Fracture Pressure Prediction Model for Vertical Wells Considering Temperature and Natural Fractures

S. L. Xia, J. Li, and H. L. Huang

Abstract Accurate prediction of formation fracture pressure before drilling can effectively reduce the occurrence of complex situations in the drilling process. When drilling in high temperature formation with natural fracture development, the current prediction model of fracture pressure is not very applicable. Therefore, by analyzing the stress state of borehole wall affected by temperature and natural fractures and combining with the fracture criterion of different borehole wall rocks, the author established the fracture pressure prediction model of high-temperature formation suitable for the development of natural fractures, and analyzed the influence of fracture strike, fracture dip Angle and borehole wall temperature changes on the fracture pressure. The research results show that: the fracture pressure increases gradually with the increase of crack inclination angle. As the angle between the fracture strike and the horizontal maximum principal stress increases, the fracture pressure first increases, then remains unchanged, and finally decreases. With the decrease of wall temperature, the fracture pressure decreases. The model was verified with field data, and the error was 2.9%, and the accuracy met the engineering requirements. The research results can provide a theoretical reference for on-site borehole stability prediction and fracturing parameter design.

Keywords Vertical well · Thermal stress · Natural fracture · Fracture pressure

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S. N. Atluri and I. Vušanović (eds.), *Computational and Experimental Simulations in Engineering*, Mechanisms and Machine Science 98,
https://doi.org/10.1007/978-3-030-67090-0_14

1 Introduction

In drilling activities, formation fracture pressure is one of the important reference data for well casing structure design, well control design and drilling fluid design. It represents the minimum mud column pressure in the well that causes fracture of open hole formation or re-opening of original fractures.

At present, the formation fracture pressure prediction models widely used in the field include Eaton method, Stephen method, Anderson method, Huang Rongzun method, etc. These models mainly consider the influence of overburden pressure, tectonic stress and pore pressure on fracture pressure, but do not consider the influence of temperature on fracture pressure [1–7]. The study by Maury [8] et al. showed that: in deep wells, it is common for the temperature of the well wall to vary between 25 and 50 °C, which can cause a thermal stress of 25 to 50 MPa on the well wall. Such a large thermal stress will have a great impact on the wellbore fracture pressure. To predict the fracture pressure of high-temperature Wells, Li [9] et al. established a new model for the calculation of fracture pressure of coupled temperature by studying the change of formation temperature caused by drilling fluid circulation and formation heat exchange and the resulting thermal stress. Deng [10] et al. proposed the calculation method of additional fracture pressure caused by temperature and natural fracture, and established a new model for calculation of fracture pressure of high-temperature and high-pressure formations with comprehensive consideration of the influence of temperature and natural fracture. However, with the increasing efforts of petroleum exploration and development in China, many new complex well conditions have emerged [11–13], and the effect of temperature on fracture pressure alone cannot meet the engineering requirements of field drilling. For example, when drilling in high-temperature formation with natural fractures, the wellbore path intersects the fracture surface, and the fracture pressure is affected not only by temperature, but also by natural fractures to a greater extent.

Aiming at the fracture pressure and prediction of high temperature formation with natural fracture development, based on the full consideration of wellbore thermal stress and the analysis of fracture stress state of wellbore fracture surface and fracture criterion under different fracture forms of wellbore, a straight well fracture pressure prediction model for high temperature formation with natural fracture development is established in this paper.

2 Thermal Stress Model of Borehole Wall

In general, the straight well ophthalmology model can be simplified as a two-dimensional plane strain model. In polar coordinates when the ground stress, pore pressure and drilling fluid column pressure are considered, the stress on the borehole wall can be expressed as [14]:

$$\sigma_{r1} = p - \delta\phi(p - p_p) \tag{1}$$

$$\sigma_{\theta 1} = (1 - 2\cos 2\theta)\sigma_H + (1 + 2\cos 2\theta)\sigma_h - p + A(p - p_p) \tag{2}$$

$$\sigma_{z1} = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\theta + A(p - p_p) \tag{3}$$

Where: σ_{r1}, $\sigma_{\theta 1}$ and σ_{z1} are respectively radial, circumferential and axial stress, MPa; p is wellbore pressure, MPa; δ is the permeability coefficient of the shaft wall, when permeability occurs, $\delta = 1$; when permeability does not occur, $\delta = 0$; ϕ is formation porosity; P_p is the original formation pore pressure, MPa; ν is Poisson's ratio of borehole wall rock, dimensionless; $A = \delta\left[\frac{\alpha(1-2v)}{1-v} - \phi\right]$, α is the effective stress coefficient.

In drilling operations, the circulation of drilling fluid usually causes the lower formation sidewall temperature to decrease and the upper formation sidewall temperature to increase. The surrounding rock will produce thermal strain due to temperature change, which will produce a heating stress field around the borehole. For homogeneous isotropic and porous thermos elastic formation, the thermal stress field around the well can be expressed by the heat conduction theory and thermos elastic mechanics as follows [15]:

$$\sigma_{rT} = \frac{E\lambda}{3(1 - v)r^2} \int_{r_w}^{r} T^f(r, t) r dr \tag{4}$$

$$\sigma_{\theta T} = -\frac{E\lambda}{3(1 - v)} \left[\frac{1}{r^2} \int_{r_w}^{r} T^f(r, t) r dr - T^f(r, t)\right] \tag{5}$$

$$\sigma_{zT} = \frac{E\lambda}{3(1 - v)} T^f(r, t) \tag{6}$$

Where, σ_{r1}, $\sigma_{\theta 1}$ and σ_{z1} are respectively radial, circumferential and axial thermal stress, MPa; E is the formation elastic modulus, MPa; λ is the thermal expansion coefficient of rock, 1/°C; $T^f(r, t) = T(r, t) - T_0$,$T(r, t)$ is the distribution function of formation temperature around the well, and at the borehole wall is T_w, °C; T_0 is the original formation temperature, °C; r is the radial distance, m.

When $r = R$, the thermal stress expression at the borehole wall can be obtained:

$$\sigma_{\mathrm{r}T} = 0 \tag{7}$$

$$\sigma_{\theta T} = \sigma_{zT} = \frac{E\lambda}{3(1 - v)} (T_w - T_0) \tag{8}$$

It can be seen that the change of borehole wall temperature only produces thermal stress in circumferential and vertical directions, and the thermal stress is positively correlated with the change of borehole wall temperature. According to the superposition criterion of small deformation stress of pore elasticity, the expression of wellbore stress distribution at high temperature can be obtained by the comprehensive Eq. (1)–(8) as follows:

$$\sigma_r = p - \delta\phi(p - p_p) \tag{9}$$

$$\sigma_\theta = (1 - 2\cos 2\theta)\sigma_H + (1 + 2\cos 2\theta)\sigma_h - p + A(p - p_p) + \frac{E\lambda}{3(1 - v)}(T_w - T_0) \tag{10}$$

$$\sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\theta + A(p - p_p) + \frac{E\lambda}{3(1 - v)}(T_w - T_0) \tag{11}$$

3 Calculation Model of Fracture Pressure Considering the Influence of Temperature and Natural Fracture Property

When the natural fractures are not completely closed or the conductivity is strong, the pressure of the liquid column in the wellbore is slightly greater than the pore pressure of the formation, which will cause leakage. Such fractures are beyond the scope of this article. This paper mainly studies the natural cracks with completely closed crack surfaces and certain cementation strength. When drilling against such natural fractures, excessively high wellbore liquid column pressure will lead to three forms of wellbore failure [16]: tensile failure of wellbore rock, shear failure along natural fractures, and tensile failure along natural fractures.

The fracture pressure prediction models for the three failure forms are respectively established as follows:

3.1 *Tensile Failure of Borehole Wall Rock*

It can be seen from Eq. (10) that when the wellbore pressure increases gradually, it will gradually decrease. When the wellbore pressure is large enough, σ_θ will decrease to a negative value. At this time, the wellbore rock is subjected to tensile action. When the tensile stress on the rock on the wellbore wall reaches its tensile strength S_t, the rock will undergo tensile failure, that is, the rock failure condition is as follows [17]:

$$\sigma_\theta = -S_t \tag{12}$$

Normally, vertical borehole wall fracture occurs at the point of minimum σ_θ. According to Eq. (10), when $\theta = 0°$ or $\theta = 0°$, σ_θ take the minimum value [18], that is:

$$\sigma_\theta = 3\sigma_h - \sigma_H - p + A(p - p_p) + \frac{E\lambda}{3(1 - v)}(T_w - T_0) \tag{13}$$

In actual working conditions, there is pore pressure in the formation, and the above formula is rewritten as:

$$\sigma_\theta = 3\sigma_h - \sigma_H - p + A(p - p_p) + \frac{E\lambda}{3(1 - v)}(T_w - T_0) - \alpha p_p \tag{14}$$

Combine Eq. (14) and Eq. (12), the fracture pressure expression of rock tensile failure when thermal stress is considered is obtained:

$$p_{m1} = \frac{3(1 - v)(3\sigma_h - \sigma_H - (A + \alpha)p_p + S_t) + E\lambda(T_w - T_0)}{3(1 - A)(1 - v)} \tag{15}$$

3.2 Shear Failure along Natural Fractures

The shear failure of the borehole wall usually occurs in the natural fracture surface and other places where the cement strength is weak. The weak surface failure criterion can be used to study the fracture pressure. The weak surface failure criterion is [19]:

$$\sigma_1 - \sigma_3 = \frac{2(C_w + \mu_w \sigma_3)}{(1 - \mu_w \cot\beta)\sin 2\beta} \tag{16}$$

Where, σ_1 and σ_2 are the maximum and minimum principal stress, respectively, MPa; C_w is the adhesion force of weak surface, MPa; μ_w is the tangent value of the internal friction angle on the weak surface, $\mu_w = \tan\varphi_w$; β is the angle between the normal direction of the weak plane and the maximum principal stress.

It can be seen from the weak surface failure criterion that when β is close to or 90°, $\sigma_1 - \sigma_2$ approaches infinity. Therefore, only when β is greater than φ_w and less than 90°, the well wall may be sheared along the fracture. In general, when natural fractures in vertical wells undergo shear failure, the rock stress of the borehole wall satisfies the relationship [20]: $\sigma_z > \sigma_r > \sigma_\theta$, that is $\sigma_1 = \sigma_z$, $\sigma_3 = \sigma_\theta$. At this time, the fracture pressure expression is:

$$p_{m2} = \frac{3(1-v)(b-a-e\cdot C_w) - e\cdot \mu_w(3(1-v)(a-A\cdot p_p) + E\lambda(T_w - T_0))}{3(1-v)(e\cdot \mu_w(A-1)-1)} \tag{17}$$

Where,

$$a = (1-2\cos 2\theta)\sigma_H + (1+2\cos 2\theta)\sigma_h$$

$$b = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\theta$$

$$e = \frac{2}{(1-\mu_w \cot\beta)}\sin 2\beta$$

3.3 Tensile Failure along Natural Fractures

Assuming that the strike of the natural fracture in the reservoir is *TR* (in the geodetic coordinate system), the dip angle is θ_{DIP}. The angle between the natural fracture and the wellbore wall is θ_1, and the horizontal maximum ground stress azimuth is θ_H. The normal stress on the surface of the natural crack is expressed as [21]:

$$\sigma_n = \sigma_z l_1^2 + \sigma_\theta l_2^2 + \sigma_r l_3^2 \tag{18}$$

where,

$$l_1 = \sin\theta_{DIP}$$

$$l_2 = \cos\theta_1 \cos\theta_{DIP} \sin(TR - \theta_H) - \sin\theta_1 \cos\theta_{DIP} \cos(TR - \theta_H)$$

$$l_3 = \cos\theta_1 \cos\theta_{DIP} \cos(TR - \theta_H) + \sin\theta_1 \cos\theta_{DIP} \sin(TR - \theta_H)$$

When the wellbore pressure is equal to the sum of the normal stress and tensile strength of the fracture plane, tensile failure of natural fractures will occur. Fracture criteria are as follows:

$$p_{m3} = \sigma_n + \sigma_t - \alpha p_p \tag{19}$$

Substituting Eq. (9), (10), (11), (18) into Eq. (19) to obtain the calculation formula of rupture pressure:

$$p_{m3} = \frac{f \cdot l_1^2 + g \cdot l_2^2 + \delta\phi p_p \cdot l_3^2 - \alpha p_p + \sigma_t}{h} \tag{20}$$

Where,

$$f = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\theta - A \cdot p_p + \frac{E\lambda}{3(1-v)}(T_w - T_0)$$

$$g = (1 - 2\cos 2\theta)\sigma_H + (1 + 2\cos 2\theta)\sigma_h - A \cdot p_p + \frac{E\lambda}{3(1-v)}(T_w - T_0)$$

$$h = 1 - A \cdot l_1^2 - (A-1)l_2^2 - (1 - \delta\phi)l_3^2$$

3.4 Calculation Model of Fracture Pressure

In actual working conditions, for different formations, borehole wall rupture can only be one of the three types. Therefore, the minimum value of the fracture pressure under the three fracture types is the fracture pressure under the actual working conditions. The comprehensive calculation model is the Eq. (21)

$$p_f = \min\{p_{m1}, p_{m2}, p_{m3}\} \tag{21}$$

Where,

$$p_{m1} = \frac{3(1-v)(3\sigma_h - \sigma_H - (A+\alpha)p_p + S_t) + E\lambda(T_w - T_0)}{3(1-A)(1-v)}$$

$$p_{m2} = \frac{3(1-v)(b - a - e \cdot C_w) - e \cdot \mu_w(3(1-v)(a - A \cdot p_p) + E\lambda(T_w - T_0))}{3(1-v)(e \cdot \mu_w(A-1) - 1)}$$

$$p_{m3} = \frac{f \cdot l_1^2 + g \cdot l_2^2 + \lambda\phi p_p \cdot l_3^2 - \alpha p_p + \sigma_t}{h}$$

4 Analysis of Factors Affecting Fracture Pressure

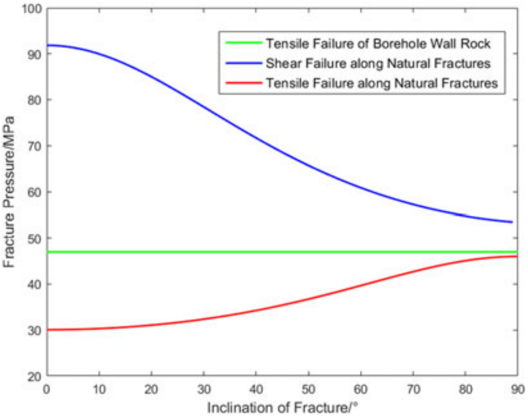
4.1 The Influence of Natural Fracture Inclination on Fracture Pressure

Taking the fracture inclination angle from 5° to 90°, the trend of fracture pressure with the fracture inclination angle is obtained (Fig. 1). With the increase of the crack inclination angle, p_{m1}(the fracture pressure of the borehole wall rock tensile failure) remains unchanged; p_{m2}(the fracture pressure of shear failure along natural fractures) decreases rapidly, but the rate of decrease gradually decreases; p_{m3} (the rupture pressure along the tensile failure of natural fractures) gradually increases, and the increase rate is small. When the crack inclination is equal to 90°, p_{m1} and p_{m3} are similar in size. This is due to the small angle between the fracture strike and the horizontal maximum principal stress selected in the calculation. When the inclination angle of the fracture surface is 90° (the fracture surface is parallel to the axis of the wellbore), p_{m1} and p_{m3} mainly depend on the circumferential stress of the well wall.

4.2 The Influence of Fracture Strike on Fracture Pressure

Taking the fracture strike from 79° to 169°, the change trend of the fracture pressure with θ_2 (the angle between the fracture trend and the maximum principal stress) is obtained (Fig. 2). As θ_2 increasing, p_{m1} remains unchanged; p_{m2} increases, and the rate of increase first increases and then decreases; p_{m3} first increases and then decreases, and the maximum value is taken at a certain angle, and the maximum value is greater than p_{m1}. Combining Eq. (21), it can be seen that the change of θ_2 will cause the change of the failure form of the borehole wall. In the calculation

Fig. 1 Trend diagram of fracture pressure changing with fracture dip Angle



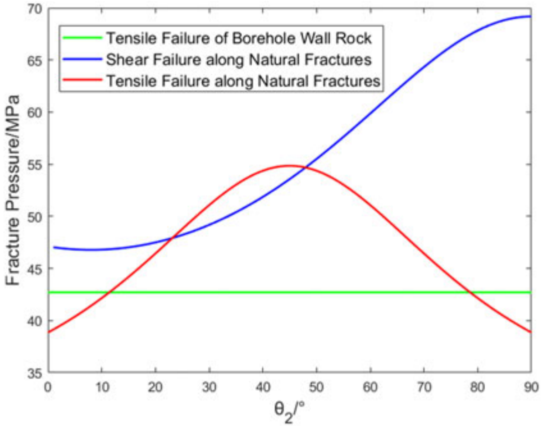


Fig. 2 Trend chart of burst pressure changing with θ_2

example in this paper, as θ_2 increases, the borehole wall fracture changes from the fracture surface tensile failure form to the rock tensile failure form, and after θ_2 increases to a certain extent, it becomes the fracture surface tensile failure form.

4.3 *Effect of Temperature on Fracture Pressure*

Taking the temperature change of the borehole wall form −50 °C to 0 °C, the trend of the change of the fracture pressure with the temperature of the well wall can be obtained (Fig. 3). As the borehole wall temperature decreases, p_{m1}, p_{m1} and p_{m3} all decrease. The fracture pressure changes linearly with the temperature of the well wall. The decrease of the well wall temperature will cause the fracture pressure to decrease, but the failure form of the borehole wall will not change.

5 Example Calculation

In a fractured formation, the natural fracture inclination is 53° and the strike is 12°. The azimuth of the maximum principal stress of the formation is 68° and the magnitude is 181.4 MPa. The minimum ground stress is 145.3 MPa, and the overburden pressure is 179.8 MPa, the pore pressure is 120.1 MPa, the rock tensile strength is 9.1 MPa. Poisson's ratio is 0.254, effective stress coefficient is 0.91, porosity is 0.132, fractured rock cohesion is 8.95, weak surface friction coefficient is 0.491, formation temperature is 150 °C. The fracture pressure measured on site is 101.4 MPa.

Use the well data to calculate the fracture pressure. The calculation results are shown in Table 1. When ignoring the influence of temperature and natural fractures, the calculated fracture pressure is 188.4 MPa, which is quite different from the

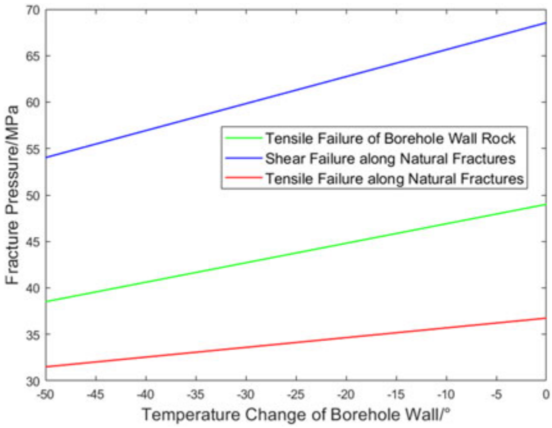


Fig. 3 The relationship between the fracture pressure and the temperature of the borehole wall

Table 1 Calculation error table of fracture pressure

Calculation conditions	Calculated value /MPa	Measured value /MPa	error
Ignore temperature and natural fracture effects	188.4	101.4	85.8%
Consider only natural fracture	123.5		21.8%
Consider natural fracture and temperature effects (models in this article)	104.3		2.9%

actual measured value, with an error of 85.8%; When only considering the influence of natural fractures, the calculated fracture pressure is 123.5 MPa, which is less different from the actual measured value, but the error is still large; When considering the effects of natural fractures and temperature on the fracture pressure at the same time, the calculated fracture pressure value is 104.3 MPa, which is close to the measured fracture pressure value of 101.4, and the error is only 2.9%. It can be seen that the error calculated by the model established in this paper is the smallest, and it is suitable for the prediction of fracture pressure in high-temperature formations with natural fractures.

6 Conclusion

In this paper, a prediction model of rupture pressure considering temperature and natural fractures is established, and a series of analyses are carried out, and the following conclusions are obtained:

(1) The thermal stress distribution around the borehole caused by the circulation of drilling fluid has nothing to do with the well circumference angle. The vertical and circumferential thermal stresses at the well wall are equal. The radial thermal stress is zero.
(2) The factors affecting the fracture pressure of high-temperature formations with natural fractures are analyzed, and the analysis results show that: as the fracture inclination angle increases, the fracture pressure gradually increases; as the angle between the fracture direction and the horizontal maximum principal stress increases, the fracture pressure first increases, then remains unchanged, and finally decreases; as the borehole wall temperature decreases, the fracture pressure decreases.
(3) Using the model in this paper to calculate the example well, the error of the calculation result is 2.9%, which meets the field requirements. Therefore, the fracture pressure prediction model established in this paper can provide a theoretical reference for on-site borehole stability prediction and fracturing parameter design.

Acknowledgements The authors gratefully acknowledge The National Natural Science Funds (Grant No. 51774304), National Science and Technology Major Project (Grant No. 2017ZX05009), the Science Foundation of China University of Petroleum, Beijing (Grant No. 2462015QZDX05), the State Key Laboratory of Petroleum Resources and Engineering.

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