Chapter 17
The Influence of Pressure Angle of Spur Gears on Bending Stress Considering the Effect of Root Fillet Radius

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Abstract Tooth breakage from excessive bending stress and surface pitting from excessive contact stress are the two primary fatigue failure modes for gears. Tooth breakage will end the gear life, so gear bending stress has to be accurately calculated for reliable operation. Many efforts have been made to increase gear bending strength including improving gear material property. With increasing demand for high power density gear applications, the need to optimize gears for minimum stress becomes increasingly important. It is imperative to understand the effect of gear parameters on bending stress in the initial stage of designing gears. Gear pressure angle is an important parameter affecting tooth bending stress. Because the critical section occurs in the gear root fillet, the root fillet radius largely affected by tool (hob) tip radius also has a great significance for improving the bending strength. The maximum tool tip radius will vary with the pressure angle changing due to the geometrical relationship of basic rack. This paper investigates the influence of pressure angle of spur gears on bending stress considering the variation of root fillet radius, and provides a recommendation on how to optimize the pressure angle for high bending strength gears. For analysis and validation of results, three methods of predicting bending stress– ISO standard, 3D-TCA (tooth contact analysis) method and Finite Element Analysis are applied and discussed.

Keywords Gear bending stress · Gear pressure angle · Root fillet radius · Gear material property

17.1 Introduction

Gears are commonly used in automotive, mechanical engineering and other industries. Nowadays, high power density gear applications are increasingly in demand, it is critical to design gears with improved load capacity. Gear tooth pitting due

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to excessive surface contact stress and root fillet fractures due to excessive tooth bending stress are two main fatigue failure modes for gears [1]. The accumulation of defects and a high concentration of stresses in micro defects are also dangerous in the event of shocks which occur during the operation of mechanisms. Micropitting on the tooth surface of the gear can cause it to break [2]. Tooth breakage causes a catastrophic failure, so gear bending stress analysis methods must be reliable and understanding the key influence parameters on bending stress is very important to improve bending strength. Pressure angle is one which plays an essential role in determining the bending stress.

A pressure angle 20° was adopted for standard gears according to ISO 53:1998(E) [3]. The standard pressure angle 20° is a compromise value and cannot meet all the needs of the different applications because of the limited load capacity on root fillet. So in many cases, non-standard pressure angles need to be designed in order to improve the gear performance.

Recently, there have been many efforts made to explore the application of non-standard pressure angle gears. Gupta [4] calculated and compared the maximum bending and contact stress for the low dedendum spur gears with different pressure angles using finite element method. Handschuh [5] investigated the effects of high pressure angle gears compared with typical gear designs, the analysis of contact and bending stress had been done on three gears- standard 3.18 module, 28 tooth and 20° pressure angle, 2.12 module, 42 tooth and 25° pressure angle and 1.59 module, 56 tooth and 35° pressure angle. Sankar [6] studied the effects of pressure angle and tip relief on the failure of a helical gear pairs. Dadhanlya [7] presented a study on the effect of pressure angle on bending stress and deformation of asymmetric involute spur gear using FEA. Oda [8] introduced a study on the effect of pressure angle, helix angle and whole tooth depth on the bending strength.

However, a very critical parameter– root fillet radius is defined by tool (hob) tip radius was not concerned in the research. Since the bending critical section occurs in the gear root fillet, root fillet radius has a great influence on gear bending strength. The root strength can be improved by using a circular fillet design or optimized fillet design according to [9–11]. And the maximum tool tip radius will vary with the pressure angle changing due to the geometrical relationship of basic rack [3]. So in this paper, the bending stress of a spur gear pair with identical dedendum and different pressure angles (14.5°, 17.5°, 20°, 22.5° and 25°) and the variation of root fillet radius are studied.

After reviewing the references, gear bending stress in spur gears can be evaluated with four methods namely, standard methods like ISO standards [12] and AGMA standards [13], 3D-TCA method, Finite Element Analysis (FEA) and experimental methods. In this paper three methods of predicting bending stress—ISO standard, 3D-TCA method and Finite Element Analysis are applied to example gear geometry and compared to make sure the results are valid.

17.2 Geometric Specifications of the Studied Gears

In this study, a spur gear pair with different pressure angles (14.5°, 17.5°, 20°, 22.5° and 25°) and the same dedendum ($h_{fp}/m_n = 1.4$) was investigated. A dedendum equal to 1.4 m_n permits the finishing tool to work without interference, while maintaining the maximum fillet radius and is recommended for high-precision gears transmitting high torques. It is also suitable for gears with tooth flanks finished by grinding [3]. The geometry parameters of the spur gears have been summarised in Table 17.1.

Tool tip radius $\rho_{fp}/m_n = 0.39$ is equivalent to a full radius form for the fillet and is the maximum fillet radius when pressure angle equal to 20° ($h_{fp}/m_n = 1.4$) [3]. To ensure consistency of root radius, only investigate the influence of pressure angle on bending stress, the tool tip radius ρ_{fp}/m_n of spur gear with pressure angle equal to 14.5°, 17.5° and 20° are defined as 0.39. When pressure angle is increased to be greater than 20°, the maximum tool tip radius ρ_{fp}/m_n cannot reach 0.39 according to the geometric relationship of basic rack showed in Fig. 17.1.

A basic rack $h_{fp}/m_n = 1.4$ gives a full radius form for the fillet as showed in Fig. 17.1. The centre of $\rho_{fp\text{-}max}$ is on the centre of the rack space. According to the geometric relationship, the maximum tool tip radius can be derived from Eq. (17.1) [3].

$$\rho_{fp}\max = \frac{\left[\frac{(\pi m)}{4} - h_{fp}\tan\alpha_p\right]}{\tan\left[\left(90^{\circ} - \alpha_p\right)/2\right]} \tag{17.1}$$

where, h_{fp} is the dedendum of basic rack.α_p is the pressure angle of the basic rack.

Table 17.1 Gear parameters for spur gear pair with different pressure angles

Parameters	α = 14.5°		α = 17.5°		α = 20°		α = 22.5°		α = 25°	
	Pinion	Wheel	Pinion	Wheel	Pinion	Wheel	Pinion	Wheel	Pinion	Wheel
Tooth number Z	39	40	39	40	39	40	39	40	39	40
Module m_n [mm]	3	3	3	3	3	3	3	3	3	3
Face width b [mm]	20	20	20	20	20	20	20	20	20	20
Dedendum h_{fp}/m_n	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
Addendum h_{ap}/m_n	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Tool tip radius ρ_{fp}/m_n	0.39	0.39	0.39	0.39	0.39	0.39	0.308	0.308	0.208	0.208
Contact ratio ε_α	2.048		1.846		1.711		1.6		1.51	

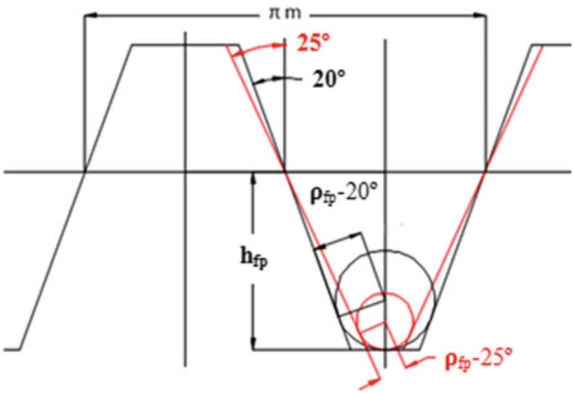


Fig. 17.1 The relationship between pressure angle and maximum tool tip radius ($h_{fp}/m_n = 1.4$)

From the Eq. (17.1), we can see that the maximum tool tip radius ρ_{fp} is correlated to pressure angle and is reduced with pressure angle increasing. The maximum tool tip radius ρ_{fp}/m_n of spur gear with pressure angle equal to 22.5° and 25° showed in Table 17.1 are determined according to Eq. (17.1) equal to 0.308 and 0.208 respectively.

17.3 Methods for Calculating Gear Bending Stress

17.3.1 ISO Standard

ISO standard is a commonly used international standard to determine gear stress. The bending stress equation in ISO 6336-3:2006 [12] based on cantilevered beam bending is given as Eq. (17.2):

$$\sigma_F = K_A K_V K_{F\alpha} K_{F\beta} \frac{F_t}{bm_n} Y_F Y_S Y_\beta Y_B Y_{DT} \tag{17.2}$$

where σ_{F0} is the nominal tooth root stress; Y_F is the form factor, which is defined in Eq. (17.3):

$$Y_F = \frac{\frac{6h_{Fe}}{m_n} cos\alpha_{Fen}}{\left(\frac{S_{Fn}}{m_n}\right)^2 cos\alpha_n} \tag{17.3}$$

Y_S is the stress correction factor. Equations (17.4), (17.5) and (17.6) define Y_S, L and q_s respectively:

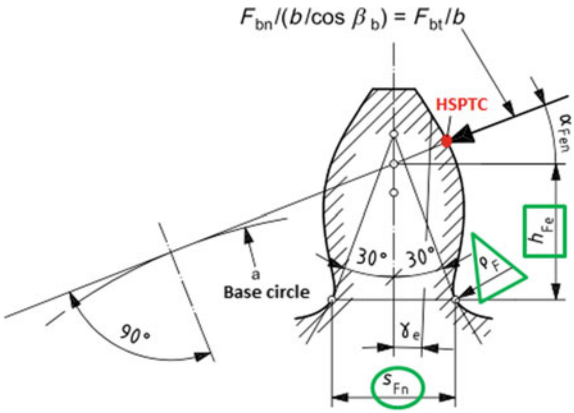


Fig. 17.2 The determination of s_{Fn}, h_{Fe} and ρ_F from [12]

$$Y_s = (1.2 + 0.13L)q_s^{\left[\frac{1}{1.21+\frac{2.3}{L}}\right]} \tag{17.4}$$

$$L = \frac{s_{Fn}}{h_{Fe}} \tag{17.5}$$

$$q_s = \frac{s_{Fn}}{2\rho_F} \tag{17.6}$$

Y_β is the helix angle factor.
Y_B is the rim thickness factor.
Y_{DT} is the deep tooth factor.
$K_A K_V K_{F\beta} K_{F\alpha}$ are the load correction factors.
s_{Fn} is the tooth root chord at the critical section.
h_{Fe} is the bending moment arm for tooth root stress relevant to load application at the outer point of single pair tooth contact.
ρ_F is the tooth root fillet radius at the critical section.
The determination of s_{Fn}, h_{Fe} and ρ_F is showed in Fig. 17.2.

17.3.2 3D-TCA Method

Analytical Tooth Contact Analysis (TCA) method is another method to predict gear tooth root bending stress. A3D-TCA software named GATES [14] (Gear Analysis for Transmission Error and Stress) which was initially developed and tested at Design Unit, Newcastle university is used to calculate gear bending stress in this study. It is an FE based analysis package, using a full 3D FEA stiffness model to estimate the gear stiffness and then using contact analysis to estimate the load distribution, stress and other functional parameters.

The bending stress determined in GATES is similar to ISO 6336 standard, calculating the bending stress at 30 degree tangent position, except the load distribution is calculated from the TCA, stiffness variation across the face width and compressive load is considered.

A critical point for bending stress determined by GATES is extended contact which occurs as the loaded tooth is restored to its original un-deflected state at the end of active profile [15]. If tip relief is not applied, the extended contact will increase the contact ratio thus lowering the actual HPSTC (for spur gear, the bending stress is calculated with load applied at the highest point of single tooth contact (HPSTC)) and then lower the actual bending stress. If too much tip relief is applied, the loading point will increases which will increase the bending stress. Therefore bending stress derived from GATES must be calculated with correct tip relief to make sure the gear loaded at exactly the same position as ISO (HPSTC). In this study, to ensure the results from GATES are valid, the correct tip relief are applied to gears.

17.3.3 Finite Element Analysis

In recent years, finite element analysis is widely used to evaluate gear stress, resulting in an abundance of published research [16, 17]. According to [16], in this study, the finite element analysis of gear bending stress is set as follows:

(1) Three teeth are used to calculate gear stress.
(2) Boundary conditions: fix the three free faces as showed in Fig. 17.3.

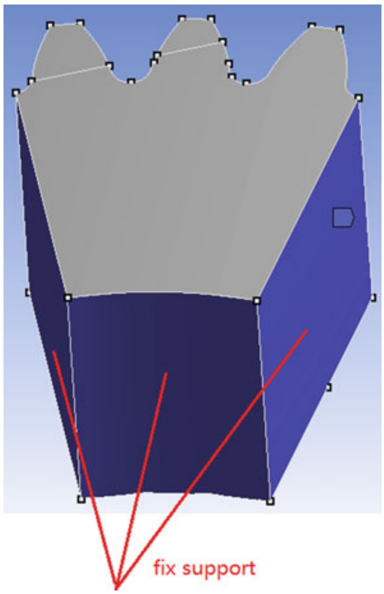


Fig. 17.3 The boundary conditions of bending stress calculation in FEA

(3) Mesh types: automatic mesh method.
(4) Element size is defined in Eqs. (17.7) and (17.8):

$$M_{\text{bending}} = 0.2\rho_{\text{F}} + 0.15 \text{ For } \rho_{\text{F}} > 0.5\text{mm} \tag{17.7}$$

$$M_{\text{bending}} = 0.366\rho_{\text{F}}^{0.614} \text{ For } \rho_{\text{F}} \leq 0.5\text{mm} \tag{17.8}$$

(5) Maximum principal stress is used to represent the gear stress.
(6) Use smaller element sizes to verify the validity of the maximum bending stress to ensure the consistency of the results.

17.4 Results and Discussions

In this paper three independent methods of calculating bending stress—ISO standard, 3D-TCA method and Finite Element Analysis are used to make sure the results are valid. The bending stresses for spur gears with different pressure angle (14.5°, 17.5°, 20°, 22.5° and 25°) from GATES and FEA are showed in Figs. 17.4 and 17.5. The bending stresses from ISO and TCA and FEA are summarized in Table 17.2 and Fig. 17.6. In this study, only bending stresses of the pinion are calculated and compared.

From Table 17.2, it can be seen that:

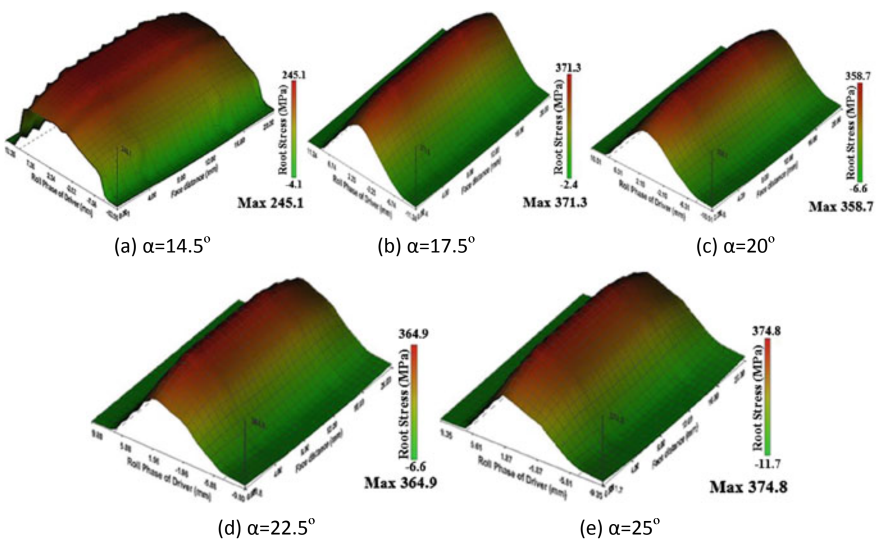
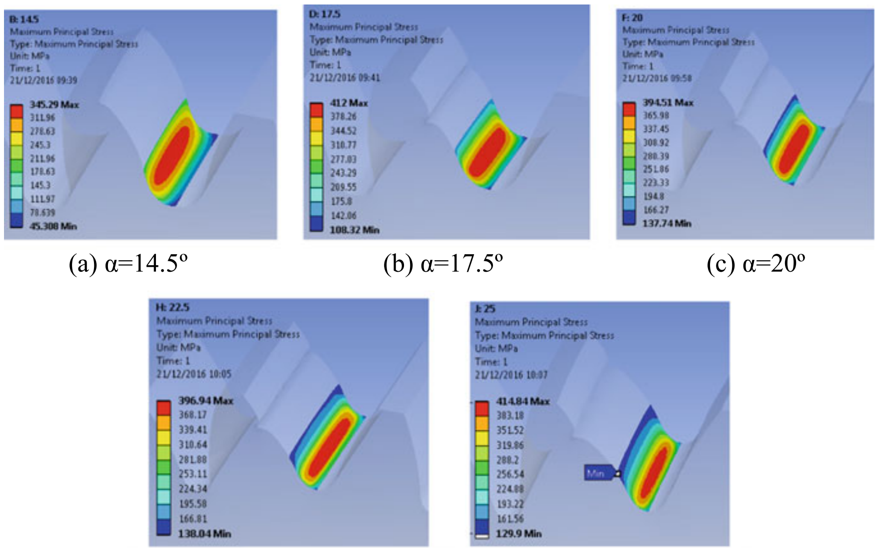


Fig. 17.4 Bending stress from GATES with tip relief for spur gear



(a) α=14.5° (b) α=17.5° (c) α=20°

(d) α=22.5° (e) α=25°

Fig. 17.5 Bending stress from FEA for spur gear

Table 17.2 Bending stresses (MPa) for gears with different pressure angle from ISO, GATES and FEA

ε_α	α	σ_{F_ISO}	$\sigma_{F\text{-}FEA}$	$\sigma_{F\text{-GATES-tip relief}}$
2.048	14.5	289.7797	345.29	245.1
1.846	17.5	394.376	412	371.3
1.711	20	381.779	394.51	358.7
1.6	22.5	391.368	396.94	364.9
1.51	25	404.729	414.84	374.8

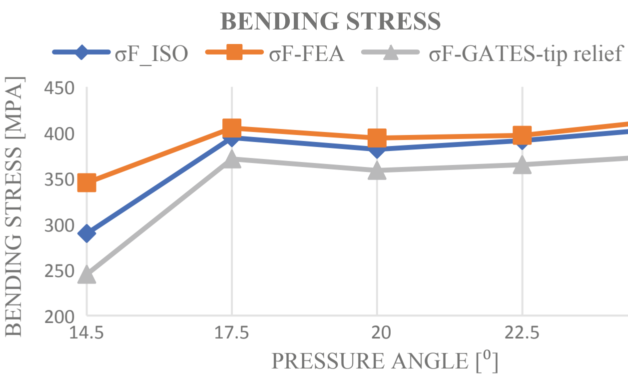


Fig. 17.6 Bending stresses for spur gears with different pressure angle from ISO, GATES and FEA

(1) The variations of bending stresses from ISO, GATES with correct tip relief and FEA with pressure angle changing have the same trends, which can be clearly seen from Fig. 17.6.
(2) The bending stress is minimum when pressure angle equal to 14.5°.
(3) Bending stress is reduced when pressure angle changing from 17.5° to 20°.
(4) While bending stress is increased when pressure angle changing from 20° to 22.5° and 25°.

There are some reasons to explain the results.

(1) The transverse contact ratio ε_α is 2.048 greater than 2.0 when pressure angle equal to 14.5°, which means there are at least 2 teeth sharing load when gear meshing as shown in Fig. 17.7. This results in lower bending stresses, without considering the effects of manufacturing deviations and misalignment.
(2) When pressure angle is increased from 17.5° to 20°, the tooth root chord at the critical section s_{Fn} is increased, the bending moment arm h_{Fe} is also slightly higher, while the root radius ρ_F is almost same. The detailed s_{Fn}, h_{Fe} and ρ_F are showed in Table 17.3, which will lead to a lower form factor Y_F and a slightly higher stress correction factor Y_s, while the decrement of form factor is greater than the increment of stress correction factor, so bending stress decreases when pressure angle changes from 17.5° to 20°.
(3) When pressure angle is increased from 20° to 22.5° and 25°, the tooth root chord at the critical section s_{Fn} and bending moment arm h_{Fe} are also increased, and the root radius ρ_F is largely reduced due to the significant declining of tool tip radius

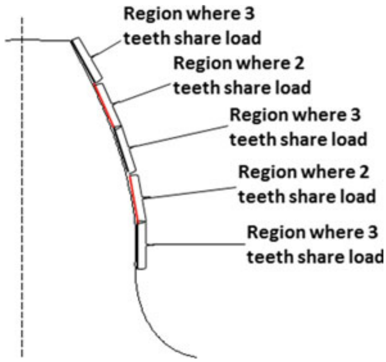


Fig. 17.7 Number of teeth sharing load condition when $2 < \varepsilon_\alpha < 3$ for spur gears

Table 17.3 The details of bending stress key factors in the calculation from ISO

α [°]	S_{Fn}/m_n	h_{Fe}/m_n	ρ_{fp}/m_n	ρ_F/m_n	q_s	Y_F	Y_S
17.5	2.053	1.047	0.39	0.585	1.755	1.503	1.842
20	2.158	1.097	0.39	0.583	1.85	1.421	1.886
22.5	2.273	1.183	0.308	0.528	2.153	1.378	1.994
25	2.405	1.28	0.208	0.463	2.599	1.329	2.138

ρ_{fp}/m_n from 0.39 to 0.308 and 0.208. The calculated S_{Fn}, h_{Fe} and ρ_F are showed in Table 17.3. The large reduction root radius ρ_F leads to significant increase in notch parameter q_s, and dramatic increases stress correction factor Y_s which is greater than the form factor Y_F reduction. So bending stress increases when pressure angle changes from 20° to 22.5° and 25°.

17.5 Conclusions

From the analysis presented in this study, the following conclusions can be drawn:

(1) The variations of bending stresses from ISO, GATES with correct tip relief and FEA with pressure angle changing have the same trends. The results in this study are valid.
(2) For basic rack $h_{fp}/m_n = 1.4$, according to the geometric relationship, the maximum tool tip radius (ρ_{fp}/m_n) of spur gear with pressure angle equal to 20°, 22.5° and 25° are equal to 0.39, 0.308 and 0.208 respectively. It can greatly affect the bending stress, so we must consider its influence when we investigate the effect of pressure angle on bending stress.
(3) Smaller pressure angle such as 14.5° in this study, can result in high contact ratio spur gears ($\varepsilon_\alpha > 2$), at least two teeth sharing the load when gears are in mesh, so it can get lower bending stress. But we must make sure this is maintained when considering manufacturing deviations, elastic deflections and misalignment of shaft and micro modifications, the gears always have transverse contact ratio greater than 2.0, otherwise, the benefit of lower bending stress will disappear.
(4) For the studied gears with $h_{fp}/m_n = 1.4$, the calculated bending stress reduced when pressure angle changing from 17.5° to 20° ($1 < \varepsilon_\alpha < 2$). And the bending stress increases when pressure angle changes from 20° to 22.5° and 25° ($1 < \varepsilon_\alpha < 2$).

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