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ISBN 978-3-030-70667-8 ISBN 978-3-030-70668-5 (eBook)
https://doi.org/10.1007/978-3-030-70668-5

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Preface

I write this preface just after watching a peaceful change in who is President of the United States, something that was not necessarily going to be the case two weeks ago. I find it propitious that after many years of working on this book I have completed it at this time.

This book follows a series of previous books dealing with complexity issues since my first book in 1991, *From Catastrophe to Chaos: A General Theory of Economic Discontinuities,* Kluwer, as well as many papers starting earlier than then. It has been ten years since my last book on this topic, which focsed on certain special topics, *Complex Evolutionary Dynamics in Urban-Regional and Ecologic-Economic Systems: From Catastrohpe to Chaos and Beyond*, Springer. This book has chapters on those topics, but is more general. Indeed, the major new material one does not find much of in those earlier books is the emphasis on "Foundations," with this focus dominating the opening chapters of this book. Indeed, originally the title of this book was planned just to be *Foundations of Complexity Economics*, but as I did bring in chapters on applications it became more appropriate to add that to the title. But the desire to try to really delve into the philosophical and mathematical underpinnings of complexity economics in a fundamental way is what has truly motivated me to write this book. I hope that I have succeeded at least somewhat in that effort.

After all these decades of me working on this broad subject, there have come to be many people who have helped me in one way or another. I shall not be able to thank all of them, but I shall do my best. For this Preface I shall start by recognizing my coauthors of works cited in this book, with this also to recognize how much this book draws on my earlier research, with much of it done in conjunction with these people. So let me thank Ehsan Ahmed, Robert Bond, David Colander, Dimitrios Dendrinos, Carl Folke, Ilaria Foroni, Mauro Gallegati, Laura Gardini, John Gowdy, Steve Guastello, Folke Günther, Georg Hartmann, Rick Holt, Cars Hommes, Heikki Isomäki, Roger Koppl, Simone Landini, Li Honggang, Antonio Palestrini, Charles Perrings, Ray Prince, the late Tònu Puu, Marina Rosser, Loraine Roy, Jamshed Uppal, and Victor Yakovenko. Of course, none of these worthy individuals should be held responsible for any errors or questionable interpretations that may be found in this book.

Beyond coauthors, many others have provided advice, hosting me for visits or speeches, data, and other forms of support. I shall keep this long list to people who did so in the last decade since my last book, although there were others who helped me in the past, with hopefully those not listed here were thanked in one of my previoius book Prefaces. For this one I thank Brian Arthur, Yuji Aruka, Rob Axtell, Erich Beinnocker, Ken Binmore, Gian-Italo Bischi, Larry Blume, Pete Boettke, Sam Bowles, Buz Brock, Jean-Philippe Bouchaud, Éric Brousseau, Bruce Caldwell, Bikas Chakrabarti, the late Carl Chiarella, Pasuale Commendatore, John Conlisk, Bob Costanza, Allin Cottril, the late James F. Crow, Paul Davidson, John Davis, Dick Day, Christophe Deissenberg, Roberto Dieci, Giovanni, Dosi, Dick Easterlin, Catherine Eckel, Euel Elliott, Gustav Feichtinger, Peter Flaschel, John Foster, Bob Frank, Herbert Dawid, Domenico Delli Gatti, Doyne Farmer, Duncan Foley, Francisco Doria, Herb Gintis, Wade Hands, Geoff Harcourt, the late Don Hester, Geoff Hodgson, Douglas Hofstadter, Xavier Gabaix, Jamie Galbraith, Geoff Harcourt, Steve Horwitz, Shashi Kant, Stephanie Kelton, Ali Khan, Alan Kirman, János Kornai, Ingrid Kubin, David Levy, Paul Lewis, Rosario Mantegna, Akio Matsumoto, Deirdre McCloskey, Claude Menard, Stan Metcalfe, Juergen Mimkes, Phil Mirowski, the late Elinor Ostrom, Sandra Peart, Mark Pingle, Jason Potts, Ben Powell, Aura Reggiani, Mario Rizzo, Neri Salvadori, the late Massimo Salzano, Claudio Sardoni, the late Tom Schelling, the late Reinhard Selten, Willi Semmler, Rajiv Sethi, Mark Setterfield, Anwar Shaikh, Ajit Sinha, Tim Smeeding, Vernon Smith, Serena Sordi, Didier Sornette, Ed Stringham, Shyam Sunder, Iryna Sushko, Nassim Taleb, Pavlina Tcherniva, Leigh Tesfatsion, Peter Turchin, Karen Vaughn, Vela Velupillai, Alessandro Vercelli, Nick Vriend, Florian Wagener, Dick Wagner, Roy Weintraub, the late Martie Weitzman, David Wolpert, Bill Wood, Randy Wray, Peyton Young, Stefano Zambelli, and Steve Ziliak.

I would also like to thank Lorraine Klimowich and all the people at Springer.

Finally, I wish to dedicate this book to my loving wife, the light of my life, Marina Vcherashnaya Rosser.

Harrisonburg, USA J. Barkley Rosser Jr
January 20, 2021

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Chapter 1
Logical and Philosophical Foundations of Complexity

1.1 Forms of Complexity

There are at least 45 definitions of *complexity* according to Seth Lloyd as reported in *The End of Science* (Horgan, 1997, pp. 303–305). Rosser Jr. (1999) argued for the usefulness in studying economics of a definition he called *dynamic complexity* that was originated by Day (1994).[1] This is that a dynamical economic system fails to generate convergence to a point, a limit cycle or an explosion (or implosion) endogenously from its deterministic parts. It has been argued that nonlinearity was a necessary but not sufficient condition for this form of complexity,[2] and that this definition constituted a suitably broad "big tent" to encompass the "four C's"[3] of *cybernetics, catastrophe, chaos, and "small tent" (now better known as heterogeneous agents) complexity*.

Norbert Wiener (1948) founded cybernetics, which relied on computer simulations and was popular with Soviet central planners and computer scientists long after it was not so admired in the West. Jay Forrester (1961), inventor of the flight simulator, founded its rival *system dynamics*, arguing that nonlinear dynamical systems can produce "counterintuitive" results. Probably its most famous application was in *The Limits to Growth* (Meadows et al. 1972), eventually criticized for its

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[1] Velupillai (2011) has labeled this view of dynamic complexity as "Day-Rosser" complexity.

[2] Strictly speaking, this is incorrect. Goodwin (1947) showed such endogenous dynamic patterns in coupled linear systems with lags. Similar systems were analyzed by Turing (1952) in his paper that has been viewed as the foundation of the theory of morphogenesis, a complexity phenomenon par excellence. However, the overwhelming majority of such dynamically complex systems involve some nonlinearity, and the uncoupled normalized equivalent of the coupled linear system is nonlinear.

[3] This coinage came from Horgan (1997, Chap. 11) who sneeringly labeled the four C's to represent *chaoplexity*, which he considered to be an intellectual bubble or fad. Rosser Jr. (1999) argued that this was a coinage like "Impressionism" that was initially an insult but can be seen as a useful characterization.

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_1

excessive aggregation. Arguably both came from *general systems theory* (von Bertalanffy, 1950, 1974), which in turn developed from *tektology*, the general theory of organization due to Bogdanov (1925-29).

Catastrophe theory developed out of broader bifurcation theory, which relies on strong assumptions to characterize patterns of how smoothly changing control variables can generate discontinuous changes in state variables at critical bifurcation values (Thom, 1975), with Zeeman's (1974) model of stock market crashes the first use of it in economics. Empirical methods for studying such models depend on multi-modal statistics (Cobb et al. 1983; Guastello 2011a, b). Due to the strict assumptions it relies upon, a backlash developed against its use, although Rosser Jr. (2007) argued this became overdone.[4]

While chaos theory can be traced back to Poincaré (1890), it became prominent after climatologist Edward Lorenz (1963) discovered *sensitive dependence on initial conditions*, aka "the butterfly effect." Applications in economics followed suggestions made by May (1976). Debates over empirical measurement and problems associated with forecasting have reduced its application in economics (Dechert, 1996).[5] It is possible to develop models that exhibit combined catastrophic and chaotic phenomena as in *chaotic hysteresis*,[6] first shown as possible in a macroeconomic model by Puu (1990), with Rosser Jr. et al. (2001) estimating such patterns for investment in the Soviet Union in the post-World War II period.

The small tent or heterogeneous agents type of dynamic complexity does not have a precise definition. Influentially, Arthur et al. (1997a) argue that such complexity exhibits six characteristics: (1) dispersed interaction among locally interacting heterogeneous agents in some space, (2) no global controller that can exploit opportunities arising from these dispersed interactions, (3) cross-cutting hierarchical organization with many tangled interactions, (4) continual learning and adaptation by agents, (5) perpetual novelty in the system as mutations lead it to evolve new ecological niches, and (6) out-of-equilibrium dynamics with either no or many equilibria and little likelihood of a global optimum state emerging. Many point to Thomas Schelling's (1971) study on a 19-by-19 Go board[7] of the emergence of urban segregation due to nearest neighbor effects as an early example.

Other forms of nonlinear dynamic complexity seen in economic models include *non-chaotic strange attractors* (Lorenz 1983), *fractal basin boundaries* (Lorenz 1983; Abraham et al. 1997), *flare attractors* (Hartmann and Rössler 1998; Rosser Jr. et al. 2003a), and more.

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[4] Arnol'd (1993) provides a clear discussion of the mathematical issues involved while avoiding the controversies.

[5] For further discussion of underlying mathematical controversies involving chaos theory, see Rosser Jr. (2000b, Mathematical Appendix).

[6] This term was coined by Abraham and Shaw (1987), and Abraham (1985) also conceived the related combined phenomenon of *chaostrophe*.

[7] It has often been claimed incorrectly that Schelling used a chess board for this study.

Other non-dynamic complexity approaches used in economics have included *structural* (Pryor 1995; Stodder 1995),[8] *hierarchical* (Simon 1962), *informational* (Shannon 1948). *Algorithmic* (Chaitin 1987), *stochastic* (Rissanen 1986), and *computational* (Lewis 1985; Albin with Foley 1998; Velupillai 2000).

Those arguing for focus on computational complexity include Velupillai (2005a, b) and Markose (2005), who say that the latter concept is superior because of its foundation on more well-defined ideas, such as *algorithmic complexity* (Chaitin 1987) and *stochastic complexity* (Rissanen 1989, 2005). These are seen as founded more deeply on the *informational entropy* work of Shannon (1948) and Kolmogorov (1983). Mirowski (2007) argues that markets themselves should be seen as algorithms that are evolving to higher levels in a Chomsky (1959) hierarchy of computational systems, especially as they increasingly are carried over computers and become resolved through programmed double-auction systems and the like. McCauley (2004, 2005) and Israel (2005) argue that such dynamic complexity ideas as *emergence* are essentially empty and should be abandoned for either more computational-based or more physics-based ones, the latter especially relying on *invariance* concepts.

At the most profound level computational complexity involves the problem of non-computability. Ultimately this depends on a logical foundation, that of non-recursiveness due to incompleteness in the Gödel sense (Church 1936; Turing 1937). In actual computer programs this manifests itself most clearly in the form of the *halting problem* (Blum et al. 1998). This amounts to the halting time of a program being infinite, and it links closely to other computational complexity concepts such as Chaitin's algorithmic complexity. Such incompleteness problems present foundational problems for economic theory (Rosser Jr. 2009a, 2012a, b; Landini et al. 2020; Velupillai 2009).

In contrast, dynamic complexity and such concepts as emergence are useful for understanding economic phenomena and are not as incoherent and undefined as has been argued. A sub-theme of some of this literature, although not all of it, has been that biologically based models or arguments are fundamentally unsound mathematically and should be avoided in more analytical economics. Instead, such approaches can be used in conjunction with the dynamic complexity approach to explain emergence mathematically and that such approaches can explain certain economic phenomena that may not be easily explained otherwise.

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[8] Structural complexity appears in the end to amount to "complicatedness," which Israel (2005) argues is merely an epistemological concept rather than an ontological one, with "complexity" and "complicatedness" coming from different Latin roots (*complecti*, "grasp, comprehend, or embrace" and *complicare*, "fold, envelop"), even if many would confuse the concepts (including even von Neumann 1966). Rosser Jr. (2004) argues that complicatedness as such poses essentially trivial epistemological problems, how to figure out a lot of different parts and their linkages.

1.2 Foundations of Computational Complexity Economics

Velupillai (2000, pp. 199–200) summarizes the foundations of what he has labeled *computable economics*[9] in the following.

> Computability and randomness are the two basic epistemological notions I have used as building blocks to define computable economics. Both of these notions can be put to work to formalize economic theory in effective ways. However, they can be made to only on the basis of two theses: the Church-Turing thesis, and the Kolmogorov-Chaitin-Solomonoff thesis.

Church (1936) and Turing (1937) independently realized that several broad classes of functions could be described as "recursive" and were "calculable" (programmable computers had not yet been invented). Turing (1936, 1937) was the first to realize that Gödel's (1931) Incompleteness Theorem provided a foundation for understanding when problems were not "calculable," called "effectively computable" since Tarski (1949). Turing's analysis introducing the generalized concept of the *Turing machine*, now viewed as the model for a rational economic agent within computable economics (Velupillai 2005b, p. 181). While the original Gödel theorem relied upon a Cantor diagonal proof arising from self-referencing, the classic manifestation of non-computability in programming is the *halting problem*: that a program will simply run forever without ever reaching a solution (Blum et al. 1998).

Much of recent computable economics has involved showing that when one tries to put important parts of standard economic theory into forms that might be computable, it is found that they are not effectively computable in any general sense. These include Walrasian equilibria (Lewis 1992), Nash equilibria (Prasad 1991; Tsuji et al. 1998), more general aspects of macroeconomics (Leijonufvud 1993), and whether a dynamical system will be chaotic or not (da Costa et al. 2005).[10]

Indeed, what are viewed as dynamic complexities can arise from computability problems that arise in jumping from a classical and continuous real number framework to a digitized, rational numbers-only framework. An example is the curious "finance function" of Clower and Howitt (1978) in which solution variables jump back and forth over large intervals discontinuously as the input variables go from integers, to non-integer rationals to irrational numbers and back. Velupillai (2005b, p. 186) notes the case of a Patriot missile missing its target by 700 m and killing 28 soldiers as "friendly fire" in Dhahran, Saudi Arabia in 1991 due to a computer's

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[9] "Computable economics" was neologized by Velupillai in 1990 and is distinguished from "computational economics," symbolized by the work one finds at conferences of the Association for Computational Economics and its journal, *Computational Economics*. The former focuses more on the logical foundations of the use of computers in economics while the latter tends to focus more on specific applications and methods.

[10] Another main theme of computable economics involves considering which parts of economic theory can be proved when such classical logical axioms are relaxed as the Axiom of Choice and the exclusion of the middle. Under such *constructive* mathematics problems can arise for proving Walrasian equilibria (Pour-El and Richards 1979; Richter and Wong 1999; Velupillai 2002, 2006) and Nash equilibria (Prasad 2005).

non-terminating cycling through a binary expansion on a decimal fraction. Finally, the discovery of chaotic sensitive dependence on initial conditions by Lorenz (1963) because of computer roundoff error is famous, a case that is computable but undecidable.

There are actually several computability based definitions of complexity, although Velupillai (2000, 2005a, b) argues that they can be linked as part of the broader foundation of computable economics. The first is the Shannon (1948) measure of information content, which can be interpreted as attempting observe structure in a stochastic system. It is thus derived from a measure of entropy in the system, or its state of disorder. Thus, if $p(x)$ is the probability density function of a set of K states denoted by values of x, then the *Shannon entropy* is given by

$$H(X) = -\sum\nolimits_{\mathrm{x}=1}{}^{\mathrm{K}} \ln\left(p(x)\right) \tag{1.1}$$

From this is it is trivial to obtain the *Shannon information content* of $X = x$ as

$$\mathrm{SI}(x) = \ln\left(1/p(x)\right) \tag{1.2}$$

It came to be understood that this equals the number of bits in an algorithm that it takes to compute this code. This would lead Kolmogorov (1965) to define what is now known as *Kolmogorov complexity* as the minimum number of bits in any algorithm that does not prefix any other algorithm $a(x)$ that a Universal Turing Machine (UTM) would require to compute a binary string of information, x, or,

$$\mathrm{K}(x) = \min|a(x)|, \tag{1.3}$$

where $|\;|$ denotes length of the algorithm in bits.[11] Chaitin (1987) would independently discover and extend this *minimum description length* (MDL) concept and link it back to Gödel incompleteness issues, his version being known as *algorithmic* complexity, which would get taken up later by Albin (1982)[12] and Lewis (1985, 1992) in economic contexts.[13]

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[11] It should be understood that whereas on the one hand Kolmogorov's earliest work axiomatized probability theory, his efforts to understand the problem of induction would lead him to later argue that information theory precedes probability theory (Kolmogorov 1983). McCall (2005) provides a useful discussion of this evolution of Kolmogorov's views.

[12] Albin liked the example of the capital aggregation problem raised by Joan Robinson (1953-54) that in order to aggregate capital one needs to already know the marginal product of capital in order to determine the discount rate for calculating present values, while at the same time one already needs to know the value of aggregate capital in order to determine its marginal product. Conventional economics attempts to escape this potentially infinite do loop by simply assuming that all of these are conveniently simultaneously solved in a grand general equilibrium.

[13] Closely related would be the *universal prior* of Solomonoff (1964) that puts the MDL concept into a Bayesian framework. From this comes the rather neatly intuitive idea that the most probable

While these concepts usefully linked probability theory and information theory with computability theory, they all share the unfortunate aspect of being non-computable. This would be remedied by the introduction of *stochastic complexity* by Rissanen (1978, 1986, 1989, 2005). The intuition behind Rissanen's modification of the earlier concepts is to focus not on the direct measure of information but to seek a shorter description or model that will depict the "regular features" of the string. For Kolmogorov a model of a string is another string that contains the first string. Rissanen (2005, pp. 89–90) defines a likelihood function for a given structure as a class of parametric density functions that can be viewed as respective models, where θ represents a set of k parameters and x is a given data string indexed by n:

$$M_k = \left\{ f(x^n, \theta) : \theta \in \mathbf{R}^k \right). \tag{1.4}$$

For a given f, with $f(y^n)$ a set of "normal strings," the *normalized maximum likelihood function* will be given by

$$f^*(x^n, M_k) = f(x^n, \theta^*(x^n)) / \left[\int_{\theta(yn)} \mathrm{f}(y^n, \theta(y^n)) \mathrm{d}y^n \right], \tag{1.5}$$

where the denominator of the right-hand side can be defined as being $C_{n,k}$.

From this the *stochastic complexity* is given by

$$-\ln f^*(x^n, M_k) = -\ln f(x^n, \theta^*(x^n)) + \ln\ C_{n,k}. \tag{1.6}$$

This term can be interpreted as representing "the 'shortest code length' for the data x^n that can be obtained with the model class M_k." (Rissanen 2005, p. 90). With this we have a computable measure of complexity derived from the older ideas of Kolmogorov, Solomonoff, and Chaitin. The bottom line of Kolmogorov complexity is that a system is complex if it is not computable. The supporters of these approaches to defining economic complexity (Israel 2005; Markose 2005; Velupillai 2005a, b) point out the precision given by these measures in contrast to so many of the alternatives.

However, Chaitin's algorithmic complexity (1966, 1987) introduces a limit to this precision, an ultimate underlying randomness. He considered the problem of a program having started without one knowing what it is and thus facing a probability that it will halt, which he labeled as Ω. He saw this randomness as underlying all mathematical "facts." Indeed, this Ω itself is in general not computable (Rosser Jr. 2020a).

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state will also have the shortest length of algorithm to describe it. Solomonoff's work was also independently developed, drawing on the probability theory of Keynes (1921).

An example of this involves a theorem of Maymin (2011) that straddles the boundary of the deep unsolved problem of whether P (polynomial) equals NP (non-polynomial) in programs,[14] thus having an unknown Ω. This theorem shows that under certain information conditions markets are efficient if P = NP, which few believe. At the edge of this da Costa and Doria (2016) use the O'Donnell (1979) algorithm that is exponential and thus not P but slowly growing so "almost P" to establish a *counterexample function* to the P = NP problem. The O'Donnell algorithm holds if P < NP is probable for any theory strictly stronger than Primitive Recursive Arithmetic, even as that cannot prove it. Such problems might appear such as in the computationally complex traveling salesman problem. Da Costa and Doria establish that under these conditions the O'Donnell algorithm behaves as an "almost P" system that implies an outcome of "almost efficient markets." This is a result that walks on the edge of the unknown, if not the unknowable.

A deeper logical issue underlying computational complexity and economics involves fundamental debates over the nature of mathematics itself. Conventional mathematics assumes axioms labeled the Zermelo-Fraenkel-[Axiom of] Choice system, or ZFC. But some of these axioms have been questioned and efforts have been made to develop axiomatic mathematical systems not using them. The axioms that have been challenged have been the Axiom of Choice, the Axiom of Infinity, and the Law of the Excluded Middle. A general term for these efforts has been *constructivist* mathematics, with systems that particularly emphasize not relying on the Law of the Excluded Middle, which means no use of proof by contradiction, has been known as *intuitionism*, initially developed by Luitzen Brouwer (1908) of fixed point theorem fame.[15] In particular, standard proofs of the Bolzano-Weierstrass theorem use proof by contradiction, with this underlying Sperner's Lemma, which in turn underlies standard proofs of both the Brouwer and Kakutani fixed point theorems used in general and Nash equilibrium existence proofs (Velupillai 2006, 2008).[16]

For mathematicians, if not economists, the most important of these debatable axioms is the Axiom of Choice, which allows for the relatively easy ordering of infinite sets. This underpins standard proofs of major theorems of mathematical economics, with Scarf (1973) probably the first to notice these possible problems. The Axiom of Choice is especially important in topology and central parts of real analysis. On the one hand, its most ultimate formulation has been shown to be false by Specker (1953). But one way out of some of these problems is by using Non-standard analysis that allows for infinite and infinitesimal real numbers (Robinson

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[14] The P = NP problem was first identified by John Nash Jr. (1955) in a letter to the US National Security Agency discussing encryption methods in cryptanalysis, which was classified until 2013. Nash said he thought it was true that P did not equal NP, but noted he was unable to prove it, and it remains unproven to this day.

[15] Ironically Brouwer's original proof of his fixed point theorem relied on ZFC axioms, with him only providing an intuitionistic alternative much later (Brouwer 1952).

[16] For authoritative logic discussions of the issues involved broadly in these constructivist alternatives, see Kleene and Vesley 1965; Kleene 1967; Bishop 1967).

1966), which allows for avoiding the use of the Axiom of Choice for proving some important theorems.

The question of the Axiom of Infinity may perhaps be most closely tied to the questions about computational complexity. The deep philosophical idea behind these constructivist approaches is that mathematics should deal with finite systems that are more realistic and more readily and easily computed. Going against this most strongly was Cantor's introduction of levels of infinity into mathematics, an innovation that led Hilbert to praise Cantor for "bringing mathematicians into paradise." But the computability critics argue that mathematical economics must fit the real world in a credible way, with efforts ongoing at constructing such an economics based on a constructivist foundation (Velupillai 2005a, b, 2012; Bartholo et al. 2009; Rosser Jr. 2010a, 2012a).

1.3 Epistemology and Computational Complexity

Regarding computational complexity, Velupillai (2000) provides definitions and general discussion and Koppl and Rosser Jr. (2002) provide a more precise formulation of the problem, drawing on arguments of Kleene (1967), Binmore (1987), Lipman (1991), and Canning (1992). Velupillai defines computational complexity straightforwardly as "intractability" or insolvability. Halting problems such as studied by Blum et al. (1998) provide excellent examples of how such complexity can arise, with this problem first studied for recursive systems by Church (1936) and Turing (1936, 1937).

In particular, Koppl and Rosser reexamined the famous "Holmes-Moriarty" problem of game theory, in which two players who behave as Turing machines contemplate a game between each other involving an infinite regress of thinking about what the other one is thinking about (Morgenstern 1935). Essentially this is the problem of n-level playing with n having no upper limit (Bacharach and Stahl 2000). This has a Nash equilibrium, but "hyper-rational" Turing machines cannot arrive at knowing they have that solution or not due to the halting problem. That the best reply functions are not computable arises from the self-referencing problem involved fundamentally similar to those underlying the Gödel Incompleteness Theorem (Rosser Sr 1936; Kleene 1967, p. 246). Aaronson (2013) has shown links between these problems in game theory and the $\mathrm{N} = \mathrm{P}$ problem of computational complexity. Such problems extend to general equilibrium theory as well (Lewis 1992; Richter and Wong 1999; Landini et al. 2020).

Binmore's (1987, pp. 209–212) response to such undecidability in self-referencing systems invokes a "sophisticated" form of Bayesian updating involving a degree of greater ignorance. Koppl and Rosser agree that agents can operate in such an environment by accepting limits on knowledge and operate accordingly, perhaps on the basis of intuition or "Keynesian animal spirits" (Keynes 1936). Hyper-rational agents cannot have complete knowledge, essentially for the same reason that Gödel showed that no logical system can be complete within itself.

However, even for Binmore's proposed solution there are also limits. Thus, Diaconis and Freedman (1986) have shown that Bayes' Theorem fails to hold in an infinite dimensional space. There may be a failure to converge on the correct solution through Bayesian updating, notably when the basis is discontinuous. There can be convergence on a cycle in which agents are jumping back and forth from one probability to another, neither of which is correct. In the simple example of coin tossing, they might be jumping back and forth between assuming priors of 1/3 and 2/3 without ever being able to converge on the correct probability of 1/2. Nyarko (1991) has studied such kinds of cyclical dynamics in learning situations in generalized economic models.

Koppl and Rosser compare this issue to that of Keynes's problem (1936, Chap. 12) of the beauty contest. In this the participants are supposed to win if they most accurately guess the guesses of the other participants, potentially involving an infinite regress problem with the participants trying to guess how the other participants are going to be guessing about their guessing and so forth. This can also be seen as a problem of *reflexivity* (Rosser Jr. 2020b). A solution comes by choosing to be somewhat ignorant or boundedly rational and operating at a particular level of analysis. However, as there is no way to determine rationally the degree of boundedness, which itself involves an infinite regress problem (Lipman 1991), this decision also ultimately involves an arbitrary act, based on animal spirits or whatever, a decision ultimately made without full knowledge.

A curiously related point here is in later results (Gode and Sunder 1993; Mirowski 2002) on the behavior of zero intelligence traders. Gode and Sunder have shown that in many artificial market setups zero intelligence traders following very simple rules can converge on market equilibria that may even be efficient. Not only may it be necessary to limit one's knowledge in order to behave in a rational manner, but one may be able to be rational in some sense while being completely without knowledge whatsoever. Mirowski and Nik-Kah (2017) argue that this completes a transformation of the treatment of knowledge in economics in the post-World war II era from assuming that all agents have full knowledge to all agents having zero knowledge.

A further point on this is that there are degrees of computational complexity (Velupillai 2000; Markose 2005), with Kolmogorov (1965) providing a widely accepted definition that the degree of computational complexity is given by the minimum length of a program that will halt on a Turing machine. We have been considering the extreme cases of no halting, but there is indeed an accepted hierarchy among levels of computational complexity, with the knowledge difficulties experiencing qualitative shifts across them. This hierarchy is widely seen as consisting of four levels (Chomsky 1959; Wolfram 1984; Mirowski 2007). At the lowest level are linear systems, easily solved, with such a low level of computational complexity we can view them as not complex. Above that level are polynomial (P) problems that are substantially more computationally complex, but still generally solvable. Above that are exponential and other non-polynomial (NP) problems that are very difficult to solve, although it remains as yet unproven that these two levels are fundamentally distinct, one of the most important unsolved problems in computer science. Above this level is that of full computational complexity associated where the minimum length is infinite, where the programs do not halt. Here the knowledge problems can only be solved by becoming effectively less intelligent.

1.4 Foundations of Dynamic Complexity Economics

In contrast with the computationally defined measures described above, the dynamic complexity definition stands out curiously as for its negativity: dynamical systems that do *not* endogenously and deterministically generate certain "well-behaved" outcomes. The charge that it is not precise carries weight. However, the virtue of it is precisely its generality guaranteed by its vagueness. It can apply to a wide variety of systems and processes that many have described as being "complex." Of course, the computationalists argue with reason that they are able to subsume substantial portions of nonlinear dynamics with their approach, as for example with the already mentioned result on the non-computability of chaotic dynamics (Costa et al. 2005).

However, most of this recent debate and discussion, especially by Israel (2005), McCauley (2005), and Velupillai (2005b, 2005c) has focused on a particular outcome that is associated with some interacting agents models within the smaller tent (heterogeneous interacting agents) complexity part of the broader big tent dynamic complexity concept. This property or phenomenon is *emergence*. It was much discussed by cyberneticists and general systems theorists (von Bertalanffy 1974), including under the label *anagenesis* (Boulding 1978; Jantsch 1982), although it was initially formalized by Lewes (1875) and expanded by Morgan (1923), drawing upon the idea of *heteropathic laws* due to Mill (1843, Book III). Much recent discussion has focused on Crutchfield (1994) because he has associated it more clearly with processes within computerized systems of interacting heterogeneous agents and linked it to minimum length computability concepts related to Kolmogorov's idea, which it makes it easier for the computationalists to deal with. In any case, the idea is of the dynamic appearance of something new endogenously and deterministically from the system, often also labeled *self-organization*.[17]

Furthermore, all of these cited here would add another important element, that it appears at a higher level within a dynamic hierarchical system as a result of processes occurring at lower levels of the system. Crutchfield (1994) allows that what is involved is symmetry breaking bifurcations, which leads McCauley (2005, pp. 77–78) to be especially dismissive, identifying it with biological models (Kaufmann 1993) and declaring that "so far no one has produced a clear empirically relevant or even theoretically clear example." The critics complain of implied *holism* and Israel identifies it with Wigner's (1960) "mystical" alienation from the solidly grounded view of Galileo.

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[17] This term has been especially associated with Bak (1996) and his *self-organized criticality*, although he was not the first to discuss self-organization in these contexts.

Now the complaint of McCauley amounts to an apparent lack of *invariance*, a lack of ergodicity or steady state equilibria, with clearly identifiable symmetries whose breaking brings about these higher-level reorganizations or transformations.

> We can understand how a cell mutates to a new form, but we do not have a model of how a fish evolves into a bird. That is not to say that it has not happened, only that we do not have a model that helps us to imagine the details, which must be grounded in complicated cellular interactions that are not understood. (McCauley 2005, p. 77)[18]

While he is probably correct that the details of these interactions are not fully understood, a footnote on the same page points in the direction of some understanding that has appeared, not tied directly to Crutchfield or Kaufmann. McCauley notes the work of Hermann Haken (1983) and his "examples of bifurcations to pattern formation via symmetry breaking." Several possible approaches suggest themselves at this point.

One approach is that of *synergetics* due to Haken (1983), alluded to above. This deals more directly with the concept of entrainment of oscillations via the *slaving principle* (Haken 1996), which operates on the principle of *adiabatic approximation*. A complex system is divided into *order parameters* that are presumed to move slowly in time and "slave" faster moving variables or subsystems. While it may be that the order parameters are operating at a higher hierarchical level, which would be consistent with many generalizations made about relative patterns between such levels (Allen and Hoekstra 1990; Holling 1992; Radner 1992), this is not necessarily the case. The variables may well be fully equivalent in a single, flat hierarchy, such as with the *control* and *state* variables in catastrophe theory models. Stochastic perturbations can lead to structural change near bifurcation points.

If slow dynamics are given by vector **F**, fast dynamics generated by vector **q**, with **A**, **B**, and **C** being matrices, and **ε** a stochastic noise vector, then a locally linearized version is given by

$$\mathrm{d}\mathbf{q} = \mathbf{A}\mathbf{q} + \mathbf{B}(\mathbf{F})\mathbf{q}\mathbf{C}(\mathbf{F}) + \boldsymbol{\varepsilon}. \tag{1.7}$$

Adiabatic approximation is given by

$$\mathrm{d}\mathbf{q} = -(\mathbf{A} + \mathbf{B}(\mathbf{F}))^{-1}\mathbf{C}(\mathbf{F}). \tag{1.8}$$

Fast variable dependence on the slow variables is given by **A + B(F)**. Order parameters are those of the least absolute value.

The symmetry breaking bifurcation occurs when the order parameters destabilize by obtaining eigenvalues with positive real parts, while the "slave variables" exhibit the opposite. Chaos is one possible outcome. However, the most dramatic situation

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[18] McCauley's argument is based on Moore's (1990, 1991a,b) study of low dimensional, iterated maps that are Turing machines without attractors, scaling properties, or symbolic dynamics. McCauley argues that this view provides a foundation for complexity as ultimate surprise and unpredictability.

is when the slaved variables destabilize and “revolt” (Diener and Poston 1984), with the possibility of the roles switching within the system and former slaves replacing the former “bosses” to become the new order parameters. An example in nature of such an emerging and self-organizing entrainment might the periodic and coordinated appearance of the slime mold out of separated amoebae, which later disintegrates back into its isolated cells (Garfinkel 1987). An example in human societies may be the outbreak of the mid-fourteenth century Great Plague in Europe, when accumulating famine and immunodeficiency exploded in a massive population collapse (Braudel 1967).

Another approach is found in Nicolis (1986), derived from the work of Nicolis and Prigogine (1977) on frequency entrainment. Rosser Jr. (1994) have argued that this can serve as a possible model for the anagenetic moment, or the emergence of a new level of hierarchy. Let there be n well-defined levels of the hierarchy, with $\mathbf{L}_1$ at the bottom and $\mathbf{L}_n$ at the top. A new level, $\mathbf{L}_{n+1}$, or *dissipative structure*, can emerge at a phase transition with a sufficient degree of entrainment of the oscillations at that level. Let there be k oscillating variables, x_j and $z_i(t)$ be an independently and identically distributed exogenous stochastic process with zero mean and constant variance, then dynamics are given by the coupled, nonlinear differential equations of the form

$$\mathrm{d}x_i/\mathrm{d}t = f_i\left(x_j, t\right) + z_i(t) + \sum\nolimits_{\mathrm{j}=1}{}^{\mathrm{k}} \int_1^{\mathrm{k}} x_j(t')\mathbf{w}_{ij}(t' + \tau)\mathrm{d}t', \tag{1.9}$$

with $\mathbf{w}_{ij}$ representing a cross-correlation matrix operator. The third term is the key, either being “on” or “off,” with the former showing frequency entrainment. Nicolis (1986) views this in terms of a model of neurons, with a *master hard nonlinear oscillator* being turned on by a symmetry breaking of the cross-correlation matrix operator when the probability distribution of the real parts of its eigenvalues exceeding zero.[19] Then a new variable vector will emerge at the $\mathbf{L}_{n+1}$ level that is y_j, which will damp or stimulate the oscillations at level $\mathbf{L}_n$, depending on whether the sum over them is below or above zero.[20] An example might be the emergence of a new level of urban hierarchy (Rosser Jr. 1994).

Regarding the relation between dynamic complexity and emergence another perspective on this has come from the Austrian School of economics (Koppl 2006, 2009; Lewis 2012; Rosser Jr. 2012a), with the idea that market economic systems spontaneously emerge, one of their deepest ideas, which they drew from the Scottish Enlightenment of Hume and Smith, as well as such thinkers as Mill (1843) and Herbert Spencer (1867-1874) who wrote on both evolution and economic sociology

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[19] In a related model, Holden and Erneux (1993) show that the systemic switch may take the form of a slow passage through a supercritical Hopf bifurcation., thus leading to the persistence for a while of the previous state even after the bifurcation point has been passed.

[20] Yet another approach involves the *hypercycle* idea due to Eigen and Schuster (1979), discussed in the next chapter.

(Rosser Jr. 2014b). This link can be found in the work of Carl Menger (1871/1981), the founder of the Austrian School. Menger posed this as follows in terms of what economic research should discover (Menger 1883/1985, p. 148):

> . . .how institutions which serve the common welfare and are extremely significant for its development come into being without a *common will* directed toward establishing them.

Menger (1892) then posed the spontaneous emergence of commodity monies in primitive societies with no fiat role by states as an important example of this.

Various followers of Menger did not pursue this approach strongly, many emphasizing equilibrium approaches not all that different from the emerging neoclassical view, which was an idea one could find in Menger's work, who is widely viewed as one of the founders of the neoclassical marginalist approach along with Jevons and Walras. The crucial figure who revived an interest in emergence among the Austrians and developed it much further was Friedrich A. Hayek (1948, 1967).[21] Hayek drew on the incompleteness results of Gödel, aware of the role of self-referencing in this, and how overcoming the paradoxes of incompleteness may involve emergence of a higher level that can understand the lower level. Curiously his awareness of this originally came from his work in psychology in his 1952 *The Sensory Order* (pp, 188–189):

> Applying the same general principles to the human brain as an apparatus of classification. It would appear to mean that, even though we may understand its *modus operandi* in general terms, or, in other words possess an explanation of the principle on which it operates, we shall never, by any means of the same brain, be able to arrive at a detailed explanation of its working in particular circumstances, or be able to predict what the results of it operations will be. To achieve this would be to require a brain of a higher order complexity, though it might still be built on the same principles. Such a brain might be able to explain what happens in our brain, but it would in turn be unable to explain its own operations, and so on.

Koppl (2006, 2009) argues that this argument applies as well to Hayek's long opposition to central planning, with a central planner facing just this problem when they attempt to understand the effect on the economy they are trying to plan of their own planning efforts.[22] This view of the importance of complexity and emergence would come to be widely influential in Austrian economics since Hayek put forward his arguments and continues to be so (O'Driscoll and Rizzo 1985; Lachmann 1986; Lavoie 1989; Horwitz 1992; Wagner 2010).

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[21] See Vaughn (1999), Vriend (2002), and Caldwell (2004) for discussion of how Hayek came to his views on complexity and emergence and how they fit with his other views.

[22] The opposition to central planning and support for spontaneous emergence of market systems from the bottom up shows up in a long debate among philosophers regarding whether emergence only works bottom up or whether it can involve top to bottom causation. Van Cleve (1990) introduces *supervention* as allowing this top down causation in emergent systems, while Kim (1999) argues that emergent processes must be fundamentally bottom up. Lewis (2012) argues that Hayek moved toward the supervention view in his later writings that also emphasized group evolutionary processes (Rosser Jr. 2014b).

1.5 Dynamic Complexity and Knowledge

In dynamically complex systems, the knowledge problem becomes the general epistemological problem. Consider the specific problem of being able to know the consequences of an action taken in such a system. Let $G(\mathbf{x}_t)$ be the dynamical system in an n-dimensional space. Let an agent possess an action set **A**. Let a given action by the agent at a particular time be given by $\mathbf{a}_{it}$. For the moment let us not specify any actions by any other agents, each of whom also possesses his or her own action set. We can identify a relation whereby $\mathbf{x}_t = f(\mathbf{a}_{it})$. The knowledge problem for the agent in question thus becomes, "Can the agent know the reduced system $\mathrm{G}(f(\mathbf{a}_{it})$ when this system possesses complex dynamics due to nonlinearity"?

First of all, it may be possible for the agent to be able to understand the system and to know that he or she understands it, at least to some extent. One reason why this can happen is that many complex nonlinear dynamical systems do not always behave in erratic or discontinuous ways. Many fundamentally chaotic systems exhibit *transiency* (Lorenz 1992). A system can move in and out of behaving chaotically, with long periods passing during which the system will effectively behave in a non-complex manner, either tracking a simple equilibrium or following an easily predictable limit cycle. While the system remains in this pattern, actions by the agent may have easily predicted outcomes, and the agent may even be able to become confident regarding his or her ability to manipulate the system systematically. However, this essentially avoids the question.

Let us consider four forms of dynamic complexity: chaotic dynamics, fractal basin boundaries, discontinuous phase transitions in heterogeneous agent situations, and catastrophe theoretic models related to heterogenous agent systems. For the first of these there is a clear problem for the agent, the existence of sensitive dependence on initial conditions. If an agent moves from action $\mathbf{a}_{it}$ to action $\mathbf{a}_{jt}$, where $|\mathbf{a}_{it} - \mathbf{a}_{jt}| < \varepsilon < 1$, then no matter how small ε is, there exists an m such that $|G(f(\mathbf{a}_{it+t'}) - G(f(\mathbf{a}_{jt+t'})| > \mathrm{m}$ for some t' for each ε. As ε approaches zero, m/ε will approach infinity. It will be very hard for the agent to be confident in predicting the outcome of changing his or her action. This is the problem of the butterfly effect or sensitive dependence on initial conditions. More particularly, if the agent has an imperfectly precise awareness of his or her actions, with the zone of fuzziness exceeding ε, the agent faces a potentially large range of uncertainty regarding the outcome of his or her actions. In Edward Lorenz's (1963) original study of this matter when he "discovered chaos," when he restarted his simulation of a three-equation system of fluid dynamics partway through, the roundoff error that triggered a subsequent dramatic divergence was too small for his computer to "perceive" (at the four decimal place).

There are two offsetting elements for chaotic dynamics. Although an exact knowledge is effectively impossible, requiring essentially infinitely precise knowledge (and knowledge of that knowledge), a broader approximate knowledge over time may be possible. Thus, chaotic systems are generally bounded and often ergodic (although not always). While short-run relative trajectories for two slightly different actions may sharply diverge, the trajectories will at some later time return toward each other, becoming arbitrarily close to each other before once again diverging. Not only may the bounds of the system be knowable, but the long-run average of the system may be knowable. There are still limits as one can never be sure that one is not dealing with a long transient of the system, with it possibly moving into a substantially different mode of behavior later. But the possibility of a substantial degree of knowledge, with even some degree of confidence regarding that knowledge is not out of the question for chaotically dynamic systems.

Regarding fractal basin boundaries, first identified for economic models by Hans-Walter Lorenz (1992) in the same paper in which he discussed the problem of chaotic transience. Whereas in a chaotic system there may be only one basin of attraction, albeit with the attractor being fractal and strange and thus generating erratic fluctuations, the fractal basin boundary case involves multiple basins of attraction, whose boundaries with each other take fractal shapes. The attractor for each basin may well be as simple as being a single point. However, the boundaries between the basins may lie arbitrarily close to each other in certain zones.

In such a case, for the purely deterministic case once one is able to determine which basin of attraction one is in, a substantial degree of predictability may ensue. Yet there may be the problem of transient dynamics, with the system taking a long and circuitous route before it begins to get anywhere close to the attractor, even if the attractor is merely a point in the end. The problem arises if the system is not strictly deterministic, if G includes a stochastic element, however small. In this case one may be easily pushed across a basin boundary, especially if one is in a zone where the boundaries lie very close to one another. Thus there may be a sudden and very difficult to predict discontinuous changes in the dynamic path as the system begins to move toward a very different attractor in a different basin. The effect is very similar to that of sensitive dependence on initial conditions in epistemological terms, even if the two cases are mathematically quite distinct.

Nevertheless, in this case as well there may be something similar to the kind of dispensation over the longer run we noted for the case of chaotic dynamics. Even if exact prediction in the chaotic case is all but impossible, it may be possible to discern broader patterns, bounds and averages. Likewise in the case of fractal basin boundaries with a stochastic element, over time one should observe a jumping from one basin to another. Somewhat like the pattern of long run evolutionary game dynamics studied by Binmore and Samuelson (1999), one can imagine an observer keeping track of how long the system remains in each basin and eventually developing a probability profile of the pattern, with the percent of time the system spends in each basin possibly approaching asymptotic values. However, this is contingent on the nature of the stochastic process as well as the degree of complexity of the fractal pattern of the basin boundaries. A non-ergodic stochastic process may render it very difficult, even impossible, to observe convergence on a stable set of probabilities for being in the respective basins, even if those are themselves few in number with simple attractors.

For the case of phase transitions in systems of heterogeneous locally interacting agents, the world of the so-called "small tent complexity." Brock and Hommes

(1997) have developed a useful model for understanding such phase transitions, based on statistical mechanics. This is a stochastic system and is driven fundamentally by two key parameters, a strength of interactions or relationships between neighboring agents and a degree of willingness to switch behavioral patterns by the agents. For their model the product of these two parameters is crucial, with a bifurcation occurring for their product. If the product is below a certain critical value, then there will be a single equilibrium state. However, once this product exceeds a particular critical value two distinct equilibria will emerge. Effectively the agents will jump back and forth between these equilibria in herding patterns. For financial market models (Brock and Hommes 1998) this can resemble oscillations between optimistic bull markets and pessimistic bear markets, whereas below the critical value the market will have much less volatility as it tracks something that may be a rational expectations equilibrium.

For this kind of a setup there are essentially two serious problems. One is determining the value of the critical threshold. The other is understanding how the agents jump from one equilibrium to the other in the multiple equilibrium zone. Certainly the second problem resembles somewhat the discussion from the previous case, if not involving as dramatic a set of possible discontinuous shifts.

Of course once a threshold of discontinuity is passed it may be recognizable when it is approached again. But prior to doing so it may be essentially impossible to determine its location. The problem of determining a discontinuity threshold is a much broader one that vexes policymakers in many situations, such as attempting to avoid catastrophic thresholds that can bring about the collapse of a species population or of an entire ecosystem. One does not want to cross the threshold, but without doing so, one does not know where it is. However, for less dangerous situations involving irreversibilities, it may be possible to determine the location of the threshold as one moves back and forth across it.

On the other hand in such systems it is quite likely that the location of such thresholds may not remain fixed. Often such systems exhibit an evolutionary self-organizing pattern in which the parameters of the system themselves become subject to evolutionary change as the system moves from zone to zone. Such non-ergodicity is consistent not only with Keynesian style uncertainty, but may also come to resemble the complexity identified by Hayek (1948, 1967) in his discussions of self-organization within complex systems. Of course for market economies Hayek evinced an optimism regarding the outcomes of such processes. Even if market participants may not be able to predict outcomes of such processes, the pattern of self-organization will ultimately be largely beneficial if left on its own. Although Keynesians and Hayekian Austrians are often seen as in deep disagreement, some observers have noted the similarities of viewpoint regarding these underpinnings of uncertainty (Shackle 1972; Loasby 1976; Rosser Jr. 2001a, b). Furthermore, this approach leads to the idea of the openness of systems that becomes consistent with the critical realist approach to economic epistemology (Lawson 1997).

Considering this problem of important thresholds brings us to the final of our forms of dynamic complexity to consider here, catastrophe theory interpretations. The knowledge problem is essentially that previously noted, but is more clearly writ large as the discontinuities involved are more likely to be large as the crashes of major speculative bubbles. The Brock-Hommes model and its descendants can be seen as a form of what is involved, but the original catastrophe theory approach brings out key issues more clearly.

The very first application of catastrophe theory in economics by Zeeman (1974) indeed considered financial market crashes in a simplified two-agent formulation: fundamentalists who stabilized the system by buying low and selling high and "chartists" who chase trends in a destabilizing manner by buying when markets rise and selling when they fall. As in the Brock-Hommes formulation he allows for agents to change their roles in response to market dynamics so that as the market rises fundamentalists become chartists, accelerating the bubble, and when the crash comes they revert to being fundamentalists, accelerating the crash. Rosser Jr. (1991) provides an extended formalization of this in catastrophe theory terms that links it to the analysis of Minsky (1972) and Kindleberger (2001), further taken up in Rosser Jr. et al. (2012) and Rosser Jr. (2020c). This formulation involves a cusp catastrophic formulation with the two control variables being the demands by the two categories of agents, with the chartists' demand determining the position of the cusp that allows for market crashes.

The knowledge problem here involves something not specifically modeled in Brock and Hommes, although they have a version of it. It is the matter of the expectations of agents about the expectations of the other agents. This is effectively the "beauty contest" issue discussed by Keynes in Chapter 12 of this *General Theory* (1936). The winner of the beauty contest in a newspaper competition is not who guesses the prettiest girl, but who guesses best the guesses of the other participants. Keynes famously noted that one could start playing this about guessing the expectations of others in their guesses of others' guesses, and that this could go to higher levels, in principle, an infinite regress leading to an impossible knowledge problem. In contrast, the Brock and Hommes approach simply has agents shifting strategies after watching what others do. These potentially higher level problems do not enter in. These sorts of problems reappear in the problems associated with computational complexity.

1.6 Knowledge and Ergodicity

A controversial issue involving knowledge and complexity involves the deep sources of the Keynes-Knight idea of fundamental uncertainty (Keynes 1921; Knight 1921). Both of them made it clear that for uncertainty there is no underlying probability distribution determining important events that agents must make decisions about. Keynes's formulation of this has triggered much discussion and debate as to why he saw this lack of a probability distribution arising.

One theory that has received much attention, due to Davidson (1982-83), is that while neither Keynes nor Knight ever mentioned it, what can bring about such uncertainty, especially for Keynes's understanding of it, is the appearance of nonergodicity in the dynamic processes underlying economic reality. In making this argument, Davidson specifically cited arguments made by Paul Samuelson (1969, p. 184) to the effect that "economics as a science assumes the ergodic axiom." Davidson relied on this to assert that failure of this axiom is an ontological matter that is central to understanding Keynesian uncertainty, when knowledge breaks down. Many have since repeated this argument, although Alvarez and Ehnts (2016) argue that Davidson misinterpreted Samuelson who actually dismissed this ergodic view as being tied to an older classical view that he did not accept.

Davidson's argument has more recently come under criticism by various observers, perhaps most vigorously recently by O'Donnell (2014-15), who argues that Davidson has misrepresented the ergodic hypothesis, that Keynes never considered it, and that Keynesian uncertainty is more a matter of short-run instabilities to be understood using behavioral economics rather than the asymptotic elements that are tied up with ergodicity. An important argument by O'Donnell is that even in an ergodic system that is going to go to a long-run stationary state, it may be out of that state for a period of time so long that one will be unable to determine if it is ergodic or not. This is a strong argument that Davidson has not succeeded in fully replying to (Davidson 2015).

Central to this is to understand the ergodic hypothesis itself and its development and limits, as well as its relationship to Keynes's own arguments, which turns out to be somewhat complicated, but indeed linked to central concerns of Keynes in an indirect way, especially given that he never directly mentioned it. Most economists discussing this matter, including both Davidson and O'Donnell, have accepted as the definition of an ergodic system that over time (asymptotically) its "space averages equal its time averages." This formulation was due to Ehrenfest and Ehrenfest-Afanessjewa (1911), with Paul Ehrenfest a student of Ludwig Boltzmann (1884) who expanded the study of ergodicity (and coined the term) as part of his long study of statistical mechanics, particularly how a long term aggregate average (such as temperature) could emerge from a set of dynamically stochastic parts (particle movements). It turns out that for all its widespread influence, the precise formulation by the Ehrenfests was inaccurate (Uffink 2006). But this reflected that there were multiple strands in the meaning of "ergodicity."

In fact there is ongoing debate about how Boltzmann coined the term in the first place. His student, Ehrenfest, claimed it was from combining the Greek *ergos* ("work") with *hodos* ("path"), while it has been argued by Gallavotti (1999) that it came from him using his own neologism, *monode*, meaning a stationary distribution, instead of *hodos*. This fits with most of the early formulations of ergodicity that analyzed it within the context of stationary distributions.

Later discussions of ergodicity would draw on two complementary theorems proven by Birkhoff (1931) and von Neumann (1932), although the latter was proven first and emphasizes measure preservation, while Birkhoff's variation was more geometric and related to recurrence properties in dynamical systems. Both involve long-run convergence, and Birkhoff's formulation showed not only measure preservation but that for a stationary ergodic system a *metric indecomposability* such that not only is the space properly filled, but that it is impossible to break the system into two that will also fully fill the space and preserve measure, a result extending fundamental work by Poincaré (1890) on how recurrence and space filling help explain how chaotic dynamics can arise in celestial mechanics.

In von Neumann's (1932) formulation let T be a measure-preserving transformation on a measure space with for every square-integrable function f on that space, $(Uf)(x) = f(Tx)$, then U is a *unitary operator* on the space. For any such unitary operator U on a Hilbert space H, the sequence of averages:

$$(1/n)\left(f + Uf + \ldots + U^{n-1}f\right) \tag{1.10}$$

is strongly convergent for every f in H. We note that these are finite measure spaces and that this refers to stationary systems, just as with Boltzmann.

Birkhoff's (1931) extension, sometimes called the "individual ergodic theorem," modifies the above sequence of averages to be:

$$(1/n)\left(f(x) + f(Tx) + \ldots + f\left(T^{n-1x}\right)\right) \tag{1.11}$$

that converge for almost every x. These complementary theorems have been generalized to Banach spaces and many other conditions.[23] It was from these theorems that the next wave of developments in Moscow and elsewhere would evolve.[24] This was the state of ergodic theory when Keynes had his debate over econometrics at the end of the 1930s with that student of Paul Ehrenfest, Jan Tinbergen.

The link between stationarity and ergodicity would come to weaken in later study, with Malinvaud (1966) showing that a stationary system might not be ergodic, with a limit cycle being an example, with Davidson aware of this case from the beginning of his discussions. However, it continued to be believed that ergodic systems must be stationary, and this remained a key for Davidson as well as being accepted by most of his critics, including O'Donnell. However, it turns out that this may break down in ergodic chaotic systems of infinite dimension, which may not be stationary (Shinkai and Aizawa 2006), which brings back the role of chaotic dynamics in undermining the ability to achieve knowledge of a dynamical system, even one that is ergodic.

Given these complications it is worthwhile to return to Keynes to understand what his concerns were, which came out most clearly in his debates with Tinbergen (1937, 1940; Keynes, 1938) over how to econometrically estimate models for forecasting macroeconomic dynamics. A deep irony here is that Tinbergen was a student of Paul Ehrenfest and so was indeed influenced by his ideas on ergodicity, even as Keynes did not directly address this matter. In any case, what Keynes objected to was the apparent absence of *homogeneity*, essentially a concern that the model itself changes over time. Keynes's solution to this was to break a time-

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[23] See Halmos (1958) for how these theorems link measure theory to probability theory.

[24] Velupillai (2013, pp. 432–433, n8) shows that while most ergodic theory has followed a frequentist formulation, the Moscow School would draw on Keynes's ideas in their approach to these issues.

series down into sub-samples to see if one gets the same parameter estimates as one does for the whole time-series. Homogeneity is not strictly identical to either stationarity or ergodicity, but it is probably the case that at the time Tinbergen, following Ehrenfest, probably assumed all three held for the models he estimated. Thus indeed the ergodic hypothesis was assumed to hold for these early econometric models, whereas Keynes was skeptical of there being a sufficient homogeneity for one to assume one knew what the system was doing over time (Rosser Jr. 2016a).

1.7 Reflexivity and the Unification of Complexity Concepts

Closely related to self-referencing is the idea of *reflexivity*. This is a term with no agreed upon definition, and it has been used in a wide variety of ways (Lynch 2000). It is derived from the Latin *reflectere*, which is usually translated to mean “bending back,” but can refer to “reflex” as in a knee jerking when tapped, not what is meant here, or more generally is linked to “reflection” as in an image being reflected, possibly back and forth many times as in the situation of two mirrors facing each other. This latter is more what the focus is here and more the type that is connected with self-referencing and all that implies. Someone who made that link strongly was Douglas Hofstadter (1979) in his *Gödel, Escher, Bach: An Eternal Golden Braid* as well as even more so later (Hofstadter 2006). For Hofstadter, reflexivity is linked to the foundations of consciousness through what calls “strange loops” of indirect self-referencing, which he sees certain prints by Maurits C. Escher as highlighting, particularly his “Drawing Hands” and also his “Print Gallery,” with many commentators on reflexivity citing “Drawing Hands,” which shows two hands drawing each other (Rosser Jr. 2020b).[25] Hofstadter argues that the foundation for his theory is the Incompleteness Theorem of Gödel, with its deep self-referencing, along with certain pieces by J.S. Bach, as well as these prints by Escher.

The term has probably been most widely used, and with the greatest variety of meanings, in sociology (Lynch 2000)). Its academic usage was initiated by prominent sociologist, Robert K. Merton (1938), who used it to pose the problem of sociologists thinking about how their studies and ruminations fit into the broader social framework, both in how they themselves are influenced by that framework in terms of biases and paradigms, but also in terms of how their studies and how they do their studies might reflect back to influence society as well. Among the sociologists the most radical uses of the concept involved sharp self-criticism wherein one

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[25] Examples of reflexivity in art are often thought to involve the *Droste Effect*, in which a work contains an image of itself within itself, clearly a matter of self-referencing. Among the earliest known examples is a painting by Giotto from 1320, *The Stefaneschi Triptych*, in which in the central panel Cardinal Stefaneschi is depicted kneeling before Saint Peter and presenting to him the triptych itself. Needless to say, even if they cease to be depicted after a finite sequence of images, such artworks exhibiting this Droste Effect imply an infinite regress of ever smaller images containing ever smaller images (Rosser Jr. 2020b).

deconstructs the paradigm and influences one is operating in to the point that one can barely do any analysis at all (Woolgar 1991), with many complaining that this leads to a nihilistic dead end. The earliest usages of the term by economists followed this particular strand of analyzing how particular economists are operating within certain methodological frameworks and how they came to do so from broader societal influences and how their work may then reflect back to influence society, sometimes even through specific policies or even ways of gathering and reporting policy-relevant data (Hands 2001; Davis and Klaes 2003).

Merton (1948) would also use the idea to propose the idea of the *self-fulfilling prophecy*, an idea that has been widely applied in economics as with the concept of sunspot equilibria (Azariadis 1981), with many seeing this as deriving originally from Keynes (1936, Chap. 12) and his analysis of financial market behavior based on the early twentieth century British newspaper beauty contests. In those contests newspapers would publish photos of young women and ask readers to rate them on their presumed beauty. The winner of such a contest was not the person who guessed which young woman was objectively the most beautiful, but rather which one received the most votes. This meant that a shrewd player of such a game was really trying to guess the guesses of the other players, with Keynes comparing this to financial markets where the underlying fundamental of an asset is less important for its market value than what investors think it is. This led Keynes even to note that this kind of reasoning can move to higher levels, trying to think what others think others think, and on to still higher levels in a potential infinite regress, a classic infinite reflection in a non-halting program. This beauty contest idea of Keynes has come to be viewed as a centerpiece of his philosophical view, implying ultimately not only reflexivity but complexity as well (Davis 2017).

Among the first to pick up on Keynes's argument and apply it to self-fulfilling prophecies in financial markets and also bringing in reflexivity as relevant to this was George Soros (1987), who would later also argue that the analysis was part of complexity economics (Soros 2013). Soros has long argued that thinking about this beauty contest-inspired version of reflexivity has been key to his own decision-making in financial markets. He sees it as explaining boom and bust cycles in markets as in the US housing bubble of the early 2000s, whose decline set off the Great Recession. He first got the term from being a student of Karl Popper's in the 1950s (Popper 1959), with Popper also an influence on Hayek (1967) in connection with these ideas (Caldwell 2013). Thus the idea of reflexivity with links to arguments about incompleteness and infinite regresses associated with self-referencing have become highly influential among economists and financiers studying financial market dynamics and other related phenomena.

We now see the possibility of linking our major schools of complexity through the subtle strange loopiness involved in indirect self-referencing at the heart of a deeper form of reflexivity. The indirect self-referencing at the heart of Gödel's incompleteness theorem is deeply linked to computational complexity in that it leads to the infinite do loops of the highest level of computational complexity in which a program never stops. The way out of incompleteness involves in effect what Davis and Klaes invoked: moving to a higher hierarchical level in which an exogenous agent or program determines what is true or false, although this opens the door to incoherence (Landini et al. 2020). The indirect self-referencing opens the door to dynamic complexity in its implications for market dynamics, with this also linking to hierarchical complexity as new levels of hierarchy can be generated. Let us consider briefly how this comes out of the fundamental Gödel (1931) theorem.

The Gödel theorem is really two theorems. The first one is the incompleteness one: any consistent formal system in which elementary arithmetic[26] can be carried out is incomplete; there are statements in the language of the formal system that can neither be proved nor disproved within the formal system. The second one addresses the problem of consistency[27]: for any consistent formal system in which elementary arithmetic can be carried out, the consistency of the formal system cannot be proved within the formal system itself. So, coherence implies incompleteness, but any attempt to overcome incompleteness by moving to a higher level involves one being unable to prove the consistency of this higher level system, with both parts of this failing due to paradoxes of (reflexive) self-referencing leading to paradoxes.

Hofstadter (2006) provides an excellent discussion of the nature of the indirectness involved in proving the main part of the theorem, which involves the use of “Gödel numbers.” These are numbers assigned to logical statements, and their use can lead to the creation of self-referencing paradoxical statements even within a system especially designed to avoid such self-referencing statements. The system that Gödel subjected this treatment to eventually generates a statement equivalent to “This sentence is unprovable” was the logical system developed by Whitehead and Russell (1910-13) specifically to provide a consistent formal foundation for mathematics without logical paradoxes. Russell in particular was much concerned about the possibility of paradoxes in set theory, such as those involving self-referencing sets. The classic problem was “Does the set of all sets that do not contain themselves contain itself?” A famous simple version of this involves “Who shaves the barber in a town where the barber only shaves those who do not shave themselves?” Both of these involve similar endless do-loops arising from their self-referencing. Whitehead and Russell attempted to eliminate these annoyances by developing the theory of types that established hierarchies of sets in ways to avoid having them refer to themselves. But then Gödel pulled his trick of establishing his numbers, which he applied to the system of Whitehead and Russell so as through indirection to generate a self-referencing statement that involved a paradox unresolvable within the system. It is rather like how the hole Escher put in the middle of his “Print Gallery” allowed for the man to look at a print on a wall in a gallery of a city that contains the gallery in which he is standing looking at it.

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[26] By “elementary arithmetic” is meant that which can be derived from Peano’s axiom set assuming standard logic of the Zermelo-Frankel type with the Axiom of Choice (ZFC).

[27] It should be noted that in his original theorem Gödel was only able to prove incompleteness for a limited form of ω-consistency. A proof for a more general form of consistency was provided by Rosser Sr (1936) who used the “Rosser Sentence” (or “trick”): “If this sentence is provable, then there is a shorter proof of its negation.” This has led some to refer to the combined theorem as the “Gödel-Rosser Theorem.”

Thus it is not surprising that the problem of self-referencing has lain at the core of much of the thinking about reflexivity from an early point, and that this thinking took on a sharper edge when various figures thought about Gödel's theorem, or even earlier about the paradoxes considered by Bertrand Russell. Linking this to understanding to complexity provides a foundation for a reflexive complexity that encompasses all the major forms of complexity.

1.8 Further Observations

In computationally complex systems the problem of understanding them is related to logic, the problems of infinite regress and undecidability associated with self-referencing in systems of Turing machines. This can manifest itself as the halting problem, something that can arise even for a computer attempting to precisely calculate even a dynamically complex system as for example the exact shape of the Mandelbrot set (Blum et al. 1998). A Turing machine cannot understand fully a system in which its own decisionmaking is too crucially a part. However, knowledge of such systems may be gained by other means.

To the extent that models have axiomatic foundations rather than being merely ad hoc, which many of them ultimately are, these foundations are strictly within the non-constructivist, classical mathematical mode, assuming the Axiom of Choice, the Law of the Excluded Middle, and other hobby horses of the everyday mathematicians and mathematical economists. To the extent that they provide insight into the nature of dynamic economic complexity and the special problem of emergence (or anagenesis), they do not do so by being based on axiomatic foundations[28] that would pass muster with the constructivists and intuitionists of the early and mid-twentieth century, much less their more recent disciples, who are following the ideal hope that "The future is a minority; the past and present are a majority," to quote Velupillai (2005b, p. 13), himself paraphrasing Shimon Peres from an interview about the prospects for Middle East peace.

There are a considerable array of models available for contemplating or modeling emergent phenomena operating at different hierarchical levels. An interesting area to see which of the approaches might prove to be most suitable may well be in the study of the evolution of market processes as they themselves become more computerized. This is the focus of Mirowski (2007) who goes so far as to argue that fundamentally markets *are* algorithms. The simple kind of posted price – spot market most people have traditionally bought things in is at the bottom of a Chomskyian hierarchy of complexity and self-referenced control. Just as newer algorithms may contain older

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[28] While this movement focuses on refining axiomatic foundations, it ultimately seeks to be less formalistic and Bourbakian. This is consistent with the history of mathematical economics, which first moved towards a greater axiomatization and formalism within the classical mathematical paradigm, only to move away from it in more recent years (Weintraub 2002).

algorithms within them, so the emergence of newer kinds of markets can contain and control the older kinds as they move to higher levels in this Chomskyian hierarchy. Futures markets may control spot markets, options markets may control futures markets, and the ever higher order of these markets and their increasing automation pushes the system to a higher level towards the unreachable ideal of being a full-blown Universal Turing Machine (Cotogno 2003).

Mirowski brings to bear more recent arguments in biology regarding coevolution, noting that the space in which the agents and systems are evolving itself changes with their evolution. To the extent that the market system increasingly resembles a gigantic assembly of interacting and evolving algorithms, both biology and the problem of computability will come to bear and will come to bear and influence each other (Stadler et al. 2001). In the end the distinction between the two may become irrelevant.

In the great contrast of computational and dynamic complexity, we see crucial overlaps involving how the paradoxes arising from self-referencing underlying computational complexity can imply the emergence so deeply associated with dynamic complexity. These interrelations may become most manifest when contemplating the mirror world of reflexivity and its endless concatenations. These are among the many considerations that lie at the foundations of complexity economics.

Chapter 2
Foundations of Complex Behavioral Economics

2.1 Overview

Herbert A. Simon developed the idea of *bounded rationality* from his earliest works (Simon 1947, 1955a, 1957), which is viewed as the foundation of modern *behavioral economics*. Behavioral economics contrasts with more conventional economics in not assuming full information rationality on the part of economic agents in their behavior. In this regard, it draws on insights regarding human behavior from other social science disciplines such as psychology and sociology, among others. Without question, one can find earlier economists who argued that people are motivated by more than mere selfish maximization. Indeed, from the very beginnings of economics with Aristotle, who put economic considerations into a context of moral philosophy and proper conduct, through the father of political economy, Adam Smith in his *Theory of Moral Sentiments* (1759), to later institutional economists such as Thorstein Veblen (1899) and Karl Polanyi (1944) who saw peoples' economic conduct as embedded within broader social and political contexts. Nevertheless, it was Simon who coined both of these terms and established modern behavioral economics.

Simon's initiatives led to a flurry of activity and research over the next few decades, much of which became more influential in business schools and management programs as the rational expectations revolution conquered most of economics during the 1970s and 1980s. Assuming bounded rationality by economic agents led him to the concept of *satisficing*, that while people do not maximize they strive to achieve set goals within constraints. This became accepted in business schools as managers were taught to achieve levels of profit acceptable to owners.

Also arising out of his discovery of bounded rationality was his interest in pursuing more deeply how people think and understand as part of their making decisions. This led him to consider how this could be studied through the use of computers. This led him to become one of the founders of the field of *artificial intelligence* (Simon 1969), and Simon more generally is regarded as one of the early

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_2

leaders of computer science more generally. But it was his concern regarding the implications of bounded rationality that led him into this nascent field.

Simon would also become a leading figure in the early development of complexity theory, particularly of hierarchical complexity theory (Simon 1962), although he only made an indirect link between this and bounded rationality. However, modern complexity theorists are much more willing to see a close and direct link between complexity of one sort or another and bounded rationality, and thus also with behavioral economics (Velupillai 2019).[1] Indeed, complexity can be seen as a, if not the, fundamental foundation for why people have bounded rationality. Complexity lies at the very heart of behavioral economics in this view, and Simon sought to understand how people decide in the face of such ineluctable complexity.

2.2 Herbert Simon and Bounded Rationality

The late Herbert A. Simon is widely considered to be the father of *modern behavioral economics*, at least it was his work to which this phrase was first applied. He was also an early theorist of complexity economics, if not the father per se, and also was one of the founders of the study of artificial intelligence in computer science. Indeed, he was a polymath who published well over 900 academic papers in numerous disciplines, and while he won the Nobel Prize in economics in 1978 for his development of the concept of *bounded rationality*, his PhD was in public admnistration and he was never in a department of economics. We must use the term "modern" before "behavioral economics" because quite a few earlier economists can be seen as focusing on actual human behavior while assuming that people do not behave fully in what we would now call an "economically rational" manner (Smith 1759; Veblen 1899).

We must at this point be clear that by "behavioral economics" we are not assuming a view similar to that of "behavioral psychology" of the sort advocated or practiced by Pavlov or B.F. Skinner (1938). The latter does not view studying what is in peoples' minds or consciousness as of any use or interest. All that matters is how they behave, particularly how they respond to respond to repeated stimuli in their behavior. This is more akin to standard neoclassical economics, which also purports to study how people behave with little interest in what is going on inside their heads. The main difference between these two is that conventional economics makes a strong assumption about what is going on inside peoples' heads: that they are rationally maximizing individual utility functions derived from their preferences using full information. In contrast, behavioral economics does not assume that

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[1] Problems arising from dynamic complexity such as sudden discontinuities and sensitive dependence on initial conditions imply extreme difficulty for agents to form rational expectations regarding future events, much less full information and complete rationality in their decisionmaking. Another source of bias is the time inconsistency implied by hyperbolic discounting (Gowdy et al. 2013).

people are fully rational and particularly does not assume that they are fully informed. What is going on inside their heads is important, and such subjects as *happiness economics* (Easterlin 2017) are legitimate topics for behavioral economics.

In any case, from the beginning of his research with his path-breaking PhD dissertation that came out as a book in 1947, *Administrative Behavior* and on through important articles and books in the 1950s (Simon 1955a, 1957), Simon saw people as being limited in both their knowledge of facts as well as in their ability to compute and solve the difficult problems associated with calculating optimal solutions to problems. They face unavoidable limits to their ability to make fully rational decisions. Thus, people live in a world of *bounded rationality*,[2] and it was this realization that led him into the study of artificial intelligence in computer science as part of his study of how people think in such a world (Simon 1969).

This led Simon to the concept of *satisficing*. People set targets that they seek to achieve and then do not pursue further efforts to improve situations once these targets have been reached, if they are. Thus a firm will not maximize profits, but its managers will seek to achieve an acceptable level of profits that will keep owners sufficiently happy. This idea of satisficing became the central key to the behavioral study of the firm (Cyert and March 1963) and entered into the management literature, where it probably became more influential than it was in economics, for quite a long time.

Some economists, notably Stigler (1961), have taken Simon's position and argued that he is actually a supporter of full economic rationality, but only adding another matter to be optimized, namely minimizing the costs of information. People are still optimizing but take account of the costs of information. However, Stigler's argument faces an unavoidable and ineluctable problem: people do not and cannot know what the full costs of information are. In this regard they face a potential problem of infinite regress (Conlisk 1996). In order to learn the costs of information, they must determine how much time they should spend in this process of learning; they must learn what the costs of learning what the costs of information are. This then leads to the next higher order problem of learning what the costs of learning what the costs of information are, and there is no end to this regress in principle.[3] In the end they must use the sorts of *heuristic* (or "rule of thumb") devices that Simon proposes that people facing bounded rationality must use in order to answer the question. Full rationality is impossible, and the ubiquity of complexity is a central reason why this is the case.

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[2] Arguably Simon was parallel on this with Broadbent (1950), who initiated studies of how limits on cognition lead to workload fatigue.

[3] Central planners faced this problem in how much time and how they spend thinking about how to plan. In the French and Russian literature this came to be known as *planification*, the process of "planning how to plan," although this term was sometimes used for planning in general as well as for dealing with the problem of aggregating micro level plans into coherent macro ones (Rosser Jr. and Rosser, 2018, p. 11).

Simon (1976) distinguishes *substantive rationality* from *procedural rationality*. The former is the sort of rationality traditionally assumed by most economists in which people are able to achieve full optimization in their decisionmaking. The latter involves them selecting procedures or methods by which they can "do their best" in a world in which such full optimization is impossible, the heuristics by which they manage in a world of bounded rationality. In this regard it is not the case that Simon views people as being outright irrational or crazy. They have interests and they generally know what those are and they pursue them. However, they are unavoidably bounded in their ability to do so fully, so they must adopt various essentially ad hoc methods to achieve their satisficing goals.

Among these heuristics that Simon advocated for achieving procedural rationality were trial and error, imitation, following authority, unmotivated search, and following hunches. Pingle and Day (1996) used experiments to study the relative effectiveness of each of these, none of which clearly can achieve fully optimal outcomes. Their conclusion was that each of these can be useful for improving decisionmaking, however, none of them is clearly superior to the others use. It is advisable for agents to several of these and to move from one to another under different circumstances, although as noted above it may be hard to know when to do that and precisely how.[4]

2.3 Imitation and the Instability of Markets

While this list of procedures that can support a boundedly rational pursuit of procedural rationality is reasonable, a point not clearly made is that excessive focus on one of these rather than others can lead to problems. Clearly following authority can lead to problems when the authority is flawed, as many unfortunate examples in history have shown. Any of these can lead to problems if too intensively followed, but one that has particularly played an unfortunate role in markets is imitation, even though it is a widely used method by many people with a long history of being evolutionarily successful. The problem is particularly acute in asset markets, where imitation can lead to speculative bubbles that destabilize markets and can lead to much broader problems in the economy, as the crisis of 2008 manifestly shows.

A long literature (MacKay 1852; Baumol 1957; Zeeman 1974; Rosser Jr. 1997) has recognized that while agents focusing on long term fundamental values of assets tend to stabilize markets by selling them when their prices exceed these fundamentals and buying when they are below those, agents who chase trends can destabilize markets by buying when prices are rising, thus causing them to rise more, and vice versa. When a rising price trend appears, trend chasers will do better in returns than fundamentalists and imitation of those doing well will lead agents who might have followed stabilizing fundamentalist strategies to follow destabilizing trend chasing

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[4] More detailed studies of this issue can be found in Allen et al. (2011).

strategies, which will tend to push the price further up. And when a bubble finally peaks out and starts to fall, trend chasers can then push the price down more rapidly as they follow each other in a selling panic.

That such a tendency to engage in trend chasing speculation is deeply rooted in the human psyche was initially established experimentally by Smith et al. (1988), with many subsequent studies supporting this observation.[5] Even in situations with a finite time horizon and a clearly identified payment that establishes the fundamental value of the asset being traded, in experimental markets it has been repeatedly shown that bubbles will appear even in these simplified and clearcut cases. People have a strong tendency to speculate and to follow each other into such destabilizing speculation through imitation. Procedures that can support procedural rationality in a world of bounded rationality can lead to bad outcomes if pursued too vigorously.

We note that such patterns regularly take three different patterns. One is for price to rise to a peak and then to fall sharply after hitting the peak. Another is for price to rise to a peak and then decline in a more gradual way in a reasonably symmetric manner. Finally, we see bubbles rising to a peak, then declining gradually for awhile, finally collapsing in a panic-driven crash. Kindleberger's classic *Manias, Panics, and Crashes* (2001) shows in its Appendix B that of 47 historical speculative bubbles, each of the first two have five examples, while the remainder, the vast majority, follow the final pattern, which requires heterogeneous agents who are not fully rational for it to occur (Rosser Jr. 1997). This shows that complexity is deeply involved in most speculative bubbles.

Figures 2.1, 2.2, and 2.3 show the time path for prices of three bubbles before, during, and immediately after the 2008 crisis. They show the three patterns described above, taken from Rosser Jr. et al. (2012). The first is for oil, which peaked at $147 per barrel in July 2008, the highest nominal price ever observed, and then crashed hard to barely over $30 per barrel in the following November. It seems that commodities are more likely to follow this pattern than other assets (Ahmed et al. 2014).

The second pattern was followed by the housing bubble, which peaked in mid-2006 according to this figure, which shows two different indexes, the Case-Shiller 10-city one and their 20-city one as well. Looking closely one can see a bit of roughness around the peak making it look almost like the third pattern, whereas in fact if one looks at housing markets in individual cities, they look as posited by this pattern, with this roughness at the national level reflecting that different cities peaked at different times, with a final round of them doing so as late as January 2007 before they all declined.

This sort of pattern historically is often seen with real estate market bubbles. The more gradual decline than in the other patterns, nearly symmetric with the increase, reflects certain behavioral phenomena. People identify very personally and intensely with their homes and as a result tend not to easily accept that their home has declined

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[5] This result contrasts with earlier work by Vernon Smith (1962) showing how with double auction markets free markets converge rapidly to equilibria.

Fig. 2.1 Oil Prices, 2000–2011

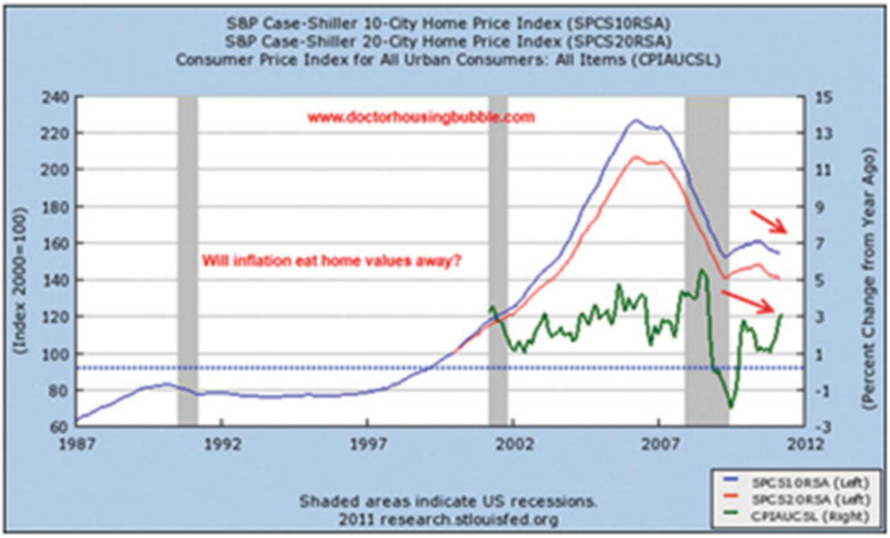
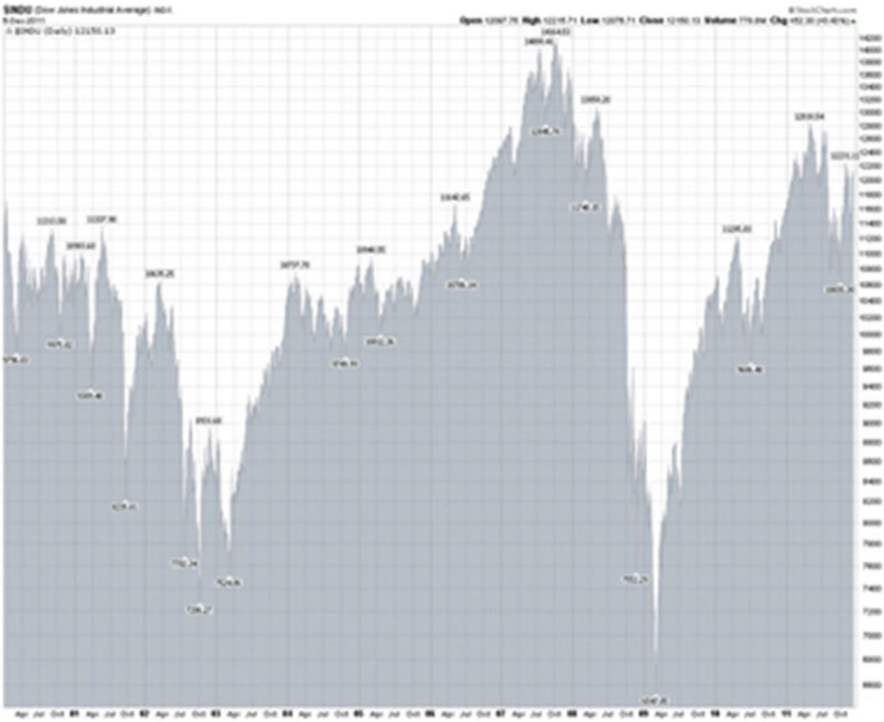


Fig. 2.2 Housing Prices in US, Case-Shiller Index, 1987–2013



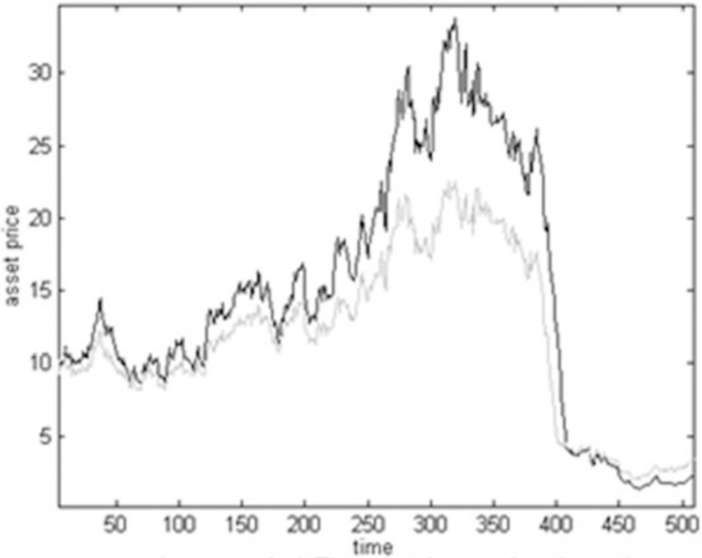
: DOW-Jones Industrial Average, 2000-2011, Daily Data

Fig. 2.3 US Stock Market Price Pattern, 2000–2011

in value whey they try to sell it during a downturn. As a result they have a tendency to offer prices that are too high and then refuse to lower their prices readily when they fail to sell. The upshot is a more dramatic decline in volume of sales on the downswing compared to the other patterns as people hang on and refuse to lower prices.

The third case shows the US stock market as exhibited by the Dow-Jones average, which peaked in October 2007, only then to crash in September 2008. Such patterns seem to be more common in markets for financial assets. Such patterns show heterogeneity of agents with different patterns of imitation, a smarter (or luckier) group that gets out earlier at the peak, followed by a less smart (or less lucky) group that hangs on hoping the price will return to rising, only to panic later en masse for whatever reason.

Finally, Figure 2.4 shows how this pattern with its *period of financial distress* (Minsky 1972) can be modeled in an agent-based model that has agents shifting from one strategy to another based on their relative successes, although not instantly (Gallegati et al. 2011). This model is based on ideas from Brock and Hommes (1997, 1998) that underlie the so-called Santa Fe stock market model (Arthur et al. 1997b).



Increase in J. The two time series share the same random numbers and same parameters but J. In the grey time series J = 0.5; in the black time series J = 3.

Fig. 2.4 Simulated Financial Distress Pattern

What triggers the delayed crash is agents running into financial constraints such as happens when individuals must meet margin calls in stock markets. The higher curve shows the pattern when agents imitate each other more strongly, as in a statistical mechanics model when there is a stronger interaction between particles.

2.4 Hierarchical Complexity and the Question of Emergence

While we can see Herbert Simon’s discovery of bounded rationality as an indirect claim to being a “father of complexity,” his most direct claim, recognized by Seth Lloyd in his famous list, is his 1962 paper to the American Philosophical Society on “The Architecture of Complexity.” In this transdisciplinary essay he deals with everything from organizational hierarchies through evolutionary ones to those involving “chemico-physical systems.” He is much concerned with the problem of the decomposability of higher-order systems into lower level ones, noting that productions ones, such as for watchmaking, as well as organizational ones, function better when such decomposability is present, which depends on the stability and functionality of the lower level systems.[6]

However, he recognizes that many such systems involve *near decomposability*, perhaps a hierarchical complexity equivalent of bounded rationality. In most of them there are interactions between the subsystems, with the broader evolution of the system depending on aggregated phenomena. Simon provides the example of a building with many rooms. Temperature in one room can change that in another, even though their temperatures may fail to converge. But the overall temperatures that are involved in these interactions are determined by the aggregate temperature of the entire building.

Simon also deals with what many consider to be the most fundamental issue involving complexity, namely that of emergence. His most serious discussion of the emergence of higher levels of hierarchical structure out of lower levels involves biological evolution, where these issues have long been most intensively discussed. He argues that how these higher levels emerged has not reflected teleological processes but strictly random processes. He also argues that even in closed systems, there need be no change in entropy in the aggregate when subsystems emerge within that system. But he also recognizes that organisms are energetically open systems, so that "there is no way to deduce the direction, much less the rate, of evolution from classical thermodynamic considerations" (Simon 1962, p. 8). However, it is the development of stable intermediate forms that is the key for the emergence of yet higher forms.

Simon does not cite this older literature, but this issue was central to the British "emergentist" literature that came out of the nineteenth century to become the dominant discourse in the 1920s regarding the broader story of biological evolution, all embedded within a broader vision fitting this within the emergence of physical and chemical systems from particles through molecules to such higher levels above biological evolution in terms of human consciousness, social systems, and yet higher systems (Lewes 1875; Morgan 1923) Simon dealt with this multiplicity of processes without drawing their interconnection as tightly as did these earlier figures. In the 1930s with the *neo-Darwinian synthesis* (Fisher 1930; Wright 1931; Haldane 1932), the emphasis returned to a near-continuous Darwinian process of gradual changes arising from the level of probabilistic changes arising from mutations at the gene level, with the gene the ultimate focus of natural selection (Dawkins 1976; Rosser Jr. 2011a, b).

While Simon avoided dealing with this issue of emergence in biological evolution in 1962, when the reductionist neo-Darwinian synthesis was at the highest level of its influence, soon the emergence view would itself re-emerge, based on multi-level evolutionary process (Crow 1955; Hamilton 1964; Price 1970). This would further develop with the study of nonlinear dynamics and complexity in such systems, with

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[6] See Rosser Jr. et al. (1994) for discussion of different forms of hierarchical relationships and emergence. Rosser Jr. (2010b) provides discussion of relations between *multidisciplinary, interdisciplinary, and transdisciplinary*.

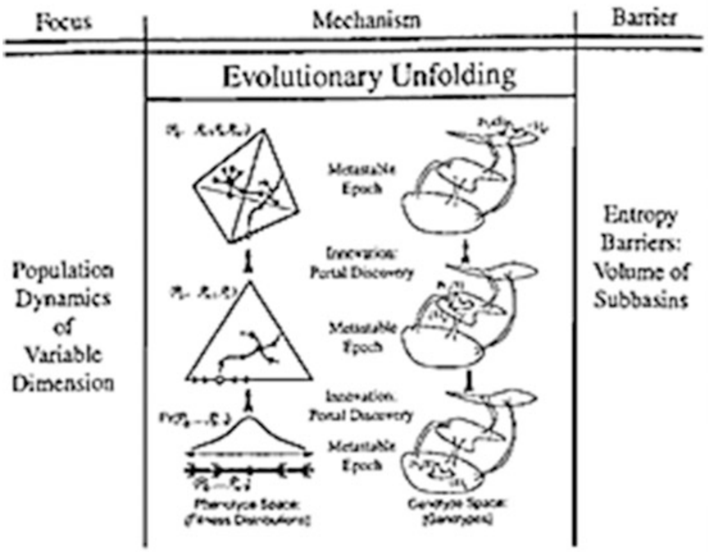

Fig. 2.5 Evolutionary Emergence

such figures as Stuart Kaufffmann (1993) and James Crutchfield (1994, 2003), who draw on computational models for their depictions of *self-organization* in biological evolutionary systems.

Figure 2.5 from Crutchfield (2003, p. 116) depicts how an initial genetic level mutation can lead to emergent effects at higher levels. On the right side are genotypes moving upwards from one basin of attraction to another, while on the left side phenotypes are also doing so in a parallel pattern. He introduces the concept of *mesoscales* for such processes, which clearly follow Simon's admonition about the necessity of stable intermediate systems emerging to support the emergence of yet higher order ones.

This view remains questioned by many evolutionists (Gould 2002). While the tradition going through catastrophe theory from D'Arcy Thompson (1917) has long argued for form arising from deep structures in organic evolution, critics have argued that such self-organizing processes are ultimately teleological ones that replicate old pre-evolutionary theological perspectives such as Paley's (1802) in which all things are in their place as they should be due to divine will. Others have criticized that such processes lack invariance principles (McCauley 2005). Others argue a more computational base for such processes (Moore 1990). There is no easy resolution of this debate, and even those advocating the importance of emergent self-organization recognize the role of natural selection. Thus, Kaufffmann (1993, p. 644) has stated, "Evolution is not just 'chance caught on a wing.' It is not just a tinkering of the ad hoc, of bricolage, of contraption. It is emergent order honored and honed by selection."

While the mechanisms are not the same, the problems of emergent self-organization apply as well to socio-economic systems. Simon's focus tended to be on organizations and their hierarchies. While he may well have sided with the more traditional neo-Darwinian synthesizers when it came to emergence of higher order structures in biological evolution, the role of human consciousness within human socio-economic systems means that the rules are different there, and the formation of higher order structures can become a matter of conscious will and planning, not mere randomness.

2.5 Bounded Rationality and Learning to Believe in Chaos

One of the greater ironies regarding bounded rationality is that it was colleagues of Herbert Simon's at Carnegie-Mellon, particularly John Muth (1961), who developed the idea of rational expectations while studying implications of bounded rationality. Muth in particular saw the assumption of rational expectations as a solution to the problems raised by bounded rationality. However, Herbert Simon would never have anything to do with this development, seeing it as a repudiation of bounded rationality. The idea that people not only know what is the true model of the economy, but that their subjective view of the probability distribution of exogenous noise in the system corresponded with the objective probability distribution of such noise, which was also conveniently Gaussian, simply was not acceptable in his view. Quite aside from the inability of boundedly rational agents to discern the "true model of the economy," he would never accept the idea that noise would be Gaussian. Indeed, he was a deep student of power law distributions that exhibit kurtosis or "fat tails" (Simon 1955b), hence he did not join his colleagues in their elation at the development of this idea.

That said, under certain circumstances it can come to pass that simple heuristic rule of thumb behaviors may do well in a world of complex nonlinear dynamics at helping boundedly rational agents mimic underlying dynamics that may even be chaotic. This can arise if agents are able to achieve *consistent expectations* or CEE (Hommes and Sorger 1998), an idea derived from work by Grandmont (1998) that had been done earlier, even though it was only published in the same year as theirs. An example of this was studied by Hommes and Rosser Jr. (2001) for fishery dynamics when these might exhibit chaotic patterns. Such patterns can arise due to the tendency of fisheries to exhibit backward-bending supply curves due to the carrying capacity limits of most fisheries. When prices go beyond a certain level that is consistent with *maximum sustained yield* the amount of fish will decline and fewer will get caught.

From Rosser Jr. (2001b), X is the biomass of fish in the fishery, with $F(X)$ being the growth rate of X, which in turn equals steady state harvest yields from the fishery, h, which in turn equals Q in the supply-demand diagram in the upper right portion of the figure. The bionomic portion is in the lower right part of the diagram and reflects a Schaeffer (1957) yield function, with r being the unconstrained natural growth rate of the fish population and K the carrying capacity of the fishery:

$$Q = h = F(X) = rX(1 - X/K). \tag{2.1}$$

This logistic is well known to be able to exhibit chaotic dynamics when in a discrete form from the work of May (1976). Following Gordon (1954) with $E =$ catch effort measured by time boats are out, $q =$ catchability per vessel per day, $C =$ cost, with constant marginal cost $= c$, $p =$ price of fish, and δ the time discount rate, then cost is given by

$$C = c/qX, \tag{2.2}$$

and the basic harvest function can be given by

$$h(X) = qEX. \tag{2.3}$$

Drawing on Clark (1990), Hommes and Rosser Jr. (2001) derived a full supply curve that varies with δ. This slopes upwards for $\delta = 0$, asymptotically approaching the output level associated with maximum sustained yield, but bends backwards for $\delta > 0.02$, reaching a maximum backward bend at $\delta = \infty$, at which point the supply curve is identical to the open access equilibrium due to Gordon (1954) given by

$$S(p) = rc/pq(1 - c/pqK), \tag{2.4}$$

with linear demand curve given by

$$D(p) = A - Bp. \tag{2.5}$$

Hommes and Rosser Jr. (2001) describe the cobweb dynamics of such a fishery under adaptive expectations by means of a discrete function

$$P_t = [A - S_\delta(p_{t-1})]/B. \tag{2.6}$$

Hommes and Rosser Jr. (2001) show that this can be chaotic for given values of δ as S varies with it. This will occur when S is backward-bending in those portions, which can also lead to catastrophic outcomes as demand shifts (Copes 1970).[7]

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[7] Rosser Jr. and Rosser (2006) consider problems of managing such catastrophic outcomes within an institutionalist framework.

The question of boundedly rational fishers arises if we allow them to base their expectations on a simple heuristic, p^e representing expected price, of a one-period autoregressive process given by

$$P^e(t) = \alpha + \beta(p_{t-1} - \alpha). \tag{2.7}$$

This AR(1) process can change according to sample autocorrelation learning in which the agents over time adjust the two control parameters, α and β, based on the performance of the fishers. Based on the CEE and assuming that the underlying chaotic dynamic for the optimizing fishery is given by an asymmetric tent map, Hommes and Rosser Jr. (2001) show that these parameters can converge on values such that this simple AR(1) heuristic will reproduce the underlying chaotic dynamic, which will be a CEE.

This is shown in Fig. 9 of Hommes and Rosser Jr. (2001), where the fishers start out catching a given level of X assuming a constant p, but as β in particular initially changes, a two-period motion appears, which then goes chaotic after later adjustment by both of the parameters occurs. This process has been called *learning to believe in chaos* We note that this dynamic remains bounded as are all chaotic dynamics, thus avoiding catastrophic collapse, a case of chaos preventing catastrophe. While this replicates to some extent standard figures showing period-doubling bifurcations to chaos, this is not one of those that involve a growth parameter varying. Rather this is a process of converging on a behavioral pattern based on autoregressive parameters adjusting in real time, not the same thing, even if it resembles it.

2.6 Behavioral Economics and Keynesian Uncertainty

Herbert Simon largely avoided directly addressing macroeconomic implications of his ideas, beyond expressing his disapproval of the rational expectations hypothesis that many claimed derived from his work, with this even being asserted as something so fundamental that it was axiomatic and could not be challenged for deep theoretical and philosophical reasons, despite its obvious and well known failure to follow empirical reality, a point that Simon was fully aware of. Given that his concept of bounded rationality violates full rational expectations, and also the deep connection with nonlinear dynamic complexity that has been presented earlier in this chapter, although not as fully as it might have been, the question arises, pushing beyond just bounded rationality to behavioral economics more broadly, what is the relationship between these ideas and the deep Keynesian (and Post Keynesian)[8] idea of fundamental uncertainty?

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[8] In contrast, post-Walrasian economics (Colander, 2006) critiques and tries to move beyond the Walrasian framework, while Post Keynesian (also called "post-Keynesian") economics tends to

The conventional view is that in 1921 Frank Knight and John Maynard Keynes both published books that established the distinction between *risk* and *uncertainty*, with Knight having clearly coined this distinction, but with Keynes's work exploring the distinction more deeply as he adopted the same terminology later (Keynes 1936; Rosser Jr. 2001a). "Risk" is quantifiable in terms of being able to identify a probability distribution that is relevant to understanding a problem. "Uncertainty" means that there is no such identifiable probability distribution. In contrast to Knight, Keynes was more aware of the possibility of various intermediate possibilities arising from inability to estimate the quantitative measure for either data availability or other reasons, as well as recognizing the difficulty of separating a variety of probability distributions possibly appropriate. This latter is a matter that has become more heavily discussed particularly since the 2008 financial crisis as the role of kurtosis or "fat tails" in financial returns has become more publicized.

The range of possibilities has been heightened by such observers as Nassim Taleb (2010) who distinguishes *grey swans* from *black swans.* The former involve probability distributions that show fat tails and are known, which can potentially explain extreme outcomes in financial markets and other situations. The latter involve true Keynesian/Knightian uncertainty, where it is impossible to assign a probability distribution, and where the events described "come out of nowhere" without any possibility of forecasting or expecting them. In this regard, Taleb argued that the 2008 crisis was a mere grey swan, an extreme outcome, that nevertheless was obviously coming and to be expected by any reasonable observer, in contrast with the October 19, 1987 crash of the stock market, 22% for the Dow-Jones average, to this day the largest one day decline ever, which was predicted by nobody and had no obvious cause, which "came out of nowhere," and which was a true black swan, an example of true and fundamental uncertainty.[9]

Rosser Jr. (1998, 2006) has argued that complexity provides a fundamental foundation for the reality of fundamental uncertainty. Paul Davidson (1996) has argued that this is not the case, that not only complexity, but such notions as Simonian bounded rationality are not proper or fundamental foundations of fundamental uncertainty. He distinguishes *ontological* uncertainty from *epistemological* uncertainty, arguing that true Keynesian uncertainty is the former based on the reality of non-ergodicity in most dynamic relations in the real world (Davidson 1982-83). In contrast he sees bounded rationality and the various variabilities arising

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admire the ideas of Keynes to varying degrees among the variety of schools of Post Keynesian thought, with Harcourt and Kreisler (2013a, b) providing an overview of these schools.

[9] There is no definitive separating these cases as even Gaussian distributions allow for extreme outcomes, if less frequently than those exhibiting kurtotic fat tails. In Tom Stoppard's (1967) *Rosencrantz and Guildentstern are Dead* the opening sequence has the ultimately doomed characters arguing about flipping coins when one of them keeps flipping heads "against all odds" 92 times in a row, an ouitcome allowed by probability distributions where the probability of a head is one half for each fair coin toss. Even Keynes accepted such a result and noted that insurance companies make profits from betting on identifiable and measurable probability distributions, even as he argued for fundamental uncertainty for many situations.

from nonlinear complex dynamics as being merely epistemological. If only people had really accurate and precise knowledge and forecasting systems, they could overcome these difficulties. Simon's emphasis on knowledge limitations and computational limitations by individuals come under special scrutiny and criticism in this regard. The foundation of bounded rationality (and complexity) is not fundamental uncertainty, but mere inability to compute and know. If only we had supercomputers with superknowledge, all would be well.

There is no ultimate resolution of this debate, although it must be noted that a major source of non-ergodicity within many systems is nonlinearity of the underlying dynamical relationships that leads to complexity. But as is well known in the econometric study of chaotic dynamics, it is profoundly difficult to distinguish deterministic chaotic dynamics from random noise (Dechert 1996). This debate faces this deep uncertainty of its own.

As it is, while behavioral economics may or may not be the foundation of true Keynesian/Knightian uncertainty, Talebian black swans, but it may provide a possible way to deal with policy in a world subject to such uncertainty from whatever source. Thus, while it remains absurdly ignored by many macroeconomists, George Akerlof's (2002) *behavioral macroeconomics* is almost certainly strongly affecting policymakers in practice, even if they do not speak openly of its influence. Real world central bankers and other macroeconomic policymakers are following heuristic behavioral patterns as recommended by the late Herbert A. Simon, even if few of them will admit to doing so.

2.7 Behavioral Economics and the Complexity of Institutional Evolution

The link between institutional economics and evolutionary economics dates to the work of Thorstein Veblen (1898). It is largely in recognition of this fact that the first organization in the United States dedicated to the study of institutional economics is called the Association for Evolutionary Economics,[10] with similar names being used in other nations for such study, including in Japan (Shiozawa et al. 2019). While it was not recognized at the time and remains little known, Veblen not only called for economics to be an *evolutionary science*, but introduced certain ideas that have since proven to be important in understanding the nature of complexity in economics, particularly that of *cumulative causation*, often,[11] thought by many to have been

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[10] In the U.S. this society has been closely associated with the so-called "old institutional economics," whereas it may be that an evolutionary approach taking into account complexity can unite the old and new approaches.

[11] It is an open debate whether or not Veblen viewed cumulative causation as necessarily implying economies of scale, although he was aware of the importance of economies of scale in industrial systems (Veblen, 1919). Setterfield (1997) recognizes Veblen's priority in introducing the concept,

introduced later by either Allyn Young (1928) or Gunnar Myrdal (1957), with the latter making the term widely known among economists. Among the various forms of complexity that are relevant to economics, cumulative causation is most obviously tied to *dynamic complexity*, which leads to increasing returns, multiple equilibria, and a variety of bifurcations in economic dynamical systems. However, it can be seen to be connected also to *computational complexity*, the main rival to dynamic complexity in economic analysis.

An important issue for the matter of how evolutionary theory relates to institutional economics in its early formulation involves Veblen's relations with John R. Commons and Joseph Schumpeter. Veblen developed ideas of Darwinian evolutionary economics in the early twentieth century in the United States, while Schumpeter is widely viewed as a strong supporter of an evolutionary approach to economic development, particularly regarding the evolution of technology, even as he criticized institutional economics and the application of biological ideas (Rosser Jr. and Rosser 2017). Also not widely known, Commons (1924) also supported an evolutionary view, although he had more of a teleological perspective on that than did either Veblen or Schumpeter, both of whom saw no necessary direction to technological evolution and change (Papageorgiou et al. 2013). Dealing with a complexity issue, Schumpeter strongly advocated a discontinuous, or *saltationalist* view of evolution (Schumpeter 1934; Rosser Jr. 1992), which Veblen agreed with regarding technological change. Regarding institutional evolution Veblen mostly saw it proceeding in a more continuous manner through cumulative causation, thus being somewhat closer to Commons on that matter, even as he argued that it was fundamentally unstable and would experience crises and breakdowns.

A central issue for institutional economics is the distinction between institutions and organizations (North 1990). This becomes central for the role of evolution in economics, in particular what is the meme that is the locus of evolutionary natural selection. In older literature the emphasis was more on organizations, such as with Commons (1934) who saw organizations competing with each other, a theme also picked up by Alchian (1950), even as Commons emphasized the deeper structures of institutions in legal systems. While organizations compete, increasingly evolutionary economists have focused on practices and routines as the more crucial memes, with this an especial theme among neo-Schumpeterian followers such as Nelson and Winter (1982).

An important element of evolutionary processes is the emergence of higher level structures out of lower level and simpler ones. This is more obvious in terms of organizations, but in institutional evolution the role of memes becomes crucial. This fits with the issue of multi-level evolution, long controversial in evolutionary theory (Heinrich 2004). Within human systems this becomes tied to cooperation, with Ostrom (1990) developing how such cooperation can arise through particular

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but argues that Young (1928) and Kaldor (1972) more clearly tied it to the phenomenon of increasing returns.

institutions. This process of emergence is linked to deep concepts of complexity, with Simon (1962) a crucial developer of this line of thought.

Understanding the complex dynamics of institutional evolution can bring about a possible reconciliation or even synthesis between the old and new institutional economics. Coase (1937) recognized that Commons originated the idea of the importance of transaction costs, the centerpiece of the new institutional economics (Williamson 1985). Mikami (2011) that has argued that the effort to minimize transactions cost can lead to complex evolutionary dynamics. This can involve Veblen's cumulative causation, recognizing how this can become linked to complex evolutionary emergence.

At the time when Thorstein Veblen was writing his most important works when the nineteenth century was turning into the twentieth, there was no clear or general awareness of what we now call *complexity*, even as many ideas we now associate with it had been floating around in various disciplines for many years, especially in mathematics and even somewhat in economics (Rosser Jr. 2009b). We have no reason to believe that Veblen was particularly aware of these strands, although evolution itself is now viewed as a complexity process par excellence (Hodgson and Knudsen 2006), which Veblen would strongly advocate.[12] In any case, central to Veblen's approach to economic evolution was his invocation of the idea of *cumulative causation*, which he was the first to introduce.[13] We must note that cumulative causation can lead to dynamic complexities through increasing returns, which Brian Arthur (1989, 1994) has argued is the central key to understanding complexity, and which Veblen recognized as present in industrial technology.

2.8 The Discontinuity Debate in Evolutionary Theory

It was Leibniz who initially coined the phrase *natura non facit saltum*, or, "nature does not take a leap." It would be picked up by Darwin himself who repeated it and applied it to his theory of natural selection, and Marshall would follow Darwin in applying to economics, repeating it in the Prefaces to all eight editions of his *Principles of Economics*. For Darwin (1859, pp. 166–167):

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[12] Veblen was not the first economist to advocate the usefulness for economics of evolutionary theory, with both Marx and Marshall doing so before he did, even as they did so from very different perspectives. The more complicating factor in all this is the fact that Darwin himself was crucially influenced by Malthus's work on population when he developed his theory of natural selection (Rosser Jr., 1992).

[13] That this is not widely known can be seen in that Business Dictionary identifies the originator of the term as Allyn Young (1928) [www.businessdictionary.com/definition/cumulative-causation.html] and Wikipedia identifies its originator (actually "Circular cumulative causation") as being Gunnar Myrdal (1957) [https://en.wikipedia.org/wiki/Circular_cumulative_causation]. Certainly Myrdal's use of the term received widespread attention.

"Natura non facit saltum...Why should not Nature take a leap from structure to structure? On the theory of natural selection we can clearly understand why she should not: for natural selection can only act by taking advantage of slight successive variations; she can never take a leap, but must advance by the shortest and slowest steps."

This was a strong statement for Darwin to make given that he did not understand the underpinnings of how the process of mutation through changes in genes worked, but indeed many evolutionary theorists since Darwin have been impressed by the idea that only minor changes in genes can occur at a time for species to be viable and survive and reproduce, thus setting up at least most evolutionary processes to be slow and gradual as asserted by Darwin. However, until the understanding of genetics was fully integrated into Darwinian theory with the neo-Darwinian synthesis in the 1930s, there was more of an opening for more noticeable discontinuous change in the Lamarckian perspective that allowed for the inheritance of acquired characteristics, and thus more rapid evolutionary change.

After the 1930s the more dramatic reassertion of the possibility for rapid change in the form of *punctuated equilibrium* would come with Eldredge and Gould (1972), whose arguments remain controversial among evolutionary biologists. However, the groundwork for their arguments was laid in the development of the neo-Darwinian synthesis itself during the 1930s, even if it was not clearly recognized at the time. A central part of the neo-Dawinian synthesis, especially as formulated by Fisher (1930), involved focusing on the gene, with natural selection operating at the level of the gene, which contrasted with theories that saw natural selection operating at higher levels on wholes. Changes at the level of a gene must be fairly small to be viable, but a method of studying this through fitness landscapes as introduced by Sewall Wright (1932) opened the door for a broader perspective, one that can be carried over to the study of institutional evolution (Mueller 2015).

A piece of groundwork always there regarding Wright's fitness landscape framework that opened the door to such saltationalist discontinuities or punctuations was that Wright from the beginning allowed for multiple local optima or equilibria within those landscapes. While he himself did not see dramatic discontinuities happening at the genetic level, he recognized that rapid environmental changes could shift the landscapes so that a former peak could fairly quickly become a valley and the nearest peak reachable by a gradient might be some distance away, which would imply some rapid evolution, if not necessarily discontinuous in genotype and phenotype.[14]

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[14] Clearly there is no definitive boundary in observing what is essentially discrete data between what is continuous and discontinuous. In biological evolution one observes different individuals across generations, and, with the exception of identical twins or clones, each individual's genotype is discretely distinct from every other's. Likewise with phenotypes, one might depict the possible variations of a certain physical characteristic on a continuous scale, but individuals will still have discrete differences from other individuals on such characteristics, even if these are very small. Thus the distinction becomes arbitrary. At the lowest level we see a discontinuous granularity, but a a higher level defenders of continuity see only gradual changes, especially in population averages, with it completely open to debate how rapid such changes must be before one can call then

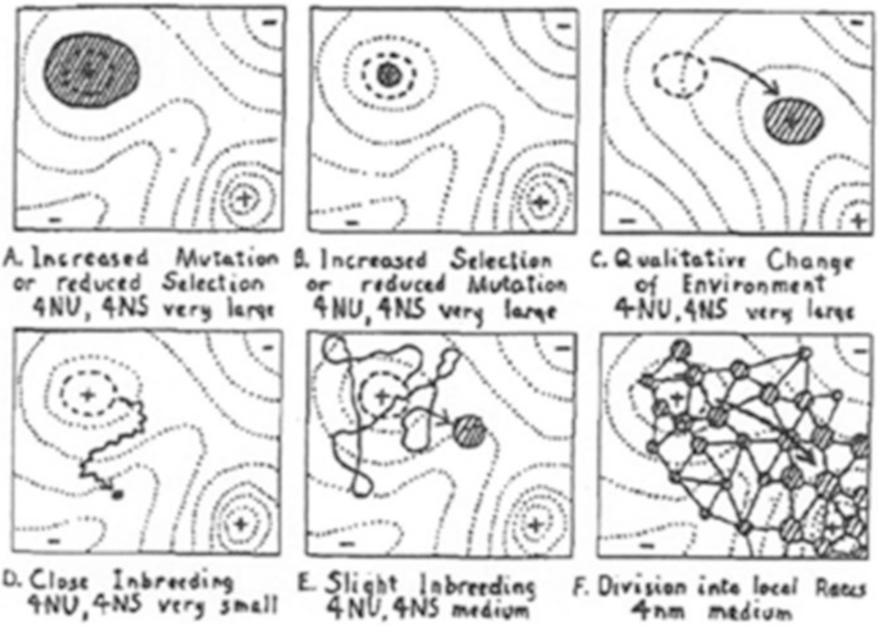

Fig. 2.6 Sewall Wright's fitness landscapes

Figure 2.6 shows Wright's original depiction of fitness landscapes and certain cases that could happen (Wright 1932, reproduced in Wright 1988, p. 110), with box C showing the case just described, a changing of the landscape due to some environmental change, which might happen quite suddenly.

Regarding the application of these ideas to economic evolution and more specifically institutional evolution, it is generally accepted that while Marshall may have agreed with Leibniz and Darwin that *natura non facit saltum*, Veblen tended to accept the idea that institutional evolution could be discontinuous, or at least that institutional equilibria were not stable and could change suddenly. Thus he declared (Veblen 1919, p. 242–243):

> "Not only is the individual's conduct hedged about and directed by his habitual relations to his fellows in the group, but these relations, being of an institutional character, vary as the institutional scene varies. The wants and desires, the end and the aim, the ways and the means, the amplitude and drift of the individual's conduct are functions of an institutional variable that is of a highly complex and unstable character."

Curiously while Schumpeter strongly supported the idea of discontinuous technological change and used the language of evolution in the context of economic development, he rejected the use of biological analogies in such discussions,

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discontinuous (see Rosser Jr., 2000a, Chap. 1, for further discussion of distinguishing continuous from discontinuous forms).

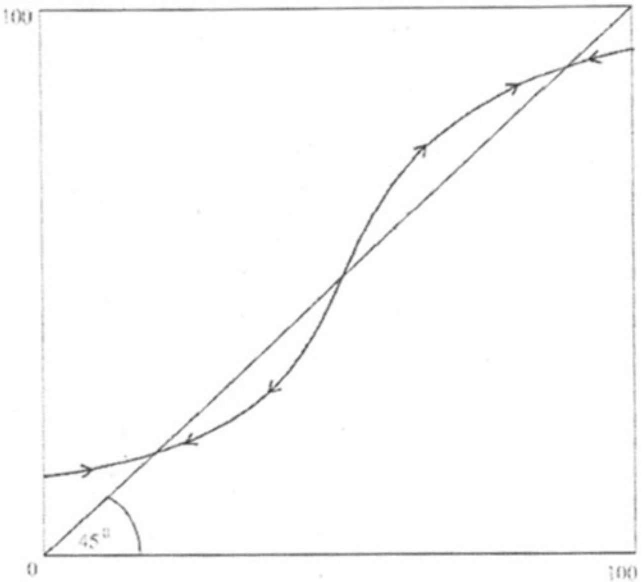

Fig. 2.7 Multiple social equilibria

declaring that (Schumpeter 1954, p. 789), "no appeal to biology would be of the slightest use." He dismissed selective mechanisms whether of a Darwinian or Lamarckian sort, using the word "evolution" in a simply developmental way (Hodgson 1993a, b).

While Wright did not spell it out, a key to the existence of multiple local equilibria in his fitness landscapes is the presence of some sort of increasing returns. This brings in Arthur's (1994) emphasis on increasing returns and its link to the existence of multiple equilibria and dynamic complexity,[15] which carries over to institutional evolution. Minniti (1995) used a variation of the Arthur et al. (1987) urn model to show how low and high crime equilibria can arise in a society, with social interactions providing positive feedbacks the key to such an outcome, with potential discontinuities arising as the amount of crime can shift very suddenly from one state to another. This is shown in Fig. 2.7 where the horizontal axis is the percent of the population who are criminals while the vertical axis shows the probability that a new entrant to society will be a criminal. Rosser Jr. et al. (2003b) applied this model informal economies in transition economies, with there also being multiple equilibria as seen by large differences in this variable among the transition economies of Eastern Europe, with the degree of inequality playing an important role as discussed in the next chapter.

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[15] The original formulation of Arthur's model was from Arthur et al. (1987) and their study of Polya urns.

2.9 Institutions, Organizations, and the Locus of Economic Evolution

If economies are evolutionary systems, then the question of what is the locus of that evolution is important. Hodgson and Knudsen (2006) argue that there are three crucial characteristics involved in truly Darwinian evolution: variability, natural selection, and inheritance. For something to qualify as a locus of evolution it must exhibit all three of these. In biological evolution the gene certainly fulfills all of these: mutation provides random variability, natural selection determines whether an organism containing a gene will survive or not, and genes pass from one organism to another through reproduction if the organism is able to survive and attract mates to effectuate this. Critics of evolutionary economics argue that there is no definitive unit or element in economies that fulfill all three of these, even if many fulfill some of them.

Given the long advocacy by institutionalist followers of Veblen for making economics an evolutionary science, these issues have been central to debates within this area. A focus on organizations has long attracted attention, with this arguably more important to Commons than to Veblen. For Commons, directed or artificial selection was more important than strictly random natural selection, and he noted that Darwin himself spent much time discussing both random natural selection as well as artificial breeding (Commons 1934, p. 657; Vanberg 1997).[16] Commons saw organizations as being subject to direction and thus appropriate objects for this sort of directed evolution, which had a goal of general human improvement. In his argument for evolution as the fundamental force in microeconomics, Armen Alchian (1950) emphasized the competition of firms, with the survival of the fittest involving which firms can come closest to maximizing profits, even if they do not know precisely how they are doing so, with firms clearly the locus of evolution.

A criticism of the idea of firms, or more generally organizations, serving as the key locus of evolution in economics is that while they are subject to random variability as they experience shocks from the system, and natural selection clearly operates in their competition with each other, with unprofitable firms failing to survive, the missing piece is that of inheritance. Firms and organizations do not essentially reproduce themselves. All they do is survive, although they may change while doing so. These changes may reflect these evolutionary forces of natural selection, but the inheritance element of their doing so must be operating at some lower level than that of the firm or organization itself.

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[16] Curiously Sewall Wright also focused on animal breeding due to his working for the US Department of Agriculture in the 1920s, where his thinking about this led him to certain of his ideas such as random drift, also known as “the Sewall Wright Effect,” sometimes seen as a violation of strict natural selection in how new species might form, although the separation of genetically distinct sub-groups of a population may happen either randomly in nature or through the conscious control and direction by humans as in animal breeding.

The leading alternative for serving as the evolutionary meme[17] is habits or practices within an organization. While they were not driven to this argument by trying to fit new institutional economics into an evolutionary framework per se, this is how North (1990) and Williamson (2000) define institutions. They are habits or practices, not organizations. This is also what Nelson and Winter (1982) came to in their search for the key to evolutionary economics, although they labeled these memes to be "routines." But prior to any of these and prior to Commons and his emphasis on organizations, Veblen identified habits, including habits of thought, as the central locus of evolution in economic institutions, declaring (Veblen 1899, pp. 190–191):

> "The situation of today shapes the institutions of tomorrow through a selective, coercive process, by acting upon men's habitual view of things, and so altering or fortifying a point of view or a mental attitude handed down from the past."

Given that as he put it the individual's conduct is "hedged about by his habitual relations with his fellows in the group," with these relations of an "institutional character," it is habits and habitual relations that are at the foundation of the evolution of institutions, even if he sees these institutions as being higher order social structures. It is the habits that are at the foundations, and habits can change, leading to new habits that may be inherited by the individuals and organizations using them.[18]

2.10 Emergence and Multi-Level Evolution

Among the ideas most strongly associated with complexity is that of *emergence*, that a higher order entity arises out of a lower level one that is not simply the sum of the parts of the lower level one, that the emergent entity is something qualitatively different. While the idea of a whole being greater than the sum of its parts has been around for a long time, a scientific formalization of it is probably due to John Stuart Mill (1843) in his discussions of logic in which he characterized situations where something qualitatively different from its parts appears as representing *heteropathic laws*. His original examples involved chemistry such has how salt appears when one combines sodium with chlorine, with salt not being at all like either of them separately. Lewes (1875) applied the term *emergence* to such phenomena. This led to the "British Emergentist" school of thought that especially in the 1920s (Morgan 1923) would apply this concept to evolution, in particular to such problems as how multi-cellular organisms arose out of uni-cellular ones. It would be applied to how larger social groups would organize themselves to act together out of previously

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[17] The term "meme" as the locus of evolution is due to Dawkins (1976), who also first proposed the idea of "universal Darwinism."

[18] How evolution of habits and norms determines tax behavior in societies is studied by Torgler (2016).

smaller separate groups, an idea clearly important in the evolution of institutions (McLaughlin 1992).

In biological evolutionary theory this view fell out of favor in the 1930s with the rise of the neo-Darwinian synthesis, which put the focus on the gene as the locus of evolution, the meme, as Dawkins (1976) labeled it. The idea that natural selection occurred at levels above the gene, at the level of "wholes" or groups, was specifically rejected (Williams 1966). The obvious counter to this in biological evolution involves the social insects (Wilson 2012), in which individuals are subordinated to the good of the colony, with the colony becoming the vehicle of evolution. Most attribute the mathematical understanding of how this can arise to the work of Price (1970) and Hamilton (1964, 1972). However, in fact, the original formalization of this understanding in terms of within-group versus between group selection was due to Crow (1955).

Let B_w be the within-group genic regression on the fitness value of the trait as defined by Wright (1951); B_b be the between-group genic regression to the fitness value; V_w be the variance among individuals within a group, and V_b be the variance among means across groups. For an *altruistic gene* one would expect B_w to be negative (that the behavior within the group damages the individual), while B_b would be positive (the behavior of the individual helps the group). From this a sufficient condition for the altruistic gene to increase in frequency is given by

$$B_b/(-B_w) > V_w/V_b. \tag{2.8}$$

Within biology there it has been widely argued that this condition rarely holds. However, it has also been recognized that it appears to hold for the social insects, and as Wilson (2012) argues, this implies that even though only a minority of species show this characteristic, they end up constituting a huge portion of the animal biomass on the earth (especially if one includes human beings in that calculation).

Indeed, this formulation can be carried over to humans to resolve the problem of cooperation versus cheating within a Prisoner's Dilemma game theoretic context (Heinrich 2004). The specific problem for humans becomes one of recognizing who is a cooperator and who is not within social groups, with successfully doing so being the condition for cooperation and a higher level coordination to come about. Considering in detail how such cooperation can arise in numerous contexts for dealing with common property resources was the central focus of the work of Ostrom (1990). This can more generally be viewed as a condition for the emergence of higher level institutions out of lower level ones.[19]

Somewhat parallel to this is a formulation of emergence in biological evolution due to Eigen and Schuster (1979) known as the *hypercycle*, which involves information preservation and transmission, tying this more to computational forms of

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[19] Sethi and Somanathan (1996) show that in such games there are multiple Nash equilibria, with some supportive and some destructive of the cooperative equilibria that are consistent with sustainable development. Rosser Jr. and Rosser (2006) extend this argument.

complexity. “the simplest system that can allow the evolution of reproducible links” (Eigen and Schuster 1979, p. 87). They define a *threshold of information content,* which if exceeded for a system will lead to a degeneration of information due to an *error catastrophe*. Above an error catastrophe there is a “disintegration of information due to a steady accumulation of errors” (Eigen and Schuster 1979, p. 25).

Let V_{m} be the number of symbols, $\sigma_{\mathrm{m}} > 1$ be the degree of selective advantage superiority of the “master copy,” and q_{m} be the quality of symbol copying. The threshold is then given by

$$V_{\mathrm{m}} < \ln \sigma_{\mathrm{m}}/(1 - q_{\mathrm{m}}). \tag{2.9}$$

Such hypercycle formation has been simulated by Mosekilde et al. (1983), and the concept has been applied to the evolution of market structures based on differential rates of learning among firms by Silverberg et al. (1988). It has also been linked with the concept of *autopoesis*, defined as the stable reproduction of a space-time structure (Varela et al. 1974).

This can be seen as linked to *self-organization* as initially formulated by Turing (1952) in the form of *morphogenesis*. When such morphogenesis involves emergence at a higher level this become *hypercyclic morphogenesis* (Rosser Jr. 1991, Chap. 6), or the *anagenetic moment* by Rosser Jr. et al. (1994). Radzicki (1990) applied such arguments to the question of the formation of institutions out of underlying chaotic dynamics.[20] Within evolution the emergence of higher hierarchical levels was also the central focus of Simon (1962).

This raises parallels within evolutionary game theoretic models of the issue of *multi-level evolution* (Heinrich 2004), with the Price-Hamilton equations providing sufficient conditions for this to occur, although the original version was due to Crow (1955). For its population, B_{w} and B_{b} are within- and between-group genetic regressions of fitness on the value of the trait, V_{w} and V_{b} are the within- and between-group genetic variances, with W the mean population fitness, then

$$\Delta C = (B_{\mathrm{w}}V_{\mathrm{w}} + B_{\mathrm{b}}V_{\mathrm{b}})/W. \tag{2.10}$$

This allows for a statement of Hamilton’s (1972) condition for an altruistic trait to increase (the equivalent of cooperation at a higher level) as

$$B_{\mathrm{w}}/(B_{\mathrm{b}} - B_{\mathrm{w}}) < r, \tag{2.11}$$

where r is the Sewall Wright coefficient of relationship (Crow and Aoki 1984). The left-hand side can be interpreted as a cost-of-fitness to benefit-minus-fitness ratio.

Another strand of emergent evolutionary processes is associated with the neo-Schumpeterian view strongly associated with Nelson and Winter (1982) and

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[20] Some of the early models of hypercycle formation required the absence of parasites. Howver, with appropriate mixing they may be stable against parasites (Boerlijst and Hogeweg, 1991).

their study of what are the key memes in evolutionary economics. They are known for their advocacy of the idea that routines are the key meme that is the locus of such evolutionary developments. Nelson and Winter themselves were less focused on this matter of emergent higher orders that become the locus of evolution, but some of their followers have pursued such ideas. In particular has been the development of the idea of *mesoeconomics* by Dopfer et al. (2004), originally due to Ng (1986). This is a level of economics that is intermediate in level between the microeconomics of the firm where the Nelson and Winter processes presumably mostly operate and the fully aggregated level of macroeconomics. The mesoeconomic level is more at the industry or sector level where a meme may have diffused across firms within a sector or even a set of related sectors. Such developments can lead to this being the most important part of the economy from the standpoint of growth and evolutionary development.

In terms of institutional evolution operating at higher levels of emergent structures, a possibly surprising supporter of this view is Austrian economist, Friedrich Hayek. This would appear to be at least partly associated with his open embrace of complexity (Hayek 1967) and especially in connection with this the concept of emergence, harking openly back to the British emergentists of the 1920s. His opening to this strand of thought came from his early work in psychology that culminated in his *The Sensory Order* (Hayek 1952). In this work he specifically saw human consciousness as an emergent property arising from the nervous system and the brain (Lewis 2012). Crucial in his formulating this was the influence of systems theory as developed by Ludwig von Bertalanffy (1950), who in turn was influenced by the *cybernetics* of Norbert Wiener (1948), thought by many to be another early form of dynamic complexity. Lying more deeply behind cybernetics was the development of the "universal system of organizations" or *tektology* of A.A. Bogdanov (1925-29), arguably a form of evolutionary institutional economics stressing emergence.[21]

Indeed, Hayek (1988) in his final work, *The Fatal Conceit*, applied his view of emergent complexity involving evolution in a higher order way, with such emergent institutional structures competing with each other and evolving as wholes competing with each other and surviving or not through a process of systemic natural selection. Some would argue that this embrace of natural selection operating at the level of higher order societal wholes constituted a contradiction with the methodological individualism of the Austrian School, although in fact in this he harked back to evolutionary ideas of the founder of that school, Carl Menger (1923) that like Hayek he fully developed late in his career.

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[21] See also Stokes (1995).

2.11 Old and New Institutional Economics from a Complex Evolutionary Perspective

The old and the new approaches to institutional economics have long been viewed as in deep conflict, with the evolutionary approach derived especially from Veblen of the old view in conflict with the greater acceptance of neoclassical economics asserted by the new, beginning with Coase (1937). Indeed, it was Veblen who initially coined the phrase "neoclassical economics," which he used in a pejorative manner to criticize the equilibrium approach of Alfred Marshall and others, so Coase's acceptance of this approach and effort to fit the new institutional economics into it would appear to be a deep conflict hard to overcome. The link between Veblen's idea of cumulative causation and modern dynamic complexity theory would seem to simply reinforce this disagreement between the approaches.

The central unifying concept of the new institutional economics is that of *transaction cost* and that minimizing this is the central core of how institutions and organizations form and develop. Whether a firm outsources an activity or carries it out within itself is determined by which of these will minimize its transaction costs as initially argued by Coase (1937), with this carried forward by Williamson (1985) and North (1990) in their more explicit formulation of the new institutional economics approach. We should note that Coase in particular, somewhat like Schumpeter, specifically rejected the direct application of biological or evolutionary ideas to his view of economics.

Even as Coase opposed the evolutionary view of the old institutional economics of Veblen, he did recognize links with parts of their views. In particular, the idea that transaction costs are important was something that he got initially from Commons (1934), with Williamson also later recognizing this source as well. As already noted, Commons took a view of institutional evolution that emphasized its directedness and its subjection to conscious human decisions, much as with the animal breeders studied by Darwin and Sewall Wright. Institutions can be consciously created by people without them simply appearing or emerging out of some mysterious dynamic process beyond human control. That this opens the door to a possible reconciliation of the old and new institutionalist approaches has been argued by Mikami (2011) who argues that even if Coase did not like biology, his views are sympathetic with sociobiology, and that the effort to minimize transaction costs can lead to a dynamic process that is complex.

2.12 Summing Up

Herbert A. Simon was the "father of behavioral economics" who formulated the concept of bounded rationality out of that. He also founded the hierarchical notion of complexity that cuts across disciplinary boundaries, which has implications for evolutionary emergence into higher level structures in nature. This goes beyond biology to a broader view of the universe, with such an emergent evolutionary process extending from emergence of atoms out of sub-atomic particles to human consciousness and beyond.

Central to understanding the complex evolution of economic institutions is fully understanding the implications of the ideas of the founder of both evolutionary economics and institutional economics, Thorstein Veblen. Particularly important was his formulation of the concept of cumulative causation, later taken up more prominently by such figures as Young, Myrdal, and Kaldor. This links to modern dynamic complexity theory through increasing returns, which leads to muiltiple equilibria and complex disequilibrium dynamics. Veblen's vision was thoroughly Darwinian in that he did not propose any directed teleological evolution in the way that favored more by fellow institutional economist, John R. Commons.

Arising from Veblen's ideas of institutional evolution is also the possibility of complex emergence of higher orders of institutions based on cooperation, linking to ideas of Herbert Simon, as well as drawing on the theory of multi-level evolution developed by biologists such as Crow, Hamilton, and Price. The existence and competition between hierarchical economic institutions also implies problems of computational complexity, again with no definite direction or outcome a likely result. This reveals the deep relations between complexity and behavioral economics.

Chapter 3
The Complex Dynamics of Social Interactions

3.1 Introduction

How large the non-observed economy (NOE) is and what determines its size in different countries and regions of the world is a much studied question (Schneider and Enste, 2000, 2002).[1] The size of this sector in an economy has important ramifications. It negatively affects a nation's ability to collect taxes to support its public sector, which can lead more economic agents to move into the non-observed sector (Johnson et al. 1997). When this sector is associated with criminal or corrupt activities it may undermine social capital and broader social cohesion (Putnam et al. 1993), which may damage economic growth (Knack and Keefer, 1997; Zak and Knack, 2001). Furthermore, as international aid programs are tied to official measures of the size of economies, these can be distorted by wide variations in the relative sizes of the NOE across different countries, especially among the developing economies.

Early studies (Guttman, 1977; Feige, 1979; Tanzi, 1980, Frey and Pommerehne, 1984) emphasized the roles of high taxation and large welfare state systems in pushing businesses and their workers into the non-observed sector. Although some more recent studies have found the opposite, that higher taxes and larger governments may actually be negatively related to the size of this sector (Friedman et al. 2000), others continue to find the more traditional relationship (Schneider, 2002; Schneider and Klinglmair, 2004).[2] Various other factors have been found to be

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[1] Many terms have been used for the non-observed economy, including informal, unofficial, shadow, irregular, underground, subterranean, black, hidden, occult, illegal, and others, with much of this terminology originating in studies in Italy (Pettinati, 1979).. Generally these terms have been used interchangeably. However, here note distinctions between some of these and thus will use the more neutral descriptor, non-observed economy, adopted for formal use by the UN System of National Accounts (SNA) (see Calzaroni and Rononi, 1999; Blades and Roberts, 2002).

[2] However, in Schneider and Neck (1993) it is argued that the complexity of a tax code is more important than its level of tax rates. Also, in Schneider and Enste (2002, pp 97–101) it is argued that

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_3

related to the NOE at the global level, including degrees of corruption, degrees of over-regulation, the lack of a credible legal system (Friedman et al. 2000), the size of the rural sector, and the degree of ethnic fragmentation (Lassen, 2007).

One factor often ignored in this mix is income inequality. The first published papers dealing empirically with such a possible relationship focused on this relationship within transition economies (Rosser Jr. et al. 2000, 2003b).[3] For a major set of the transition economies they found a strong and robust positive relationship between income inequality and the size of the non-observed economy. The first of these also found a positive relationship between changes in these two variables during the early transition period while the second only found the levels relationship still holding significantly after taking account of several other variables. The most important other significant variable was a measure of macroeconomic instability, specifically the maximum annual rate of inflation a country had experienced during the transition.

Here the hypothesis of a relationship between the degree of income inequality and the size of the non-observed economy is extended to the global data set studied by Friedman et al. (2000). Macroeconomic variables are considered that they did not include and also an index of trust as a measure of social capital. A main conclusion is that the finding of earlier studies carries over to the global data set: income inequality and the size of the non-observed economy possess a strong, significant, and robust positive correlation. No other variable shows up as consistently similarly related, although a corruption index does for some specifications. However, inflation is not significantly correlated for the global data set, in contrast to findings for the transition countries, and neither is per capita GDP. In contrast with Friedman et al, measures of regulatory burden and lack of property rights enforcement are weakly negatively correlated with the size of the non-observed economy but not significantly so. However, lack of property rights enforcement is strongly negatively correlated with corruption, and regulatory burden is also under some specifications. The finding of Friedman et al. (2000) that taxation rates are negatively correlated with the size of the non-observed economy holds only insignificantly in multiple regressions.

In addition, which variables are correlated in multiple regressions with income inequality, levels of corruption, and trust are considered. In a general formulation the two variables that are significantly correlated with income inequality are a positive relation with the size of the non-observed economy and the regulatory burden, with a

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for low income countries higher tax rates might reduce the share of the shadow economy as some government is needed to establish official markets.

[3] Lewis Davis (2007) notes the theoretical model of Rauch (1993) that hypothesizes such a relationship in development in conjunction with the Kuznets curve. During the middle stage of development inequality increases as many poor move to the city and participate in the "underemployed informal economy," a concept that follows the discussion of de Soto (1989), although this resembles more the "underground" economy as defined later here. Rauch does not provide empirical data and his theoretical model differs from the one presented here and involves a different mechanism as well. Rosser Jr. et al. (2007) initially extended this beyond the transition economies to a broader global data ser.

negative relation with taxation rates significant at the ten per cent level. Regarding the corruption index, the variables significantly correlated with it are negative relations with property rights enforcement and trust. Trust is significantly negatively related to corruption but counterintuitively is positively related to the size of the non-observed economy, although their bivariate relation is negative.

Beyond these more specific empirical findings (and related policy implications), there is a more general methodological issue to consider. It contributes to the emerging paradigm that emphasizes the role of social interactions of heterogeneous agents in complex economic systems as being important to consider in addition to the more conventional analysis that focuses solely upon individual incentives. That such a clear implication of the conventional approach as that higher taxes should be associated with greater involvement in the non-observed economy may be nullified by the effect of such social interactions is strong evidence of this conclusion.

3.2 Labor Returns in the Non-Observed Economy

Whereas Friedman et al. (2000) focus upon decisions made by business leaders, let us consider decisions made by workers regarding which sector of the economy they wish to supply labor to. This allows us to see clearly the issue of social interactions involved in the formation of the non-observed economy that tend to be left out in such discussions. Focusing on business leaders' decisions does not explain why income distribution might enter into the matter, and it may be that the use of such an approach in much previous literature explains why researchers have avoided the hypothesis we find to be so compelling. However, factors such as social capital and social cohesion seem related to the degree of income inequality and thus need to be recognized.

We need to clarify the use of terminology. As noted in footnote 1 above, most of the literature in this field has not distinguished between such terms as "informal, underground, illegal, shadow," and so forth in referring to economic activities not reported to governmental authorities (and thus not generally appearing in official national and income product accounts, although some governments make efforts to estimate some of these activities and include them). In Rosser Jr. et al. (2000, 2003b) the terms "informal" and "unofficial" were respectively used, with it argued that all of these labels meant the same thing. However, it must be recognized there that there were different kinds of such activities and that they had different social, economic, and policy implications, with some clearly undesirable and others potentially desirable from certain perspectives, e.g. businesses only able to operate in such a manner due to excessive regulation of the economy (Asea, 1996).[4]

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[4] Another positive aspect of non-observed economic activity of any sort arises from multiplier effects on the rest of the economy that it can generate (Bhattacharya, 1999).

Rosser Jr. et al. (2007) used the term "Non-Observed Economy" (NOE), which will be will be used here and which was introduced into the United Nations System of National Acccounts (SNA) in 1993 (Calzaroni and Rononi, 1999), and which has become accepted in policy discussions within the OECD (Blades and Roberts, 2002) and other international institutions. The SNA further subdivides the NOE into three broad categories: *illegal*, *underground*, and *informal* (Calzaroni and Rononi, 1999). There are further subdivisions of these regarding whether their status is due to statistical errors, underreporting, or non-registration, which we shall not discuss further.

The illegal sector consists of activities that would be in and of themselves illegal if officially reported, e.g. murder, theft, bribery, and so forth. Some of corruption fits into this category, but not all. By and large these activities are viewed as unequivocally undesirable on social, economic, and policy grounds. Underground activities are those that are not illegal per se, but which are not reported to the government in order to avoid taxes or regulations. Thus they become illegal, but only because of this non-reporting of them. Many of these may be desirable to some extent socially and economically, even if the non-reporting of them reduces tax revenues and may contribute to a more corrupt economic environment. Finally, informal activities are those that take place within households and do not involve market exchanges for money. Hence they would not enter into national income and product accounts by definition, even if they were to be reported. They are generally thought to occur more frequently in rural parts of less developed countries and to be largely beneficial socially and economically. Although the broader implications of these different types of non-observed economic activity vary considerably, they all result in no taxes being paid to the government on them.

Although not necessary for positive relations between our main variables, income inequality, corruption, and the size of the NOE, conditions under which multiple equilibria arise as discussed in Rosser Jr. et al. (2003b) are of interest. This idea draws on a considerable literature, much of it in sociology and political science, which emphasizes positive feedbacks and critical thresholds in systems involving social interactions. Schelling (1978) in economics and Granovetter (1978) in sociology noted such phenomena, with Crane (1993) discussing cases involving negative social conduct spreading rapidly after critical thresholds are crossed. Putnam et al. (1993) suggested possible multiple equilibria in discussing the contrast between northern and southern Italy in terms of social capital and economic performance. Although Putnam emphasizes participation in civic activities as key in measuring social capital, others focus more on measures of generalized trust, found to be strongly correlated with economic growth at the national level (Knack and Keefer, 1997; Zak and Knack, 2001; Svendsen, 2002). Given that Coleman (1990) defines social capital as the strength of linkages between people in a society, it can be related to social cohesion and potentially lower transactions costs in economic activity.

The concept of social capital is controversial. Early advocates of the idea included Bourdieu (1977) and Loury (1977). Major overviews can be found in Woolcock (1998), Dasgupta (2000), Svendsen and Svendsen (2004), with Durlauf and

Fafchamps (2005) providing a more critical perspective. The latter note that different observers provide conflicting definitions of the concept with confused measures and econometric estimates. They note especially the problem of "negative social capital," that strong links within certain sub-groups, such as the mafia, may be inimical to economic growth. Putnam (2000) distinguishes between "bridging" social capital and "bonding" social capital. The former consists of links throughout society generally, the kind that presumably reduce transactions costs of economic activity. The latter are between individuals within a sub-group of society, the sort that could be inimical to general economic growth, although not necessarily to the incomes of the members of the group and might correspond more to the negative social capital of Durlauf and Fafchamps.[5] We shall assume that measures of generalized trust serve as proxies for the more economically productive, bridging social capital.

Dasgupta (2000, pp. 395-396) provides three alternative conceptualizations at the aggregate level for the operation of social capital, which he identifies with trust. The first has it operating through total factor productivity

$$Y = Af(K, N), \tag{3.1}$$

where Y is total output, A is total factor productivity, K is aggregate physical capital, and N is labor force. A is a positive function of bridging social capital, seen as lowering transactions costs through generalized trust. Dasgupta finds the evidence for this weak, at least for East Asia. The second approach distinguishes human capital, H, and sees it being influenced along with physical capital by the lowering of transactions costs through social capital

$$Y = Af(B(K, H), N), \tag{3.2}$$

where B now captures the social network externalities of social capital. Dasgupta reports for this as well that evidence is weak for B contributing substantially to economic growth in newly industrializing countries. Finally Dasgupta postulates that social capital works through both human capital and labor via C,

$$Y = Af(K, CN(H, N)). \tag{3.3}$$

Dasgupta then argues that it is not possible to clearly distinguish between these hypotheses. However, here I shall consider (3.3) to be the more appropriate representation and further consideration will assume that the social externality element will operate through its impact on labor directly (we shall not worry about physical capital directly).

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[5] Lassen (2007) argues that ethnic divisions break down social capital and can open the door to a larger informal economy. Bjørnskov (2006) provides a fuller set of elements involved in social capital.

Rosser Jr. et al. (2000, 2003b) argue that the link between income inequality and the size of the NOE is a two-way causal relationship, running principally through breakdowns of social cohesion and social capital. Income inequality leads to a lack of these, which in turn leads to a greater tendency to drop out of the observed economy due to social alienation. Zak and Feng (2003) find transitions to democracy easier with greater equality. Going the other way, the weaker government associated with a large NOE reduces redistributive mechanisms and tends to aggravate income inequality.[6] Bringing corruption into this relation simply reinforces it in both directions. Although no one prior to Rosser Jr. et al. (2000) directly linked income inequality and the NOE, some did so indirectly. Thus, Knack and Keefer (1997) noted that both income equality and social capital were linked to economic growth and hence presumably to each other. Putnam (2000) shows among the states in the United States that social capital is positively linked with income equality but is negatively linked with crime rates.

The formal argument in Rosser Jr. et al. (2003b) drew on a model of participation in mafia activity due to Minniti (1995). That model was in turn based on ideas of positive feedback in Polya urn models due to Arthur et al. (1987; see also Arthur, 1994). The basic idea is that the returns to labor of participating in NOE activity are increasing for a while as the relative size of the NOE increases and then decrease beyond some point. This can generate a critical threshold that can generate two distinct stable equilibrium states, one with a small NOE sector and one with a large NOE sector. In the model of criminal activity the argument is that law and order begins to break down and then substantially breaks down at a certain point, which coincides with a substantially greater social acceptability of criminal activity. However, eventually a saturation effect occurs and the criminals simply compete with each other leading to decreasing returns. Given that two of the major forms of NOE activity are illegal for one reason or another, similar kinds of dynamics can be envisioned.

Let N be the labor force; N_{noe} be the proportion of the labor force in the NOE sector; r_j be the expected return to labor activity in the NOE sector minus that of working in the observed sector for individual j, and a_j be the difference due solely to personal characteristics for individual j of the returns to working in the NOE minus those of working in the observed economy, with this capturing both the human capital and social capital effects on the individual. Let us assume that this variable is uniformly distributed on the unit interval, $j \in [0, 1]$, with a_j increasing as j increases, ranging from a minimum at a_{o} and a maximum at a_1. Furthermore, this difference in

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[6] This effect is seen further from studies showing that tax paying is tied to general trust and social capital. Scholz and Lubell, 1998; Slemrod, 1998). Anderson et al. (2004) provide experimental evidence of links between equality and the willingness to provide for public goods. Although not explicitly mentioning income distribution, Schneider and Enste (2002) emphasize “tax morality” as a factor in paying taxes, and they recognize that the perceived fairness of a tax system influences this. If general trust and income equality increase tax morality, then they could increase the paying of taxes.

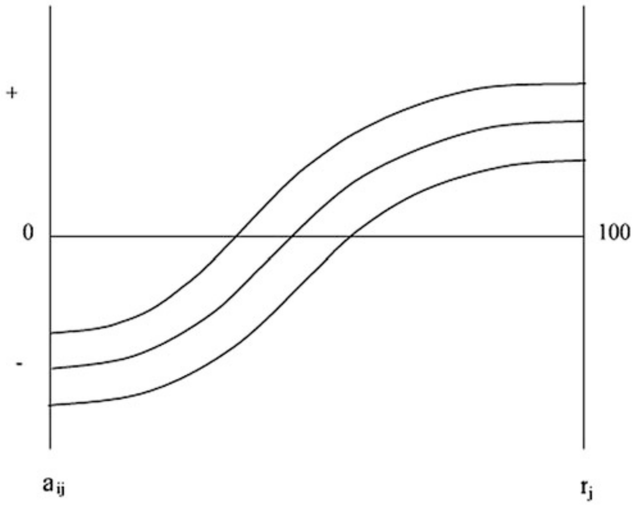

Fig. 3.1 Relative returns to working in non-observed sector for three separate individuals (vertical axis) as a function of percent of economy in non-observed sector (horizontal axis)

returns between the sectors follows a cubic function. With all parameters assumed positive this gives the return to working in the NOE sector for individual *j* as

$$r_j = a_j + \left(-\alpha N_{\text{noe}}{}^3 + \beta N_{\text{noe}}{}^2 + \gamma N_{\text{noe}}\right), \tag{3.4}$$

with the term in parenthesis on the right hand side equaling $f(N_\mu)$. Figure 3.1 shows this for three individuals, each with a different personal propensity to work in the NOE sector.

Broader labor market equilibrium is obtained by considering stochastic dynamics of the decisionmaking of potential new labor entrants. Let $N' = N + 1$; $q(noe) =$ probability a new potential entrant will work in the NOE sector, $1 - q(\text{noe}) =$ probability new potential entrant will work in observed sector, with $\lambda_{\text{noe}} = 1$ with probability q (noe) and $\lambda_{\text{noe}} = 0$ with probability $1 - q(\text{noe})$. This implies that

$$q(\text{noe}) = [a_1 - f(N_{\text{noe}})]/(a_1 - a_0). \tag{3.5}$$

Thus after the change in the labor force the NOE share of it will be

$$N'_{\text{noe}} = N_{\text{noe}} + (1/N)[q(\text{noe}) - N_{\text{noe}}] + (1/N)[\lambda_{\text{noe}} - q(\text{noe})]. \tag{3.6}$$

The third term on the right is the stochastic element and has an expected value of zero (Minniti, 1995, p. 40). If $q(\text{noe}) > N_{\text{noe}}$, then the expected value of $N'_{\text{noe}} > N_{\text{noe}}$. This implies the possibility of three equilibria, with the two outer ones stable and the intermediate one unstable. This situation is depicted in Figure 3.2.

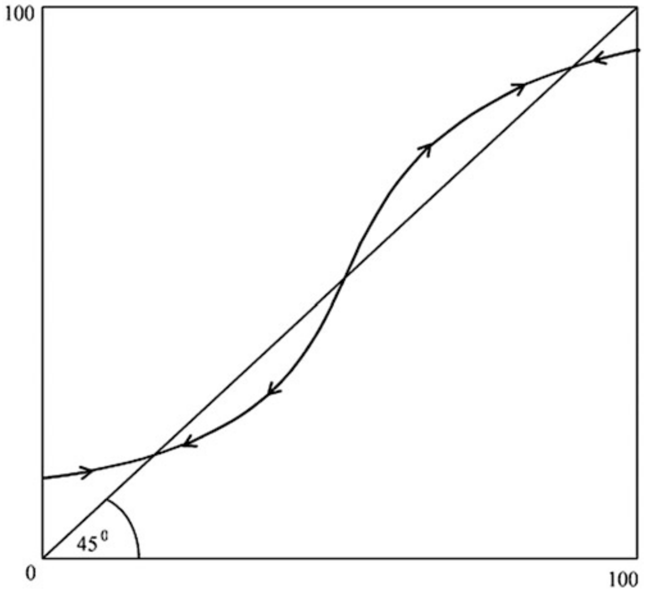

Fig. 3.2 Probability a new labor market entrant will work in the non-observed sector, $q(u)$, (vertical axis) as a function of the percent of labor in the non-observed sector (horizontal axis)

The argument can be summarized by positing that the location of the interval [a_0, a_1] rises with an increase in either the degree of income inequality, in the level of corruption in the society, or in an increase in the gap between bridging and bonding social capital. Such an effect will tend to increase the probability that that an economy will be at the upper equilibrium rather than at the lower equilibrium and if it does not move from the lower to the higher it will move to a higher equilibrium value. In other words, we would expect that either more income inequality or more corruption will result in a larger share of the economy being in the non-observed portion. However, in using trust as the main indicator of social capital, the relationship is ambiguous as it will depend on what kind of social capital it reflects. If it reflects bridging social capital, then we would expect more trust to lead to less activity in the NOE, whereas if it reflects bonding social capital it may well do the opposite.

Furthermore, one can expect there to be mutual interactions among several of these. The non-observed economy can be expected to increase inequality through reducing tax revenues available for redistribution. We also expect a strong feedback from it to corruption, with all of these potentially affecting social capital in various ways.

Finally, other variables that may interact with these and each other, including broader institutional, policy, or macroeconomic factors described below, must be considered.

3.3 Variables and Data Sources

Here I shall review part of the empirical analysis by Rosser Jr. et al. (2007), in which eight variables are considered: a measure of the share of the NOE sector in each economy, a Gini index measure of the degree of income inequality in each economy, an index of the degree of corruption in each economy, real per capita income in each economy, inflation rates in each economy, a measure of the tax burden in each economy, a measure of the enforcement of property rights, a measure of the degree of regulation in each economy, and a degree of generalized trust.[7] This set of variables produced equations for all the dependent variables with high degrees of statistical significance based on the F-test. Results for the 1992–93 and for 2000 were estimated using OLS estimates. There are problems with measuring each of these variables.

Without question the hardest of these to measure is the relative share of an economy that is not observed. The essence of the problem is that one is trying to observe that which by and large people do not wish to have observed. Thus there is inherently substantial uncertainty regarding any method or estimate, and there is much variation across different methods of estimating. Schneider and Enste (2000) provide a discussion of the various methods that have been used. However, they argue that for developed market capitalistic economies the most reliable method is one based on using currency demand estimates. An estimate is made of the relationship between GDP and currency demand in a base period, then deviations from this model's forecasts are measured. This method, due to Tanzi (1980), is widely used within many high income countries for measuring criminal activity in general. Given that most of the currency demand models assume that tax rates measure the underground economy effect, this complicates their use for testing that variable.[8]

Schneider and Enste recommend the use of electricity consumption models for economies in transition, a method originated by Lizzera (1979) because of the instability of financial relationships during economic transition. Kaufmann and Kaliberda (1996) and also Lackó (2000) have made such estimates for transition economies, with these providing the basis for the earlier work by Rosser Jr. et al. (2003b). Kaufmann and Kaliberda's estimates are similar in method to the currency

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[7] Other variables have been included in other tests, including unemployment rates, aggregate GDP, a fiscal burden measure, and a general economic freedom index.. However, neither of the first two was significant and they were not in other studies as well. Real per capita GDP presumably is a better measure than aggregate anyway. Regarding fiscal burden, this is the same as the tax burden measure except that it includes the level of government spending. Most literature supports the idea that the tax aspect is the more important part of this and our results would support this. Finally the overall economic freedom index contains five sub-indexes, three of which are already being used individually. Also one index going into it is a measure of "black market activity," which looks like another measure directly of non-observed economic activity, or at least an important portion of it. So this variable has too many direct correlations with other variables to be of use.

[8] In the current economy looking at cash demand may not work so well given the rise of cryptocurrencies and their use for criminal activities (Norgaard, 2020).

demand one except that a relationship is estimated between GDP and electricity use in a base period, with deviations later providing the estimated share of the NOE. Lackó's approach differs in that she model's household electricity consumption relations rather than electricity usage at the aggregate level. Of course many forms of underground economic activity do not involve the use of electricity, and electricity production technology can change over time in ways complicating such estimates.

Another approach is MIMIC, or multiple indicator multiple cause, first used in this context by Frey and Pommerehne (1984) and used by Loayza (1996) to make estimates for various Latin American economies. This method involves deriving the measure from a set of links between presumed underlying variables and presumed indicators. This method has the problem that it in effect already presumes to know what the relationships are, so that one will get biased results for testing it on any of the presumed underlying variables.[9]

One more method is to look at discrepancies in national income and product accounts data between GDP estimates and national income estimates. Schneider and Enste list several other methods that have been used. However these four are the ones underlying the numbers we use in our estimates.

While some alternatives to some of their other variables are used, the measures of the NOE that Friedman et al. (2000) use are used for the 1992–93 estimates that are most directly comparable with their study. These in turn are taken from tables appearing in an early version of Schneider and Enste (2000). They have 69 countries listed and for many countries provide two different estimates. By and large for OECD countries they use currency demand estimates, mostly due to Schneider (1997) or Williams and Windbeck (1995) or Bartlett (1990), with averages of the estimates provided when more than one is available. For transition economies electricity consumption models are used, mostly from Kaufmann and Kaliberda, with a few from Lackó. Electricity consumption models are also used for the more scattered estimates for Africa and Asia, with most of these estimates drawn on work of Lackó as reported in Scheider and Enste. For Latin America most of the estimates come from Loayza (1996) who used the MIMIC method. However for some countries electricity consumption model numbers are available, due to Lackó and reported by Schneider and Enste. Finally the national income and product accounts discrepancy approach was the source for one country, Croatia, also as reported in Schneider and Enste. Here the estimate is selected from those available based on the prior arguments regarding which would be expected to be most accurate. Most of these numbers are for the early to mid-1990s.

For 2000 numbers provided by Schneider and Klinglmair (2004) are used. A substantial portion of these numbers are based on the DYMIMIC extension of the

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[9] The originators of the MIMIC approach were Zellner (1970) and Goldberger (1972). Breusch (2005) shows that the use of it for some underground economy estimates leads to very fragile results, an outcome that may be more general than just for the MIMIC method. MIMIC stands for "multiple indicators, multiple causes" and DYMIMIC simply adds "dynamic" to the front of that.

MIMIC method. This makes for difficulties in comparing our results for the two different data points and for any studies of dynamic relations between them, which generally showed mostly non-significant results.[10] Unfortunately there were fewer country numbers available for this year, with the set consisting mostly of ones from the OECD and the transition economies. This variable became the main limiting one for 2000 data set, which had only 21 countries for all variables.

Although not as difficult to measure as the NOE, income inequality is a variable that is somewhat difficult to measure, with various competing approaches. The Gini coefficient is the most widely available number across different countries, although it is not available for all years for most countries. Furthermore there are different data sources underlying estimates of it, with the surveys in higher income countries generally reflecting income whereas in poorer countries they often reflect just consumption patterns. For most of the transition countries for 1992–93 estimates constructed by Rosser Jr. et al. (2000) are used, however for the other countries numbers provided by the *UN Human Development Report* for 2002 or 2003 are used, which are also for various years in the 1990s. Of the 69 countries studied in Friedman et al. (2000) there are three for which no Gini coefficient data are available, Argentina, Cyprus, and Hong Kong. Hence they are not included in these estimates.

The measure of corruption is an index used by Friedman et al. (2000) that comes from Transparency International (1998). It should be noted that the scale used for this index is higher in value for less corrupt nations and ranges from one to ten. This is in contrast to our NOE and Gini coefficient numbers, which rise with more NOE and more inequality. Thus, a positive relation between corruption and either of those other two variables will show up as a negative relationship for our variables. For 2000 numbers updated from the same source are used.

Real per capita GDP numbers come from *UN Human Development Report* for 2001 and are for the year 2000. The inflation rate estimate is from the same source but is an average for the 1990–2000 period. The measure of tax burden comes from Heritage Foundation's *2001 Index of Economic Freedom* (O'Driscoll Jr. et al. 2001). This combines an estimate based on the top marginal income tax rate, the marginal tax rate faced by the average citizen and the top corporate tax rate and ranges from one (low tax burden) to 5 (high tax burden). This number increases as the taxation burden increases. The measure of property rights enforcement comes from O'Driscoll Jr. et al. (2001) and ranges from one (high property rights enforcement) to five (low property rights enforcement). The measure of regulatory burden is also from O'Driscoll Jr. et al. (2001) and ranges from one (low regulatory burden) to five (high regulatory burden). Obviously there is a considerable amount of subjectivity involved in many of these estimates. After taking account of these variables so far, the usable data set is reduced from 69 to only 52.

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[10] In a personal communication (2005), Dominick Enste notes that while the DYMIMIC method may have advantages as an estimate of the NOE, the way that other variables enter into its measurement may make it less well suited for use in checking on the independent significance of those variables in explaining the determinants of the NOE.

Finally, the measure of trust for 1992–93 is the index used in the World Values Survey (Inglehart et al. 1998), which varies from zero to 100, with higher meaning more trust. Although they study 43 "societies," many of these are sub-sections of the nations observed here, such as the city of Moscow and Northern Ireland. In the end, when the numbers from this sort are combined with those listed above one is left with only 32 of the original 69 countries, with the set heavily dominated by OECD and transition countries. Thus, in order to capture a broader view, regressions both with and without the trust variable are considered For 2000 numbers for this index used were provided personally by Ronald Inglehart, for which year estimates for many more countries were available.[11]

3.4 Empirical Findings

Prior to OLS multiple regressions for the 1992–93 data, the correlation matrix for these nine variables generally foreshadows the regression results, with a few exceptions. Using the larger 52 nation set without trust, for each of the three other main dependent variable the independent variables that prove to be statistically significant in the OLS regressions also have a high absolute value in the correlation matrix with the dependent variable. The two exceptions are that lack of property rights enforcement and regulatory burden appear strongly correlated with the NOE, but not so in the multiple regression. But their relations with corruption are the highest bivariate correlations in the matrix, foreshadowing that corruption may carry their effect in some multiple regressions. The main outlier comes when we bring in trust and the data set is reduced to 32 nations. Trust is negatively correlated with the NOE in the correlation matrix but seems to be positively related with it in the multiple regression at the ten per cent level.

In the OLS regression without the trust variable in which the measure of the non-observed economy is the dependent variable and the other seven variables are the independent ones. Most statistically significant is the corruption index, so at the 5 percent level, with it the most strongly correlated in the correlation matrix. The expected positive relationship between these two (shown by a negative sign) holds. The other significant variable at the 5 percent level is the Gini coefficient. The qualitative results seen here show up consistently in other regressions with these and other variables in various combinations.

Another shows the same regression but with the trust variable included as an independent variable and with the number of observations reduced by 20 because of the unavailability of the trust index for those countries. The Gini index continues to be significant, even more strongly so than in the previous regression. Corruption is no longer significant, although is nearly so at the ten per cent level. However, a

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[11] Discussion with several interlocutors suggest that these estimates have many problems. Nevertheless, they were probably the best such numbers available for such a wide set of countries.

peculiar result is that trust is positively related to NOE and significantly so at the ten per cent level. This could be that the trust number is picking up “bonding” as well as “bridging” social capital, possibly consistent with this result.

Following the arguments of McCloskey and Ziliak (1996) the size of the coefficients for these two statistically significant variables are large enough to be economically significant as well. In the larger regression, the presumed ceteris paribus relations would be that a 10 percent increase in the Gini coefficient would be associated with a 6 percent increase in the share of GDP in the non-observed economy, while a 10 percent increase in the rate of corruption (change in index value of one point) would be associated with 4 percent increase in the share of GDP in the non-observed economy. These are noticeable relationships economically, although one must be careful about making such, extrapolations as these.[12]

However, one finding for the transition nations does not carry over to the global data set. This is the statistically significant relationship between inflation and the size of the NOE, which even carried over to the growth of the NOE as well. A possible explanation of this is that during the period of observation the transition economies experienced much higher inflation than most of the rest of the world, with Ukraine reaching a maximum annual rate of more than 10,000 percent. This high inflation was strongly related to the general process of institutional collapse and breakdown that happened in those countries then.

One finding of Friedman et al. (2000) is not confirmed, their finding that taxation burden is negatively correlated with the size of the NOE significantly. The correlation matrix shows a negative bivariate correlation of −0.45, but in the larger regression this becomes a weakly positive and statistically insignificant relation, while in the Table in a further regression it is weakly negative but insignificant relation. The likely explanation for the contrast between this finding and that of Friedman et al. (2000) is that there is a strong negative relation between taxation burden and income inequality, at least in the larger data set as seen in the largest regression. .In the multiple regression this dominates. The more important factor here is income inequality, and when it appears in an equation the statistical significance (and even the sign found) disappears. Thus, that Friedman et al. (2000) left out income distribution in their various estimates appears to have profoundly distorted their findings. The relation is not statistically significant in either direction in a more fully specified model.

Then there is the OLS regression results for the smaller set of variables but with the Gini coefficient as the dependent variable. The size of the NOE is statistically significant at the 5 percent level, although not at the 1 percent level. Even more statistiarelly significant, holding strongly at the 1 percent level, is tax burden, which is negatively correlated. It would appear that these tax burdens result in noticeable income redistribution, or if they do not, then nations with more equal income

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[12] There have been some spectacular examples of nations having dramatic increases in both inequality and the size of their non-observed economy, with what happened in Russia between 1989 and 1993 especially notable.

distributions are more willing to tolerate higher tax rates. As in earlier regressions, the inflation measure also does not show up as statistically significant as holds for the other variables.

Regarding economic significance the relation from the NOE to income inequality appears to be somewhat weaker than going the other way. Thus, a 10 percent increase in the share of the non-observed economy in GDP would only be associated with about a 2 percent increase in the Gini coefficient. The taxation burden appears to be economically significant, with a 20 percent increase in tax burden leading to a 40 percent decline in Gini coefficient.

Another regression brings in trust to this estimate for the smaller 32 countries data set. While the NOE continues to be a significant variable, taxation is now only significant at the ten percent level, with regulatory burden now becoming significant at the 5 percent level, with it negatively correlated with inequality. Also, our macroeconomic variables come back into play somewhat, with the deflator being significant at the 10 percent level and positively correlated with inequality.

Then consider results for trust as the dependent variable, which is only available for the 32 observations data set. The most significant variable is corruption at the 1 percent level, which has the expected sign. An anomalous result is that NOE is significant at the 10 percent level, but with an unexpected positive sign, overturning the bivariate relation between these two variables in the correlation matrix. A surprising result is that the hypothesis that equality would drive trust does not hold up fully. The sign is as expected, but just missing being significant at the 10 percent level. Thus, curiously, inequality seems to more directly related to NOE than the hypothesized intermediary, social capital as measured by trust, although this may be due to the smaller data set available with the trust variable.

Then consider the correlation matrix for the variable set for 2000, with generally similar results compared to the earlier period. There are OLS regressions on the full variable set for each of the main dependent variables, with only one each shown given that the limiting variable for this period is the NOE variable. Unfortunately there are only 21 countries in this data set, confined to OECD and transition economies.

The one probably of greatest interest has NOE as the dependent variable. The results are reasonably consistent with the 1992–93 estimates earlier, but with some additional variables significant. Thus, inequality is significant again at the 5 percent level with our expected positive sign, and trust is again significant with a positive sign and at the 1 percent level. As before, this latter undoes the sign observed in the correlation matrix. The two additional variables that are significant are corruption, which is positively related as expected and at the 10 percent level of significance, along with inflation, which is counterintuitively negatively related with NOE and significant at the 5 percent level, strongly contrasting with findings just for the transition economies.

For one with the Gini coefficient as the dependent variable, the basic story of the two-way relationship between the NOE and inequality continues to hold up, with the NOE positive and significant at the 5 percent level. Also, the influence of inflation is even stronger, and is positively related at the 1 percent level. Unlike the earlier data set, trust is now a significant variable, negatively related to inequality and significant at the 1 percent level. Also different from the earlier estimate are that the taxation and regulation variables are no longer significant, although taxation continues to have a negative sign.

A serious problem for these estimates is potential endogeneity of various variables with each other, with many possibilities available. An effort to deal with this involved several possible simultaneous equations formulations using two-stage least squares.[13] Unfortunately the results from these estimates were generally weak, raising questions about the robustness of the findings.

3.5 Conclusions

The finding of Rosser Jr. et al. (2000, 2003b, 2007) that there appears to be a significant two-way relationship between the size of the non-observed economy (or informal or unofficial economy) and income inequality is tentatively confirmed when the data set is expanded to include nations representing a more fully global sample based on OLS regressions, but does not retain significance in simultaneous equations formulations or in estimates of changes in variables between the two time periods. The finding of Friedman et al. (2000) that there is a strong relationship between the size of the non-observed economy and the level of corruption in an economy is more weakly confirmed, and may be a significant two-way relationship, although somewhat stronger in going from corruption to the non-observed economy than the other way. This weakens in the runs with trust that cover only 32 countries for 1992–93, but is stronger for 2000. That the maximum annual rate of inflation to be important in the size of the non-observed economy holds for the transition economies it does not hold for larger national data sets..

The finding not confirmed from the Friedman, Johnson, Kaufmann, and Zoido-Lobatón study is that of a negative relationship between higher taxes and the size of the non-observed economy. These results find no statistically significant relationship, in between this view and the alternative more traditional view that argues that higher taxes drive people into the non-observed economy. The failure of the Friedman et al. (2000) to include any measures of income inequality may explain this contrast and shows the importance of social interactions.

Their findings that the non-observed economy increases with lack of property rights enforcement and regulatory burdens is not directly found for either time period. However, there are strong relations between these and corruption for the broader data set without the trust variable in 1992–93 and for property rights enforcement with the trust variable, with corruption strongly linked with the non-observed economy, suggesting perhaps that this is the pathway through which these variables have their effect. However, these relationships did not hold up at all

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[13] See Rosser Jr. et al. (2007) for further discussion of this issue.

in 2000, although these variations may reflect the varying sets of countries used, with the 21 country 2000 set being limited to OECD and transition economies, whereas the larger of the 1992–93 sets at 52 countries, without the trust variable, includes many less developed countries.

Using trust a our measure of social capital led to somewhat confusing results that may reflect conflicts between bonding social capital between sub-groups versus bridging social capital across groups, although presumably generalized trust should represent the latter. In any case it had an unexpected positive and significant relation with the non-observed economy for both time periods, more consistent with it as measure of bonding social capital. While insignificant with inequality in 1992–93 it was significantly and negatively related to inequality in 2000, consistent with most literature. Regarding corruption it was significant in both time periods with the expected negative sign. The NOE and corruption were the significant variables determining trust in 1992–93, retaining their signs, while in 2000 inequality was significantly negative and inflation was curiously significantly positive.

Efforts to test the robustness of these results using two-stage least squares on each of the data sets and OLS do not hold up well, warning of a fragility found by both Durlauf and Fafchamps (2005) and Breusch (2005) regarding studies of both social capital and the non-observed economy. Problems and uncertainties regarding much of the data, especially for the estimates of the size of the non-observed economy, are probably substantial contributors to this lack of robustness.

While these results should be used cautiously in making policy recommendations, they do reinforce the warning delivered in Rosser Jr. and Rosser (2001): international organizations concerned about the negative impacts on revenue collection in various countries of having large non-observed sectors should be cautious about recommending policies that will lead to substantial increases in income inequality. Fiscal austerity programs to reduce budget deficits that focus on reducing egalitarian transfer programs may backfire into a situation of reduced revenues. Sharply increasing inequality may well have the counterproductive outcome of increasing the size of the non-observed economy and corruption, thus reducing tax revenues and more broadly engendering a decline of social capital and general social cohesion, a deep finding showing how a conventionally expected result can be found not to hold when dynamically complex social interactions are accounted for.

Chapter 4
Econophysics, Entropy, and Complexity

4.1 The Origins and Nature of Econophysics

The term *econophysics* was neologized in 1995 at the second Statphys-Kolkata conference in Kolkata (formerly Calcutta), India by the physicist H. Eugene Stanley, who was also the first to use it in print (Stanley et al. 1996a). Mantegna and Stanley (1999, pp. viii–ix) define "the multidisciplinary field of econophysics" as "a neologism that denotes the activities of physicists who are working on economics problems to test a variety of new conceptual approaches deriving from the physical sciences" Chakrabarti 2005, p. 225).

The list of such problems has included distributions of returns in financial markets (Mantegna 1991; Levy and Solomon 1997; Bouchaud and Cont 1998; Gopakrishnan et al. 1999; Lux and Marchesi 1999; Sornette and Johansen 2001; Farmer and Joshi 2002; Li and Rosser 2004) the distribution of income and wealth (Drăgulescu and Yakovenko 2001; Bouchaud and Mézard 2000; Chatterjee et al. 2007; Yakovenko and Rosser Jr. 2009), the distribution of economic shocks and growth rate variations (Bak et al. 1993; Canning et al. 1998), the distribution of firm sizes and growth rates (Stanley et al. 1996b; Takayasu and Okuyama 1998; Botazzi and Secchi 2003), the distribution of city sizes (Rosser Jr 1994; Gabaix 1999), and the distribution of scientific discoveries (Plerou et al. 1999; Sornette and Zajdenweber 1999), among other problems, all of which are seen at times not to follow normal or Gaussian patterns that can be described fully by mean and variance. The main sources of conceptual approaches from physics used by the econophysicists have been from models of statistical mechanics (Spitzer 1971), geophysical models of earthquakes (Sornette 2003), and "sandpile" models of avalanches, the latter involving self-organized criticality (Bak 1996). An early physicist to assert the essential identity of statistical methods used in physics and the social sciences was Majorana (1942), who has been viewed by some econophysicists as a precursor..

A common theme among those who identify themselves as econophysicists is that standard economic theory has been inadequate or insufficient to explain the

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_4

non-Gaussian distributions empirically observed for various of these phenomena, such as "excessive" skewness and leptokurtotic "fat tails" (McCauley 2004; Chatterjee and Chakrabarti 2006; Lux 2009). The emergence of econophysics followed fairly shortly on the influential interactions and discussions that occurred between groups of physicists and economists at the Santa Fe Institute (Anderson et al. 1988; Arthur et al. 1997a), with some of the physicists involved in these discussions also becoming involved in the econophysics movement.

Now we come to a great curiosity and irony in this matter: some of the main techniques used by econophysicists were initially developed by economists (with many others developed by mathematicians), and some of the ideas associated with economists were developed by physicists. Thus, in a sense, these efforts by physicists resemble a bringing of coals to Newcastle, except that it must be admitted that many economists either forgot or never knew of these issues or methods. This is true of the most canonical of such models, the Pareto distribution (Pareto 1897).

4.2 The Role of the Pareto Distribution

If there is a single issue that unites the econophysicists it is the insistence that many economic phenomena occur according to distributions that obey scaling laws rather than Gaussian normality. Whether symmetric or skewed, the tails are fatter or longer than they would be if Gaussian, and they appear to be linear in figures with the logarithm of a variable plotted against its cumulative probability distribution. They search for physics processes, most frequently from statistical mechanics, that can generate these non-Gaussian distributions that obey scaling laws.

The canonical (and original) version of such a distribution was discovered by the mathematical economist and sociologist, Vilfredo Pareto, in 1897. Let N be the number of observations of a variable that exceed a value x with A and α positive constants. Then

$$N = Ax^{-\alpha}. \tag{4.1}$$

This exhibits the scaling property in that

$$\ln(N) = \ln A - \alpha \ln(x). \tag{4.2}$$

This can be generalized to a more clearly stochastic form by replacing N with the probability that an observation will exceed x. Pareto formulated this to explain the distribution of income and wealth and believed that there was a universally true value for α that equaled about 1.5. More recent studies (Clementi and Gallegati 2005) suggest that it is only the upper ends of income and wealth distributions that follow such a scaling property, with the lower ends following the lognormal form of the Gaussian distribution that is associated with the random walk, originally argued for the whole of the income distribution by Gibrat (1931), a point further studied by Yakovenko and Rosser Jr (2009), Shaikh (2016), and Shaikh and Jacobo (2020).

The random walk and its associated lognormal distribution is the great rival to the Pareto distribution and its relatives in explaining stochastic economic phenomena. It was only few years after Pareto did his work that the random walk was discovered in a Ph.D. thesis about speculative markets by the mathematician Louis Bachelier (1900), five years prior to Einstein using it to model Brownian motion, its first use in physics (Einstein 1905). Although the Pareto distribution would have its advocates for explaining stochastic price dynamics (Mandelbrot 1963), the random walk would become the standard model for explaining asset price dynamics for many decades, although it would be asset returns that would be so modeled rather than asset prices themselves directly as Bachelier did originally. As a further irony, it was a physicist, M.F.M. Osborne (1959), who was among the influential advocates of using the random walk to model asset returns. It was the Gaussian random walk that would be assumed to underlie asset price dynamics when such basic financial economics concepts as the Black-Scholes formula would be developed (Black and Scholes 1973). Letting p be price, R be the return due to a price increase, B be debt, and σ be the standard deviation of the Gaussian distribution, then Osborne characterized the dynamic price process by

$$\mathrm{d}p = Rp\mathrm{d}t + \sigma p\mathrm{d}B. \tag{4.3}$$

Meanwhile, a variety of efforts were made over a long time by physicists, mathematicians, and economists to model a variety of phenomena using either the Pareto distribution or one its relatives or generalizations, such as the stable Lévy (1925) distribution, prior to the clear emergence of econophysics. Alfred J. Lotka (1926) saw scientific discoveries as following this pattern. George Zipf (1941) would see city sizes as doing so. Benoit Mandelbrot (1963) saw cotton prices doing so and was inspired to discover fractal geometry from studying the mathematics of the scaling property (Mandelbrot 1963, 1997). Ijiri and Simon (1977) saw firm sizes also following this pattern, a result more recently confirmed by Axtell (2001).

4.3 The Role of Statistical Mechanics

Also, economists would move to use statistical mechanics models to study a broader variety of economic dynamics prior to the emergence of econophysics as such. Those doing so included Hans Fōllmer (1974), Lawrence Blume (1993), Steven Durlauf (1993), William Brock (1993), Duncan Foley (1994) and Michael Stutzer (1994), with Durlauf (1997) providing an overview of an even broader set of applications. However, by 1993 the econophysicists were fully active even if they had not yet identified themselves with this term.

While little of this work explicitly focuses on generating outcomes consistent with scaling laws, it is certainly reasonable to expect that many of them could. It is true that the more traditional view of efficient markets with all agents possessing full information rational expectations about a single stable equilibrium is not maintained in these models, and therefore the econophysics critique carries some weight. However, many of these models do make assumptions of at least forms of bounded rationality and learning, with the possibility that some agents may even conform to the more traditional assumptions. Stutzer (1994) reconciles the maximum entropy formulation of Gibbsian statistical mechanics with a relatively conventional financial economics formulation of the Black-Scholes options formula, based on Arrow-Debreu contingent claims (Arrow 1974). Brock and Durlauf (2001) formalize heterogeneous agents socially interacting within a utility maximizing, discrete choice framework.[1] Neither of these specifically generates scaling law outcomes, but there is nothing preventing them from doing so potentially.

While some econophysicists seek to integrate their findings with economic theory, as noted above many seek to replace conventional economic theory, seeing it as useless and limited. An irony in this effort is that it has been argued that conventional neoclassical economic theory itself was substantially a result of importing nineteenth century physics conceptions into economics, with not all observers approving of this (Mirowski 1989a). The culmination of this effort is seen by many as being Paul Samuelson's *Foundations of Economic Analysis* (1947), whose undergraduate degree was in physics at the University of Chicago. Samuelson himself noted approvingly that Irving Fisher's 1892 dissertation (1920) was partly supervised by the pioneer of statistical mechanics, J. Willard Gibbs (1902), and as far back as 1801, Nicholas-François Canard (1969) conceived of supply and demand as ontologically being contradicting "forces" in a physics sense. So the interplay between economics and physics has been going on for far longer and is considerably more complicated than is usually conceived.

Most of this deeper historical background of going back and forth is not known to current econophysicists. This has led sometimes to arguments being made that are potentially problematical on various grounds. These have been discussed in a critical papery by Gallegati et al. (2006) called "Worrying Trends in Econophysics." The trends they identified included a lack of knowledge of previous literature (especially in economics), a tendency to believe that universal empirical regularities can be found in economics that probably are not there in contrast to what one finds in much of physics, a tendency to use unrigorous statistical methodologies sometimes little better than simply looking at figures, and finally using questionable theoretical foundations such as assuming conservation principles in situations where they are unlikely to hold. McCauley (2008) replied to their critique, making the strong case that economic theory is so flawed it should simply be rejected wholesale in favor of

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[1] While most financial economic modeling done by econophysicists has drawn on models derived from statistical mechanics, a rival has been models based on geophysical earthquake models (Sornette, 2003). See also Rosser Jr. (2008b).

ideas coming from physics. Rosser Jr (2008a, b) considered this debate and notes that indeed economists often make assumptions that are not true, although there are clearly limits to how unreal assumptions can be in a useful model. He also argues that a way to resolve this is to have more research done jointly by physicists and economists, with there having been some development of that.

4.4 Econochemistry and Econobiology

Curiously but unsurprisingly given the tremendous attention given to the new econophysics movement, it has spawned imitators since 2000 in the form of *econochemistry* and *econobiology*, although these have not had nearly the same degree of development. The former term is the title of a course of study established at the University of Ulm by Barbara Mez-Starke and was used to describe the work of Hartmann and Rössler (1998) at a conference in 2002 in Urbino, Italy (see also Padgett et al. (2003) for a more recent effort). The latter term first appeared in Hens (2000), although McCauley (2004, pp. 196–199) dismisses it as not a worthy competitor for econophysics. Nevertheless, there has long been a tradition among economists of advocating drawing more from biology for inspiration than from physics (Hodgson 1993a, b), going back at least as far as Alfred Marshall's famous declaration that economics[2] is "a branch of biology broadly interpreted" (Marshall 1920, p. 637), even as Marshall's actual analytical apparatus arguably drew more from physics than from biology.

4.5 Econophysics and Entropy

> "I have come over the years to have some impatience and boredom with those who try to find an analogue of the entropy of Clausius or Boltzman or Shannon to put into economic theory. It is the *mathematical* structure of *classical* (phenomenological, macroscopic, nonstochastic) *thermodynamics* that has isomorphisms with *theoretical economics*."—Paul A. Samuelson, (1990, p. 263)
>
> ". . .throughout his [Samuelson's] career. . .the master of scientific rhetoric, continuously hinting at parallels between neoclassical theory and twentieth century physics, and just as consciously denying them, usually in the same article."—Philip Mirowski, (1989b, p. 186]

The problematic role of entropy in econophysics is highlighted by the quotations presented above: that the arguably most influential economist of the twentieth century, Paul A. Samuelson, played it both ways regarding the role of the concept of entropy in the development of economic theory, and more broadly the role of

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[2] For a more complete discussion of the relations between econophysics, econochemistry, and econobiology within the transdisciplinary perpective, see Rosser Jr. (2010b).

physics in economics. While he regularly ridiculed applications of the entropy concept in economics, he more powerfully than any other economist imposed concepts drawn on physics onto standard neoclassical economics, including that important part of econophysics, statistical mechanics, a contradiction pointed out so forcefully by Mirowski.

The term “econophysics” was introduced verbally by H. Eugene Stanley at a conference in Kolkata in 1995, and at length in print by Mantegna and Stanley (1999) who identified it with physicists applying ideas from physics into economics. This formulation becomes problematic when we understand that people educated as physicists have long been doing this, with Samuelson himself a leading example, along with the one of those who received a Nobel Prize in economics before him, Jan Tinbergen (1937), whose major professor was Paul Ehrenfest, who formulated the “ergodic hypothesis” with his wife (1911), drawing on the work of his major professor, Ludwig Boltzmann (1884).[3] Boltzmann linked the study of statistical mechanics to the concept of entropy as developed initially by Clausius (1867), who in turn was inspired by the work on the thermodynamics of steam engines by Carnot in 1824. The simplest formulation of the Law of Entropy took the form of the Second Law of Thermodynamics: that in a closed thermodynamical system entropy increases.

Given that these borrowings from physics into economics have long predated the more recent movement of physicists to apply their models to economics, we shall expand the concept of economics irrespective of whether these applications were done by people who were primarily physicists, primarily economists, or who were arguably both, with many important economists having originally trained as physicists, with Tinbergen as the student of Ehrenfest being an example.

Regarding the specific application of the entropy idea into economics and hence as a form of econophysics, we shall distinguish between two basic approaches. One may be labeled ontological while the other can be viewed as metaphorical, although some involved in this have at times confused these two such as the energeticists, Helm, Winiarski, and Ostwald, as described by Mirowski (1989a). In the ontological formulation, the foundation of the economy is seen being physical and biological processes driven by energy, with the Second Law of Thermodynamics serving as a key organizing principle for this foundation, with Georgescu-Roegen (1971) the most influential exponent of this idea, even as he was also a critic of it, as noted by Rosser Jr. (1991). This view follows more the tradition of Carnot and Clausius, but also depends on the work of Boltzmann as modified by Gibbs (1902) in statistical mechanics, with the Boltzmann-Gibbs formulation of the Law of Entropy. This approach has its greatest advocates among ecological economists, some of whom speak of this view as representing “biophysical economics” (Christensen 1989).

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[3] See Rosser Jr. (2016a) for further discussion of the development of the ergodic hypothesis and its relationship to economics. Rosser Jr. (2016b) considers the role of entropy in econophysics in more detail.

The metaphorical approach draws more on the information formulation of entropy due to Shannon and Weaver (1949), with applications across finance and equilibrium theory, many of these more closely tied to modern econophysics. Ultimately these two concepts of entropy share common mathematics of probability distributions of logarithms of products of possible states of the world, even as they have considerably different applications. While much of modern econophysics is more concerned with other matters such as power law distributions of variables, the entropy concept enters into many applications of econophysics, with important new approaches to economics relying on these more metaphorical formulations.[4]

4.6 Unity of the Core Entropy Concepts

The most widely used form of the Boltzmann equation for entropy is on his grave, although he never wrote it down in that way (Uffink 2014). It involves *W,* the thermodynamic probability of an aggregate state of a system of gas molecules, with *k* the Boltzmann constant, and *S* being entropy. It takes the form

$$S = k \ln W. \tag{4.4}$$

Given *N* microscopic states of the system, the probability of a gas molecule being in the ith state is N_i/N. *W* is then given by (Chakrabarti and Chakraborty 2006)

$$W = N!/\Pi N_i!. \tag{4.5}$$

This means that Boltzmann entropy can be rewritten as

$$S = k \ln\ (N!/\Pi N_i!). \tag{4.6}$$

Basic Shannon entropy is given by *H* of the probability distribution of states of informational uncertainty for message i. of $H(p_1 \ldots p_n)$. This then equals (Shannon and Weaver 1949; Renyi 1961)

$$H(p_1 \ldots p_n) = -k \Sigma p_i \ln p_i \tag{4.7}$$

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[4] We note that there are now a variety of extensions of the more basic Boltzmann-Gibbs and Shannon versions of entropy, including Renyi (1961), and Tsallis (1988) (this latter more closely tied to the study of power law distributions), with various efforts at generalizing these being made such as by Thurner and Hanel (2012). However, we shall not focus on these and note that most of these reduce to the simpler forms asymptotically as certain modifying parameters approach infinity, even as we recognize that they may well be useful for future applications. See Rosser Jr. (2016b) for further discussion.

Recognizing that $p_i = N_i/N$, the basic unity of these two concepts appear as N increases, which leads the Boltzmann formula in (4.6) to approach (Tsallis 1988; Thurner and Hanel 2012)

$$S = -kN\Sigma p_i \ln p_i \tag{4.8}$$

which means that in the limit as N approaches infinity, Boltzmann entropy is proportional to Shannon entropy.

4.7 Ontological Entropy and Econophysics as the Fundamental Limit to Growth

The ontological approach to econophysics derives from the direct and foundational role of energy in the economy, not merely for industrial production or providing for electricity or transportation, but at the ecological or biophysical level, that of solar energy driving the global biosphere. This is more a return to the Carnot and Clausius view of thermodynamics, where the continued incoming of solar energy shows the openness of the earth's system that allows it to avoid the law of entropy as long as the sun lasts (Georgescu-Roegen 1971; Rosser Jr. 1991).[5] However, that arriving solar energy itself is finite and thus provides a direct limit on economic activity that depends on the ecosystems through which the solar energy dissipates in the food chains that are driven by that energy. In addition, Georgescu-Roegen extended this argument to broader material resource inputs, arguing that they are also subject to a form of the law of entropy as well that provides further limits on the economy. More broadly for him (Georgescu-Roegen 1971, p. 281) "the economic process consists of a continuous transformation of low entropy into high entropy, that is, into *irrevocable waste*, or, with a topical term, into pollution."

While variations of this argument have become highly influential, especially in ecological economics as with Martinez-Allier (1987), it has faced sharp criticisms as well. Thus, Gerelli (1985) argues that the scale of the solar input is such that it is orders of magnitude beyond really limiting the world economy, with many other more mundane constraints more relevant in the short run. Nordhaus (1992) estimated entropy to be as many as 12 orders of magnitude below technology as a limit to growth, with Young (1994) weighing in similarly. In that regard the drawdown of stored energy sources and their limits such as with fossil fuels may be more relevant with the pollution from using them even more limiting as with such outcomes as

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[5] Georgescu-Roegen (1971) in particular strongly relied on the argument of Schrōdinger (1945, Chap. 6) regarding how life is ultimately an anti-entropic process based on organisms being open systems able to draw in both matter and energy while they live, with in this sense the death of organisms representing the ultimate victory of entropy. An alternative is to more directly follow Carnot and Clausius in emphasizing the role of the steam engine in the modern economy as in Cockshott et al. 2009).

climate change arising from the burning of such fuels releasing their stored carbon dioxide. Other critics have emphasized the limitless ingenuity of the human mind such as Julian Simon, who argued that (1981, p. 347) “those who view the relevant universe as unbounded view the second law of thermodynamics as irrelevant to the discussion.”

Another important figure in this line of argument was Alfred J. Lotka (1925), the father of the concept of predator-prey cycles. Lotka argued that the law of entropy is a deep driving force in evolution, a source of a teleological directedness of the process towards greater complexity. He saw this as the fundamental physical foundation of biology that needed to be studied mathematically, and he in turn saw the economy as deriving from the ecosystem as the more recent ecological economists have. Ironically Lotka was a tremendous influence on Paul Samuelson, who cited him prominently in his magnum opus, *Foundations of Economic Analysis* (1947), although more for his categorization of the stability conditions of linear systems rather than for his arguments regarding the law of entropy or its relation to the economy.

4.8 Ontological Entropy and the Energy View of Economic Value

Closely related to arguing that energy flows dissipating as the law of entropy operates are the foundation of the economy is the idea that either energy or some measure of entropy should be the basis for measuring value in an economy. This was first proposed by “energeticist” physicists of the late nineteenth and early twentieth centuries. Thus Helm (1887) and Winiarski (1900) argued that gold was “socio-biological energy.” Closer to the entropy argument was Ostwald (1908) whose view was that conversion factors based on the physical availability of specific forms of energy was the key to fundamental value determination. Extending this, Julius Davidson (1919) argued that the law of diminishing returns in economics[6] was ultimately based on the law of entropy. Much later Davis (1941) would argue that the utility of money was a form of “economic entropy,” although Lisman (1949) noted that this was not operationally equivalent to how the law of thermodynamics works in physics, and Samuelson (1972) simply dismissed these arguments as being “crackpot.”

Interestingly some of those who supported the idea of entropy playing a fundamental ontological role in economics also had issues with such approaches to value. Lotka (1925, p.355) noted that,

“The physical process is a typical case of ‘trigger action’ in which the ratio of energy set free to energy applied is subject to no restricting general law whatsoever

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[6] The law of diminishing (marginal) returns or productivity is probably the only so-called “law” in economics for which no counterexample has been found.

(e.g. a touch of the finger upon a switch may set off tons of dynamite). In contrast with the case of thermodynamics conversion factors, the proportionality factor is here determined by the particular mechanism employed."

Likewise for Georgescu-Roegen (1971), while he saw entropy as the ultimate limit to growth, he did not see it as all that useful for determining value, which he saw as ultimately coming from utility. Thus, nobody wants the low entropy poisonous mushroom and some people value more highly the high entropy beaten egg to the low entropy raw egg. These are matters of utility, and while Georgescu-Roegen did not see utility (or marginal utility to be more precise) as the sole source of value as did the subjectivist theorists of the Austrian School, he certainly saw it as very important and was a major developer of modern utility theory early in his career.[7]

4.9 Metaphorical Entropy and General Equilibrium Value

Moving to the heart of economics, entropy has been proposed as an alternative to the conventional Arrow-Debreu explanation of value. That standard view has equilibrium being a vector of prices that are fixed points. The entropic alternative recognizes the reality of a stochastic world in which equilibrium is better depicted as a probability distribution of prices as prices are never the same everywhere at any point in time for any commodity except as measure zero accident. An early expression of this idea is due to Hans Fōllmer (1974). A fuller development of this has been due to Foley (1994), later extended by Foley and Smith (2008).

The basic Foley (1994) model involves strong assumptions such as that all possible transactions within an economy have equal probability. However his solution involves a statistical distribution of behaviors in the economy where a particular transaction is inversely proportional to the exponential of its equilibrium entropy price, with this coming from a maximum Botlzmann-Gibbs entropy set of shadow prices. Walrasian general equilibrium is a special case of this model when "temperature" is zero. The more general form lacks the usual welfare implications, and it allows for the possibility of negative prices as in the case of Herodotus auctions (Baye et al. 2012).[8] However, Foley emphasizes the crucial role of constraints in this approach, something shared with the Arrow-Debreu model.

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[7] Rosser Jr. (2008a) provides further discussion of this debate.

[8] Herodotus described a marriage auction in Babylon with descending prices for potential brides. The most desirable would go for positive prices, but the auction allowed for negative prices for the least desirable potential brides. This contrasts with most societies where there is either a positive bride price or a positive groom price, more often described as a "dowry." The problem of negative prices is often obfuscated by declaring two separate markets, such as one to supply water when it is scarce and a different one to remove it when it is flooding. But the Babylonian bride market described by Herodotus makes it clear that there can be unified markets with both positive and negative prices.

Let there be m commodities, n agents of type k who achieve a transaction x of which there are is $h^k[x]$ proportion of agents type k out of r who do transaction x out of an offer set A, of which there are mn. *Multiplicity* of an assignment for n agents assigned to S actions, each of them s, is given by:

$$W[n_s] = n!/(n_1!\ldots n_s!\ldots n_S!). \tag{4.9}$$

Shannon entropy of this multiplicity is given by:

$$H\{h^k[x]\} = -\Sigma_{k=1}{}^r W^k \Sigma_{x\varepsilon Ak} h^k[x]\, x = 0. \tag{4.10}$$

Maximizing this entropic formulation subject to the appropriate feasibility constraints, which if non-empty, gives the unique canonical Gibbs solution:

$$H^k[x] = \exp[-\Pi x]/\Sigma_x \exp[-\Pi x], \tag{4.11}$$

where Π are vectors of the entropy shadow prices.

4.10 Entropy Between Econophysics and Sociophysics

Another metaphorical use of entropy concepts has been in conjunction with that close relative of econophysics, *sociophysics*. Initially coined by Galam et al. (1982), it follows the neologism *sociodynamics* as developed by Weidlich and Haag (1980). A major emphasis of this sociophysics is on modeling group dynamics including herding. A solution favored by Weidlich and Haag (1983) is the master equation, used especially for studying migration patterns, among other phenomena. When constraints do not uniquely solve the stochastic model of this equation, an nth order Markov process can emerge as the unique maximum entropy solution (Lee and Pressé 2012).

While not as developed as econophysics, sociophysics has followed its founding by Galam along with Weidlich and Haag along a variety of paths, with Chakrabarti et al. (2008) providing a fine overview of these investigations. Both the possibilities of applying the entropy concept to this approach have been studied in depth by Mimkes (2008), who also strives to extend his analysis to all of the social sciences. In his formulation we see a return to the question of ontological versus metaphorical applications of the entropy concept as Mimkes ties entropy to the fundamental nature of the production function. While this conjures up the vision of Georgescu-Roegen (1971) where the actual processes of the economy are fundamentally a working out of the Second Law of Thermodynamics, Mimkes eventually retreats to a more metaphorical application where it is the mathematical formulation of entropy as a descriptive device for data on distributional outcomes in the economy that is the prime focus of the analysis. While he invokes and implies the deeper ontological perception, the more metaphorical approach wins out in the end. However, there is no reason why a further developed sociophysics may not yet involve more seriously the ontological approach.

4.11 Metaphorical Entropic Financial Modeling

On the title page of his *Foundations of Economic Analysis* (1947), Paul Samuelson famously quoted Gibbs as saying, "Mathematics is a language." That it certainly is. But in the case of Shannon entropy, as well as financial models based on entropy mathematics, it is metaphor rather than linguistic ontology.

Drawing on much discussion from various econophysicists, Schinkus (2009) argues that econophysicists are more inclined than regular economists to approach data without preconceptions regarding distributions or parameter values, although they may be more inclined to draw on ideas from physics, with entropy among those in connection with financial modeling. Thus, Dionosio et al. (2009, p. 161) argue that:

> "Entropy is a measure of dispersion, uncertainty, disorder and diversification used in dynamic process, in statistics and information theory, and has been increasingly adopted in financial theory."

Applications of the law of entropy using Shannon entropy or Boltzmann-Gibbs distributions easily fit into explaining or modeling distributions that rely on lognormality, which are easily consistent with Gaussian approaches. While we know that ultimately these entropies are essentially identical mathematically, the real difference is that one we believe is driven to maximization as a law of physics whereas in the more metaphorical ones observing an extremum for entropy is simply a useful mathematical condition.

Someone drawing on both of the main measures of entropy in order to develop core financial theory in the form of the Black–Scholes options pricing formula (1973) is Michael J. Stutzer (1994, 2000). In the second of these he used Shannon entropy for his generalization of the link, after pointing out that Cozzolino and Zahneri (1973) had used Shannon entropy to derive lognormal stock price distributions, the same year that Black and Scholes (1973) published their result without directly relying on any entropy mathematics. For his generalization Stutzer (2000) posed the problem in discrete form as considering a stock market price process given by

$$\Delta p/p = \mu\Delta t + \sigma\surd\Delta \mathrm{t}\Delta \mathrm{z}, \tag{4.12}$$

where p is price, t is the time interval, and the second term on the right hand side is the random shock, with these distributed $\sim N(0, \Delta t)$. With Q as quantity, $r\Delta t$ the riskless rate of return, and P the actual conditional risk density distribution, a central focus is the conditional risk neutral given by $\mathrm{d}Q/\mathrm{d}P$.

From these one considers the relative entropy minimizing conditional risk neutral density that in effect maximizes order

$$\arg\min_{\mathrm{d}Q/\mathrm{d}P} \int \log\ \mathrm{d}Q/dP\, dQ, \tag{4.13}$$

subject to a martingale restriction given by

$$r\Delta t - E[(\Delta p/p)(dQ/dP] = 0, \tag{4.14}$$

From this he shows that when asset returns are IID with normally distributed shocks as given above, the martingale product density formed from the relative entropy minimizing conditional risk is that used to calculate the Black-Scholes option pricing formula. He recognizes that this does not easily generalize to non-Gaussian distributions such as the power law ones much studied by econophysicists, suggesting a weaker approach using Generalized Auto Regressive Conditional Heteroskedastic (GARCH) processes.

4.12 More Metaphor, the Anti-Entropic Minsky Bubble Process

As discussed above, entropy maximization implies Gaussian stochastic dynamics. These are not consistent with power law distributions seen in financial market returns or in wealth distributions. A likely source for this difference is the tendency to anti-entropic bubble dynamics that can be described by the Minsky process (Minsky 1972, 1982; Kindleberger 2001; Rosser Jr. 1991). Rather than evening out irregularities, a speculative bubble can heighten deviations from long run equilibrium outcomes, whether of a stochastic entropic sort as modeled by Foley and others or a deterministic Walrasian general equilibrium. Positive feedback dynamics arising from momentum or noise traders drive prices to extremes away from these equilibria temporarily, generally ending with some sort of crash. These extreme movements lead to the kurtotic fat tails that appear in financial asset return dynamics so ubiquitously (Lux 2009).

Minsky (1972) argued that these dynamics emerge endogenously through psychological mechanisms wherein agents become complacent regarding risk during periods of entropic equilibrium with Gaussian distributions predominating in response to exogenous shocks. They proceed through stages of increasing risk taking, wherein leverage ratios rise and bubbles emerge. The final stage of this process involves Ponzi dynamics that have become unhinged from fundamentals fully. Wealth has risen dramatically with the speculative bubble prices, but in the end the bubble crashes, usually in a dramatic Minsky Moment when panic takes over and agents sell the asset en masse (Kindleberger 1972). With this the dynamic returns prices into the longer run entropic equilibrium zone, the “Revenge of Entropy.”

It has been well understood that such dynamics have historically generally taken one of three different forms (Rosser Jr. 1991, Chap. 5). All three of these cases are shown in this book in Figs. 2.1, 2.2, 2.3 (Rosser Jr. et al. 2012) for assets that exhibited each of them during the period of the Great Recession of 2007–2009, although the dynamics are shown for a longer time, with one of them (US housing) peaking prior to the broader financial crash that ushered in the Great Recession.

The first case involves a price that rises in an accelerating fashion, only to suddenly crash after peaking in a dramatic Minsky Moment. For the period of the financial crisis, this is well exhibited by oil prices, which peaked in July, 2008 at $147 per barrel only to decline sharply to around $30 per barrel in November, 2008 see Fig. 2.1, this book). Such patterns are often seen in commodity price speculative bubbles

The second case involves a more gradual rise in prices that then declines also in a gradual way after the peak is reached. Such a case can be argued not to possess a Minsky Moment proper in the sense of a sudden crash associated with a panic, although there may be panicky emotions involved for agents in such a bubble and its decline. The example from the financial crisis is that if US housing, whose prices began to rise in 1998 and then peaked in 2006, declining thereafter for several years as shown in Fig. 2.2 (this book). Indeed real estate seems more prone to exhibit such a pattern, and one explanation that seems to hold especially for residential real estate is that people refuse to sell immediately in the downturn, believing that the prices are "unfair" and "too low," leading them to rent out their housing if they must move or simply refusing to sell. Such patterns thus tend to show a fall in volume of sales during the decline more than a rapid decline in price, which falls as eventually people give up and accept the lower prices.

The third case is historically the most common as documented by Kindleberger (2001, Appendix B). It involves prices rising to a peak, then declining for awhile in a gradual way during a "period of financial distress" (Minsky 1972), then at a later time experiencing the Minsky Moment and crashing hard. During the financial crisis most financial asset markets showed this pattern, with Fig. 2.3 (this book) showing the example of the US stock market as measured by the Dow-Jones Industrial Average. It peaked in October, 2007, but then crashed in September, 2008, a full 11 months later after going through a period of a more erratic decline.

Such dynamics cannot be modeled by assuming homogeneous agents. At the peak, smart or lucky "insiders" sell out to less smart or lucky "outsiders" who continue to hang on to the asset, much as the homeowners in the second case refuse to sell their homes initially as the price declines. The Minsky Moment finally arrives when panic hits this group of agents and they sell en masse in the crash. Even though this is by far the most common pattern of speculative bubbles in history, there have been few efforts to model this. Such an effort was made by Gallegati et al. (2011) in an agent-based model ultimately derived from the Brock-Hommes approach (Brock and Hommes 1997). In this framework behavior of heterogeneous agents is qualitatively determined by a contagion parameter and a willingness to change behavior parameter. The mechanism of the delayed crash after the period of financial distress came from a wealth constraint such as agents encounter in asset markets with margin calls. When price falls below a certain level they may be forced to sell. Figure 2.4

(this book) shows a simulation that shows the general pattern and also shows the impact of an increase in the strength of the contagion parameter, which moves the peak higher and delays it slightly (Gallegati et al. 2011).

4.13 Modeling Wealth and Income Distribution Dynamics Using Statistical Mechanics

Studying wealth and income distribution dynamics we find that the relationship between entropy-based non-power law distributions and power law distributions plays a central role in the modeling of these dynamical systems. In particular it increasingly looks as if while wealth dynamics largely reflect power law distributions, income distribution dynamics may be a combination, with entropy-related Boltzmann-Gibbs distributions best explaining income distribution for the poorest 97–98 percent, whereas a Pareto power law distribution may do better for the top level of income, where wealth dynamics may play a more important role (Drăgulescu and Yakovenko 2001; Yakovenko and Rosser Jr. 2009).

Awareness of the possibility of using entropy ideas in the measurement of income distribution began with economists looking for generalizations of the various competing measures that have been used for studying income distributions. Thus in 1981, Cowell and Kugal (1981) sought a generalized axiomatic formulation for additive measures of income distribution. They found that by adding two axioms to the usual approach they were able to show that a generalized entropy approach could subsume the widely studied Atkinson measure (1970) and Theil measure (Bourguignon 1979). While the Atkinson measure has been more widely used and is able to distinguish skewness of tails, the Theil may have more generality. Bourguignon (1979) shows that it is the only decomposable "income-weighted" inequality measure that is zero homogeneous. Cowell and Kugal (1981) show that adding a sensitivity axiom to their others yields the Theil index as the only one that is derivable from a generalized entropy concept.

These early discussions also involved strong claims regarding the difficulties of linking entropy measures with power law distributions, claims that now look to be overdone to some extent. Thus we find Montroll and Schlesinger (1983, p. 209) claiming that:

> "The derivation of distributions with inverse power tails from maximum entropy formalism would be a consequence only of an unconventional auxiliary condition that involves the specification of the average of a complicated logarithmic function."

This statement may be overdone, although indeed logarithmic functions are involved in the relationship between the two, which is not surprising given that entropy measures are essentially logarithmic.

The power law distribution approach dominates discussion of wealth distribution dynamics, as it does financial market dynamics. The father of this approach was Vilfredo Pareto (1897), who was initially trained as an engineer, but then became a socio-economist as his theory involved the relationship between social classes over time. Very appropriately Pareto's original motivation and focus of study was in fact income distribution. He claimed a universal truth associated with an estimated income distribution parameter. He was wrong, especially given that his theory fits better wealth distributions rather than income distributions, as noted above. Pareto argued incorrectly that his supposedly universal coefficient for the power law explanation of income distribution fit into his theory of the "circulation of the elites," in which nothing could be done to equalize income because the political process would simply involve substituting one power elite for another with no noticeable change in the income distribution. But we must recognize that he formulated this view at the end of the nineteenth century, when there had been a century of no major changes in the socio-economic structure anywhere. Needless to say, not too long afterwards there were large changes in the distribution (Piketty 2014), even as his method went "underground," only to be revived for other uses such as describing urban metropolitan size distributions (Auerbach 1913).

The modern concern with income distribution based on power law physics concepts from Pareto was due to a sociologist, John Angle (1986). After the appearance of current econophysics, many stepped forward to apply power law distributions to study the dynamics of wealth distributions. Drawing on the work of Pareto, who mistakenly thought he had found a universal coefficient for income distribution, econophysicists found that current wealth distributions fit Pareto's power law view (Bouchaud and Mézard 2000; Chakraborti and Chakrabarti 2000; Solomon and Richmond 2002).

At this point the question needs to be considered as to whether we are dealing with ontological as opposed to "merely" metaphorical models in these matters. We know that there are stochastic tendencies for wealth and income dynamics, but it is not at all obvious that the various apparent imperatives for entropy maximization or minimization are actually driving outcomes. Nevertheless many studying these matters see thermodynamical processes underlying basic tendencies of wealth and income distribution dynamics. Such processes are not quite as direct as the ontological direction based on Carnot's steam engines, but derive from broader tendencies of wealth and income distribution dynamics occurring in the absence of substantial changes in public policy regarding distributional policies.

Pareto was mistaken in his original proposal. He thought that he had found a universal law of income distribution that fit with his theory of the "circulation of the elites," within which it did not matter which elite group was ruling society, the underlying distribution of income would not change. He was wrong. The legacy of his approach has been in the study of wealth distributions, where his presentation of power laws is now understood to explain wealth distributions rather than income distributions.

The Pareto distribution is given by:

$$N = A/x^{\alpha}, \tag{4.15}$$

where N is the number of observations above x, and A and α are constants. This includes as special cases a wide variety of other forms that underlie many econophysics models. The special case when $\alpha = 1$ leads to "Zipf's Law," (Zipf

1941), widely viewed to describe urban size distributions as well as many others, although how far this "law" applies is a matter of ongoing debate.

Yakovenko and Rosser Jr. (2009) present a unified income distribution analysis combining an entropic Boltzmann-Gibbs formulation for lower income distribution with a Paretian power law distribution for the highest levels of income. The model makes a heroic assumption of conservation of money or income or wealth, which empirically is not unreasonable for the United States since the mid-1970s for median levels, even as the top strata have seen growing levels. But this fits with the combination of a lognormal entropic model for the majority of the population with regard to income, even as the top level of the income distribution seems to follow a wealth dynamic following a Paretian power law distribution.

Assuming a conservation of money, *m*, the entropically based Boltzmann-Gibbs equilibrium distribution is given by the probability, *P*, that the level will be *m*, given by:

$$P(m) = ce^{-m/T_m}, \tag{4.16}$$

where *c* is a normalizing constant, and T_m is the "money temperature" in thermodynamic terms, which is equal to the money supply per capita. This describes the lower portion of the income distribution.

Assuming a fixed rate of proportional money transfers with this equal to γ, the stationary distribution of money (income) is related to the Gamma distribution form that differs from the Boltzmann-Gibbs by having a power-law prefactor, m^{β},where

$$\beta = -1 - \ln 2/\ln(1-\gamma). \tag{4.17}$$

This relates the Boltzmann-Gibbs form to a power law equivalent more simply than supposed by Montroll and Schlesinger (1983). This formulation that shows the connection between the two conceptualizations of wealth and income distributions is given by:

$$P(m) = cm^{\beta}e^{-m/T}. \tag{4.18}$$

This represents the stationary distribution, but allowing *m* to grow stochastically disconnects the outcome from the maximum entropy solution (Huang 2004). The stationary distribution under these conditions becomes a mean-field case governed by a Fokker-Planck equation, which is neither Boltzmann-Gibbs nor Gamma, but is a version of a generalized Lotka-Volterra distribution, with *w* the wealth per person, *J* is the average transfer between agents, and σ is the standard deviation, and is

$$P(w) = c\left[\left(e^{-J/\sigma\sigma w}\right)/\left(w^{2+J/\sigma\sigma}\right)\right]. \tag{4.19}$$

So it is possible to combine an entropic Boltzmann-Gibbs formulation for the lower part of the income distribution with a power law form for its upper end, which corresponds to the wealth dynamics formulation deriving ironically from Pareto,

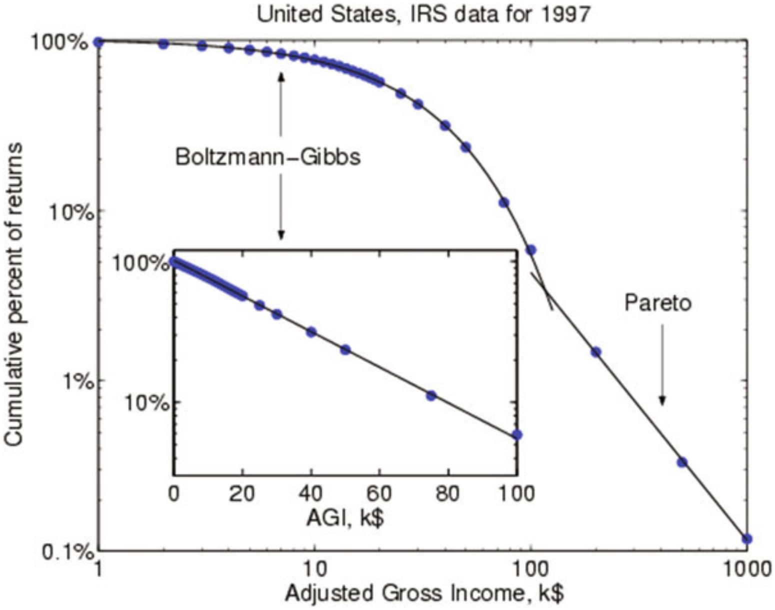

Fig. 4.1 Log-log United States income distribution, Boltzmann-Gibbs and Pareto sections, 1997, from Yakovenko (2013, Fig. 5)

given that he originally thought his conceptualization was a universal law of income distribution. His formulation would be countered soon after by Bachelier (1900), but we now see the two conjoined to provide an empirical explanation of income distribution that has deep roots in Marxist and other classical economic formulations regarding socio-economic class dynamics (Cockshott et al. 2009; Shaikh 2016; Shaikh and Jacobo 2020).

Figure 4.1 shows such a distribution in its log-log form for the US income distribution in 1997, with the Boltzmann-Gibbs portion, covering the lower 97 percent of the income distribution being nonlinear on the left-hand side, while the Paretian portion is linear in logs on the right-hand side covering the top 3 percent of the income distribution (Yakovenko 2013, Fig. 5).

4.14 Crashing Bubbles and the Revenge of Metaphorical Entropy

We now consider more specifically how the financial market dynamics interact with the income and wealth distribution dynamics in the course of speculative bubbles following a Minsky process. A notable aspect of a major bubble is that it raises the wealth and income of the top portion of the income and wealth distribution hierarchy

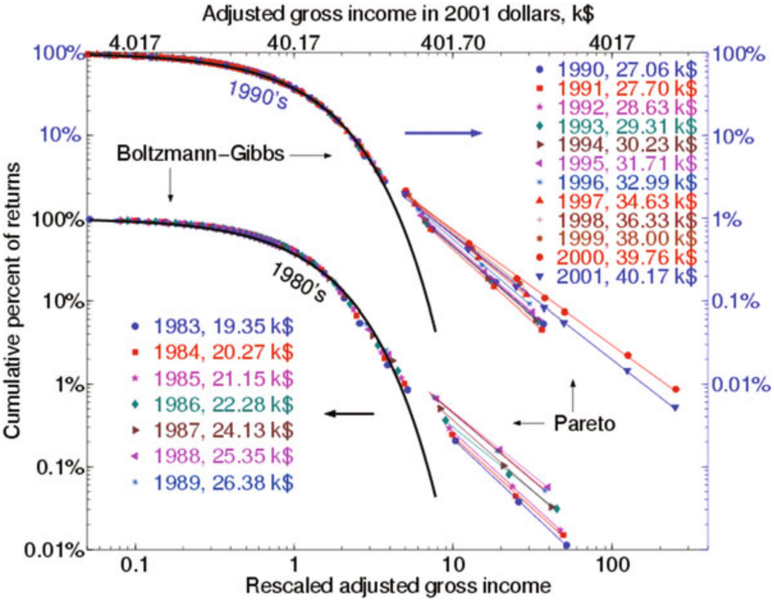

Fig. 4.2 Annual log–log US income distribution, 1983–2001, from Yakovenko (2013, Fig. 6)

compared to the rest. This is associated with the anti-entropic dynamics of the process and is reversed when the bubble disappears in a crash, the "revenge of entropy."[9] This should show up during a bubble as an upward movement of the Paretian portion that will also move its boundary with the Boltzmann-Gibbs portion of the distribution to the left.

We do not have the data for the most recent financial crisis, nor do we have it for the Great Depression, another period that followed a major financial crash that has been posited to have sharply reduced wealth and somewhat equalized the income distribution, although wealth levels did decline substantially, the Great Depression bringing about the end of the "Gilded Age" (Smeeding 2012). Events during the 2007–2009 Great Recession are more complicated in part because several different bubbles were involved, with the crash of the housing bubble heavily impacting the middle class while the stock market and derivatives market crashes more heavily affected the wealthy. Thus at its bottom point in 2009, the US stock market had fallen by more than half its value. Total wealth declined by the end of 2009 by 50 percent. Of this, wealth for the top 10 percent fell by 13 percent while the wealth of the top 1 percent fell by 20 percent (Smeeding 2012). However, the stock market recovered rather rapidly, more so than in the 1930s or even after 2000, whereas the US housing market recovered much more slowly, thus leading to an outcome where while wealth inequality was probably reduced for a period of time during

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[9] See Rosser Jr. (2020c) for further discussion.

2008–2009, it almost certainly rose after that as those at the top gained from the recovery of the stock market while those in the middle were held back by the continuing problems in the US housing market. The Minsky process was at work, but in a more complicated manner than in some other historical situations.

However, supporting evidence, if weak, can be seen from considering the end of the dotcom bubble in 2000. This can be seen in Fig. 4.2 (Yakovenko 2013, Fig. 6), which shows the log–log relation for US income distribution for the years 1983–2001. In general, one sees little movement of the Boltzmann-Gibbs portion, but small annual changes of the Paretian part, reflecting steadily increasing inequality over time. However, there is one exception in this figure, what happened between 2000 and 2001, the last years shown, with 2000 the end of the dotcom bubble. In this case we see a reversal, with the 2001 Paretian portion lying below the 2000 portions. This would be consistent with our story of a revenge of entropy following the crash of the fairly substantial dotcom bubble of the late 1990s.

Chapter 5
Econophysics and Entropy in Dynamically Complex Urban/Regional Systems

> "…classical thermodynamics is…the only physical theory of universal content which I am convinced, within the framework of the framework of applicability of its basic concepts will never be overthrown."—Albert Einstein, quoted in Rifkin (1981, p. 44)

5.1 Opening Observations

Since at least the early efforts of Alan Wilson (1967, 1970), the idea of using the law of entropy to assist in modeling the development of urban and regional spatial structural patterns has been influential. To understand how this has been done and how useful it is as an approach, we must first consider the various formulations of that law that have been made. The full development of the idea is associated with the second law of thermodynamics due principally to Boltzmann (1884), although drawing on earlier work by Carnot (1824) and Clausius (1867). Jaynes (1957) prepared this approach for application in economics with Georgescu-Roegen (1971) also providing a deep perspective. Later, Shannon (1948) would extend this to the study of information patterns. Rosser Jr. (2016b) argues that within economic systems the former is most appropriate when ontological thermodynamic forces are objectively driving the dynamics of a system. The latter is more important as a metaphorical tool when a similar mathematical pattern arises.[1]

A way in which the first may generate structural patterns is through the operation of energy in the system, given that the second law of thermodynamics is about how energy dissipates through closed systems. Energy is crucial in transportation, so it should not be surprising that as transportation costs enter into determining such

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[1] Purvis et al. (2019) argue for a third type of entropy, "figurative," which suggests an increasing disorder or randomness. However, here this form will be considered to be subsumed into the other two, especially in the first Samuelson (1972) provides a critique of some uses of entropy in economic models as well as Kovalev (2016).

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_5

spatial patterns we might see the law of entropy in its objective form as relevant to shaping such patterns, and indeed, transportation costs have been seen as central in shaping urban and regional spatial patterns dating back to von Thūnen (1826). Drawing on a proposal by Reilly (1931) and work by Weaver (1948). Wilson (1967, 1970, 2010) would use the assumption of minimizing transportation costs to model a complex system of spatial distribution of rent-maximizing activities. Another early effort along similar lines was due to Medvekov (1967).

Many applications of entropy or urban and regional models would follow the metaphorical approach based on the Shannon's (1948) information entropy. An early effort along these lines was due to Chapman (1970) for a model of spatial concentration or dispersion of activities and also Batty (1976). Likewise this has underpinned models of urban sprawl (Cabral et al. 2013). Indexes of degrees of racial segregation have been based on such measures (Mora and Ruiz-Castillo, 2011). Likewise, measures for land-use diversity have been based on such entropy (Walsh and Webber, 1977).

Rather returning to fundamental thermodynamic formulation have been efforts to model ecological sustainability of urban and regional systems based on their patterns of energy usage. Assessing carbon footprints is due to Wackernagel and Rees (1996). More direct applications including using the concept of exergy are due to Balocco, Paeschi, Grazzini, and Basosi (2000). Marchinetti, Putselli, and Tierzi (2006) consider such models under within the complex systems dynamics of dissipative systems (Prigogine 1980).

An alternative stresses anti-entropic forces associated with agglomeration for modeling patterns of urban hierarchy[2] reflecting power law distributions initiated by Pareto (1897), supported by Singer (1936) and Gabaix (1999). A special case is the rank-size rule due to Auerbach (1913) and Zipf (1941), supported by Batten (2001), Nitsch (2005), and Berry and Okulicz-Kozaryn (2012).

Finally models of complex self-organization of urban and regional structure reflecting interactions between entropic and anti-entropic elements have been developed by many including Papageorgiou and Smith (1983), Weidlich and Haag (1987), Krugman (1996), Portugali (1999), Gabaix and Ioannides, 2004), and Rosser Jr. (2011a). These interactions can trigger the irregularities in dynamic paths that mark dynamically complex systems, which urban and regional systems surely are.

The law of entropy, or second law of thermodynamics, thus becomes that in a closed system entropy increases, which was first formulated by Clausius (1867), who also stated the classical fist law of thermodynamics that in a closed system the amount of energy is constant, with this more fully developed by Ludwig Boltzmann (1884). The inspiration for this development came from the study of steam engines by Sadi Carnot (1824). He made the initial crucial observation of the first law, which would be crucial to understanding the impossibility of a perpetual motion machine. Carnot formulated that the work of a steam engine came from the transformation of

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[2] Formal modeling of agglomeration in urban systems is due to Fujita (1988) and Krugman (1991) based on Dixit and Stiglitz (1977). See also Fujita et al. (1999).

heat energy from a hotter source to a cooler sink and recognized a maximum efficiency for this transformation.[3] It was from this understanding that Clausius derived his conceptualization, later adumbrated by Boltzmann.

The most important variation of this would the metaphorical one measuring informational entropy due to Shannon (1948) and Shannon and Weaver (1949). While these two forms of entropy apply to very different situations with no ontological law of entropy operating with regard to Shannon's metaphorical informational entropy, they are fundamentally related.[4]

This fundamental unity extends to later variations and generalizations of the entropy concept as developed by Renyi (1961), Tsallis (1968), and Thurner and Hanel (2012). This latter links to a development in Russia of the "new entropy" that links to ergodicity theory where entropy is seen as an isomorphism between Bernoulli states (Kolmogorov, 1958; Sinai, 1959; Ornstein, 1970).

5.2 The Wilson Model

The most influential modeler of urban and regional systems to use the concept of entropy has been Sir Alan G. Wilson (1967, 1969, 1970, 2000, 2010). His original main model was of the spatial distribution of flows of retail activity, based on a model of Reilly (1931). The space is partitioned by origins I and destinations j (often a central place) so that S_{ij} is a matrix of money flows from origins I to retails sites j. Then the entropy to be maximized subject to budget constraints of the flows is given by

$$\mathrm{Max}\, S = -\Sigma S_{ij}\ \ln S_{ij}, \tag{5.1}$$

where for benefits of a retail site given by W_j and costs of going from an origin to a retail site given by c_{ij} this will give a rent-maximizing spatial distribution

$$S = \Sigma W_j \exp\left(c_{ij}\right). \tag{5.2}$$

This then could be further modified by specifying more activities with population levels and types of retail outlets. In principle this is broadly consistent with the

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[3] Carnot's book was long hard to find a copy of and long had little direct influence on the development of steam engines, although eventually its implication that having a greater difference in temperature between the source and the sink could increase the efficiency of such engines was acted on by people such as Joseph Diesel in developing improved steam engines (Georgescu-Roegen, 1971).

[4] The distinction between ontological and metaphorical entropy is due to Rosser Jr. (2016b) and discussed in the previous chapter. Lotka (1922) argued that evolution is fundamentally driven by an ontological thermodynamic process based on the law of entropy. Brooks et al. (1989) see metaphorical information entropy as useful for understanding biological evolution.

original von Thūnen (1826) model of ring-patterned rent around a central place, although Wilson rarely stressed this point.

This basic model due to Wilson has since gone through many modifications and extensions, including many by Wilson himself, often with various coauthors. Thus while Wilson originally assumed that transport costs grow linearly with the log of benefits, both may be logarithmic, which might be true for a model of long trips involved in interurban transport, with other functional forms possible as constraints get adjusted accordingly (Haynes and Phillips, 1987).

The model has also been extended to various other applications. Thus Rees and Wilson (1976) and Rogers (2008) placed this into models of migration flows. Straussfogel (1991) used it in studies of suburbanization. In models of trade flows, input-output relations can be introduced into integrated models (Kim et al. 1983; Roy and Flood, 1992).

While the basic model assumed discrete zones, Angel and Hyman (1976) extended entropy-maximizing to continuous space representations. Problems of empirical estimation arise in connection with aggregation and spatial structure in models of spatial interaction (Batty and Skildar, 1982). Econometric models of spatial autocorrelation in this framework have been developed (Berry et al., 2008) as well as broader forms of spatial interaction (Fischer and Griffith, 2008).

Greater emphasis on a metaphoric Shannon information entropy approach was due to Snickars and Weibull (1977). Fotheringham (1983) applied this for the case of competing destination zones. Smith and Hsieh (1997) introduced a Markov equivalent. Anas (1984) links utility maximization and entropy maximization in these models using a multinomial logit model. Wilson (2010) argues that these approaches are consistent with the "disorganized complexity" interpretation of Shannon's information entropy approach as posited by Weaver (1948). This contrasts with the initial approach of Wilson (1967, 1970) that pursued an entropy approach drawing more on Botlzmann.

A substantial expansion of this framework within the Boltzmann framework was due to Harris and Wilson (1978) who introduced slow dynamics into the model. This took the form of introducing elements derived from Lotka (1925) and Volterra (1938), with Wilson (2008) labeling the result of this combination of Boltzmann, Lotka, and Volterra the "BLV approach." The slow dynamics allow for growth depending on the profitability of given locations, with the related fast dynamics being shorter term equilibrium adjustment dynamics. This setup provided a basis for considering models of catastrophic bifurcations and cascades (Wilson, 1981; Batty, 2009) as well as chaotic dynamics (May, 1973; Rosser Jr., 1991).[5]

This would eventually lead to a broader consideration of how the Wilson model fits into a broader complexity framework, especially linking with Weaver's (1948) distinction between organized and disorganized forms of complexity. For this, entropy can be seen as providing a key organizing principle (Wilson, 2006) relying

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[5] For alternative systems providing similar possibilities see Allen and Sanglier (1979) and Nijkamp and Reggiani (1988), with Rosser Jr. (2011a) providing a broad overview.

on the BLV approach. This has even been proposed to provide an explanation of how entropic based models of lower level flows can provide a foundation for scale free power law distributions of the distributions of settlement area sizes (Dearden and Wilson, 2009), which we shall consider below as associated with anti-entropic organizational principles.

5.3 Variations on Entropic Spatial Distribution Models

While Wilson's work inspired a large effort by many people as seen in the previous section, others also used various entropic measures to study spatial distributions in urban and regional systems of various things. One line of research was inspired by applying the Theil (1972) index, which is based on the Shannon information entropy measure. Among the first to do so was Batty (1974). The basic spatial version of the Theil index where H is the index, n is the number of zones, and p_i is the probability that variable x appears in zone I, is given by

$$H_n = [\Sigma p_i \log(1/p_i)]/\log n. \tag{5.3}$$

This entropy measure can vary from 0 to 1, with the latter indicating a fully equal distribution across the spatial zones, at maximum entropy, and 0 indicating a total concentration in one zone, or a maximum degree of inequality and anti-entropy. This index has been widely applied across many social and natural sciences.

Batty's (1974) variation of this, which he called *spatial entropy*, involves considering what happens as the size of the zones shrinks, also implying an increasing number of them. If Δx_i is zone size then the Batty spatial entropy index is given by

$$H = (\lim \Delta x_i \to 0) - \Sigma p_i \log(p_i/\Delta x_i). \tag{5.4}$$

This formulation is very similar to one proposed by Bailey (1990) for measuring *social entropy*, with again the focus on degrees of similarity or equality across social groups or zones.

Among the more direct applications of this for urban systems has been in studying urban sprawl (Cabral et al. 2013). One line has been to measure the degree of fragmentation of ownership. Miceli and Sirmans (2007) argues that this discourages development as real estate developers prefer less dispersed patterns of ownership. Scattered patterns associated with urban sprawl lead to a form of monopoly power that manifests itself through the holdout problem. More broadly urban sprawl is seen as contributing to a variety of social and environmental problems, with higher costs of infrastructure and even greater public health problems (Brueckner, 2000; Nechyba and Walsh, 2004; Frenkel and Ashkenazi, 2007).

While most observers see urban sprawl as posing major problems, it has its defenders. Thus Wassmer (2008) argues that sprawl increases satisfaction with housing and schools, lower crime rates, and greater convenience of car travel, although the latter is a target of those arguing sprawl exacerbates environmental problems. Cabral et al. (2013) see this as a matter of tradeoffs. Higher spatial entropy levels are demanding for transport and infrastructure, while lower levels increase levels of inequality and social economic fragmentation.

Unsurprisingly information entropy measures have been used to measure degrees of racial segregation in urban areas for both residences and schools (Mora and Ruiz-Castillo, 2011). While probably the most commonly used measured in these studies is the Theil index shown in Eq. (5.3) above and first proposed for studying school segregation by Theil and Finizza (1971), with applications such as studying segregation in the San Francisco Bay area (Miller and Quigley, 1990). However, Mora and Ruiz-Castillo argue for the superiority of the de-normalized form of this known as the mutual information index, also due to Theil (1971), which may be more useful for studying decomposability by schools.

Yet further spatial applications include measuring diversity of land use patterns (Walsh and Webber, 1977) and spatial settlement distributions (Medvekov, 1967) as well as spatial patterns of population distribution (Chapman, 1970). Purvis et al. (2019) provide an overview of many of these applications.

5.4 Thermodynamically Sustainable Urban/Regional Systems

Most of the models discussed in the previous two sections have relied on the metaphorical information concept of entropy coming from Shannon and Weaver, with the possible exception of Wilson's development of slow dynamics that draws more directly on Boltzmann. However, another strand of entropic analysis of urban and regional systems relies more on the original ontological approach in which an urban or regional system is seen as being driven by thermodynamics in its original physical sense involving energy transfers and transformations following the Second Law of Thermodynamics. Among those pursuing such an approach have been Rees (1992), Balocco et al. (2004), Zhang et al. (2006), Marchinetti, Pulselli, and Tierzi (2006), and Purvis et al. (2019).

The focus of most of this research is particularly on the ecological sustainability of urban and regional systems, with viewing them as open dissipative systems experiences inflows and outflows of energy and materials (Georgescu-Roegen, 1971; Prigogine, 1980). While for closed systems entropy increases, with open systems entropy can either increase or decrease if energy and materials flow into the system. This was indeed the Schrōdinger (1945) argument about life, that it involves an anti-entropic process whereby living things draw in energy and create order and structure as long as they live. A specific term for anti-entropy is *exergy* (Rant, 1956).

Let us then distinguish three concepts: total entropy or S_{total}, inside entropy or S_i, and outside entropy or S_{o}. These are related dynamically according to

$$\mathrm{d}S_{\text{total}}/\mathrm{dt} = \mathrm{d}S_i/\mathrm{dt} + \mathrm{d}S_{\text{o}}/\mathrm{d}t, \text{with } \mathrm{d}S_i/\mathrm{d}t > 0. \tag{5.5}$$

However, $\mathrm{d}S_o/\mathrm{d}t$ can be either positive or negative, so if it is negative and has an absolute value exceeding that of the absolute value exceeding that of S_i, then total entropy may decline as the system generates order as it draws in energy and materials, only to export them as waste and disorder, with entropy increasing outside the system. As Wackernagel and Rees (1996) put it, "Cities are entropic black holes," with this raising serious questions about their sustainability as they generate large ecological footprints.

Exergy is often defined as being the maximum amount of useful work possible to reach a maximum entropy state, which means it must be zero if a maximum entropy state is achieved. Rant's (1956) original formulation was in the context of chemical engineering. If B is exergy, U is internal energy, P is pressure, V is volume, T is temperature, S is entropy, μ_i is the chemical potential of component i, and N_i is the moles of component i, then Rant's formulation is given by

$$B = U + PV - TS - \Sigma\mu_i N_i. \tag{5.6}$$

This implies, ceteris paribus, that

$$\mathrm{d}B/\mathrm{d}t \leq 0 \leftrightarrow \mathrm{d}S/\mathrm{d}t \geq 0, \tag{5.7}$$

which highlights the interpretation of exergy as being anti-entropy.[6]

An application of this using a modification of Rant's equation due to Moran and Sciubba (1994) has been done by Balocco et al. (2004). They study the exergy involved in building construction and real depreciation in the town of Castelnuovo Beardenga near Siena, Italy. This involves also using input-output relations involved with the construction industry. They conclude that more recent buildings are not as efficient as older ones, with those built in 1946–1960 providing the highest sustainability.

Following both Wackernagel and Rees as well as Balocco, Paeschi, Grazzini, and Basosi, and also Haken (1988) and Svirizhev (2000), Zhang et al. (2006) engage in an ambitious effort to apply entropy concepts to the study of sustainable development of Ningbo, China, a city of nearly 6 million somewhat south of Shanghai in Zhejiang province. Their effort combines both ontological measures of entropy as well as metaphoric information ones as they break their analysis into four parts. The first two are tied to development and are *sustaining input entropy* and *imposed output energy*, which are basically determined by production. The second two are considered to be part of the metabolism of the urban system, *regenerative metabolism* and *destructive metabolism,* which are tied to the generation of pollution and its cleanup. This becomes a measure of harmony with the environment. The outcome of the first gives the developmental degree while the second gives the harmony degree. They estimate these for the 1996–2003 period and find that these two measures were generally going in opposite directions, with the developmental degree rising

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[6] It is also sometimes known as *negentropy*, for "negative entropy."

(associated with declining entropy) as the harmony degree was declining (associated with rising entropy). This poses the problem of sustainability of urban development in China quite sharply.

Marchinetti, Pulselli, and Tierzi (2006)[7] consider this approach from a more general level, drawing on ideas due to Morin (1995) regarding autonomy versus dependence of systems on their environment, while using the dissipative structures approach of open systems associated with Prigogine (1980). They see urban systems evolving between extremes of autarchy and globalization. However, they argue that in the end neither of these extremes is sustainable, In their advocacy of a balanced path they emphasize how urban and regional systems are ecosystems that operate on the basis of energy flows (Odum, 1969) within a set of complex wholes emerging from a set of interacting micro-level components (Ulanowicz, 2012).

5.5 Anti-Entropic Processes in Urban/Regional Systems

Pushing against this entropic version of the structure of urban and regional systems is a power law version of such structuring, at least for certain cases and situations. Arguably this is dealt with in the entropy framework, given the matter of the balance between exergy and entropy in urban and regional systems. Most of the systems and measures up until now have involved essentially internal relations or distributions within urban or regional systems. But when one considers higher level distributional systems the entropy relation may break down or even become completely irrelevant.

One way that anti-entropic forces can manifest themselves is by the appearance of power law distributions (Rosser Jr., 2016b), with substantial evidence that city sizes may follow such distributions (Gabaix, 1999). Pareto (1897) identified the concept of power law distributions. For P is population, r is rank, and A and α are constants, then

$$rPr^{\alpha} = \mathrm{A}, \tag{5.8}$$

which can be put into log–log form, which is linear,

$$\ln r = \ln \mathrm{A} - \alpha(\ln Pr). \tag{5.9}$$

We note that for the special case of $\alpha = 1$, the population of entity of rank r becomes

$$P_r = P_1/r, \tag{5.10}$$

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[7] Ironically Marchettini and coauthors are in the same institute at the University of Siena as Balocco and coauthors, but neither group cites the work of the other.

Which was labeled the *rank-size rule* by Auerbach (1913) and would later come to be known as Zipf's Law, argued to hold for many distributions (Zipf, 1941).[8]

The issue of whether or not city size distributions follow Zipf's Law and thus obey the rank-size rule has been a matter of ongoing debate since Auerbach (1913) first proposed it and Lotka (1925) questioned it. Some, especially urban geographers (Berry and Okulicz-Kozaryn, 2012) have argued that it is a universal law. Others, more often economists, have questioned it, arguing that there is no clear reason why it should be followed, even if city sizes may well exhibit power law distributions (Batten, 2001; Fujita et al. 1999), although Gabaix (1999) argues that Zipf's Law arises in the limit if Gibrat's Law holds that growth rates are independent of city sizes.

Batten (2001) in particular shows US city size distributions exhibiting power law distributions from 1790 to the present, even if not exactly the rank-size rule (with the fact that Los Angeles is substantially larger than half the size of New York an example why it might not hold). Nitsch (2005) carried out a meta-study of past empirical studies, observing a wide range of findings across studies, but when looking at them in the aggregate they found a mean of $\alpha = 1.08$, quite close to the Zipf value. Berry and Okulicz-Kozaryn (2012) argue that the variations in estimates are due to not using consistent measures of urban regions across studies, and if the largest such measures are used of megalopolises, then Zipf's Law and the rank-size rule holds fully. In any case, whether it does or not, the evidence is strong that city size distributions are power law distributed, showing a domination by anti-entropic forces for this part of urban and regional systems.

A possible foundation for these anti-entropic processes that can generate power law distributional outcomes is economies of scale, long known to be a foundation also of economic complexity (Arthur, 1994). Urban systems in particular can exhibit as many as three different kinds of economies of scale: internal firm level economies (Marshall, 1879), localization economies involving external agglomeration between firms in a single industry (Marshall, 1919), and urbanization economies that involve external agglomeration economies spilling across industries (Hoover and Vernon, 1959).

Rigorous models of how increases in agglomerative tendencies can overcome congestion effects can destabilize an equilibrium of equal population distribution, essentially a maximum entropy outcome, and lead to the rise of urban concentrations are due to Papageorgiou and Smith (1983) and Weidlich and Haag (1987). However these models have since been superseded by "new economic geography" ones that emphasize economies of scale arising in within monopolistic competition as analyzed by Dixit and Stiglitz (1977). While Fujita (1988) initiated using this for modeling urban and regional systems, Krugman's (1991) approach received the most attention and influence (Rosser Jr., 2011a).

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[8] Gabaix and Ioannides (2004) argue that Kuznets (1955) first provided a formal way to estimate power law distributions for urban sizes.

5.6 Complexity, Entropy, and Self-Organization of Urban/Regional Systems

This brings us to a realization that the interaction between entropic and anti-entropic forces within urban and regional systems can generate complexity that underlies emergence of higher ordered structural patterns through self-organization as bifurcation points are encountered within nonlinear dynamics of the systems that lead to morphogenetic structural transformations (Rosser Jr., 1990, 1991; Krugman, 1996; Portugali, 1999). This can be seen by looking at how these systems operate from the perspective of dynamic complexity, which Day (1994) defined as systems endogenously not converging on a steady state or exponential growth. Such complexity is known to take four forms: cybernetics, catastrophe theory, chaos theory, and agent-based complexity (Rosser Jr., 1999). All these forms can be seen to have operated within urban and regional systems.

The most important model of urban dynamics based on a cybernetics was due to Forrester (1961) in his *Urban Dynamics*, although he labeled his approach to be part of *systems dynamics* theory. This involved a set of nonlinear difference equations with complicated interconnections with each other involving positive and negative feedback effects. When simulated it exhibited structural breaks and sudden changes at certain points, with the system too complicated for discovering these by analysis, rather requiring simulation instead.

Much more widespread have been studies of structural changes in urban and regional and more general spatial systems using catastrophe theory. Amson (1974) initiated the use of catastrophe theory in such systems, examining rent and "opulence" (attractiveness) determinants of urban density using a cusp catastrophe model. Mees (1975) modeled the revival of cities in medieval Europe as a butterfly catastrophe. Wilson (1976) modeled transportation modal choice as a fold catastrophe, and drawing on the entropic retail model, Poston and Wilson (1977) did so for retail center size.[9] Isard (1977) initiated the study of agglomeration effects bringing about the sudden emergence of cities in models balancing urban and rural areas using the cusp catastrophe, with Casetti (1980) and Dendrinos (1980) following. Dendrinos (1978, 1979) used higher order catastrophe models to study industrial-residential dynamics and slum formation in cities. Puu (1979, 1981) did so as well to study structural changes in regional trading patterns. Nijkamp and Reggiani (1988) showed how an optimal control model of nonlinear dynamic spatial interaction can generate a catastrophe theoretic interpretation.

The application of chaos theory to the study of complex urban and regional dynamics was initiated by Beaumont, Clarke, and Wilson (1981) for intraurban residential and retail dynamics, again drawing on the entropic intraurban model.

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[9] Wilson (1981) provides an early overview of many of these models. Dendrinos and Rosser Jr. (1992) show how many are linked.

White (1985) combined this model with ideas from synergetics (Haken, 1983)[10] to show self-organization arising from chaotic fluctuations near bifurcations points. A series of papers and books emphasized interregional migration or more general population dynamics (Rogerson, 1985; Day et al. 1987; Dendrinos, 1982; Dendrinos and Sonis, 1990). Another area of study was chaotic dynamics in interregional business cycle models (Puu, 1989, 1990). There have also been studies of chaotic dynamics in extended versions of the new economic geography core-periphery models based on monopolistic competition (Currie and Kubin, 2006; Commendatore et al. 2007).

Finally, it turns out that the very initiation of agent-based complexity models came out of efforts to model the emergence of racial segregation in cities by Schelling (1971, 1978). These changes can be measured by entropic methods. Curiously, Schelling did not use either analytic models or computer simulation, but instead played a game on a 19 by 19 Go board with black and white stones, simply assuming small local differences in desires to live next to people like one or not. A high entropy beginning of integration ends up with emergence of a low entropy segregated pattern. Schelling's model has been studied since in many variations and contexts and found to be highly robust. Zhang (2004) considered it as an evolutionary game on a lattice torus, while Fagiolo et al. (2007) as a network model. Such models have a similarity to the cybernetics models, except that they more clearly rely on generating higher-order self-organization emerging from low level agents interacting with each other according to strictly local effects, a foundational complexity approach.

5.7 Further Observations

It is completely natural that both entropy and complexity are deeply involved in the dynamics and spatial structures of urban and regional systems. The spatial nature of such systems opens them to having local neighborhood effects being very important, which is foundational for advanced views of complexity and the ubiquity of external agglomeration effects underlie nonlinearities that furthermore lead to dynamic complexities of various sorts, including catastrophic discontinuities and chaotic dynamics.

As open systems, complexity is further enhanced by the dissipative nature of urban and regional systems. They are subject to the competition between entropic and anti-entropic forces that interact to stimulate complex dynamics. This is especially the case for the ontological thermodynamics of urban and regional systems operating as ecosystems.

However, metaphorical entropy measures based on Shannon information entropy have proven useful in understanding and modeling a variety of aspects of urban and regional systems. This includes both spatial patterns as well as sociological

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[10] Extensions of this to fractal synergetic models of self-organization of urban hierarchies are due to Fotheringham et al. (1989) and Rosser Jr. (1994).

structures such as racial segregation, which have also been found to exhibit complex dynamics as with the Schelling model. Few areas of economics or the broader social sciences exhibit so many instances of complex dynamics enhanced by entropic forces as urban and regional systems.

This interaction calls for a new world view. As Jeremy Rifkin (1981, p. 256) puts it, "In the end, our individual present rests forever in the collective soul of the unfolding process itself. To conserve as best we can the fixed endowment that was left to us, and to respect as best we can the natural rhythm that governs the becoming process, is to express our ultimate love for all life that preceded us and all life that will follow."

Chapter 6
Complex Ecological-Economic Systems and Their Governance Issues

6.1 Introduction: Ostrom, Complexity, and Governance

The late Elinor Ostrom was the person who most clearly saw through the supposed dilemma called the “tragedy of the commons” (Hardin 1968; Ostrom 1990). It was widely argued that managing common property resources was an impossible proposition, that either common property is privatized in some way or else there will be an inevitable tendency for the resource to be overharvested, possibly to complete destruction or exhaustion. Such outcomes were seen as inevitable outcomes of prisoner dilemma games where agents using common property resources will fail to cooperate üwith one another and instead seek to get as much of the resource for themselves as soon as possible. However, she understood from early in her work (Ostrom 1976) that people seek to work out arrangements for managing common property resources. As she studied this phenomenon over time she came to realize that different groups pursue different solutions. This led her to pose the concept of polycentricity and the importance of institutional diversity around the world, based on local circumstances and cultures (Ostrom 2005, 2012).

Also over time she came to understand that the challenge of managing common property resources becomes more difficult when the governance system inevitably becomes part of a complex ecologic-economic system (Ostrom 2010a, b). Indeed, it is often the human intervention into a natural system that introduces the complexity in the system, the ecologic-economic system. This induced complexity makes those managing it that more responsible for what they do.

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_6

6.2 Complex Fishery Dynamics

The classic tragedy of the commons for fisheries was first posed by Gordon (1954), who incorrectly identified it as a problem of common property, while nevertheless identifying the inefficient overharvesting that can occur in an open access fishery. However, even when efficiently managed, fisheries may exhibit complex dynamics, particularly when discount rates are sufficiently high. Just as species can become extinct under optimal management when agents do not value future stocks of the species sufficiently, likewise in fisheries, as future stocks of fish are valued less and less, the management of the fishery can become to resemble an open access fishery. Indeed, in the limit, as the discount rate goes to infinity at which point the future is valued at zero, the management of the fishery converges on that of the open access case. But well before that limit is reached, complex dynamics of various sorts besides catastrophic collapses may emerge with greater than zero discount rates, such as chaotic dynamics.

We shall now lay out a general model based on intertemporal optimization to see how these outcomes can arise as discount rates vary, following Hommes and Barkley Rosser Jr (2001).[1] We shall start considering optimal steady states where the amount of fish harvesting equals the natural growth rate of the fish as given by the Schaeffer (1957) yield function.

$$h(x) = f(x) = rx(1 - x/k), \tag{6.1}$$

where the respective variables are the same as in Chap. 2: x is the biomass of the fish, h is harvest, $f(x)$ is the biological yield function, r is the natural rate of growth of the fish population without capacity constraints, and k is the carrying capacity of the fishery, the maximum amount of fish that can live in it in situation of no harvesting, which is also the long-run bionomic equilibrium of the fishery.

We more fully specify the human side of the system by introducing a catchability coefficient, q, along with effort, E, to give that the steady state harvest, Y, also is given by

$$h(x) = qEx = Y. \tag{6.2}$$

We continue to assume constant marginal cost, c, so that total cost, C is given by

$$C(E) = cE. \tag{6.3}$$

With p the price of fish, this leads to a rent, R, that is

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[1] For further discussion, see Rosser Jr (2001b, 2011a, Chap. 9), and Foroni et al. (2003).

$$R(Y) = pqEx - C(E). \tag{6.4}$$

So far this has been a static exercise, but now let us put this more directly into the intertemporal optimization framework, assuming that the time discount rate is δ. All of the above equations will now be time indexed by t, and also we must allow at least in principle for non-steady state outcomes. Thus

$$dx/dt = f(x) - h(x), \tag{6.5}$$

with $h(x)$ now given by (6.2) and not necessarily equal to $f(x)$. Letting unit harvesting costs at different times be given by $c[x(t)]$, which will equal c/qx, and with a constant $\delta > 0$, the optimal control problem over $h(t)$ while substituting in (6.5) becomes

$$\max \int_0^\infty e^{-\delta t}(p - c[x(t)])(\, f(x) - dx/dt)dt, \tag{6.6}$$

subject to $x(t) \geq 0$ and $h(t) \geq 0$, noting that $h(t) = f(x) - dx/dt$ in (6.6). Applying Euler conditions$'$ gives

$$f(x)/dt = \delta = [c'(x)f(x)]/[p - c(x)]. \tag{6.7}$$

From this the optimal discounted supply curve of fish will be given by

$$x(p,\delta) = k/4\left\{1 + (c/pqk) - (\delta/r) + \left[(1 + (c/pqk) - (\delta/r))^2 + (8c\delta/pqkr)\right]^{1/2}\right\}. \tag{6.8}$$

This entire system is depicted in Fig. 6.1 (Rosser Jr 2001b, p. 27) as the Gordon-Schaefer-Clark fishery model.

The most dramatic aspect of this model is the backward-bending supply curve that arises, with Copes (1970) being the first to explain this possibility for fisheries, strongly supported by Clark (1990). One can see that a gradual increase in demand in this situation can lead to a sudden increase in price and a catastrophic collapse of output.

We note that when $\delta = 0$, the supply curve in the upper right quadrant of Fig. 6.1 will not bend backwards. Rather it will asymptotically approach the vertical line coming up from the maximum sustained yield point at the farthest point to the right on the yield curve in the lower right quadrant. As δ increases, this supply curve will start to bend backwards and will actually do so well below $\delta = 2\%$. The backward bend will continue to become more extreme until at $\delta = \infty$ the supply curve will converge on the open access supply curve of

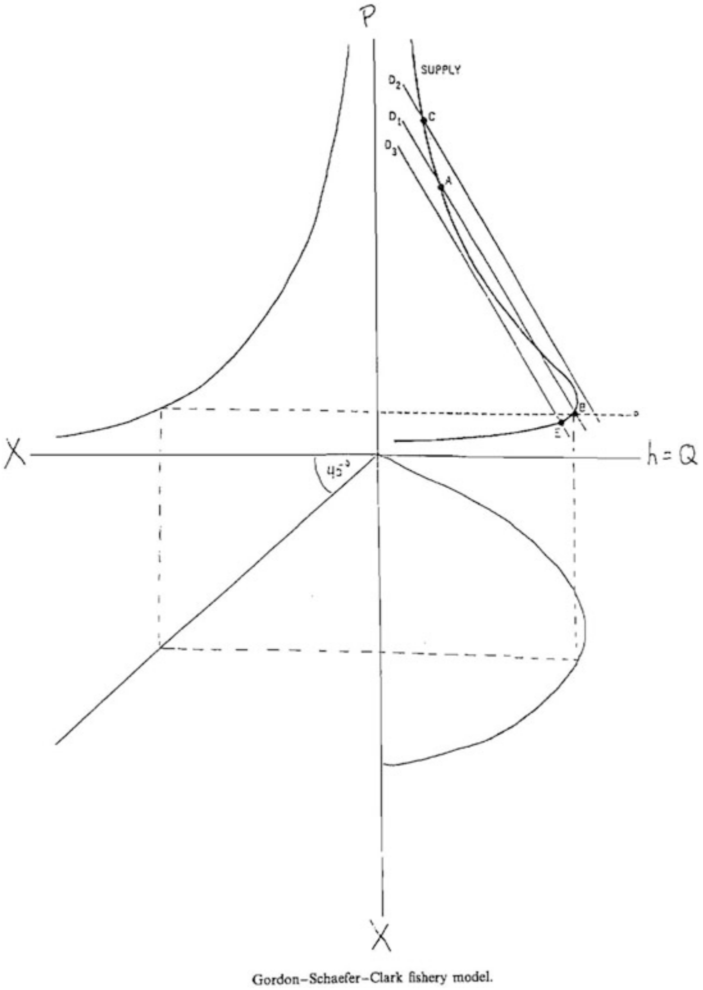

Fig. 6.1 Gordon-Schaefer-Clark fishery model

$$x(p,\infty) = (rc/pq)(1 - c/pqk). \tag{6.9}$$

It should be clear that the chance of catastrophic collapses will increase as this supply curve bends further backwards and the possibility for multiple equilibria increases, so that a smooth increase in demand can lead to a catastrophic increase in price and collapse of quantity. So, even if people are behaving optimally, as they become more myopic, the chances of catastrophic outcomes will increase.

Regarding the nature of the optimal dynamics, Hommes and Rosser Jr (2001) show that for the zones in which there are multiple equilibria in the backward-bending supply curve case, there are roughly three zones in terms of the nature of the optimal outcomes. At sufficiently low discount rates, the optimal outcome will simply be the lower price/higher quantity of the two stable equilibrium outcomes. At a much higher level the optimal outcome will simply the higher price/lower quantity of the two stable equilibria. However, for intermediate zones, the optimal outcome may involve a complex pattern of bouncing back and forth between the two equilibria, with the possibility of this pattern being mathematically chaotic arising.[2]

To study their system, Hommes and Rosser Jr (2001) assume a demand curve of the form

$$D(p(t)) = A - Bp(t), \tag{6.10}$$

with the supply curve being given by (6.8). Market clearing is then given by

$$p(t) = [A - S(p(t), \delta]/B. \tag{6.11}$$

This can be turned into a model of cobweb adjustment dynamics by indexing the p in the supply function to be one period behind the p being determined, with Chiarella (1988) and Matsumoto (1997) showing chaotic dynamics in generalized cobweb models.

Drawing on data from Clark (1985, pp. 25, 45, 48), Hommes and Barkley Rosser Jr (2001) assumed the following values for parameters: $A = 5241$, $B = 0.28$, $r = 0.05$, $c = 5000$, $k = 400{,}000$, $q = 0.000014$ (with the number for A coming from $A = kr/(c - c^2/qk)$). For these values they found that as δ rose from zero at first a low price equilibrium was the solution, but starting around $\delta = 2\%$ period-doubling bifurcations began to appear, with full-blown chaotic dynamics appearing at around $\delta = 8\%$. When δ rose above 10% or so, the system went to the high price equilibrium.

6.3 Complexity Problems of Optimal Rotation in Forests

Some complexities of forestry dynamics have long been known in connection with the matter of spruce-budworm dynamics (Ludwig, Jones, and Holling, 1978).[3] In order to get at related sorts of dynamics arising from unexpected patterns of forest benefits as well as such management issues as how to deal with forest fires and patch

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[2] This is below the range that chaotic dynamics emerge in Golden Rule growth models (Nishimura and Yano 1996). Chaotic dynamics appear in the non-optimizing model of a halibut fishery with a backward-bending supply curve (Conklin and Kohlberg 1994). Doveri et al. (1993) showed this for more generalized multiple-species aquatic ecosystems. Zimmer (1999) argued that chaotic cycles are more likely to appear in laboratories due to noise in natural environments, while Allen et al. (1993) argue that chaotic dynamics in a noisy environment may help a species to survive.

[3] See also Holling (1965) for a foreshadowing of this argument. For broader links, Holling (1986) argued that these spruce-budworm systems in the Canadian forests could be affected by “local surprise” or small events in distant locations, such as the draining of crucial swamps in the US Midwest on the migratory paths of birds that feed on the budworms.

size, as well as the basic matter of when forests should be optimally cut, we need to develop a basic model (Rosser Jr 2005). We shall begin with the simplest sort of model in which the only benefit of a forest is the timber to be cut from it and consider the optimal behavior of a profit-maximizing forest owner under such conditions.

Irving Fisher (1907) considered what we now call the "optimal rotation" problem of when to cut a forest as part of his development of capital theory. Positing positive real interest rates he argued that it would be optimal to cut the forest (or a tree, to be more precise) when its growth rate equals the real rate of interest, the growth rate of trees tending to slow down over time. This was straightforward: as long as a tree grows more rapidly than the level of the rate of interest, one can increase one's wealth more by letting the tree grow. Once its growth rate is set to drop below the real rate of interest, one can make more money by cutting the tree down and putting the proceeds from selling its timber into a bond earning the real rate of interest. This argument dominated thinking in the English language tradition for over half a decade, despite some doubts raised by Alchian (1952) and Gaffney (1957).

However, as eloquently argued by Samuelson (1976), Fisher was wrong. Or to be more precise, he was only correct for a rather odd and uninteresting case, namely that in which the forest owner does not replant a new tree to replace the old one, but in effect simply abandons the forest and does nothing with it (or perhaps sells it off to someone else). This is certainly not the solution to the optimal rotation problem in which the forest owner intends to replant and then cut and replant and cut and so on into the infinite future. Curiously, the solution to this problem had been solved in 1849 by a German forester, Martin Faustmann (1849), although his solution would remain unknown in English until his work was translated over a century later.

Faustmann's solution involves cutting sooner than in the Fisher case, because one can get more rapidly growing younger trees in and growing if one cuts sooner, which increases the present value of the forest compared to a rotation period based on cutting when Fisher recommended.

Let p be the price of timber, assumed to be constant,[4] $f(t)$ be the growth function of the biomass of the tree over time, T be the optimal rotation period, r be the real interest rate, and c the cost of cutting the tree, Fisher's solution is then given by

$$pf'(T) = rpf(T), \tag{6.12}$$

which by removing price from both sides can be reduced to

$$f'(T) = rf(T), \tag{6.13}$$

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[4] This is a nontrivial assumption, with a large literature existing on the use of option theory to solve for optimal stopping times when the price is a stochastic process (Reed and Clarke 1990). Arrow and Fisher (1974) first suggested the use of option theory to deal with possibly irreversible loss of uncertain future forest values.

which has the interpretation already given: cut when the growth rate equals real rate of interest.

Faustmann solved this by considering an infinite sum of discounted earnings of the future discounted returns from harvesting and found this to reduce to

$$pf'(T) = rpf(T) + r\left[(pf(T) - c)/\left(e^{rT} - 1\right)\right]. \tag{6.14}$$

which implies a lower *T* than in Fisher's case due to the extra term on the right-hand side, which is positive and given the fact that *f*(*t*) is concave. Hartman (1976) generalized this to allow for non-timber amenity values of the tree (or forest patch of same aged trees to be cut simultaneously),[5] assuming those amenity values can be characterized by $g(t)$ to be given by

$$pf'(T) = rpf(T) + r\left[(pf(T) - c)/\left(e^{rT} - 1\right)\right] - g(T). \tag{6.15}$$

An example of a marketable non-timber amenity value that can be associated with a privately owned forest might be grazing of animals, which tends to reach a maximum early in the life of a forest patch when the trees are still young and rather small. Swallow et al. (1990) estimated cattle grazing amenity values in Western Montana to reach a maximum of 12.5 years, with the function given by

$$g(t) = \beta_0 \exp(-\beta_1 t), \tag{6.16}$$

Rosser Jr (2005) showed that this case reached a global maximum at 76 years, slightly longer than the 73 years of the Faustmann solution, but it indeed exhibits multiple local optima, reflecting nonlinearities in these forestry dynamics (Rosser Jr 2005; 2011a, b, 2013; Vincent and Potts 2005).[6]

More frequently this $g(t)$ function involves matters not so easily appropriated by a private owner, in short, externalities. Some government forest owners try to incorporate these into planning efforts, with this something long done by the United States Forest Service, which uses public hearings to gauge public sentiment regarding alternative land uses in its planning for national forests (Johnson et al. 1980; Bowes and Krutilla 1985). Among those are hunting and fishing, which sometimes

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[5] A more general model based on Ramsey's (1928) intertemporal optimization that solves for the optimal profile of a forest was initiated by Mitra and Wan Jr. (1986), This approach took seriously Ramsey's invocation of a zero discount rat in which case management converges on the maximum sustained yield solution, with Khan and Piazza (2011) studying this from the standpoint of classical turnpike theory.

[6] The existence of these multiple equilibria opens the possibility for capital theoretic paradoxes as the real rate of interest varies (Rosser Jr 2011b). Prince and Rosser Jr (1985) studied the implications of this for benefit-cost analysis, with this holding potentially for the George Washington National Forest case discussed in this paper below. See Asheim (2008) for an application to the case of nuclear power.

both private and public owners can get some payments by users, if for public forests more indirectly through hunting and fishing licenses.

Less easily captured are broader biodiversity issues, especially involving endangered species (Perrings et al. 1995). This has been a difficult issue in many developing nations, where systems have been established to try to provide economic benefits for local populations for preserving such species, with in some nations ecotourism a method for this. This becomes more difficult in situations where aboriginal rights have been violated in the past (Kant 2000; Gram 2001).[7]

Carbon equestration is an externality of forests getting more attention, with less frequent cutting tending to aid this (Alig et al. 1998), especially given that standard timber harvesting often involves burning underbrush and unused limbs, not to mention that timber harvesting also can also increase soil erosion and flooding (Plantinga and Wu, 2003). But younger trees may absorb more CO_2 and replacing one species with others may also improve this (Alavalapati et al. 2002). All of this may also interact with biodiversity efforts in various ways (Caparrøs and Jacquemont 2003).

A good example of these complexities has been studied for the George Washington National Forest in Virginia and West Virginia drawing on information in its planning process (FORPLAN, Johnson et al. 1980). There one finds hunting-related multiple maxima tied to deer that reach a peak 8 years after a clearcut, with wild turkeys and grouse reaching a maximum at about 25 years after a clearcut (and this also the maximum for vegetative diversity), and bears reaching a maximum after about 60 years, with this setting up conflicts over cutting more frequently in some parts of the forest to please deer hunters and much less to even no cutting in other parts to favor bear hunters, both of these powerful interest groups pressuring decision makers for that forest (Rosser Jr 2005, 2011a, 2013).

If a forest is not strictly a subsistence one and thus has at least one product sold in a market, then for a fixed land area, a forest may a backward-bending long-run supply curve for that product, particularly if it is timber. Empirical observations support the possible existence of such situations, including a study of smallholder timber sales from the edge of the Amazon rain forest (Amacher et al. 2009). They found strongly negative and statistically significant elasticities of supply for timber in their sample for plots with secure tenure, although for ones with insecure tenure the curve slopes upward. The authors offer little argument for why this result should occur, partly as they are mostly concerned with other issues such as the role of credit and the presence or not of the Transamazonian highway. The little explanation they do provide invokes the model of the backward-bending supply curve of individual labor rather than that of fisheries. “The timber price effect follows from the fact that the smallholder may have predetermined revenue targets that timber sales are intended to help meet” (Amacher et al. 2009, p. 1796).

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[7] For more detailed discussions of the special problems of tropical deforestation and rights of indigenous peoples, see Barbier (2001); Kahn and Rivas (2009).

As it is, theoretical models of the possibility of backward-bending supply curves of timber have been developed in the past, inspired in particular by the work of Colin Clark on such curves for fisheries. The first to do so was Hyde (1980). Even more strongly inspired by Clark (1985, 1990), Binkley (1993) developed a formal model based on the Faustmann model,[8] also presenting tentative evidence in support of it from the long run supply of loblolly pines in the US Southeast. Needless to say, these cases open up the possibility of the sort of complex dynamics already discussed for the fishery case.

Using the variables already defined, we present Binkley's model below, adding $\pi(t)$ for the net present value of the future stream of timber receipts, which the forest owner will seek to maximize. In contrast to our earlier discussion, price will be allowed to change, although we shall eschew using option theory. This forest may contain trees or stands of varying ages. In any given year, some tree or stand will reach the optimal rotation age, T, and will be harvested. Supply will be in per unit land area terms.

The forest owner seeks to maximize

$$\pi(t) = -c + pf(t)e^{-rt} + \pi(t)e^{-rt}. \tag{6.17}$$

The first order condition for solving this is to find $d\pi/dt = 0$, which is given by

$$f'(t)/[f(t) - c/p] = r/(1 - e^{-rt}). \tag{6.18}$$

This implies a long-run supply relationship between price and optimal rotation age, T, as given by

$$S(p) = f(T(p))/T(p). \tag{6.19}$$

From this one gets a non-monotonic supply curve as a function of T that goes from zero to zero as T increases, with a maximum sustained yield (MSY) at an intermediate value of T given by

$$1/T = f'(T)/f(T). \tag{6.20}$$

From this it is possible to derive the relationship between price and optimal rotation age, T, which appears in (6.20) as given by

$$p = c/\{\, f(T) - f'(t)[(1 - e^{-rt})/r]\}. \tag{6.21}$$

This is summarized in Fig. 6.2.[9]

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[8] Yin and Newman (1999) confirmed the basic model, although also showing that aggregate supply curves allowing for variable land will be upward-sloping.

[9] Variables in the figure are those used by Binkley, translating to this paper as $v = f$, $t = T$, and $l = r$.

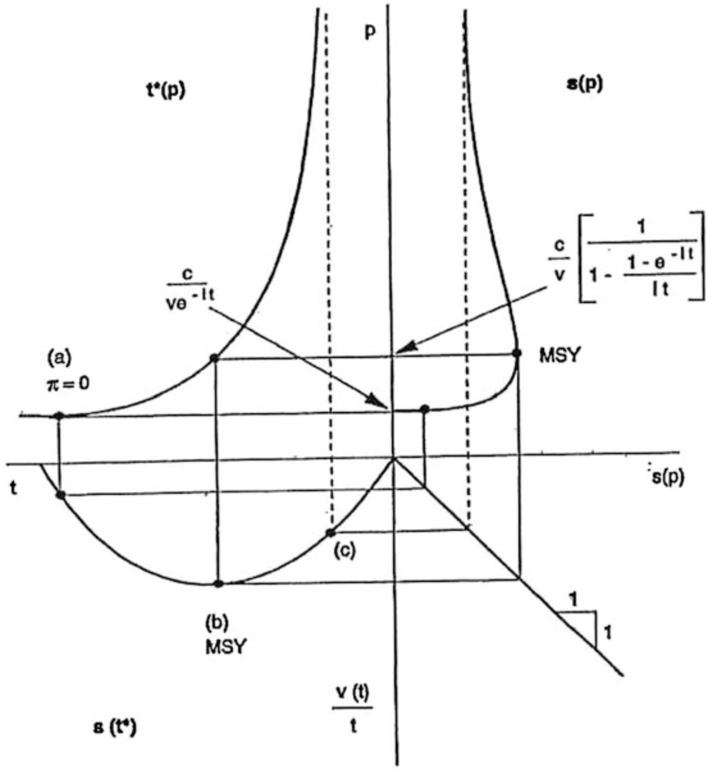

Fig. 6.2 The backward-bending supply curve of timber

There are parallels to the backward-bending supply curve of fish presented above, but also some differences. Crucial to both is the assumption of a maximum carrying capacity. Both effectively have only three figures, with one quadrant just a 45 degree line, between rotation age for the forest and fish biomass for the fishery. Both have a non-monotonic function that lies behind the backward bend of the supply curve, the Schaefer yield function of steady state harvest and fish biomass for the fishery and between rotations age and timber supply for the forest. In both, the maximum supply point is associated with the MSY point.

In both the upward sloping portion of the supply curve is associated with the "outer" portion of the relevant yield function beyond the MSY point. For the fishery there are lots of fish there, easily caught at low prices. For the forest this is the longer rotation periods when the trees are larger. On the other side of MSY is the backward-bending portion of the supply curve. For the fishery there are few fish, thus expensive to catch. For the forest, this is associated with a much shorter rotation period in which the trees are small when cut, thus producing less timber over time.

Binkley summarizes the situation in his conclusion thusly (1993, p. 178):

> "High stumpage prices imply not only that the output from the forest has a high value, but also that capital in the form of growing stock has a high opportunity cost. At high prices, it is optimal to conserve on the use of capital and therefore to reduce the growing stock inventory by reducing the rotation age."

6.4 Complexities of Climate-Economy Systems

It has long been argued that climatic systems just by themselves are chaotic, with Lorenz (1963) posing the butterfly effect initially specifically in connection with modeling climate, and with this being a main reason that it is difficult to do weather forecasting beyond a few days for a specific location. However, even if climate by itself is not chaotic and the economy by itself is not chaotic, a coupled system of the two may well be (Rosser Jr 2002, 2020d).

In particular, Chen (1997) has shown how such a system can arise. He assumes a two-sector economic model with agriculture and manufacturing that is closed by a CES utility function for a homogenous agent and with labor the only economic input. There is a two-way interaction with climate, drawing on a model due to Henderson-Sellers and McGuffie (1987). Hotter climate reduces agricultural production while increased manufacturing heats the climate due to pollution. Under certain parameter values of this model, chaotic dynamics emerge, even though neither system by itself is chaotic.

Rosser Jr (2020d) considers further a model that extends an analysis of *flare attractors* in economic systems, with these initially used to study autocatalytic reactions such as flares in physical chemistry (Rōssler and Hartmann 1995). This is arguably part of the not-so-well developed *econochemistry*. The underlying mathematics derive from Milnor attractors (Milnor 1985) that are continuous but nowhere differentiable and exhibit "riddled basins." Rōssler (1976) used this approach to develop his continuous chaotic attractor and then extended this in Rōssler et al. (1995). Hartmann and Rössler (1998) applied this model to entrepreneurial activities and Rosser Jr. et al. (2003a) applied it to examining asset price volatility. Rosser Jr (2020d) further applied this to a coupled climate-economic system that can provide the sort of kurtotic climate outcomes studied by Weitzman (2009, 2011, 2012, 2014) and Rosser Jr (2011a).

In this model the economic part derives from a model of Day (1982) that is a modified Solow growth model that faces limits to capital expansion, possibly due to environmental limits. This sets it up for a logistic formulation that resembles the model of May (1976) known to generate chaotic dynamics. This economic model is then posed in a regional setup with interacting inputs to climate that can lead to kurtotic "flares." The basic economic model has a labor exponent of α, a capital exponent of β, y is per capita output, k is the capital-labor ratio, population growth rate is λ, and m is the "capital-congestion coefficient." Thse modified production function is

$$F(k) = \beta k^{\beta}(m - k)^{y}. \tag{6.22}$$

Assuming a consistent savings rate, the capital ratio implies the following difference growth equation:

$$K_{t+1} = \alpha\beta k_t{}^{\beta}(m - k_t)^{y}/(1 + \lambda). \tag{6.23}$$

Following May (1976), Rosser Jr. et al. (2003a) assumed values that guarantee a chaotic dynamic assuming a constant capital share, given by

$$\mathrm{A}\beta/(1 + \lambda) = 3.99 = k_{t+1}/(1 - k_t). \tag{6.24}$$

In contrast to earlier formulations, the heterogeneous entities are locations rather than agents. They are driven by a reaction function B, with parameters b and a critical value of k that is a, beyond which there will be a substantial increase in temperature, a "flare." A full outburst depends on a sufficient number of locations passing their critical value, with $1 > a > 0$. With $c > 0$ and location of type l out of n, s is overall demand, the general form of this reaction function is given by:

$$B^{l}{}_{t+1} = b^{i}{}_{t} + b^{l}{}_{t}\left(a^{l} - k^{l}{}_{t}\right) - cb^{(i)2}{}_{t} + cs_{t}. \tag{6.25}$$

In this system the first term is an autoregressive component; the second is the switching term; the third provides a stabilizing component, while the fourth is the destabilizing element coming from the buildup of previous trends, with the overall demand given by:

$$S_{t+1} = b^{1}{}_{t} + b^{2}{}_{t} + \ldots + b^{n}{}_{t}. \tag{6.26}$$

In Rosser Jr. et al. (2003a) assuming certain values of these parameters allow a simulation that provides a sequence of outcomes exhibiting scattered Kurtotic outbursts consistent with the Weitzman scenario for global warming.

6.5 Stability, Resilience, Complexity of Ecosystems Revisited and Policy

It has been argued that there is a relationship between the diversity of an ecosystem and its stability, although this was later found not to be true in general, with indeed mathematical arguments existing suggesting just the opposite May 1973). It was then suggested by some that the apparent relationship between diversity and stability in nature was the other way around, that stability allowed for diversity. More broadly, it was argued that there is no general relationship, with the details of relationships within an ecosystem providing the key to understanding the nature of

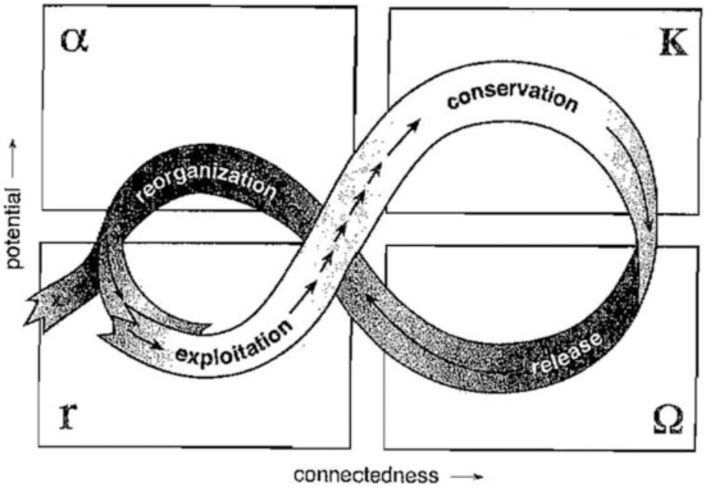

Fig. 6.3 Cycle of the four ecosystem functions

the stability of the system, although certainly declining biodiversity is a broad problem with many aspects (Perrings et al. 1995).

Out of this discussion came the fruitful insight by C.S. Holling (1973) of a deep negative relationship between stability and resilience. This relationship can be posed as a conflict between local and global stability: that greater local stability may be in some sense purchased at the cost of lesser global stability or resilience. The palm tree is not locally stable as it bends in the wind easily in comparison with the oak tree. However, as the wind strengthens, the palm tree's bending allows it to survive, while the oak tree becomes more susceptible to breaking and not surviving. Such a relationship can even be argued to carry over into economics as in the classic comparison of market capitalism and command socialism. Market capitalism suffers from instabilities of prices and the macroeconomy, whereas the planned prices and output levels of command socialism stabilize the price level, output, and employment. However, market capitalism is more resilient and survives the stronger exogenous shocks of technological change or sudden shortages of inputs, whereas command socialism is in greater danger of completely breaking down, which indeed happened with the former Soviet economic system.

This recognition that ecosystems involve dynamic patterns and do not remain fixed over time, led Holling (1992) to extend his idea to more broadly consider the role of such patterns within maintaining the resilience of such systems, and also to consider how the relationships between the patterns would vary over time and space within the hierarchical systems (Holling and Gunderson 2002; Holling et al. 2002; Gunderson et al. 2002a, b). This resulted in what has come to be called the "lazy eight" diagram of Holling, which is depicted in Fig. 6.3 (Holling and Gunderson 2002, p. 34) and shows a stylized picture of the passage of a typical ecosystem through four basic functions over time.

This can be thought of as representing a typical pattern of ecological succession on a particular plot of land.[10] Conventional ecology focuses on the *r* and *K* zones, corresponding to *r*-adapters and *K*-adapters. So, if an ecosystem has collapsed (as in the case of a forest after a total fire), it begins to have populations within it grow again from scratch, doing so at an r rate through the phase of exploitation. As it fills up, it moves to the *K* stage, wherein it reaches carrying capacity and enters the phase of conservation, although as noted previously, succession may occur in this stage as the precise set of plants and animals may change at this stage. Then there comes the release as the overconnected system now become low in resilience collapses into a release of biomass and energy in the Ω stage, which Gunderson and Holling identify with the "creative destruction" of Schumpeter (1950). Finally, the system enters into the *α* stage of reorganization as it prepares to allow for the reaccumulation of energy and biomass. In this stage soil and other fundamental factors are prepared for the return to the *r* stage, although this is a crucially important stage in that it is possible for the ecosystem to change substantially into a different form, depending on how the soil is changed and what species enter into it, with an example of the shift from buffalo-grass and grama to rattlesnake bush and tumbleweed in the US Southwest a possibility as described by Leopold (1933)

This basic pattern can be seen occurring at multiple time and space scales within a broader landscape as a set of nested cycles (Holling 1986, 1992). An example drawn on the boreal forest and also depicting relevant atmospheric cycles is depicted in Fig. 6.4 (Holling et al. 2002, p. 68). One can think in terms of the forest of each of the levels operating according to its own "lazy eight" pattern as described above. Such a pattern is called a *panarchy*.

Increasingly policymakers come to understand that it is resilience rather than stability per se that is important for longer term sustainability of a system. In the face of exogenous shocks and the threat of extinction of species (Solé and Bascompte 2006), special efforts must be made to approach things adeptly. Costanza et al. (1999) propose seven principles for the case of oceanic management: Responsibility, Scale-Matching, Precautionary, Adaptive Management, Cost Allocation, and Full Participation. Of these, Rosser Jr (2001b) suggests that the most important are the Scale-Matching and Precautionary Principles, with Wilson et al. (1999) especially emphasizing the scale perception and matching problem as deeply crucial.

Scale-matching means that the policymakers operate at the appropriate level of the hierarchy of the ecologic-economic system. Following Ostrom (1990) and Bromley (1991), as well as Rosser Jr (1995) and Rosser Jr. and Rosser (2006), the idea is to align both property and control rights at the appropriate level of the hierarchy. Managing a fishery at too high a level can lead to the destruction of fish species at a lower level (Wilson et al. 1999).

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[10] We note here the definition often used of an "ecosystem" as being a set of interrelated biogeochemical cycles driven by energy. In terms of scale, these can range from a single cell to the entire biosphere. Thus we have a set of nested ecosystems that may operate at various levels of aggregation.

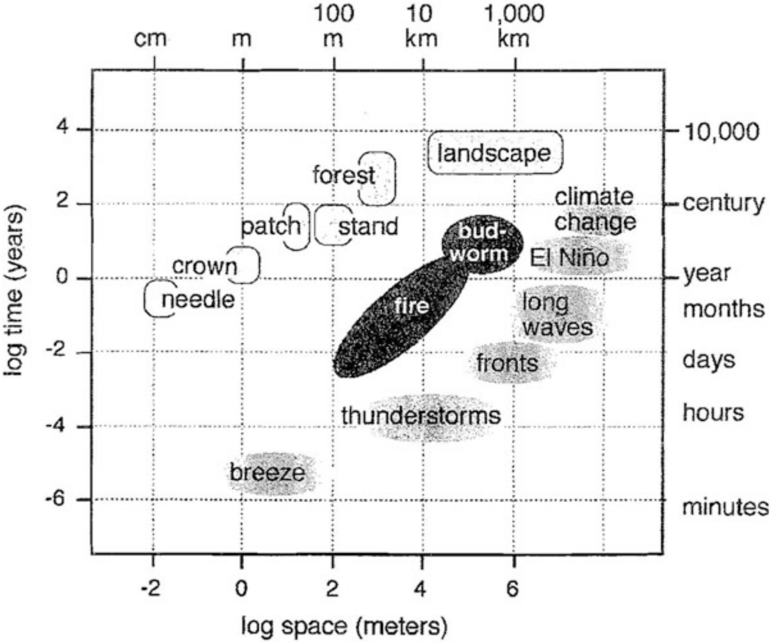

Fig. 6.4 Time and space scales of the boreal forest and the atmosphere

Assuming that appropriate scale-matching has been achieved, and that a functioning system of property rights and control has been established, the goal of managing to maintain resilience may well involve providing sufficient flexibility for the system to be able to have its local fluctuations occur without interference while maintaining the broader boundaries and limits that keep the system from collapsing. In the difficult situation of fisheries, this may involve establishing reserves (Lauck et al. 1998; Grafton et al. 2009 or system of rotational usage (Valderarama and Anderson 2007). Crucial to successfully doing this is having the group that manages the resource able to monitor itself and observe itself (Sethi and Somanathan 1996), with such self-reinforcement being the key to success in the management of fisheries for certain as in the case of the lobster gangs of Maine (Acheson 1988) and the fisheries of Iceland (Durrenberger and Palsson 1987). Needless to say, all of this is easier said than done, especially in the case of fisheries where the relevant local groups are often quite distinct socially and otherwise from those around them and thus tending to be suspicious of outsiders who attempt to get them to organize themselves to do what is needed (Charles 1988).

Property rights and control rights may not coincide (von Ciriacy-Wantrup and Bishop 1975), with control of access being the key to governing the commons. Without control of access, property rights are irrelevant. The work of Ostrom and others makes clear that property rights may take a variety of forms. While these alternative efforts often succeed, sometimes they do not, as the failure of an early effort to establish property rights in the British Columbia salmon fishery demonstrates (Millerd 2007). Some common property resources have been managed successfully for centuries, as in the case of the Swiss alpine grazing commons (Netting 1976), whose existence has long disproven the simple version of the "tragedy of the commons" as posed by Garrett Hardin (1968).

The policy problems become more difficult when different levels of hierarchy are important in the dynamics of an ecologic-economic system, especially when nonlinear complex dynamics are operative at these important multiple levels. Policies may need to be implemented at different levels, but with these consistent with each other to be effective. This problem becomes probably clearest in returning to considering the global climate issue, which indeed ranges from the almost minutely local to the fully global.

A further complication due to the complexities associated especially with chaotic dynamics is that when a system is decomposed from the global to the regional or local level, it may be subject to severe effects due to sensitive dependence on initial conditions. Thus, Massetti and Lorenzo (2019) have considered in detail the regional level forecasts from simulations of global level climate models using the United Nations IPCC for projecting possible future climate outcomes. In particular they ran simulations slightly varying initial starting values for certain variables and indeed found substantial sensitive dependence for regional level predictions. Thus for the west-central portion of the United States some projections would have substantial warming while others actually found cooling happening, even as the global average temperature showed warming, again for starting values only slightly apart. This replicates the old result for climate models found by Lorenz (1963). Needless to say, this seriously complicated knowing what to do at more local levels for such situations.

These multi-layered complexities involve deep uncertainties about all the matters noted above and more. These include ongoing debates about underlying science issues, as well as the full nature of the interactions between the economic and climatological aspects. The elements of this involve chaotic dynamics subject to sensitive dependence on initial conditions, which makes the whole matter much more difficult to understand. All this leads to the inability of any observer or agent to reliably know how the system operates in full detail reliably. This implies that it would be wise to involve heuristic rules of thumb based on bounded rationality as crucial parts of policy in such highly complex situations (Rosser Jr. and Rosser 2015).

Chapter 7
Complexity and the Future of Economics

7.1 The Evolution of Economics

The neoclassical era in economics has ended. Based on the views presented in this book, I think an argument can be made that it has been replaced by the *the complexity era*[1]. This new era has not arrived through a revolution. Instead, it has evolved out of the many strains of neoclassical work, along with work done by less orthodox mainstream and heterodox economists. It is the wave of the future.

Imagine for a moment that one were looking at the economics profession in England in 1890. One would say that Alfred Marshall, with his blend of historical and analytical economics, was the economics of the future; Walras and Edgeworth, both of whom adopted a more mathematical approach, would be considered minor players. Now fast forward to the 1930s—Marshall is seen as a minor player, while the mathematical approach of Walras and Edgeworth has become the foundation for Samuelson's cutting-edge economics (although Marshall has continued to be cited somewhat since). Now imagine economics in 2050. Much of what is currently done in economics will not be cited or even considered important. Some parts of economics, which today are considered minor, will be seen as the forerunners of what economics will become.

The point of this comparison is to make clear that to judge the relevance of economic contributions one must be forward-looking. One must have a vision of what economics will be in the future, and judge research accordingly. Current journal publication and citation metrics don't do that; they have a status-quo bias because they are backward looking, and thus encourage researchers to continue research methods and approaches of the past, rather than developing approaches of

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[1] Regarding the "end of neoclassical economics," see Colander (2000a), with Veblen (1898) coining the term "neoclassical" pejoratively at the same time he argued for economics to adopt an evolutionary approach. For identifying its successor as being the "complexity era" see Holt et al. (2011).

J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5_7

the future. They are useful, obviously, because they show current activity, but they are only part of the picture. Articles dotting i's and crossing t's, even ones that are cited relatively often in the short term, are far less important than articles that strike out in new directions. These are the ones that will change the direction of economics and be remembered in future history of economic thought texts.

Any literature assessment has to be based on a judgment about the future direction of economics. If one does not, one is, by default, accepting the judgment that the current approach in the profession will continue. But for the future of economics—there will be more acceptance that the economy is complex, and the profession, over time, will adopt certain kinds of technical mathematical, analytical and statistical tools to deal with that complexity. Models based on a priori assumptions will decrease, and be replaced by empirically driven models and assumptions. Behavioral economics will expand; experiments will become part of economist's tool kit, as will complex technical tools such as cluster analysis, ultra metrics, and dimensional analysis. This increasing complexity will be accompanied by a division of labor—theorists and statisticians will be become more and more specialized, but they will be complemented by economists who have a broad overview of where economics is going, and are trained in applying economics. Economics will stop trying to answer grand questions such as whether the market is preferred to command and control, or if the market is efficient, and answer smaller questions such as what market structure will achieve the ends that policy makers are trying to achieve.

Arguably the term "complexity" has been overused and over hyped, so this vision is not of a grand complexity theory that pulls everything together. It is a vision that sees the economy as so complicated that simple analytical models of the aggregate economy—models that can be specified in a set of analytically solvable equations—are not likely to be helpful in understanding many of the issues that economists want to address. Thus, the Walrasian neoclassical vision of a set of solvable equations capturing the full interrelationships of the economy that can be used for planning and analysis is not going to work. Instead, analysis should be based on experimental and empirical data. From there *we build up*, using whatever analytic tools we have available. This is different from the old vision where economists mostly did the opposite—starting at the top with grand mathematical theories of a Bourbakist axiomatic sort, and then working down.

The complexity vision not only connects the various research threads that will be the future of economics; it is also provides the best way to look at the economics profession itself—the economics profession as an evolving complex system that has competing forces operating at all times. It is a profession that can only be understood as a system in constant change and flux.

7.2 More on the Nature of Complexity

Adopting a complexity vision does not require choosing among the many specific definitions of complexity. However, a useful general definition of a complex system comes from Herbert Simon (1962, p. 267):

> Roughly by a complex system I mean one made up of a large number of parts that interact in a non-simple way. In such systems, the whole is more than the sum of the parts, not in an ultimate metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole. In the face of complexity, an in-principle reductionist may be at the same time a pragmatic holist.

Simon then goes on to emphasize how this definition leads to a focus on the hierarchical structure of systems and emphasizes that he is drawing on older literatures, particularly general systems theory (von Bertalanffy 1974), which he sees as including the work of economist Kenneth Boulding (1978) with cybernetics (Wiener 1948), and information theory (Shannon and Weaver 1949). Of these, cybernetics can be seen as a foundational form of dynamic complexity, while information theory can be seen as a foundational form computational complexity.

The emphasis on the problem of the whole and the parts raises two central issues in economics and for more recent approaches to complexity. One is the problem of the relationship between micro and macro in economics, which calls to mind the old problem of Keynes's "fallacy of composition". Walrasian approaches to macroeconomics have attempted to avoid this problem through the use of representative agent models. Others have proposed dealing with this problem through the invocation of an intermediate zone between the micro and the macro, the "meso," which is seen as crucial to evolutionary dynamics of a complex economy (Ng 1980; Dopfer et al. 2004). Further development of this approach has been due to Potts (2000), Metcalfe and Foster (2004), Dopfer (2005), Shiozawa (2004), Shiozawa et al. (2019), and Rosser Jr. (2021), with Hodgson (2006) arguing that Darwinian evolution is the most fundamental of all complex systems, drawing deeply on Veblen (1898) who first clearly argued for economics to adopt an evolutionary approach.

Simon's general definition also has the virtue of being close to the original meaning of the word "complex" as found in the *Oxford English D*ictionary (OED 1971, p. 492) where it is first defined as "a whole, comprehending in its compass a number of parts," from the Latin "complectere," meaning "to encompass, embrace, comprehend, comprise." Among its partial synonyms is "complicated," although, as Israel (2005) points out, this comes from a different Latin root, "complicare," meaning "to fold together" or "interwoven". Israel takes the strong position that this latter is a merely epistemological concept while the former is fundamentally ontological, complaining that such figures as von Neumann (1966) mistook them as identical, although this is arguably an overly strong position.

A final virtue of this general definition is that it encompasses one of the current cutting edge areas of economics—the behavioral and experimental approaches, which are not identical. Some who follow these approaches do not consider the complexity view to be all that relevant to what they do (Ken Binmore and Matthew Rabin for example, even as these two disagree strongly with each other on certain matters (Colander et al. 2004a)). However, at the foundation of behavioral economics is the concept of *bounded rationality*, introduced originally by Herbert Simon. It is not just Simon, but many since who have seen complexity as implying that rationality must be bounded (Sargent 1993; Arthur et al. 1997a; Rosser Jr. and Rosser 2015), and thus is lying at the foundation of behavioral economics, with Sent (1997) discussing the relation between the views of Sargent and Simon.

Looking forward a crucial part of dynamic complexity economics is the heterogeneous interacting agents approach. This approach emphasizes dispersed and interacting heterogeneous agents (Arthur et al. 1997a; Tesfatsion, 2006; Hommes 2021). For many economists this is what they mean when they refer to “complexity models.” However, as discussed earlier in this book, dynamic complexity competes with computational complexity as the most important approach to complexity economics.

Advocates of the computational complexity approach (Albin and Foley 1998; Velupillai 2000, 2005a, b, 2009; Markose 2005) argue that its greater precision makes it a superior vehicle for scientific research in economics. It must be admitted that there is some truth to this. Nevertheless, the vast majority of research in economics that identifies itself with complexity tends to be more of the dynamic variety described above. Furthermore, this definition is certainly less useful when we consider the question of the economics profession itself as a complex evolving system. Here we consider that the first two definitions provide a more useful construct for analysis than this admittedly challenging and substantial view of complexity, which we expect has the potential for important future research in the area of economic complexity. Not only is the economics profession a set of hierarchies, but it also evolves through a set of local interactions among dispersed networks of influence.

7.3 What is Cutting Edge Complexity Work?

The definitions of complexity are important because they provide a way to integrate the different strains of modern economics into a single unifying theme—the theme of complexity. The acceptance by the economics profession that the economy is complex signals a new openness to ideas from other disciplines and making it a more transdisciplinary field. Some current work falling into this broad tent complexity approach includes the following:

- Evolutionary game theory is redefining how institutions are integrated into the analysis.
- Ecological economics is redefining how nature and the economy are viewed as interrelating in a transdisciplinary formulation.
- Behavioral economics is redefining how rationality is treated.

- Econometric work dealing with the limitations of classical statistics is redefining how economists think of empirical proof.
- Complexity theory is offering a way of redefining how we conceive of general equilibrium and economic dynamics more broadly.
- Agent based computational economic (ACE) analysis is providing an alternative to analytic modeling.
- Experimental economics is changing the way economists think about empirical work, with this being the principal method by which behavioral economics is studied.

These changes are ongoing and have, in varying degrees, entered the mainstream. As that has happened, there have been a broader set of changes in how mainstream economics sees itself. Modern economics is more willing to accept that the formal part of economics has limited applicability. It is also far more willing to question the special status of economics over the other fields of inquiry and to integrate the methods of other disciplines into their methods, with Loasby (1989) and Colander (1995) arguing this is more consistent with a Marshallian rather than a Walrasian approach.

Each of these different strains has certain characteristics that are quite different from what is presented in economic textbooks. In most textbooks today one gets the impression that economics has not changed much during the last 50 years. Essentially, one learns a paradigm that develops a simple analytic deductive model, sometimes called the Max U model. The microeconomics taught in these texts is some variation on the Max U model presented with little contextual flavor that characterized Marshall’s use of it. The Max U model presented in the standard text focuses almost entirely on efficiency and optimization, assuming agents are rational, selfish, and are operating in an environment that arrives at a unique equilibrium.

The MaxU model has been explored to death and, from a cutting-edge view, is no longer of much interest. (That doesn’t mean it doesn’t still have considerable relevance. There are still many practical applications that warrant research; however from a cutting edge standpoint, we’ve pulled about all we can from it.) That is why a major part of the new cutting-edge work moves beyond these assumptions. While it does not deny the usefulness or insight provided by that model, it does not see a model based only on these assumptions as sufficient, and is therefore pushing the envelope on each of those assumptions. Some examples of how cutting-edge work is questioning these neoclassical assumptions would be the following:

- Cutting-edge economics researchers are expanding the meaning of rationality to include a much broader range of agent actions that reflect actual actions; in the new approach, individuals are purposeful (incentives still matter) but are not necessarily formally rational. The new research considers the behavioral foundations of actions, using experiments to determine what people actually do, rather than simply basing their arguments on what people rationally should do, with Payne et al. (1993) integrating psychology into this. The work in game theory by such economists as Peyton Young (1998) is pushing rationality to its limits to demonstrate the importance of the expectations and information environment in

people's decisions. The cutting edge work that is being done here is going beyond the traditional definition of rationality, with extended versions of Herbert Simon's bounded rationality increasingly being accepted.
- Cutting-edge researchers are moving away from a narrow view of selfishness. While textbook economics generally assumes that agents who care only about themselves, the new work is trying to come to grips with the more realistic sense of individuals who, while they are self-interested, are also social beings, concerned about others and deriving happiness from interacting with others.
- Cutting-edge researchers are moving away from the assumption of a unique equilibrium, and are dealing with complex systems that have multiple equilibria, path dependence, and no clear-cut answer. A complex economy does not have a single equilibrium; it has many basins of attraction. The question researchers ask is which basin is sustainable. In this work equilibrium is not a state of the economy; the economy is continually in flux.

Combined, these changes can be summarized as a movement from an economics of *rationality, selfishness, and equilibrium* to an economics of *purposeful behavior, enlightened self-interest, and sustainability.* Cutting-edge work helps to move that transformation along.

7.4 Changes in Research Methods

Another aspect of cutting-edge work that is consistent with the complexity era involves changes in research methods that can serve as a catalyst for many changes in the profession. For example, advances in computing technology have led to new approaches such as agent-based modeling. This allows economists to analyze complicated systems, with more complicated interactions between the agents, out of which higher-order structures may emerge or self-organize. Also, instead of assuming optimal behavior, economists are using lab, field and natural experiments to determine what people actually do. As economists have started to use these new techniques they are taking notice of institutions, since the incentives embodied in those institutions are often central in understanding people's behavior.

This change is being accompanied by a change in the deductive nature of economic reasoning. The new work is based more on empirical inductive reasoning, and far less on pure deductive reasoning. As this is happening, the math being used in economic analysis is becoming less the Bourbakian math of "theorem-proof," and more applied mathematics, which is designed to come up with answers about policy issues, and not just talk about general issues (Weintraub 2002). Set theory and calculus, which come to definite results, are being replaced by game theory, which seldom comes to a definite conclusion independent of the precise structure of the game. For example, current work on auctions combines insights from game theory with experimental results, which are then used in practice (Banks et al. 2003).

Similarly, information economics is used in designing efficient algorithms for search engines.

7.5 Cutting Edge Complexity Work and Modern Macroeconomics

Interestingly, these cutting edge changes in micro theory toward inductive analysis and a complexity approach have not occurred in macroeconomics. In fact, the evolution of macroeconomic thinking in the United States has gone the other way. By that, we mean that there has been a movement away from a rough and ready macro theory that characterized the macroeconomics of the 1960s toward a theoretically analytic macro theory based on abstract, representative agent models that rely heavily on the assumptions of equilibrium. This macro work goes under the name New Classical, Real Business Cycle, and the Dynamic Stochastic General Equilibrium (DSGE) theory, and has become the mainstream in the U.S.

In part, this development is understandable. The macro theory prevalent in the 1960s claimed a much stronger theoretical foundation than was warranted, and many of the conclusions it came to were supported by neither empirical evidence nor theory. However, while the new theoretical models have done a good job in eliminating the old theory, it is less clear as to what the new theoretical work has added to our understanding of the macroeconomy. At best, the results of the new macro models can be roughly calibrated with the empirical evidence, but often these new models do no better than any other model, and the only claim they have to being preferred is aesthetic—they have micro foundations. However, it is a strange micro foundation—a micro foundation based on assumptions of no heterogeneous agent interaction, when, for many people, it is precisely the heterogeneous agent interaction that leads to central characteristics of the macro economy. This is the essential insight of Keynes' fallacy of composition.

Of course we have seen efforts to introduce heterogeneous agents into the DSGE context, with this leading to the appearance of Heterogeneous Agent New Keynesian (HANK) models. However, often as in Krusell and Smith Jr. (1998) these models do not involve direct interactions between agents. Rather one gets an interval of an infinite number of agents varying on a particular parameter, with, in effect, that interval acting like the representative agent of other DSGE models. This does not lead to a complexity approach to macro modeling. Such an approach will have macro outcomes emerging from a set of behaviorally based interacting heterogenous agents, with a good example being Delli Gatti et al. (2008).

The interesting cutting-edge work in macro is not in the theoretical developments organized around representative agent micro foundations, but the work that views macroeconomy as a complex system. In this work, one sees the macroeconomy as being endogenously organized. The issue is not why there are fluctuations in the macro economy, but why is there so little instability where complex interactions could generate chaos, although chaotic dynamics do stay within bounds consistent with the “corridor of stability” idea of Leijonhufvud (1973, 2009), which resembles the “resilience-stability tradeoff” studied by Holling (1973) in ecology. The belief that one could develop a micro foundation for macroeconomics without considering the feedback of the macro system on the individual is beyond belief. While it may still make sense to push analytic macro theory as far as one can, to see whether it will provide any insights, in the short term, such analytic extensions of pure theoretical models based on assumptions that are far from reality offer little hope for policy guidance. In the absence of a pure theoretical foundation, macro policy is best based more on statistical models that pull as much information as possible from the data. Empirical macro precedes theoretical macro.

7.6 Complexity Economics and the Debate over Heterodox Economics

The basic argument of this chapter that complexity economics is not only a crucial part of the cutting edge of economic research but in fact substantially underpins the broader future of economics was made in a strong form initially by Colander et al. (2004a) and Colander et al. (2004b), with the first of those a book mostly of interviews with “cutting edge economists,” all but one of whom were located in the United States,[2] with this not planned but simply came about out of convenience given we are all based in the U.S. This would be followed up by a similar book largely of interviews focusing on European economists and economics (Rosser Jr. et al. 2010),[3] with one planned for Asia that never happened, although arguably in Japan there is a tradition that has led to such an independent and locally developed such approach (Morris-Suzuki 1989; Ikeo 2014; Shiozawa 2004; Shiozawa et al. 2019; Rosser Jr. 2021).

The second item is an article largely derived from the opening chapter of the book that laid out the framework we had going into the interviews, in which the theme of complexity was a recurring theme. This paper, published in the *Review of Political Economy*, would attract the most attention (and citations) of all these works and set off a considerable debate to be discussed below, with several of our later works focusing heavily on this debate (Colander et al. 2007–08, 2010; Rosser Jr. et al. 2013).

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[2] Those interviewed in the (Colander et al. 2004a) US-based book were Deirdre McCloskey, Ken Binmore, Herb Gintis, Bob Frank, Mat Rabin, William (“Buz”) Brock, Duncan Foley, Richard Norgaard, and Rob Axtell wirh Peyton Yong, with ex post overviews by Ken Arrow and Paul Samuelson.

[3] Those interviewed in the (Rosser Jr et al. 2010) Europe-based book were Alan Kirman, Ernst Fehr, Cars Hommes, Mauro Gallegati with Laura Gardini, Geoff Hodgson, Joan Martinez-Allier, and Robert Boyer, with ex post overviews by János Kornai and Reinhard Selten.

An issue going back decades actually as one can surmise if one has read this book all the way through to here is that for much of this time ideas associated with complexity economics were not always easily accepted by mainstream economists. The papers often appeared in oddball journals, with some exceptions, or in arguably oddball books, even though in a number of cases these papers and books would later become heavily cited and widely respected and influential. This led us to think seriously about the nature of how economics evolves and how new ideas or approaches develop and enter into economics, moving from some fringe and ridicule to eventually ending up in textbooks, with one of us, David Colander, having long worn the hat of both an economic educator (Colander 2000b) and a historian of economic thought (Colander 2000c), as well as tying these concerns to ideas of complexity economics and even applying them to economics itself as a field (Colander et al. 2009; Colander 2015; Holt and Rosser Jr. 2018).

A centerpiece of this process and debate involves the role of *heterodox economics* and its relationship to non-heterodox economics, with to what extent do new ideas emerge from heterodox economists and how is it that when "successful" they move more into the mainstream. This issue was very live in our first interview book (Colander et al. 2004a) in which indeed those we interviewed themselves differed on how they viewed themselves regarding their status in the profession, with some viewing themselves as clearly heterodox (Duncan Foley) while others viewed themselves as more in the mainstream (Ken Binmore). This pushed us to think harder about what was going on here.

What we came up with was to bifurcate the question to a degree, and to argue that there is an intellectual aspect to it and a sociological aspect to it, with there being three categories under consideration: *orthodoxy, heterodoxy, and mainstream* (although confronting this one of our interviewees, Herb Gintis, joked not totally unseriously that he likes to think of himself as a "homodox economist"). We decided that orthodoxy is an intellectual category, mainstream is a sociological category, but heterodoxy is both, which is where much of the trouble arises. Orthodox economics in its pure form is the old "neoclassical economics" that Colander argued (Colander 2000a) has died, that economics described by the trinity of rationality, greed, and equilibrium. Its purest manifestation was at the University of Chicago for decades, although at a more fundamental level its hardest line exponents were long based in the "sacred zip code" in Cambridge, Massachusetts at Harvard and especially at MIT, with Paul Samuelson as perhaps the supreme godfather, whom we interviewed along with Ken Arrow for the end of our first book after letting them see our other interviews. As it is, even at these bastions this old orthodoxy no longer reins, and all sorts of formerly unacceptable approaches, especially behavioral economics, now infest the hallways and offices.

Mainstream is a sociological category. It is really people, those in charge of the economics profession, those at the top schools, running the top journals, controlling funding for research, and so on. We noted that even by soon after 2000 or so there were quite a few such people in these positions, including Nobel Prize winners, whose ideas were not strictly orthodox, with people like George Akerlof and Vernon Smith sticking out as examples, although Smith has not generally been at top schools. This would also include a few players from earlier who have been heavily cited in this book as important in developing complexity economics, such as Herbert Simon. All of these won Nobel Prizes and are or were highly respected, but also have long felt somewhat at odds with the hard core of "the Establishment," even as they looked to more serious outsiders as part of that orthodox "Establishment." They are or were "mainstream," but not "orthodox." This was our key claim, and the one that brought much criticism down upon our heads.

This key claim had another part to it, the claim that in contrast to the other two main categories, heterodoxy is both an intellectual and sociological category. Thus heterodox economists are both intellectually opposed to and critical of the old orthodox economics, and they are also not in the top schools and find it hard to publish in top journals, feeling discriminated against and even oppressed. In some cases this has led to them failing to get tenure at various institution due to their troubles publishing sufficiently in sufficiently prestigious journals and otherwise suffering professionally.

Understandably this has led to resentment and anger by many, with some of this arguably justified. For many of these self-identified heterodox economists, the enemy is "the orthodox mainstream," and they abreact to this identifying some of the mainstream economists as "non-orthodox." To these harder core heterodox economists, these erstwhile non-orthodox mainstream are if not outright sellouts, then people who have played a game to make themselves acceptable to those in charge buy not challenging vigorously enough orthodoxy (Lavoie 2012; Lee 2012). That they may be making their ideas accepted to some degree by the mainstream and even old orthodox simply shows that they are assimilating to the mainstream and orthodox, not that they are succeeding in getting the mainstream to accept their ideas and even arguably redefine the nature of orthodoxy. As it is, even among those critical of our formulation there are differences. Thus Marc Lavoie (2012) recognizes a group he calls "dissenters" who are in effect our group of non-orthodox mainstreamers, whereas the harder line Fred Lee (2012) basically dismissed this whole category, arguing that taking them seriously or trying to be like them was simply giving in to domination by orthodoxy and giving up on heterodoxy.

Needless to say, among the heterodox have arisen over time many different schools of thought. This is not the place to get into any detailed discussion of all of these, although throughout this book at times ideas of one or another of them have been called upon or invoked, including Marxist, Austrian, Post Keynesian, evolutionary, institutionalist, behavioral, ecological, and more, especially when their approaches seemed open to or in congruence with elements of complexity economics. Indeed, the origins of many ideas in complexity economics clearly came out of one or another of these schools at particular points in time, and arguably the strongest proponents of some of those ideas remain still firmly identified with one or another of these schools.

Of course a great irony is that each of these schools of thought themselves have developed their own internal orthodoxies and leading individuals and journals and locations that claim authority to define the school and who is in it or not in it, with the result that heresies arise within even these schools leading to the development of sub-schools that can become so numerous and differentiated one from another by such obscure debates that outsiders find it difficult if not impossible to figure out what is going on or who is what. The wars among the Marxists were among the most famous, and involved at times literally wars and people literally killing each other, as Stalin's assassination of Trotsky most dramatically demonstrated. Austrians are split between Misesians and Hayekians. The divisions among Post Keynesians are especially numerous, with Paul Davidson long holding a dominating position in the U.S. as founding editor of the *Journal of Post Keynesian Economics* while European based rival groups such as the neo-Ricardian Sraffians argued vigorously against his views and those of others. The various schools of the heterodox came to have their own sub-heterodox. In some of these battles some sub-schools are friendlier to complexity ideas than others, with Hayekians more so among Austrians and so-called Kaldorians among Post Keynesians also more so, just to give two examples.

These debates and differences of view have even been present among the three coauthors I have cited here on this matter, myself, David Colander, and Ric Holt. Dave has long taken the harder line of in effect criticizing the heterodox for not trying harder to get along with the mainstreamers, not trying to use "more honey" rather than "more vinegar," which has tended to bring more criticism down on his head from some heterodox, as he has often been very public and articulate about these views to an almost "in your face" way with some heterodox, much to the annoyance of the latter. I have been probably the one more at the other end, more sympathetic to the complaints by many heterodox regarding their being rejected and oppressed and discriminated against, with Ric being the one who often was diplomatically making peace between Dave and me when we worked together. It may be that I personally felt more heterodox, being at a not particularly prestigious state university and for a long time feeling isolated and ignored.

But Dave argued that for all those attitudes I became a mainstreamer, especially after the 1991 publication of my first book, *From Catastrophe to Chaos: A General Theory of Economic Discontinuities,* which became a success after it came out, going into three printings and receiving favorable reviews and lots of citations, even though it had been rejected by 13 publishers before Kluwer took it up at the behest of Zac Rolnik there. My position especially changed when I became editor in 2001 of the *Journal of Economic Behavior and Organization*, which has long been viewed as being "heterodox but respectable," a fine line to walk. Founded ,by Dick Day, it indeed was an early outlet of many complexity ideas, including chaos theory as well as game theory, behavioral economics, and new institutionalist economics. While in the 1980s much of this work was unpublishable in the top journals, that has changed, with leaders of these fields winning Nobel Prizes and such material now published in top journals and even getting into graduate textbooks. This even included to some extent ideas I expressed in that 1991 book, which is now viewed as a reference volume by many. Dave put it to me that I had become mainstream, whether I liked it or not, because "the top people respect what you do," and also because many of the ideas that I have worked on that were viewed as heterodox have become, well, respectable. Indeed, arguably this is a part of how economics more broadly has entered the complexity era.

I close this section by noting an old joke I heard from Dave Colander that he first heard from Abba Lerner. "But look," the Rabbi's wife remonstrated, "when one party to the dispute presented their case you said 'you are right' and then when the other party presented their case you again said 'you are quite right.' Surely they both cannot be right." To which the Rabbi answered, "My dear, you are quite right!"

7.7 Complexity Economics and Public Policy

If indeed the future of economics is to be heavily influenced by ideas from complexity economics, then for many the proof of the pudding boils down to how useful is it for informing public policy discussions and formulations. This is a matter of ongoing dispute and controversy. Much of this has involved especially the use of heterogeneous agent modeling of the sort discussed earlier in this book that was especially strongly associated with the Santa Fe Institute, where arguably the focus has more recently been upon behavioral economics and game theory than upon that particular sort of modeling. Of course, as Rosser Jr. and Rosser (2015) argue and has been argued above in this book, there are strong links between complexity economics and behavioral economics, with the central role of Herbert Simon in the early development of both a strong sign of this.

It must also be recognized that large parts of each do not particularly belong to the other. But indeed, if the old orthodoxy was highlighted by a trinity of rationality, greed, and equilibrium, both standard behavioral economics and complexity economics challenge all three of those, so it is not surprising that there is considerable overlap, and it is not surprising indeed again, that the journal I edited from 2001 to 2010, the *Journal of Economic Behavior and Organization* (and the one I now edit, the *Review of Behavioral Economics*) have both been major outlets for both approaches, including their overlap.

One area where there is frustration on the part of many complexity-oriented economists has been felt has involved macroeconomics, discussed above. There has been a major push to adopt interacting heterogeneous agent modeling at such crucial policymaking entities as central banks, but aside from study going on at some of them, these have not won the day or been substantially adopted. It is widely reported that at the US Federal Reserve three different kinds of models are used to advise policymakers: DSGE models, structural models that are essentially complicated derivations from the ISLM approach, and atheoretical models based on vector autoregressive methods. While full-blown interacting heterogeneous agent models have not joined this triumvirate reportedly, each of these has absorbed elements of complexity economics. As noted above, DSGE models have changed to include multiple agents as well as some nonlinearities and even essentially ad hoc behavioral fixes. There may be less of this going on with the older structural models, but the VAR-derived models have long incorporated nonlinear methods of various sorts, with there being a long interaction between complexity and nonlinear econometrics and time series approaches. There has also been an incorporation into all three kinds of models of financial factors, with these parts of the models also often involving various complexity elements. Indeed, at some banks, there is much modeling of networks of financial relationships (Haldane 2013), clearly a complexity approach, if one only touched upon in this book.

More broadly, while Brock and Colander (2000) made an initial stab at a more general approach, Colander and Kupers (2014) try to go beyond conventional formulations and provide a provocative stance, even as it almost certainly has its limits. It effectively relies upon emphasis on emergence of structure and order out of "bottom up" rather than "top down" approaches, emphasizing spontaneity and creativity to seek new and innovative solutions to entrenched problems. They came together while participating in a conference about climate policies. There was a split between those who advocated largely market-oriented policies and those who advocated largely government regulation-oriented policies. They were unhappy with this simplistic dichotomy and sought for a complexity-oriented alternative, which led them to their emphasis on bottom up policies that might well involve both markets and governments.

Their approach is summarized in the following (Colander and Kupers 2014, p. 21):

> "In the complexity policy frame, one starts with a recognition that there is no ultimate compass for policy other than a highly educated common sense. Scientific models provide, at best, half-truths. In our view, the education of that common sense very much includes a basic appreciation of complexity, as well as of humanities, mathematics, and others. Policy compasses are created and evolve, they are fallible products of a particular time and place, and must be treated as such. The nature of the relation between market and government, as well as top-down versus bottom-up solutions, as well as the property that policy itself is part of the complex system, is posited pretty clearly in the following … the duality of market versus government is a product of the standard economic policy frame itself. Within a complexity frame, both the more actrive top-down "government" solution and the less active bottom-up are seen as having evolved from the bottom up. Within this frame, the policy solution is an element of the system, not outside it."

Invoking "metapolicy," they avoid advocating specific policies. However, they provide some examples of what they like. An example is the "shared space" system of traffic control in the town of Drachten, the Netherlands, developed by Hans Monderman. When one drives into Drachtem one finds no stop signs or street lights or even sidewalks. Yet traffic flows smoothly and with few accidents. It helps that Drachten is not a large city where such a system simply may not work. This may look like a semi-anarchist "no government market fundamentalism," but they argue that is not the case. This is because this system depends on an existing institutional framework: a preexisting system of myriad rules and regulations, drivers' licenses, car safety standards, a broader legal framework, and more. Thus it is not a spontaneous anarcho-capitalism, but a carefully framed and bounded system that allows for the emergence of order. As they also note, "In the complexity frame, a well-functioning market is a consequence of previous and successful government metapolicy" (Colander and Kupers 2014, p. 25).

Another related issue they get into is one that Rosser Jr. (2001a, 2020e) has also addressed, namely the relationship between the views of Keynes and Hayek and how each of them relate to complexity, with Hayek (1967) having specifically discussed complexity and taking it seriously in his later years, while Keynes never specifically addressed it. For Colander and Kupers they see some overlap of the views of the two, even as on many issues they clearly differed sharply, with indeed Keynes looking more like the top-down government-intervention advocate against Hayek the advocate of bottom-up market-based spontaneous order. Pretty clearly Hayek fits their approach with this approach, so the question becomes where does Keynes fit in with this?

One response they make is that the most famous piece of top-down advocacy by Keynes involved the Great Depression, which he viewed as a "one-off" special case. Otherwise he generally favored bottom-up approaches. They point out the friendly letter Keynes (1944) wrote to Hayek (1944) when he published his *The Road to Serfdom* in which he expressed his "moral and philosophical sympathy" for Hayek's arguments. Even so, the letter itself recognized their differences, with Keynes arguing that "...we almost certainly want more [planning]. But the planning should take place in a community in which as many people as possible, both leaders and followers, share your moral position" (Colander and Kupers 2014, p. 40). They claim this shows Keynes supporting bottom-up solutions, but that would "minimize government intervention into the market, but still achieve socially desirable ends" (ibid.). However, pretty obviously others might find them stretching a bit on this point.

As it is on this matter of Keynes and Hayek and their connection with complexity, I see their overlap coming from a different direction. This would be the old bugaboo of fundamental uncertainty, which has been discussed above in this book. Keynes (1921) first made this argument that such uncertainty involves the non-existence of a probability distribution in his *Treatise on Probability*, but brought it back later in his *General Theory* (1936) and some other works. Many have seen this as implying a complexity view of the economy (Davis 1994, 2017).

Hayek did not address this specifically using probability theory, but in his discussion of complexity (Hayek 1967) it is there fitting in with his dismissal of a tendency to a long-run equilibrium and his preference for a constantly evolving economy marked by spontaneous emergence of order. A broader argument of Austrians more generally related to uncertainty is how this opens the door for the important role of entrepreneurs who operate crucially within such a profoundly uncertain environment. When pushed Keynes might be more inclined to fall back on government to rein in and limit the uncertainty, while Hayek might be more inclined to trust the spontaneous order arising from unfettered markets, but they share an understanding of the deep nature of the dynamic processes of the economy that it is complex.

7.8 The Paradox of Economics as a Complex Adaptive System

The question of whether or not the future of economics is to be fundamentally complexity economics or not has a curiously paradoxical aspect. A theme among many complexity economists is that the economics profession is itself a complex adaptive system. It is characterized by the sorts of nonlinearities and positive feedbacks that Brian Arthur (1994) emphasized as the central elements of complex systems. Ironically these characteristics present contradictory forces, one for instability and one for stability.

Positive feedback effects are most famously known as undermining equilibrium. They imply a non-convexity that removes one of the standard assumptions made when one uses a fixed point theorem to prove existence of an equilibrium. In a market if there are increasing returns then if one firm gets larger than others, its long-run average costs may fall below those of others allowing it to undercut its competitors so that they may come to be unable to earn a non-negative profit, which in turn in the end can lead to a natural monopoly as the competitors end up driven out of business eventually, assuming that there is no limit to those economies of scale.

But this outcome brings us to the paradoxical aspect: if indeed there are these unlimited economies of scale, one can end up in a situation where indeed there is an entrenched monopoly that cannot be ousted by newly entering competitors unless there is a fundamental change in technology or some other element of the system that allows for the potentially new entrant to be able to break down this system. But the system can become deeply entrenched and hard to profoundly change. Thus a complex adaptive system might well end up becoming an essentially stagnant and conservative one, stuck in its ways, with all changes simply reinforcing its stasis as positive feedback effects simply drive it deeper and deeper into the condition it has achieved.

So it is that David Colander sees the economics profession having tendencies to simply reinforce itself in an existing state despite being battered by outside forces of change. Some of this pessimism has come from seeing developments in macroeconomics since the financial crisis and Great Recession, when the DSGE model continued to hold sway in a dominant position among practicing policymaking macroeconomists at central banks and in academia, although one that has been tweeked to some degree by ad hoc changes of the sorts mentioned above. Thus he argues (Colander 2015, p. 230): “There are now some discussions in the texts of macro-prudential policy, zero lower bounds, structural stagnation (although much of that discussion goes under the name, secular stagnation), quantitative easing, and even some mention of Minsky moments. But in the underlying macro model of a stable economic system composite aggregate rationality remains.”

Furthermore, drawing on Piketty’s work (Piketty 2014), trends to greater and greater income and wealth inequality seem to be deeply entrenched and hard to overcome or halt, much less reverse. Obviously this is not a simple or straightforward story, and competing trends can coexist at different levels. Thus at the global level we see a trend to increasing aggregate equality due to rising incomes in the two largest nations, China and India, even as we have seen increasing inequality inside most nations, thus undermining the optimism of Simon Kuznets (1955) regarding the implications for income inequality of long-run economic development. Nevertheless this is not inevitable, quite aside from the possibility of major revolutionary political economic upheaval as we saw in the early part of the twentieth century. So some of the most unequal nations, notably some in Latin America, have seen some movement towards greater income equality, if not dramatic (Rosser Jr and Rosser, 2019, Chaps. 18–19). The inequality trend is not inevitable or impossible to overcome.

However, getting back to the economics profession itself, especially in the United States, which dominates the world's economic profession increasingly (Rosser Jr. et al. 2010), this tendency to dynamic self-reinforcement and entrenchment in a path-dependent sort of way may be manifesting itself. Colander particularly sees this operating through the educational system, with the system's conservatism enhanced by what he calls "the 15 percent rule," the idea that leading textbooks cannot change by more than 15 percent at a time due to the unwillingness of established faculty in a field to change their class notes too frequently.

But in the case of the economics profession in particular in response to the financial crisis and the Great Recession we saw an ironically peculiar process in effect. Despite widespread calls for fundamental change coming from many quarters, the crisis generated incentives for the profession not to change, with these incentives reinforcing self-satisfaction and inertia. It operated in the following way according to him: "The larger the crisis, the more students want to hear what economics has to say, more sign up for economics, and more revenue flows into economics, reinforcing the institutional structure. This leads the profession to respond: 'Why change what we are doing? We are doing quite well, thank you" (Colander 2015, p. 234).

Thus we have this paradox that the complex adaptive nature of the economics profession with its increasing returns dynamics ends up enhancing its tendency to stasis and not changing in a fundamental way. The move into a full complexity era may continue, but it is extremely hard to overturn the apple cart and dramatically change the way things are done, to move to a fundamentally new and different kind of economics so. But then, it is the nature of dynamically complex systems to generate surprises with new forms emerging unexpectedly when one least expects them to do, even as we have seen in the grandest and most important of all complex systems, the evolutionary process, which certainly operates in the economics profession as it does in the larger socio-economic system and the even larger ecologic-economic system in which we all live.

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J. B. Rosser, Jr., *Foundations and Applications of Complexity Economics*,
https://doi.org/10.1007/978-3-030-70668-5

Subject Index

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https://doi.org/10.1007/978-3-030-70668-5