Mathematical Model of the Throughput of an IP Network Switching Node with a Non-constant Amount of Space in the Router RAM

Pavel Dunayev, Yermek Sarsikeyev, Olga Galtseva, and Gufana Narimanova

Abstract This paper offers a mathematical model that allows determining the performance and throughput of an IP network node under conditions when the amount of space in the router RAM is not constant, that is, in the case of overloads and a number of other factors. The Cisco Packet Tracer software package is used as a network emulator. The results of mathematical and simulation modeling are compared.

1 Introduction

IP network routers are important elements of the city's telecommunications infrastructure. They influence the bandwidth of the transmission channel. Their parameters, settings, amount of RAM (random access memory), etc. to a great extent determine the efficiency of the entire telecommunications network. Due to insufficient throughput of the IP node, the throughput of the entire network decreases, the packet service time increases, which increases the delay time of the IPTD (Internet Protocol Packet Transfer Delay) IP packet and the IPDV (Internet Protocol Packet delay variation) delay variation [1, 2]. According to the authors of the Chapter, based

P. Dunayev · Y. Sarsikeyev
Energy Department, JSC Kazakh Agrotechnical University Named After S. Seifullin,
62 Pobeda Avenue, 010000 Nur-Sultan, Republic of Kazakhstan
e-mail: dunayev.kz@mail.ru

Y. Sarsikeyev
e-mail: sarsikeyev.ermek@yandex.ru

O. Galtseva (✉)
School of Non-Destructive Testing, National Research Tomsk Polytechnic University,
30 Lenin Avenue, Tomsk, Russia 634050
e-mail: rabota2013tpu@mail.ru

G. Narimanova
Department of Innovation Management, Tomsk State University of Control Systems and
Radioelectronics, 40 Lenin Avenue, Tomsk, Russia 634050
e-mail: guftana@mail.ru

on the above, increasing and evaluating the bandwidth of an IP node is a very urgent task.

Most of the known methods for assessing the throughput of IP networks are based on models in the form of a QN (Queuing Network), in which nodes display IP packet delays [3–5].

There are several methods for modeling telecommunications networks: physical, analytical, simulation, and combined. With the physical modeling method, the performance of the systems under study is measured in real time, i.e. an experiment is being performed. The result of this method is a model with high adequacy in a real system or network. Despite the advantages, this method has a drawback, i.e. the high cost of equipment for creating a model [6].

An analytical model is a set of analytical expressions that reflect functional dependencies between the parameters of a real system during its operation. Such models are used for simple systems with no requirements for high accuracy of the results obtained.

A simulation model is a computer program that reproduces events occurring in a real system. The result of the simulation model is the collected statistical data on important network characteristics [6].

The results of the experimental verification prove the correctness of the approach when drawing up the mathematical model, particularly, of taking into account the number of RAM space and the size of an IP packet. This condition distinguishes the constructed model from standard methods based on the QN.

2   Theoretical Part

In reality, IP networks show a situation when routers process a large number of IP packets of various sizes. The service or processing time for these packets depends on the switching matrix used. The switching matrix is the basis of any router since it is used to transmit packets from the input data port to the output one. Switching can be performed in several ways: memory switching, bus switching, and connection network switching (Fig. 1).

More complex interconnecting networks perform switching in several stages, which ensures simultaneous transmission of packets from different input ports to the same output port through a switching matrix [7].

The router may process 2 million or more packets per second [8–11].

For example, there may be either 5 thousand 64-byte IP packets or 10 thousand 32-byte ones at the same time. It is logical to assume that at all other things being equal if only 32-byte IP packets are used on routers, the performance will increase by 2 times since the amount of RAM space will 2 times increase. As is known, the throughput is measured in bits/s, thus, when talking about the number of packets per second, the authors mean the hardware performance. Therefore, with the declared hardware performance, the throughput should increase as the IP packet size increases.
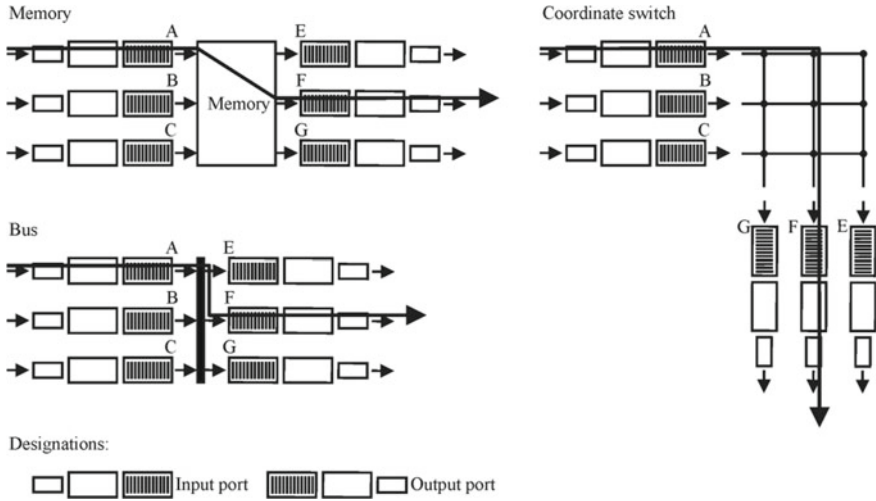
Fig. 1 Three ways to switch data

Thus, the count of the percentage of different sizes of information packets is very important for a more accurate calculation of the parameters of the IP network switching node.

Let us consider the simplest mathematical model of the performance of an IP network switching node.

Let us assume that there are two sizes of IP packets being service by the router: 1024 and 32 bytes each. The standard amount of RAM for the router selected for research, for the Cisco 2811 router in this case, is 256 MB. Therefore, the router can process 250 thousand 1024-byte packets within a certain time or 8 million 32-byte packets, or 200 thousand 1024-bytes packets and 1 million 600 thousand 32-byte packets, etc. Let us assume that the router's RAM is used as efficiently as possible. We also assume that the delay time is equal for all IP packet sizes (10–20 ms) [12].

Now, let us take a situation when the amount of space in RAM is not a constant but a discrete random variable. In this case, it is impossible to calculate the throughput of the switching node accurately by using standard methods.

If the case IP packets of the same size are sent, the performance may be calculated as follows

$$C_{IP} = M_R k_n \frac{1}{t_{serv}} \text{ [Packets/s]}, \tag{1}$$

where M_R is the amount of space in RAM or the number of packets being served, taking into account the size (250 thousand 1024-byte packets, 8 million 32-byte packets), k_n is the RAM inefficiency factor, and t_{serv} is the packet processing time (the delay introduced by the router).

For now, let us simplify by not taking into account the inefficiency factor of RAM use and assume it to be equal to one. When using 1024-byte IP packets, the performance will be equal to

$$C_{IP} = M_R \frac{1}{t_{serv}} = 250,000 \times \frac{1}{0.02} = 12.5 \times 10^6 \text{ [Packets/s]},$$

and in the case of using 32-byte IP packets

$$C_{IP} = M_R \frac{1}{t_{serv}} = 8,000,000 \times \frac{1}{0.02} = 400 \times 10^6 \text{ [Packets/s]}.$$

As can be seen, the performance of the switching node increased by 32 times when using packets of a smaller information size because the RAM space 32 times increased.

Let us determine what the performance will be if the router receives packets of different sizes at the same time. To be briefer, let us denote them "A" and "B".

Let us assume that the information flow of IP packets arriving at the router looks like this (from right to left)

$$\text{AABBBABAB} \ldots,$$

where "A" is a 1024-byte packet and "B" is a 32-byte packet.

We will consider the occurrence of the current IP packet as an event independent from the occurrence of the previous one, and the information flow itself is infinite.

The probability that a randomly selected packet will be "A" is equal to P_A, and for the "B" packet, it is P_B.

Let us consider two hypotheses, H_1 and H_2. To simplify the calculations, let us take the following:

H_1—5 consecutive packets are of the "B" type; H_2—2 out of 5 packets are of the "A" type. In this case, the probability of hypotheses is $P(H_1) = P_B^5$, and $P(H_2) = 1 - P_B^5$. Since the total probability of hypotheses is equal to one, it means that they form a complete group of events.

Since the amount of space in RAM in the accepted case is a random variable, let us find its mathematical expectation

$$M = \sum_i X_i P_i = 3(1 - P_B^5) + 5P_B^5 = 3 + 2P_B^5.$$

By substituting the amount of space in RAM with its mathematical expectation in expression (1), we obtain the following.
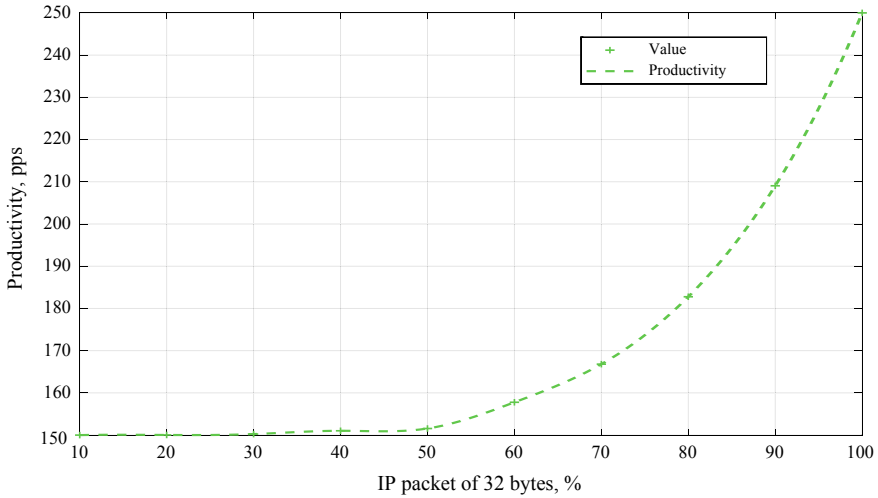
$$C_{IP} = \frac{1}{t_{serv}}[3 + 2P_B^5],$$

Fig. 2 Switching node productivity when servicing type "B" IP packets

and in the general case, if the inefficiency factor of use of the router RAM servicing space is taken into account, we obtain the following

$$C_{IP} = \frac{1}{t_{serv}}\left[3k_{250}\left(1 - P_B^5\right) + 5k_{8000}P_B^5\right] = \frac{1}{t_{serv}}\left[3k_{250} + (5k_{8000} - 3k_{250})P_B^5\right]$$

With 10% of processing of "B" packets, the performance is equal to $C_{IP} = 150.001$ [Packets/s].

As a percentage of the increase in the "B" IP packet service, the performance of the switching node is as follows (Fig. 2).

3 Experimental Part

Actually, it is almost impossible to verify the resulting mathematical model. In this regard, to verify the obtained mathematical model, a simulation model built in the Cisco Packet Tracer 7.3.0 data network simulator was used (Fig. 3).

The modeling process included several stages:

1. The transfer of the 1024-byte packet flow from PC2 to PC3 was provided with the help of a traffic generator.
2. An echo request (ping) with a 32-byte packet was sent from PC1 to PC3.
3. The echo request was repeated each time the number of 32-byte packets increased by 10%.
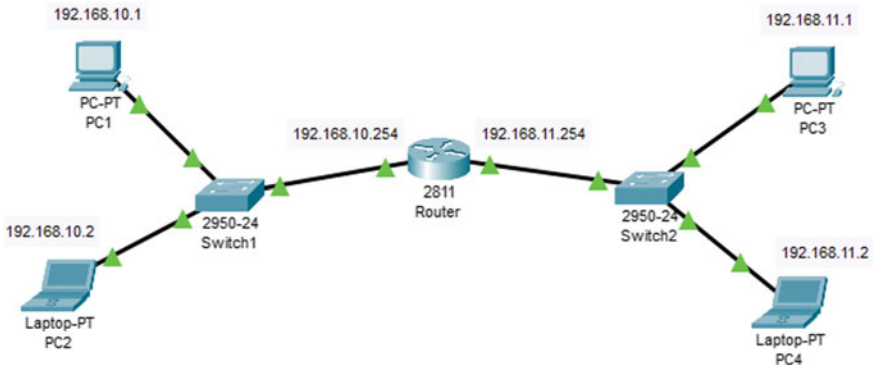
Fig. 3 Simulation model

A graphical representation of the results of the router performance and throughput simulation is shown in Fig. 4.

As can be seen from Figs. 2 and 4, the results of mathematical and simulation modeling of the router performance coincide, which indicates the adequacy of the obtained mathematical model. With 50% of 32-byte packets, the router performance and throughput are almost unchanged.

As a result of the simulation and its verification, let us derive a formula for determining the router throughput in a general form

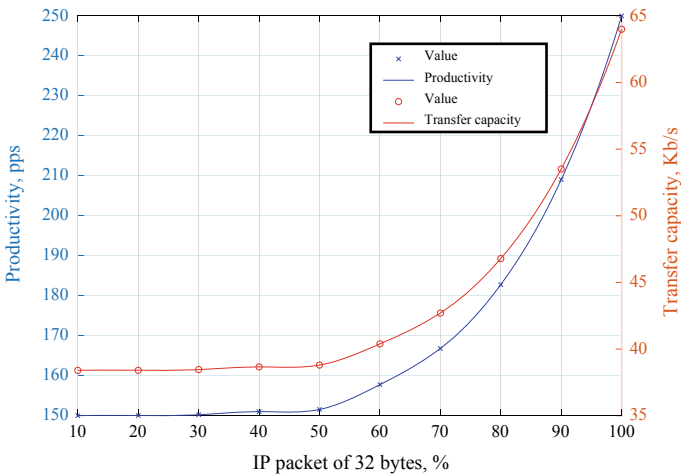$$B_{IP} = 8 \sum_i \frac{M_R k_{ni}}{\max(t_{serv,i})} P(H_i) \text{ [Bit/s]}, \tag{2}$$

Fig. 4 Productivity and transfer capacity of the switching node in the simulation

where H_i is the hypothesis concerning the combination of IP packets of various size, $R(H_i)$ is the probability of the *Hi* hypothesis, M_{Ri} is the amount of space in RAM or the number of packets being serviced, $\max(t_{serv,i})$ is the maximum time delay introduced by the router.

4 Conclusion

Using this model, it is possible to calculate the performance and throughput of a router at a non-constant amount of RAM space. It should be noted that this model considers the switching node of an IP network only with sequential descent of packets. It also does not include a number of factors that significantly affect the router throughput, such as the method used to switch packets in the switching matrix of the router and queue formation [7], the size of the router buffer [13–16], and so on.

The development of a method for calculating the throughput of IP network switching nodes, taking into account the influencing factors, as well as mathematical and simulation modeling, are fundamental methods in the design and use of telecommunications networks.

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