

Equity Market Segmentation in Risk-Based Portfolio Construction Techniques



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Abstract Risk-based portfolio construction models—i.e., strategic asset allocation approaches that do not make use of expected returns as inputs, such as optimal risk parity, most diversified portfolio, global minimum variance, and equal weighting—have reached a widespread use in the asset management industry. The aim of this research is to verify how these techniques reach different results depending on whether the global equity market is subdivided on the basis of a sector breakdown, carried out according to the industry sectors of each company, or in function of a geographical breakdown, carried out according to the listing market of each company. An empirical analysis, applied on a representative sample of global equity market indexes calculated by MSCI, is implemented by making use of the typical and most advanced statistical and financial evaluation measures. This comparative analysis reaches consistent results, showing a significant preference for the sector breakdown compared to the geographical one. In conclusion, this outcome can be ascribed to the segmentation of the equity market into sector indexes characterized by better external differentiation and stronger internal coherence. Moreover, sector indexes are characterized by a lower degree of concentration in comparison to the geographical ones.

Keywords Risk-based strategies · Sector indexes · Geographical indexes · Risk parity

1 Introduction

The typical strategic asset allocation begins with the segmentation of the investment universe into distinct asset classes, each one composed of financial assets with homogeneous risk-return profiles. The definition of the asset classes is significant,

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since the forecasting of the market variables is based on the expectations regarding the macroeconomic scenario, in order to forecast the trend of each market sector.

Equity asset classes are commonly identified by geographical or sector criteria (Basile and Ferrari 2016). Geographical criteria assume that stocks listed in the same market are more correlated, because firms operate with the same currency, interest rate curve, economic policy, and country risk. Sector criteria, instead, assume that stocks of companies in the same economic sector are correlated, as the industry determines their sensitivity to the economic cycle and factors such as developments in production processes, technological advancements, competition, infrastructural needs, consumer preferences, and commodities market.

This study is aimed to the inquiry of the segmentation technique most appropriate for risk-based portfolio construction methods in the equity market and an empirical analysis is carried out in order to reach this goal. This research provides an innovative point of view in the field of risk-based techniques, because the existing literature (Roncalli 2014; Scherer 2015; Braga 2016) has not investigated the potential presence of different portfolio performances based upon the segmentation of their assets. In other words, previous research has been focused on the theoretical and empirical implementation of risk-based asset allocations, but has not taken into account the qualitative aspects of the inputs in the portfolio construction process, i.e., whether the grouping of assets into homogeneous economic segments or geographical markets may determinate an impact on the ex-post portfolio risk-adjusted performance. The results of this empirical analysis complement the existing literature, showing a clear preference for the sector segmentation of assets in risk-based techniques, therefore providing a reference to scholars and practitioners alike for the primary decision in portfolio construction, i.e., the technique of segmentation of the investable universe of assets.

The remainder of this study is organized into four sections: Sect. 2 reviews the scientific literature on risk-based techniques of portfolio construction; Sect. 3 explains the methodology implemented and the dataset; Sect. 4 provides the results and their interpretation; and Sect. 5 concludes the study.

2 Risk-Based Strategies: Literature Review

The typical feature of risk-based strategies is the exclusion of the expected returns from their inputs; therefore, they are also known as μ -free strategies. The employment of these techniques requires only the estimation of risk measures, which are the sole inputs needed for asset allocation (Braga 2016). This feature makes these techniques very parsimonious in terms of inputs measurement and, as a consequence, guarantees a limitation of estimation risk.

Their rationale can be traced to the previous research about estimation error (Best and Grauer 1991). In particular, Chopra and Ziemba (1993) have demonstrated that an agent with average risk aversion can suffer losses eleven times higher due to an error in the estimation of expected returns compared to an identical error of

variances. Despite the advantage caused by the simplified estimation of inputs, some scholars have expressed concern about these techniques, given their lack of a defined objective function (Scherer 2011; Lee 2011). Risk-based strategies can be further updated and integrated into other portfolio construction techniques, as, for example, with the Black-Litterman approach (Haesen et al. 2014; Jurczenko and Teiletche 2018).

The following subsections provide details about the most used risk-based techniques, such as optimal risk parity, global minimum variance, most diversified portfolio, and equally weighting.

2.1 The Optimal Risk Parity Portfolio

The theoretical rationale of risk parity has been delineated in a formal way by Maillard et al. (2010) for the first time, after some initial studies by practitioners (Qian 2005, 2006; Neurich 2008). It is founded on the theory of risk budgeting, which states that the process of portfolio construction is based upon the allocation of risk, instead of asset allocation (Denault 2001). Therefore, optimal risk parity prevents the concentration of risk in a minority of large exposures to a small group of assets.

Risk allocation is defined as such that each asset provides the same ex-ante contribution to risk, i.e., an equal contribution to the total portfolio risk (Roncalli 2014; Scherer 2015), and, therefore, this strategy does not omit any asset of the investment universe from the portfolio. Moreover, the allocation of negative risk budgets to one or more assets would concentrate the risk exposure to the other components; for this reason, the relative risk budgets are required to be non-negative.

2.2 The Most Diversified Portfolio

The asset allocation of the most diversified portfolio is the result of the optimization, subject to the usual budget and no short sell constraints, of the diversification ratio (Chouiefaty and Coignard 2008), i.e., the ratio between the weighted average of the standard deviations of the returns of portfolio assets and the standard deviation of the portfolio returns. The weights of some assets can be set equal to zero, therefore, in this model, it is not granted that all the assets available in the investment universe are included in the portfolio. Moreover, this technique does not follow the model of risk budgeting and thus this approach does not guarantee ex-ante a homogeneous allocation of risk.

2.3 The Global Minimum Variance Portfolio

The global minimum variance portfolio is the only risk-based strategy that identifies a portfolio lying on the ex-ante efficient frontier (Clark et al. 2013). Actually, the formula of portfolio variance itself is the objective function and thus variances and covariances of assets are the only inputs required by this model. The fact that expected returns are ignored by the portfolio construction is the reason underlying the inclusion of this portfolio into the category of risk-based strategies.

The optimization process is subject to the no short selling and budget constraints and the solution can allow the presence of weights equal to zero, thus the global minimum variance portfolio can exclude some of the assets. The minimization of portfolio risk is reached when all the assets included have the same marginal risk.

2.4 The Equally Weighted Portfolio

The equally weighted portfolio is a heuristic approach, which attributes the same allocation to each asset; thus, it is also known as 1/N strategy and naïve diversification. This method of portfolio construction has often been studied by behavioral finance (Windcliff and Boyle 2004; Abate 2014), due to the lack of a formal objective function that would justify its use.

The equal-weighted strategy adopts the 1/N rule to asset allocation, while the optimal risk parity technique uses the same rule with regards to risk allocation. Moreover, the inclusion of assets is another feature that these two strategies have in common, in fact they both ensure that the full set of available assets is always included in the portfolio. As a consequence, naïve diversification can be implemented without taking into account any sample moment; at the same time, it is regarded a risk-based technique, because its construction rule provides an effective risk diversification.

While the available literature has reached mixed results with regards to the ex-post performances of equally weighted portfolios (Kritzman et al. 2010), some empirical studies, such as DeMiguel et al. (2009), have measured statistically superior results if compared to apparently more advanced techniques.

3 Data and Methodology

In order to compare the efficiency of the two techniques of subdivision of the equity asset class, it is necessary to measure the out-of-sample performances and the statistical moments of the different portfolio construction strategies. In the empirical analysis, which has been implemented with a rolling-window procedure, portfolios are constructed on the basis of the time series of monthly returns of the indexes of the

global stock market, denominated in euro, in excess of the 12-month Euribor rate, employed as a proxy of the risk-free rate. This rate has been chosen because this analysis is carried out from the perspective of an investor resident in the Euro area.

3.1 Statistical Properties of the Sample

The MSCI All Country World Index (ACWI) has been selected as the proxy of the global equity asset class. Following the sector approach, it is subdivided into 11 sector indexes; the geographical approach, instead, identifies six geographical indexes. While more geographical benchmarks could have been selected, but this choice has been avoided because some portfolios could have allocated a large share of their investments to areas of limited importance in the current configuration of the global stock markets. Moreover, a lower number of indexes limits the estimation error. The indexes of this sample are gross total return, i.e., they include the reinvestment of dividends gross of taxes, and have been weighted by free-float. The time series of monthly returns runs from November 1998 to October 2018, for a total of 240 months. This time window has been selected in order to estimate the parameters for the different portfolio construction models on a series of 60 months, which comprises distinct trends in stock markets, including the presence of extreme or rare events. The out-of-sample returns of risk-based strategies are estimated on the remaining 180-months window.

Table 1 provides the statistical sample moments of the full panel of data. As it can be noticed, only one series is subject to positive skewness and all the distributions show leptokurtosis, i.e., the phenomenon also known as “fat tails.” Given that equity markets throughout the world are usually highly correlated, the absence of negative correlation coefficients was expected. The presence of asymmetric and leptokurtic distributions requires the testing of their deviations from the Gaussian distribution, which has been carried out taking into account: normality, autocorrelation, heteroscedasticity, and stationarity (Tsay 2010). Table 2 summarizes the outcomes of these tests, which identify clear deviations from the hypotheses of normality, typical of traditional portfolio management. Therefore, risk-based models should be more reasonable, given that they require less estimates and do not limit themselves to the Gaussian distribution.

3.2 The Implementation of the Empirical Analysis

Portfolio construction strategies are applied on both the sector and the geographical criteria of segmentation of the market portfolio, to identify which one provides superior risk-adjusted performances and preferable statistical properties. All the portfolios are subject to the usual no short-selling and budget constraints (Braga 2016), while no additional constraints on asset or risk allocation have been imposed,

Table 1 Sample moments of the indexes' excess returns

	Benchmark	Expected return (%)	Standard deviation (%)	Skewness	Kurtosis
Sector	MSCI ACWI/Consumer Discretionary	0.48	4.82	-0.46	4.76
	MSCI ACWI/Consumer Staples	0.45	2.96	-0.86	4.19
	MSCI ACWI/Energy	0.56	5.34	-0.11	3.62
	MSCI ACWI/Financials	0.32	5.14	-0.64	5.73
	MSCI ACWI/Health Care	0.44	3.52	-0.58	3.22
	MSCI ACWI/ Industrials	0.48	4.69	-0.90	5.77
	MSCI ACWI/Information Technology	0.59	6.90	-0.38	4.45
	MSCI ACWI/Materials	0.57	5.32	-0.61	5.48
	MSCI ACWI/Real Estate	0.48	4.85	-0.87	6.74
	MSCI ACWI/Telecom Services	0.18	4.66	-0.37	4.90
	MSCI ACWI/Utilities	0.33	3.37	-0.92	4.14
Geographical	MSCI Emerging Markets	0.81	5.10	-0.53	4.73
	MSCI Europe ex UK	0.35	4.77	-0.54	4.27
	MSCI Japan	0.26	4.88	0.11	3.46
	MSCI North America	0.43	4.20	-0.67	4.33
	MSCI Pacific ex Japan	0.53	4.08	-0.68	4.54
	MSCI United Kingdom	0.29	3.93	-0.63	3.74

Source: Author's analysis on Morningstar Direct data

because their inclusion would mitigate the characteristic features of the models, which would reduce the significance of this comparison. Portfolios are rebalanced every three months, thus, with a total time series of 240 monthly returns and the 60 returns used for the first in-sample estimates, 60 portfolios for each strategy have been constructed, with a total of 180 out-of-sample monthly returns. Given the different levels of turnover of the models employed, transaction costs have been taken into account. A higher turnover causes more costs at each rebalancing date and, as a consequence, it reduces net returns. A transaction cost of 0.2% is applied to each index, given the relatively high liquidity of the index-tracking financial instruments available in the markets, such as exchange-traded funds. The models object of this empirical analysis are:

- Equally weighted portfolio
- Global minimum variance portfolio
- Maximization of the Sharpe ratio (for comparison with risk-based models)
- Minimization of the conditional Value at Risk (VaR) at the 95% confidence level
- Most diversified portfolio

Table 2 Tests of deviations from the Gaussian distribution of the indexes' excess returns

Sector	Benchmark	Jarque-Bera test ^a		Lilliefors test ^a		Ljung-Box test		Engle's ARCH test		Ljung-Box test on χ^2		ADF test ^a	
		stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value
	MSCI ACWI/ Consumer Discretionary	39.42	0.001***	0.08	0.001***	10.79	0.0556*	18.67	0.002***	23.71	0.000***	-6.28	0.001***
	MSCI ACWI/ Consumer Staples	43.70	0.001***	0.07	0.003***	6.61	0.251	24.02	0.000***	22.76	0.000***	-5.61	0.001***
	MSCI ACWI/ Energy	4.30	0.091	0.04	0.500	2.52	0.773	11.00	0.052*	8.80	0.117	-6.64	0.001***
	MSCI ACWI/ Financials	91.16	0.001***	0.08	0.002***	17.63	0.004***	53.50	0.000***	102.35	0.000***	-6.35	0.001***
	MSCI ACWI/ Health Care	13.72	0.007***	0.09	0.001***	5.76	0.331	14.54	0.013**	19.25	0.002***	-6.30	0.001***
	MSCI ACWI/ Industrials	108.96	0.001***	0.10	0.001***	14.75	0.012**	24.58	0.000***	37.49	0.000***	-6.03	0.001***
	MSCI ACWI/ Information Technology	26.83	0.001***	0.09	0.001***	6.44	0.266	61.97	0.000***	95.07	0.000***	-5.57	0.001***
	MSCI ACWI/ Materials	76.68	0.001***	0.06	0.076*	12.10	0.033**	29.71	0.000***	32.05	0.000***	-6.93	0.001***
	MSCI ACWI/ Real Estate	170.77	0.001***	0.09	0.001***	18.84	0.002***	42.89	0.000***	77.90	0.000***	-6.25	0.001***
	MSCI ACWI/ Telecom Services	41.64	0.001***	0.08	0.001***	17.77	0.003***	38.78	0.000***	78.06	0.000***	-5.49	0.001***
	MSCI ACWI/ Utilities	47.26	0.001***	0.10	0.001***	5.76	0.330	10.49	0.063*	13.17	0.022**	-5.30	0.001***

(continued)

Table 2 (continued)

Geographical	Benchmark	Jarque-Bera test ^a		Lilliefors test ^a		Ljung-Box test		Engle's ARCH test		Ljung-Box test on χ^2		ADF test ^a	
		stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value	stat	p-value
	MSCI Emerging Markets	41.12	0.001***	0.06	0.056*	15.46	0.009***	11.41	0.044**	14.98	0.011**	-6.42	0.001***
	MSCI Europe ex UK	27.67	0.001***	0.07	0.012**	10.37	0.066*	21.53	0.001***	35.07	0.000***	-5.36	0.001***
	MSCI Japan	2.61	0.224	0.04	0.345	15.85	0.007***	5.72	0.334	5.33	0.378	-5.85	0.001***
	MSCI North America	35.52	0.001***	0.09	0.001***	8.63	0.125	31.05	0.000***	52.23	0.000***	-5.99	0.001***
	MSCI Pacific ex Japan	42.37	0.001***	0.08	0.002***	7.03	0.218	13.78	0.017**	15.19	0.010***	-5.97	0.001***
	MSCI United Kingdom	21.42	0.001***	0.06	0.024**	6.82	0.235	26.42	0.000***	39.19	0.000***	-5.63	0.001***

Source: Author's analysis on Morningstar Direct data

^a The p-values of Jarque-Bera test, Lilliefors test and ADF test are bounded in the interval [0.0010, 0.5000] ***, **, and * represent statistical significance levels of 1%, 5% and 10%, respectively

- Optimal risk parity, employing the conditional VaR (95%) as its measure of risk
- Optimal risk parity, employing the standard deviation as its measure of risk

4 The Results of the Analysis

4.1 The Statistical Features of the Portfolios

Table 3 provides the statistical features of the out-of-sample excess returns of each portfolio. As a first consideration, it can be noted that these techniques provide returns significantly in excess of the risk-free rate, giving the opportunity to investors of allocating their wealth also in relatively aggressive strategies. All the distributions have negative skewness and kurtosis in excess of 3, in agreement with the indexes of the selected asset classes. The tests of deviations from normality employ the same techniques used for the equity benchmarks and their results are summarized in Table 4.

Table 3 Sample moments of the portfolios' excess returns

	Strategy	Expected return (%)	Standard deviation (%)	Skewness	Kurtosis
Sector	Equally weighted	0.60	3.60	-1.12	6.46
	Global minimum variance	0.61	2.74	-1.01	4.99
	Max Sharpe ratio	0.73	3.61	-0.97	4.78
	Minimum conditional VaR 95%	0.56	2.91	-0.92	4.48
	Most diversified portfolio	0.65	3.22	-1.18	6.08
	Risk parity conditional VaR 95%	0.61	3.39	-1.22	6.63
	Risk parity standard deviation	0.61	3.37	-1.19	6.47
Geographical	Equally weighted	0.56	3.47	-1.06	5.81
	Global minimum variance	0.60	3.40	-1.25	6.21
	Max Sharpe ratio	0.85	4.42	-0.80	6.24
	Minimum conditional VaR 95%	0.58	3.52	-1.10	5.97
	Most diversified portfolio	0.45	3.29	-0.90	5.41
	Risk parity conditional VaR 95%	0.56	3.40	-1.14	6.02
	Risk parity standard deviation	0.56	3.41	-1.10	5.91

Source: Author's analysis on Morningstar Direct data

Most diversified portfolio	67.87	0.001***	0.10	0.001***	16.53	0.006***	21.75	0.001***	37.06	0.000***	-5.69	0.001***
Risk parity conditional VaR 95%	107.17	0.001***	0.11	0.001***	15.84	0.007***	21.29	0.001***	36.18	0.000***	-5.38	0.001***
Risk parity standard deviation	99.63	0.001***	0.11	0.001***	16.90	0.005***	23.52	0.000***	40.79	0.000***	-5.36	0.001***

Source: Author's analysis on Morningstar Direct data
^aThe *p*-values of Jarque-Bera test, Lilliefors test and ADF test are bounded in the interval [0.001, 0.500]
 ***, ** and * represent statistical significance levels of 1%, 5% and 10%, respectively

The Jarque-Bera test rejects the presence of normality and a similar result is shown by the Lilliefors test, which identifies only one case in which it is accepted. The Ljung-Box test indicates six portfolios not subject to autocorrelation. Moreover, Engle's ARCH test and the Ljung-Box test on the squared residuals identify heteroscedasticity in all the tested time series. According to the Dickey-Fuller ADF test, all the portfolios are stationary. In conclusion, these tests replicate the same results of the ones carried out on the indexes, both geographical and sector.

4.2 *Comparative Analysis*

The first aspect considered in the comparison of the two market segmentation techniques is the assessment of portfolio risk, because it is the core of risk-based models. This analysis is carried out by measuring the standard deviation and the conditional VaR at 95% confidence level. Efficiency is also taken into consideration, given that rational economic agents seek to select the asset allocation with the highest risk-adjusted performance. The appraisal of portfolios' efficiency is based on two measures typical of the asset management industry, i.e., the Sharpe ratio and the Sortino ratio, and, given the presence of non-normal returns, on the conditional Sharpe ratio (95%), calculated as the ratio between the mean return in excess of the risk-free rate and the conditional VaR (95%). Finally, due to the rejection of the hypothesis of normality, it is necessary to evaluate also skewness and kurtosis.

According to the standard deviation (Table 3), the sector segmentation appears to be preferable to the geographical one, as indicated by the comparative results for each asset allocation technique, except for naïve diversification. Surprisingly, the techniques which seek to maximize the Sharpe ratio reach out-of-sample suboptimal results. The conditional VaR (95%) of each portfolio for both the segmentation techniques (Table 5) confirms that the sector criterion is systematically preferable to the geographical one. More in detail, the two methods of subdivision of the global equity market portfolio reach opposite results when they are employed in techniques seeking the minimization of the conditional VaR (95%). Actually, the portfolio built by making use of sector indexes has lower ex-post risks than almost every other asset allocation technique, except the global minimum variance portfolio. The minimization strategy of the conditional VaR (95%) applied on geographical indexes, instead, achieves a severely negative result, showing empirically the inconsistency of the ex-post asset allocation if compared to its inputs.

When efficiency is measured by the Sharpe ratio, the sector segmentation is significantly preferable by a rational investor. Moreover, the strategy of portfolio construction maximizing the ex-ante Sharpe ratio reaches a positive result out-of-sample, but the most efficient asset allocation is reached by the global minimum variance portfolio made of sector indexes. Given that the distributions are not Gaussian, the Sharpe ratio is not an optimal measure of efficiency. To overcome this limitation, the Sortino ratio is employed too. In fact, it can be regarded as the

Table 5 Risk-adjusted performance measures of the portfolios' excess returns

	Strategy	Sharpe ratio	Sortino ratio	Conditional VaR (95%)	Conditional Sharpe ratio (95%)
Sector	Equally weighted	0.17	0.23	-9.02%	0.07
	Global minimum variance	0.22	0.32	-6.67%	0.09
	Max Sharpe ratio	0.20	0.29	-8.75%	0.08
	Minimum conditional VaR 95%	0.19	0.28	-7.01%	0.08
	Most diversified portfolio	0.20	0.28	-8.10%	0.08
	Risk parity conditional VaR 95%	0.18	0.25	-8.66%	0.07
	Risk parity standard deviation	0.18	0.25	-8.56%	0.07
Geographical	Equally weighted	0.16	0.22	-8.92%	0.06
	Global minimum variance	0.18	0.24	-8.98%	0.07
	Max Sharpe ratio	0.19	0.29	-10.20%	0.08
	Minimum conditional VaR 95%	0.17	0.23	-9.32%	0.06
	Most diversified portfolio	0.14	0.19	-8.51%	0.05
	Risk parity conditional VaR 95%	0.17	0.23	-8.95%	0.06
	Risk parity standard deviation	0.16	0.23	-8.87%	0.06

Source: Author's analysis on Morningstar Direct data

excess return compared to the minimum acceptable return (MAR) per unit of downside risk. In this study, the MAR is the risk-free rate.

Table 5 provides the values of this asymmetric risk-adjusted performance measure. Notwithstanding its distinct theoretical framework, the ranking is the same reached with the Sharpe ratio. Therefore, also in this case, the sector segmentation appears to be significantly preferable to the geographical criterion. Economic agents' aversion to tail risk (i.e., negative returns in the extreme left of the distribution) can be taken into account by the conditional Sharpe ratio (95%). Table 5 reports the values of this measure, which exhibit a clear resemblance to those of the two preceding measures of risk-adjusted performance. It can be noticed that the higher degree of efficiency of the sector criterion is confirmed again. Table 3 provides the levels of skewness. The fact that this moment is not included in any asset allocation technique causes a certain randomness in the results; thus, in this case, it is not possible to identify a preference for a criterion of equity market segmentation. Like for the skewness, also kurtosis has not been taken into account as an input in any of the portfolio construction strategies object of this analysis. Yet, sector indexes seem to provide a better result with regards to this extreme risk measure (Table 3).

According to the outcomes of the empirical analysis, the techniques that minimize a risk measure have reached better results when employing sector indexes. More in detail, the global minimum variance portfolio constructed with sector indexes has shown the best performance, thanks to the diversification typical in this segmentation criterion. In contrast, the techniques of the optimal risk parity have not reached the top ranks of the different evaluation measures taken into account. Nevertheless, it can be noted that also for these strategies the sector segmentation is preferable to the geographical one. These findings imply that portfolio managers should group their assets according to their economic sector in order to aim at superior risk-adjusted performances.

5 Conclusions

These evidences provided by the empirical analysis can be evaluated taking into account the theory of the optimal risk parity strategy, which gives a primary importance to the estimation risk. It applies strict constraints on the asset allocation, given that all the assets must provide the same percentage risk contribution ex-ante and thus none can be excluded from the portfolio. The constraints have been set in order to avert the concentration of risk and to limit the amount of transaction costs. If, as in this analysis, the estimation risk is not high, these constraints do not allow the construction of efficient portfolios in terms of risk-adjusted performance.

This analysis reaches substantially coherent results with respect to the two techniques of segmentation of the equity asset class, highlighting a significant preference for the employment of sector indexes. This conclusion is due to the classification of the equity market into sector benchmarks that enjoy a better external differentiation and a higher degree of internal coherence. In addition, they are also characterized by a lower concentration. Actually, this feature guarantees that the results of the risk-based techniques are not caused by the relative performance of the geographical areas with higher market capitalization, as, instead, is the case of the geographical segmentation.

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