The Role of Logic in Science

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Logic is the science of proof. But, proof of what? "…since people have tried to prove obvious propositions they have found that many of them are false", noted Russell astutely (Russell 1963, 61). Historical answers to our question are, therefore, quite diverse, despite what anachronistic studies of logic may imply. They comprise the long and complex story of the development of logic from Aristotle to Gödel , reflecting the time-old rear action battle fought by classic epistemologists against skepticism. Popper's reconstruction of logic as the theory of refutation, has helped us realize that the skeptic has effectively won this historical battle, and that securing empirical knowledge by logical means alone (be they inductive, analytic or transcendental) is a futile effort. And it helped us move forward to the cleaner concept of logic as expressing the most basic methodological procedures accepted in science. As such, disagreements between logicians are metaphysical, or heuristic controversies about the proper (fruitful and convenient) methodological rules for conducting science.

Even before Aristotle's revolutionary invention of the logical variable philosophers did their best to reason convincingly and even quite systematically, and of course they sometimes succeeded. Parmenides used what we nowadays would call analytical reasoning in an attempt to prove his incredible theory, and he titled it "*piston logon*" (proven assertion). Zeno, his student, has provided us with rough yet brilliant versions of *reductio ad absurdum* of the opposite theory (the Democritian claim that the cosmos contains a void, and that it, therefore, allows for motion and time). Famously, and quite brilliantly, the sophist Gorgias ridicules the pretentiousness of their argumentation style by demonstrating that it can easily be applied to Parmenides' own theory, thus yielding its refutation as well as the refutation of its negation (an exercise that Plato himself repeats in a enigmatic manner, without

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discussing its meaning, in his "Parmenides"). Gorgias thus exposed ancient epistemology's dead-end. Socrates heroically refused to allow such defeatism. Although he typically admits ignorance in matters epistemological he nevertheless recommends relentlessly attempting to refute the knowledge claims of his interlocutors, declaring successful refutations to be genuine epistemic achievements. Having no logical variables to his disposal he utilizes a repetitive style known as Socratic *elenchus* which, if it was abstracted and generalized, would have amounted to what we nowadays call *modus tollens*. Finally, participants of Plato's later dialogues (notably "The Sophist" and "The Statesman") rekindle Eleatic pretentiousness, as they seek to establish definitions of various subject matters by a procedure of logical analysis known as *diaeresis*. If this procedure were to be generalized, it would have provided us with the following logical rule: x is either a or b; it is not a, therefore it is b.

In this lively environment of the pre-history of logic brilliant philosophers search and test various argumentation styles and semi-explicit debate procedures that will later on be united under the study of Logic. Three very distinct approaches stand out here, and they are worth mentioning because they set the stage for all future controversies about the status of logic. The first, which we can call "sophistic" and even "relativistic", declares all argumentation procedures epistemically barren albeit (sometimes) rhetorically effective: they are, as Gorgias had claimed, mere manifestations of rhetorical wizardry, nothing more. The second approach, which we can call "reluctantly skeptic", declares logical enquires into the nature of reality a worthy heroic effort. It stresses that refutation of a theory is a genuine epistemological achievement, as it validly demonstrates its falsity. But it forbids us to derive from it the absolute truth of its negation. For a refutation too can, one day, be refuted. As Socrates has put it: the oracle declaring him the wisest of all men must be right, but only because he (Socrates) is aware of his own ignorance. The third approach towards these argumentation procedures should be titled "dogmatic". It seeks to establish incontestable foundations for science, and then validly reason in an attempt to derive the rest of knowledge from them. This attitude is implied by the Eleatics and by the mature Plato, and was adopted, developed and systematized by Aristotle . Its influence upon the history of logic cannot be underestimated, as it shaped its relationship with science and metaphysics for over two millennia, until Frege and Russell, Tarski and Popper had altogether changed our view of the matter, ridding logic from the impossible burden of proving empirical science, returning us to a more Socratic point of view.

For in Aristotle it is clear that dialectics is not merely a mode of argumentation that allows one to carefully formulate informative theoretical conjectures, but also a mean to establish them as uncontestable basic truths, aka 'essential definitions', which will function as premises for scientific (apodeictic) syllogisms. This applies equally for inductive syllogisms, in Aristotle, for Aristotle regards induction as a type of dialectical argument (*An. Pos. 71a4*). Now, since today we take it as a matter of course that this task—logically proving informative theoretical knowledge—is unfeasible, we must be very careful when inspecting the situation: we must not allow our utmost respect for Aristotle to blur our realization that he has committed what nowadays we regard as a highly influential philosophical error. Indeed respect

for Aristotle is typically so great that it is remarkably rare to find in the learned literature attempts to reconstruct his error. I have endeavored to analyze it in great detail (Bar-Am 2008). Let me try and briefly sum up things for you here.

Both Plato and Aristotle were greatly impressed by the fact that seemingly immediate and incontestable observations presuppose, or at least seem to presuppose, a great deal of theoretical knowledge of universals and their taxonomical hierarchies, which we perhaps were unaware of at first, but that we can easily become aware of by critical, logical analysis, by dialectics and/or induction (*An. Pos. 71a6*). For example, the observation "this rose is red", which seems so immediate as to be incontestable when facing a red rose, seems to presuppose the taxonomical knowledge "Red is a Color", which subtly conflates empirical knowledge (the existence of red things in our cosmos is of course a contingent fact) and an analytical appearance (clearly, red is a color, as every English speaker knows). Similarly, (or, more accurately, misleadingly similarly) the famous dialectical inquiry performed by Socrates and the slave in Meno, based on a drawing in the sand, leads the slave (and us readers) to recognize that we were implicitly all along in possession of an apriori notion of a (semi-general) case of the Pythagorean theorem.

The general epistemological idea behind these examples (which is shared by Plato and Aristotle) is that there exists a grand matrix of universals, of natural kinds, that orders them according to their proper taxonomical relations, which is somehow presupposed by our observations, and which is obtainable, exposable and extractable, by logical analysis, by dialectics and induction. This grand matrix of universals, Aristotle argues, is empirical science in its entirety. Logic, for him, is therefore not merely the method of expressing the grand matrix by means of apodeictic syllogisms, but also the method of exposing and establishing it by means of dialectical and inductive syllogisms. This point is central: the isomorphism between the method of expressing the taxonomical relations that science comprises (apodeictic syllogisms) and the method of exposing and establishing them as essential definition (dialectical and inductive syllogisms) is the heart of Aristotle's epistemology, indeed it is so central to it, that it features the opening remarks of his Prior Analytics (*An. Pr. 24a23–24b13*), the opening remarks of his Posterior Analytics (*An. Pos. 71a4-9*), as well as the opening remarks to his Topics (*Top. 100a25–100b24*). It is the birth of the myth that the skeptic can be answered by logical means alone, for we can somehow, to use Aristotle's own words, "prove the universal from the self evident nature of the particular" (*An. Pos. 71a6*).

Aristotle's theory that the grand matrix of science is extractable by logical means alone, (from our observations by induction, and from our critical inquiries by dialectics, that is by a logical analysis of our concepts) is perhaps the most influential epistemology ever formulated. It is also very vague. For Aristotle never made it clear how exactly the process is to be performed and why it guarantees the obtainment of empirical truth. Clearly, induction may lead us astray: we speculated that all swans are white until Tasmania was discovered. And just as clearly, if you and I conclude a dialectical conceptual inquiry with the conclusion that absolute speed cannot exist (as Leibniz had done), this does not make it into an empirical fact, as Einstein had

shown. Induction and logical analysis may, perhaps, be excellent tools for formulating conjectures, but they are no tools for proving them. And so, the greatest minds in the history of philosophy have endeavoured to break up and reconstruct the missing pieces in Aristotle's claim that science can be proven on logical reasoning alone, or at most, on logical reasoning and uncontestable immediate experiences.

Consider Leibniz for example. Greatly impressed by the Aristotelian idea that logical analysis of concepts may lead to the intuitive recognition of essential definitions, he sharpened Aristotle's rough notion of proof in an attempt to improve and complete Aristotle's program. He explicitly suggested that, essentially, all empirical truths are analytic, and that demonstrating that they are analytic is tantamount to proving them. Perhaps, he added, we find it difficult to currently realize all this because our current conceptual framework is not yet isomorphic to the grand matrix of being, the cosmic taxonomy of natural kinds, but should we succeed in constructing the perfect language, a semantic framework that would perfectly correspond to the grand matrix of universals, we would be able to prove that all empirical truths are analytic. He left us dozens of drafts that are supposed to detail how such a language would look like. But of course he never constructed one.

Or consider Kant. Overwhelmed by Hume's (rather trivial) observation that inductive arguments are not really isomorphic to apodeictic syllogisms, indeed that, strictly speaking, they are invalid inferences, he was nevertheless greatly impressed by Aristotle's statement that the observation of particulars presupposes a great deal of abstract theoretical knowledge. He thus endeavored to extract this theoretical knowledge from our experiences, a process that he titled "transcendental logic", and which he never actually describes or details. Although the bombastic name may somewhat intimidate us, we should note here in passing that there is nothing particularly transcendental about such an endeavor: the inference "x is impossible unless y is true; x is possible; hence y is true" is a rather basic case of *modus tollens*. Its premises are nothing more than empirical conjecture, as Salomon Maimon had observed.

And consider George Boole . He formulated the first extensional logical system, thus destroying by fiat the grand Aristotelian plan to logically establish empirical theoretical knowledge. Still, he tried to utilize his brilliant new logic to secure empirical science by probability and induction. However, in Boole it is already very clear that knowledge of the various probabilities used in inductive inference is an extralogical, empirical matter, and hence that it is not logical reasoning alone that secures the foundations of science.

But it is Frege, as Agassi observes (Agassi 2018), who deserves credit as the first modern logician proper: he is so in virtue of the fact that he was the first to have abandoned altogether the Aristotelian program—securing empirical science—replacing it with the far more modest one of securing arithmetic by logical means alone. Then Russell discovered his paradox, and Gödel had sealed matters for this new program too, by demonstrating that, strictly speaking, it cannot be performed.

The bankruptcy of the two justificationist programs brings us back to the crucial question of the desired place of logic in science. What is its role there? What service does it provide to the scientist? Today it is hard to fathom, but until Popper arrived at the scene logic was solely a tool in the hands of dogmatists: a tool for establishing some truth (empirical or arithmetic), for justifying it. However, as Popper insisted, the only way to learn something about a universal empirical statement from a singular statement, is by observing that the latter refutes the former. Thus logic becomes refutation theory. As refutation theory logic finally returns to the Socratic role of methodology proper, methodology without guarantees for success, methodology that is free of its historical epistemological burdens, and indeed, as Bartley insisted, methodology that can one day be modified, at least in principle.

Consequently, in Popper, for the first time we find a view of science that is anti-foundationalist. It does not proceed from first principles (from essential definitions, or from immediate experiences), but rather from problems, that is from the challenge of explaining inconsistencies between our theories and our experiences. Indeed, as Popper had observed (echoing Aristotle) observation reports are theory-laden, and so (contra Aristotle) the theory which they presuppose, even when it is a priori, is conjectural too. This also freed logicians from the impossible burden of justifying induction: it is not merely that the task is now openly admitted as essentially hopeless, as Hume has already done, but it becomes essentially uninteresting, as science has no use for it.

Popper also famously used logic that is the theory of refutations, as a tool for demarcating science. This use was problematic since it sometimes gave the misleading impression that Popper intended to declare some theories (e.g. Newtonian physics) as more easily refutable than others (e.g. Adler's psychology) (Popper [1959] 2005). Clearly, refutability is not a property of theories. Rather critical mindedness is a property of speakers: the more critical minded we are towards a given theory, the more scientific it becomes. This way, logic is not a tool for demarcating science from pseudo-science, but rather an aid in distinguishing between the rational and the dogmatic.

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