

# Kinds of Unfinished Description with Implication for Natural Science



J. Rowan Scott 

**Abstract** The Reductive Sciences and Complexity Sciences employ micro-scale first rigorous ‘bottom-up’ reductive Logic as well as the mathematics of Symmetry and Group theory when modeling systemic change, causal relationships and mechanisms. *Four* kinds of ‘unfinished description’ result from this approach. First, the reductive micro-scale first assumption fails to replicate the complexity of natural evolutionary processes. Second, the reductive ‘bottom-up’ metaphor obscures significant facets of a more complicated natural evolutionary Logic. Third, abstract, rigorous, ‘bottom-up’ reductive Logic is susceptible to *undecidable reductive propositions* revealing *formal reductive incompleteness* and its *implications*, which include necessary *meta-consideration* in determining *reductive logical consistency*. Fourth, the powerful mathematics of Symmetry and Group Theory is not sufficient when mathematically modeling causal relationships in Nature. Consequently, the *Reductive Scientific Narrative* creates a ‘comprehensive’ description of causal relationships in Nature and evolution that is fundamentally ‘unfinished’. Explaining and then ‘correcting’ each of the four kinds of ‘unfinished description’ illuminates a novel path that can more closely approximate the natural system and move Reductive Science and Complexity Science toward a deeper consilience.

**Keywords** Micro-scale · Multi-scale · ‘bottom-up’ reductive logic · Formal reductive incompleteness · Meta-consideration · Consistency · Symmetry · Group theory · Self-similarity · Fractals · Meta-construction · Meta-reductive paradigms

## 1 Introduction

When modeling systemic change and causal relationships in Nature, the Reductive Sciences begin with a reductive account involving micro-scale entities, and then applies rigorous ‘bottom-up’ reductive Logic as well as the applied mathematics of Symmetry and Group theory [18]. There is an active debate, which polarizes the

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Reductive Sciences and the developing Complexity Sciences, regarding the usefulness of rigorous ‘bottom-up’ reductive Logic when modeling the complexity of the natural order [26]. While doubting the usefulness of ‘bottom-up’ reductive Logic [9] the Complexity Sciences generally presume that Symmetry and Group theory provide a useful applied mathematical approach to defining causal relationships.

It is, however, possible to spell-out *four* inter-related kinds of ‘unfinished or partial scientific description’ associated with ‘bottom-up’ reductive Logic *and* the mathematics of Symmetry and Group theory when these are used in the context of theoretical, experimental and mathematical modeling of causal relationships in Nature and evolution.

### 1.1 *Four Descriptive Deficiencies*

**First.** The *reductive account begins with a natural micro-scale entities first assumption*, which fails to recognize Nature and evolution may self-organize on *multiple* scales at once. Proposing an alternative *meta-reductive natural multi-scale co-evolving entities assumption* as an alternative offers a theoretically and experimentally accessible *hypothesis*, which, if it finds support, could reveal the *reductive, begin with micro-scale entities first assumption*, is actually a ‘special case’ within a more comprehensive model of natural system evolution possible under a *meta-reductive natural multi-scale co-evolution assumption*.

**Second.** The *reductive ‘bottom-up’ metaphor* accepts an implicit or explicit *postulate* that the Universe and natural evolution self-organize from an *initial* sea of *micro-scale* quantum states. Every other *subsequent* form of hierarchically organized mid-scale or macro-scale complexity is *assumed* to result from ‘bottom-up’ self-organization of quantum micro-scale states and processes. If *meta-reductive natural multi-scale co-evolution* finds theoretical and experimental support, then, the *reductive ‘bottom-up’ metaphor* will become a ‘special case’ within a more comprehensive meta-reductive metaphor capable of describing and modeling *natural multi-scale co-evolution*. A much more complicated co-evolutionary metaphor may be required, capable of describing, at a minimum, multi-scale primary ‘bottom-up’, ‘top-down’ causal relationships as well as secondary ‘lateral’ or ‘horizontal’ causal relationships.

**Third.** Rigorous, formal, *reductive Logic* is susceptible to well-formed and logically true argument composing potential *contradiction* and *self-referencing paradox* [4]. This realization inevitably leads to the insight that reductive Logic and argument can contain *undecidable reductive propositions*, which reveal *formal reductive incompleteness* as well as a need for *meta-consideration* in determining the *consistency* of reductive logical argument [23, 25].

This discovery opens up a previously unrecognized path toward resolution of questions about the usefulness of reductive Logic in natural science. Rather than *polarization* between *reductive* and *anti-reductive* positions in debates regarding the usefulness of reductive Logic, it may be possible to demonstrate that polarized

pro- and anti- positions, which either accept or reject reductive Logic in the task of modeling complex causal relationships, are *both poorly formulated* arguments.

If reductive Logic is susceptible to *formal reductive incompleteness* and its implications, then, addressing the *shared* failure to recognize formal incompleteness could reveal novel ways to *preserve* ‘bottom-up’, rigorous, strong, formal reductive Logic as a ‘special case’ within a more complex *iterative* implementation of meta-reductive Logic associated with *natural multi-scale co-evolution*, a *more complicated multi-scale causal metaphor* and *formal reductive incompleteness*. What is *undecidable* within current ‘bottom-up’ reductive Logic may be *decidable* within an adapted multi-scale meta-reductive Logic.

The above facets of *multi-scale co-evolution*, a *multi-scale causal metaphor* and *formal reductive incompleteness* suggest a novel concern for any adapting natural science: There is a hypothetical possibility that Nature and co-evolution contain identifiable *unresolvable natural instantiations* and *natural evolutionary incompleteness*. As well, Nature may instantiate an identifiable pattern of causally significant *natural incompleteness driven novelty generation*. These unusual structures and processes would be expected to appear in relation to natural *transitions*, where an established consistent systemic order, transits into impending systemic inconsistency, which then self-organizes a meta-systemic resolution with the emergence of a novel meta-consistent, meta-order.

Thus, hypothetically, the evolving natural system may have an inherent, iterated and inevitable, co-evolutionary relationship with natural co-evolutionary incompleteness and its implications. *Multi-scale co-evolution*, *reductive logical incompleteness* and *natural evolutionary incompleteness*, all require novel approaches and adaptations of reductive Logic, which are poised to transform theoretical understanding in *both* Reductive Science and Complexity Science.

**Fourth.** The applied mathematics of Symmetry and Group theory [18] may not be sufficient to model causal relationships in a Universe in which multi-scale co-evolution and inherent natural incompleteness are manifest. A more comprehensive model of causal relationships may require a synthesis of Symmetry and Group theory *with* Self-similarity and Fractals. Self-similarity is a ‘special kind’ of symmetry that focuses on invariance across multiple scales of causally significant self-organization [20]. Symmetry and Group theory may become a ‘special case’ within a more comprehensive meta-reductive model of causal relationships in natural systems where Symmetry and Self-similarity are integrated.

The presence of an unrecognized relationship shared by all *four* of the interconnected kinds of ‘unfinished’ or ‘partial’ scientific description and modeling *distorts* the comprehensive *Reductive Scientific Narrative*; and, *distorts* the frequently incommensurate story told in the *Scientific Narrative of Complexity Science* [27]. An integrated understanding of the four kinds of ‘unfinished’, ‘partial’ or ‘incomplete’ scientific description, could lead to a novel consilience between Reductive Science and Complexity Science. Within such a synthesis, mind and consciousness can assume a novel assimilated relationship *in* Nature and evolution.

In order to proceed, a clear sense of the modern reductive scientific approach is required, from which insights about the four kinds of ‘unfinished description’ can be developed.

## 2 The Modern Reductive Scientific Narrative

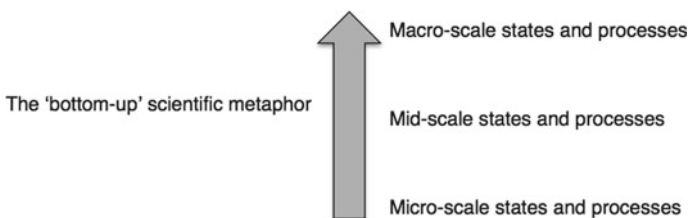
The ‘bottom-up’ metaphor of rigorous reductive scientific Logic makes an *assumption* that Nature, the Universe inflating in the Big Bang, natural evolution, self-organization and the emergence of complexity, are *naturally composed* from *micro-scale states and processes*.

Consistent with the micro-scale assumption, the comprehensive Reductive Scientific Narrative states that approximately 14.5 billion years of natural evolution in our Universe, originates from a tiny point of origin, which inflates and undergoes a series of *phase changes*, which are defined by successive states and processes engaging in cycles of change. An initial, hyper-dense, small, perhaps cool and simple state and process, transitions into an inflating, extremely hot, high-energy, state and process. Then, the ‘whole’ expands and transforms again, everywhere at once, creating a sea of diverse ‘parts’ or quantum micro-scale states and processes, which rapidly interact and slowly cool, generating increasingly complicated states and processes within the early quantum universe [5]. Early micro-scale quantum states and processes in our Universe, continue to evolve and progressively self-organize, creating the mid-scale and macro-scale emergent hierarchy of complexity [10] (Fig. 1).

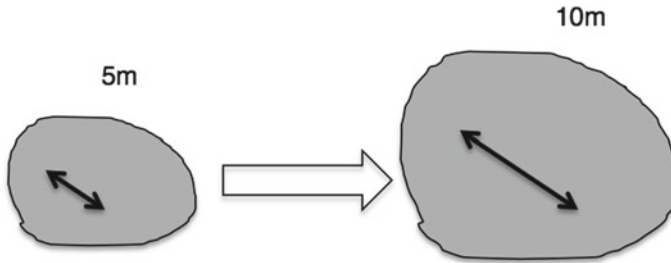
Depictions of the Big Bang suggest this statement can describe the very early inflation from a tiny local energy state:

*‘The Universe rapidly expands everywhere at once’.*

The accelerating expansion of the observable Universe involves increased distance between large cosmic objects with time. The *scale* of space changes with time while the gravitationally bound objects in space-time do not move within space and do not show the same expansion as the larger scale objects that are gravitationally unbound. The Big Bang model of cosmic expansion, therefore, only applies to large-scale objects roughly the size of galaxy clusters or greater. The expansion is therefore a *metric expansion*, which alters the size and geometry of space-time. ‘Dark energy’ has been suggested as a way to explain the accelerating expansion [12].



**Fig. 1** Micro-scale first evolution and the ‘bottom-up’ scientific metaphor



**Fig. 2** Metric expansion in bread dough and the universe

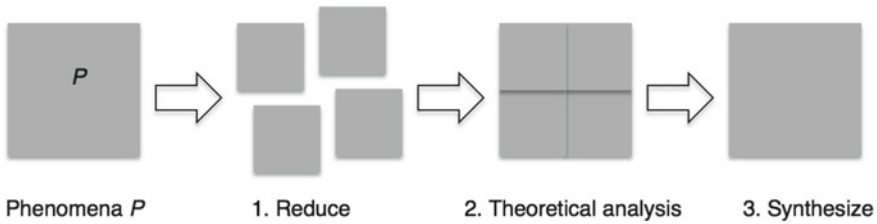
The focus of attention in a description of cosmic expansion is on *increasing distance* and *metric measures of distance* between large-scale objects. An *analogy* often offered, involves a rising loaf of *raisin bread*, in which warm yeast growth and multiplication *expands and increases the space* between raisins *everywhere at once* (Fig. 2).

The Reductive Sciences, the reductive scientific method [1] and ‘bottom-up’ reductive scientific Logic [31], suggest one should seek understanding of natural causal relationships, cosmic inflation, natural evolution and raisin bread using a *three-step* scientific process [21].

- (1) *First, select a ‘whole’* phenomenon of interest and *reduce* it, by breaking it down or deconstructing the ‘whole’ into its *smallest* scientifically accessible components or ‘*parts*’, until you arrive at the most fundamental, tiniest ‘*parts*’ that reductive thought and the available scientific tools can access.
- (2) *Second, theoretically analyze* each reduction of the original complexity, creating as comprehensive a *model* as possible.
- (3) *Third, ensuring you leave nothing out, begin synthesizing* a ‘bottom-up’ reductive conception, starting from the smallest scientifically accessible elements or ‘*parts*’, moving ‘upward’ toward the ‘*whole*’. Ultimately, through ‘bottom-up’ synthesis, arrive at a reductive account of the original ‘*whole*’ phenomenon.

As a further component of the *final step* and *synthesis*, it is recommended that the entire process be *experimentally* and/or *computationally* revisited. Starting with the smallest accessible reduced entities or components contained within a carefully selected semi-isolated situation known to enable the natural interaction of the necessary entities or components; the scientist experimentally demonstrates that the interacting ‘*parts*’ can *spontaneously recreate* the ‘*whole*’ phenomenon. If possible, also create a mathematical or computationally equivalent model, in which mathematically rigorous statements or computationally equivalent ‘*parts*’ interact. In such a semi-isolated context, demonstrate that a scientist and mathematician can *create* a mathematically precise or computationally equivalent version of the ‘*parts*’ composing, interacting and recreating the ‘*whole*’ phenomenon and its relationships (Fig. 3).

A reductive scientific and mathematical analysis of *presumed cause and effect relationships* within Nature, approaches selected phenomena of interest, *P*, with a



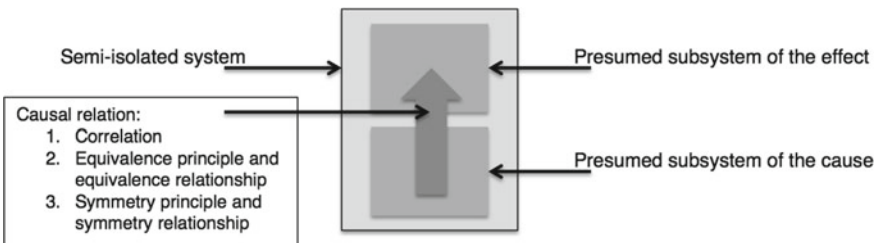
**Fig. 3** The three-step reductive scientific approach

‘bottom-up’ reductive focus on micro-scale states and processes that can be *mathematically* approached employing the demanding mathematics of *Symmetry* and *Group Theory* [18]. Symmetry is a two-part concept linking something that stays the same and does not change, with something related that does change and does not stay the same. Symmetry Groups involve, a set of elements, including an inverse for every element and an identity element, as well as an associative binary operation. Sequences of operations performed upon any series of elements linked by the binary operator, are associative if the sequence always produces the same outcome, whatever the grouping, as long as the order remains the same  $[(a \times b) \times c = a \times (b \times c)]$  [30] (Fig. 4).

In a *selected semi-isolated* context, a presumed *subsystem of the cause* is differentiated from a presumed *subsystem of the effect*. Observation and experimental set-ups identify and detail *correlation* between the presumed cause and effect subsystems (an apparent mutual relation or connection between the two subsystems), then *equivalence relations* between the presumed causal subsystems (demonstrable invariance or similarity between the two subsystems), and finally, specifies any *preserved symmetry* shared by the presumed causal subsystems (demonstrable invariance and associated variance between the two subsystems).

The Symmetry Group concept allows the scientist or mathematician to abstract simplified meaningful pattern from the overall complexity of the two sub-systems of cause and effect differentiated within the complex sequences of state and process, which may otherwise involve complicated, changing, elements, entities, compositions, interactions and systemic relationships.

Ultimately, assuming nothing significant has been left out, the careful application of reduction, theoretical analysis and synthesis; experimental exploration and precise mathematical analysis, involving reductive Logic, Symmetry and Group



**Fig. 4** The semi-isolated experimental context causal relationships

theory, natural science arrives at the most precise and exact definition of a causal relationship in Nature that reductive science and applied mathematics are capable of achieving.

In modern reductive natural science, using ‘bottom-up’ reductive Logic, Symmetry and Group theory, there are only *three* known kinds of *causal mechanism* underlying every natural phenomenon where scientists might choose to tease out the presence of a causal relationship. Two causal mechanisms are *classical* and one is *quantum*. In the words of Nicolas Gisin [7]:

“All correlations observed in science, outside quantum physics that is, can be accounted for either by an influence of one event on another (explanation of the first type), or by common local causes ... (explanation of the second type). In both types of explanation, the said influence or common cause propagates continuously from point to point through space, and in a precise sense, all such explanations are local. ... (Quantum) physics provides us with a third possible explanation”.

Non-local Quantum correlations (explanation of the third type) involve quantum *entanglement*, invoking the philosophical puzzle, the paradoxes and the complicated Quantum dynamics of action at a distance, with no local mechanism [19] (Fig. 5).

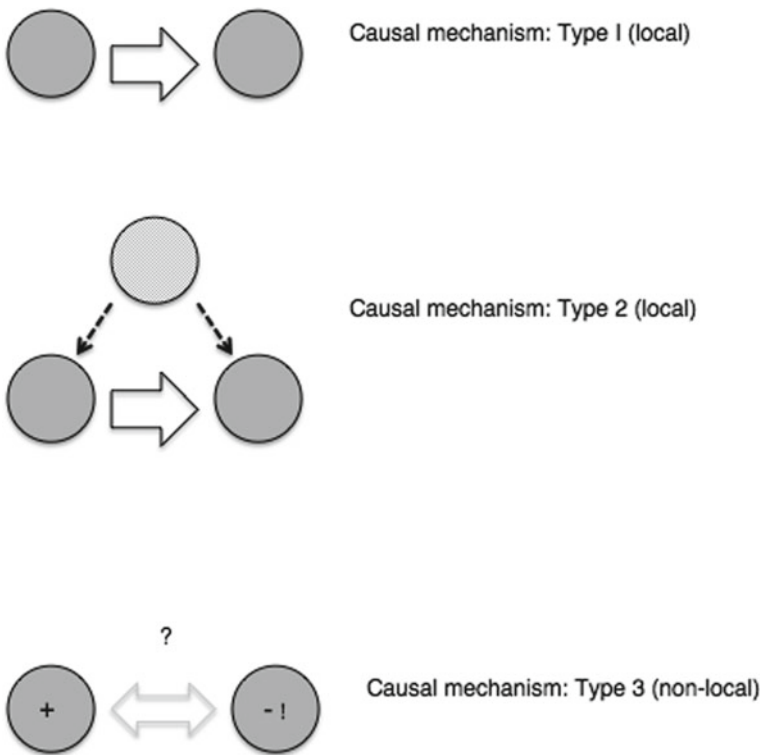


Fig. 5 Causal mechanisms in reductive science

### 3 A Thought Experiment

Just as it is possible for a scientist to discover, after careful scientific study of a natural causal relationship that something significant has been left out; so too the modern comprehensive Reductive Scientific Narrative created by the collective collaboration of a vast scientific and mathematical community, may have left something significant out of our understanding of Nature.

The further discussion in this paper explores *four* significant kinds of ‘unfinished’, ‘partial’ or ‘incomplete’ scientific and mathematical description. The discussion is composed in the form of a *thought experiment* [3].

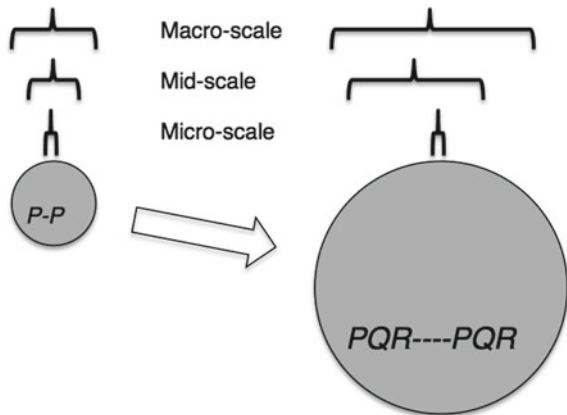
The thought experiment begins by offering a *slightly modified* scientific description of the Big Bang and the initial moments of creation in our Universe. The altered description is presented in this statement:

*‘The Universe rapidly expands and evolves, everywhere at once, on every scale at once’.*

The few extra words introduced into this modified description (‘evolved’ and ‘on every scale at once’) can make a significant difference to the overall theoretical understanding of the evolving natural system and causal relationships in Nature (Fig. 6).

The modified phrase goes beyond interest in *metric measures of distance* between large objects. The addition of the word, ‘*evolves*’, introduces awareness that the raisins in the raisin bread, or the gravitationally bound regions involving smaller scale phenomena that do not show accelerating expansion, are local islands or regions where natural system evolution explores self-organization, and energy-matter and space-time create hierarchically organized complexity. The addition of the phrase, ‘*on every scale at once*’, focuses attention, not just on metric measures of distance, but on the *differentiation of micro-scale, mid-scale and macro-scale* phenomena within complex regions of the Universe. *Evolution and differentiation of scale* across gravitationally bound and unbound regions, suggests a multifaceted hypothesis and

**Fig. 6** Metric expansion, multi-scale differentiation and evolution, in a universe that expands and evolves, everywhere at once, on every scale at once





an alternative scientific methodological approach for modeling causal relationships in Nature.

The transformations and implications of the slightly modified statement are spelled-out in the thought experiment. The following discussion will examine how a natural science influenced by the adapted phrase might go about approaching theoretical puzzles, related experimental contexts and specific selected phenomenon of interest.

### 3.1 *Micro-scale or Multi-scale? the Multi-scale Hypothesis*

In a Universe that rapidly *expands and evolves, everywhere at once, on every scale at once*; the metaphorical *bread dough* and *raisins* in the raisin bread, or the gravitationally unbound and bound regions in the Universe, *evolve, in lock-step, everywhere at once, on every scale at once.*

The phrase '*everywhere at once on every scale at once*' suggests a scientific hypothesis that transforms the reductive conception of *micro-scale evolution* and the *reductive, micro-scale entities first assumption*, into a kind of 'unfinished description'. The *reductive micro-scale first assumption* is translated and positioned as a component and a 'special case' in an integrated model of *meta-reductive multi-scale, multi-directional co-evolution*. Multi-scale co-evolution entails multi-directional co-evolution. It is impossible to consider these two adaptations of reductive thought independently. Nevertheless, this section will concentrate on the multi-scale adaptation while the next section of the paper will address multi-directional co-evolution.

In multi-scale co-evolution, causal relationships must be looked at in relation to expanding inter-related, local and non-local, multi-scale causal spheres or manifolds. Multi-scale spheres or manifolds occur everywhere at once on every scale at once. A multi-scale sphere or manifold must encompass any 'part' and a multi-scale sphere or manifold must encompass any conception of the 'whole'. Multiple scales co-evolve, self-organize, emerge and engage in the natural construction of hierarchically organized complexity, in relation to any 'part' or in relation to any conception of the 'whole'. Meta-reductive epistemology predicts that multi-scale, multi-directional, co-evolution effectively correlates and inter-relates *everywhere at once on every scale at once*, in the metaphor of rising bread dough or in the ontological instantiation of the Universe.

The phrase '*everywhere at once on every scale at once*' therefore implies linked and correlated micro-scale, mid-scale and macro-scale relationships must inflate, expand, differentiate and co-evolve *together* within the rising bread dough, or within the inflating, expanding and co-evolving Universe. The micro-scale, mid-scale and macro-scale 'parts' *and* the micro-scale, mid-scale and macro-scale 'whole', of the *raisin bread* or the 'whole' Universe, *co-evolve*, in an integrated multi-directional co-evolutionary *synchronic* and *diachronic* relationship. In *meta-reductive, multi-scale, multi-directional, co-evolution* the 'whole' must be considered in relation to the 'parts' and the 'parts' must be considered in relation to the 'whole'. The historical

reductive focus on micro-scale, unidirectional, evolution becomes a ‘special-case’ in the meta-reductive conceptualization.

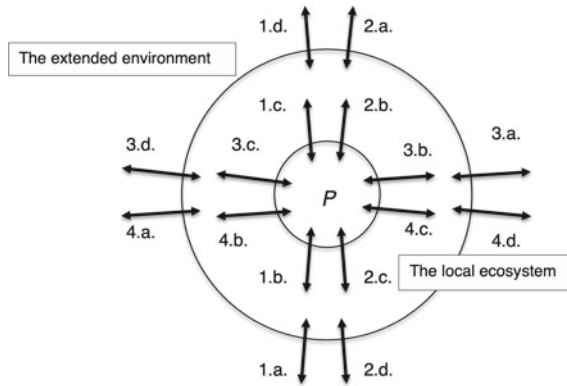
The historical reductive scientific pursuit of a semi-isolated phenomenon of interest, focuses attention on micro-scale first ‘bottom-up’ subsystems of presumed cause and effect and then proceeds to construct the phenomena of interest with a unidirectional metaphorical focus on ‘upward’ causal organization: this extremely successful reductive approach becomes a ‘special-case’ in the meta-reductive account. The meta-reductive account places any *selected phenomenon of interest* within an *adapted* meta-reductive semi-isolated situation that assumes multi-scale, multi-directional, co-evolution. The multi-scale, multi-directional assumption of co-evolution necessarily leads to an adaptation of ‘bottom-up’ reductive Logic. Singular ‘bottom-up’ reductive Logic is transformed into a ‘special-case’ and meta-reductive form, which is then capable of handling multi-scale, multi-directional, co-evolution, through an iterated multiple application of reductive Logic exploring multiple scales and multiple directions of developing causal relationship.

In the meta-reductive account causal mechanisms and causal relationships can be examined in relation to any specific phenomena of interest composed within a semi-isolated meta-reductive sphere or manifold. A blend of reductive and meta-reductive conceptualization is required. The instantiated phenomena of interest can be described within a semi-isolated meta-reductive sphere or manifold as an abstract *entity, state and process*, involving a *composition* of ‘parts’, as well as the *interaction* of those ‘parts’, formulating a semi-isolated ‘whole’. *Hierarchical* organization, causal mechanisms and relationships may be present within the meta-reductive sphere or manifold; however, these must be considered through the lens of multi-scale, multi-directional, co-evolution with the reductive ‘bottom-up’ account treated as a ‘special-case’.

The construction so far could describe a selected phenomenon such as a simple quantum entity or it could describe an evolved Complex Physical System (CPS) or Complex Adaptive System (CAS) [23]. These selections are all now viewed through the lens of meta-reductive conception. Meta-reductive, multi-scale, multi-directional, co-evolution is presumed to apply in relation to a ‘simple’ micro-scale system involving a quantum particle/wave, and also presumed to apply in relation to more ‘complex’ Meta-Reductive Complex Physical Systems (MCPS) or Meta-Reductive Complex Adaptive Systems (MCAS).

The selected phenomena of interest contained within the meta-reductive semi-isolated sphere or manifold is then explored in relation to a matrix of potential causally significant *relationships* that surround the semi-isolated system. The meta-reductive matrix of causal relationships *potentially* has causal impact *within* the defined semi-isolated sphere or manifold as well as *beyond* a defined semi-isolated sphere or manifold. The meta-reductive matrix of causal relationships causally implicates the local ecosystem or the extended environment within which a defined semi-isolated sphere or manifold sits.

In Fig. 7 the ‘special case’ of a ‘bottom-up’ reductive examination of the micro-scale enabling of a selected phenomenon of interest is encompassed within 1.a. and



**Fig. 7** Possible meta-reductive relationships associated with a given selected phenomena *P*. The selected phenomenon *P* is in the sphere of a meta-reductive semi-isolated system. The semi-isolated system is in the meta-reductive sphere of a local eco-system. The selected phenomenon *P* and the local eco-system are in an extended meta-reductive environment

1.b. in the meta-reductive framework and  $4 \times 4$  matrix of possible causal relationships. The matrix surrounds the selected phenomenon of interest within a field of potential sources of causal input, circular causal events, feedback loops, and potential causal relationships. The selected phenomenon of interest can be influenced by, participate in, or precipitate causal relationships within the  $4 \times 4$  meta-reductive matrix.

The  $4 \times 4$  matrix of co-evolutionary, multi-scale, multi-directional causal relationships include:

- (1) Multi-scale relationships arising ‘bottom-up’ involving the *extended environment* and *local ecosystem* that influence, participate with, or precipitate the *organizing* or *organized* selected phenomena *P*. The phenomenon *P* is not characteristically considered to be an integral part of the system making up the extended environment or local ecosystem:
  - (a) *Enabling* ‘bottom-up’ causal relationships arising in the *extended environment* that make possible the process *organizing* phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘bottom-up’ enabling causal relationships arising in the extended environment.
  - (b) *Enabling* ‘bottom-up’ causal relationships arising in the *local ecosystem* that make possible the process *organizing* selected phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘bottom-up’ enabling causal relationships arising in the local ecosystem.
  - (c) *Engaging* ‘bottom-up’ causal relationships arising from the local ecosystem that influence the *organized* phenomena *P*. The organized phenomenon *P* can feedback and influence the ‘bottom-up’ engaging causal relationships arising from the local ecosystem.

- (d) *Engaging* ‘bottom-up’ causal relationships arising from the extended environment that influence the *organized* phenomena *P*. The organized phenomenon *P* can feedback and influence the ‘bottom-up’ engaging causal relationships arising from the local ecosystem.
- (2) Multi-scale relationships arising ‘top-down’ involving the *extended environment* and *local ecosystem* that influence, participate with, or precipitate the *organizing* or *organized* selected phenomena *P*. The phenomenon *P* is not characteristically considered to be an integral part of the system making up the extended environment or local ecosystem:
- (a) *Enabling* ‘top-down’ causal relationships arising from the *extended environment* that influence and make possible the process *organizing* phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘top-down’ enabling causal relationships arising in the extended environment.
  - (b) *Enabling* ‘top-down’ causal relationships arising in the *local ecosystem* that make possible the process *organizing* selected phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘top-down’ enabling causal relationships arising in the local ecosystem.
  - (c) *Engaging* ‘top-down’ causal relationships arising from the local ecosystem that influence the *organized* phenomena *P*. The organized phenomenon *P* can feedback and influence the ‘top-down’ engaging causal relationships arising from the local ecosystem.
  - (d) *Engaging* ‘top-down’ causal relationships arising from the extended environment that influence the *organized* phenomena *P*. The organized phenomenon *P* can feedback and influence the ‘top-down’ engaging causal relationships arising from the local ecosystem.
- (3) Multi-scale relationships arising from ‘lateral’ causal relationships in the *extended environment* and *local ecosystem* that influence, participate with, or are precipitated by, the *organizing* or *organized* selected phenomena *P*. *Lateral causal relationships* involve a co-developing, complex system in which the phenomenon *P* is considered to be an integral part:
- (a) *Enabling* ‘lateral’ causal relationships arising in the *extended environment* that make possible the process *organizing* phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘lateral’ enabling causal relationships arising in the extended environment of which *P* is a component.
  - (b) *Enabling* ‘lateral’ causal relationships arising in the *local ecosystem* that make possible the process *organizing* phenomena *P*. The organizing phenomena *P* can feedback and influence the ‘lateral’ enabling causal relationships arising in the local ecosystem of which *P* is a component.
  - (c) *Engaging* ‘lateral’ causal relationships arising from the *local ecosystem* that influence and participate with the *organized* phenomena *P*. The organized

phenomenon *P* can feedback and influence the ‘lateral’ engaging causal relationships arising from the local ecosystem.

- (d) *Engaging* ‘lateral’ causal relationships arising from the *extended environment* that influence and participate with the *organized* phenomena *P*. The organized phenomenon *P* can feedback and influence the ‘lateral’ engaging causal relationships arising from the extended environment.
- (4) Multi-scale ‘horizontal’ causal relationships involving the *organizing* or *organized* selected phenomena *P*, actively participate in influencing, participating with, or precipitate events in the *local ecosystem* and *extended environment*. *Horizontal causal relationships* involve phenomena *P* as an integral component of a co-developing, complex system:
- (a) *Enabling* ‘horizontal’ causal relationships arising from the *organizing* phenomena *P* that influence the *extended environment* of *P*. The extended environment can feedback and influence the ‘horizontal’ enabling causal relationships arising from the *organizing* phenomena *P*.
  - (b) *Enabling* ‘horizontal’ causal relationships arising from the *organizing* selected phenomena *P* that influence the *local ecosystem* of *P*. The local ecosystem of which *P* is a component can feedback and influence the ‘horizontal’ enabling causal relationships arising from the *organizing* phenomena *P*.
  - (c) *Engaging* ‘horizontal’ causal relationships arising from the *organized* phenomena *P* that influence and participate in the local ecosystem of *P*. The local ecosystem can feedback and influence the ‘horizontal’ engaging causal relationships arising from the *organized* phenomenon *P*.
  - d) *Engaging* ‘lateral’ causal relationships arising from the *organized* phenomena *P* that influence and participate in the extended environment of *P*. The extended environment can feedback and influence the ‘horizontal’ engaging causal relationships arising from the *organized* phenomenon *P*.

‘*Simple*’ non-local and local causal mechanisms purport to account for *all* the fundamental causal mechanisms in Nature and evolution defined by Reductive Science. In a model of *multi-scale, multi-directional, co-evolution*, composed by an adjacent possible Meta-Reductive Science, there may be further, more complicated, naturally organized and hierarchically related causal mechanisms. These may involve more ‘*complex*’, *multi-scale, multi-directional systemic causal mechanisms* associated with *complex, systemic, multi-scale, multi-directional, co-evolutionary causal relationships*.

More complex systemic causal mechanisms may already be known but not yet categorized in a coherent manner. The *reductive incompleteness hypothesis* (which will be stated below) defines a *fundamental incompleteness driven novelty generating causal mechanism* that can only be visualized, in theory, or detected in Nature, in association with recognized natural evolutionary incompleteness. The incompleteness driven novelty generating causal mechanism may form complex *combinations* with other complex systemic causal patterns, such as non-linear processes, stochastic

processes, Lamarckian processes, systemic emergence and transient or sustained complementarity.

David Bohm suggested integrated ‘part’ and ‘whole’ evolution could be defined as ‘explicate’ and ‘implicate’ evolution, respectively [14]. A model of integrated *multi-scale co-evolution* presents a more complicated picture. In *multi-scale co-evolution*, inflating and expanding energy-matter and space-time, *opens* integrated ‘whole’ and ‘part’ causal spheres, manifolds or domains, which involve phenomena, relationships and mechanisms occurring *everywhere at once on every scale at once*.

Bohm’s vision describes a *unitary* sequence and trajectory of integrated *implicate* (whole) and *explicate* (part) evolution. In *multi-scale, multi-directional, co-evolution*, an integrated sequence of intersecting trajectories of multi-scale co-evolution, involves causal mechanisms and relationships in inter-related spheres, manifolds or domains. *Within* each domain, multi-scale, multi-directional, co-evolution of ‘whole’ and ‘part’ co-occurs, with self-organizing entities, composition, interaction and relationships. As well, at the edge of any domain, multi-scale, multi-directional, co-evolution involves interactive mechanisms and relationships occurring *at or beyond* the *boundary* of each causal domain. These relational events co-occur in the local and extended environment, where expansive multi-scale co-evolution of ‘whole’ and ‘part’, self-organize further *extended* entities, compositions, interactions and relationships that reach *beyond the boundary* of any given or selected causal domain.

The multi-scale conception leads to a *multi-directional hypothesis* (already employed but stated below). In order to approximate the complicated causal relationships a novel multi-directional scientific metaphor must be developed. The *multi-scale, multi-directional, co-evolutionary* perspective transforms the *reductive ‘bottom-up’ metaphor* into a ‘special case’ within a *multi-directional metaphor* creating a complex *spherical* causal map and matrix of possible relationships associated with any definable, selected phenomenon of interest within a particular causal domain.

### ***3.2 Bottom-Up or Multi-directional? the Multi-directional Hypothesis***

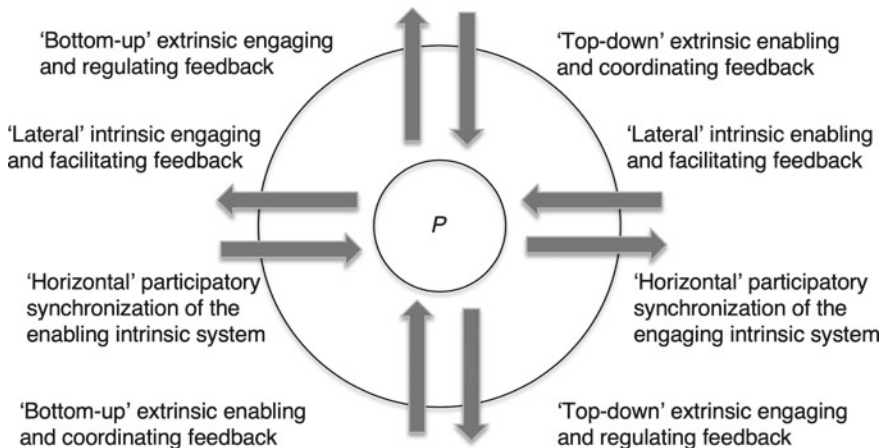
The multi-scale co-evolution hypothesis predicts a related multi-directional hypothesis along with a related metaphor. If, the natural system *expands and evolves everywhere at once on every scale at once*, then, the reductive ‘bottom-up’ metaphor and unitary ‘bottom-up’ formal reductive Logic will produce an *insufficient, unfinished, partial, informally incomplete* picture of Nature and the natural Logic of evolution. The *reductive ‘bottom-up’ metaphor* can, however, become a ‘special-case’ within a *multi-scale co-evolutionary, multi-directional metaphor*. Unitary ‘bottom-up’ application of reductive Logic must also become a ‘special case’ within a spherical causal matrix describing *multiple* causal relationships. The spherical causal matrix

involves multiple, iterative, multi-directional applications of reductive Logic associated with each of the multiple descriptions developed in the multi-scale co-evolution hypothesis.

To capture the possible multi-directional causal relationships in a *metaphor* describing a multi-scale co-evolutionary causal sphere, manifold or domain; the ‘special-case’ of ‘bottom-up’ reductive Logic mapping ‘bottom-up’ enabling relationships must be coordinated with other iterative and multi-directional applications of reductive Logic. This forms an integrated matrix of metaphorical ‘bottom-up’ and ‘top-down’ extrinsic enabling relationships along with interactive ‘bottom-up’ and ‘top-down’ coordinating and regulating feedback [6]. The ‘bottom-up’ and ‘top-down’ relationships must then be integrated with ‘lateral’ intrinsic enabling and engaging relationships and facilitating feedback, as well as ‘horizontal’ participatory relationships in which the phenomena synchronizes the intrinsic enabling and engaging systems. Multiple, multi-directional iterations of reductive Logic are then required to map the interactions and relationships in a meta-reductive description of a natural evolving system (examples will be developed later in the text) (Fig. 8).

A matrix of iterated reductive logical applications allows for detailed examination of metaphorical ‘bottom-up’, ‘top-down’ and ‘lateral’ or ‘horizontal’, co-evolving, multi-directional causal relationships within and beyond the boundary of any co-evolutionary causal sphere, manifold or domain.

In the *center* of the spherical map of a selected phenomenon *P*, or at the *boundary* of the central sphere and the peripheral extended causal relationships associated with the multi-scale environment, ‘bottom-up’, ‘top-down’ and secondary ‘lateral’ or ‘horizontal’ relationships, may other-organize or self-organize, entities, compositions, interactions and relationships involving particular complex phenomena that can co-evolve *positive* and *negative feedback* as well as *circular causation*.



**Fig. 8** Possible metaphorical meta-reductive relationships associated with a given selected phenomena *P*

The *multi-scale hypothesis* and the *multi-directional hypothesis*, predict complex points of *intersection* within multi-scale causal spheres, manifolds or multi-scale causal domains. The description of the intersections between multiple trajectories of multi-directional, primary and secondary causal accounting, might arrive at simple unidirectional or complicated multi-directional, reductive statements or natural instantiations that might provide clear evidence of reductive formal incompleteness and natural evolutionary incompleteness.

These considerations take us into a deeper look at the *Reductive Formal Incompleteness Hypothesis*.

### 3.3 Complete or Incomplete? Reductive Formal Incompleteness

A formal logical and mathematical proof of formal reductive incompleteness impacting ‘bottom-up’ reductive Logic has recently been attempted [25]. ‘Unfinished description’ can therefore involve ‘bottom-up’ formal reductive Logic. This particular kind of ‘partial’ scientific description will be addressed first in relation to modern scientific ‘bottom-up’ reductive Logic, and then in relation to an imagined, integrated meta-reductive Logic that can model multi-scale co-evolution and multi-directional causation. This is presented in the form of a series of theoretical hypotheses. In relation to ‘bottom-up’ reductive Logic:

- (1) The *Reductive Formal Incompleteness Hypothesis*: Modern, scientific, ‘bottom-up’ reductive Logic can formulate *contradiction* and *self-referencing paradox*. To avoid the threat of inconsistency in the application of formal reductive Logic, where necessary, such contradictory or paradoxical reductive statements can be defined as *undecidable reductive propositions* indicating *formal reductive incompleteness*. Reductive formal incompleteness demands *meta-consideration* whenever determining *reductive logical consistency* [23].
- (2) The *Meta-Reductive Meta-Construction Hypothesis*: An implication of formal reductive incompleteness and necessary meta-consideration predicts *meta-construction* of adjacent possible adapted forms of reductive Logic capable of *resolving* previously identified *undecidable reductive propositions*. As well, meta-construction might lead to *meta-reductive sciences* that employ adapted reductive Logic formulated in *adjacent possible Meta-Reductive Scientific Paradigms*. Meta-construction can *preserve* ‘bottom-up’ reductive Logic as a ‘special case’ within the structure of an adapted Logical system Meta-Reductive Science and Meta-Reductive Paradigm [24]. Modern Reductive Thought could then become a ‘special case’ within adapted Meta-Reductive Thought [25, 26].
- (3) The *Natural Evolutionary Incompleteness Hypothesis*: ‘Bottom-up’ reductive Logic is susceptible to formal incompleteness, and ‘bottom-up’ reductive Logic *may* describe attributes of Nature and the evolutionary Logic of natural evolving systems. Therefore, Nature and natural evolution *may* manifest properties and



definable patterns consistent with *natural evolutionary incompleteness*. This may take the form of transient or preserved states and processes that manifest as ‘unresolvable natural instantiations’— ‘unresolvable’ natural forms are likely to exhibit properties that ‘*are and are not*’ consistent with the *existent order* within which they belong.

- (4) The *Natural Meta-Construction Hypothesis*: Unresolvable natural instantiation and natural evolutionary incompleteness may be *resolved* when *one* natural order becomes embedded and encompassed within an emerging natural order and meta-system, which exhibits its own pattern of meta-consistency [24]. A *natural meta-consistent resolution* may also occur when *two* co-evolving, ordered and consistent regimes intersect, compete, collaborate or integrate.
- (5) The *Inevitability of Incompleteness Hypothesis*: It is *inevitable* that any scientific Logic purporting to model the natural order or an evolving meta-order, will ultimately exhibit its own pattern of formal incompleteness. It is also inevitable that any natural order manifesting a resolution of natural incompleteness will ultimately reveal its own pattern of natural evolutionary incompleteness.

Defining these hypotheses in theoretical and experimentally accessible terms would allow their investigation in relation to Nature and could provide substantial support for the four kinds of ‘unfinished’ description involving ‘bottom-up’ reductive Logic and their resolution. It would also provide support for necessary meta-construction of adapted forms of scientific Logic, adapted meta-reductive sciences and adapted Meta-Reductive Paradigms. Adapted paradigms could produce adapted Meta-Reductive Scientific Narratives that include a pervasive awareness of reductive Logical incompleteness and the ubiquitous presence of natural evolutionary incompleteness.

**Can Reductive Logic Adapt? Meta-Reductive Logic.** To succeed the meta-reductive perspective must preserve ‘bottom-up’ reductive Logic as a ‘special case’ and adapt ‘bottom-up’ reductive Logic in order to confront meta-reductive, multi-scale, multi-directional, incompleteness driven, co-evolution.

The rigor of ‘bottom-up’ reductive Logic can be *preserved* and *encompassed* as a ‘special case’ in a more comprehensive, multi-descriptive, iterative, meta-reductive Logic. Every *iteration* of reductive Logic employed in the multiple descriptions of a meta-reductive, multi-scale and multi-directional perspective can be internally rigorous, strong, reliable and consistent; pursuing the goal of well-formed, logical, reductive statements targeting causal mechanisms and relationships.

However, the structure of preserved ‘bottom-up’ reductive Logic must also be *adapted* particularly in relation to defining, exploring and determining the *limits* and *implications* associated with the *formal incompleteness of reductive Logic*. A meta-reductive Logic therefore needs to be capable of handling *formal reductive incompleteness* by, perhaps, adapting reductive Logic into a *paraconsistent* form [13]. Such an adapted Logic would include *undecidable reductive propositions* declaring the presence of *formal reductive incompleteness*, thus defining a *limit* on any particular iterative implementation of reductive Logic.

In a paraconsistent meta-reductive Logic, *meta-consideration* is the only route to determining *reductive consistency*. A paraconsistent reductive Logic must be capable of effectively exploring the *onset of incompleteness* in any single iterative application of reductive Logic in the context of meta-reductive, multi-scale, multi-directional, incompleteness driven, co-evolution. As well, an adapted paraconsistent reductive Logic must be able to model the *complex edge of incompleteness* where multiple iterative implementations of reductive Logic intersect and interact in the context of meta-reductive, multi-scale, multi-directional, incompleteness driven, co-evolution. The intention of a paraconsistent, meta-logic would be to model how abstract scientific Logic and the natural Logic of Nature go about *resolving* and *deciding* previously unresolvable natural instantiations and undecidable reductive propositions.

Any paraconsistent Logic capable of modeling meta-reductive, multi-scale, multi-directional, incompleteness driven, co-evolution, would assume the inevitability of eventually running into its own undecidable reductive propositions.

Novel definitions and scientifically accessible hypotheses must inevitably appear in relation to the development of a meta-reductive paraconsistent Logic. For instance, self-organization, emergence and complementarity must also be redefined and explored in the framework of meta-reductive, multi-scale, multi-directional, incompleteness driven, co-evolution.

The next task involves spelling-out the relationship shared by reductive formal incompleteness and another kind of ‘unfinished and partial description’ associated with the mathematics of Symmetry and Group theory, which appears when this form of abstract and applied mathematics is employed in the task of modeling causal mechanisms and relationships in Nature.

### 3.4 *The Synthesis of Symmetry and Self-similarity*

In focusing attention on a scientific understanding of causal relationships in Nature through the applied mathematics of Symmetry and Group theory [18], natural science ignores a fundamental and significant conceptual link between Symmetry and Self-similarity that could advance the understanding of causal relations in natural evolving systems.

Symmetry is a *two-part* mathematical relationship involving *invariance* in relation to *variance*. *Self-similarity* is a *special* kind of symmetry in which *invariance* is related to *variance* across a range of *scale* [20, 33]. Symmetry and Group theory can be integrated with Self-similarity and Fractals and this synthesis can significantly modify the scientific approach to modeling causal relationships in Natural settings.

**Defining a Fundamental Unit of Self-organization.** In order to achieve the synthesis, a *fundamental unit of systemic evolutionary self-organization* needs to be defined that can link the abstract conception of Symmetry and Self-similarity; which then suggests an accessible hypothesis regarding Symmetry and Self-similarity in natural evolving systems and a deeper reason for the remarkable usefulness of mathematics in natural science [34].

A hypothetical *abstract fundamental unit of systemic organization* linking Symmetry and Self-similarity would need to be simple enough that it spells-out a fundamental *associative binary operation* allowing a consistent definition for a fundamental evolutionary Symmetry Group.

In order to integrate Symmetry and Self-similarity in causal modeling, the *same abstract fundamental unit of systemic organization* would need to also spell-out an *equivalent fundamental iterative algorithmic operation* consistent with a definable evolutionary Multi-Fractal.

To be scientific, the hypothesized *abstract fundamental unit of systemic organization* must be both theoretically and experimentally accessible. Ultimately, its existence in Nature must be repeatedly verified, thus, demonstrating that the abstract epistemological unit is *related to an ontological* manifestation in the form of a *natural fundamental evolutionary unit of systemic organization*. The scientific task, therefore, is to demonstrate that the abstract theoretical structure and process is a good approximation of a natural evolutionary structure and process (Fig. 9).

A candidate for a joint *binary associative* (Group theory) and *iterative relation* (Multi-Fractal) capable of fulfilling these expectations can be composed from a very simple binary relationship. The binary ‘change or no change’, or, the binary ‘difference that does or does not make a difference’ [2] offers a good starting point.

Within any *consistent established natural order*, *one half of the binary* can be stated: *change* can occur that makes *no difference* to the structure and rules of an established order and *change* can occur that makes no difference to the *consistency* of an established order. The *other half of the binary* can be stated: a *change* can occur that *does make a difference* to the structure and rules of an *established order* and does make a difference to the *consistency* of an established order.

The *binary* ‘change or no change’, ‘does or does not’, ‘is or is not’, when associated with the *consistency of an established natural order*, provides a fundamental binary

1. Defining a fundamental evolutionary meta-reductive binary *associative operation* and binary *iterative relation*, requires *meta-consideration* of the *consistency* of any naturally established order.

*Binary:  $\Delta$  or  $\Delta^0$  in the Consistency  $\mathbb{C}$  in an established order  $\theta$*

2. A Meta-reductive Evolutionary Symmetry Group and Evolutionary Multi-Fractal can be based on the meta-reductive binary.

3. The Evolutionary Multi-Fractal Symmetry Group reveals a self-similar epistemological and ontological unit of complex organization, which again requires *meta-consideration* of the *consistency* associated with any naturally established order.

a) A self-similar epistemological unit of self-organization: The *Meta-Reductive Complex Natural History*,

b) A self-similar ontological unit of self-organization: The *Meta-Reductive Complex Natural Cycle of Existence*.

**Fig. 9** A fundamental evolutionary binary operation; and, 2. The Evolutionary Multi-Fractal Symmetry Group; and, 3. Hypothesized complex units of organization

operation or abstract choice that can form the core of an *associative and iterative relationship*. Such a binary operation or binary choice results in the construction of a Symmetry Group and Multi-Fractal that responds to the presence and the implications of formal incompleteness.

The construction of a Symmetry Group and Multi-Fractal, where the associative binary operation and iterative relation is related to logical consistency, can only be achieved by composing the necessary structure in the space of *meta-consideration* and *meta-construction*. The Symmetry Group and Multi-Fractal must also be sensitive to formal reductive incompleteness in any abstract logic or mathematical system model of sufficient complexity. The Symmetry Group and Multi-Fractal must therefore initially exist *abstractly*, only in the epistemological world of self-reflective and self-referencing mind, consciousness and meta-consideration of change or no change in consistency. *However*, the composed Symmetry Group and Multi-Fractal may hypothetically describe an experimentally accessible ontological natural instantiation involving symmetry operations and self-similar iteration, manifest in the Universe and flow of complex natural system evolution.

In order to avoid contradiction and paradox, formal reductive incompleteness and natural evolutionary incompleteness demand that the *consistency* of any *algorithmic pattern* involving *natural systemic order* must be determined as a *meta-consideration*, which occurs outside the natural systemic order. Consciousness can assume the position of a meta-level system capable of making the determination of consistency in relation to a given sequence in the natural order. Where an individual consciousness *is* the system of interest, then another individual consciousness must assume the position of meta-consideration in relation to determining logical system consistency. *Meta-consideration* avoids contradiction and paradox and preserves, protects and sustains the abstract meaning and coherence of the inherent Logic associated with a selected abstract system or a selected natural evolving system.

While the operation, choice or decision regarding *consistency or inconsistency* is *binary*, formal incompleteness predicts that the possible states of the system from which the binary decision is abstracted, can involve *three* abstract options. These are: (1) A *consistent order* with an *established Logic*; (2) A *transitional order* in which there are *unresolved or undecided* statements which *protect the system's Logic and consistency*; and, (3) An *inconsistent order* with a *failed Logic*. These *three* possibilities are related to *three* Logic states: *true* (decidable and consistent); *undecidable* (consistent only because a statement threatening contradiction, paradox and inconsistency is left undecided); and *false* (decided and inconsistent).

The *binary* decision regarding *consistency or inconsistency* can also be linked with potential adjacent possible *meta-constructions*. In the abstract, meta-constructions involve additional axioms added to a Logic system or system model that can *resolve* or *decide* a previously *undecidable* proposition or statement. The equivalent in a natural system, involves a natural *meta-construction* that moves a *transitional order* into a *resolved meta-order*. In a transitional order, there are *unresolvable instantiations* and *undecided properties* that have not yet made manifest their potential action, interaction, relationship or implication. In a resolved meta-order, the same previously unresolvable instantiation and undecided property has transformed and

become manifest in its emergent properties within the resolved meta-system. The unresolvable instantiation becomes an entity, takes part in a composition, participates in an interaction, or engages in a relationship with entities that are not part of the *prior established consistent order*. The unresolvable instantiation is now revealed by its behavior and properties to be a resolvable instantiation that is an integral part of an *emergent order with its own inherent meta-consistency*.

Within a *natural transitional order*, a *change* occurs in the form of an energy-matter and space-time, self-organized entity, composition, interaction or relationship, which is contained within a local and extended environment. The *change 'is and is not consistent with the established order'*. Such a transitional state and process can only be *unresolved* within the existing natural order and *undecided* in a Logic-based model of the natural order. A transitional state and process in a scientific model of a natural system would need to be left undecided and undecidable, as it threatens contradiction and self-referencing paradox, by having properties that are both *consistent* with the established order and *not consistent* with the established order. The transitional form exhibiting natural evolutionary incompleteness must be left unresolved and undecided. The status, as a transitional entity, cannot, however, be established, until the extended emergent meta-order begins to self-organize *around* the transitional property that *is not consistent* with the previously established order but is consistent within itself.

As an *emergent meta-order* self-organizes and establishes its own meta-Logic and meta-consistency, a transitional 'is and is not' state and process will reveal its capacity to participate in meta-consistent or meta-constructed relationships.

Within the meta-order the *binary decision* and *binary operation* involving consistency will again appear. The potential for meta-level incompleteness is inevitable.

A simple and fundamental link can now be made between the concepts defining Symmetry, Group theory, Self-similarity and Fractals.

**Defining a Fundamental Evolutionary Symmetry Group.** A fundamental evolutionary Symmetry Group would involve:

- (a) A set of *elements* (defined by specific examples of *binary 'change or no change in consistency'*, *in relation to an established consistent order*; ie., there exists an instantiated natural order, involving state and process, energy-matter and space-time phenomenon, entity, composition, interaction and relationship in which the inherent Logic can be spelled-out and the consistency determined as a meta-consideration);
- (b) An *inverse* for every element (an *element* is defined by a *binary 'change or no change* in relation to consistency, and the presence of a binary choice can *exist or not-exist*—Nature can erase evidence of historical events or destroy the present moment of states and processes);
- (c) An *identity* element (defined as a *binary change* which precipitates '*no change in consistency*' and '*no change in existence or non-existence*, leaving the element, the order and the consistency, *unchanged*—a binary change that makes no difference); as well as,

- (d) An *associative binary operation* ('change or no change in the consistency of an established order', is the fundamental binary operation): The three choices associated with incompleteness therefore are: 'is consistent therefore decided', 'is and is not consistent therefore undecided' and 'is not consistent therefore decided'.

Sequences of binary operations performed upon any series of elements linked by the binary operator, are *associative*, if the sequence always produces the same outcome, whatever the grouping, as long as the order remains the same  $[(a \times b) \times c = a \times (b \times c)]$ . In this case, the *binary operation* and *any sequence of elements*, in the form of an instantiated evolutionary sequence of state and process, energy-matter and space-time phenomenon, entity, composition, interaction and relationship, sustains the evolutionary sequence and outcome of, change and no-change, existence and non-existence, order and disorder, as well as, unresolved transition and meta-order. The sequence and outcome are sustained no matter how consciousness might choose to group, to self-reflect and meta-consider; and no matter how Nature might naturally cluster, group, or instantiate the sequence. If the sequence is associative, the outcome of the entire sequence, which might include the self-referential emergence of consciousness, will remain the same.

Focusing on a binary operation that is sensitive to inherent incompleteness and also is constructed on a relationship involving consistency, which necessarily requires meta-consideration in its determination; it becomes possible to construct an evolutionary Symmetry Group that is *associative*. *Consciousness* does *not* have a special role in any part of the natural evolutionary sequence that emerges from the Symmetry Group. However, consciousness can *choose* to go beyond abstract description, clustering and grouping of an invariant evolutionary sequence, by spelling-out Nature's simple rules, laws and patterns associated with evolving symmetry relationships as well as intentionally or unintentionally engaging in non-local or local *interactions* with the evolutionary sequence [11], thus altering the course of co-evolution.

**Defining a Fundamental Evolutionary Multi-Fractal.** A fundamental Multi-Fractal can now be composed that can be integrated with the fundamental Symmetry Group, by using the same fundamental unit of self-organization:

The properties of a Fractal include, *self-similarity*, *iteration* and *scaling*. In the abstract, a *fractal* is a *geometric form* created by an *iterated system*. The Mandelbrot set, for instance, is a fractal with some logical depth but little or no effective complexity (the length of a concise description of a set of entities regularities).

The Mandelbrot set can be *reduced* to a simple equation:

$$f_c(z) \stackrel{\text{def}}{=} z^2 + c$$

Despite its iterative simplicity and lack of effective complexity, it produces an infinite and vastly complex geometric object.

If the process of natural evolution spells out a potentially increasingly complex and repeating *regularity*, then natural evolution and the repeating regularity can be taken as an *iterated system* and the products of complexification can be taken as similar to

a potentially increasingly *complex geometric form*. Thus, change, evolution, self-organization, the emergence of a hierarchy of complexity as well as the entire sequence of the evolving Universe, can be described as a *multi-fractal system*, one which does exhibit a *very high order of effective complexity* in a region *intermediate between total order and complete disorder* [22].

The *regularity* can be described by a self-similar, iterated and scaled, *binary sequence of operations*, involving, ‘*change or no change in the consistency of an established order*’.

$$f_{\mathbb{C}}(\theta) \stackrel{\text{def}}{=} \theta(\Delta_{\mathbb{C}} \vee \Delta_{\mathbb{C}}^0) + \mathbb{C}$$

A function ( $f$ ) operates on the consistency ( $\mathbb{C}$ ) of an existent order ( $\theta$ ). An iterated binary operation ( $\Delta_{\mathbb{C}} \vee \Delta_{\mathbb{C}}^0$ ) defined by: change ( $\Delta$ ) in consistency ( $\mathbb{C}$ ) or ( $\vee$ ) no change ( $\Delta^0$ ) in consistency ( $\mathbb{C}$ ) operates on an existent order ( $\theta$ ) and ( $+$ ) the constant abstract meta-consideration of the consistency ( $\mathbb{C}$ ) associated with the given order ( $\theta$ ). The value  $+\mathbb{C}$  refers to invariant contexts where proven epistemological meta-consideration of logical consistency maps onto logical and experimentally provable ontological consistency. The value  $-\mathbb{C}$  refers to variant contexts where proven epistemological meta-consideration of logical inconsistency maps onto logical and experimentally provable ontological inconsistency ( $-\mathbb{C}$  represents meta-consideration of an interesting hypothetical and very different scientific context—however, it is *not* the context we are presently exploring).

Once again, there are *three* choices relevant in each self-similar, iterated and scaled representation of the binary sequence of operations and elements: (1) The ‘change or no change is consistent with the established order’; (2) the ‘change or no change is consistent with the established order but only by having potentially contradictory or paradoxical properties that are not yet fully instantiated, manifest or resolved, in relation to a potential adjacent possible emerging meta-system and meta-order, within which the unresolved properties would be defined as resolved and consistent; and, (3) The ‘change or no change is not consistent with the established order’.

The evolutionary Multi-Fractal is then defined by:

- (a) *Self-similarity*: The abstract elements and binary operations, associated with consistency and incompleteness, can be related to natural energy-matter, space-time, entity, composition, interaction and relationship involving any level or degree of hierarchical complexity.
- (b) *Iteration*: The sequence of binary operations, acting on elements associated with consistency and incompleteness, can be any length or degree of complexity and the abstract nature of the iteration will be sustained.
- (c) *Scaling*: The abstract elements, the sequence of binary operations and the relationship with consistency and incompleteness, can be related to any natural system involving micro-scale, mid-scale or macro-scale state and process.

The proposed evolutionary Multi-Fractal can be further spelled-out and integrated with other scientific work. For instance, the fractal dimension of our Universe can be defined more precisely and the relationship shared by the self-similar, iterative, scaled

fractal evolutionary pattern and simple rules, laws and diverse, complex, real world patterns that emerge within the sequence of Nature's fractal evolution. The synthesis of the fundamental evolutionary Symmetry Group and the fundamental evolutionary Multi-Fractal, can integrate Symmetry and Self-similarity in the analysis of causal relationships in Nature. The integration produces a scientific narrative supported by a Complex Multi-Fractal Symmetry Group.

**Symmetry, Self-similarity and Meta-Reductive Causal Relationships.**

Symmetry demands analysis of causal relation through exploration of *correlation*, *equivalence relations* and *symmetry relations* in selected subsystems of cause and effect. Self-similarity would demand analysis of causal relation in selected subsystems of cause and effect, through *multi-scale*, *self-similar correlations*, *multi-scale self-similar equivalence relations* with subsequent determination of *significant multi-scale self-similar causal relationships*. The exploration of symmetry and self-similarity in relation to causal relationships in Nature would make it possible to spell-out both 'simple' and 'complex' causal mechanisms; as well as possible to abstract and quantify the rate at which Nature and evolution increase detail in the form of simple or complex symmetry groups [30] while increasing complexity and the fractal dimension of energy-matter states and processes in relativistic space-time [20].

The abstract, simple, *fundamental unit of systemic evolutionary self-organization* can be linked sequentially into more complex abstract units of self-organization that can also exhibit high degrees of symmetry and self-similarity. A linked sequence, for instance, could be conceived and defined in relation to an *epistemological Complex Natural History* and an *ontological Complex Cycle of Existence*, which can describe the history and cycle of existence of simple or complex, evolving, micro-scale, mid-scale or macro-scale, energy-matter, space-time phenomena [24].

Two further hypotheses arise from the integration of Symmetry with Self-similarity and a further integration of the dynamics of incompleteness, in relation to the scientific task of defining causal mechanisms and relationships in Nature:

- (1) An *Incompleteness Driven Novelty Generating Hypothesis*: predicts natural *incompleteness driven novelty generation* could function as a *ubiquitous fundamental complex systemic causal mechanism* in natural evolving systems.
- (2) A *Hypothetical Hierarchy of Simple and Complex Causal Mechanisms* can be composed: where non-local and local causal mechanisms may belong within a more complicated description of causal relationships defined under Symmetry, Self-similarity and Incompleteness.
  - (a) Non-local and local causal mechanisms can be defined as *simple causal mechanisms* discovered under the mathematical umbrella of Symmetry.
  - (b) A fundamental *Incompleteness Driven Novelty Generating Causal Mechanism* could be defined as a fundamental *complex causal mechanism* allowing transition into incompleteness and novelty generation.
  - (c) The mathematics of Self-similarity, when used in examining causal relationships, might lead to theoretical and experimental categorization of more *complex causal mechanisms* beyond the *Incompleteness Driven Novelty*



*Generating Causal Mechanism.* Complex causal mechanisms could self-organize from *simple* non-local or local causal mechanisms as above and the *Incompleteness Driven Novelty Generating Causal Mechanism* combined with other secondary more complex causal mechanisms.

- (i) Simple *primary* non-local and local causal events may enter an *edge of incompleteness*, where the complex systemic *incompleteness driven novelty generating causal mechanism* could combine with *secondary* and more complicated kinds of causal relationship.
- (ii) Complex causal relationships associated with incompleteness are already found within descriptions of reductive ‘bottom-up’ evolution but will also be found within the description of multi-scale, multi-directional, multi-relational, co-evolution.
- (iii) Examples of *secondary* complex systemic causal mechanisms that could combine at the *edge of incompleteness* with *incompleteness driven novelty generation*, include, but is not limited to: phase change; non-linear or chaotic process; stochastic system evolution; Lamarckian system evolution; bidirectional intersection with developing systemic duality or complementarity; simple conservative (reductive) emergence or more complex forms of novel or radical emergence (meta-reductive), consistent with multi-scale, multi-directional, co-evolution.

### 3.5 *Can the Scientific Method Adapt? Meta-Reductive Methodology*

Two perspectives on ‘part’ and ‘whole’ relationships provide a framework for considering how to adapt reductive scientific theoretical and experimental methodology in the face of formal reductive incompleteness.

The ‘part’ perspective is consistent with modern Reductive Science. A selected phenomenon of interest, *P*, can be the ‘raisin’ in the metaphor of raisin bread, such as an energy-matter *state* in space-time. A scientist can expand the range of their interest by examining a *sequence* of *states*, thus exploring the *process* of *P*. If the scientist chooses to further expand the range of their interest, then a full synchronic and diachronic, *Four-Dimensional Life Cone Trajectory* could be modeled for phenomenon. The *Four-Dimensional Life Cone Trajectory* is a hypothetical meta-reductive *epistemological* structure, which composes a *Complex Natural History* describing the entire sequence of states and processes comprising the existence of phenomenon *P*. It is intended to model an *ontological* structure and process, defined as the *Complex Natural Cycle of Existence* of the selected phenomenon *P*. A semi-isolated experimental examination of *P* could *reduce* the overall system to cause and effect subsystems, which exhibit observable, predictable or repeated pattern and causal relationships. The sum of the experimental observations allows the scientist to piece together the entire narrative story of the phenomenon *P*.

The ‘whole’ perspective is again consistent with modern Reductive Science. In this case, the selected phenomenon of interest  $P$  is, the ‘whole’ multiplying expanding yeast, the flour and other ingredients including the raisins making up the ‘whole’ raisin bread dough. This is the perspective of modern Cosmology. The ‘whole’ perspective is *limited* by the *impossibility* of ever repeating  $P$ . Science, however, is not limited in its ability to abstractly semi-isolate the system, reduce it, then replicate and make predictions from conceptual, computational or mathematical models of  $P$ . A *Four-Dimensional Life Cone Trajectory* composing the *Complex Natural History* and the *Complex Natural Cycle of Existence* of selected phenomenon  $P$  can be pieced together. Again, a narrative story of  $P$  can be composed.

A Meta-Reductive *modification* of the reductive scientific *experimental* approach to  $P$  or  $P'$  encompasses the reductive semi-isolated system as a ‘special case’ in a meta-reductive extended semi-isolated system.

A *reductive semi-isolated system and its presumed cause and effect subsystems*, are positioned *inside* an encompassing, extended, *meta-reductive semi-isolated environment with its presumed cause and effect subsystems*. A selected phenomenon of interest  $P$  or ‘part’ is contained in a reductive semi-isolated system.  $P$  is then encompassed within  $P'$  or the ‘whole’. The ‘whole’  $P'$  is located in an encompassing, extended, meta-reductive semi-isolated system, which holds the ‘special case’ of the reductive semi-isolated system and ‘part’  $P$ ; and is further subdivided into a local environment  $E$ , and an extended environment  $U$ , representing the ‘whole’ Universe. Attention can oscillate between the two integrated perspectives involving phenomenon  $P$  or  $P'$  and  $P$  and  $E$  or  $U$  (Fig. 10).

This modification of semi-isolated theoretical and experimental methodology is already in use in science but it is not formalized in a *meta-reductive framework* in which *formal reductive incompleteness* and *multi-scale co-evolution* are addressed.

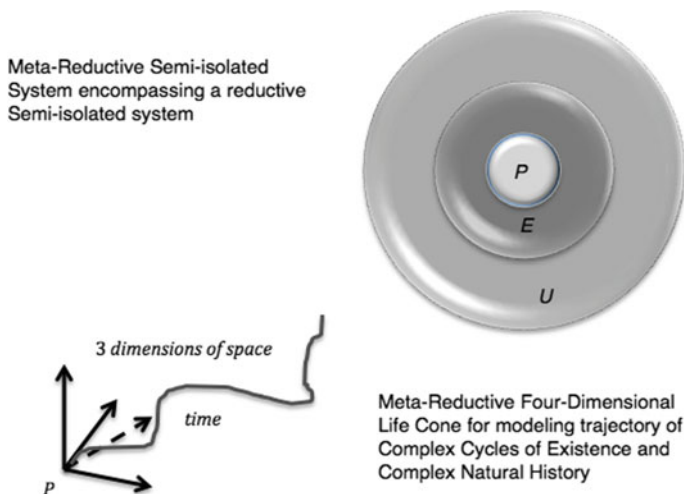


Fig. 10 Meta-reductive semi-isolated system and the four-dimensional life cone

## 4 Conclusion

This brief outline is but a scant sketch of some of the implications for Reductive Natural Science of multi-scale, multi-directional co-evolution and formal reductive incompleteness. It is intended to capture the interest of anyone who can see the merit of going beyond the bounds of the Reductive Scientific Paradigm. Unknown Meta-Reductive Paradigms sit in the adjacent possible of the Reductive Paradigm, waiting to be explored and offering potential roads to a deeper understanding of Nature and a necessary consilience between the Reductive Sciences and the Complexity Sciences.

A meta-reductive perspective is poised to offer potential novel resolutions to unsolved problems in many branches of natural science. For instance, a Meta-Reductive Matrix of Causal Mechanisms and Relations, offers an alternative way to view phenomena such as quantum entanglement, wave collapse and non-local phenomena within Quantum physics [19]. A meta-reductive causal matrix could also offer a novel way to view the still unresolved boundary between Quantum physics and Relativity theory [15]. A meta-reductive definition of complementarity, involving causally significant symmetry, asymmetry and self-similarity, can reach across many scales of self-organization and many orders of complexity. Complementarity within fundamental physics might then fit within a hierarchy of complementarity, concerning phenomena throughout the hierarchy of complexity and Nature [8].

The meta-reductive perspective preserves 'bottom-up' reductive Logic, by encompassing it as a 'special case' within a meta-reductive paraconsistent Logic, which is capable of modeling undecidable dynamics as well as capable of modeling multi-scale and multi-directional co-evolution. However, the application of reductive Logic is substantially altered in the meta-reductive perspective. The *iterative* application of reductive Logic within a paraconsistent, meta-reductive context reinforces the scientific application of abstract logical and mathematical formal system modeling, allowing for a more immediate and direct connection between scientific thought and modern applied mathematical thought.

By iteratively applying formal system reasoning in modeling undecidable dynamics in multi-scale, multi-directional co-evolution, it is possible to generate accessible scientific and applied mathematical hypotheses, which can link and integrate phenomena across a wide range of scales from micro-scale, to mid-scale, to macro-scale in the natural self-similar evolving system. For instance, scientifically describing undecidable dynamics and natural, patterned, algorithmic, co-evolution in micro-scale quantum systems could theoretically and experimentally access an important *multiple system equivalence*. This descriptive *equivalence* involves, iterated *formal system models*, which can capture undecidable dynamics and multi-scale and multi-directional co-evolution; iterated *dynamical system models* that can also capture undecidable dynamics and equivalent undecidable, multi-scale, multi-directional, co-evolving dynamics in chaotic systems and in non-linear dynamics

[21]; and iterated *information theoretical system models*, which again could equivalently model undecidable dynamics and multi-scale, multi-directional co-evolution in an information theoretical context.

The iterated multiple system equivalence could be used to spell-out, in formal, dynamical and information theoretical terms, a more detailed analysis of how natural differentiation, synthesis and fabrication of self-organized phenomena can arise from binary ‘bits’ of evolution, involving emerging order, existing order and consistency, unresolvable and undecidable dynamics, incompleteness driven novelty generation and emerging patterns of meta-order and meta-consistency. The theoretical and mathematical equivalence of formal, dynamical and information theoretical system modeling [25], could deepen our understanding of the self-similar, iterative and scaled properties within the natural system [22, 24], ranging from the micro-scale puzzles of quantum physics, to the mid-scale complexity of brain/mind and consciousness and on to the macro-scale intricacy of the whole Universe. Multi-scale, multi-directional, co-evolution, described by a novel meta-reductive, iterated, paraconsistent Logic could further explore this important mathematical system equivalence and in so doing might advance the integration of the Reductive Sciences with the Complexity Sciences.

The meta-reductive study of *change*, introduces an awareness of ‘resolvable and unresolvable’ natural phenomena and ‘decidable and undecidable’ abstract theoretical conceptions associated with *causal mechanisms* and *dynamic relationships* in natural systems. Consequently, the meta-reductive understanding provides a novel categorization of ‘simple’ non-linear and linear causal mechanisms, a fundamental *incompleteness driven novelty generating causal mechanism*, and then ‘complex’ interactive meta-reductive causal mechanisms and relationships.

Meta-reductive natural science and mathematics could more effectively synthesize the applied mathematics of *symmetry* and *self-similarity* allowing for an integrated understanding of causal mechanisms and relationships, as well as a deeper awareness of the usefulness of Symmetry Groups and Fractal mathematics in the scientific modeling of causal relationships in natural systems.

A meta-reductive paradigm offers a unique perspective from which to consider the relationship between, Nature’s simple rules such as recently pursued by Wolfram in relation to Quantum physics and Relativity theory [35], general physical laws and self-organized, emergent patterns within naturally evolving systems. The meta-reductive conception of ontological Complex Cycles of Existence and epistemological Complex Natural Histories, the meta-reductive proposal of a fundamental binary unit of self-organization with an associated fundamental evolutionary Symmetry Group and Multi-Fractal, offers a novel way to integrate underlying simple rules with general physical law, and bring the Reductive Sciences and the Complexity Sciences into a closer inter-disciplinary relationship. The preservation and advancement of formal reductive Logic composed in a meta-reductive framework involving hypothesized natural, multi-scale, multi-directional, incompleteness driven co-evolution creates the potential for an integrated meta-reductive perspective capable of addressing many unresolved scientific problems. Among these anomalous and outstanding problems, a meta-reductive model of a Complex Physical System

(CPS) or Complex Adaptive System (CAS) might be better positioned to address the complexity [28, 32] and the paradoxes of brain, mind [29] and consciousness [16, 17]. A task for another paper.

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