Generic Framework for Optimization of Local Dissemination in Wireless Networks

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Abstract. We present a generic framework to compute transmission schedules for a comprehensive set of well-known local dissemination problems in Wireless Networks. In our framework, we formulate the communication restrictions to overcome while solving those problems as a mathematical optimization program, where the objective function is to minimize the transmissions schedule length. The program is solved by standard methods which may yield partial solutions. So, the method is iterated until the solution is complete. The schedules obtained achieve the desired dissemination under the general affectance model of interference. We prove the correctness of our model and we evaluate its efficiency through simulations.

1 Introduction

The algorithmic problem of disseminating information in ad-hoc wireless communication networks (for instance, embedded in the Internet of Things) has been well studied. Depending on the field of application, challenges such as communication-link interference or network-node limitations yield different models, but always the desired dissemination is an instance of the following general problem. Some nodes are the source of one or many data packets, and the goal is to deliver those packets to some destination nodes, possibly through multiple hops. The specific meaning of *some* defines the multiple versions of dissemination. Yet, to the best of our knowledge, the various versions of dissemination have been mostly studied independently until now.

Given that *local* dissemination is the atomic part of any network-wide communication task, in this work, we focus on different variants of the problem of passing information to nodes that are within reach of the sources of such information in *one* hop. Even in the local context, depending on whether it is enough to receive from (resp. send to) one or more neighboring nodes, application requirements yield different types of dissemination. Moreover, transmission

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to all neighboring nodes may be required to happen simultaneously, or may be enough to do it along multiple rounds of communication. Some of these local problems are known in the literature as *Local-broadcast* [10] (transmit to all neighbors in one slot), *Wake-up* [6,8] (receive from at least one neighbor), and *Link-scheduling* [12,13] (transmit through an input set of links). We define formally all the local dissemination problems studied in Sect. 2.

Motivated by current data-link layer technologies, we focus on settings with one task per node, which restricts Link-scheduling to one instance of each originator node in the input set of links[1].

We adopt the notation used in the Link-scheduling literature for our node-centered tasks. The set of nodes such that the local dissemination task needs to be solved for each of them is called the set of ***requests***. Once the task is completed for a given node u, we say that u has been ***realized***, or ***pending*** otherwise. Our framework is generic also with respect to the set of requests. That is, our methods can be applied to solve the dissemination problems on sets of requests that are proper subsets of the network nodes. We notice that this is not a simple reduction of the problem to a smaller subgraph. While solving for a subset, all the other nodes still participate (and produce interference). The definitions in Sect. 2 reflect this generalization.

We do not assume any underlying communication infrastructure. That is, ***transmitters*** (i.e. source nodes) attempt to deliver a ***message*** (i.e. the information to deliver) by radio broadcast but, if two or more nodes transmit at the same time, mutual interference may prevent the ***receivers*** from getting the message. To take into account this phenomenon, we study local dissemination under a general model of interference called ***affectance***. As in [15,16] we parameterize affectance with a real value $0 \leqslant a(u,(v,w)) \leqslant 1$ that represents the affectance of each transmitter u on each link (v,w).

Affectance is a general model of interference in the sense that comprises other particular models studied before (cf. [16]). Moreover, previous models do not accurately represent the physical constraints in real-world deployments. For instance, in the Radio Network model [2] interference from non-neighboring nodes is neglected. Signal to Inteference-plus-Noise Ratio (SINR) [5,19] is a simplified model because other constraints, such as obstacles, are not taken into account. Yet, should the application require the use of Radio Network or SINR models, a simple instantiation of the affectance matrix allows the application of our generic framework, as we show below.

Customarily, we assume that time is slotted and we call the sequence of transmit/not-transmit states of the nodes a ***transmissions schedule***.

Related Work. Before [15,16], the generalized affectance model was introduced and used only in the context of one-hop communication, more specifically, to

[1] If a Link-scheduling input contains multiple instances of the same originating node, representing different links/packets outgoing from that node, we can simply create virtual copies of the node. We keep the assumption of different link originators for the easy of presentation of the generic framework.

link scheduling by Kesselheim [12,13]. He also showed how to use it for dynamic link scheduling in batches. This model was inspired by the affectance parameter introduced in the more restricted SINR setting [5]. They give a characteristic of a set of links, based on affectance, that influence the time of successful scheduling these links under the SINR model.

We note that interference measures for link scheduling cannot immediately be applied to local broadcast or wake up. Intuitively, the reason is that link scheduling is a link-oriented task whereas local broadcast and wake up are node-oriented. For instance, specific classes of power assignments (e.g. linear) are not well defined when a node has to transmit through many links simultaneously. So, later on, the interference characteristic was generalized in [15,16], called the maximum average tree-layer affectance, to be applicable to multi-hop communication tasks such as broadcast, together with another characteristic, called the maximum path affectance.

The Wake Up, Local Broadcast, Link Scheduling, and other local dissemination problems have been thoroughly studied under various models [6,8,10,12,13]. In the SINR model, single-hop instances of broadcast in the ad-hoc setting were studied by Jurdzinski et al. [7,9] and Daum et al. [3], who gave several deterministic and randomized algorithms working in time proportional to the diameter multiplied by a polylogarithmic factor of some model parameters. To the best of our knowledge, ours is the first formulation for these and other problems under the affectance model of interference.

Our Results and Approach. The main contribution of this work is the design of a *generic framework to compute transmission schedules for a comprehensive set of local dissemination problems.*

We start by formulating the communication restrictions to overcome in solving each of those problems in one mathematical optimization program, where the objective function is to minimize the transmissions schedule length. The formulation so obtained is an Integer Linear Program (ILP). The model obtained can be instantiated on each of the problems as needed by removing constraints. The local dissemination problems studied may require multiple rounds of communication for non-trivial network topologies. Note that our ILP entails an optimization over many rounds of communication, rather than a simple repeated application of one-round optimizations.

Even the seemingly simpler problem of deciding whether a given ILP with binary variables has a feasible solution, regardless of the objective function, is well known to be NP-complete (cf. 0–1 Integer Programming [11]). Since the optimization version asks to minimize the value of the objective function, subject to all the constraints, it is also NP-complete. So, to apply our method in networks of significant size, LP-relaxation and Randomized rounding [4] are applied. That is, we relax the domain of the variables of the ILP to real numbers in the $[0, 1]$ interval, and we round the values in the solution at random.

Due to rounding, the schedule obtained may not solve the dissemination problem under consideration for all the requests. Thus, we repeat the above steps iteratively updating the set of requests until all are realized. That is, our generic framework tolerates multiple applications of the ILP (if needed) reducing the set of requests in each iteration, but with all the network nodes participating in the schedule and, hence, introducing interference.

Our approach provides a versatile engineering solution for a variety of fundamental communication problems in one tool. Specifically, given the network topology and the affectance of nodes on links, one can solve the mathematical formulation adequately tailored for the problem of interest using our framework, and use the transmission schedules obtained. The method requires knowledge of all affectance values. These values may be obtained geometrically for the Radio Network, SINR or similar models of interference, or may be measured in the field in advance for the most general model. Moreover, affectance may be even obtained by the network nodes as in Conflict Maps (CMAP) [20], where nodes probe the network to build a map of conflicting transmissions.

Evaluation. We apply our methods on two network topologies with obstacles. One of them is based on a real-world floor plan of an office building, the other is a simple square grid with obstacles spaced at regular intervals. Physical measurements of interference capture all the signal-attenuation factors that are present in the specific physical medium where the network is deployed. Distance, reflection, scattering, and diffraction all have an impact on signal attenuation in an environment with obstacles. Customarily, we simulated those effects computing attenuation as the inverse of the distance raised to the path-loss exponent. We considered boundary cases of high- and low interference. The distance was computed assuming that the signal sorts the obstacles by going around them.

In our experimental evaluations, we observed that the number of iterations that our method must be applied to obtain a transmissions schedule is constant with respect to the network size, even if the set of requests is all the nodes. Given that the cases studied are natural instances of real-world network deployments, these results show the effectiveness of our methods in practice.

To the best of our knowledge, this is the first comprehensive tool to compute local dissemination schedules for Wireless Networks under a general model of interference.

Roadmap. In Sect. 2 we specify the details of the affectance and network models. In Sect. 3 we specify our generic framework, including the ILP formulation in Sect. 3.1 and the proof of correctness in Sect. 3.2. In Sect. 4 we present our simulation results.

2 Model and Problems

We model the Wireless Network topology as a graph $G = (V, E)$, where V is a set of n nodes and E is the set of communication links among such nodes. That is, for each pair of nodes $u, v \in V$, the ordered pair $(u, v) \in E$ if an only if v is able to receive a radio transmission from u directly (if there is no interference). Without loss of generality, we assume that time is slotted so that the length of one slot is enough to achieve such communication, provided that interference from other communications is low enough as defined later.

Following [16], we model interference as ***affectance*** of nodes on links. That is, we define a matrix A of size $|V| \times |E|$, where $a(u,(v,w))$ quantifies the interference that a transmitting node $u \in V$ introduces to the communication through link $(v,w) \in E$. We normalize affectance to the range $[0,1]$, that is, $0 \leqslant a(u,(v,w)) \leqslant 1$. The aim of the affectance matrix is to apply our framework to any interference scenario, given that the affectance values are part of the input. Hence, we do not fix any specific values even though, for instance, $a(u,(u,v))$ is naturally 0.

For convenience, we denote $a_{V'}((v,w))$ as the affectance of a set of nodes $V' \subseteq V$ on a link $(v,w) \in E$, and $a_{V'}(E')$ as the affectance of a set of nodes $V' \subseteq V$ on a set of links $E' \subseteq E$. In this model definition, we do not restrict affectance to a specific function, as long as its effect is additive, that is,

$$a_{V'}((v,w)) = \sum_{u \in V'} a(u,(v,w))$$
$$a_{V'}(E') = \sum_{(v,w) \in E'} a_{V'}((v,w)) \ .$$

Under the above affectance model, a ***successful transmission*** is defined as follows. For any pair of nodes $v, w \in V$ such that $(v,w) \in E$, a transmission from v is received at w in a time slot t if and only if: v transmits in time slot t, and $a_{\mathcal{T}(t)}((v,w)) < 1$, where $\mathcal{T}(t) \subseteq V$ is the set of nodes transmitting in time slot t. The event of a non-successful transmission, that is when the affectance is at least 1, is called a ***collision***. We assume that a node listening to the channel cannot distinguish between a collision and background noise present in the channel in absence of transmissions.

The affectance model defined subsumes any other interference model as long as the impact of interference is additive. For instance, in the Radio Network model where a node receives a transmission at a given time t if and only if exactly one of the neighbors of w is transmitting at time t, for $u, v, w \in V$ and $u \neq v$ the affectance matrix is the following:

$$A(u,(v,w)) = \begin{cases} 0 \text{ if } (u,w) \notin E \ , \\ 1 \text{ otherwise} \ . \end{cases}$$

On the other hand, consider the SINR with uniform power assignment model in [5] where a node receives a transmission if and only if the following holds for a parametric threshold β':

$$\frac{P/d_{uv}^{\alpha}}{N + \sum_{w \neq u} P/d_{wv}^{\alpha}} > \beta' \ .$$

In the latter, P is the transmission power level, N is the background noise, d_{uv} is the Euclidean distance between nodes u and v, α denotes the path-loss exponent.

Then, the affectance matrix is

$$A(u,(v,w)) = \frac{P/d_{uw}^{\alpha}}{P/(\beta' d_{vw}^{\alpha}) - N} .$$

The proof of the latter is a simple application of the SINR model definition and it is left to the full version of this work for brevity.

2.1 Local Dissemination Problems

In this work, we study the following local dissemination problems. Recall that, with respect to the usual definition of these problems in the literature, ours parameterize the problem on subsets of network nodes, called a set of requests.

- ***Wake Up***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the wake-up problem is solved at time slot t if, for every node $v \in R$, there exists some time slot $t' \leqslant t$ and some link $(u,v) \in E$ such that there was a successful transmission through (u,v) in t'. As a worst-case scenario definition, we assume that no nodes wake-up spontaneously.
- ***Link Scheduling***: Given a Wireless Network as defined, a set of requests $R \subseteq V$, and a set of link-requests $\mathcal{R}$ such that link $(u,v) \in \mathcal{R}$ if and only if $u \in R$, the link-scheduling problem is solved at time slot t if, for every node $u \in R$ and every link $(u,v) \in \mathcal{R}$, there exists some time slot $t' \leqslant t$ such that there was a successful transmission through (u,v) in t'.
- ***Local Broadcast***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the local-broadcast problem is solved at time slot t if, for every node $v \in R$, there exists some time slot $t' \leqslant t$ such that for every link $(v,w) \in E$ there was a successful transmission through (v,w) in t'. As a worst-case scenario definition, we assume that all links outgoing a node have to be scheduled in the same time slot.

We also consider extensions of the above known problems to the following generalizations.

- ***Receive-One***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the problem is solved at time slot t if, for every node $v \in R$, there exists some time slot $t' \leqslant t$ and some link $(u,v) \in E$ such that there was a successful transmission through (u,v) in t'. (Equivalent to wake-up.)
- ***Transmit-One***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the problem is solved at time slot t if, for every node $v \in R$, there exists some time slot $t' \leqslant t$ and some link $(v,u) \in E$ such that there was a successful transmission through (v,u) in t'.
- ***Receive-All***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the problem is solved at time slot t if, for every node $v \in R$, and for every link $(u,v) \in E$, there exists some time slot $t' \leqslant t$ such that there was a successful transmission through (u,v) in t'.

– ***Transmit-All***: Given a Wireless Network as defined and a set of requests $R \subseteq V$, the problem is solved at time slot t if, for every node $v \in R$, and for every link $(v, u) \in E$, there exists some time slot $t' \leqslant t$ such that there was a successful transmission through (v, u) in t'. (Equivalent to local broadcast if all links are scheduled in the same time slot.)

3 Generic Framework

In the following, a transmissions schedule is denoted as a matrix $X_V = (x_{ut})_{u \in V, t \in \mathbb{N}}$, where $x_{ut} \in \{0, 1\}$. We denote as $|X_V|$ the number of columns of X_V where $x_{ut} = 1$ for some $u \in V$, called the length of the schedule. Also, let $P \in$\{Receive-one, Receive-all, Transmit-one, Transmit-all, Local-broadcast, Link-scheduling\} be one of the problems defined in Sect. 2.

The generic framework (described in Algorithm 1) includes the application of our ILP (cf. Sect. 3.1) to the particular problem to solve. The variables in the ILP are restricted to be either 0 or 1. The problem of deciding whether a given ILP with binary variables has a feasible solution, regardless of the objective function, is known as 0-1 Integer Programming, and it is known to be NP-complete [11]. Hence, the optimization version, where the objective function is minimized subject to all the constraints, is also NP-complete. That is, unless P=NP, it would take an impractical amount of time to solve the ILP for networks of significant size.

```
input : network graph G = (V, E), affectance matrix A, set of requests R,
        problem P and, if P = Link-scheduling, set of link-requests 𝓡 such that
        ∀u ∈ R : ∃(u, v) ∈ E : (u, v) ∈ 𝓡 and
        ∀(u, v) ∈ 𝓡 : ∀w ∈ V : w ≠ v ⇒ (u, w) ∉ 𝓡
output: transmissions schedule X_V that solves P for R
while R ≠ ∅ do
    instantiate the ILP of Sect. 3.1 to compute X_V that solves P for R
    relax the integrality constraints to reals in [0, 1] (i.e. ILP → LP)
    solve the LP to obtain a matrix X'_V = (x'_ut)_{u∈V,t∈ℕ}, where x'_ut ∈ [0, 1]
    set x_ut ← 1 with probability x'_ut, or x_ut ← 0 otherwise
    verify the solution and remove all realized nodes from R
end
```

Algorithm 1: Generic Framework for Optimization of Local Dissemination in Wireless Networks.

To make it practical, our framework includes the application of standard approximation methods [4]. Specifically, LP-relaxation and rounding [18]. The solution of the LP can be obtained in polynomial time [14], but the solution values are reals in $[0, 1]$. To obtain integers in $\{0, 1\}$ as required by a transmissions schedule, we apply randomized rounding.

The integer assignments for the LP decision variables after rounding are a transmissions schedule, but due to rounding they may not preserve some of the constraints in the original ILP. In other words, the schedule may not solve the problem for all requests. An option would be to de-randomize the rounding step using the method of conditional probabilities, but given the number of constraints it would be computationally prohibitive. Thus, we include in our framework a final step when we verify the schedule obtained to identify the nodes that have been realized, and we iterate the method on the pending nodes. The total schedule length is the sum of the lengths of the sequence of schedules computed over this iterative process. Our simulations (cf. Sect. 4) show that in practice the number of iterations does not depend on the network size, and in fact it is very small.

In the following sections, we specify the details of our ILP formulation of local dissemination problems under affectance, and we prove its correctness.

3.1 Integer Linear Program Formulation

Definitions

- Indices:
  u, v, w: network nodes, $u, v, w \in V$.
  (v, w): directed network link, $(v, w) \in E$.
  t: time slot, $t \in [T]$.
- Input parameters:
  $a_u((v, w))$: affectance of node u on link (v, w), $0 \leqslant a_u((v, w)) \leqslant 1$.
  T: a large positive integer constant not less than the schedule length.
  $R \subseteq V$: set of requests.
  $\mathcal{R} \subseteq E$: set of link-requests, where $\forall u \in R : \exists (u, v) \in E : (u, v) \in \mathcal{R}$ and $\forall (u, v) \in \mathcal{R} : \forall w \in V : w \neq v \Rightarrow (u, w) \notin \mathcal{R}$.
- Decision variables:
  $x_{ut} = 1$ if node u transmits in time slot t, otherwise $x_{ut} = 0$.
- Auxiliary variables:
  $x_t = 1$ if some node transmits in time slot t, otherwise $x_t = 0$.
  $y_{vwt} = 1$ if total affectance on link (v, w) at time t is less than 1, otherwise $y_{vwt} = 0$.
  $z_{vwt} = 1$ if there is a successful transmission in link (v, w) at time t, otherwise $z_{vwt} = 0$.
  $z_{vt} = 1$ if there are successful transmissions in all links outgoing from v at time t, otherwise $z_{vt} = 0$.
  $\check{z}_{vt} = 1$ if there is a successful transmission in some link outgoing from v at time t, otherwise $\check{z}_{vt} = 0$.

Objective Function

The objective function is simply to minimize the length of the schedule. That is, to minimize the number of time slots when some node transmits.

Minimize

$$\sum_{t \in [T]} x_t$$

subject to the constraints that follow.

Transmission-Indicator Constraints

The following constraints restrict x_t to be an indicator of transmissions at time t. Given that x_t is restricted to be binary, Constraint 1 restricts $x_t = 0$ if $\sum_{u \in V} x_{ut} = 0$, and Constraint 2 restricts $x_t = 1$ if $\sum_{u \in V} x_{ut} > 0$:

$$\forall t \in [T] : x_t \leqslant \sum_{u \in V} x_{ut} \tag{1}$$

$$\forall t \in [T] : n x_t \geqslant \sum_{u \in V} x_{ut} \ . \tag{2}$$

Affectance-Indicator Constraints

The following constraints restrict y_{vwt} to be an indicator of "low" affectance on link (v, w) at time t. Given that y_{vwt} is restricted to be binary, Constraint 3 restricts $y_{vwt} = 1$ if $\sum_{u \in V} a_u((v, w)) x_{ut} < 1$, and Constraint 4 restricts $y_{vwt} = 0$ if $\sum_{u \in V} a_u((v, w)) x_{ut} \geqslant 1$:

$$\forall (v, w) \in E : \forall t \in [T] : \sum_{u \in V} a_u((v, w)) x_{ut} - 1 \geqslant -y_{vwt} \tag{3}$$

$$\forall (v, w) \in E : \forall t \in [T] : \sum_{u \in V} a_u((v, w)) x_{ut} - 1 < (n - 1)(1 - y_{vwt}) \ . \tag{4}$$

1-Link Successful-Transmission Constraints

The following constraints restrict z_{vwt} to be an indicator of successful transmission in link (v, w) at time t. Given that z_{vwt} is restricted to be binary, Constraint 5 restricts $x_{vt} = 1$ if $z_{vwt} = 1$, Constraint 6 restricts $y_{vwt} = 1$ if $z_{vwt} = 1$, and Constraint 7 restricts that it must be $y_{vwt} = 0$ or $x_{vt} = 0$ if $z_{vwt} = 0$:

$$\forall (v, w) \in E : \forall t \in [T] : z_{vwt} \leqslant x_{vt} \tag{5}$$

$$\forall (v, w) \in E : \forall t \in [T] : z_{vwt} \leqslant y_{vwt} \tag{6}$$

$$\forall (v, w) \in E : \forall t \in [T] : z_{vwt} \geqslant y_{vwt} + x_{vt} - 1 \ . \tag{7}$$

All-Outlinks Successful-Transmission Constraints
The following constraints restrict z_{vt} to be an indicator of successful transmission in *all* links outgoing from v at time t. Given that z_{vt} is restricted to be binary, Constraint 8 restricts $z_{vt} = 1$ if $\sum_{w \in out(v)} z_{vwt} = |out(v)|$, and Constraint 9 restricts $z_{vt} = 0$ if $\sum_{w \in out(v)} z_{vwt} < |out(v)|$:

$$\forall v \in V : \forall t \in [T] : (1 - z_{vt}) \leqslant |out(v)| - \sum_{w \in out(v)} z_{vwt} \tag{8}$$

$$\forall v \in V : \forall t \in [T] : |out(v)|(1 - z_{vt}) \geqslant |out(v)| - \sum_{w \in out(v)} z_{vwt} \ . \tag{9}$$

Some-Outlink Successful-Transmission Constraints
The following constraints restrict $\check{z}_{vt}$ to be an indicator of successful transmission in *some* link outgoing from v at time t. Given that $\check{z}_{vt}$ is restricted to be binary, Constraint 10 restricts $\check{z}_{vt} = 1$ if $\sum_{w \in out(v)} z_{vwt} > 0$, and Constraint 11 restricts $\check{z}_{vt} = 0$ if $\sum_{w \in out(v)} z_{vwt} = 0$:

$$\forall v \in V : \forall t \in [T] : |out(v)|\check{z}_{vt} \geqslant \sum_{w \in out(v)} z_{vwt} \tag{10}$$

$$\forall v \in V : \forall t \in [T] : \check{z}_{vt} \leqslant \sum_{w \in out(v)} z_{vwt} \ . \tag{11}$$

Integrality and Range Constraints

$$\forall v \in V : \forall t \in [T] : x_{vt} \in \{0, 1\} \tag{12}$$

$$\forall t \in [T] : x_t \in \{0, 1\} \tag{13}$$

$$\forall (v, w) \in E : \forall t \in [T] : y_{vwt} \in \{0, 1\} \tag{14}$$

$$\forall (v, w) \in E : \forall t \in [T] : z_{vwt} \in \{0, 1\} \tag{15}$$

$$\forall v \in V : \forall t \in [T] : z_{vt} \in \{0, 1\} \tag{16}$$

$$\forall v \in V : \forall t \in [T] : \check{z}_{vt} \in \{0, 1\} \ . \tag{17}$$

Problem-Specific Constraints

– The model is completed with one of the constraints that follow, depending on the specific problem studied.
– Receive-one: there is at least one time slot when w receives, that is:

$$\forall w \in R : \sum_{t \in [T]} \sum_{v \in in(w)} z_{vwt} \geqslant 1. \tag{18}$$

– Receive-all: there is at least one time slot when w receives from v:

$$\forall w \in R : \forall v \in in(w) : \sum_{t \in [T]} z_{vwt} \geqslant 1 \ . \tag{19}$$

– Transmit-one: there is at least one time slot when some neighbor of v receives from v:

$$\forall v \in R : \sum_{t \in [T]} \sum_{w \in out(v)} z_{vwt} \geqslant 1 \ . \tag{20}$$

– Transmit-all: there is at least one time slot when w receives from v:

$$\forall v \in R : \forall w \in out(v) : \sum_{t \in [T]} z_{vwt} \geqslant 1 \ . \tag{21}$$

– Local-broadcast: there is at least one time slot when all out-neighbors of v receive:

$$\forall v \in R : \sum_{t \in [T]} z_{vt} \geqslant 1 \ . \tag{22}$$

– Link-scheduling there is at least one time slot when w receives from v:

$$\forall v \in R : \forall (v, w) \in \mathcal{R} : \sum_{t \in [T]} z_{vwt} \geqslant 1 \ . \tag{23}$$

3.2 Correctness

Lemma 1. *The indicator variables in the Integer Program of Sect. 3.1 are well defined.*

Proof. We prove that each indicator variable is 1 if and only if the corresponding event occurred. For each new variable, we use that previous variables are well defined.

– x_{vt}, for $v \in V$ and $t \in [T]$: it is by definition $x_{vt} = 1$ if and only if node v transmits in time slot t.
– x_t, for $t \in [T]$: indicates that node v transmits at time t.

$$\exists u \in V : x_{ut} = 1 \Rightarrow \sum_{u \in V} x_{ut} \geqslant 1, \text{ using Constraint 2,}$$
$$\sum_{u \in V} x_{ut} \geqslant 1 \wedge n x_t \geqslant \sum_{u \in V} x_{ut} \Rightarrow n x_t \geqslant 1, \text{ using Constraint 13,}$$
$$n x_t \geqslant 1 \wedge x_t \in \{0, 1\} \Rightarrow x_t = 1 \ .$$

$$\forall u \in V : x_{ut} = 0 \Rightarrow \sum_{u \in V} x_{ut} = 0, \text{ using Constraint 1,}$$
$$\sum_{u \in V} x_{ut} = 0 \wedge x_t \leqslant \sum_{u \in V} x_{ut} \Rightarrow x_t \leqslant 0, \text{ using Constraint 13,}$$
$$x_t \leqslant 0 \wedge x_t \in \{0, 1\} \Rightarrow x_t = 0 \ .$$

– y_{vwt}, for $(v, w) \in E$ and $t \in [T]$: indicates low affectance on link (v, w) at time t. Using Constraints 3 and 14 we get:

$$\sum_{u \in V} a_u((v,w))x_{ut} < 1 \wedge \sum_{u \in V} a_u((v,w))x_{ut} - 1 \geqslant -y_{vwt} \Rightarrow 1 - y_{vwt} < 1,$$
$$1 - y_{vwt} < 1 \wedge y_{vwt} \in \{0,1\} \Rightarrow y_{vwt} = 1 \ .$$

Using Constraints 4 and 14 we obtain:

$$\sum_{u \in V} a_u((v,w))x_{ut} \geqslant 1 \wedge$$
$$\sum_{u \in V} a_u((v,w))x_{ut} - 1 < (n-1)(1 - y_{vwt}) \Rightarrow (n-1)(1-y_{vwt}) > 0$$
$$(n-1)(1-y_{vwt}) > 0 \wedge y_{vwt} \in \{0,1\} \Rightarrow y_{vwt} = 0.$$

– z_{vwt}, for $(v, w) \in E$ and $t \in [T]$: indicates a successful transmission in link (v, w) at time t. That is, it indicates whether the affectance on (v, w) is low and v transmits. Using Constraints 7 and 15, we get

$$x_{vt} = 1 \wedge y_{vwt} = 1 \wedge z_{vwt} \geqslant y_{vwt} + x_{vt} - 1 \Rightarrow z_{vwt} \geqslant 1$$
$$z_{vwt} \geqslant 1 \wedge z_{vwt} \in \{0,1\} \Rightarrow z_{vwt} = 1 \ .$$

On the other hand, using Constraints 5 and 15, we have

$$x_{vt} = 0 \wedge z_{vwt} \leqslant x_{vt} \wedge z_{vwt} \in \{0,1\} \Rightarrow z_{vwt} = 0 \ .$$

And using Constraints 6 and 15,

$$y_{vwt} = 0 \wedge z_{vwt} \leqslant y_{vwt} \wedge z_{vwt} \in \{0,1\} \Rightarrow z_{vwt} = 0 \ .$$

– z_{vt}, for $v \in V$ and $t \in [T]$: indicates a successful transmission in all links outgoing from v at time t. Using Constraints 8 and 16, we obtain

$$\sum_{w \in out(v)} z_{vwt} = |out(v)| \wedge (1 - z_{vt}) \leqslant |out(v)| - \sum_{w \in out(v)} z_{vwt} \Rightarrow (1 - z_{vt}) = 0$$
$$(1 - z_{vt}) = 0 \wedge z_{vt} \in \{0,1\} \Rightarrow z_{vt} = 1 \ .$$

Using Constraints 9 and 16, we get

$$\sum_{w \in out(v)} z_{vwt} < |out(v)| \wedge$$
$$|out(v)|(1 - z_{vt}) \geqslant |out(v)| - \sum_{w \in out(v)} z_{vwt} \Rightarrow |out(v)|(1 - z_{vt}) > 0$$
$$|out(v)|(1 - z_{vt}) > 0 \wedge z_{vt} \in \{0,1\} \Rightarrow z_{vt} = 0.$$

– $\check{z}_{vt}$, for $v \in V$ and $t \in [T]$: indicates a successful transmission in some link outgoing from v at time t. Using Constraints 10 and 17, we obtain

$$\sum_{w \in out(v)} z_{vwt} > 0 \wedge |out(v)|\check{z}_{vt} \geqslant \sum_{w \in out(v)} z_{vwt} \Rightarrow |out(v)|\check{z}_{vt} > 0$$
$$|out(v)|\check{z}_{vt} > 0 \wedge \check{z}_{vt} \in \{0,1\} \Rightarrow \check{z}_{vt} = 1 \ .$$

Using Constraints 11 and 17, we get

$$\sum_{w \in out(v)} z_{vwt} = 0 \wedge \check{z}_{vt} \leqslant \sum_{w \in out(v)} z_{vwt} \Rightarrow \check{z}_{vt} \leqslant 0$$

$$\check{z}_{vt} \leqslant 0 \wedge \check{z}_{vt} \in \{0,1\} \Rightarrow \check{z}_{vt} = 0 \ .$$

□

Theorem 1. *The Integer Program of Sect. 3.1 is correct.*

Proof. To prove the correctness of our formulation it is enough to prove that, for each of the communication problems studied, if the corresponding constraint is true the problem is solved, and viceversa. We include such proof for the Receive-one problem. For the other problems the proof is similar.

Constraint 18 is true if, for each node $w \in R$, there is at least one time slot $t \in T$ and one node $v \in in(w)$ for which the indicator variable $z_{vwt} = 1$. By Lemma 1, if $z_{vwt} = 1$ there is at least one time slot when w receives, as required by the Receive-one problem.

On the other hand, the Receive-one problem is solved when, for each node $w' \in R$, there is at least one time slot t' when node w' receives successfully from at least one of its neighbors. Consider one of those neighbors $v' \in in(w')$. In that case, by Lemma 1 we know that $z_{v'w't'} = 1$. Hence, Constraint 18 is true. □

4 Simulations

In this section we present applications of our generic framework to network deployments. We study two network topologies including obstacles: a grid and a layer network. We note that the cases studied are an illustration of our methods applied to networks that frequently appear in real world deployments, rather than examples of worst-case scenarios.

As a layer-network, i.e. a bipartite graph on a partition transmitters-receivers, we used as a model of obstacles the floorplan of the School of Computer Science and Information Systems at Pace University (see Fig. 1). We considered nodes installed in the intersections of each square of four ceiling panels. We focus on one layer of this network going across various offices. For simplicity, to evaluate performance as n grows, we replicated the same office multiple times in a layer.

The walls of these offices have a metallic structure. Hence, each office behaves as a Faraday cage blocking radio transmissions (specially millimeter wave). Consequently, most of the radio waves propagate through doors (which are not metallic). We fixed the radio transmission power to be large enough to reach five grid cells, so that transmissions from layer to layer are possible. Given the offices dimensions, transmitters within an office are connected to all receivers. On the other hand, the interference to other offices in the same layer is approximated by adding ten grid cells for each office of distance. The resulting topology can be seen in Fig. 2.

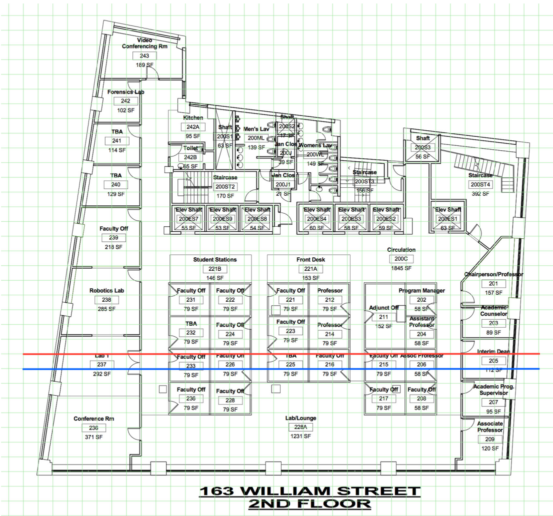


Fig. 1. A layer of the network grid.

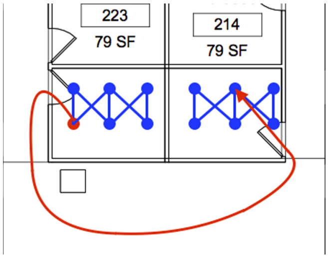


Fig. 2. Affectance example.

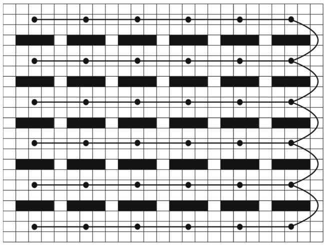


Fig. 3. Square grid.

In the second case studied, nodes are deployed in a square grid, but with a more intricate placement of obstacles among them, as shown in Fig. 3. In this case the range of communication is assumed to be 4 grid cells (measured in Manhattan distance for simplicity) so that connected nodes form paths, which we assume to be connected in one end by some other means.

Physical measurements of interference capture all the signal-attenuation factors present in the specific physical medium. In an environment with obstacles, those factors include distance, reflection, scattering, diffraction, etc. Customarily for synthetic inputs, we computed attenuation as the inverse of the distance between transmitter and receiver raised to the path-loss exponent α. To evaluate low- and high-interference scenarios, we considered boundary cases of $\alpha = 6$ and $\alpha = 2$ respectively [17].

The separation between transmitter and receiver was measured in Manhattan distance, assuming that the signal sorts the obstacles by going around them. Then, assuming a uniform transmission-power assignment, the affectance of each node u on each link (v, w) was computed as the ratio of the attenuation between u and w over the attenuation on (v, w).

For the network topologies described, and for various values of n, we applied our generic framework instantiated in each of the six local dissemination problems studied, using as a worst case scenario $R = V$. We measured the length of the schedules obtained and the number of iterations our framework needed to obtain the solution for all nodes. To solve the corresponding LP's we used IBM ILOG CPLEX Optimization Studio V12.8.0 in Java, on the Pace University

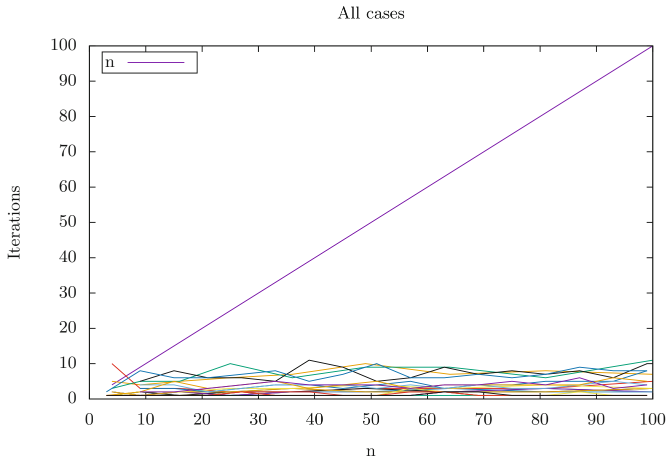


Fig. 4. Framework iterations for all cases studied.

Seidenberg School of CSIS Dell HPC cluster. (Head node with dual 12core Xeon processors, 192 GB memory, and 8 × 2.4TB HDs, and two GPU Compute nodes each with dual 12core Xeon processors, 384 GB memory, and 3 × NVIDIA Tesla V100 32 G Passive GPUs, with a Red Hat Enterprise Linux environment.)

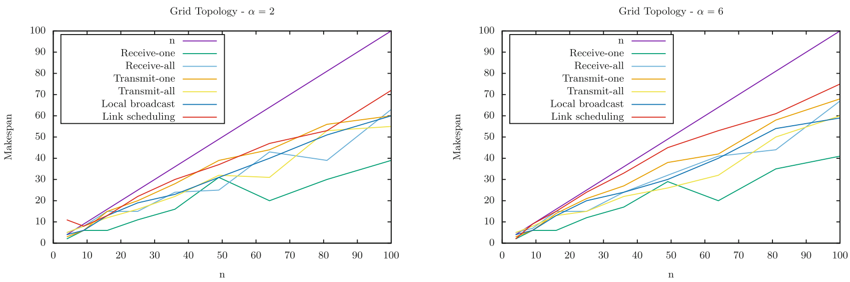
The results of our evaluations are discussed in the following section.

5 Discusion of Results and Conclusions

In this work, we present a generic framework to compute transmission schedules to solve a comprehensive set of local dissemination problems frequently studied for Wireless Networks. Our framework provides an engineering solution with theoretical guarantees of correctness. Based on measurements of interference in the specific deployment area, one can obtain transmission schedules for any of the problems studied with one tool.

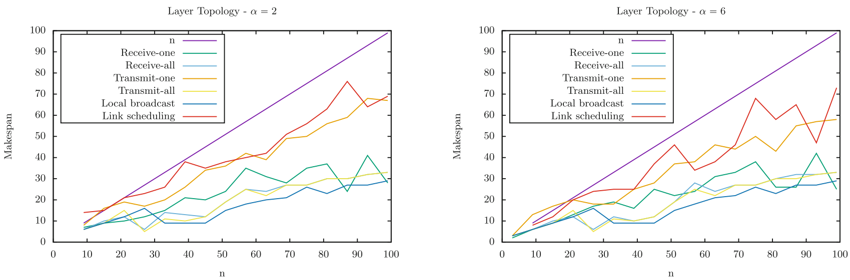
The practicality of our framework is shown by evaluating the number of iterations of LP-solver application until the solution is complete. It can be seen in Fig. 4 that the number of iterations remains constant when the network size grows, for all problems, topologies, and path-loss exponents studied, even though the set of requests used for the simulations was $R = V$. The length of the schedules obtained for the variety of problems studied, as the network size grows, under low- and high-interference, for two typical network topologies, and in a typical setting with obstacles are shown in Figs. 5 and 6.

To the best of our knowledge, this is the first comprehensive tool to compute local dissemination schedules for Wireless Networks under a general model of interference. A possible improvement, suggested by one of the reviewers and an interesting open direction, relates to the IP formulation - aimed to make it simpler and algorithmically more tractable.



(a) $\alpha = 2$. (b) $\alpha = 6$.

Fig. 5. Schedule length for grid topology.



(a) $\alpha = 2$. (b) $\alpha = 6$.

Fig. 6. Schedule length for layer topology.

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