Mechanical Behavior of Drillstring with Oscillator During Sliding Drilling

Xiaolei Shi, Wenjun Huang, and Deli Gao

Abstract In the process of drilling operation, the mechanical behavior of the drill string is complex and the failure problem is prominent, and friction is an important factor restricting the extension of the borehole. Based on the vibration mechanism and basic knowledge of drill string mechanics, this paper introduces the action of oscillator comprehensively considers the vibration function form of excitation force excited by oscillator, drill string friction model, drill string end constraint conditions and continuity conditions, establishes a drill string dynamic model with oscillator and uses finite difference method to solve it. Then, the above model is verified by an example, and the drag reduction effects with and without oscillators are analyzed and compared. Finally, sensitivity analysis is carried out to evaluate the drag reduction effects of various parameters. The results show that the fluctuation are more obvious for the displacement of drill string and the hook load with oscillator, and the effect of reducing the average friction coefficient is significant, which is more conducive to the transfer of axial force. When the energy excited by the vibrator is the same, the vibration function form of large amplitude and small frequency excitation force is beneficial to load transfer and can reduce the average friction coefficient of the drill string. When other parameters are constant, increasing the amplitude or increasing the number of oscillators is conducive to improving the drag reduction efficiency of drill string, increasing the penetration rate and increasing the borehole extension distance. This study has important guiding significance for the design optimization of drill string and safe operation on site.

Keywords Drillstring · Vibration mechanism · Oscillator · Friction reduction · Load transfer

X. Shi · W. Huang · D. Gao (✉)
MOE Key Laboratory of Petroleum Engineering, China University of Petroleum, Beijing 102249, China
e-mail: gaodeli@cup.edu.cn

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1 Introduction

In recent years, with the rapid development of drilling technology, the depth of drilling has been continuously extended. More and more complex structure wells such as horizontal wells and extended reach wells have become the focus of exploration and development. Compared with conventional well types, these new complex structure wells are facing many technical problems, especially in the long horizontal section where the friction torque is relatively large, which is prone to serious propping up and sticking, thus causing serious drilling accidents and affecting the normal drilling of oil and gas wells. In order to effectively reduce the friction torque between the drill string and the borehole wall and reduce the propping pressure of the drill string, the average friction force of the whole drill string should not only be reduced, but also the static friction force of the drill string should be changed into dynamic friction force. Starting from the vibration mechanism of the drill string, it is of great significance to develop suitable vibration tools and establish a reasonable drill string vibration drag reduction model for in-depth theoretical analysis to improve the extension limit of the well and the drilling velocity.

At present, the vibration drag reduction tools abroad are mainly mechanical vibration type and hydraulic type, mechanical vibration type is represented by Agitator hydraulic oscillator, and hydraulic type is represented by FDR tool. The Agitator hydraulic oscillator [1, 2] can generate an impact force of 3.45–4.14 MPa according to the difference of frequency, amplitude, outer diameter and length. Through axial vibration, the sliding friction of the drilling tool can be reduced by 75%–80%. On the premise of obtaining the same mechanical drilling tool, the required weight on bit with the tool is about 60% of that without the tool, and all MWD will not be disturbed by Agitator tool, which can ensure the smooth transfer of MWD signals. Meanwhile, axial vibration generated by Agitator tool will not damage the bearings and teeth of drill bit. Agitator hydraulic oscillator has been applied in many oil fields around the world and has achieved good results. It can greatly reduce the friction of sliding drilling and improve the penetration rate. The FDR tool [3] was developed by RF-Rogaland Research. The tool is mainly composed of a two-way moving hydraulic cylinder with an inner valve and an outer valve. It has been comprehensively tested in a large displacement well with a diameter of 177.8 mm, and has been tested in a coiled tubing drilling with an outer diameter of 44.5 mm and a length of 600 m. The results show that the friction resistance of coiled tubing can be reduced by 90% and the drag reduction effect is obvious. In order to solve the problem of axial force transfer in extended reach wells, Sinopec has developed a hydraulic oscillator and developed a SLZDDQ172 hydraulic pulse oscillation tool [4]. The tool has an outer diameter of 172 mm, a working frequency of 15 Hz, a working pressure consumption of 3–4 MPa, and a displacement of 10–30 L/s. It has good field application effect, can increase the penetration rate by more than 20%, and has good adaptability to PDC bits, cone bits, etc. However, the research and development of the new turbine hydraulic oscillator [5] is still at the laboratory test stage, and no on-site experiment has been carried out. Most of the other vibration drag reduction tools introduced by others [6–9] still learn from the principle of Agitator tools and make improvements.

The research on theoretical model is relatively mature at present. Johancsik et al. [10] established a soft rope model using model analysis method and predicted the pulling force of directional well drill string by computer programming method, which provided theoretical basis for well trajectory design and so on. It was obtained that the friction coefficient of drilling was between 0.25 and 0.4 when sliding drilling was performed. Ho [11] established a new stiffness model that considered the three factors of drill string rigidity, centralizer position and annulus clearance was established and revised the original model considering the influence of bottom hole assembly. Considered the influence of joints on the basis of predecessors, Gao et al. [12] calculated the critical force of helical buckling of tubular string in inclined straight section, the contact force between tubular string and borehole wall distributed along helical section and the average lateral contact force of post-buckling and proposed a new method to calculate the critical force of sinusoidal buckling, proving that when the drill string undergoes helical buckling, there will be a large friction between the drill string and borehole wall. Miska et al. [13] established an improved soft rod dynamic model based on the movement of drill pipe in 2D and 3D boreholes. The model can better calculate the stress of moving drill pipe or casing. The actual design calculation can be solved by using numerical calculation method and computer code programming. Wang et al. [14] proposed a new friction reduction technology called "earthworm-like drilling" based on "soft-string" model, which can improve the load transfer and extended-limit. Wang et al. [15] considered the transformation and decomposition mechanism of static friction force and dynamic friction force, and established a drag reduction model which was solved by second-order finite difference method and analyzed the vibration behavior and parameters under different axial load transfer and tool surfaces. In order to investigate the friction reduction mechanism of the hydraulic axial vibratory tool in coiled tubing drilling, Zhang et al. [16] proposed a dynamic model by introducing the Coulomb friction term to the wave equation. According to the wave equation, Liu et al. [17] established a FEA model by considering the coupling of the drillstring-wellbore contract and the action of AOT, which was verified analytically and experimentally. The results show that the friction-reduction is mainly influenced by drillstring material properties and well-bore parameters. Mahjoub et al. [18] proposed a new modeling approach to accurately model the effect of the AOT on drilling operations without the need to carry out resource-intensive and time-consuming dynamic computation. The results show that once the influence length and the maximum displacement are calculated, an effective friction coefficient is estimated as the mean value of the instantaneous friction coefficient and used in a stiff-string torque and drag model. Omojuwa et al. [19] presented an analytical model to predict the behavior of axial oscillation-supported drillstings working under downhole and surface conditions. The model is useful to perform parametric studies, downhole data correlation and placement study of AOTs along the drillstring in horizontal and extended-reach wells.

Through investigation, it can be found that previous studies have independently analyzed vibration drag reduction tools or drag reduction models based on the vibration mechanism of drill string, ignoring the friction between the drill string and the borehole wall, or replacing the friction with Coulomb friction. Few people have carried out mechanical analysis on drill string with oscillator and studied its drag reduction effect. Starting from the vibration mechanism of the drill string, this paper analyzes the process of the drill string from static to motion. Considering the action of the oscillator on the drill string, a bilinear hysteretic friction model is adopted to establish the dynamic model of the drill string with the oscillator. The new model is verified by field data. Sensitivity analysis of the key parameters of the drill string or oscillator can accurately evaluate the drag reduction effect of the oscillator.

2 Theoretical Model

Before establishing the model, the following assumptions need to be made on the model:

(1) Both the drill string and the inner wall of the borehole are rigid, and the cross-sectional area of the drill string is always circular.
(2) The drill string is in uniform contact with the borehole wall without rotation.
(3) Only consider axial vibration and ignore the lateral and torsional vibration of the drill string.
(4) The bilinear hysteretic restoring force model is adopted to depict the friction between tubular string and wellbore.

A micro-element segment with an arc length of ds is selected as the research unit on the entire drill string. The unit is connected to the adjacent micro-element segment through a spring-like oscillator, and θ is the inclination angle of the drill string. To study its vibration response during the drilling process, establish the schematic diagram shown in Fig. 1.

2.1 Differential Equation

As shown in Fig. 1 above, the drill string is composed of multiple micro-element segments, and the force analysis of one of the micro-element bodies is performed. Under static conditions, the drill string lies flat on the borehole wall, relying on the weight of the drill string, static friction and bottom weight to maintain balance. The force equation of the micro-element body of this section of the drill string can be written as:

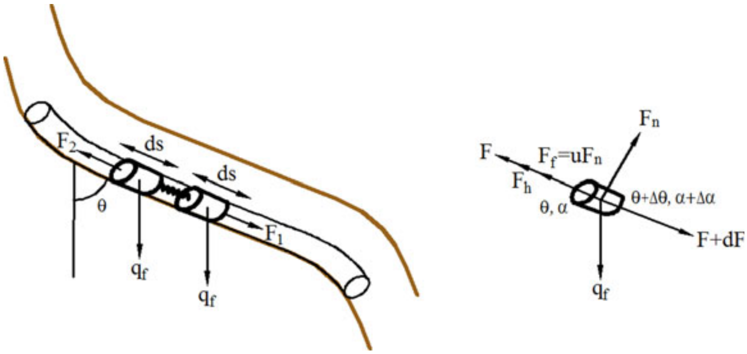

Fig. 1 Stress state of differential element of drillstring while drilling

$$F + \mathrm{d}F + \rho gA\cos\theta ds - F - \mu\rho gA\sin\theta ds = 0 \quad (1)$$

During the vibration process, the oscillator can apply regular excitation force to the drill string. The drill string also needs to overcome the viscous force of drilling fluid. At this time, the static friction force between the drill string and the borehole wall is converted into dynamic friction force. Under the action of the weight, friction force, weight of bottom and viscous force of the drill string, the force equation of the drill string micro-element body in this section can be written as follows:

$$F + \mathrm{d}F + \rho gA\cos\theta ds - F - \pi DC\frac{\partial U}{\partial t}ds - F_f ds = \rho Ads\frac{\partial^2 U}{\partial t^2} \quad (2)$$

Where U, u, ρ, D, A are axial displacement function, friction coefficient, density, annulus hydraulic diameter and cross-sectional area of drillstring, respectively; F_f is the friction force between drillstring and borehole wall (casing); s is the distance from any point of the drillstring to the top of the section; C is the viscosity coefficient of drilling fluid; t is time; g is gravitational acceleration; θ is angle of inclination.

2.2 Friction Model

As we all know, vibration is a natural phenomenon in nature. If divided according to the nature of the motion equation in the vibration process, the contact between the drill string and the borehole wall will produce nonlinear vibration, considering the elastic nature of the contact surface and the dynamic response characteristics of the external disturbance excitation to the system. Iwan et al. [20] put forward a bilinear hysteretic restoring force model, which equates the dry friction surface with a spring and an ideal Coulomb friction pair connected in series. the nonlinear restoring force F_f with memory characteristics is approximately described by a double fold line model. The constitutive relation in incremental form can be written as follows:

$$\begin{cases} dF_f = \frac{k_s}{2}\left[1 + \operatorname{sgn}\left(F_s - \left|F_f\right|\right)\right]dx \\ k_s = \frac{F_s}{x_s} \end{cases} \tag{3}$$

Where x_s is the limit value of elastic deformation when the contact surface of the dihedral slides macroscopically; F_s is the memory restoring force when slipping; x is the relative displacement deformation of the two ends of the hysteresis link; k_s is the linear stiffness before the dry friction link slips.

3 Calculation Method

3.1 Finite Difference Method

The finite difference method can be used to solve the above problems. In the process of solving differential equations by finite difference method, independent variables take discrete values. We transform the differential equations of continuous variables into difference equations in discrete variable grids, and determine the function values at each grid point with initial values or boundary values. The specific method is to discretize the differential equations first, and then solve the difference equations.

Since, $F = EA\frac{\partial U}{\partial s}$ the explicit central difference scheme of Eq. 1 and Eq. 2 can be expressed as:

$$\frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{\Delta s_i^2} = \frac{\mu_i^j \rho g \sin\theta - \rho g \cos\theta}{E} \tag{4}$$

$$\frac{E}{\rho}\frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{\Delta s_i^2} + g\cos\theta - \frac{\pi DC}{\rho A}\frac{U_i^{j+1} - U_i^j}{\Delta t} - \frac{F_{f_i}}{\rho A} = \frac{U_i^{j+1} - 2U_i^j + U_i^{j-1}}{\Delta t^2} \tag{5}$$

Where E is elastic modulus; Δt is the time interval, the superscript "j" in U_i^j represents the i-th time point.

3.2 Initial Condition

In general, the calculation of finite difference equations requires initial conditions. In another word, when the right side of Eq. 1 and Eq. 2 is set to zero, the initial displacement needs to satisfy the equations. The discretization scheme of the initial displacement can be expressed as:

$$U_i^1 = u_{initial} \tag{6}$$

The values of $u_{initial}$ are obtained by solving Eq. (4).

The discretized scheme of initial velocity condition is expressed as:

$$\frac{U_i^2 - U_i^0}{2\Delta t} = v_{initial} \tag{7}$$

Note that, the term U_j^0 in Eq. 5 can be eliminated by combing Eq. 5 and Eq. 7 while j = 1.

3.3 Boundary Condition

The top of the drillstring is tied to hook, so the axial displacement of the top of drillstring is equal to the vertical displacement of hook. Then, the top boundary condition is expressed as:

$$U_1^j = u_{hook} \tag{8}$$

For the drillstring installed with oscillator, an additional excitation force will be applied to the drillstring. Therefore, the boundary conditions at the upper and lower ends of the oscillator can be expressed as the axial force as:

$$(EA)_{i-1}\frac{U_i^j - U_{i-2}^j}{2\Delta s_{i-1}} = F_t + FF(t), (EA)_i\frac{U_{i+1}^j - U_{i-1}^j}{2\Delta s_i} = F_t \tag{9}$$

Where F_t is the axial force at the joint of oscillator and drillstring; *FF(t)* is the excitation force applied by the vibrator with time.

In the sliding drilling process, the axial force on bit is determined with bit-rock interaction model. Here, the axial force on the bottom of drillstring is equal to drill bit, namely

$$(EA)_n\frac{U_{n+1}^j - U_{n-1}^j}{2\Delta s_n} = Wob(t) \tag{10}$$

Where *Wob(t)* is the weight-on-bit which changes with time. Note that, the term U_{n+1}^{j} in Eq. 10 can be eliminated by combing Eq. 5 and Eq. 10 while i = n.

3.4 Continuous Condition

When two or more than two kinds of drillstrings are adopted, the relevant parameters such as tubular diameter, weight, etc. are discrepant for different drilling strings. Then the continuous conditions on the node connecting to different tubular strings should be satisfied. To be specific, the axial displacement and equivalent axial forces on adjacent segments are continuous, namely

$$U_{i-1}^{j} = U_{i}^{j} \tag{11}$$

$$(EA)_{i-1}\frac{U_{i}^{j} - U_{i-2}^{j}}{2\Delta s_{i-1}} = (EA)_{i}\frac{U_{i+1}^{j} - U_{i-1}^{j}}{2\Delta s_{i}} \tag{12}$$

4 Results and Discussions

On the basis of the above model and calculation method, the mechanical behaviors of drillstring in sliding drilling operation are studied. A horizontal well is drilled of which the depth of kick off point is 500 m, inclination angle of the horizontal section is π/2 and well depth is 3000 m. A 5-1/2″drill pipe is adopted of which drilling string weight in air per unit length is 28.03 kg/m, Bingham drill fluid is used of which the density is 1.2 g/cm^3. The rotation speed is 50 rad/s, the rate of penetration is 15 m/h, and four oscillators are arranged below the kick off point. The excitation frequency of the oscillator is 5 Hz, and the excitation load changes in sine function with respect to time.

In the numerical simulation, to reveal the transition between sticking friction and sliding friction, the time interval should be very small. The time interval is set to 4e −4 s, segment length is set to 3 m, operation time is set to 30 s. It takes about 30 s to obtain the results.

4.1 Vibration Behaviors

As shown in Fig. 2, during the sliding drilling operation, when the drill bit breaks rock at the set drilling speed, the axial displacement of the drillstring increases. When the drillstring starts to move, the displacement increases more rapidly. As the

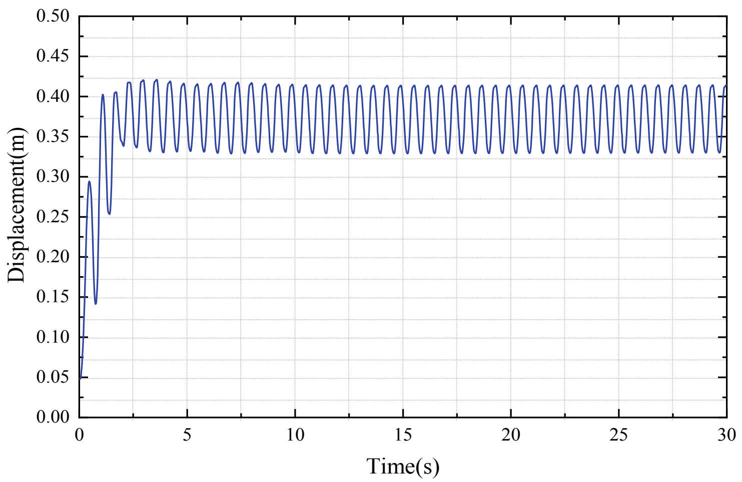

Fig. 2 Displacement change of drillstring at 600 m hole depth in sliding drilling operations

time increases, the movement of the drillstring enters a stable state, the displacement oscillates around its average value.

As shown in Fig. 3, during the sliding drilling operation, the friction coefficient of the drillstring always shows a regular change. Generally speaking, because the vibration of the drillstring is nonlinear, and the friction coefficient is opposite to the direction of the movement of the drillstring, the change of the friction coefficient is not constant. When the drillstring moves in the positive direction, it always transitions from the static friction coefficient to the dynamic friction coefficient in the form of a parabola, and then returns to the static friction coefficient with the same law, and finally changes the direction of movement and reciprocates with the same law. In other words, the friction coefficient first decreases and then increases, and the direction of change continues to decrease and then increases, and reciprocates.

As shown in Fig. 4, the velocity of the drillstring presents a relatively regular movement pattern, the drillstring is in an unstable state at the beginning of the movement, and the fluctuation of the drillstring velocity is not stable. With the increase of time, the velocity change amplitude gradually stabilizes, and the velocity always reciprocates around the zero point.

As shown in Fig. 5, since the top of the drillstring is always controlled by the hook, the load change at the hook can reflect the working conditions of the bottom hole drillstring and bit in real time. At the beginning of drilling, the hook load is relatively small. With the increase of time, combined with the action of the downhole oscillator, the hook load increases rapidly and reaches a stable state. The load always oscillates around an average value. It can be seen that during the sliding drilling process, the increase in the hook load is more obvious, indicating that axial friction is decreased a lot and sliding drilling process is more smooth.

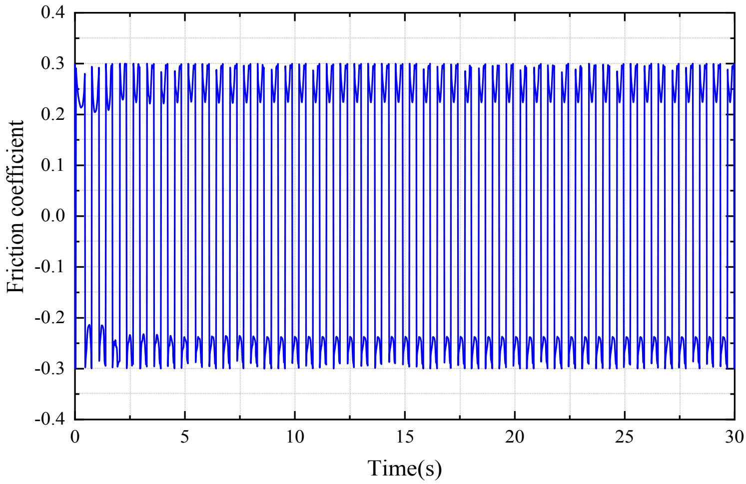

Fig. 3 Friction coefficient change of drillstring at 600 m hole depth in sliding drilling operations

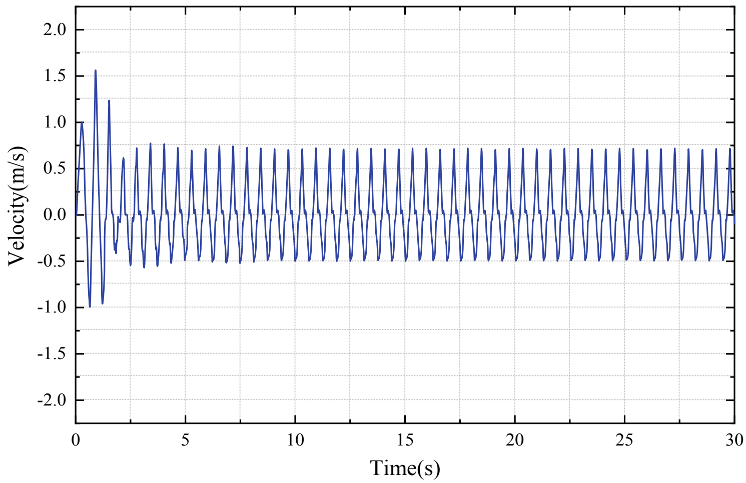

Fig. 4 Velocity change of drillstring at 600 m hole depth in sliding drilling operations

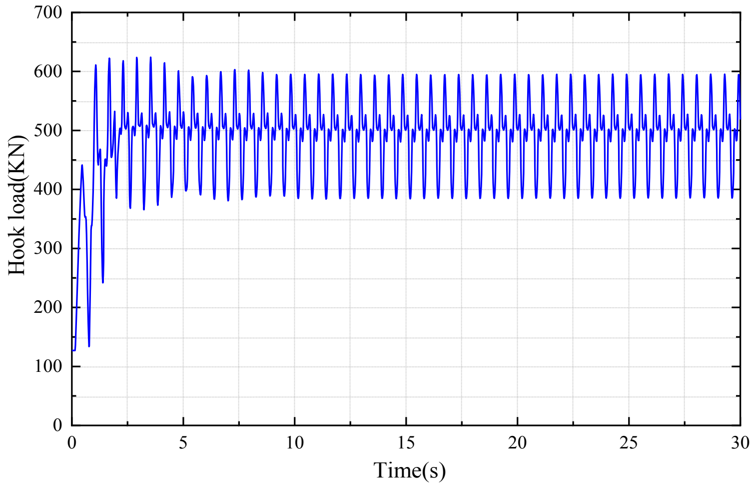

Fig. 5 Change of hook load in sliding drilling operations

4.2 Drag Reduction of Oscillator

Before optimizing the parameters, the mechanical behaviors of the drillstring with and without oscillator are first analyzed. To simulate the no oscillator case, the excitation force of the oscillator is set to zero in the above model.

The calculation results with or without oscillator are shown in Figs. 6, 7, 8. Figure 6 and Fig. 7 show similar change trends. When the oscillator is installed on the drillstring, the displacement of the drillstring and the hook load increase greatly, showing a more violent fluctuation pattern. But the displacement of the drillstring and the hook load without the oscillator basically have no change, only a slight jump at the beginning of the movement, and then it is stable. This indicates that the drillstring with oscillator is excited by the excitation force, the load transfer is more obvious, and the drillstring has been in motion; The comparison between the average friction coefficients of the drillstring with and without the oscillator are given in Fig. 8. The average friction coefficient of the drillstring with the oscillator will be greatly reduced, especially at the installation location of the oscillator. The decrease in the average friction coefficient of the drillstring is very obvious. The analysis of the above results shows that the load transfer on drillstring with oscillator is much better than that without oscillator, and the average friction coefficient is greatly reduced.

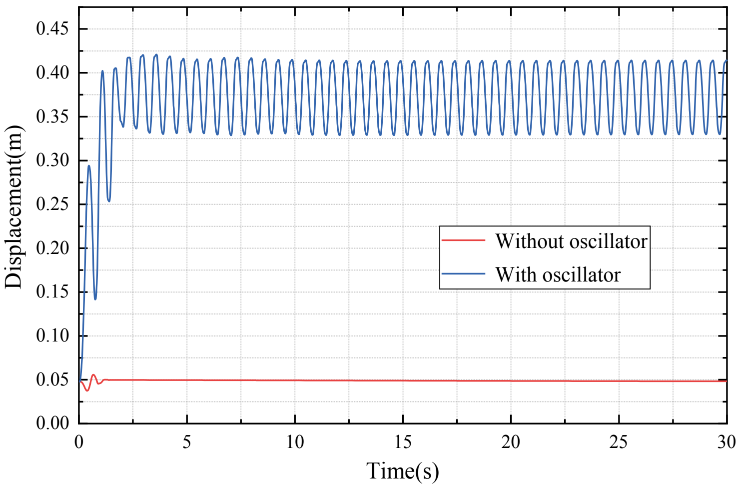

Fig. 6 Change of drillstring displacement in sliding drilling operations

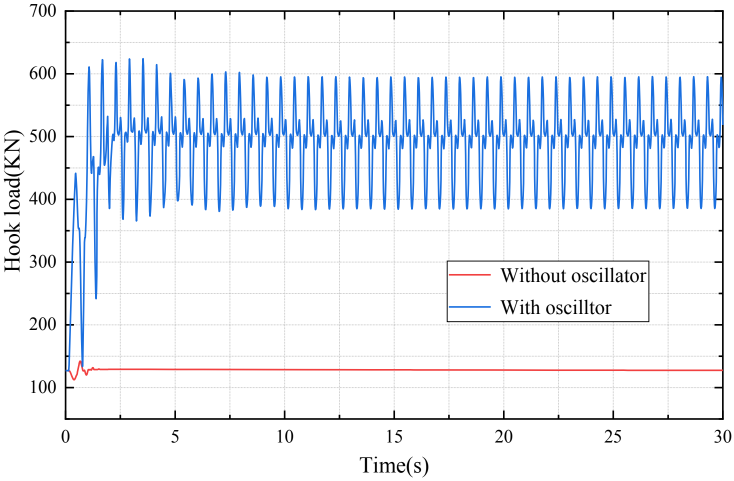

Fig. 7 Change of hook load under sliding drilling operation

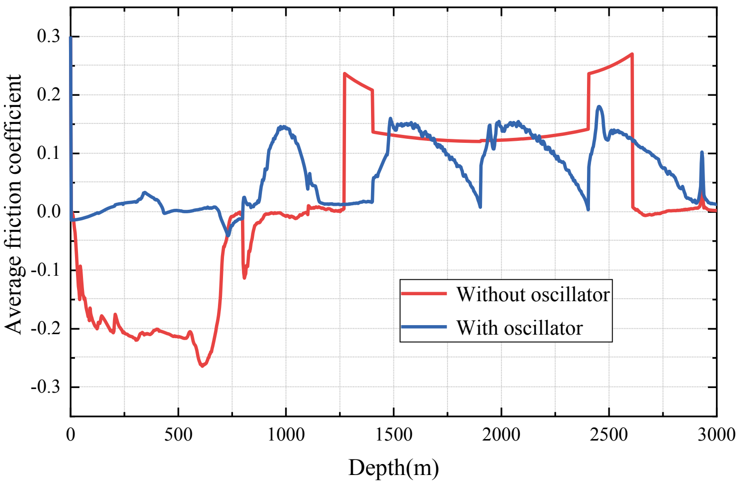

Fig. 8 Change of average friction coefficient of drillstring in sliding drilling operations

4.3 Oscillator Optimization

Through the above analysis, we have known that the drill string with a oscillator can play a friction reduction effect during the drilling process, but the acquisition of the relevant indicators of the oscillator is not clear, so the following parameters will be optimized to maximize the effect of friction reduction from the form of excitation force generated by oscillator, amplitude variation and oscillator number.

The evaluation of the friction reduction effects under different vibration forms of the excitation force is shown in Figs. 9, 10, 11. As shown in Fig. 9, oscillators with different amplitudes and vibration frequencies generate the same energy in one cycle to ensure that the oscillator can provide the same load in the same cycle. As shown in Fig. 10, the vibration force of larger amplitude has a greater impact on the hook load. The vibration function form has basically no effect on the average hook load, because the oscillator can generate the same vibration energy in the same period. As shown in Fig. 11, when the vibration energy is the same, the vibration function form with larger amplitude and smaller frequency has the most obvious effect on reducing the average friction coefficient. But when the amplitude increases and the frequency decreases to a certain degree, the drag reduction effect stabilizes in a good state.

The evaluation of the amplitude reduction effect on friction reduction is shown in Fig. 12 and Fig. 13. As shown in Fig. 12, the larger the vibration force amplitude, the larger the average hook load and the fluctuation amplitude of hook load, indicating that large vibration force amplitude is the key factor improving the axial force transfer along the tubular string; as shown in Fig. 13, the amplitude increases, the average friction coefficient decreases, indicating that the large vibration force amplitude has a significant effect on reducing the average friction coefficient.

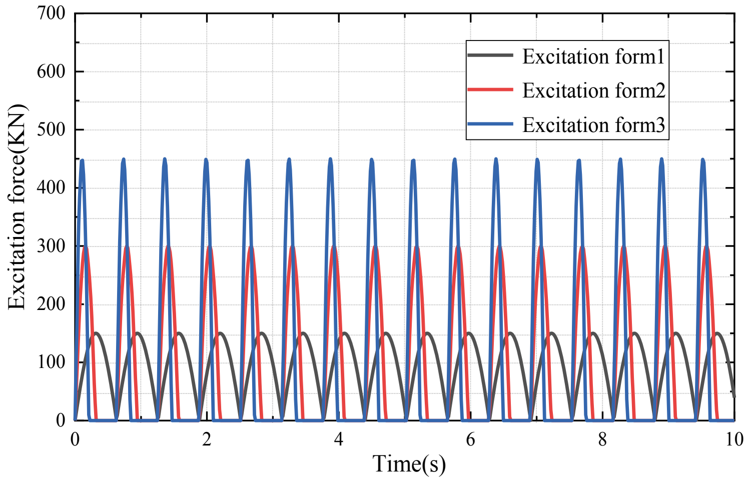

Fig. 9 The action form of the oscillator with excitation force

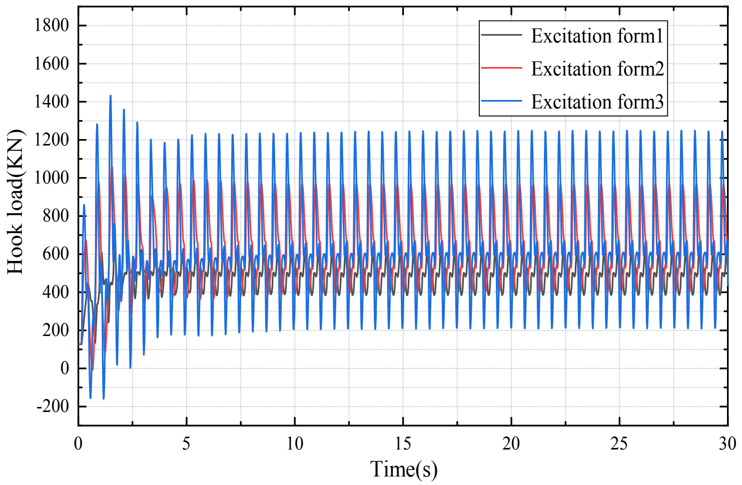

Fig. 10 Effect of the excitation force form on hook load

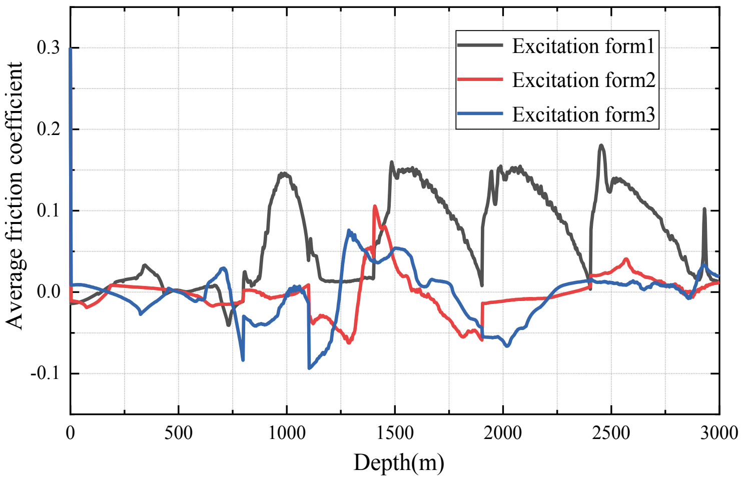

Fig. 11 Effect of the excitation force form on the average friction coefficient

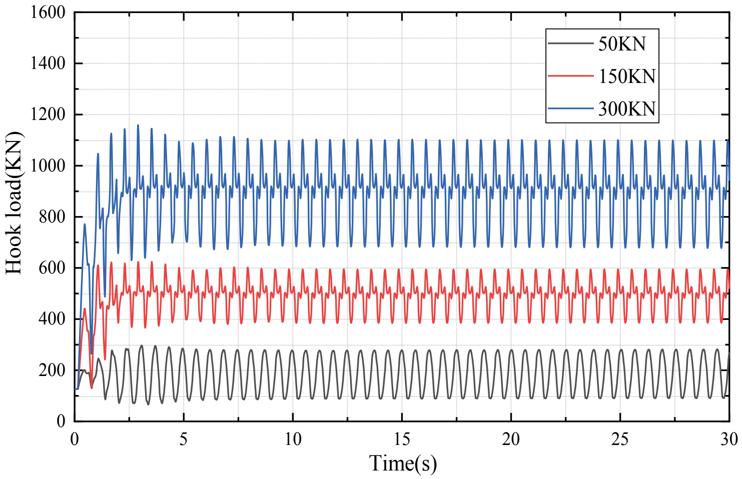

Fig. 12 Effect of vibration amplitude on hook load

However, if the amplitude is too large, the drillstring is subjected to alternating loads, which may lead to the risk of slippage of the drill pipe joint and fatigue damage of the drillstring.

The evaluation of the number of oscillators on friction reduction effect is shown in Fig. 14 and Fig. 15. As shown in Fig. 14, the more oscillators there are, the more violent the hook load fluctuation is, and the average hook load increases significantly, indicating that the number of oscillators is crucial to the transmission of

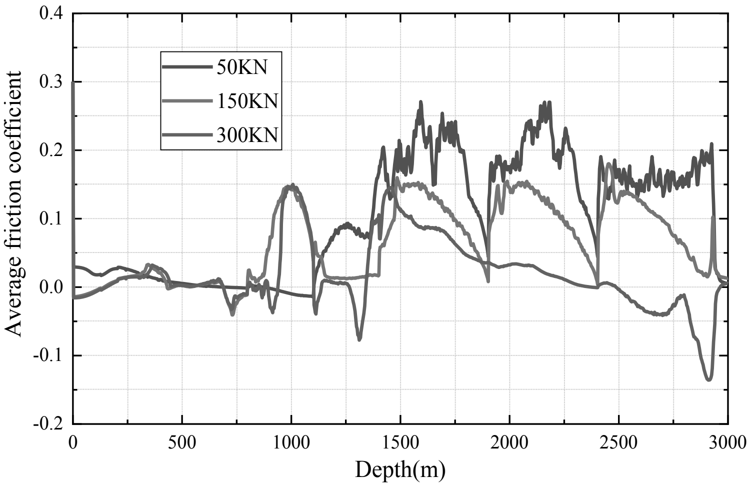

Fig. 13 Effect of vibration amplitude on average friction coefficient

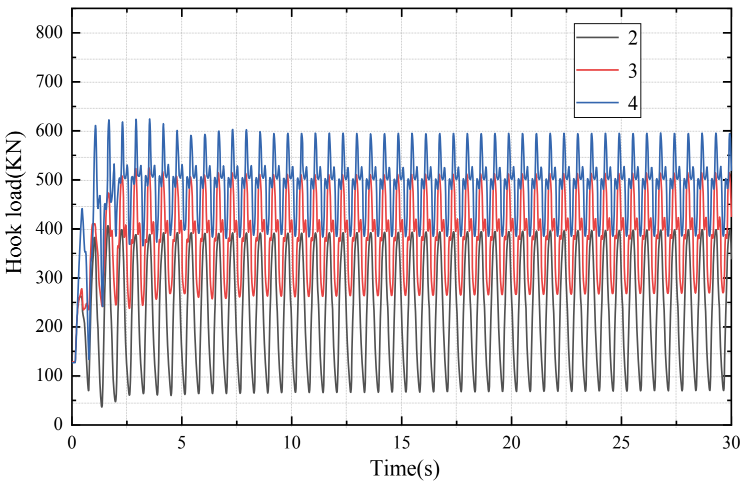

Fig. 14 Effect of the oscillator number on hook load

axial force of the drillstring. As shown in Fig. 15, the more oscillators, the lower the average friction coefficient of the drillstring, indicating that multiple oscillators can significantly decrease the average friction coefficient. But if too many oscillators are installed, may lead to large hydraulic power loss and high risk of tubular fatigue.

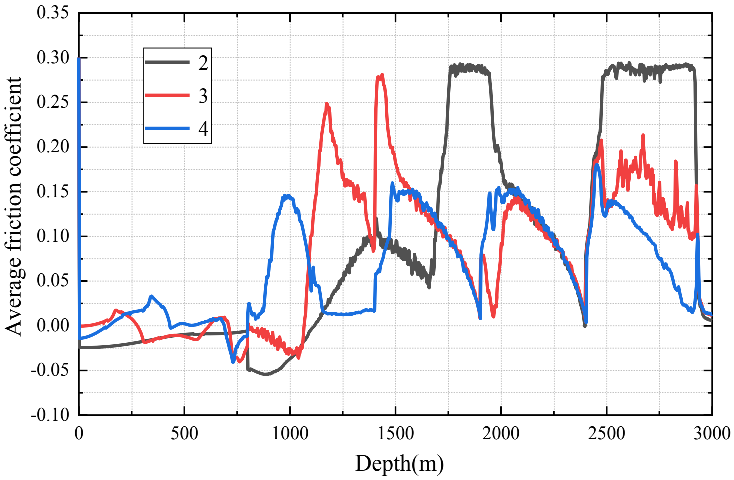

Fig. 15 Effect of the oscillator number on average friction coefficient

5 Conclusions

From above analyses we can see obtain the following conclusions:

(1) A dynamic model of drillstring with drag reduction oscillators is established based on the bilinear hysteretic restoring force model, which provides the basis for mechanical analysis and optimal design of drillstring with drag reduction oscillators.
(2) Comparing the results of drill string with oscillator and without oscillator, the displacement of the drill string with oscillator, the hook load fluctuation is more obvious, and the effect of reducing the average friction coefficient is significant, which indicates that installing oscillator in the drilling process is beneficial to the transmission of axial force.
(3) With the increase of drag reduction oscillator number and their vibration force amplitudes, the friction factor and axial friction are both decreased. However, too large vibration force amplitude will lead to drillstring fatigue and the hydraulic energy loss increasing with the drag reduction oscillator number. Therefore, there are optimal values predicted for vibration force amplitude and drag reduction oscillator number.

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