Challenges and Opportunities from Translingual Research on Multilingual Mathematics Classrooms

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Abstract This chapter provides a commentary on multilingual mathematics classroom research over the last two decades in order to look toward the next decades with a focus on issues of policy and practice. I select some of the theoretical nuances and concerns that have shaped the domain with respect to the critical relationships between: (1) monolingually oriented educational policies and progress in multilingual mathematics classroom research; and between (2) this progress and its implications for mathematics teacher education policies and pedagogies. To this end, I undertake a threefold interpretation of progress in the research domain. I argue that three meta-theoretical concerns have challenged, not without frictions and back-and-forth fluctuations, monolingually oriented policies, practices, and theories. I start with domain research grounded on language as tool of communication and on codeswitching as encoder of accurate meaning in multilingual mathematics teaching and learning. I follow with research that interrogates the ideal of meaning accuracy, and then end with the most recent line of translingual domain research with implications for the broader field and the work of teacher educators and researchers.

Keywords Multilingual mathematics classroom research · Monolingual policies · Multilingual learners · Translanguaging practices · Translingual position

1 Monolingual Policies and Practices, Multilingual Learners

Research in multilingual mathematics classrooms has become more and more common in the field of mathematics education, with some of the studies bringing up claims of possible generalization to a diversity of educational contexts and content areas. This chapter provides a meta-theoretical commentary on research and guiding ideas specifically created in the field of mathematics education over the last two

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decades in order to look toward the next decades with a focus on issues of policy and practice that can inform work in other educational fields. I do not review the research literature in any systematic sense, but rather select some meta-theoretical concerns and nuances that have shaped the domain concerning the critical relationships between: (1) monolingually oriented educational policies and progress in multilingual mathematics classroom research; and between (2) this progress and its implications for mathematics teacher education policies and pedagogies.

Three points are particularly influential and have consequences for the interpretation of the two abovementioned critical relationships. First, I have carried out most of my research and developmental work in multilingual mathematics classrooms of Catalonia. In this autonomous region in Spain, Europe, Catalan is the language of instruction since 1985. Law allows families the choice of education in Spanish, the other official language, although this rarely happens due to the high status and generalized use of Catalan. This is not the case, for example, in France, where the language policy categorizes Catalan as regional and includes it in the not very popular model of bilingual education available for regional languages. Second, I am trilingual myself, or a double second-language learner. I was brought up speaking Catalan, found Spanish out of the home at school in the late 1970s, and then found English in the process of becoming a researcher, while also for some years being a secondary school teacher who taught mathematics to learners of diverse cultural groups in the 1990s. Third, I have learned immensely from work with colleagues on the discussion of language-in-education policies and multilingual mathematics classroom practices in Catalonia (e.g. Gorgorió & Planas, 2001), Arizona (e.g. Planas & Civil, 2013), South Africa (e.g. Planas & Phakeng-Setati, 2014), and Greece (e.g. Chronaki & Planas, 2018). This network continues to stimulate my thinking and research, not without acknowledging the singularities and sociopolitical backgrounds across contexts. The parallelisms between the power exercised by state and colonizing languages in public domains including education (by means of the privileging of one language) are many. Nonetheless, the landscapes that emerge from the colonial periods of exclusion of the African languages in South Africa are very different from those that emerge from the nationalist state projects throughout Europe and the United States and the policies of exclusion of non-state languages spoken by the majority of people in certain regions.

In the following sections, I start by arguing that "one classroom, one language, one mathematics" policies, pedagogies, and ideologies have complicated the research on multilingual mathematics teaching, learning, and assessment, by making it more difficult to notice the epistemic function of languages other than the language of instruction and of mathematical cultures other than school mathematics. The "one classroom, one language, one mathematics" tradition actually forms a significant part of the foundation for most classroom research in the field of mathematics education. A generation of scholars (myself included) have walked a long journey to view languages, language use, and curricular content in the multilingual mathematics classroom beyond the countable alternation of separate languages. Still today, state policies, pedagogies, and ideologies of monolingual normativity narrow the lenses through which we think about mathematics teaching and learning, and what can comprise mathematically relevant language use in the distinct research contexts.

Despite this, increased conceptual refinements of language as social, support the progressive uncovering of translanguaging practices across languages and cultures in school mathematics teaching and learning. These practices illustrate the varied and creative possibilities of multilingual language use that put communication and participation at the center and eventually challenge or resist norms of linguistic accuracy. In this chapter, I use translanguaging to refer to situations of practice explained as "the deployment of a speaker's full linguistic repertoire without regard for watchful adherence to the socially and politically defined boundaries of named (and usually national and state) languages" (Otheguy et al., 2015, p. 283). At the ideological-theoretical level, I adopt the term translingual to describe the position that captures and defends the language practices of all those who use the linguistic, cultural, and social resources at their disposal to produce and investigate mathematics teaching and learning. Such a position has implications for policies and pedagogies of mathematics teacher education, but also for methodological practices in the broader research field and for research and practice in related educational fields.

2 Monolingually Oriented Educational Policies

In this section, I begin to address the critical relationship between monolingually oriented educational policies and progress in research on multilingual mathematics classrooms, a relationship that can also be read as the story of progress of this research in spite of or because of these policies. In the research field of mathematics education, and compared to what happens with the tradition of researching the role and use of theories (e.g. Planas & Schütte, 2018), there is not a strong tradition with regard to the role and use of policies. It is often explained that the theories we choose either widen or narrow what we do and see, and how we think. A similar relational perspective can be applied to the policies surrounding the classroom sites in which we plan, design, and develop our studies. As occurs with theories, progress in the research of specific policies and in the understanding of the role these policies play in the research processes mediates the construction of the field directions. Multilingual mathematics classroom research must hence be analyzed with regard to the theories undertaken and the policies that shape education.

The study of language policies as a distinct field can be traced back to the early study of the struggle with the state about whose knowledge, experiences, and ways of using language are legitimate in Fishman et al. (1968), and in Ruiz (1984). These pioneering works support the link of language policies with social processes that guide and regulate, in school education, how teachers mediate classrooms and instruction with multilingual learners, and how researchers decide what can be researched that is realistic (and which is more likely to receive research funding and institutional acknowledgment). Accordingly, policies that influence practice influence the spaces of research on that practice as well. Language policies and monolingual ideologies enter pedagogy and research in multilingual mathematics classrooms in the form of language choice, but also through the tacit establishment of norms such as talking,

writing, or reading one language at a time, which ultimately narrow the effective use of all the linguistic, cultural, and social resources at the disposal of practitioners and researchers.

Monolingually oriented educational policies and research traditions that, for example, see multilingual speaking and writing as abnormal, specifically influence multilingual mathematics classroom research, its performance, and its communication. In international journal articles that mention the original languages involved in lesson data, for example, sentences about space restrictions on not presenting them or on not doubling the length of the transcripts are typically accepted, with the resulting monolingual representation of the classroom sites. Even more decisively, policies and ideologies enter the domain through the basic questions we are able to ask about language in the multilingual classroom, the claims and data we are able to hold and collect, the conclusions we are able to draw from those data, and the directions we are able to maintain and foresee.

The language-in-education policies and the studies in collaboration in my major research context over the last two decades co-illustrate the general case of monolingual orientations entering and challenging multilingual mathematics classroom research in the form of the choices as to what to ask, claim, and conclude. As in other world regions, the educational policies in Catalonia fabricate classroom multilingualism in terms of separate languages and linguistic differences among groups of speakers and learners. In the organizational attempt to reduce linguistic differences before entering the regular classroom, there are Catalan language support classes or "special classes" in the schools for learners who are new arrivals (for details, see Newman et al., 2013). Vallcorba (2010), the past Director of the "Plan for Language and Social Cohesion" in education, recalls the language policy principles that declare Catalan as the language of instruction at all levels of education, and as the normal vehicle of expression in internal school activities and in those of external projection.

Regardless of how teachers put these policy principles into practice, we find the representation of the regular classroom as a place where the language of instruction is the "normal vehicle of expression," which resonates with the representation of language as a neutral tool of communication. The fact that the school is the only place of use of Catalan for most learners of immigrant families in areas of poverty is disregarded, and the possibility of multilingual translanguaging practices for learning and teaching subject content. The local policy actually portrays the language support classes as "the" resource in spite of their problematic functioning at several practical levels. More often than not, these classes are: converted into school spaces for keeping mainstream learners labeled as disruptive out of the regular classroom; guided by pedagogies of curricular remedial arrangements and simplified language across school subjects; and conducted by teachers who are themselves new arrivals to the school. All these remind us of the lack of professional preparation on contents of language for subject-specific teaching and learning in the region, which is a feature more widely documented across regions (Essien et al., 2016), whose consequences affect education of different content areas. I will return to this topic on teacher education later in the chapter.

3 Progress in Multilingual Mathematics Classroom Research

Perhaps, because of my first-hand experiences as a second-language learner and a secondary school mathematics teacher, my earliest studies aimed to explore multilingual practices in the participation of learners from minority linguistic and cultural groups in mathematical lessons of the regular and the support classrooms (Gorgorió & Planas, 2001). Despite the local policy oriented to make learners monolingual in school work, I knew that these learners' languages functioned as resources in classroom peer work (not so much in the whole group), similarly to what seemed to happen in Arizona (Civil, 2007), South Africa (Setati & Adler, 2000), and Greece (Chronaki, 2009). The representational lenses at that time were therefore not too narrow, but still not sufficiently wide to ask questions and collect data about multilingual practices other than translation or codeswitching. While the sociocultural stance for school teaching and learning and for multilingualism explained the mediational role of switching languages, the ideals of mathematical and linguistic accuracy remained present. More discussion and lesson observation would be necessary to see, legitimate, and analyze the complex sociolinguistic interactions and translanguaging practices in the classroom work.

Below, I present three meta-theoretical concerns in the domain research that have challenged, not without frictions and back-and-forth fluctuations, monolingually oriented policies, practices, and theories. I draw on instances of classroom data and reflections from three studies to develop a threefold interpretation of progress in the domain. The presentation of this interpretation is, in turn, twofold because, for each concern or step, I first introduce evidence from my own empirical research and then relate it to evidence from other studies in the domain literature. Even though I model these concerns separately, using different studies in time, they are dependent on each other and on a diversity of theoretical refinements of language as social (Planas, 2018; Planas et al., 2018). I start with research grounded on the ideal of language as a tool of communication that legitimated the efforts to study the practice (shortened in the literature as codeswitching) of using two or more languages to encode accurate meaning in classroom mathematical conversations. I then follow with domain research that epitomizes the political dimension of language as social through the critique of the ideals of communicational tool and accurate meaning, and of universal school mathematics. The third meta-theoretical concern is domain research that gives visibility to classroom practices of translanguaging that acts as a catalyst against the presumed highest epistemic value of the "one classroom, one language, one mathematics" ideology.

In the regular classrooms of the three studies I use to show my arguments, all the learners with stories of immigration shared at least one common language with teachers and peers. This composition was a result of policies of segregation with Latin American migrants mostly living in some districts of Barcelona, the main city in my region. The planning of newer research projects was shaped by these policies, and by the illusion of classroom sites of simplified methodological and

translation work to make sense of the languages and cultures of the learners who did not belong to the majority group. Still today, the local funding policies contemplate the category of technical assistant for specific languages (Amazigh, Bangla, Urdu…), to be contracted for relatively short amounts of time which do not allow them to become interpreters or researchers with whom to discuss meaning in data. Given my capacity for "self-translating" in Catalan and Spanish, the option of work with bilingual sites was a relief at that time. I was somehow limited by a language ideology centered on the naturalization of some languages over others, and on the importance of attending to the details of language separation and "correct" linguistic form, that did not allow me to see and recognize fluid moves across multiple languages in the classroom interactions of my data.

The field of critical linguistics has thoroughly examined and uncovered the (un)articulated language ideologies (of the particular research communities, of researchers, of the school, of teachers, learners, and families) that constrain the vision of the "multi" and discriminate non-standardized languages, cultures, forms of knowledge, and practices (Blommaert, 1999). This notion of language ideologies is a key to understand the powerful role of language in making and changing the world, specifically the worlds inside and outside schools and classroom research, through the naturalization of common-sense ideas such as the need for a hierarchy of bounded, named languages traversed by norms of linguistic purism and other notions of language correctness. Whether or not aware, mathematics education researchers, mathematics teachers, and mathematics teacher educators are not immune to these conflicting, received language ideologies of construction, recognition, and imposition of an official culture and language norms. These are ideologies and norms conducive to the enforcement of a homogenous culture of school mathematics and of schooling, and to the devaluation of all other cultures and languages and of their speakers.

3.1 First Concern: Asking Questions About Codeswitching

Planas and Setati (2009) document multilingual spontaneous practices of learners with minimal pedagogical intervention from teachers. In that study, the collection and analysis of classroom data were aimed at investigating "how much" Catalan and Spanish each Latin American migrant learner had spoken during five lessons, and then at "counting" the shifts between Catalan [C] and Spanish [S]. There were two research questions: (1) Do Spanish-dominant bilingual students in Catalan classrooms switch languages during mathematical activity? (2) If so, what are some of the factors that seem to promote the language switching with a group of these students in the context of specific lessons? An assumption framing these questions was that the contrast between mathematical participation in small and whole groups was a consequence of the language mostly spoken in the interaction. We corroborated this assumption and concluded that the learners switched to the home language as soon as the conceptual demands in peer mathematical talk increased. At that time, classroom work in which

learners used their languages for mathematical communication in creative, hybrid ways was not studied. The decision of examining language shifts rather echoed ideologies of linguistic purism and language separation. This is a literal piece of the data in peer work (p. 43):

Máximo:	[C] Hem de decidir les fletxes que dibuixem i ja està./We need to decide the arrows that we draw and that's all
Eliseo:	[C] Primer pensem les fletxes, després les dibuixem i després en parlem./First we think about the arrows, then we draw them and then we talk about it
Máximo:	[S] Esta idea de las flechas no es fácil. Tenemos que imaginar los diferentes movimientos que existen dentro del tornado./This idea of the arrows is not easy. We have to imagine the different movements that exist within the tornado
Eliseo:	[S] Una flecha tiene que ser una línea recta para que el tornado baje. Tenemos la *t* para la translación./An arrow needs to be a straight line for the tornado to go down. We have the *t* for the translation

The contents under discussion in the excerpt above are part of the unit called "Our dynamic planet," which included a variety of paper-and-pencil mathematical activities that encouraged learners to pose questions and solve problems about Euclidean geometrical transformations in real contexts. In the lesson of the excerpt, the teacher wanted the learners to think about and graphically represent on the plane the composition of spatial transformations such as translations, rotations, homotheties, and symmetries. The central task in this lesson was "How can you mathematically represent a tornado?", and learners started drawing arrows to represent linear motion. In all the lessons, there was an initial open-ended question presenting the task that had more than one answer.

Sociocultural studies that have approached the multilingual mathematics classroom from a communicational perspective well document the attention to characteristics of alternating or shifting languages. Moschkovich (2002) with Latino bilingual learners in California, and Setati and Adler (2000) with multilingual learners in South African townships, characterized codeswitching as a strategy for convergence toward the academic register of school mathematics in the English language of instruction. Shifting languages was therefore an action with meaning in the interactional processes and not just a simple expression of choice. This field-based characterization of codeswitching helped the domain to understand some of the mathematically relevant functions and dynamic forms of language common to mathematics lessons in research contexts with monolingually oriented policies and pedagogies. In this way, it could be claimed that codeswitching is a "natural" unproblematic consequence of what multilingual learners do with language in the mathematics classroom, even when the educational policies and the classroom norms are differently oriented and refrain learners from flexibly using their languages to express and develop their thinking. It was also claimed that multilingual learners within the mathematics classroom behave as people who speak more than one language generally do, and that codeswitching is not necessarily a symptom of lexical gaps, linguistic difficulties, or deficient language abilities of specific groups of learners.

The location of switching languages on the same communicational continuum as other language practices in multilingual talk reinforced the moves away from remedial views of learners' codeswitching. Multilingual mathematical interaction was now closer to being understood in relation to the multiple linguistic forms available in language for the development of the learner's ability to engage with the content. For this to happen, nonetheless, the interrogation of the ideal of codeswitching as an encoder of both communication and accurate meaning would be necessary. Domain researchers were leaving behind the vision of the languages of multilingual learners as sources of difficulties and were starting to see them as sources of mathematical meaning.

3.2 Second Concern: Asking Questions About Languaging

Planas (2014) documents classroom codeswitching viewed as related to the generation of mathematical learning opportunities in peer work, and hence conceptualized as learning resources and sources of mathematical meaning rather than indicators of linguistic difficulties or lexical gaps. The initial research question of that study centered on the potential of switching languages for classroom mathematics learning and meaning making. In a period in which I had not yet started reading about translanguaging and translingual research, an unexpected finding was very revealing. I could see Latin American learners who codeswitched to talk about the language ("the Spanishes and Catalans") they were producing and about their ways of using it during mathematical work. It seemed as if they were acting on language or languaging by talking about their linguistic innovations and those of the others in the small group. Some of these languaging processes interacted with talk in ways that seemed to unravel mathematical meaning and understanding; thus, they could be read as an expression of language as a resource for mathematics learning. An example reproduced in that paper shows learners who language (act on language) in Catalan (italics) and Spanish (non-italics) to invent and use two pair names without genuine meaning in the school mathematics for numerical consecutiveness. This is a literal piece of part of the transcript for that example (p. 61):

Anna:	*Puc donar exemples, com 3 + 5, 3.5 + 4.5… És sumar 1.* [*I can give examples, like 3 + 5, 3.5 + 4.5… It's adding 1*]
Juan:	*Sí, sumar 1, però els,* los números tienen que ser *no continuats*, como 3, 4, 5, 6… [*Yes, adding 1, but the*, the numbers need to be non-continued, like 3, 4, 5, 6…]
Carmen:	*Continuats?* Es *consecutius!* [{smiles} *Continued?* It's *consecutive*!]
Juan:	*Conse… Mira, així ho entendrem millor.* 3.5 y 4.5 son *continuats.* [*Conse… Look, this way we will understand it better.* 3.5 and 4.5 are *continued*]
Anna:	*Són consecutius per la resta, és 1.* [*They are consecutive because of the difference, it's 1*]

(continued)

(continued)

Juan:	*Però són decimals…* son *continuats*, no es que se siguen. [*But they are decimals…* they are continued, they don't follow each other]
Carmen:	*Sí, han de ser consecutius i que…* se sigan. [*Yes, they need to be consecutive and…* follow each other]

An important conclusion was that the discussion around the created terms enabled learners to interact in fluent ways and prompted mathematical reasoning on the classroom task. Learners could rather have focused on abnormal language use and put concerns of linguistic or meaning accuracy above the discussion, negotiation, and naming of a concept of numerical consecutiveness imagined for rational numbers. Instead, they engaged in creative mathematical and linguistic processes. It is far from easy for this to happen and to be seen in the context of naturalized norms that enforce the adequacy of specific school mathematics in the language of instruction, which tend to limit classroom talk and mathematical curiosity. In my role as researcher, I was now ready to focus on mathematics learning beyond evidence of adjustment to prescriptive school mathematics and normative talk. The resulting conclusions about the pedagogic realization of multilingual languaging are common to a stream of sociocultural studies under the approach of language as a resource that is closer to seeing processes of mathematics learning beyond linguistic and meaning accuracy and prescriptive curricula. Martínez (2018) with bilingual participants in language immersion classes in Colombia and in the United States, and Phakeng et al. (2018) with data from South African, Indian, and Catalonian trilingual classrooms, also exemplify languaging events with language inventions and innovative talk about language and some of the positive effects on mathematical work and learning.

The approach in Planas (2014) took a contrastive stance to the approach in Planas and Setati (2009) in that it included the explicit vision of codeswitching as a resource in processes of talk about language and mathematics, and the possibility of unpredictable mathematical meaning construction and language use. Nonetheless, it remained limited by monolingual orientations in a number of subtle ways, such as the choice of how to transcribe mathematical talk. The learners in the data did not speak the mathematical symbols for numerals as I made them appear in the transcripts. I had not considered how learners said *3 + 5, 3.5 + 4.5, 3, 4, 5, 6* or *3.5 and 4.5* in peer work as if that was not relevant data, or the embedded processes of making mathematical meaning could be guessed. The use of italic and non-italic fonts for numerals, as if the meaning encoded belonged to Catalan or Spanish, was theoretically and politically sensitive, since it reflected the representation of a neutral mathematical language with autonomous existence. From the mathematical meaning perspective, it is however very different to say "three and half and three plus one and half," or "three point five and four point five," or even "three with five and four with five," all of which are possibilities coming out of *3.5 and 4.5*. The omission of how learners talked symbols in peer work had undermined the communication of processes of mathematical meaning making and their mathematical ability to discuss numerals and properties like being consecutive in combination with number subsets.

3.3 *Third Concern: Asking Questions About Translanguaging*

Today, the conclusion that translation and codeswitching are just some of the many multilingual language forms available along the communicational continuum supports domain researchers in the ability to see mathematical, linguistic, and semiotic creations of significance in the interrogation of linguistic and meaning accuracy and universal school mathematics. This includes the coinage of the "trans" terminology in the research domain that functions to mean the conceptualization of the language of (school) mathematics, and not only language or the language of learners or the language of teachers, as diverse and fluid. Accordingly, practices of translanguaging refer to the creative use of language for mathematics teaching and learning as classroom participants make sense of their worlds and identities in relation to the languages and cultures of mathematics of the others. A typical situation of translanguaging is when bilingual migrant learners retain their home languages for peer work discussions, and produce linguistic forms that mix the academic language of instruction and the everyday home languages to talk in ways that create broader spaces for interrogation and understanding of mathematical meaning.

The research questions in Planas and Chronaki (2021) directly relate translanguaging to multilingual hybrid spaces that challenge restrictive views of school mathematics and of linguistic behavior. That study shows how translanguaging in mathematical talk displays linguistic creations through combined forms of everyday and academic Catalan and Spanish to challenge the institutionalization and implementation of naturalized mathematical and non-mathematical meanings. Just as codeswitching had appeared to be a common practice of the multilingual learners in the mathematics classroom rather than an aspiration, linguistic translanguaging also appeared to be an existing condition of mathematical conversations among multilingual peers. In an example, the struggles for meaning around "baixar" [going down] and "saltar" [jumping] became part of the mathematical talk to solve the particularization of a Fibonnaci-type problem by counting the possibilities of climbing a staircase through combinations of one and two step-sizes that add up exactly to ten. This problem, together with another Fibonacci-type problem on seating arrangements with rows of desks in a classroom, were posed to work on divisibility properties and reflect on the mathematical fact that every positive integer can be expressed as a sum of distinct Fibonacci numbers. This is a literal piece of the transcript, now with the linguistic expressions for numerals (p. 159):

Maria:	*Sempre es baixa, no t'estàs parat*	*You are always going down, you don't stand still*
Leo:	Pero a veces no bajas, saltas. Y a veces solo bajas	But sometimes you don't go down, you jump. And sometimes you only go down
Maria:	*Ada, tu ho tens clar?*	*Ada, does this make sense to you?*

(continued)

(continued)

Ada:	*Sí, baixar*	*Yes, going down*
Ton:	*Et deixes combinacions d'uns i dosos*	*You're missing combinations of ones and twos*
Leo:	He empezado pero hay mucho que bajar y saltar. Al menos treinta. Si la escala fuera más corta…	I began but there is too much to go down and to jump. At least thirty. If the staircase was shorter…
Ton:	*Umm… Si fos tres, seria: u, u, u; dos, u; u, dos… i dos, dos impossible. Ara ve quatre*	*Umm… If it was three, it would be: one, one, one; two, one; one, two… and two, two impossible. Now it's four*

At this point of the progress in multilingual mathematics classroom research, several issues arise. Compared to the spontaneity of learners' translanguaging, translingual pedagogies and translingual research methodologies are complex in contexts of one culture of mathematics and one language of instruction. Even though we can be attentive and sensitive to the politics of our choices in research, we can involuntarily continue to prescribe linguistic differentiation and mathematical universalism, and hence limit the representation of some of the learners' abilities in the data and findings. In Planas and Chronaki (2021), for example, the original lesson transcripts again use italic and non-italic fonts to distinguish Catalan and Spanish; a choice that resonates with a monolingual view of bilingualism and a tacit allusion to language separation. While this choice can be an object of critique, the analytical focus on mathematical content and on processes of mathematical meaning positively aligns with a translingual stance in the research domain. We find similar methodological-analytical concerns in the work of Gándara and Randall (2019) with multilingual mathematics learners' translanguaging in the Democratic Republic of the Congo, and in the work of Garza (2017) with Latino mathematics learners who are bilingual in the United States. Both studies address the latent and sometimes productive tension in classroom talk between how much attention to give to mathematics and to language, and specifically how much attention to give in the research process to the politics of representing diverse languages and mathematical cultures through transcripts that unintentionally suggest issues of bright lines between languages.

4 Implications of Translingual Research for Mathematics Teacher Education

I have already explained that the educational policies dictating the representations of multilingualism in my major research context entail problematic strategies of space separation (i.e. regular and special or language support classes), of curricular separation (i.e. remedial simplified subject contents in the support classes), and of teacher separation (i.e. less-experienced teachers assigned to the support classes). Similar arrangements remain problematic in other world contexts like Arizona, South Africa, or Greece. I now reflect upon the possibilities translingual research offers for policies

and pedagogies of mathematics teacher education that can make a change in school mathematics education in a direction different to the one traced by monolingually oriented ideologies.

Essien et al. (2016), and Rangnes and Meaney (2020) show the influence of monolingual orientations on mathematics teacher education policies and pedagogies across countries, and suggest research work to deploy pedagogies of "one classroom, one language, one mathematics" also in place in the teacher education institutions. These authors argue for the inclusion of curricular language content for subject-specific teaching and learning in settings of teacher education and professional development. Elsewhere (Planas, 2021), for sites of mathematics teacher professional development, I present the instructional principle of critically distinguishing and choosing or producing instances of lexical elaboration in classroom teacher talk for the overt communication of conceptual meaning within the algebra of equations. Whereas this study does not directly deal with the multiple languages and mathematical cultures of the learners in the classrooms selected for developmental work, it examines the argument for lexical elaboration in articulation with the argument for drawing on fluid language practices in favor of processes of mathematical meaning making. In this chapter, I additionally claim the possibility of envisaging translanguaging itself as a teaching pedagogy or resource that can help student teachers, teachers, and teacher educators to perform their diverse mathematical identities and those of the others in the teacher education modules. Otherwise, if we keep teacher education practices uncritically aligned with monolingual policies and pedagogies, we will prevent (student) teachers from creatively using their languages and mathematical cultures in the thinking of school mathematics teaching, and we will dissuade them from valuing learners' translanguaging.

Just as in the multilingual mathematics classroom, the dominance of monolingually oriented policies and pedagogies does not totally constrain mathematics teacher training. There is room in the teacher education modules for sociolinguistic hybrid spaces of interaction in which translanguaging can take place, and in which more than one language can be chosen to talk and write about more than one culture of mathematics expressed in more than one language. As a mathematics teacher educator in my university, I have been searching for such translanguaging spaces in the curriculum. My research work in multilingual mathematics classrooms has facilitated this search. Linguistic data with school learners and teachers translating, codeswitching, and more generally translanguaging in mathematical interaction have inspired pedagogic work in mathematics teacher education modules. This includes the opening of new learning spaces with the student teachers to experiment with their languages and cultures of mathematics, and the reflection on how school learners also creatively experiment with their languages and cultures of mathematics in the school classroom.

The issue of mixing languages and challenging normative school mathematics is very sensitive in my training context. For some of the future teachers, the material texts with school learners moving between Catalan and Spanish and inventing mathematical meaning through terms like "non-continued" to discuss a possible concept of consecutiveness in the school register of rational numbers, are first experienced

as aberrant uses of the language of instruction and of school mathematics. While student teachers are usually favorable to opening space to learners' participation, it is complex to convince them of the pedagogical, epistemic, and political role of translanguaging in the mathematics classroom. For them, the claim that creative processes of mathematical and linguistic meaning are positive for moving mathematical thinking and learning is somehow counterintuitive. It is also difficult for them to notice the risks of undermining some of the learners' mathematical ideas behind the "one classroom, one language, one mathematics" normativity. However, once they start understanding the nature of the relationship between mathematical thinking and translanguaging, they start to be aware that they are in the module as future teachers of mathematics that will teach mathematics, not the language of instruction or the facts of mathematics only, to a diversity of learners.

What is clear from my observations in the university modules is that linguistic hybrid texts of school lesson data offer an excellent basis on which to discuss what it means to be a mathematics teacher in the multilingual classroom. The discussion on these texts helps to interrogate and deconstruct professional identities while paying attention to the struggles between language policies and the communication and learning of mathematics. In a session with preservice teachers in February 2020 in which I proposed work on the complete episode on consecutive numbers, partially reproduced in the previous section, the major goal was to move our thinking from issues of policy and normativity to issues of mathematics learning and school mathematics. One of the student teachers, Lidia, was a vehement defender of prescriptive mathematical meaning:

> *Consecutive numbers mean something very exact and they* [school learners] *need to know. They cannot start inventing, and talking as they want as if that was not a classroom and there was no teacher.* I see what you say, *that they did that to find ways to solve the mathematical problem, and that they succeeded. But this is…* too risky. *They may not be so lucky with the next mathematical problem.* [My translation of my notes in Catalan on Lidia's talk]

Lidia equated the exactness in the school definition of a mathematical concept with the pedagogic conditions of the language processes for making mathematical meaning and explaining the concept. In my reaction, I asked her the meaning of consecutive numbers, or the meaning that she viewed as exact. Her response offered the opportunity to keep working with all the student teachers on the interrogation of static representations of school mathematics and language in mathematical meaning making. She codeswitched to tell us that two numbers are consecutive when "*són enters i se succeeixen* uno a uno" [*are integers and succeed* one by one]. From there, we started the discussion on the differentiation between "successive" and "consecutive" in school numeracy, and put it in contrast to the synonymy of the two pair words in everyday Catalan. Another student teacher, Òscar, interestingly codeswitched to ask:

> *Could we use a longer definition* [of consecutive numbers], *with words like sub…* subsequent *or every next, so that the definition is more an explanation of the mathematical idea?* [My translation of my notes in Catalan on Òscar's talk]

The distinction suggested between knowing a mathematical definition and producing a mathematical explanation functioned to open up interactional spaces for exploring and valuing Lidia's creative use of language as a resource in her explanation of the mathematical concept. More generally, the pedagogic emphasis on the positive opportunities of sharing nuances in mathematical meaning resulted in the interrogation of school mathematical definitions as finite products, compared to mathematical explanations, which limit the learner agency in the thinking and learning process. While offering a critique of the learners in the school episode and taking the role of the definer of mathematical consecutiveness, Lidia had produced an example of translanguaging to initiate the mathematical explanation of the concept against the boundaries of exact or accurate meaning in school mathematics. In discussing the language for numerical consecutiveness not as static definitive text but as fluid in the process of mathematical meaning construction, we see the pedagogic tension between putting limits on language creativity and increasing opportunities of mathematical understanding.

It takes a process of education and of self-interrogation and self-questioning of cultural boundaries for future teachers to see that they are already living in spaces of continuous translanguaging, and that this is potentially good for their preparation as school teachers of mathematics. More time, more agents, and more developmental actions are necessary for teachers and future teachers to make sense of professional identities that challenge language policies and naturalized views of school mathematics. This reflection is also applicable to researchers, and the process that it takes to see the meaning and implications of monolingual and monocultural orientations for research work.

5 The Translingual Position in the Research Field of Mathematics Education

Throughout this chapter, we have seen some of the lessons that multilingual mathematics classroom research can offer to policy, practice (specifically mathematics teacher education), and research in mathematics education. I have drawn together various threads of the argument about the relationships between monolingually oriented educational policies and progress in multilingual mathematics classroom research. Moreover, I have reflected on how these relationships can mediate the emergence of translanguaging pedagogies in mathematics teacher education and, more generally, the adoption of a translingual position that captures and defends the language practices of all those who use the linguistic, cultural, and social resources at their disposal to produce and investigate mathematics teaching and learning. The translingual position encourages the flexible use of languages because it views them as complementary resources that enrich one another and the educational experiences of all participants regardless of their home languages and cultures. Such a position, however, is not easy to undertake. It confronts institutional policies and

programs, and instructional materials with a strong monolingual stance across many world contexts in which teachers and teacher educators are differently aware of the pedagogic and epistemic richness of the multilingualism in their classrooms. There are also many challenges with respect to how the translingual position can enter the broader research field of mathematics education and the work of its researchers. In this final section, the focus moves to some of the implications for mathematics education research of interpreting and integrating findings related to the most recent translingual research on multilingual mathematics classrooms. The consideration of effects of the translingual position in terms of policy, practice, and research is completed in this way. Classroom research on language and mathematics has certainly experienced profound shifts from a logic in which languages are distinct from each other, to a logic that considers languages for how they are creatively negotiated and used in mathematical interaction and communication. A growing body of literature accordingly represents the translingual position in multilingual mathematics classroom research. Here, researchers look at how the languages of the learners, of the teachers, and of mathematics cross boundaries to meet, and interrogate translation or codeswitching thought to encode univocal meaning. The broader research field and even part of the specific research domain, however, have not readily adopted this position. More often than not, studies in mathematics education continue to imply work that assumes the fixity of mathematical meaning in processes of translation and of representation of data for conversation with the international scientific community. The issues that our research work needs to address are many in order to develop methodological approaches aligned with the identification, analysis and recognition of translanguaging as common in data and in research practice.

The opportunity for reflexivity generated by translingual research in multilingual mathematics classrooms and mathematics teacher education settings may help field researchers to see the political and ethical role we all have in the material we generate, and in the hybrid spaces we make possible or constrain in our research projects, decisions, and processes. Since the claim of Morgan (2007) that all mathematics classrooms are multilingual in some sense, the world and its classrooms have become more diverse, but illusions of monolingualism and monoculturalism keep moving the research field. The translingual position might facilitate looking across languages and cultures to capture the meanings produced within and by the research process, rather than seeing meaning as static and uncritically tied to assumptions of "one classroom, one language, one mathematics." Such an approach might also facilitate an understanding that even in apparently monolingual research contexts, the ways we use and see language drive data representation and meaning in the production of claims, findings, and reality.

Mathematics education research cannot remain aloof to the theoretical-methodological issues and positions around language, communication, and culture that the practices and consequences of translating data and of assuming unchanged meaning raise. In this regard, a first lesson from translingual research in multilingual mathematics classrooms is that processes of linguistic translation always involve processes of mathematical meaning making and valuing. A second lesson is that

working in more than one language, rather than necessarily implying the experiencing of lexical gaps or linguistic difficulties, can result in spaces for language innovation and newer meaning with the potential for increased mathematical interaction and thinking with the "others." The issues at stake are hence beyond mathematics education research in multilingual sites, and have implications for the quality of the data and findings arising from monolingually oriented studies. The politics and ethics of translation are complex and the task of representation in research is fundamentally problematic. It is not trivial and absent of consequences how the translated representation of the language of some learners may cover home cultures of mathematics, how the translated representation of the language of non-English speaking researchers may cover or devaluate ways of thinking, or how the representation of spoken language in written language may be thought of as untranslated and univocal.

All researchers in mathematics education, not only those explicitly working in or with multilingual sites, should develop responsible and ethical awareness of the many nuances that exist in language and in word meaning, and of the many ways in which language and meaning can be used to create spaces of understanding with implications for interaction with other researchers. In this respect, a third lesson from translingual research in multilingual mathematics classrooms is that, even when it seems that the same language is being spoken, the politics and ethics of language require questions about meaning, communication, and understanding. The experience of progress in the research domain makes me think that multilingual mathematics classrooms and mathematics teacher education that is language responsive will provide field researchers, who apparently live and work in monolingual sites, with tools to address the challenges and opportunities arising from language-based mathematics education research. A crucial challenge is the co-construction of progress and reflexivity in contexts framed by the illusion of becoming a learner under monolingual and monocultural conditions. These concerns around capitalizing on translingual research make total sense in other educational fields that deal with teaching and learning in science and science teacher education classrooms.

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