8.9228	6.8460	0.1014
13	8.9228	7.3640	0.1014
14	8.9228	7.8826	0.1014
15	8.9228	8.4019	0.1014

From Figs. 1, 2 and 3, we can realize the effect of S on performance measures as following:

1. The expected number of customers in the system $E[N]$ has no change when the maximum inventory level S increases.
2. The expected number of items in inventory $E[I]$ increases when the maximum inventory level S increases.
3. The probability that the server is idle b_0 has no change when the maximum inventory level S increases.

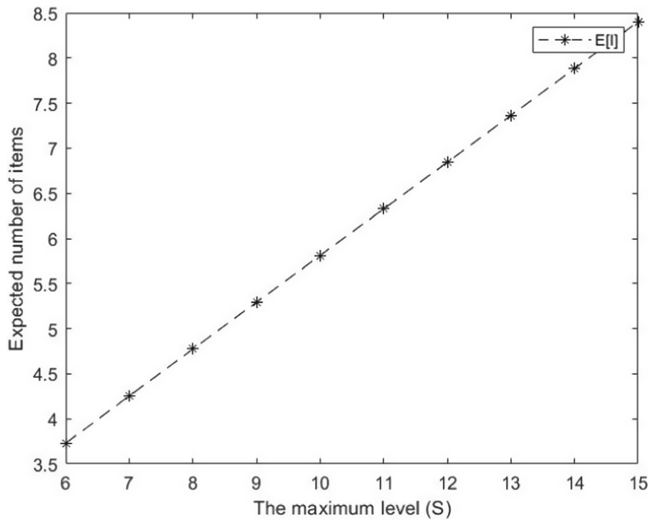

Fig. 2. Effect of S on expected number of items in inventory

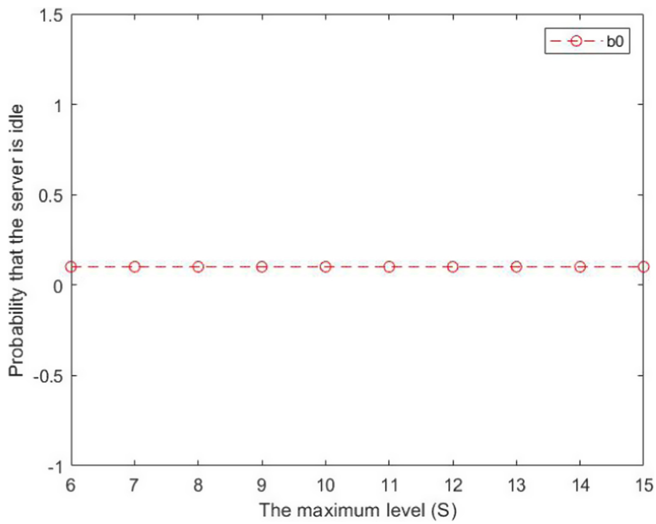

Fig. 3. Effect of S on probability that the server is idle

6 Conclusion

In this paper, we analyse an $MAP/PH/1$ queueing inventory model under (s, S) policy with lead time positive and with common life time for inventoried items. In the case of expiry of the common life time for the inventoried items, the supply of items is immediately in local purchase to bring the inventory level to the maximum inventory level S. Different performance measures are estimated under Steady state condition. In this paper, we study the effect of maximum inventory level S on the performance measures of the system numerically. We realize that firstly, the expected number of customers in the system $E[N]$ has no change when the maximum inventory level S increases. Secondly, the expected number of items in inventory $E[I]$ increases when the maximum inventory level S increases. Finally, the probability that the server is idle b_0 has no change when the maximum inventory level S increases.

Acknowledgement. The first author acknowledges the Indian Council for Cultural Relations (ICCR) (Order No: 2019-20/838) and Ministry of Higher Education (MOHE), Order No: 2019/35 in Sultanate of Oman for their supports.

References

1. Maqbali, K.A.K.A.L., Joshua, V.C., Krishnamoorthy, A.: On a single server queueing inventory system. In: Vishnevskiy, V.M., Samouylov, K.E., Kozyrev, D.V. (eds.) DCCN 2020. LNCS, vol. 12563, pp. 579–588. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-66471-8_44
2. Divya, V., Krishnamoorthy, A., Vishnevsky, V.M., Kozyrev, D.V.: On a queueing system with processing of service items under vacation and N-policy with impatient customers. Queueing Models Serv. Manage. **3**(2), 167–201 (2020)
3. He, Q.M.: Fundamentals of Matrix-Analytic Methods. Springer, New York (2014). https://doi.org/10.1007/978-1-4614-7330-5
4. Krishnamoorthy, A., Jose, K.P., Narayanan, V.C.: Numerical investigation of a PH/PH/1 inventory system with positive service time and shortage. Neural Parallel Sci. Comput. **16**, 579–592 (2008)
5. Krishnamoorthy, A., Shajin, D.: MAP/PH/1 retrial queueing-inventory system with orbital search and reneging of customers. In: Rykov, V.V., Singpurwalla, N.D., Zubkov, A.M. (eds.) ACMPT 2017. LNCS, vol. 10684, pp. 158–171. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-71504-9_15
6. Krishnamoorthy, A., Raju, N.: N-policy for (s, S) perishable inventory system with positive lead time. Korean J. Comput. Appl. Math. **5**(1), 253–261 (1998)
7. Krishnamoorthy, A., Varghese, R., Lakshmy, B.: Production inventory system with positive service time under local purchase. In: Dudin, A., Nazarov, A., Moiseev, A. (eds.) ITMM 2019. CCIS, vol. 1109, pp. 243–256. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-33388-1_20
8. Nair, S.S., Jose, K.P.: A MAP/PH/1 production inventory model with varying service rates. Int. J. Pure Appl. Math. **117**(12), 373–381 (2017)
9. Shajin, D., Krishnamoorthy, A., Dudin, A.N., Joshua, V.C., Jacob, V.: On a queueing-inventory system with advanced reservation and cancellation for the next K time frames ahead: the case of overbooking. Queueing Syst. **94**(1), 3–37 (2020)
10. Shajin, D., Krishnamoorthy, A., Manikandan, R.: On a queueing-inventory system with common life time and Markovian lead time process. Oper. Res. 1–34 (2020)
11. Stewart, W.J.: Probability, Markov Chains, Queues, and Simulation: The Mathematical Basis of Performance Modeling. Princeton University Press, Princeton (2009)