

Units for Magnetic Quantities



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Abstract The centimeter-gram-second (CGS) system of units was adopted by the pioneers of electromagnetism in the nineteenth century. By the early twentieth century, two limitations of the CGS system became apparent: its inability to gracefully incorporate the electrical units common in engineering and inconvenient factors of 4π in electromagnetic equations. Giovanni Giorgi was most responsible for the development of the rationalized meter-kilogram-second-ampere system, which evolved into the International System of Units (SI). In 2019, the SI was redefined in terms of seven defining constants of nature, which set the value of the elementary charge. A direct consequence is that the value of the magnetic constant, the permeability of vacuum, is no longer fixed in the SI. Some conversions from CGS electromagnetic units to SI units in an updated conversion table thus involve the redefined permeability of vacuum, whereas other conversions require only powers of 10 and factors of 4π . The effect on magnetism and magnetic measurements is more philosophical than practical.

Keywords Magnetism · Magnetism history · Magnetic units · Electromagnetic units · International System of Units · Giorgi system · Permeability of vacuum · Magnetic constant · Conversion table · Units of measure · Magnetic quantities · International Bureau of Weights and Measures

1 The Centimeter-Gram-Second System of Units

In 1873, the same year that James Clerk Maxwell published the first edition of *A Treatise on Electricity and Magnetism*, the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, under the leadership of William Thomson (later known as Lord Kelvin), presented its first report at the 43rd

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meeting of the British Association for the Advancement of Science. It formally recommended the adoption of the centimeter-gram-second (CGS) system of units [1].

The following year, noting that “students usually find peculiar difficulty in questions relating to units,” the Committee commissioned a book to explain the new CGS system and give examples of its application to physical measurements [2]. The book, authored by the Committee’s secretary, Joseph David Everett, contained an appendix that reproduced the Committee’s first report to the British Association [3].

However, the appendix omitted the dissent for the record by Committee member George Johnstone Stoney, who objected that “the centimetre was recommended as the unit of length against my earnest remonstrance,” stating that “it is far too small.” Stoney predicted that “the metre must in the end be accepted as the standard unit of length” [1] (the British spelling “metre” is used in the original). Indeed, the Committee’s recommendation reversed the decision of its predecessor, the British Association’s Committee for Standards of Electrical Resistance, which had adopted the meter-gram-second (MGS) system [4, 5]. But by 1873, the CGS system was preferred over the MGS system because it had the advantage “of making the value of the density of water practically equal to unity” [1].

The CGS system is an “absolute” system, that is, one based on the fundamental mechanical units of length L , mass M , and time T . Thus, the quantities in the electrostatic (ESU) and electromagnetic (EMU) subsystems of CGS all resolve to whole or fractional powers of centimeters, grams, and seconds. For example, the dimensions for magnetic moment in EMU are $L^{5/2} M^{1/2} T^{-1}$, with units $\text{cm}^{5/2} \cdot \text{g}^{1/2} \cdot \text{s}^{-1}$. Although magnetic moment has no named unit in EMU (recourse is often made to writing “emu” as a pseudo-unit), the units for magnetic moment correspond to those for the ratio of ergs per gauss: $\text{cm}^2 \cdot \text{g} \cdot \text{s}^{-2} / \text{cm}^{-1/2} \cdot \text{g}^{1/2} \cdot \text{s}^{-1}$. (The name “erg” was recommended as the unit for work and energy by the British Association in 1873. The name “gauss” was assigned, initially, to magnetic field strength by the International Electrical Congress in 1900 and, later, to magnetic flux density by the International Electrotechnical Commission in 1930.)

It was the intent of the British Association’s Committee for the Selection and Nomenclature of Dynamical and Electrical Units that “one definite selection of three fundamental units be made once for all” so “that there will be no subsequent necessity for amending it” [1].

It was not to be.

2 The Rationalized Meter-Kilogram-Second-Ampere System

One of Oliver Heaviside’s many accomplishments was the reformulation of Maxwell’s cartesian equations in compact vector calculus notation. He believed that the factor of 4π in electromagnetic equations was simply an illogical convention, and he made a strong case for rationalization of the CGS system, that is, removal of the irrational number 4π in most equations, including those of Maxwell [6].

Giovanni Giorgi viewed rationalization as an optional but convenient adjunct to a four-dimensional, meter-kilogram-second (MKS) system, in which the fourth, electromagnetic unit was initially not specified [7]. Giorgi respectfully submitted preprints of his papers to Heaviside, who was 21 years his senior and quite famous. Heaviside was skeptical, as evidenced by his notations on Giorgi's correspondence, currently in the archives of the International Electrotechnical Commission [8]. In Giorgi's typewritten letter of 11 March 1902 to Heaviside, he outlined the differences between their two systems: "My object was in fact not only to get rid of the 4π , but to bring the practical electrical units into agreement with a set of mechanical units of reasonable size, and then to have a system which is absolute and practical at the same time." Years later, Giorgi extended the classical definition of an absolute system of units by noting the equivalence of mechanical and electrical energy and thus applied the coveted "absolute" adjective to his four-dimensional MKS system [9].

The meaning of the permeability of vacuum μ_0 was central to Giorgi's system [10]. He noted, "In my system, $[\mu_0]$ is not a numeric, nor do I assume any special value for it; it is a physical quantity, having dimensions, and to be measured by experiment" [11]. Thus, he regarded both μ_0 and the permittivity of vacuum ϵ_0 as subject to experimental refinement, with $\mu_0 \approx 1.256 \times 10^{-6}$ henries per meter and $\epsilon_0 \approx 8.842 \times 10^{-12}$ farads per meter, and both subject to the condition that $(\mu_0 \epsilon_0)^{-1/2}$ is equal to the speed of light $c \approx 3 \times 10^8$ m/s. He noted that his four-dimensional system "is neither electrostatic nor electromagnetic, because neither the electric nor the magnetic constant of free ether is assumed as a fundamental unit" [12].

Opposition to the full adoption of Giorgi's system was led by Richard Glazebrook, a former student and intellectual heir of Maxwell, who served as the chair of the Symbols, Units, and Nomenclature (SUN) Commission of the International Union of Pure and Applied Physics. The SUN Commission accepted the three-dimensional MKS as a parallel system but with μ_0 as just a fixed scaling factor with respect to the CGS system [10].

3 The International System of Units

Eventually, in 1954, the 10th General Conference on Weights and Measures (CGPM) approved the ampere as the fourth base unit, thereby formalizing the "MKSA" practical system of units. In 1960, the 11th CGPM adopted the name *Système International d'Unités*, with the abbreviation "SI," for the practical system of units. In the SI, the "definition of the ampere was based on the force between two current carrying conductors and had the effect of fixing the value of the vacuum magnetic permeability μ_0 (also known as the magnetic constant) to be exactly $4\pi \times 10^{-7} \text{ H}\cdot\text{m}^{-1} = 4\pi \times 10^{-7} \text{ N}\cdot\text{A}^{-2}$," [13].

On 16 November 2018, in Versailles, France, the 26th CGPM adopted the most significant change in units of measure since 1954. It went into effect on

20 May 2019, World Metrology Day. The revised SI fixed the values of formerly measurable constants: the Planck constant, h ; the elementary charge, e ; the Boltzmann constant, k ; and the Avogadro constant, N_A , thereby, individually or in combination, redefining the units kilogram, ampere, kelvin, and mole. The cesium 133 hyperfine transition frequency, $\Delta\nu_{\text{Cs}}$; the luminous efficacy of radiation of frequency 540×10^{12} Hz, K_{cd} ; and the speed of light in vacuum, c , had already been fixed by the CGPM in 1967, 1979, and 1983, respectively, which defined the units second, candela, and meter [13].

The motivation for the use of defining constants is explained carefully in the 9th edition of the *SI Brochure* [[13], pp. 125–126]:

Historically, SI units have been presented in terms of a set of—most recently seven—base units. All other units, described as derived units, are constructed as products of powers of the base units.

Different types of definitions for the base units have been used: specific properties of artefacts such as the mass of the international prototype for the unit kilogram; a specific physical state such as the triple point of water for the unit kelvin; idealized experimental prescriptions as in the case of the ampere and the candela; or constants of nature such as the speed of light for the definition of the unit metre.

To be of any practical use, these units not only have to be defined, but they also have to be realized physically for dissemination. In the case of an artefact, the definition and the realization are equivalent—a path that was pursued by advanced ancient civilizations. Although this is simple and clear, artefacts involve the risk of loss, damage or change. The other types of unit definitions are increasingly abstract or idealized. Here, the realizations are separated conceptually from the definitions so that the units can, as a matter of principle, be realized independently at any place and at any time. In addition, new and superior realizations may be introduced as science and technologies develop, without the need to redefine the unit. These advantages—most obviously seen with the history of the definition of the metre from artefacts through an atomic reference transition to the fixed numerical value of the speed of light—led to the decision to define all units by using defining constants.

The choice of the base units was never unique, but grew historically and became familiar to users of the SI. This description in terms of base and derived units is maintained in the present definition of the SI, but has been reformulated as a consequence of adoption of the defining constants.

Instead of the definition of the ampere fixing the value of μ_0 , the 2019 revision of the SI defines the ampere in terms of the fixed value of e . As a result, the value of μ_0 must be determined experimentally. Similarly, the permittivity of vacuum $\epsilon_0 = 1/(\mu_0 c^2)$ must be determined experimentally (as it was before c was fixed in 1983). The product $\mu_0 \epsilon_0 = 1/c^2$ remains exact. The experimental value of μ_0 is now based on that of the dimensionless fine-structure constant α , the coupling constant of the electromagnetic force: $\mu_0 = 2h\alpha/ce^2$, where h is the newly fixed Planck constant, c is the fixed speed of light in vacuum, and e is the newly fixed elementary charge (equal to the absolute value of the electron charge). The relative standard uncertainties in μ_0 , ϵ_0 , and α are identical [14].

It was reasonable to fix the value of e instead of μ_0 because, by the 1990s, the realization of the ampere was by Ohm's law, the Josephson effect for voltage, and the quantum Hall effect for resistance (both in terms of the 1990 recommended values of e and h [15]), not by the force on currents in parallel wires. A definition

of the ampere and the kilogram in terms of fixed values of e and h , respectively, brought the practical quantum electrical standards into exact agreement with the SI [13].

4 Conversion Factors

Conversion tables are helpful for magnetics researchers who want to compare data appearing in published articles. The need will diminish with time as the SI becomes universal for instruction in electromagnetism. Magnetics researchers who currently measure in SI units and analyze using SI equations do not have to worry about conversion factors, but even they occasionally need to refer to published data in EMU.

Units of measure have been examined and reexamined vigorously. The monograph by Silsbee is noteworthy for its completeness [16]. The appendixes in the textbooks by Jackson [17] and Coey [18] are good resources. Few articles deal specifically with units for magnetic properties. Bennett et al. published a conversion guide especially for magnetics in which they pointed out, to the surprise of many, that “emu” is not actually a unit [19]. During an evening panel discussion on magnetic units at the 1994 Joint Magnetism and Magnetic Materials—International Magnetics Conference, different perspectives were advanced by seven practitioners [20], some of whom recapitulated their recent articles or prefaced their future articles on the subject [21, 22, 23].

In the MKSA system and the SI of 1960, μ_0 served both as a conversion factor and as a means for rationalization with respect to EMU. Thus, the 2019 revision of the SI, which made μ_0 an experimental constant, has consequences for magnetics. A conversion guide for magnetic quantities from EMU to SI may now distinguish between conversions based on an experimental determination of μ_0 and conversions based on rationalization of EMU. As first noted by Davis, conversion factors to CGS systems, such as EMU, which made use of the exact relation $\{\mu_0/4\pi\} \equiv 10^{-7}$, are no longer exactly correct after the SI revision of 2019 [24] (The curly brackets mean that one removes the units associated with the quantity within.)

Table 1 is a conversion guide from EMU to SI that reflects the redefinition of the SI. Conversion factors formerly based on the fixed permeability of vacuum $\{\mu_0\} \equiv 4\pi \times 10^{-7}$ are here replaced explicitly by the symbol $\{\mu_0\}$. However, factors based only on the conversion of centimeters to meters, grams to kilograms, and rationalization of EMU retain the factor of 4π ; for example, the sum of the three axial demagnetizing factors of an ellipsoid is 4π in EMU and unity in the SI.

Magnetism in the SI is concordant with the Sommerfeld constitutive relation $B = \mu_0(H + M)$ for magnetic flux density B , magnetic field strength H , and magnetization M . However, magnetic polarization J and magnetic dipole moment j , derived from the Kennelly convention, $B = \mu_0H + J$, are also recognized. In both conventions, B and H have units different from each other.

Table 1 Conversion of units for magnetic quantities. In the right column, $\{\mu_0\}$ refers to the numerical value of μ_0 , the recommended value of which may change slightly over time. Factors of 4π originate from the conversion of unrationalized EMU to rationalized SI units. In the absence of units, a dimensionless quantity is labeled with its associated system of units (EMU or SI). The arrows (\rightarrow) indicate correspondence, not equality. From [10], after [25]

SI Symbol	SI Quantity	Conversion from EMU and Gaussian Units to SI Units ^(a)
Φ	Magnetic flux	$1 \text{ Mx} = 1 \text{ G}\cdot\text{cm}^2 \rightarrow 10^{-8} \text{ Wb} = 10^{-8} \text{ V}\cdot\text{s}$
B	Magnetic flux density, magnetic induction	$1 \text{ G} \rightarrow 10^{-4} \text{ T} = 10^{-4} \text{ Wb/m}^2$
μ	Permeability ^(b)	$1 \text{ (EMU)} \rightarrow \{\mu_0\} \text{ H/m} = \{\mu_0\} \text{ N/A}^2 = \{\mu_0\} \text{ Wb/(A}\cdot\text{m)}$
H	Magnetic field strength, magnetizing force	$1 \text{ Oe} \rightarrow 10^{-3} \{\mu_0\} \text{ A/m}$
m	Magnetic moment	$1 \text{ erg/G} = 1 \text{ emu} \rightarrow 10^{-3} \text{ A}\cdot\text{m}^2 = 10^{-3} \text{ J/T}$
j	Magnetic dipole moment	$1 \text{ erg/G} = 1 \text{ emu} \rightarrow 10^{-3} \{\mu_0\} \text{ Wb}\cdot\text{m}$
M	Magnetization, volume magnetization	$1 \text{ erg/(G}\cdot\text{cm}^3) = 1 \text{ emu/cm}^3 \rightarrow 10^3 \text{ A/m}$ $1 \text{ G} \rightarrow 10^{-4} \{\mu_0\} \text{ A/m}$
J, I	Magnetic polarization, intensity of magnetization	$1 \text{ G} \rightarrow 10^{-4} \text{ T} = 10^{-4} \text{ Wb/m}^2$
σ	Specific magnetization, mass magnetization	$1 \text{ erg/(G}\cdot\text{g)} = 1 \text{ emu/g} \rightarrow 1 \text{ A}\cdot\text{m}^2/\text{kg}$
χ	Susceptibility, volume susceptibility	$1 \text{ (EMU)} \rightarrow 4\pi \text{ (SI)}$
χ^v, χ^m	Specific susceptibility, mass susceptibility	$1 \text{ cm}^3/\text{g} \rightarrow 4\pi \times 10^{-3} \text{ m}^3/\text{kg}$
w, W	Energy product, volume energy density ^(c)	$1 \text{ erg/cm}^3 \rightarrow 10^{-1} \text{ J/m}^3$
N, D	Demagnetizing factor	$1 \text{ (EMU)} \rightarrow (4\pi)^{-1} \text{ (SI)}$

^(a) EMU are the same as Gaussian units for magnetostatics: Mx = maxwell, G = gauss, Oe = oersted. SI: Wb = weber, T = tesla, H = henry, N = newton, J = joule.

^(b) In the SI, relative permeability $\mu_r = \mu/\mu_0 = 1 + \chi$. In EMU, permeability $\mu = 1 + 4\pi\chi$. Relative permeability μ_r in the SI corresponds to permeability μ in EMU.

^(c) In the SI, $w [\text{J/m}^3] = B [\text{T}] \cdot H [\text{A/m}] = \mu_0 [\text{Wb/(A}\cdot\text{m)}] \cdot M [\text{A/m}] \cdot H [\text{A/m}]$. In EMU, $w [\text{erg/cm}^3] = (4\pi)^{-1} B [\text{G}] \cdot H [\text{Oe}] = M [\text{erg/(G}\cdot\text{cm}^3)] \cdot H [\text{Oe}]$.

In EMU, $B = H + 4\pi M$, where B and H have the same units with different names, gauss (G) and oersted (Oe). As has been noted, “the magnetization, when written as $4\pi M$, is also in gauss and may be thought of as a field arising from the magnetic moment. When magnetization is expressed simply as M (the magnetic moment m per unit volume), its units are $\text{erg}\cdot\text{G}^{-1}\cdot\text{cm}^{-3}$. In terms of base units, $\text{erg} = \text{cm}^2\cdot\text{g}\cdot\text{s}^{-2}$ and $\text{G} = \text{cm}^{-1/2}\cdot\text{g}^{1/2}\cdot\text{s}^{-1}$; therefore, $\text{erg}\cdot\text{G}^{-1}\cdot\text{cm}^{-3}$, the units for M , are dimensionally but not numerically equivalent to G^{-1} ” [21].

In the table, dimensionless quantities are labeled with their associated system of units (EMU or SI) to distinguish them. In magnetic materials with permeability μ , $B = \mu H$, where μ is dimensionless in EMU. The conversion of dimensionless volume susceptibility χ from EMU to SI is based on the correspondence between $\mu = 1 + 4\pi\chi$ in EMU and relative permeability $\mu_r = \mu/\mu_0 = 1 + \chi$ in SI; that is, $4\pi\chi$ (EMU) corresponds to χ (SI); $\{\mu_0\}$ is not involved. This also follows from the definition $\chi = M/H$, in both EMU and SI, and $4\pi\chi$ (EMU) having units of gauss per oersted (dimensionless). The conversion of specific (mass) susceptibility follows from that of volume susceptibility.

The SI redefinition of the ampere implies that the EMU abampere (the prefix “ab” means “absolute”) does not convert exactly to 10 amperes, as was similarly footnoted by Quincey and Brown in relation to the abcoulomb and coulomb [26]. This affects the conversion of magnetic field strength H from oersteds (the named unit for gilberts per centimeter, which corresponds to $(4\pi)^{-1}$ abamperes per centimeter) to amperes per meter by requiring the use of $\{\mu_0\}$. Alternatively, the

conversion factor of $10^{-4}/\{\mu_0\}$ in the table may be considered to arise from the equivalence of oersted and gauss in EMU, the conversion of gauss to tesla, and the relationship $B = \mu_0 H$ in vacuum. The same factor is used in the table for the conversion of magnetization M , when formulated as $4\pi M$ in gauss, to amperes per meter.

Conversions based on transformations from gauss to tesla and erg to joule do not involve $\{\mu_0\}$. For example, magnetization in gauss converts to magnetic polarization in tesla without involvement of $\{\mu_0\}$. However, magnetic moment, when expressed in EMU as erg per gauss (or “emu”), converts to magnetic dipole moment in weber meters with a required factor of $\{\mu_0\}$.

5 Epilogue

While the accepted value of $\{\mu_0\}$ will change slightly over time with changes in the experimental fine-structure constant α , $\{\mu_0\}$ is currently equal to $1.256\,637\,0621 \times 10^{-6} \pm 0.000\,000\,0019 \times 10^{-6}$, based on the latest quadrennial adjustment to the fundamental physical constants by the International Science Council’s Committee on Data [27]. That is, the value of $\{\mu_0\}$ is equal to $4\pi \times 10^{-7}$ to nine significant figures. Thus, the distinction between $\{\mu_0\}$ and $4\pi \times 10^{-7}$ is largely philosophical and hardly practical; their difference is much smaller than the total uncertainty in any magnetic measurement.

In the revised SI, it is compelling to regard B as the primary magnetic field vector, μ_0 as an experimental constant, and H as an arithmetically derived auxiliary vector [10]. For displays of measurement data, the symbol B_0 could be used for applied magnetic field in units of tesla, much as $\mu_0 H$ is sometimes used, where B_0 is distinguished from the flux density B in magnetic materials. Magnetic volume susceptibility χ should remain defined as M/H (dimensionless), not M/B_0 , because M/H is embedded historically in EMU, the MKSA system, and the SI.

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