Transitional Intermittency in a Flat Plate Boundary Layer Subjected to Elevated Free-Stream Turbulence

Nguyen Thanh Tung and Dmitry Sboev

Abstract The results are presented of experiments on verification of the recently proposed statistical model of laminar-turbulent transition in a boundary layer subjected to elevated free-stream turbulence. It turned out to be that not all of the assumptions in this theory were completely justified. However, the most important proposition on the threshold character of the turbulent spots' generation after the streamwise streaks reached certain amplitude was confirmed in all tests.

Keywords Boundary layer · Free stream turbulence · Streamwise streaks · Intermittency

1 Introduction

A transition prediction remains one of the main problems in the physics of turbulence. It is even truer for transition under influence of elevated free-stream turbulence (FST). The empirical correlations [1] are commonly used for predictions until now. Recently Ustinov suggested [2] a new statistical theory of intermittency in a transitional boundary layer under influence of elevated FST. The present contribution is aiming to an experimental validation of this theory.

The theory is based on the assumption of Poisson process of turbulent spots generation due to a secondary instability of streamwise streaks. The laminar streaks growth is assumed as algebraic $u_{rms}^2 = cX$, where the u_{rms} is root-mean-square velocity fluctuations, the X is streamwise coordinate and c is some constant of proportionality. The streaks development is represented as the one-dimensional Gaussian random process. The streaks breakdown starts as their instant amplitude rises above some

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critical value a_c. With help of the well-known result of theory of random processes for the frequency of the outliers above a_c the following relation for the intermittency function F is obtained

$$F(\gamma) = -\ln(1-\gamma) = ARe^n\left(\frac{u_{rms}}{a_c}\right)^4 \exp\left(-\frac{a_c^2}{2u_{rms}^2}\right), \quad (1)$$

where γ is intermittency, Re is the Reynolds number based on X. The meaning of the exponent n is explained below. In this formula the u_{rms} is taken on the laminar parts of flow. The kinematic parameter A depends only on kinematic parameters of spots propagation as well as on the streak's longitudinal length scale L. In the flat plate experiment [3] it was found that L grows as square root of X

$$L = a * X^{1/2}, \quad (2)$$

where the $a*$ is constant of proportionality. The theory of optimal disturbances and the linear theory of receptivity predict the linear growth of L

$$L = aX. \quad (3)$$

In the experiments [4] it was shown that the streaks' longitudinal scales evolves from the linear dependence (3) at the early stages of their development to the nonlinear one (2) at the late stages. An exponent n at the Reynolds number Re in (1) is also depends on the law of streamwise scale L, $n = 1/2$ is for the linear law and $n = 1$ for the nonlinear one.

It is useful to represent the intermittency function as

$$G = \ln\left(-\frac{F(\gamma)}{u_{rms}^4 Re^n}\right) = -\frac{1}{2}a_c^2\left(\frac{1}{u_{rms}^2}\right) + A\frac{1}{a_c^4} \quad (4)$$

In the following the notation G will be used for the linear normalization of (4) with $n = 1/2$ and $G*$ for the $n = 1$ case. The constant a_c is the same for both cases. The parameters a_c, A and $A*$ should be determined from experiment. M.V. Ustinov has obtained the values $a_c = 0.31$ and $A = 10.4$ for the linear normalization of (3), (4) after processing of flat plate experimental data available to him.

The theory described is semi-empirical by its nature and based on the following main assumptions: the algebraic growth of streaks, their gaussianity, a definitive law of the streak's longitudinal scales and the threshold character of breakdown. All of them were checked in some details in the course of present experiments.

2 Experimental Setup

The experiments were conducted in the MIPT open return wind tunnel AT-3 at free stream velocities $U_0 = 4$–14 m/s. The test section has octagonal cross-section, the width 800 mm and length 1200 mm. Now the wind tunnel is about 80 years old. The model was a 1.2 m long, 10 mm thick flat plate with the semicylindrical (diameter 4 mm) leading edge. The leading edge arrangement was taken from [5] due to its technological simplicity. The coordinate system used has its origin on the leading edge, X is streamwise axis, and the Y is wall-normal coordinate. The hot-wire measurements were performed with single-wire probe mounted on traverse. The movement of the wire was carried out manually along the X axis and by means of PC-controlled linear stage along Y axis.

A FST was generated by the several two-plane and woven grids installed in the slot at the beginning of the test section at the distance 200 mm from the leading edge. The grids' wire diameters d and mesh sizes M are given in the Table 1. The FST levels Tu at the leading edge ranges from 1.7–4.9%. The FST decayed downstream as X^{-b} with $b = 0.50$–0.87. The turbulence integral scale Λ was 4.9–5.9 mm in all regimes. It turned out to be difficult in this set-up to get a broad variation in Λ as it was intended initially. By the choosing of U_0 and Tu values it was possible to adjust a transition zone position over the flat plate length.

A flat plate has a flap to adjust a stagnation line on its upper side. To avoid a leading edge separation a flap was so positioned that slightly accelerated boundary layer was created with a Hartree parameter $\beta = 0.03$–0.05. This boundary layer is still close to the Blasius solution. The velocity distribution over the model was close controlled in the each regime. In Fig. 1 an example of the mean flow evolution from the laminar to turbulent state is shown. A good agreement between the measured profiles and the Falkner-Skan solution in laminar parts of flow is obvious.

Table 1 Parameters of the experiments

Regime	Grid, $d \times M$ (mm)	U_0 (m/s)	Tu (%)	β	a_c	A
R1	G1, 0.5 × 9.8	10.47	1.84	0.035	**0.32**	**12.01**
R2		11.40	1.86	0.029	**0.31**	**12.71**
R3		13.96	1.88	0.026	**0.28**	**13.06**
R4	G2, 1.7 × 14	5.70	4.00	0.032	**0.35**	**0.89**
R5		6.67	4.02	0.046	**0.35**	**1.45**
R6		10.84	3.34	0.410	**0.33**	**5.56**
R7	G3, 0.75 × 6.75	11.15	1.69	0.032	**0.29**	**60.76**
R8	G6, 1.2 × 16	3.85	3.16	0.035	**0.37**	**14.07**
R9		7.64	4.19	0.054	**0.36**	**2.48**
R10		9.74	4.55	0.028	**0.35**	**1.02**
R11		12.71	4.91	0.044	**0.33**	**3.08**

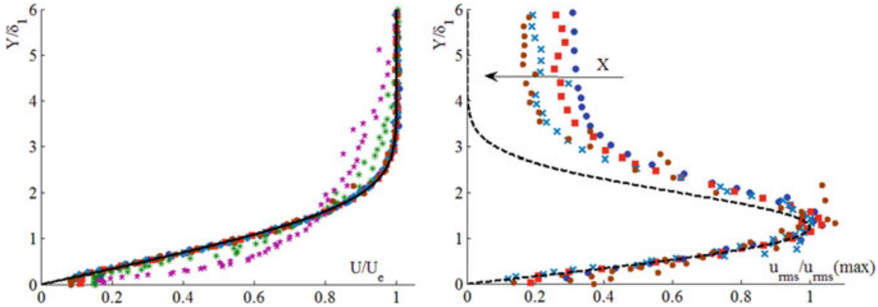

Fig. 1 The mean velocity profiles (left) and u_{rms} profiles (right) in the R2 regime. The solid and dashed lines is the Falkner-Skan solution for U and $U(\mathrm{d}U/\mathrm{d}Y)$ respectively, $\beta = 0.029$, U_e is external velocity and δ_1 is the displacement thickness

A streaks formation is demonstrated in Fig. 1, where the profiles of the u_{rms} for the laminar parts of flow are plotted. The experimental points were smoothed by a spline and normalized with the maxima of smoothed curves. The profiles have a very characteristic bell-like shape with a single maximum in the middle part of the boundary layer. Also the curve of so called "breathing mode" $U(\mathrm{d}U/\mathrm{d}Y)$ is plotted, which is a good representation of the theoretical streak profile. As it was expected, the agreement between the "breathing mode" and experimental profiles is good in the lower half of the boundary layer. In the upper half the influence of decaying external turbulence is seen.

For intermittency measurements a detector function was chosen in form of the envelopes of first and second derivatives of band-pass filtered signal u. Using of envelopes allows to dismiss a holding time parameter in the computations of the indicator functions. The cumulative intermittency distributions as functions of the threshold value were constructed. To choose the threshold level a method of [6] was implied. In some regimes, when it was difficult to fit a straight line to a cumulative intermittency distribution, the original method [7] of finding of the region of maximum curvature as threshold value was used. The examples of intermittency profiles through the boundary layer are plotted in Fig. 2.

3 Results

The algebraic growth of streaks. The some examples of u_{rms} development over streamwise coordinate are shown in Fig. 3. The data was taken from the positions of maxima in wall-normal profiles. In all regimes shown the transition onset is located at the middle or downstream part of the plate. The cases R2 and R7 at high velocities with moderate *Tu* demonstrates the well-known linear growth of u^2_{rms}

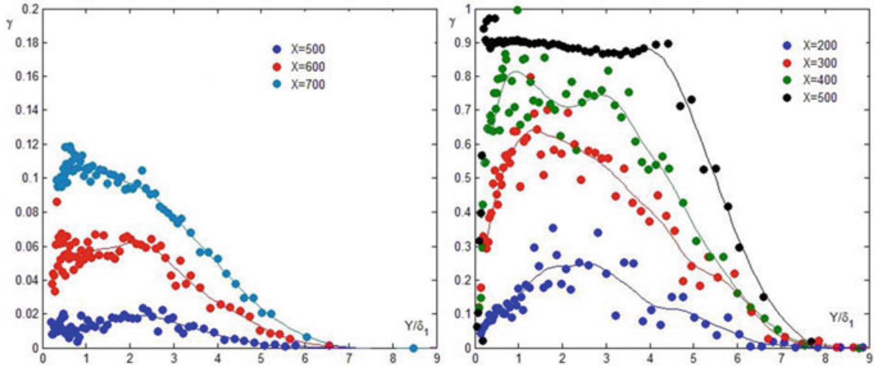

Fig. 2 The profiles of intermittency in the regimes R8 (left) and R5 (right), *X* is in mm

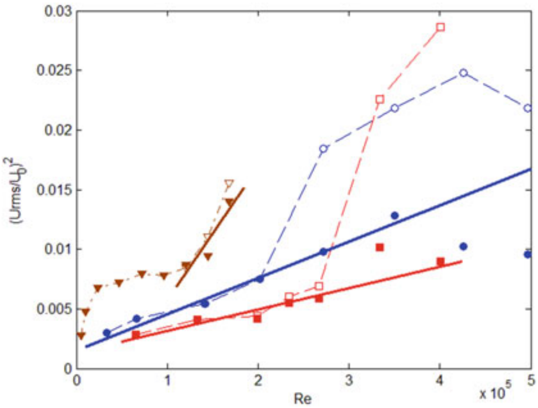

Fig. 3 Downstream evolution of u^2_{rms} in the regimes R2 (blue), R7 (red) and R8 (brown). Open symbols corresponds to a full signal, the solid ones to laminar parts of a signal. The solid lines are the linear approximations of fluctuations on the laminar parts

with Reynolds number $Re = U_0X/\nu$. Also on the laminar parts of the intermittent signal the fluctuations seems to be follow this tendency at least at the early stages of transition.

The regime R8 is somewhat anomalous. In all others regimes with this grid G6 a transition starts very close to the leading edge within a distance of first 100 mm. Because of the low *Re*, the growth of u^2_{rms} in case R8, after a brief rapid amplification near the leading edge, is slowed down and starts again very close to the transition onset. It is possible that in R8-R11 cases the nonlinear receptivity (e.g. a vorticity stretching mechanism by the blunt leading edge) had a strong influence on disturbances development.

In some cases the fluctuations of laminar parts of flow begins to decay inside of zone of laminar flow destruction. For evaluation of functions *G* in these transitional regions the extrapolations of u^2_{rms} dependencies from the laminar boundary layer were used just like it was done in [2] (see Fig. 3, R2 and R7). For the aforementioned regimes R8-R11 the data from the early stages of turbulent spots development were

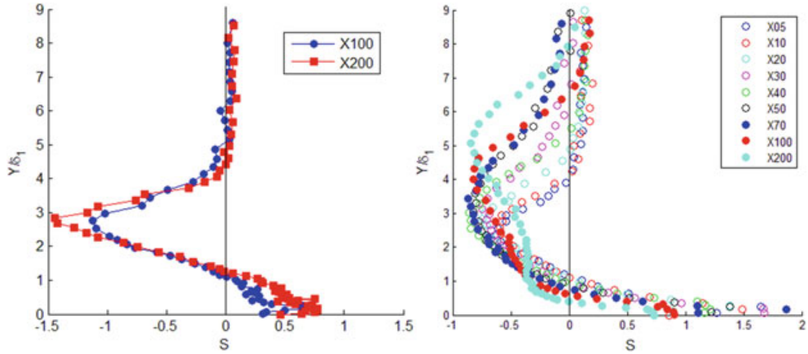

Fig. 4 The profiles of skewness S of the velocity fluctuations' probability density functions for the regimes R1 (laminar boundary layer, left) and R11 (transitional, right). X is in mm

fitted with cX (Fig. 3, R8) and extrapolated downstream if needed. So in some cases the prehistory of the disturbances development was ignored.

Gaussianity of fluctuations. It is known (e.g. [8, 9]) that the fluctuations' field in a boundary layer subjected to moderate FST is far from the gaussian one. The gaussianity of probability density function can be evaluated with the help of skewness S. The Gauss distribution has zero skewness. The profiles of S for the nonlinear cases R8 and R11 are given in Fig. 4. At the upstream positions (for example in the case R1) they are typical for all regimes. The S is positive near the wall and has a negative minimum near the edge of a boundary layer. The position of zero-crossings located at $Y/\delta_1 = 1.2$–1.5, almost at the same height as the maxima in Y-profiles of u_{rms}. Thus only near the middle part of laminar boundary layer and at the early stage of transition the fluctuations u could be approximately considered as gaussian. The beginning of turbulent spots generation leads to severe deformations of Y-profiles of S, zero-crossing points moves closer to the wall and assumption of gaussianity in [2] becomes no more valid at all.

The streak's longitudinal scale. Introducing the wave number $\alpha = 2\pi f/U_0$ and normalizing the power spectral density in the manner of [3] it is possible to show that nondimensional wavenumber spectra E should collapse if the α and E scaled with the δ_1 for nonlinear law (2) or with X for the linear one (3). The nonlinear scaling was for the first time demonstrated in the experiments [3]. In present study, as well as in [4], it was found that in general a change from the linear to nonlinear evolution of L occurs. Such example is shown in Fig. 5. The spectra of E while nondimensionalized with δ_1 does not collapse at the upstream stations, on another hand the spectra normalized with X collapses. At the $X = 400$ mm in the case R1 the first turbulent spots appears. In some regimes, just like in the aforementioned case R8, the nonlinear development of L starts from the very beginning of the streaks development. This diversity is not accounted for in the theory [2]. However, it only will affect the kinematic parameter A.

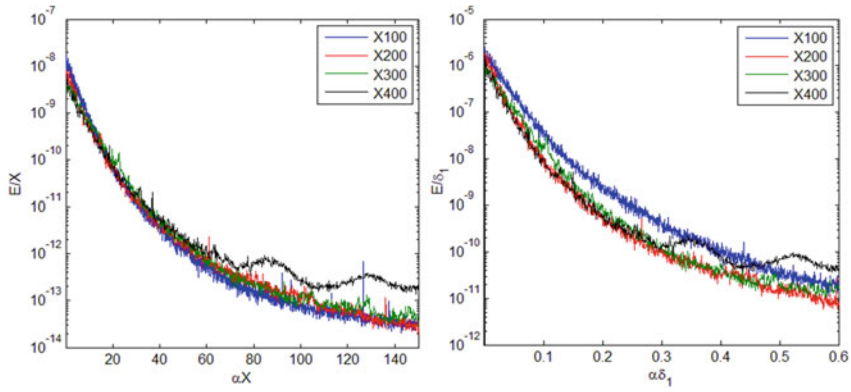

Fig. 5 Wavenumber spectra E nondimensionalized using the linear law (3) (left) and the nonlinear law (2) (right), regime R1, X is in mm

The intermittency functions G, G* and F. The linear G and nonlinear G^* functions evaluated with the Eq. (4) is plotted in Fig. 6 for the regimes with the late transition onset. As predicted the linear fits agrees very well with the experimental points. However nor G functions, nor G^* functions do not collapses into a single straight line as it was proposed in [2]. Meanwhile, for the all regimes the negative slopes of the linear fits are very close to each other. The values of threshold constant a_c for the linear function G is given in the Table 1. The mean value of a_c in the Table 1 is 0.33 that is close to the value 0.31 obtained in [2] from the experiments [3]. For the nonlinear normalization G^* the values of a_c lays between 0.27 and 0.32 with the mean 0.30. It is follows from these data that for whole range of regimes covered, the streaks' instance threshold amplitude to produce a turbulent spot is nearly the same.

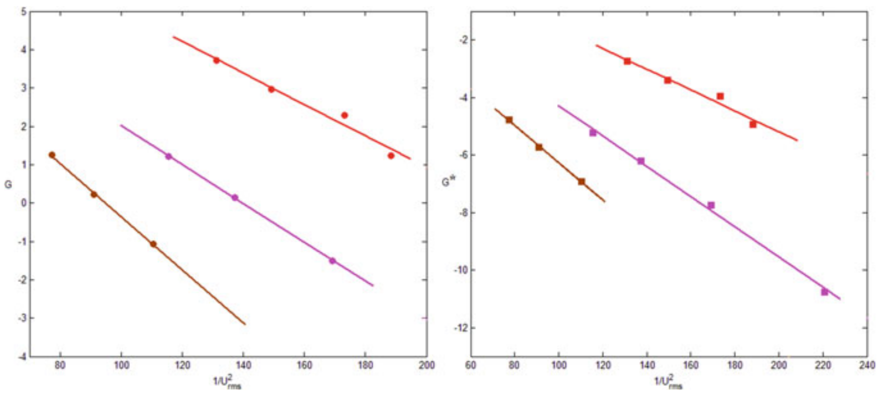

Fig. 6 The intermittency functions $G(1/u^2_{rms})$ (left) and $G^*(1/u^2_{rms})$ (right) for the regimes R1 (magenta), R7 (red) and R8 (brown)

Its value is about 30–35% of free-stream velocity and independent of longitudinal scales of streaks.

It should be noted that the theory under consideration does not specify an exact mechanism of spot formation. The theory only states that in the stationary random process of spots' birth and development the streak's instant amplitude required for new spot formation should be above a certain threshold. The first ever spot that needed to start this random process in the given region of flow could be produced by completely different means.

The fact that nor *G* nor *G** functions do not collapses into a single straight line is explained by the variations in kinematic parameter *A*. Because the turbulent spot's propagations velocities do not depend on the way of its birth, these variations are mainly could be explained by the aforementioned variations in the development of longitudinal scales of streaks. The values of *A* obtained for the linear normalization *G* are given in the Table 1. The scatter of *A* for nonlinear normalization is almost the same.

Because it is impossible from the data obtained to choose between two functions *G* and *G** it was decided to use the linear normalization to obtain the intermittency functions *F* from the Eq. (1). Due to the scatter in A the Eq. (1) was applied to the each set of data separately. In calculations of *F* the a_c and *A* values from the Table 1 for the each regime were used respectively. The results are shown in the Fig. 7. In most cases the agreement between the experiments and Eq. (1) is reasonably good. It is worth to mention, that the agreement is unexpectedly good for the nonlinear regimes R6, R10 and R11, when the transition starts in immediate vicinity of the blunt leading edge and regions of pure laminar streaks development is almost absent.

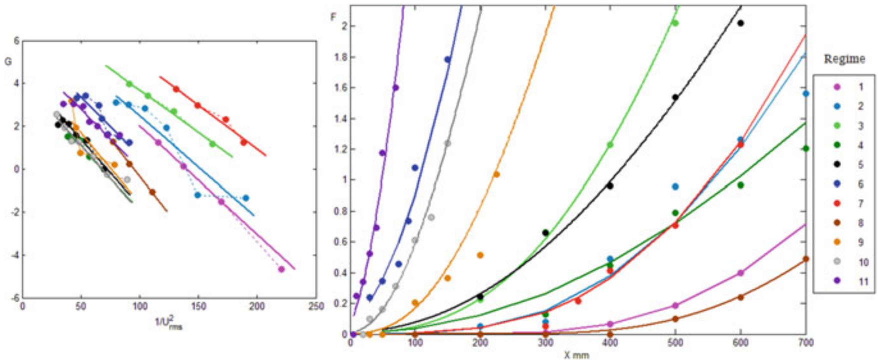

Fig. 7 The intermittency functions *G* (left) and the measured (points) and calculated (curves) intermittency functions *F* (right) obtained for all regimes

4 Conclusions

The statistically stationary process of the streaks transformation into the spots has a threshold character. The streak threshold level a_c for the birth of a turbulent spot in the present setup does not depend on a test regime and is about 30–35%.

When appropriately calibrated, the Ustinov's semiempirical statistical model of transition at high FST level gives the reasonable predictions. It performed well even in the cases of transition onset at the leading edge despite the fact that one of the main assumptions of gaussianity of the streaks development was not always hold.

Some of the assumptions behind the model weren't being justified in these experiments. In particular, the kinematic parameter A was found to be dependent on a test regime. To obtain a prediction method on basis of the model, the kinematic parameter needs to be calibrated in a more accurate manner.

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