Chapter 13
The Development of Communication in Chinese Mathematics Curricula

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Abstract This chapter examines the development related to mathematical communication abilities in math syllabus and curriculum standards at the junior high level in China since 1902. This chapter analyses curriculum documents in China from 1902 to 2011 using keyword frequency analysis and text analysis. The study found that mathematical communication abilities in curriculum standards over the past hundred years are defined in four ways: teacher-student communication, student-self communication, student-student communication, and student-text communication. The analysis of the changes to the curriculum requirements provides a better understanding of mathematical communication abilities in China and offers insights on the key factors that affect the development of students' mathematical communication abilities.

Keywords Mathematical communication abilities · Math syllabus · Curriculum standards · Types of communication · Teacher-student communication · Student-self communication · Student-student communication · Student-text communication · Keyword frequency analysis · Text analysis

13.1 Introduction

With the growing usage of mathematics in modern society, *mathematical communication ability* has become an important part of math competency. Niss (2015) explained the following:

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> Considering the fact that anyone who is learning or practising mathematics has to be engaged, in some way or another, in receptive or constructive communication about matters mathematical, either by attempting to grasp others' written, oral, figurative or gestural mathematical communication or by actively expressing oneself to others through various means, a mathematical communication competency is important to include. (p. 40)

Mathematical communication is the process in which students learn and use mathematical language to communicate and understand the world, such as using specific mathematical symbols and terminologies. With mathematical communication abilities, students are expected to build common sense regarding mathematics (Shi, 1998; Niss, 2003; Xu, 2013). As future citizens, students need to achieve certain levels of mathematical communication abilities. However, such abilities are not innate. The higher a student's grade level is, the more complicated and instructive his or her mathematical communication abilities are. It is imperative for educators to establish a set of explicit, detailed and measurable mathematical communication abilities to evaluate students' current communication ability levels and to promote their mathematical communication abilities.

In China, syllabus and curriculum standards play an important role in guiding curriculum writing, teaching and learning. The latest mathematics standards contain modified requirements of mathematical communication abilities for students. The analysis of the changes in the requirements provides a better understanding of mathematical communication abilities in China and offers us insights on the key factors that affect the development of students' mathematical communication abilities.

13.2 Literature Review

13.2.1 Definition of Mathematical Communication

Communication is a process of receiving and communicating through language, symbols, diagrams and artistic forms, which requires listening, speaking, reading and writing as the main means. In many curriculum standards, mathematical communication abilities entail the processes of receiving and expressing. For example, the German mathematics standards state that mathematical communication abilities include the understanding of mathematical text or expression as well as the written or verbal communication of mathematical thinking and solutions. Reading and understanding mathematical texts is a process of *receiving*, while interpreting and presenting mathematical ideas in written or oral form belongs to the *expressing* process (Kultusministerkonferenz, 2004). The German standards require that students be able to receive, understand and evaluate mathematical facts as well as present one's own mathematical ideas and assess and correct others' ideas (Xu, 2007). The United Kingdom's national curriculum guide requires students to understand and interpret mathematics in multiple representations and to communicate mathematics with confidence in the most appropriate way (U.K. Department of Education, 2007). Students should be able to choose the most effective way to communicate in different contexts. Students are also required to provide explanations and assess the correctness of expressing. Such processes involve an understanding of mathematical information and help develop students' mathematical thinking.

Some standards define *mathematical communication abilities* with a focus on either the process of *receiving* or the process of *expressing*. For example, the *Curriculum and Evaluation Standards for School Mathematics* in the United States has a focus on the process of expressing. It requires students to "reflect upon and clarify their thinking about mathematical ideas and relationships, and formulate mathematical definitions and express generalisations discovered through investigation, and to express mathematical ideas orally and in writing" (National Council of Teachers of Mathematics, 1989, p. 140). Singapore's secondary school syllabus also focuses on the expressing process. It states that a critical skill in education is the ability to use mathematical language to express the process of mathematical thinking and argumentation accurately, concisely and logically (Singapore Ministry of Education, 2011). Recently, Singapore has paid more attention to mathematical communication and has mentioned that "communication of mathematics is necessary for the understanding and dissemination of knowledge within the community of practitioners as well as general public" (Singapore Ministry of Education, 2019, p. 6). The *Mathematics Curriculum Standards for Compulsory Education* in China (Ministry of Education of the People's Republic of China, 2012) lists four requirements as mathematical communication abilities:

1. Students will be able to communicate about their own algorithms and processes to solve the problem and to express their own ideas.
2. Under the guidance of teachers, students will be able to choose the appropriate strategy to solve the problem through communicating with others.
3. Students will be able to explain and communicate the statistical results and make simple assessments and predictions based on the results.
4. Students will be able to rethink the whole process of mathematical participation, to write a report or short paper about the research process and results, and to communicate so as to further obtain mathematical practice experience.

The sequence of the four requirements implies the assumption that a good receiving process serves as the basis for the improvement of the expressing skill.

13.2.2 Classification of Mathematical Communication

Students with strong mathematical communication abilities can explain a large amount of quantitative data encountered in daily life and make reasonable evaluations of the data. They can also fully reflect on their own problems and understand arguments from others. Niss (2003) defined *mathematical communication* as involving two processes. The first process is to understand the mathematical meaning of the texts presented in various representations, including written, visual or verbal. The second process is to present one's own mathematical ideas in multiple representations at different levels of precision.

The Common Core State Standards for Mathematical Practice (National Governors Association and Council of Chief State School Officers, 2010) include eight standards that apply to students from kindergarten to 12th grade. Students should be able to perform the following important tasks: make sense of problems, reason abstractly, construct arguments and critique the reasoning of others, construct mathematical models, use appropriate tools, attend to precision, make use of structure and look for and express regularity in repeated reasoning. Communication is key to many of these tasks. To construct mathematical models, students must construct representations of mathematical thinking—a crucial element of communication. To construct *arguments, critique* the reasoning of others, *attend to precision* or *express* regularity in repeated reasoning, students must be able to clearly communicate their mathematical thinking. Mathematical communication skills include mathematical dialogue, writing and reading.

Mathematical dialogue is the conversation of mathematics between two or more persons. It is a two-way process involving listening and speaking. For example, a teacher-student dialogue and dialogue among students in the classroom are mathematical dialogues. Regarding the purpose of student dialogue in mathematics classrooms, Pimm (1987) categorised mathematical dialogue as *mathematical dialogue with others* and *mathematical dialogue of self-reflection*. Students use mathematical dialogue with others to convey their own mathematical ideas. Through self-reflective mathematical dialogue, students can effectively organise their own thinking and clarify mathematical meanings and ideas, thus gaining further understanding of mathematics. For example, when solving a mathematical problem, students read the mathematical questions repeatedly to clarify or correct the problem-solving model. The repeated reading method indicates that self-reflective dialogue can promote student reflection on mathematical thinking. Self-reflective dialogue is implicit and serves as the basis of conversations with others.

Mathematical writing is an important complement to verbal communication. When students write in mathematics, they are actively involved in the process of absorbing mathematical knowledge, developing mathematical understanding, and improving math-learning attitudes. Common mathematical writing in classes includes diary writing and explanatory writing. One type of diary writing asks students to reflect on the entire learning process by debriefing the math they have learned. Clarke et al. (1993) conducted a study on diary writing for 4 years with a focus on mathematical debriefing. They asked seventh-grade students to write a math diary with three prompts at the end of each math class. The three prompts were as follows: What did you do in class? What did you learn? What were the examples and questions? The purpose of explanatory writing is to describe and explain the process of solving a mathematical problem or the validation of a mathematical solution to a given question. Shield and Galbraith (1998) studied two explanatory writing tasks: (1) writing a letter to a classmate who'd missed the class to explain what was learned in the class and (2) writing to help a student who had difficulties with the math in class.

Mathematical reading involves reading and understanding texts containing words, forms, figures, illustrations, timetables, etc. (Organisation for Economic Co-operation and Development, 2009). In mathematical reading, students need to process and transition among multiple representations, including symbols, diagrams, graphics and forms. It is a nonlinear process and is the main difference between mathematical reading and other reading (Bosse & Faulconer, 2008).

13.3 Research Question

This chapter examines the requirements related to mathematical communication abilities in math syllabus and curriculum standards at the junior high level since 1902. Two research questions are explored:

1. What are the definitions of mathematical communication abilities in math syllabus and curriculum standards used throughout the past 100 years in China?
2. What are the changes in requirements for mathematical communication abilities in math syllabus and curriculum standards?

To answer the two research questions, we reviewed literature and analysed curriculum documents in China from 1902 to 2011. Findings illustrate the changes in defining mathematical communication abilities and the requirements for student mathematical communication abilities in China.

13.4 Research Methods

13.4.1 Objects of Content Analysis

The data for this study are math syllabus and curriculum standards at the junior high level in China from 1902 to 2011. In particular, the documents from 1902 to 2000 were selected from the *Collection of primary and secondary curriculum standards and syllabus of the twentieth century in China (Mathematics volume)*, published by People's Education Press and edited by Curriculum and Teaching Materials Research Institute. The curriculum documents after 2000 were selected from *Mathematics Curriculum Standards for Full-Time Compulsory Education (Experimental version)* (Ministry of Education of the People's Republic of China, 2001) and *Mathematics Curriculum Standards for Full-Time Compulsory Education (2011 version)* (Ministry of Education of the People's Republic of China, 2012).

13.4.2 Procedures of Content Analysis

13.4.2.1 Content Analysis

The content analysis method was used to analyse documents. Mayring (2015) simplified content analysis into three steps: deletion, interpretation and structuring. Texts were assessed with predetermined criteria and were coded in both inductive and deductive classifications. Frequency of the keywords was counted. In this study, we first filtered documents with the keyword expression. An analysis framework was then developed to code the filtered documents.

13.4.2.2 Analysis Framework of Mathematical Communication

Mathematical communication abilities are a set of abilities revolving around receiving mathematical information through reading and understanding of mathematical texts and expressing mathematical ideas in written or verbal form (including mathematical thinking processes, problem-solving strategies and mathematical answers).

There are three types of mathematical communication: teacher-student communication, student-student communication and student-text communication (Nührenbörger & Steinbring, 2009). Teacher-student communication is a conversation led by the teacher, usually with a rapid introduction, and passively received by the students. In such a conversation, the teacher dominates the delivery of mathematical concepts and mathematical thinking. Student-student communication entails conversations involving various levels of mathematical understandings and practices. Participating students are open to communicate their mathematical ideas, no matter the correctness or completion of the mathematical idea. Student-text communication is the communication with mathematical texts, such as solving mathematical problems, reading textbooks and learning mathematical concepts (Nührenbörger & Steinbring, 2009). In addition, students' self-communication and reflection is becoming more and more important, and should be an important part of mathematical communication ability. In the present study, four types of communication were investigated (see Fig. 13.1).

Based on the information processing theory, mathematical communication is a process of receiving, processing and expressing (Zeng & Lian, 2017). Figure 13.2 shows the various activities involved in the three phrases of the mathematical

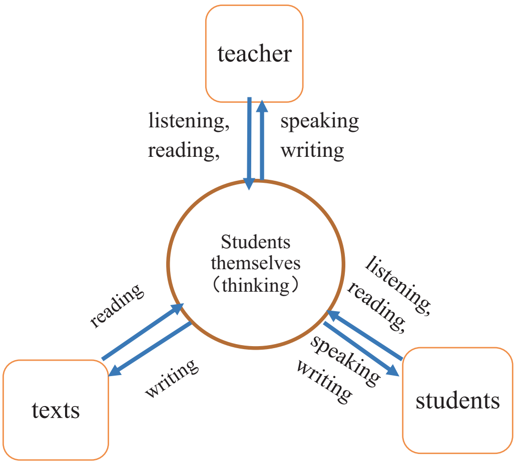

Fig. 13.1 The four types of mathematical communication

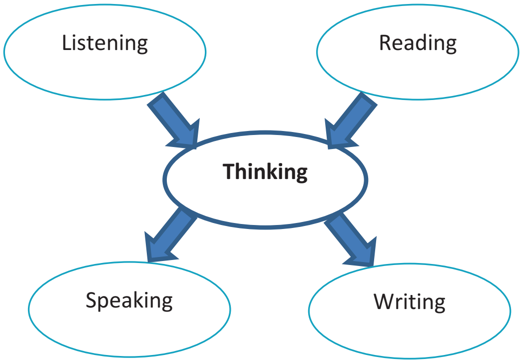

Fig. 13.2 The process of mathematical communication

communication process. Receiving exists in all three types of mathematical communication: teacher-student, student-student and student-text. Processing is mainly implicit self-reflective thinking and communication within an individual student. Expressing is the process of a student presenting mathematical ideas in verbal or written form after receiving and processing mathematical information.

13.4.2.3 Coding Framework for Mathematical Communication

As discussed in Chap. 3, the cognitive requirements in the process of mathematical communication include three levels: reproduction, connection and reflection. Reproduction is when students express or present simple mathematical content and recognise information embedded in short mathematical texts. Connection is the transfer of others' mathematical thinking from one carrier to another and students' explanations of their thinking processes, solutions and results briefly and logically. Reflection is the process of understanding the meaning of complex mathematical texts, comparing and judging others' mathematical thinking, and expressing one's own inspection and reflection on the learning process. The coding system of mathematical communication was developed using the following analysis framework (Table 13.1).

Every single sentence from the curriculum documents was a coding unit. For example, the sentence "using the trajectory method to solve the drawing problem" was one coding unit. The content area mentioned in this sentence is geometry, coded as A3. The context of mathematical communication was coded as B2, since it is an educational context. The communication form was coded as C3, which is student-based communication. The cognitive requirement is a conversion (D21). Thus, the code for this sentence is A3B2C3D21. In a case where a sentence involved multiple mathematical contexts or cognitive requirements, all suitable codes were applied to the sentence. Two researchers who had background knowledge and experience in curriculum content analysis independently coded the same 20 sentences randomly selected from the curriculum documents. Comparison revealed that 90.7% of the coding results were consistent. The researchers discussed and reconciled the

Table 13.1 The coding framework for mathematical communication

Dimension		Code	Description
(A) Content domains		A0	Comprehensive requirements
		A1	Arithmetic
		A2	Algebra
		A3	Geometry
		A4	Probability and statistics
(B) Communication context		B1	Personal context
		B2	Educational context
		B3	Social context
(C) Communication types		C1	Student-teacher: The teacher asks students to answer questions and discuss the process of mathematics, mathematical thinking and mathematical methods with other students. It is mainly about the process by which students receive and understand information
		C2	Student-self: Students answer questions and give results by accepting information and carefully thinking and expressing mathematical conjectures or feelings about the speech of mathematical topics
		C3	Student-text: Communication occurs between students and texts when students do mathematical problems, review textbooks and learn mathematical concepts
		C4	Student-student: Students express their opinions to the communication objects (teachers, peers or texts) and use relevant mathematical knowledge and concepts to prove their ideas, convince and understand the objects of communication, listen to the mathematical ideas and strategies of communication objects, understand their methods of thinking, analyse the mathematical views expressed by others and judge others' abilities to express, listen and absorb others' ideas. It includes processes of acceptance, processing and expression
(D) Cognitive domains	(D1) Recognise & imitate	D11	Recognise: Be able to identify and select information from short mathematical texts
		D12	Imitate: Be able to clearly express simple mathematical facts, such as understanding of simple mathematical content
	(D2) Connect & transform	D21	Transform: Recognise and select information from mathematical texts and understand its significance and be able to convert the mathematical ideas of others from one carrier (chart, text, symbol, object or action, etc.) to another, so as to facilitate further understanding
		D22	Connect: Be able to express the thinking process, the solution and the result in a brief and logical way and be able to explain the explanation (correct or wrong) of the mathematical text made by others

(continued)

Table 13.1 (continued)

Dimension		Code	Description
	(D3) Reflect & extend	D31	Reflect: Comprehend the meaning of complex mathematical texts and compare and judge other people's mathematical ideas
		D32	Extend: Be able to fully present the process of a complex solution and argumentation; be able to compare, evaluate and correct the understandings of others; be able to flexibly transform the carrier of mathematical ideas and select the optimal expression carrier according to the specific situation; and be able to express the examination and reflection of the learning process so that the problem-solving process is rational, complete, concise and harmonious

remaining 9.3% of the coding results and reached an agreement in the end. Then all relevant sentences ($N = 306$) were coded by both researchers.

13.5 Results

We analysed curriculum and syllabus standards from 1902 to 2011 using keyword frequency analysis and text analysis. Findings were categorised into five time periods: 1902–1922, 1923–1951, 1952–1977, 1978–2000 and 2001–2011. The division of the time periods was based on the year when one curriculum reform started (see Chap. 1).

13.5.1 The Emergence of Mathematical Communication Abilities: 1902 to 1922

From 1902 to 1922, China reformed school curricula, mirroring academic systems in Japan, Germany, and America (Curriculum and Teaching Materials Research Institute, 2001). The phrase *mathematical communication ability* was not used in the syllabi or standards during this period of time (Fig. 13.3). However, some of the statements in these texts imply that the required mathematical communication abilities at that time were abilities regarding teacher-student communication and student-self communication. For example, *Middle School Rules Approved by Emperor*, published in 1904, pointed out that teachers should "teach the bookkeeping . . . and then teach plane geometry and three-dimensional geometry, and also teach algebra" (Curriculum and Teaching Materials Research Institute, 2001, p. 206) so that students could "know the application of knowledge of bookkeeping" and "the format of the calculation table" (Curriculum and Teaching Materials Research Institute, 2001, p. 206). Although the term *communication* was not used, the statement "teachers should teach" indirectly indicated that students needed to receive mathematical

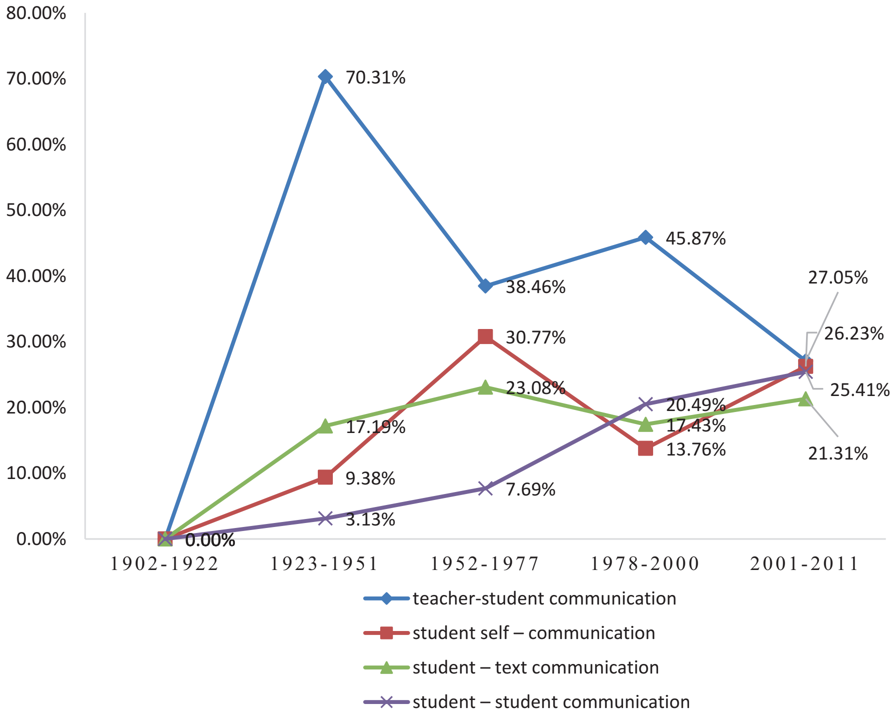

Fig. 13.3 Changes of mathematical communication ability requirements in curriculum standards

information and study it. After that, students needed to talk with themselves to grasp mathematical knowledge and understanding.

13.5.2 The Rise of Mathematical Communication Abilities: 1923 to 1951

In 1923, the Ministry of Education of China released revised curriculum standards for primary, middle and high schools. The new curriculum standards listed requirements for mathematical communication abilities, such as requiring a teacher to guide, question, and teach mathematics to students. After that point, mathematical communication abilities started to become explicitly required in curriculum standards.

From 1923 to 1951, the keywords related to mathematical communication abilities in curriculum standards included *oral answering*, *asking*, *discussion*, *critical questioning* and *explanation*. The different types of communication – including teacher-student communication, student-self communication, student-student communication and student-text communication – appeared in the documents. Among these, teacher-student communication had the largest percentage (70.31%) of relevant sentences coded using the coding framework for mathematical communication. Student-text communication accounted for 17.15%. The percentages of student-self communication and student-student communication were less than 10% (9.38% and 3.13%, respectively).

The percentages show that during this period of time, the curriculum standards emphasised the importance of teacher-student communication in mathematics teaching. Students were expected to receive mathematical information from the guidance of teachers. The standards required students to process mathematical information and express mathematical ideas according to the way trained by teachers. There was little emphasis on student-self communication and student-student communication.

13.5.3 Student-Oriented Mathematical Communication Requirement: 1952 to 1977

From 1952 to 1977, the keywords that reflected mathematics communication in the curriculum standards were *posing mathematics questions* and *Q&A lectures*. The requirement of *expressing one's ideas in mathematical language* was listed in the standards for the first time.

The percentages of relevant sentences which focused on student-self communication, student-student communication and student-text communication increased. As Fig. 13.3 shows, student-self communication increased from 9.38% during 1923 to 1952 to 30.77% during 1952 to 1977. Student-text communication increased gradually. Student-text communication consists of students' interactions with textbooks, mathematical problems and other written mathematical texts. At this stage, mathematical communication requirements were oriented around students' behaviours; they emphasised that students should deal with written mathematical information and express their ideas to others.

13.5.4 The Emphasis of Student-Student Communication: 1978 to 2000

Since 1978, curriculum standards increased the emphasis on communication among students, stating that students should be able to express their views in mathematical language to others and discuss with each other. The proportion of student-student communication in curriculum standards increased from 7.69% to 20.49% (Fig. 13.3). For example, in 1988, the mathematical syllabus listed "expressing one's thoughts and opinions concisely" as one of the purposes of schooling (Curriculum and Teaching Materials Research Institute, 2001, p. 553). In 1992, the syllabus put forward that "students have the ability to expound their thoughts and conceptions using mathematical language correctly" (Curriculum and Teaching Materials Research

Institute, 2001, p. 605). In 2000, teacher-student interactions and student-student interactions were prioritised in the curriculum standards.

The shift from teacher-student communication to student-oriented communication in curriculum standards shows the increasing recognition of student-centred learning in mathematics. Students are expected to express their mathematical ideas to teachers or classmates. They should use relevant mathematical knowledge and abilities to prove their ideas and convince others. At the same time, students are required to listen to others to understand their mathematical ideas, strategies and ways of thinking.

Some examples of the keywords related to mathematical communication abilities during this period included *explanation using examples*, *heuristic teaching*, and *explaining mathematical ideas*. The frequency of keywords focused on mathematical communication abilities in curriculum standards offered us some insights on the emphasis of student-student communication from 1978 to 2000 (Fig. 13.4). All three phases of the mathematical communication process can be found in the curriculum standards. Teacher-student communication is found in teachers' understanding, guiding, and conducting heuristic teaching as students receive mathematical information. Student-self communication is found in students reflecting and thinking on the information they received. Student-student communication is found in students questioning, expressing, and communicating their mathematical ideas to others.

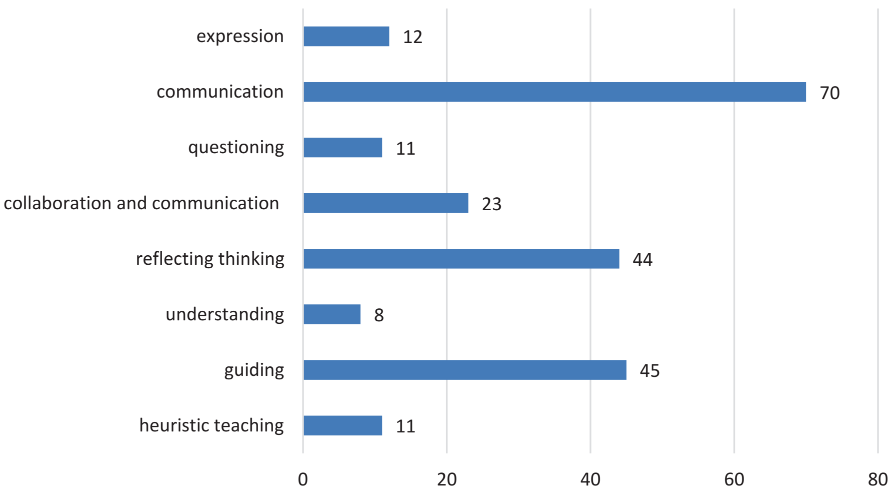

Fig. 13.4 Frequency of requirements for mathematical communication abilities in curriculum standards

13.5.5 Collaboration-Oriented Mathematical Communication: 2001 to 2011

From 2001 to 2011, attention to the phases of mathematical communication changed. The requirement for *expressing* mathematical ideas increased from 34.48% to 43.48%, while attention given to *receiving* mathematical information decreased from 53.45% to 31.06% (Fig. 13.5). Some examples of the keywords on mathematical communication in curriculum standards included *inspirational teaching*, *communication and interaction*, *communicating with mathematical languages*, *collaboration* and *questioning*.

The focus on collaboration and communication was one significant feature during this period of time. The terms *collaboration* and *communication* appeared 23 times in the curriculum standards. The *Mathematics Curriculum Standards for Compulsory Education (2011 version)*, published in 2012 (Ministry of Education of the People's Republic of China, 2012), highlighted that the goal of mathematical communication is to learn to communicate with others.

13.5.6 Other Changes in Requirements for Mathematical Communication Abilities

Since there was no clear expression on mathematical communication abilities in curriculum standards from 1902 to 1922, the changes in requirements on mathematical communication abilities presented here are from 1923 to 2011. We looked at the

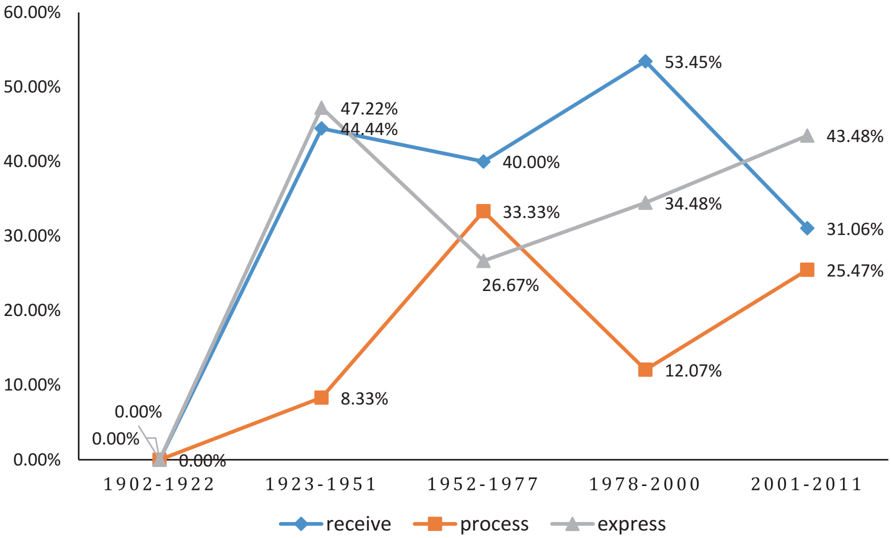

Fig. 13.5 Changes in the phases of mathematical communication in curriculum standards

changes from three perspectives: mathematical content areas, cognitive requirements and communication contexts.

13.5.6.1 Changes in Terms of Mathematical Content Areas

Throughout nearly 100 years, the requirements for mathematical communication abilities in different mathematical content areas have changed dramatically. Some mathematical content received almost 15 times more attention in 2011 as compared to 1923, while the emphasis on some math content dropped 20% (Fig. 13.6).

As shown in Fig. 13.6, the *comprehensive requirements* content area received the most attention from 1923 to 2011. This illustrates that mathematical communication skills are a set of comprehensive abilities, such as mathematical reasoning and mathematical representation, which cannot be achieved overnight (Cai & Xu, 2016). The content areas that increased the most in attention were probability and statistics. The percentage of requirements for mathematical communication abilities in probability and statistics increased from 2% (in the period from 1923 to 1951) to 31% (in the period from 2001 to 2011). Such a huge increase reflected the changes of requirements for the teaching and learning of probability and statistics in curriculum standards. With the rapid development of economy in China, people likely realised the importance of attaining certain probability knowledge, such as the difference between uncertainty thinking and mathematical certainty thinking,

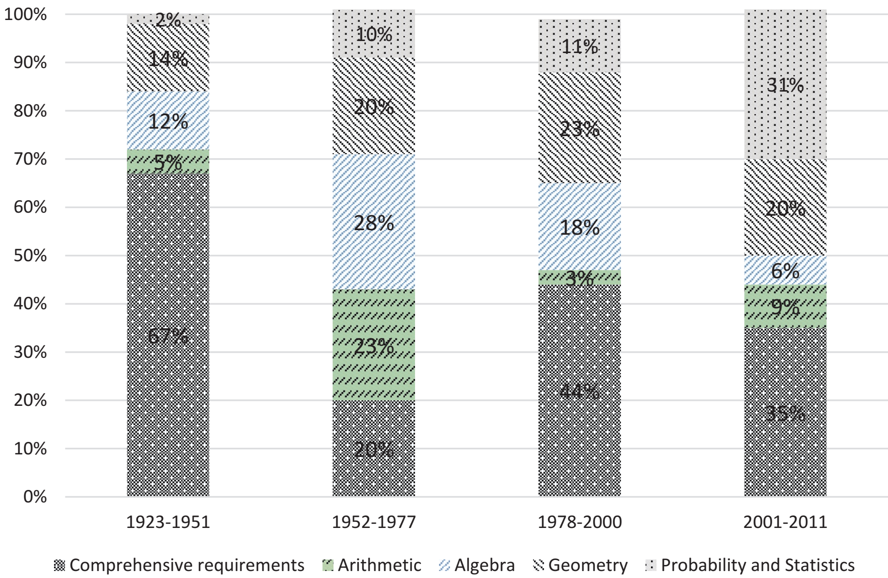

Fig. 13.6 Changes in requirements for mathematical communication abilities based on content areas

statistical thinking and inductive inference in probability statistics (Ministry of Education of the People's Republic of China, 2018). The standards also stated that teachers should let students experience simple data collection and organising processes. Student would then understand some data collection methods such as surveys and assessments and could present the results in various representations, such as texts, pictures and tables. Students would engage in activities such as collecting, describing and analysing data; evaluating and communicating; understanding the necessity of sampling; and experiencing the use of samples to make estimations or predictions. Students would accumulate relevant mathematical-activity experience in collaborating and communicating with others.

In mathematical content areas such as algebra, arithmetic and geometry, there were few changes in the requirements. When examined in detail, most of the changes were to requirements for basic abilities such as reading tables, performing calculations, and validating solutions. China issued a series of notices and notifications to adjust the teaching requirements on various content areas between 1952 and 1977. Although more than 70% of the teaching requirements focused on algebra, arithmetic and geometry, there were few requirements on mathematical communication abilities. For instance, when solving fraction equations, students were required to test whether there was an extraneous root. No discussion was needed (Curriculum and Teaching Materials Research Institute, 2001, p. 360).

13.5.6.2 Changes in Cognitive Demands

A total of 306 coding units with a focus on cognitive demands were analysed. As mentioned in the methods section, we categorised three levels of cognitive demands: recognise and imitate (level 1), connect and transform (level 2), and reflect and extend (level 3). In general, there was an increased requirement for the high-level cognitive demands throughout the past 80 years (Fig. 13.7).

The percentage that referred to the highest-level cognitive demands for mathematical communication increased almost 20% over the past 80 years. In the period from 1923 to 1951, only 27% of the 306 units in curriculum documents related to the level 3 cognitive demands, but in 2001, the percentage of level 3 reached 43%, which was the highest among all three levels. The percentage of level 1 cognitive demands decreased from 35% to 18% over the past 80 years, except for an unexpected rise to 59% in the period of 1952 to 1977. Similar changes happened to level 2. From 1923 to 2011, the percentage of *connect and transform* related to mathematical communication remained fairly stable at about 40%, except for a dramatic drop to 12% during the period of 1952 to 1977. Further studies could be conducted to explore the potential reasons for the substantial changes during that period.

After coding and classifying the 306 units in curriculum documents, we compiled the results in Fig. 13.8.

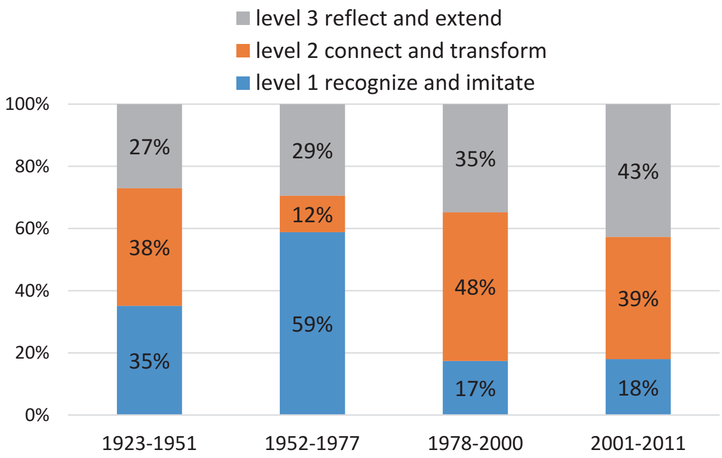

Fig. 13.7 Changes in requirements for mathematical communication abilities in terms of cognitive demands

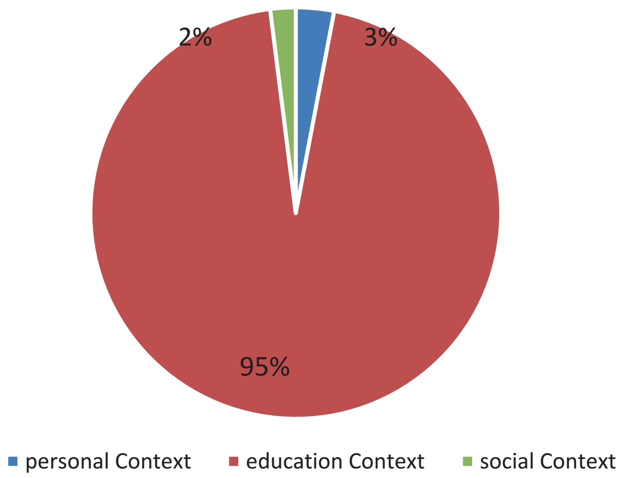

Fig. 13.8 Distribution of communication contexts

The majority of the mathematical communication context in curriculum standards is the educational context (95%). The personal and social contexts mainly appeared after 2001. Within the educational context, almost half of contexts require teacher-student communication (48.80%). 21.31%, 11%, and 18.9% of educational contexts initiate student-text communication, student-student communication, and student-self communication respectively. Since teacher-student communication plays a dominant role in mathematical communication, it is vital to promote teacher-student communication to support students' mathematical learning.

13.6 Conclusion

In general, we found that mathematical communication abilities in curriculum standards over the past hundred years were defined in four types: teacher-student communication, student-self communication, student-student communication, and student-text communication. The development of the definitions and requirements of mathematical communication abilities in China went through five phases: the emergence of mathematical communication abilities from 1902 to 1922; the rise of mathematical communication abilities from 1923 to 1951; student-oriented mathematical communication abilities from 1952 to 1977; the emphasis of student-student communication from 1978 to 2000; and collaboration-oriented mathematical communication from 2001 to 2011.

Mathematical communication is defined as a process of receiving, processing and expressing mathematical information and ideas. Among all four types of mathematical communication abilities, teacher-student communication plays a dominant role in curriculum documents in China. Starting from the curriculum reform in 1952, there was a shift from teacher-student communication to student-oriented communication in the curriculum standards requirements. More emphasis was placed on student-student, student-self, and student-text communication.

In thinking of the research question regarding changes to the requirements for mathematical communication abilities, we found considerable changes to the requirements in terms of mathematical content areas, cognitive demands, and communication contexts. Over the past hundred years, there has been a substantial increase in the requirements for mathematical communication abilities in probability and statistics and high-level cognitive demands (e.g., level 3, reflect and extend). The percentages of mathematical communication abilities requirements for the comprehensive requirements content area and educational context remain at half or above.

With the development of mathematics curricula, the standards have put emphasis on the requirements of mathematical communication abilities comprehensively. The four objectives of the current mathematics curriculum for compulsory education all have a focus on mathematical communication abilities. For example, in relation to objectives of problem-solving, students "should experience problem-solving collaboratively with others and explain their own thinking ways . . . and communicate with others and can understand others' thinking ways and conclusions" (Ministry of Education of People's Republic of China, 2012, p. 14). In addition, the objectives of emotion, attitudes and values include requirements for mathematical communication abilities – namely, students should "dare to express their own ideas. . . develop habit for collaborative communication" (p. 15).

The current curriculum standards place high demands on mathematical communication. After implementing the mathematics curriculum, the development of students' mathematics communication ability has reached the curriculum goal to a certain extent. The next chapter will investigate and analyse the students' mathematical communication ability.

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