Design of Coupled FIR Filters for Solving the Nuclear Reactor Point Kinetics Equations with Feedback

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Abstract A new method of solving the nuclear reactor point kinetics equations with feedback is presented in this chapter. In small nuclear reactors, the reactor power transients are estimated by solving the stiff point kinetics equations with feedback. Here, a new computational method is developed using finite impulse response (FIR) filters for solving the stiff point kinetics equation with feedback. The point kinetics equations are converted into convolution equation by applying discrete Z transform. The power and precursor concentrations, appearing in the point kinetics equations, are written in terms of convolution equation with different impulse response functions. The impulse response functions characterize the FIR filter. This method is applied to estimate the transients in few benchmark thermal reactors for different types of reactivity perturbations with temperature feedback, i.e., step, ramp, and oscillatory reactivity inputs. This method has high stability, i.e., a small change in the time step of the order of 5 or 10 does not lead to large error in the solution. The transients estimated by this method are compared with other standard methods and they are found to be in good agreement.

Keywords Finite impulse response · Reactor · Transient

1 Introduction

The power transients in nuclear reactors are estimated by solving the time-dependant neutron diffusion equation in three dimensions. For small reactors, the point kinetics equations are sufficient in predicting the power transients caused by reactivity perturbations. The prediction of reactor power under reactivity perturbation is important from the safety point of view. The point kinetics equations describe the space-independent time-evolution of nuclear reactor power and precursor

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concentrations under reactivity perturbation. The point kinetics equations are stiff differential equations, and they require a very small time step to solve the equations. There are various methods to solve the point kinetics equations. Aboanber and Nahla [1, 2] developed the analytical inversion method for solving the point reactor kinetics equations with temperature feedback. Nahla [3] applied Taylor's Series Method (TSM) for solving the point kinetics equations. Aboanber [4] and Nahla [5] developed the analytical exponential method and the generalized Runge–Kutta method for solving the point kinetics equations. Li et al. [6] presented the better basis function (BBF) method for solving the point kinetics equations. Recently the modified exponential time differencing method was developed [7] to solve the point kinetics equations using large time step. The major constraint in solving the stiff point kinetics equations is the proper selection of time step. In most of the cases, a small change in the time step may lead to large error in the solution of point kinetics equation.

In the present work, a new computational method is developed using the finite impulse response (FIR) filters for solving the reactor point kinetics equations with feedback. According to this new computational method, the power and precursor concentrations, appearing in the point kinetics equations, are written as convolution integrals. The convolution integrals are solved using discrete Z transform. By applying inverse Z transform, the power and precursor concentrations are written as simple convolution equation with different impulse response functions. The impulse response functions characterize the FIR filters. Here, the impulse response functions are chosen according to the type of reactivity perturbation. By appropriately choosing the impulse response functions, the FIR filters can be designed for solving the point kinetics equations with feedback. The impulse response functions are different for power and precursor concentrations. The impulse response functions are found to be stable and possess finite radius of convergence. This new computational method is applied to estimate the nuclear reactor power transient in few benchmark thermal reactors for different types of reactivity perturbations, i.e., step, ramp, and oscillatory. In all the cases, the estimated power transient is found to be in good agreement with the standard methods. The advantage of this computational method is that the power transient can be estimated using large time step without losing accuracy, and this method has high stability, i.e., a change in the sampling time interval by a factor of 5 or 10 does not alter the solution to a larger extent. In all the cases, the estimated power transient, for various types of reactivity perturbations with feedback, is found to be in good agreement with the reference results. A scheme to choose the sampling time interval is also discussed.

2 Point Kinetics Equations and FIR Filters

Consider the point kinetics equations [8] describing the nuclear reactor power transient:

$$\frac{dp(t)}{dt} = \left(\frac{\rho(t) - \beta}{\Lambda}\right) p(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{1}$$

$$\frac{dC_i}{dt} = \left(\frac{\beta_i}{\Lambda}\right) p(t) - \lambda_i C_i, \quad (i = 1, 2, \ldots 6) \tag{2}$$

In the above Eqs. (1) and (2), p is the power, Λ is the prompt neutron generation time, β_i is the effective fraction of the ith group of delayed neutrons, β is the total effective fraction of delayed neutrons $\left(\beta = \sum_{i=1}^{6} \beta_i\right)$, and λ_i and C_i are the decay constant and precursor concentration of the ith group of the delayed neutron. The initial conditions of the point kinetics equations are given as $p(t = 0) = p_0$, $c_i\ (t = 0) = \frac{\beta_i}{\Lambda \lambda_i} p_0$, where p_0 is the steady state power before the introduction of any external reactivity. In the above equation, $\rho(t) = \rho_{\text{ex}}(t) + \rho_{\text{fb}}(t)$ is the net reactivity acting on the reactor, $\rho_{\text{ex}}(t)$ is the external reactivity, and $\rho_{\text{fb}}(t)$ is the feedback reactivity. In the case of constant reactivity insertion (without feedback), $\rho(t) = \rho_{\text{ex}}(t) = \rho_0$, and the solution of Eqs. (1) and (2) can be written as:

$$p(t) = \sum_{i=1}^{6} \lambda_i \int_{-\infty}^{t} e^{\left(\frac{\rho_0 - \beta}{\Lambda}\right)(t-\tau)} C_i(\tau)\, d\tau \tag{3}$$

$$C_i(t) = \left(\frac{\beta_i}{\Lambda}\right) \int_{-\infty}^{t} e^{-\lambda_i (t-\tau)} p(\tau)\, d\tau \tag{4}$$

Equations (3) and (4) are rewritten as:

$$p(t) = \sum_{i=1}^{6} \lambda_i \int_{-\infty}^{0} e^{\left(\frac{\rho_0 - \beta}{\Lambda}\right)(t-\tau)} C_i(\tau)\, d\tau + \sum_{i=1}^{6} \lambda_i \int_{0}^{t} e^{\left(\frac{\rho_0 - \beta}{\Lambda}\right)(t-\tau)} C_i(\tau)\, d\tau \tag{5a}$$

$$C_i(t) = \left(\frac{\beta_i}{\Lambda}\right) \int_{-\infty}^{0} e^{-\lambda_i (t-\tau)} p(\tau)\, d\tau + \left(\frac{\beta_i}{\Lambda}\right) \int_{0}^{t} e^{-\lambda_i (t-\tau)} p(\tau)\, d\tau \tag{5b}$$

It is assumed that before the application of reactivity perturbation, i.e., $t \leq 0$, the reactor is at constant power, i.e., $p(t) = p_0$, $C_i(t) = C_0$ and net reactivity acting on the reactor is zero. Under this assumption, Eqs. (5a) and (5b) are rewritten as:

$$p(t) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)t} + \sum_{i=1}^{6} \lambda_i \int_0^t e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)(t-\tau)} C_i\,(\tau)\,d\tau \tag{6a}$$

$$C_i(t) = C_0 e^{-\lambda_i t} + \left(\frac{\beta_i}{\Lambda}\right) \int_0^t e^{-\lambda_i (t-\tau)}\, p\,(\tau)\,d\tau \tag{6b}$$

The integrals appearing in Eqs. (6a) and (6b) are convolution integrals. Using Z transform [9], the convolution integrals (Eqs. 6a and 6b) are written as:

$$p(t) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)t} + T_s \sum_{i=1}^{6} \lambda_i g(Z) C_i(Z) \tag{7a}$$

$$C_i(t) = C_0 e^{-\lambda_i t} + T_s \left(\frac{\beta_i}{\Lambda}\right) h(Z) p(Z) \tag{7b}$$

where T_s is the sampling period,

$$\begin{aligned} &g(Z) = \sum_{n=0}^{\infty} g\,[n]\,z^{-n}, \quad h(Z) = \sum_{n=0}^{\infty} h\,[n]\,z^{-n}, \quad p(Z) = \sum_{n=0}^{\infty} p\,[n]\,z^{-n}, \\ &C_i(Z) = \sum_{n=0}^{\infty} C_i\,[n]\,z^{-n}, \quad g\,[n] = e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)n} \text{ and } h_i\,[n] = e^{-\lambda_i n} \end{aligned} \tag{8}$$

Using Eq. (8) and making use of inverse Z transform [9], the power and precursor concentrations (Eqs. 7a and 7b) are written as:

$$p(n) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)n} + T_s \sum_{i=1}^{6} \lambda_i \sum_{m=0}^{n} g\,[n-m]\,C_i\,[n] \tag{9}$$

$$C_i(n) = C_0 e^{-\lambda_i n} + T_s \left(\frac{\beta_i}{\Lambda}\right) \sum_{m=0}^{n} h_i\,[n-m]\,p\,[n] \tag{10}$$

Equations (9) and (10) are the representation of finite impulse response (FIR) filters. In the above equations, $g\,[n] = e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)n}$ is the impulse response function for calculating the power (step reactivity without feedback) and $h_i\,[n] = e^{-\lambda_i n}$ is the impulse response function for calculating the precursor concentration. Here, the FIR filters, (Eqs. 9 and 10), are coupled, i.e., to calculate $p(n)$, the value of $C_i[n]$

is required and to calculate $C_i[n]$, the value of $p(n)$ is required. The power and precursor concentration are obtained from coupled FIR filters as follows. First, an initial guess about $p(n, n > 1)$ is assumed and this is used to get the value of $C_i(n)$. This $C_i(n)$ is again used to get the value of $p(n)$. This process is repeated iteratively till the values of $p(n)$ and $C_i(n)$ are converged. The coupled form of realization of FIR filters for solving the point kinetics equations (Eqs. 9 and 10) with one group of delayed neutron precursor is shown in Fig. 1.

Denoting $\sum_{n=0}^{\infty} g[n-m]\, C_i[n] = g[n] * C_i[n] = C_i[n] * g[n] = y_1[n]$ and $\sum_{n=0}^{\infty} h_i[n-m]\, p[n] = h_i[n] * p[n] = p[n] * h_i[n] = y_{2i}[n]$, the power and precursor concentrations (Eqs. 9 and 10) are rewritten as:

$$p(n) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)n} + T_s \sum_{i=1}^{6} \lambda_i y_1[n] \tag{11}$$

$$C_i(n) = C_0 e^{-\lambda_i n} + T_s \left(\frac{\beta_i}{\Lambda}\right) y_{2i}[n] \tag{12}$$

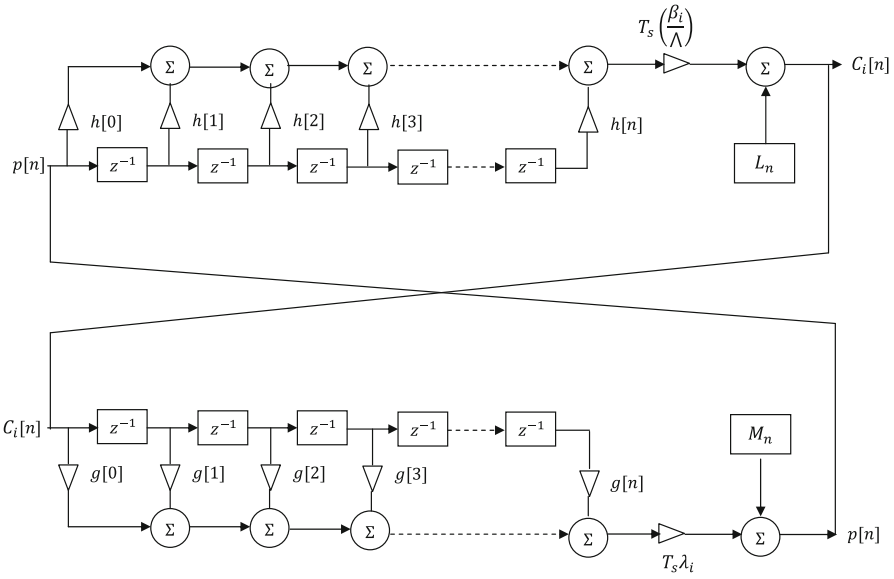

Fig. 1 Realization of coupled FIR filters for solving the point kinetics equations for step reactivity without feedback (assuming one group delayed neutron precursor). $L_n = C_0 e^{-(\lambda_i T_s)n}$ and $M_n = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)T_s n}$

Equations (11) and (12) do not satisfy the initial boundary condition, i.e., to satisfy the initial condition, the impulse response functions, $y_1[n]$ and $y_{2i}[n]$, are improved such that:

$$\tilde{y_1}[n] = y_1[n] - \frac{1}{2}\left[g[n]\,C_i[0] + g[0]\,C_i[n]\right] \tag{13}$$

$$\tilde{y_{2i}}[n] = y_{2i}[n] - \frac{1}{2}\left[h_i[n]\,p[0] + h_i[0]\,p[n]\right] \tag{14}$$

Using the improved impulse response functions, (Eqs. 13 and 14), the FIR filter representations of power and precursor concentrations are given as:

$$p(n) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)n} + T_s \sum_{i=1}^{6} \lambda_i \tilde{y_1}[n] \tag{15}$$

$$C_i(n) = C_0 e^{-\lambda_i n} + T_s \left(\frac{\beta_i}{\Lambda}\right) \tilde{y_{2i}}[n] \tag{16}$$

3 Selection of Sampling Time Interval T_s

For step reactivity (constant input) insertions ($|\rho_0| < \beta$) without feedback, the impulse response functions for power and precursor concentrations are found to be $g[n] = e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)T_s n}$ and $h_i[n] = e^{-\lambda_i T_s n}$, respectively. In this case, the radius of convergence of the impulse response function $g[n]$ is given by $|Z| > e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)T_s}$ and the radius of convergence of $h_i[n]$ is given by $|Z| > e^{-\lambda_i T_s}$ for the precursor concentration "i". For minimum sampling time interval, the radius of convergence is 1 and for maximum sampling time interval, the radius of convergence is 0. In this way, the radius of convergence lies between zero and one, i.e., $0 < |Z| < 1$. This is shown in Figs. 2a, b for power and precursor concentrations. By increasing the number of terms in the summation in Eqs. (9) and (10), the power and precursor concentrations can be accurately estimated. In other words, for a given transient duration, by choosing small sampling time interval, power and precursor concentrations can be estimated accurately. This is equivalent to choosing the radius of convergence nearer to one. Hence by fixing the radius of convergence nearer to one, the sampling time interval, T_s, can be estimated. In the present case, the radius of convergence is fixed as 0.9 and the sampling time interval, for power, is found to be $T_s = \frac{\log_e(0.9)}{\left(\frac{\rho_0-\beta}{\Lambda}\right)}$. In a similar way, the sampling time interval for precursor

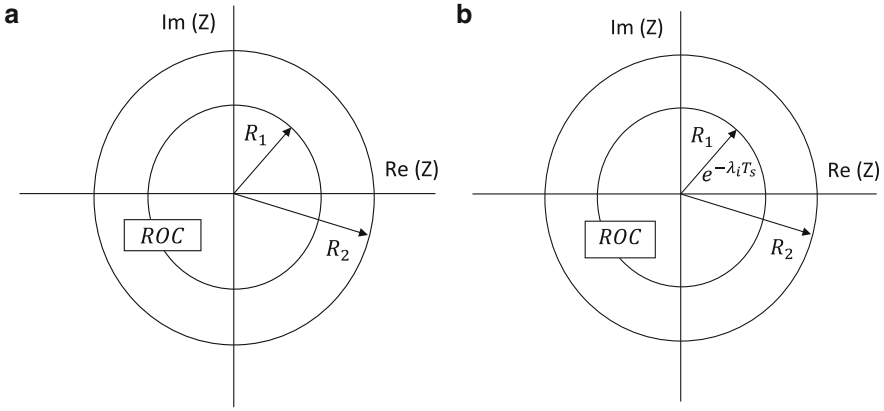

Fig. 2 (**a**) Region of convergence (ROC) of impulse response function $g[n]$ for power under step reactivity of insertion ($|\rho_0| < \beta$) without feedback. The region of convergence is $0 < |Z| < 1$, $R_1 = e^{\left(\frac{\rho_0 - \beta}{\Lambda}\right)T_s}$, and $R_2 = 1$. (**b**) Region of convergence (ROC) of impulse response function $h_i[n]$ for precursor concentration. The region of convergence is $0 < |Z| < 1$, $R_1 = e^{e^{-\lambda_i T_s}}$ and $R_2 = 1$

concentration (using $h_i[n]$) is found to be $T_s(i) = \frac{\log_e(0.9)}{\lambda_i}$. The minimum of T_s and $T_s(i)$ is taken as the sampling time interval.

4 Numerical Results

4.1 Transient from Step Reactivity Without Feedback

Consider the power transients of the thermal reactor described by [3]. The decay constants of the neutron precursors and the delayed neutron fractions of the thermal reactor are taken as $\lambda_1 = 0.0127\,s^{-1}$, $\lambda_2 = 0.0317\,s^{-1}$, $\lambda_3 = 0.115\,s^{-1}$, $\lambda_4 = 0.311\,s^{-1}$, $\lambda_5 = 1.4\,s^{-1}$, $\lambda_6 = 3.87\,s^{-1}$, $\beta_1 = 0.000285$, $\beta_2 = 0.0015975$, $\beta_3 = 0.00141$, $\beta_4 = 0.0030525$, $\beta_5 = 0.00096$, $\beta_6 = 0.000195$, and $\Lambda = 5.0 \times 10^{-4}$ s. Step reactivities $\rho_0 = -1\$$, $\rho_0 = -0.5\$$, $\rho_0 = +0.5\$$ and $\rho_0 = 1.0\$$ are inserted and the resulting power transient is computed using coupled FIR filters. Table 1 shows the values of the power transients obtained from coupled FIR filters along with the exact values given by Nahla [3]. The absolute errors, $|(X_{cal} - X_{exact})|$, are shown in Table 1. From the Table 1, it is observed that the coupled FIR method is capable of estimating the transient to a good accuracy. It is also shown in Table 2 that as the sampling time interval is changed by a factor of 10 or 20, the error in the estimation of power transient is small, indicating that this method has high stability against the change in the sampling time interval. The impulse response functions for power and precursor concentrations are found

Table 1 The power estimated by the coupled FIR filters and the exact values (Nahla [3])

Reactivity	Time	Exact value	Coupled FIR method ($T_s = 1.0$e-3s)	Absolute error
−1.0$	1.00	0.43333	0.43691	0.00358
	10.0	0.23611	0.23687	0.00076
−0.5$	1.00	0.60705	0.61044	0.00339
	10.0	0.39607	0.39701	0.00094
+0.5$	1.00	2.51149	2.46761	0.04388
	10.0	14.2150	14.0498	0.16520
+1.0$	0.50	10.3562	10.3531	0.00310
	1.00	32.1448	32.1356	0.00920

Table 2 The absolute error in the estimation of power transient as the sampling time interval (T_s) is varied

T_s	Reactivity	Time	Exact value	Coupled FIR method	Absolute error
0.001 s	+1.0$	1.0	32.1356	32.1835	0.04790
0.01 s	+1.0$	1.0	32.1356	31.8037	0.37980
0.02 s	+1.0$	1.0	32.1356	31.4398	0.74370

to be $g[n] = e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)n}$ and $h_i[n] = e^{-\lambda_i n}$, respectively. In this case, the radius of convergence of the impulse response function $g[n]$ is given by $|Z| > e^{\left(\frac{\rho_0-\beta}{\Lambda}\right)}$ and the radius of convergence of $h_i[n]$ is given by $|Z| > e^{-\lambda_i}$ for the precursor concentration "i". In general, the radius of convergence of $h_i[n]$ can be taken to be $|Z| > e^{-\lambda_0}$, where λ_0 is the minimum value of decay constant of the precursor group.

4.2 Transient from Step Reactivity with Temperature Feedback

Consider another example of thermal reactor described by Nahla [3] with the following parameters: $\lambda_1 = 0.0124\,\mathrm{s}^{-1}$, $\lambda_2 = 0.0305\,\mathrm{s}^{-1}$, $\lambda_3 = 0.111\,\mathrm{s}^{-1}$, $\lambda_4 = 0.301\,\mathrm{s}^{-1}$, $\lambda_5 = 1.13\,\mathrm{s}^{-1}$, $\lambda_6 = 3.0\,\mathrm{s}^{-1}$, $\beta_1 = 0.00021$, $\beta_2 = 0.00141$, $\beta_3 = 0.00127$, $\beta_4 = 0.00255$, $\beta_5 = 0.00074$, $\beta_6 = 0.00027$, and $\Lambda = 5.0 \times 10^{-5}$ s. A step reactivity $\rho_0 = 0.5$\$ is inserted, and the temperature rise ($T(t)$) with power ($p(t)$) in the reactor is given by:

$$\frac{\partial T(t)}{\partial t} = 0.05\ p(t)\ ^{\circ}\mathrm{C/s}$$

The feedback reactivity is given by [3]:

$$\frac{\partial \rho_{\mathrm{fb}}}{\partial T} = -5.0 \times 10^{-5} \left(\frac{\Delta k}{k}\right) /^{\circ}\mathrm{C}$$

Table 3 The peak power computed using coupled FIR filter ($T_s = 1.0\text{e-}3$) for various step reactivity insertions with temperature feedback

Reactivity	Peak power		Time (s) of occurrence of peak power	
	Coupled FIR filter	TSM	Coupled FIR filter	TSM
+0.5$	44.429	45.754	28.07	28.29
+1.0$	808.0851	807.8765	0.954	0.953
+1.2$	8020.365	8020.848	0.323	0.317
+1.5$	43,023.16	43,021.00	0.174	0.168
+2.0$	167,844.6	167,739.00	0.103	0.098

The estimated peak power is compared with Taylor Series Method (TSM) (Nahla [3])

With temperature feedback, the power and precursor concentration are given by:

$$p(n) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)n} + T_s \sum_{i=1}^{6} \lambda_i \sum_{m=0}^{n} g\,[n-m]\, C_i\,[n] + T_s \sum_{m=0}^{n} g\,[n-m] \left(\frac{\rho_{fb}\,[n]\, p\,[n]}{\Lambda}\right) \tag{17}$$

$$C_i(n) = C_0 e^{-\lambda_i n} + T_s \left(\frac{\beta_i}{\Lambda}\right) \sum_{m=0}^{n} h_i\,[n-m]\, p\,[n] \tag{18}$$

The peak power and the time of occurrence of peak power, under the temperature feedback, are estimated using the coupled FIR filters for various step reactivity insertions. The results are given in Table 3 along with that obtained using Taylor series method (TSM) [3].

4.3 Transient from Ramp Reactivity Without Feedback

4.3.1 Transient from Positive Ramp Reactivity

Consider an example of thermal reactor described by Nahla [5], with the following parameters: $\lambda_1 = 0.0127\,s^{-1}$, $\lambda_2 = 0.0317\,s^{-1}$, $\lambda_3 = 0.115\,s^{-1}$, $\lambda_4 = 0.311\,s^{-1}$, $\lambda_5 = 1.4\,s^{-1}$, $\lambda_6 = 3.87\,s^{-1}$, $\beta_1 = 0.000266$, $\beta_2 = 0.001491$, $\beta_3 = 0.001316$, $\beta_4 = 0.002849$, $\beta_5 = 0.000896$, $\beta_6 = 0.000182$, and $\Lambda = 2.0 \times 10^{-5}$ s. A positive ramp reactivity of the form $\rho(t) = (0.25\$)t/s$ and $\rho(t) = (0.5\$)t/s$ is inserted in the reactor, the transient following this reactivity is estimated by coupled FIR filter, and the result is compared with that of GAEM method [5]. The results are given in Tables 4 and 5. In this case, the power and precursor concentrations are given by:

Table 4 The power transient computed using coupled FIR filter (T_s = 1.0e-3) for ramp reactivity 0.25$/s

Time	Coupled FIR method (T_s = 5.0e-5s) 0.25$/s	GAEM	Absolute error
0.25	1.070897	1.069541	0.001356
0.50	1.159835	1.156694	0.003141
0.75	1.271031	1.265331	0.005700
1.00	1.411403	1.401981	0.009422

The power transient is compared with the GAEM method (Nahla [3]). The absolute error is shown

Table 5 The power is computed using coupled FIR filter (T_s = 1.0e-3) for ramp reactivity 0.50$/s, and it is compared with the GAEM method (Nahla [3])

Time	Coupled FIR method (T_s = 5.0e-5s) 0.5$/s	GAEM	Absolute error
0.25	1.152200	1.149200	0.00300
0.50	1.377465	1.368927	0.00853
0.75	1.727601	1.707600	0.02000
1.00	2.322041	2.275271	0.04677

The absolute error is shown

$$p(n) = p_0 e^{\left(\frac{-\beta}{\Lambda}\right)n} + T_s \sum_{i=1}^{6} \lambda_i \sum_{m=0}^{n} k\,[n-m]\, C_i\,[n] + T_s \sum_{m=0}^{n} k\,[n-m] \left(\frac{\rho_{ex}\,[n]\, p\,[n]}{\Lambda}\right) \tag{19}$$

$$C_i(n) = C_0 e^{-\lambda_i n} + T_s \left(\frac{\beta_i}{\Lambda}\right) \sum_{m=0}^{n} h_i\,[n-m]\, p\,[n] \tag{20}$$

In the above equations (Eqs. 19 and 20), the impulse response function $k\,[n] = e^{\left(\frac{-\beta}{\Lambda}\right)n}$ and $\rho_{ex}(t) = 0.1\beta t$. In this case, the radius of convergence of the impulse response function $k[n]$ is given by $|Z| > e^{\left(\frac{-\beta}{\Lambda}\right)}$, and the radius of convergence of $h_i[n]$ is given by $|Z| > e^{-\lambda_i}$ for the precursor concentration "i".

4.3.2 Transient from Negative Ramp Reactivity

Consider another example of thermal reactor described by Li et al. [6], with the following parameters: $\lambda_1 = 0.0127\,s^{-1}$, $\lambda_2 = 0.0317\,s^{-1}$, $\lambda_3 = 0.115\,s^{-1}$, $\lambda_4 = 0.311\,s^{-1}$, $\lambda_5 = 1.4\,s^{-1}$, $\lambda_6 = 3.87\,s^{-1}$, $\beta_1 = 0.000266$, $\beta_2 = 0.001491$, $\beta_3 = 0.001316$, $\beta_4 = 0.002849$, $\beta_5 = 0.000896$, $\beta_6 = 0.000182$, and $\Lambda = 2.0 \times 10^{-5}$ s. A negative ramp reactivity of the form $\rho(t) = -0.1\$\ t/s$ is inserted in the reactor, the transient following this reactivity is estimated by coupled

Table 6 The power is computed using coupled FIR filter (T_s = 1.0e-3) for negative ramp reactivity, −0.1$/s, and the power transient is compared with the GAEM method (Nahla [3])

Time	Coupled FIR method	TSM	Absolute error
2.0	0.786412	0.791955	0.00554
4.0	0.604639	0.612976	0.00834
6.0	0.464981	0.474027	0.00905
8.0	0.360466	0.369145	0.00868
10.0	0.282778	0.290636	0.00786

The absolute error is shown

Table 7 The power is computed using coupled FIR filter for sinusoidal reactivity insertion $\rho(t) = 0.001 \sin(4\pi t)$ and compared with the modified ETD method (Mohideen Abdul Razak and Devan [7])

Time	Coupled FIR method (T_s = 1.0e-4s)	Modified ETD method	Absolute error
0.0	1.000000	1.0000000	0.000000
0.4	0.876102	0.8828488	0.006747
0.8	0.932204	0.9334387	0.001235
1.2	1.123357	1.1070115	0.016345
1.6	1.182720	1.1615297	0.02119
2.0	1.002299	0.9992939	0.003005
2.4	0.880687	0.8857330	0.005046
2.8	0.937168	0.9366979	0.00047
3.2	1.129228	1.1108737	0.018354
3.6	1.188583	1.1653616	0.023221
4.0	1.007087	1.0024811	0.004606
4.4	0.884874	0.8885748	0.003701
4.8	0.941632	0.9397346	0.001897
5.0	1.009291	1.0039612	0.00533

The absolute error is shown

FIR, and the result is compared with that of Taylor Series Method [3]. The results are shown in Table 6.

4.4 Transient from Oscillatory Reactivity

The power transients caused by a sinusoidal reactivity insertion are analyzed here for the thermal reactor described by Li et al. [6]. The delayed neutron precursor parameters are given as follows: $\lambda_1 = 0.0127\,s^{-1}$, $\lambda_2 = 0.0317\,s^{-1}$, $\lambda_3 = 0.115\,s^{-1}$, $\lambda_4 = 0.311\,s^{-1}$, $\lambda_5 = 1.4\,s^{-1}$, $\lambda_6 = 3.87\,s^{-1}$, $\beta_1 = 0.000266$, $\beta_2 = 0.001491$, $\beta_3 = 0.001316$, $\beta_4 = 0.002849$, $\beta_5 = 0.000896$, $\beta_6 = 0.000182$, and $\Lambda = 2.0 \times 10^{-5}$ s. A sinusoidal reactivity of the form $\rho(t) = 0.001 \sin(4\pi t)$ is inserted in the reactor, the transient following this reactivity is estimated by coupled FIR method, and the result is compared with that of the modified exponential time differencing method [7]. The estimated power transient is given in Table 7.

5 Conclusion

A new computational method for estimating the nuclear reactor power transients using finite impulse response (FIR) filter is developed and presented. The nuclear power transients, in small reactors, are estimated by solving the point kinetics equations. According to this method, the stiff point kinetics equations are written as convolution integrals. The convolution integrals are converted into simple algebraic equations using discrete Z transform. Here, the power and precursor concentrations are written as simple algebraic equations. This method has less computational effort in estimating the transients. The impulse response functions, involved in the convolution, characterize the FIR filters. Here, the reactor power and precursor concentrations are represented by two different FIR filters. The impulse response function is different for different types of reactivity perturbation. The impulse response functions are found to be stable, and they have finite radius of convergence. This method is applied to estimate the power transient of thermal reactor for step (constant) reactivity perturbation with temperature feedback. The power transients estimated with temperature feedback are found to be in good agreement with standard results. In a similar manner, the method is also applied to estimate the power transients for ramp reactivity input. The estimated power transients under ramp reactivity perturbation are found to be in good agreement with reference results. It is also shown that this method has high stability, i.e., any change in the time step by a factor of 10 or 20 will not lead to large error in the estimation of power. From the comparisons of results, it can be concluded that this method is capable of estimating the reactor power transients for various types of reactivity perturbations with feedback. This method can be easily designed and implemented for estimating the power transient with feedback. A scheme to choose the sampling time interval for solving the stiff point kinetics equations is also established.

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