Review of Symmetry-Breaking Options on Mathematical Programming Models with Rolling Horizons Procedure

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Abstract The rolling horizons procedure is widely used both in industry and in scientific research for the resolution of mathematical programming models. It allows reducing the size of the models to be solved in the times allowed with the available computational capacities. It takes into consideration the closest information with less uncertainty. But programming models can have symmetries, when they have variables that can be **permuted** without changing the structure of the problem. These symmetries increase the search spaces for possible solutions, increasing the need for computation and presenting alternative solutions with equivalent results in the objective function. The symmetry can generate different solutions with equivalent values in the objective function but that can imply very different results in the long term with the rolling horizons procedure. This paper presents the proposed actions to symmetry break that have been applied to the rolling horizons procedure. The work provides the different proposals identified to break the symmetry.

Keywords Supply chain management · Rolling horizons · Symmetry · Mixed integer linear programming

1 Introduction

The rolling horizons procedure is a frequent tool in industry and academic environments [7, 35]. Practical application is identified in inventory management, production planning, scheduling/sequencing, plant location, machine replacement, cash management, capacity expansion and wheat trading/storage [5]. Its use can support decisions in uncertain environments and transform the resolution of a long-horizon problem into a sequential resolution of short-horizon problems [9, 17, 26, 30, 31, 39]. But it is a heuristic method, where it should be said that the best planning operations proposals that are obtained on each rolling horizon are not necessarily the same

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planning operations proposals that would be found in the solution of the entire time horizon [4].

The rolling horizons are a representation of the industrial reality [7]. Companies make decisions about the operations planning from forecasts and orders, their ongoing situation, and the available capacity [16]. The information is usually updated in the following periods along with the results of the planned planning. The new information available may have variations within the expected ranges (stochastic tools, fuzzy, sensitivity analysis can be used to simulate it), or totally unexpected (require contingency plans for accidents or catastrophes) or higher than forecast ranges but expected (reactive plan in real time together with preventive actions could be used). This new information allows updating the planning for the following periods, by modifying the previous plan or launching a new planning recalculation [24]. The companies recalculate their planning according to information updates, although they try not to make changes in the near periods, in order to reduce the nervousness or planning instability and costs [32]. The competitiveness of the company lies in the balance between its operating costs and its ability to react to changes.

These problems frequently have symmetries in their mathematical programming models. Alternative solutions can be generated with similar results in the objective function. Variables that can be exchanged without changing the structure of the problem [1]. These symmetries slow down the search algorithms of the best solution to the objective function that complies with the modeling constraints. The symmetries increase the options that the branch and bound algorithms must solve [33]. The symmetry in a problem increases the search space size for the algorithms. Equivalent solutions can be exchanged [3], where different solutions are proposed for the same objective value [38]. This may make it more difficult to demonstrate the optimality of the problem solutions and, therefore, increase the computation time [25]. Jans [13] showed that eliminating the formulation symmetry can be useful to accelerate computational times. It reduces the amount of search needed to solve the problem [10]. Margot [18] comments that breaking symmetry can turn a computationally intractable problem into one that is easily solved.

In the procedure of rolling horizons in mathematical programming models, symmetry can become especially relevant. The different solutions within the allowed tolerance or calculation time can imply large differences in the following planning horizons. The proposed solutions for the model can vary significantly from one computer equipment to another.

To the best of our knowledge, we have not found a review of the literature on the treatment of symmetry when using the rolling horizons procedure. Therefore, a systematic review of the literature on measures to avoid symmetry in models with the rolling horizons procedure is proposed.

The rest of the paper is structured as follows: first, a short description of the review methodology is introduced; second, a brief discussion of the results is presented; finally, the paper ends with the conclusion and future works.

2 Review Methodology

The review has been carried out following the systematic literature review protocol presented and used by Marín et al. [19] and Rius-Sorolla et al. [28, 29].

The steps proposed are as follows: set the goal, select type of reference and database, search filter and manage reference, extract information of selected reference, and write the report.

Therefore, it uses a transparent procedure, to find, evaluate, and synthesize the results of relevant research. The procedures are explicitly defined in advance to ensure that the exercise is transparent and can be replicated.

Our goal is to identify actions to avoid symmetry in the application of the rolling horizons procedure.

The search was carried out on selecting the references that contain the terms "symmetry" and "rolling horizon" in the database of *Scopus* and *Web Of Science* (WoS) with access from Universitat Politècnica of València. The extracted works have been those that deal with both concepts. A snowball strategy has also been applied to the first identified references in order to include other related jobs. In Table 1, the publication of the reviewed references can be identified.

3 Results

According to Margot [18], the main approaches to deal with symmetries in integer linear programming are symmetry breaking inequalities, perturbation, fixing variables, and pruning of the enumeration tree. It also can be reformulated the problem so that it has a reduced amount of symmetry, or even none at all [10].

The actions identified to break the symmetry are:

- Add new restrictions to order equivalent elements [12, 14, 22, 25]. If a product can be done on several machines, it is established that it should be done in the first machine. Lexicographic perturbation Eq. (1),

$$x_i \leq x_{i+1} \, \forall i \tag{1}$$

- Sort the machines according to some natural logic, such as varying the setup costs per machine, or decreasing the total costs per machine or decreasing resources capacity, or with a weighting factor in the objective function increasing parameter values with product index [8, 13].
- Set new restrictions that avoid those undesired solutions, without eliminating feasible solutions [6]. One way is by setting variables to reduce the feasible solution space [21, 34, 36, 37]. A binary variable is fixed on some periods, as can be seen in Eq. (2).

$$\delta_t = 1, \, \forall t \geq t_s \tag{2}$$

Table 1 Publication of the selected references

Publication	References
International Journal of Production Research	4
Computers and Operations Research	3
Informs Journal on Computing	2
International Journal of Production Economics	2
Transportation Science	2
Annals of Operations Research	1
Computers and Chemical Engineering	1
Computers and Industrial Engineering	1
European Journal of Operational Research	1
Expert Systems with Applications	1
International Journal of Electrical Power & Energy Systems	1
International Journal of Healthcare Technology and Management	1
International Journal of Innovative Computing, Information and Control	1
International Journal of Sustainable Engineering	1
Journal of Marine Science and Engineering	1
Journal of Rail Transport Planning & Management	1
Lecture notes	1
Management and Production Engineering Review	1
Mathematical Problems in Engineering	1
Modern Physics Letters B	1
Nuclear Physics B—Proceedings Supplements	1
Operations Research	1
Transportation Research Part C: Emerging Technologies	1
Transportation Research Part E: Logistics and Transportation Review	1
Urban Rail Transit	1
	33

- Give continuity with the previous period, like if the oven was working in the previous period, the setup is eliminated [23], as can be seen in Eq. (3).

$$Z_{yt} \leq Z_{y(t-1)} \, \forall y, t > 1 \tag{3}$$

The actions identified in the systematic literature review attempt to break the symmetries produced by equivalent demands in different products [3], by the existence of equivalent resources in the models, such as parallel machines [2, 8, 11, 13] or a combination of both [38] or routes with equal costs [15].

4 Conclusion and Future Work

This work presents the results of a systematic literature review on actions to break the symmetry in models that apply the rolling horizons procedure.

It highlights the importance of breaking symmetry in order to avoid unwanted solutions or speed up the resolution process. The identified symmetries focus on demands related to equivalent products and operations that can be performed on equivalent resources. The identified symmetries relate to product, resources, operations and time indexes in the mathematical programming models [27].

The actions identified to break the symmetry have been grouped into four categories. And they are those that give a lexicographic order, varying the parameters of the model to establish a differentiation and setting variables in certain conditions to eliminate undesired solutions and restriction between periods to give an order.

Future research should be done to extend the symmetry breaking action to another index as the time relative index of the rolling horizons procedure. In addition, a specific pruning option could be developed for the rolling horizons procedure.

References

1. Alemany J, Magnago F, Moitre D, Pinto H (2014) Symmetry issues in mixed integer programming based Unit Commitment. Int J Electr Power Energy Syst 54:86–90. https://doi.org/10.1016/j.ijepes.2013.06.034
2. Bard JF, Purnomo HW (2005) Hospital-wide reactive scheduling of nurses with preference considerations. IIE Trans 37(7):589–608. https://doi.org/10.1080/07408170590948468
3. Bohlin M, Gestrelius S, Dahms F, Mihalák M, Flier H (2016) Optimization Methods for Multistage Freight Train Formation. Transp Sci 50(3):823–840. https://doi.org/10.1287/trsc.2014.0580
4. Cao Y (2015) Long-Distance procurement planning in global sourcing. Ecole Centrale Paris. Accessed from https://tel.archives-ouvertes.fr/tel-01154871/
5. Chand S, Hsu VN, Sethi S (2002) Forecast, solution, and rolling horizons in operations management problems: a classified bibliography. Manuf Service Oper Manag 4(1):25–43. https://doi.org/10.1287/msom.4.1.25.287
7. Denton BT, Miller AJ, Balasubramanian HJ, Huschka TR (2010) Optimal allocation of surgery blocks to operating rooms under uncertainty. Oper Res 58(4-part-1):802–816. https://doi.org/10.1287/opre.1090.0791
6. de Sampaio RJBB, Wollmann RRGG, Vieira PFGG, Sampaio RJB, De Wollmann RRGG, Vieira PFGG (2017) A flexible production planning for rolling-horizons. Int J Prod Econ 190(2016):31–36. https://doi.org/10.1016/j.ijpe.2017.01.003
8. Ewaschuk CM, Swartz CLE, Zhang Y (2018) An optimization framework for scheduling of converter aisle operation in a nickel smelting plant. Comput Chem Eng 119:195–214. https://doi.org/10.1016/j.compchemeng.2018.08.024
9. Garcia-Sabater JP, Maheut J, Garcia-Sabater JJ (2009) A capacitated material requirements planning model considering delivery constraints: a case study from the automotive industry, pp 378–383. https://doi.org/10.1109/ICCIE.2009.5223806
10. Gent IP, Petrie KE, Puget JF (2006) Symmetry in constraint programming, pp 329–376. https://doi.org/10.1016/S1574-6526(06)80014-3

11. Gicquel C, Minoux M, Dallery Y (2011) Exact solution approaches for the discrete lot-sizing and scheduling problem with parallel resources. Int J Prod Res 49(9):2587–2603. https://doi.org/10.1080/00207543.2010.532927
12. Hooshmand F, MirHassani SA, Akhavein A (2017) A scenario-based approach for master surgery scheduling under uncertainty. Int J Healthcare Technol Manag 16(3/4):177. https://doi.org/10.1504/IJHTM.2017.088849
13. Jans R (2009) Solving lot-sizing problems on parallel identical machines using symmetry-breaking constraints. INFORMS J Comput 21(1):123–136. https://doi.org/10.1287/ijoc.1080.0283
14. Lamsal K, Jones PC, Thomas BW (2017) Sugarcane Harvest Logistics in Brazil. Transp Sci 51(2):771–789. https://doi.org/10.1287/trsc.2015.0650
15. Lima RM, Grossmann IE, Jiao Y (2011) Long-term scheduling of a single-unit multi-product continuous process to manufacture high performance glass. Comput Chem Eng 35(3):554–574. https://doi.org/10.1016/j.compchemeng.2010.06.011
16. Luo L, Luo Y, You Y, Cheng Y, Shi Y, Gong R (2016) A MIP model for rolling horizon surgery scheduling. J Med Syst 40(5):127. https://doi.org/10.1007/s10916-016-0490-9
17. Lv Y, Zhang J, Qin W (2017) A genetic regulatory network-based method for dynamic hybrid flow shop scheduling with uncertain processing times. Appl Sci 7(1):23. https://doi.org/10.3390/app7010023
18. Margot F (2010) Symmetry in integer linear programming. 50 Years of Integer Programming 1958–2008: From the Early Years to the State-of-the-Art. Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-540-68279-0
19. Marin-Garcia JA, Ramirez Bayarri L, Atares Huerta L (2015) Protocol: Comparing advantages and disadvantages of Rating Scales, Behavior Observation Scales and Paired Comparison Scales for behavior assessment of competencies in workers. A systematic literature review. Working Papers Oper Manag 6(2):49. https://doi.org/10.4995/wpom.v6i2.4032
20. Medina-Lopez C, Marin-Garcia JA, Alfalla-Luque R (2010) Una propuesta metodológica para la realización de búsquedas sistemáticas de bibliografía (A methodological proposal for the systematic literature review). Working Papers Oper Manag 1(2). https://doi.org/10.4995/wpom.v1i2.786
21. Moniz S, Barbosa-Póvoa AP, de Sousa JP (2014) Simultaneous regular and non-regular production scheduling of multipurpose batch plants: A real chemical–pharmaceutical case study. Comput Chem Eng 67:83–102. https://doi.org/10.1016/j.compchemeng.2014.03.017
22. Moosavi A, Nikfarjam A (2019) A multi-path routing-inventory problem for a closed-loop supply chain considering the heterogeneous fleet of vehicles. Int J Sustain Eng 1–15. https://doi.org/10.1080/19397038.2019.1566412
23. Motta Toledo CF, da Silva Arantes M, Hossomi YBM, Almada-Lobo B (2016) Mathematical programming-based approaches for multi-facility glass container production planning. Comput Oper Res 74:92–107. https://doi.org/10.1016/j.cor.2016.02.019
24. Rafiei R, Gaudreault J, Bouchard M, Santa-Eulalia L (2012) A reactive planning a pproach for demand-driven wood remanufacturing industry: a real-scale application. CIRRELT, vol 71
25. Raknes NT, Ødeskaug K, Stålhane M, Hvattum LM, Raknes NT, Ødeskaug K et al (2017) Scheduling of maintenance tasks and routing of a joint vessel fleet for multiple offshore wind farms. J Marine Sci Eng 5(1):11. https://doi.org/10.3390/jmse5010011
26. Ramezanian R, Fallah Sanami S, Shafiei Nikabadi M (2017) A simultaneous planning of production and scheduling operations in flexible flow shops: Case study of tile industry. Int J Adv Manuf Technol 88(9–12):2389–2403. https://doi.org/10.1007/s00170-016-8955-z
27. Rius-Sorolla G, Maheut J, Estelles-Miguel S, Garcia-Sabater JP (2017) Protocol: Systematic Literature Review on coordination mechanisms for the mathematical programming models in production planning with decentralized decision making. WPOM-Working Papers Oper Manag 8(2):22. https://doi.org/10.4995/wpom.v8i2.7858
28. Rius-Sorolla G, Maheut J, Coronado-Hernandez JR, Garcia-Sabater JP (2018a) Lagrangian relaxation of the generic materials and operations planning model. CEJOR. https://doi.org/10.1007/s10100-018-0593-0

29. Rius-Sorolla G, Maheut J, Estellés-Miguel S, Garcia-Sabater JP (2018b) Coordination mechanisms with mathematical programming models for decentralized decision-making: a literature review. CEJOR. https://doi.org/10.1007/s10100-018-0594-z
30. Rius-Sorolla G, Maheut J, Garcia-Sabater JP (2018c) Distributed programming production planning with a shared capacity coordinated by a Lagrangian relaxation model on a rolling horizon. In: Conference handbook #euro2018valencia. Valencia (Spain), p 246
31. Rodriguez MA, Montagna JM, Vecchietti A, Corsano G (2017) Generalized disjunctive programming model for the multi-period production planning optimization: An application in a polyurethane foam manufacturing plant. Comput Chem Eng 103:69–80. https://doi.org/10.1016/j.compchemeng.2017.03.006
32. Sahin F, Narayanan A, Robinson EP (2013) Rolling horizon planning in supply chains: review, implications and directions for future research. Int J Prod Res 51(18):5413–5436. https://doi.org/10.1080/00207543.2013.775523
33. Sherali HD, Smith JC (2001) Improving discrete model representations via symmetry considerations. Manage Sci 47(10):1396–1407. https://doi.org/10.1287/mnsc.47.10.1396.10265
34. Suerie C (2005) Time continuity in discrete time models : new approaches for production planning in process industries. Springer. Retrieved from https://www.scopus.com/record/display.uri?eid=2-s2.0-53849140089&origin=resultslist&sort=plf-f&src=s&nlo=&nlr=&nls=&sid=9fb4c59cd32f2b0c32fafe3f748cb9d2&sot=a&sdt=a&sl=36&s=ALL%28+symmetry+AND+%22rolling+horizon%22%29&relpos=35&citeCnt=6&searchTerm=
35. Tiacci L, Saetta S (2012) Demand forecasting, lot sizing and scheduling on a rolling horizon basis. Int J Prod Econ 140:803–814. Elsevier. https://doi.org/10.1016/j.ijpe.2012.02.007
36. Xiao J, Zhang C, Zheng L, Gupta JNDD (2013) MIP-based fix-and-optimise algorithms for the parallel machine capacitated lot-sizing and scheduling problem. Int J Prod Res 51(16):5011–5028. https://doi.org/10.1080/00207543.2013.790570
37. Xiao Y, Xie Y, Kulturel-Konak S, Konak A (2017) A problem evolution algorithm with linear programming for the dynamic facility layout problem—A general layout formulation. Comput Oper Res 88:187–207. https://doi.org/10.1016/j.cor.2017.06.025
38. Zamarripa M, Marchetti PA, Grossmann IE, Singh T, Lotero I, Gopalakrishnan A et al (2016) Rolling horizon approach for production−distribution coordination of industrial gases supply chains. Ind Eng Chem Res 55(9):2646–2660. https://doi.org/10.1021/acs.iecr.6b00271
39. Zulkafli NI, Kopanos GM (2017) Integrated condition-based planning of production and utility systems under uncertainty. J Cleaner Prod 167:776–805. https://doi.org/10.1016/j.jclepro.2017.08.152