Chapter 2
Computing in Early Civilizations

Key Topics
Babylonian mathematics
Egyptian civilization
Greek and Roman civilization
Counting and numbers
Solving practical problems
Syllogistic logic
Algorithms
Early ciphers

2.1 Introduction

The pace of change and innovation in western society over the last 20–30 years has been phenomenal. There is a proliferation of sophisticated technology such as computers, smart phones, the Internet, the World Wide Web, social media, and so on. Software is pervasive and is an integral part of automobiles, airplanes, televisions, and mobile communication. The pace of change is relentless, and communication today is instantaneous with video calls, text messaging, mobile phones, and e-mail. Today people may book flights over the World Wide Web as well as keeping in contact with family members in any part of the world. In previous generations, communication often involved writing letters that took months to reach the recipient. Communication improved with the telegraph and the telephone in the late nineteenth century, but today it is instantaneous.

The new technologies have led to major benefits[1] to society and to improvements in the standard of living for many citizens in the western world. It has also reduced the necessity for humans to perform some of the more tedious or dangerous manual tasks, as computers may now automate many of these. The increase in productivity due to the more advanced computerized technologies has allowed humans, at least in theory, the freedom to engage in more creative and rewarding tasks.

[1] The new technologies are of major benefit to society, but it is essential to move toward more sustainable development to ensure the long-term survival of the planet. This involves finding technological and other solutions to reduce greenhouse gas emissions as well as moving to a carbon neutral way of life. The environmental crisis is a major challenge for the twenty-first century.

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Societies have evolved over millennia and some early societies had a limited vocabulary for counting: for example, “one, two, three, many” is associated with a number of primitive societies, and indicates limited numerate and scientific abilities. It suggests that the problems dealt with in this culture were elementary. These primitive societies generally employed their fingers for counting, and as humans have five fingers on each hand and five toes on each foot, then the obvious bases would have been 5, 10, and 20. Traces of the earlier use of the base 20 system are still apparent in modern languages such as English and French. This includes phrases such as “three score” in English and “quatre vingt” in French.

The decimal system (base 10) is used today in western society, but the base 60 was common in early computation circa 1500 BC. One example of the use of base 60 today is still evident in the subdivision of hours into 60 minutes, and the subdivision of minutes into 60 seconds. The base 60 system (i.e., the sexagesimal system) is inherited from the Babylonians [Res:84], and the Babylonians were able to represent arbitrarily large numbers or fractions with just two symbols.

The achievements of some of these early civilizations are impressive. The archeological remains of ancient Egypt such as the pyramids at Giza and the temples along the Nile such as at Karnak and Abu Simbel are inspiring. These monuments provide an indication of the engineering sophistication of the ancient Egyptian civilization. The objects found in the tomb of Tutankhamun[2] are now displayed in the Egyptian museum in Cairo and demonstrate the artistic skill of the Egyptians.

The Greeks made major contributions to western civilization including contributions to Mathematics, Philosophy, Logic, Drama, Architecture, Biology, and Democracy.[3] The Greek philosophers considered fundamental questions such as ethics, the nature of being, how to live a good life, and the nature of justice and politics. The Greek philosophers include Parmenides, Heraclitus, Socrates, Plato, and Aristotle. The Greeks invented democracy, and their democracy was radically different from today’s representative democracy.[4] The sophistication of Greek

[2] Tutankhamun was a minor Egyptian pharaoh who reigned after the controversial rule of Akhenaten. Howard Carter discovered Tutankhamun’s intact tomb in the Valley of the Kings. The quality of the workmanship of the artifacts found in the tomb was extraordinary, and a visit to the Egyptian museum in Cairo is memorable.

[3] The origin of the word “democracy” is from demos (δημος) meaning people and kratos (κρατος) meaning rule. That is, it means rule by the people. It was introduced into Athens following the reforms introduced by Cleisthenes. He divided the Athenian city-state into thirty areas. Twenty of these areas were inland or along the coast and ten were in Attica itself. Fishermen lived mainly in the ten coastal areas; farmers in the ten inland areas; and various tradesmen in Attica. Cleisthenes introduced ten new clans where the members of each clan came from one coastal area, one inland area on one area in Attica. He then introduced a Boule (or assembly) which consisted of 500 members (50 from each clan). Each clan ruled for $^{1}/_{10}$ th of the year.

[4] The Athenian democracy involved the full participations of the citizens (i.e., the male adult members of the city-state who were not slaves), whereas in representative democracy, the citizens elect representatives to rule and represent their interests. The Athenian democracy was chaotic and could also be easily influenced by individuals who were skilled in rhetoric. There were teachers (known as the Sophists) who taught wealthy citizens rhetoric in return for a fee. The origin of the word “sophist” is the Greek word σοφος meaning wisdom. One of the most well-known of the

architecture and sculpture is evident from the Parthenon on the Acropolis and the Elgin marbles[5] that are housed today in the British Museum, London.

The Hellenistic[6] period commenced with Alexander the Great, and led to the spread of Greek culture throughout most of the known world. The city of Alexandria became a center of learning during the Hellenistic period. Its scholars included Euclid who provided a systematic foundation for geometry, and his famous work "The Elements" consists of thirteen books.

There are many words of Greek origin that are part of the English language. These include words such as psychology which is derived from two Greek words: psyche (ψυχη) and logos (λογος). The Greek word "psyche" means mind or soul, and the word "logos" means an account or discourse. Other examples are anthropology derived from "anthropos" (ανθροπος) and "logos" (λογος).

The Romans were influenced by Greeks culture, and following Rome's defeat of the Greek city-states, many Greeks became tutors in Rome, as the Roman's recognized the value of Greek culture and knowledge. The Romans built aqueducts, viaducts, and amphitheatres; they developed the Julian calendar; formulated laws (lex); and maintained peace throughout the Roman Empire (pax Romano). The ruins of Pompeii and Herculaneum demonstrate their engineering excellence. The Roman numbering system is still employed in clocks and for page numbering in documents, but it is cumbersome for serious computation. The collapse of the Roman Empire in Western Europe led to a decline in knowledge and learning in Europe. However, the eastern part of the Roman Empire continued at Constantinople until the Ottomans conquered it in 1453 AD.

2.2 The Babylonians

The Babylonian[7] civilization flourished in Mesopotamia (in modern Iraq) from about 2000 BC until about 500 BC. Various clay cuneiform tablets containing mathematical texts were discovered and deciphered in the nineteenth century [Smi:23]. These included tables for multiplication, division, squares, cubes, and square roots; measurement of area and length; and the solution of linear and quadratic equations. The late Babylonian period (c. 500 BC) includes work on astronomy.

The Babylonians recorded their mathematics on soft clay using a wedge-shaped instrument to form impressions of the cuneiform numbers. The clay tablets were

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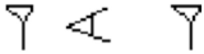
sophists was Protagoras. The problems with Athenian democracy led philosophers such as Plato to consider alternate solutions such as rule by philosopher kings. This is described in Plato's Republic.

[5] The Elgin marbles are named after Lord Elgin who moved them from the Parthenon in Athens to London in 1806. The marbles show the Pan-Athenaic festival that was held in Athens in honor of the goddess Athena after whom Athens is named.

[6] The origin of the word Hellenistic is from Hellene ('Ελλην) meaning Greek.

[7] The hanging gardens of Babylon were one of the seven wonders of the ancient world.

then baked in an oven or by the heat of the sun. They employed just two symbols (1 and 10) to represent numbers, and these symbols were then combined to form all other numbers. They employed a positional number system[8] and used the base 60 system. The symbol representing 1 could also (depending on the context) represent 60, 60^2, 60^3, etc. It could also mean $^1/_{60}$, $^1/_{3600}$, and so on. There was no zero employed in the system, and there was no decimal point (no "sexagesimal point"), and therefore, the context was essential.



The example above illustrates the cuneiform notation and represents the number 60 + 10 + 1 = 71. They used the base 60 system for computation, and one possible explanation for this is the ease of dividing 60 into parts as it is divisible by 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. They were able to represent large and small numbers and had no difficulty in working with fractions (in base 60) and in multiplying fractions. They maintained tables of reciprocals (i.e., $^1/_n$, $n = 1, \ldots 59$ apart from numbers like 7, 11, etc., which are not of the form $2^\alpha 3^\beta 5^\gamma$ and cannot be written as a finite sexagesimal expansion).

Babylonian numbers may be represented in a more modern sexagesimal notation [Res:84]. For example, 1;24, 51, 10 represents the number $1 + {}^{24}/_{60} + {}^{51}/_{3600} + {}^{10}/_{216000} = 1 + 0.4 + 0.0141666 + 0.0000462 = 1.4142129$ and is the Babylonian representation of the square root of 2. The Babylonians performed multiplication as the following calculation of (20) × (1;24, 51, 10), that is, 20 × sqrt(2) illustrates:

$$20 \times 1 = 20$$

$$20 \times ;24 = 20 * {}^{24}\!/_{60} = 8$$

$$20 \times {}^{51}\!/_{3600} = {}^{51}\!/_{180} = {}^{17}\!/_{60} = ;17$$

$$20 \times {}^{10}\!/_{216000} = {}^{3}\!/_{3600} + {}^{20}\!/_{216000} = ;0,3,20$$

Hence, 20 × sqrt (2) = 20; + 8; + ;17 + ;0, 3, 20 = 28;17, 3, 20

The Babylonians appear to have been aware of Pythagoras's theorem about 1000 years before the time of Pythagoras. The Plimpton 322 tablet (Fig. 2.1) records various Pythagorean triples, that is, triples of numbers (a, b, c) where $a^2 + b^2 = c^2$. It dates from approximately 1700 BC.

They developed an algebra to assist with problem solving, which allowed problems involving length, breadth, and area to be discussed and solved. They did not employ notation for representation of unknown values (e.g., let x be the length and y be the breadth), and instead, they used words like "length" and "breadth." They

[8] A positional numbering system is a number system where each position is related to the next by a constant multiplier. The decimal system is an example: for example, $546 = 5 \times 10^2 + 4 \times 10^1 + 6$.

were familiar with and used square roots in their calculations, as well as techniques to solve quadratic equations.

They were familiar with various mathematical identities such as $(a + b)^2 = (a^2 + 2ab + b^2)$ as illustrated geometrically in Fig. 2.2. They also worked on astronomical problems, and they had mathematical theories of the cosmos to predict when eclipses and other astronomical events would occur. They were also interested in astrology, and they associated various deities with the heavenly bodies such as the planets, as well as the sun and moon. Various clusters of stars were associated with familiar creatures such as lions, goats, and so on.

The Babylonians used counting boards to assist with counting and simple calculations. A counting board is an early version of the abacus, and it was usually made of wood or stone. The counting board contained grooves, which allowed beads or stones to be moved along the groove. The abacus differed from counting boards in that the beads in abaci contained holes that enabled them to be placed in a particular rod of the abacus.

2.3 The Egyptians

The Egyptian civilization developed along the Nile from about 4000 BC, and the pyramids at Giza were built during the Fourth Dynasty around 3000 BC. The Egyptians used mathematics to solve practical problems such as measuring time, measuring the annual Nile flooding, calculating the area of land, book keeping and accounting, and calculating taxes. They developed a calendar circa 4000 BC, which

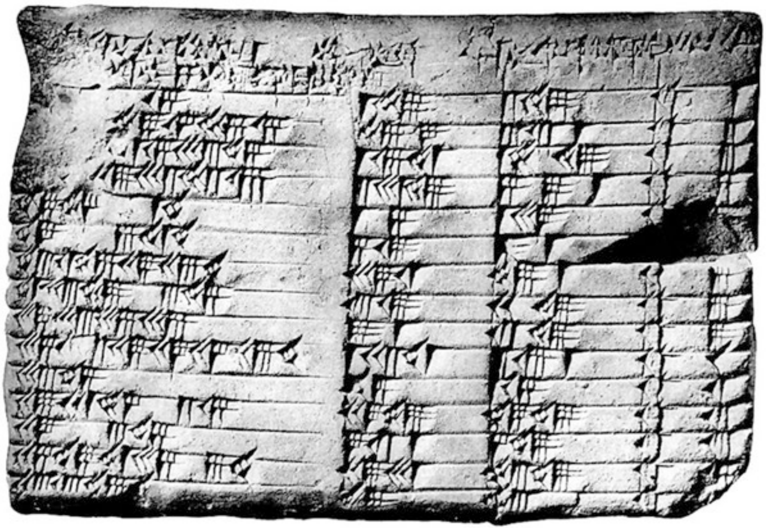


Fig. 2.1 The Plimpton 322 tablet

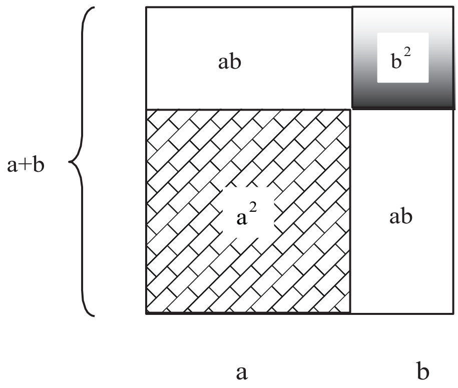


Fig. 2.2 Geometric representation of $(a + b)^2 = (a^2 + 2ab + b^2)$

consisted of 12 months with each month having 30 days. There were five extra feast days to give 365 days in a year. Egyptians writing commenced around 3000 BC, and it is recorded on the walls of temples and tombs.[9] A reed-like parchment termed "papyrus" was also used for writing, and there are three Egyptian writing scripts namely hieroglyphics, the hieratic script, and the demotic script.

Hieroglyphs are little pictures and are used to represent words, alphabetic characters as well as syllables or sounds, and there were also special indeterminate signs to indicate the type of reading to be used. Champollion deciphered hieroglyphics with his work on the Rosetta stone, which was discovered during the Napoleonic campaign in Egypt, and it is now in the British Museum in London. It contains three scripts: Hieroglyphics, Demotic script, and Greek. A key part of the decipherment was that the Rosetta stone contained just one name "Ptolemy" in the Greek text, and this was identified with the hieroglyphic characters in the cartouche[10] of the hieroglyphics. There was just one cartouche on the Rosetta stone, and Champollion inferred that the cartouche represented the name "Ptolemy." He was familiar with another multilingual object, which contained two names in the cartouche. One he recognized as Ptolemy and the other he deduced from the Greek text as "Cleopatra." This led to the breakthrough in translation of the hieroglyphics [Res:84], and Champollion's knowledge of Coptic (where Egyptian is written in Greek letters) was essential in the deciphering

The Rhind Papyrus is a famous Egyptian papyrus on mathematics. The Scottish Egyptologist Henry Rhind purchased it in 1858, and it is a copy created by an Egyptian scribe called Ahmose.[11] It is believed to date to 1832 BC. It contains

[9] The decorations of the tombs in the Valley of the Kings record the life of the pharaoh including his exploits and successes in battle.

[10] The cartouche surrounded a group of hieroglyphic symbols enclosed by an oval shape. Champollion's insight was that the group of hieroglyphic symbols represented the name of the Ptolemaic pharaoh "Ptolemy."

[11] The Rhind papyrus is sometimes referred to as the Ahmose papyrus in honor of the scribe who wrote it in 1832 BC.

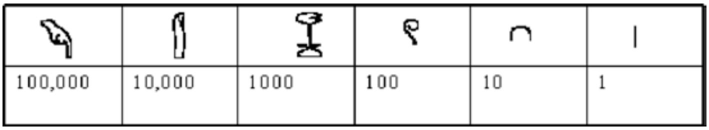


Fig. 2.3 Egyptian numerals

Fig. 2.4 Egyptian representation of a number

examples of all kinds of arithmetic and geometric problems, and students may have used it as a textbook to develop their mathematical knowledge. This would allow them to participate in the pharaoh's building program.

The Egyptians were familiar with geometry, arithmetic, and elementary algebra. They had formulae to find solutions to problems with one or two unknowns. A base 10 number system was employed with separate symbols for one, ten, a hundred, a thousand, a ten thousand, a hundred thousand, and so on. These hieroglyphic symbols are represented in Fig. 2.3.

For example, the representation of the number 276 in Egyptian Hieroglyphics is given in Fig. 2.4.

The addition of two numerals is straightforward and involves adding the individual symbols, and where there are ten copies of a symbol it is then replaced by a single symbol of the next higher value. The Egyptian employed unit fractions (e.g., $1/n$ where n is an integer). These were represented in hieroglyphs by placing the symbol representing a "mouth" above the number. The symbol "mouth" represents part of. For example, the representation of the number $^{1}/_{276}$ is given Fig. 2.5.

The papyrus included problems to determine the angle of the slope of the pyramid's face. The Egyptians were familiar with trigonometry including sine, cosine, tangent, and cotangent, and they knew how to build right angles into their structures by using the ratio 3:4:5. The papyrus also considered problems such as the calculation of the number of bricks required for part of a building project. They were familiar with addition, subtraction, multiplication, and division. However, their multiplication and division was cumbersome as they could only multiply and divide by two.

Suppose they wished to multiply a number n by 7. Then, $n \times 7$ is determined by $n \times 2 + n \times 2 + n \times 2 + n$. Similarly, if they wished to divide 27 by 7 they would note that $7 \times 2 + 7 = 21$ and that $27 - 21 = 6$ and that therefore the answer was $3\ ^{6}/_{7}$. Egyptian mathematics was cumbersome and the writing of it was long and repetitive. For example, they wrote a number such as 22 by 10 + 10 + 1 + 1.

The Egyptians calculated the approximate area of a circle by calculating the area of a square $^{8}/_{9}$ of the diameter of a circle. That is, instead of calculating the area in terms of our familiar πr^2 their approximate calculation yielded $(^{8}/_{9} \times 2r)^2 = {}^{256}/_{81}\ r^2$

or 3.16 r^2. Their approximation of π was $^{256}/_{81}$ or 3.16. They were able to calculate the area of a triangle and volumes.

The Moscow papyrus is a well-known Egyptian papyrus, and it includes a problem to calculate the volume of the frustum. The formula for the volume of a frustum of a square pyramid[12] was given by $V = {}^{1}/_{3}\, h(b_1^2 + b_1 b_2 + b_2^2)$, and when b_2 is 0, then the well-known formula for the volume of a pyramid is given: that is, $^{1}/_{3}\, hb_1^2$.

2.4 The Greek and Hellenistic Contribution

The Greeks made major contributions to western civilization including mathematics, logic, astronomy, philosophy, politics, drama, and architecture. The Greek world of 500 BC consisted of several independent city-states, such as Athens and Sparta, and various city-states in Asia Minor. The Greek polis (πολις) or city-state tended to be quite small, and it consisted of the Greek city and a certain amount of territory outside the city. Each city-state had its own unique political structure for its citizens: some were oligarchs where political power was maintained in the hands of a few individuals or aristocratic families; others were ruled by tyrants (or sole rulers) who sometimes took power by force, but who often had a lot of support from the public. These included people such as Solon, Peisistratus, and Cleisthenes in Athens.

The reforms by Cleisthenes led to the introduction of the Athenian democracy. Power was placed in the hands of the citizens who were male (women or slaves did not participate). It was an extremely liberal democracy where citizens voted on all of the important issues. Often, this led to disastrous results as speakers who were skilled in rhetoric could exert significant influence (e.g., the disastrous Sicilian expedition during the Peloponnesian war). This led Plato to advocate austere rule by philosopher kings rather than democracy, and Plato's republic was influenced by the ideals of Sparta.

Early Greek mathematics commenced approximately 500–600 BC with work done by Pythagoras and Thales. Pythagoras was a philosopher and mathematician who had spent time in Egypt becoming familiar with Egyptian mathematics. He was born on the island of Samos (off the coast of Turkey), and he later moved to Croton in the south of Italy. He formed a secret society known as the Pythagoreans, and they included men and women who believed in the transmigration of souls and that

Fig. 2.5 Egyptian representation of a fraction

[12] The length of a side of the bottom base of the pyramid is b_1 and the length of a side of the top base is b_2

number was the essence of all things. They discovered the mathematics for harmony in music, with the relationship between musical notes being expressed in numerical ratios of small whole numbers. Pythagoras is credited with the discovery of Pythagoras's theorem, although the Babylonians probably knew this theorem about 1000 years before Pythagoras. The Pythagorean society was dealt a major blow[13] by the discovery of the incommensurability of the square root of 2, that is, there are no numbers p, q such that $\sqrt{2} = {}^{p}/_{q}$.

Thales was a sixth century (BC) philosopher from Miletus in Asia Minor (Turkey) who made contributions to philosophy, geometry, and astronomy. His contributions to philosophy are mainly in the area of metaphysics, and he was concerned with questions on the nature of the world. His objective was to give a natural or scientific explanation of the cosmos, rather than relying on the traditional supernatural explanation of creation in Greek mythology. He believed that there was single substance that was the underlying constituent of the world, and he believed that this substance was water. He also contributed to mathematics [AnL:95], and Thales's theorem in Euclidean geometry states that if A, B and C are points on a circle, where the line AC is a diameter of the circle, then the angle $<ABC$ is a right angle.

The rise of Macedonia led to the Greek city-states being conquered by Philip of Macedonia in the fourth century BC. His son, Alexander the Great, defeated the Persian Empire, and he extended his empire to include most of the known world. This led to the Hellenistic Age where Greek language and culture spread to the known world. Alexander founded the city of Alexandra, and it became a major center of learning in Ptolemaic Egypt.[14] However, Alexander's reign was very short as he died at the young age of 33 in 323 BC.

Euclid lived in Alexandria during the early Hellenistic period. He is considered the father of geometry and the deductive method in mathematics. His systematic treatment of geometry and number theory is published in the thirteen books of the Elements [Hea:56]. It starts from five axioms, five postulates, and twenty-three definitions to logically derive a comprehensive set of theorems. His method of proof was generally constructive, in that as well as demonstrating the truth of a theorem the proof would often include the construction of the required entity. He was also used indirect proof (a nonconstructive proof) to show that there are an infinite number of primes:

1. Suppose there is a finite number of primes (say n primes).
2. Multiply all n primes together and add 1 to form N.

$$\left(\mathrm{N} = p_1 * p_2 * \ldots. * p_n + 1\right)$$

[13] The Pythagoreans were a secret society and its members took a vow of silence with respect to this discovery. However, one member of the society is said to have shared the secret result with others outside the sect, and the apocryphal account is that he was thrown into a lake for his betrayal and drowned. They obviously took Mathematics seriously back then!

[14] The ancient library in Alexandria was once the largest library in the world. It was built during the Hellenistic period in the third century BC and destroyed by fire in 391 A.D.

3. N is not divisible by p_1, p_2, ….., p_n as dividing by any of these gives a remainder of one.
4. Therefore, N must either be prime or divisible by some other prime that was not included in the list.
5. Therefore, there must be at least $n + 1$ primes.
6. This is a contradiction (it was assumed that there are n primes).
7. Therefore, the assumption that there is a finite number of primes is false.
8. Therefore, there is an infinite number of primes.

Euclidean geometry included the parallel postulate (the fifth postulate). This postulate generated interest, as many mathematicians believed that it was unnecessary and could be proved as a theorem. It states that:

Definition 2.1 (Parallel Postulate)
If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

This postulate was later proved to be independent of the other postulates with the development of non-Euclidean geometries in the nineteenth century. These include the hyperbolic geometry discovered independently by Bolyai and Lobachevsky and elliptic geometry as developed by Riemann. The standard model of Riemannian geometry is the sphere where lines are great circles.

The material in the Euclid's Elements is a systematic development of geometry starting from the small set of axioms, postulates, and definitions, leading to theorems derived logically from the axioms and postulates. There are some jumps in reasoning, and the German mathematician, David Hilbert, later added extra axioms to address this. Euclidean geometry contains many well-known mathematical results such as Pythagoras's theorem, Thales's theorem, Sum of Angles in a Triangle, Prime Numbers, Greatest Common Divisor and Least Common Multiple, Euclidean Algorithm, Areas and Volumes, Tangents to a point, and Algebra.

The Euclidean algorithm is one of the oldest known algorithms, and it is used to determine the greatest common divisor of two numbers a and b. It is presented in the Elements, but it was known well before Euclid. The formulation of the *gcd* algorithm for two natural numbers a and b is as follows:

1. Check if b is zero. If so, then a is the *gcd*.
2. Otherwise, the $gcd\ (a, b)$ is given by $gcd\ (b, \text{a mod } b)$.

It is also possible to determine integers p and q such that $ap + bq = gcd\ (a, b)$.

The proof of the Euclidean algorithm is as follows. Suppose a and b are two positive numbers whose gcd is to be determined, and let r be the remainder when a is divided by b.

1. Clearly $a = qb + r$ where q is the quotient of the division.
2. Any common divisor of a and b is also a divider or r (since $r = a - qb$).
3. Similarly, any common divisor of b and r will also divide a.

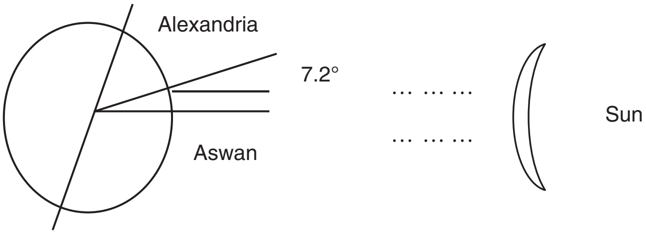


Fig. 2.6 Eratosthenes measurement of the circumference of the earth

4. Therefore, the greatest common divisor of *a* and *b* is the same as the greatest common divisor of *b* and *r*.
5. The number *r* is smaller than *b* and we will reach $r = 0$ in finitely many steps.
6. The process continues until $r = 0$.

Comment 2.1
Algorithms are fundamental in computing as they define the procedure by which a problem is solved. A computer program implements the algorithm in some programming language.

Eratosthenes was a third century BC Hellenistic mathematician and scientist who worked as librarian in the famous library in Alexandria. He was the first person to estimate of the size of the circumference of the earth. His approach to the calculation was as follows (Fig. 2.6):

1. Eratosthenes believed that the earth was a sphere.
2. On the summer solstice at noon in the town of Syene (ancient name of Aswan[15]) on the Tropic of Cancer in Egypt the sun appears directly overhead.
3. He assumed that rays of light came from the sun in parallel beams and reached the earth at the same time.
4. At the same time, in Alexandria, he had measured that the sun would be 7.2° south of the zenith.
5. He assumed that Alexandria was directly north of Aswan.
6. He concluded that the distance from Alexandria to Aswan was $^{7.2}/_{360}$ of the circumference of the earth.
7. The distance between Alexandria and Aswan was 5000 stadia (approximately 800 km).
8. He established a value of 252,000 stadia or approximately 39,6000 km (the actual circumference at equator is 40,075 km).

Eratosthenes's calculation was an impressive result for 200 BC. The errors in his calculation were due to:

[15] The town of Aswan is famous today for the Aswan high dam, which was built in the 1960s. There was an older Aswan dam built by the British in the late nineteenth century. The new dam led to a rise in the water level of Lake Nasser and flooding of archaeological sites along the Nile. Several sites such as Abu Simbel and the island of Philae were relocated to higher ground.

1. Aswan is not exactly on the Tropic of Cancer (it is actually 55 km north of it).
2. Alexandria is not exactly north of Aswan (there is a difference of 3° longitude).
3. The distance between Aswan and Alexandria is 729 km not 800 km.
4. Angles in antiquity could not be measured with absolute precision.
5. The angular distance is actually 7.08° and not 7.2°.

The first century BC Stoic philosopher and polymath, Posidonius, also calculated the circumference of the earth using the star Canopus, and he arrived at a similar figure (240,000 stadia or 39,0000 km). Eratosthenes also calculated the approximate distance to the moon and sun, and he also produced maps of the known world. He developed a useful algorithm for determining all of the prime numbers up to a specified integer, and this is known as the *Sieve of Eratosthenes*. The steps in the algorithm are as follows:

1. Write a list of the numbers from 2 to the largest number to be tested. This first list is called A.
2. A second List B is created to list the primes. It is initially empty.
3. The number 2 is the first prime number, and it is added to List B.
4. Strike off (or remove) all multiples of 2 from List A.
5. The first remaining number in List A is a prime number, and this prime number is added to List B.
6. Strike off (or remove) this number and all multiples of it from List A.
7. Repeat steps 5 through 7 until no more numbers are left in List A.

Comment 2.2
The Sieve of Eratosthenes method is a well-known algorithm for determining prime numbers.

Archimedes was a mathematician and astronomer who lived in Syracuse, Sicily. He discovered the law of buoyancy known as Archimedes's principle:

The buoyancy force is equal to the weight of the displaced fluid.

He is believed to have discovered the principle while sitting in his bath, and he was so overwhelmed with his discovery that he rushed out onto the streets of Syracuse shouting "Eureka," forgetting to put on his clothes.

The weight of the displaced liquid is proportional to the volume of the displaced liquid. Therefore, if two objects have the same mass, the one with greater volume (or smaller density) has greater buoyancy. An object will float if its buoyancy force (i.e., the weight of liquid displaced) exceeds the downward force of gravity (i.e., its weight). If the object has exactly the same density as the liquid, then it will stay still, neither sinking nor floating upwards.

For example, a rock is generally a very dense material, and so it usually does not displace its own weight. Therefore, a rock will sink to the bottom as the downward weight exceeds the buoyancy weight. However, the weight of a buoyancy device is significantly less than the liquid that it would displace, and so it floats at a level where it displaces the same weight of liquid as the weight of the object.

Archimedes also made good contributions to mathematics including an approximation to π, contributions to the positional numbering system, geometric series, and to maths physics. He also solved several interesting problems: for example, the calculation of the composition of cattle in the herd of the Sun god by solving a number of simultaneous *Diophantine equations* (named after Diophantus). The herd consisted of bulls and cows, with one part of the herd consisting of white, second part black, third spotted, and the fourth brown. Various constraints were then expressed in Diophantine equations, and the problem was to determine the precise composition of the herd.

He calculated the number of grains of sands in the known universe, and challenged the prevailing view this was too large to be determined. He developed a naming system for large numbers, as the largest number in use at the time was a myriad myriad (100 million), where a myriad is 10,000. He developed the laws of exponents: that is, $10^a 10^b = 10^{a+b}$, and his calculation of the upper bound includes not only the grains of sand on each beach but on the earth filled with sand and the known universe filled with sand. His final estimate of the upper bound of the number of grains of sand in a filled universe was 10^{64}.

It is possible that he may have developed the odometer,[16] which could calculate the total distance traveled on a journey. An odometer is described by the Roman engineer Vitruvius around 25 BC. It employed a wheel with a diameter of 4 feet, and the wheel turned 400 times in every mile.[17] The device included gears and pebbles and a 400-tooth cogwheel that turned once every mile, and caused one pebble to drop into a box. The total distance traveled was determined by counting the number of pebbles in the box.

Aristotle was born in Macedonia and he became a student of Plato at Plato's academy in Athens in the fourth century BC. (Fig. 2.7). Aristotle later founded his own school (known as the Lyceum) in Athens, and he was also the tutor of Alexander the Great. He made contributions to biology, logic, politics, ethics, and metaphysics.

His starting point to knowledge acquisition was the senses, as he believed that these were essential to acquire knowledge. His position is the opposite of Plato who argued that the senses deceive and should not be relied upon. Plato's writings are mainly in dialogs involving his former mentor Socrates.[18]

Aristotle made important contributions to formal reasoning with his development of syllogistic logic. Syllogistic logic (also known as term logic) consists of

[16] The origin of the word "odometer" is from the Greek words οδος (meaning journey) and μετρον meaning (measure).

[17] The figures given here are for the distance of one Roman mile. This is given by $\pi 4 \times 400 = 12.56 \times 400 = 5024$ (which is less than 5280 feet for a standard mile in the Imperial system).

[18] Socrates was a moral philosopher who deeply influenced Plato. His method of enquiry into philosophical problems and ethics was by questioning. Socrates himself maintained that he knew nothing (Socratic ignorance). However, from his questioning, it became apparent that those who thought they were clever were not really that clever after all. His approach obviously would not have made him very popular with the citizens of Athens. Socrates had consulted the oracle at Delphi to find out who was the wisest of all men, and he was informed that there was no one wiser than him. Socrates was sentenced to death for allegedly corrupting the youth of Athens, and he was forced drink the juice of the hemlock plant (a type of poison).

reasoning with two premises and one conclusion. Each premise consists of two terms and there is a common middle term. The conclusion links the two unrelated terms from the premises. For example:

Premise 1	All Greeks are Mortal
Premise 2	Socrates is a Greek.
Conclusion	Socrates is Mortal

Fig. 2.7 Plato and Aristotle

The common middle term is "Greek," which appears in the two premises. The two unrelated terms from the premises are "Socrates" and "Mortal." The relationship between the terms in the first premise is that of the universal, that is, anything or any person that is a Greek is mortal. The relationship between the terms in the second premise is that of the particular, that is, Socrates is a person that is a Greek. The conclusion from the two premises is that Socrates is mortal, that is, a particular relationship between the two unrelated terms "Socrates" and "Mortal."

The example above is an example of a valid syllogistic argument. Aristotle studied the various possible syllogistic arguments and determined those that were valid and those that were invalid. There are several candidate relationships that may potentially exist between the terms in a premise. These are listed in Table 2.1.

In general, a syllogistic argument will be of the form:

$$\frac{\begin{array}{c}\mathrm{S}\,x\,\mathrm{M}\\ \mathrm{M}\,y\,\mathrm{P}\end{array}}{\mathrm{S}\,z\,\mathrm{P}}$$

where x, y, and z may be universal affirmation, universal negation, particular affirmation, and particular negation. Syllogistic logic is described in more detail in [ORg:20]. Aristotle's work was highly regarded in classical and medieval times, and Kant believed that there was nothing else to invent in Logic.

An early form of propositional logic that was developed by Chrysippus[19] in the third century BC. Aristotelian logic is of historical interest today, and it has been replaced by propositional and predicate logic.

Ptolemy was a second century AD Hellenistic mathematician and cartographer, and he created a table of chords (essentially equivalent to a table of values of the sine function). He also produced maps of the inhabited world and a geocentric model of the universe.

Table 2.1 Syllogisms: relationship between terms

Relationship	Abbr.
Universal affirmation	A
Universal negation	E
Particular affirmation	I
Particular negation	O

[19] Chrysippus was the head of the Stoics in the third century BC.

2.5 The Romans

Rome is said to have been founded[20] by Romulus and Remus about 750 BC. Early Rome covered a small part of Italy, but it gradually expanded in size and importance. It destroyed Carthage[21] in 146 BC to become the major power in the Mediterranean. The Romans colonized the Hellenistic world, and they were influenced by Greek culture and mathematics. Julius Caesar (Fig. 2.9) conquered the Gauls in 58 BC.

The Gauls consisted of several disunited Celtic[22] tribes. Vercingetorix succeeded in uniting them, but he was defeated by at the siege of Alesia in 52 BC.

The Roman number system uses letters to represent numbers (Fig. 2.8) and a number consists of a sequence of letters. The evaluation rules specify that if a large number follows a smaller number, then the smaller number is subtracted from the large: for example, IX represents 9 and XL represents 40. Similarly, if a smaller number followed a larger number, they were generally added: for example, MCC represents 1200. They had no zero in their system.

The use of Roman numerals was cumbersome, and an abacus was often employed for calculation. An abacus consists of several columns in which pebbles are placed. Each column represented powers of 10, that is, 10^0, 10^1, 10^2, 10^3, etc. The column to the far right represents one; the column to the left 10; next column to the left 100; and so on. Pebbles (calculi) were placed in the columns to represent different

Fig. 2.8 Roman numbers

I = 1
V = 5
X = 10
L = 50
C = 100
D = 500
M = 1000

[20] The Aeneid by Virgil suggests that the Romans were descended from survivors of the Trojan War, and that Aeneas brought surviving Trojans to Rome after the fall of Troy.

[21] Carthage was located in Tunisia, and the wars between Rome and Carthage are known as the Punic wars. Hannibal was one of the great Carthaginian military commanders, and during the second Punic war, he brought his army to Spain, marched through Spain and crossed the Pyrenees. He then marched along southern France and crossed the Alps into Northern Italy. His army also consisted of war elephants. Rome finally defeated Carthage and leveled the city.

[22] The Celtic period commenced around 1000 BC in Hallstatt (near Salzburg in Austria). The Celts were skilled in working with iron and bronze, and they gradually expanded into Europe. They eventually reached Britain and Ireland around 600 BC. The early Celtic period was known as the "Hallstatt period," and the later Celtic period is known as the "La Téne" period. The La Téne period is characterized by the quality of ornamentation produced. The Celtic museum in Hallein in Austria provides valuable information and artifacts on the Celtic period. The Celtic language has similarities to the Irish language. However, the Celts did not employ writing, and the Ogham writing developed in Ireland was developed in the early Christian period.

Fig. 2.9 Julius Caesar

numbers: for example, the number represented by an abacus with 4 pebbles on the far right; 2 pebbles in the column to the left; and 3 pebbles in the next column to the left is 324. The calculations were performed by moving pebbles from column to column.

Merchants introduced a set of weights and measures (including the *libra* for weights and the *pes* for lengths). They developed an early banking system to provide loans for business, and commenced minting coins around 290 BC. The Romans also made contributions to calendars, and Julius Caesar introduced the Julian calendar in 45 BC. It has a regular year of 365 days divided into 12 months, and a leap day is added to February every 4 years. However, too many leap years are added over time, and this led to the introduction of the Gregorian calendar in 1582.

Caesar employed a substitution cipher (Fig. 2.10) on his military campaigns to ensure that important messages were communicated safely. This involves the substitution of each letter in the plaintext (i.e., the original message) by a letter a fixed number of positions down in the alphabet. For example, a shift of 3 positions causes the letter B to be replaced by E, the letter C by F, and so on. The Caesar cipher is easily broken, as the frequency distribution of letters may be employed to determine the mapping. The cipher is defined as follows:

The process of enciphering a message (i.e., plaintext) involves mapping each letter in the plaintext to the corresponding cipher letter. For example, the encryption of "summer solstice" involves:

Plaintext :	Summer Solstice
Cipher Text	vxpphu vrovwleh

Alphabet Symbol	abcde fghij klmno pqrst uvwxyz
Cipher Symbol	dfegh ijklm nopqr stuvw xyzabc

Fig. 2.10 Caesar Cipher

The decryption involves the reverse operation: that is, for each cipher letter, the corresponding plaintext letter is identified from the table.

Cipher Text vxpphu vrovwleh
Plaintext : Summer Solstice

The encryption may also be done using modular arithmetic. The numbers 0–25 represent the alphabet letters, and addition (modula 26) is used to perform the encryption. The encoding of the plaintext letter *x* is given by:

$$c = x + 3 \left(\mathrm{mod}\, 26\right)$$

Similarly, the decoding of a cipher letter represented by the number *c* is given by:

$$x = c - 3 \left(\mathrm{mod}\, 26\right)$$

The emperor Augustus[23] employed a similar substitution cipher (with a shift key of 1). The Caesar cipher remained in use up to the early twentieth century. However, by then, frequency analysis techniques were available to break the cipher. The Vigenère cipher uses a Caesar cipher with a different shift at each position in the text. The value of the shift to be employed with each plaintext letter is defined using a repeating keyword.

2.6 Islamic Influence

Islamic mathematics refers to mathematics developed in the Islamic world from the birth of Islam in the early seventh century up until the seventeenth century. The Islamic world commenced with the prophet Mohammed in Mecca and spread throughout the Middle East, North Africa, and Spain. The Golden Age of Islamic civilization was from 750 AD to 1250 AD, and during this period, enlightened caliphs recognized the value of knowledge and sponsored scholars to come to Baghdad to gather and translate the existing world knowledge into Arabic.

[23] Augustus was the first Roman emperor and his reign ushered in a period of peace and stability following the bitter civil wars. He was the adopted son of Julius Caesar and was called Octavian before he became emperor. The earlier civil wars were between Caesar and Pompey, and following Caesar's assassination, civil war broke out between Mark Anthony and Octavian. Octavian defeated Anthony and Cleopatra at the battle of Actium in 31 BC.

This led to the preservation of the Greek texts during the Dark ages in Europe. Further, the Islamic cities of Baghdad, Cordoba, and Cairo became key intellectual centers, and scholars added to existing knowledge (e.g., in mathematics, astronomy, medicine, and philosophy), as well as translating the known knowledge into Arabic.

The Islamic mathematicians and scholars were based in several countries in the Middle East, North Africa, and Spain. Early work commenced in Baghdad, and the mathematicians were also influenced by the work of Hindu mathematicians, who had introduced the decimal system and decimal numerals. Among the well-known Islamic scholars are Ibn Al Haytham, a tenth century Iraqi scientist; Mohammed Al Khwarizmi (Fig. 2.11), a ninth Persian mathematician; Abd Al Rahman al Sufi, a Persian astronomer who discovered the Andromeda galaxy; Ibn Al Nazis, a Syrian who did work on circulation in medicine; Averroes, who was an Aristotelian philosopher from Cordoba in Spain; Avicenna, who was a Persian philosopher; and Omar Khayman, who was a Persian Mathematician and poet.

Many caliphs (Muslim rulers) were enlightened and encouraged scholarship in mathematics and science. They set up a center for translation and research in Baghdad, and existing Greek texts such as the works of Euclid, Archimedes, Apollonius, and Diophantus were translated into Arabic. The Islamic scholar, Al-Khwarizmi, made contributions to early classical algebra, and the word algebra comes from the Arabic word "*al jabr*" that appears in a textbook by Al Khwarizmi. The origin of the word algorithm is from "Al Khwarizmi."

Education was important during the Golden Age, and the Al Azhar University in Cairo (Fig. 2.12) was established in 970 AD, and the Al-Qarawiyyin University in Fez, Morocco, was established in 859 AD. The Islamic World has created beautiful architecture and art, including the ninth century Great Mosque of Samarra in Iraq; the tenth century Great Mosque of Cordoba; and the eleventh century Alhambra palace and fortress complex in Grenada.

Fig. 2.11 Mohammed Al Khwarizmi

The Moors[24] invaded Spain in the eighth century AD, and they ruled large parts of the Peninsula for several centuries. Moorish Spain became a center of learning, and this led to Islamic and other scholars coming to study at the universities in Spain. Many texts on Islamic mathematics were translated from Arabic into Latin, and these were invaluable in the renaissance in European learning and mathematics from the thirteenth century. The Moorish influence[25] in Spain continued until the time of the Catholic Monarchs[26] in the fifteenth century. Ferdinand and Isabella united Spain, defeated the Moors in Andalusia, and expelled them from Spain.

Fig. 2.12 Al Azhar University, Cairo

[24] The origin of the word "Moor" is from the Greek work μυοροζ meaning very dark. It referred to the fact that many of the original Moors who came to Spain were from Egypt, Tunisia, and other parts of North Africa.

[25] The Moorish influence includes the construction of various castles (*alcazar*), fortresses (*alcazaba*), and mosques. One of the most striking Islamic sites in Spain is the palace of Alhambra in Granada, and it represents the zenith of Islamic art.

[26] The Catholic Monarchs refer to Ferdinand of Aragon and Isabella of Castile who married in 1469. They captured Granada (the last remaining part of Spain controlled by the Moors) in 1492.

The Islamic contribution to algebra was an advance on the achievements of the Greeks. They developed a broader theory that treated rational and irrational numbers as algebraic objects, and moved away from the Greek concept of mathematics as being essentially Geometry. Later, Islamic scholars applied algebra to arithmetic and geometry and studied curves using equations. This included contributions to reduce geometric problems such as duplicating the cube to algebraic problems. Eventually, this led to the use of symbols in the fifteenth century such as:

$$x^n.x^m = x^{m+n}.$$

The poet Omar Khayman was also a mathematician who did work on the classification of cubic equations with geometric solutions. Other scholars made contributions to the theory of numbers: for example, a theorem that allows pairs of amicable numbers to be found. Amicable numbers are two numbers such that each is the sum of the proper divisors of the other. They were aware of Wilson's theory in number theory: that is, for p prime then p divides $(p - 1)! +1$.

The Islamic world was tolerant of other religious belief systems during the Golden Age, and there was freedom of expression provided that it did not infringe on the rights of others. It began to come to an end following the Mongol invasion and sack of Baghdad in the late 1250s and the Crusades. It continued to some extent until the conquest by Ferdinand and Isabella of Andalusia in the late fifteenth century.

2.7 Chinese and Indian Mathematics

The development of mathematics commenced in China about 1000 BC, and it was independent of developments in other countries. The emphasis was on problem solving rather than on conducting formal proofs. It was concerned with finding the solution to practical problems such as the calendar, the prediction of the positions of the heavenly bodies, land measurement, conducting trade, and the calculation of taxes.

The Chinese employed counting boards as mechanical aids for calculation from the fourth century BC. Counting boards are similar to abaci and are usually made of wood or metal, and contained carved grooves between which beads, pebbles, or metal discs were moved. The abacus is a device, usually of wood having a frame that holds rods with freely sliding beads mounted on them. It is used as a tool to assist calculation, and it is useful for keeping track of the sums, the carry, and so on of calculations.

Early Chinese mathematics was written on bamboo strips and included work on arithmetic and astronomy. The Chinese method of learning and calculation in mathematics was *learning by analogy*. This involves a person acquiring knowledge from observation of how a problem is solved, and then applying this knowledge for problem solving to similar kinds of problems.

They had their version of Pythagoras's theorem and applied it to practical problems. They were familiar with the Chinese remainder theorem, the formula for finding the area of a triangle, as well as showing how polynomial equations (up to degree ten) could be solved. Other Chinese mathematicians showed how geometric problems could be solved by algebra, how roots of polynomials could be solved, how quadratic and simultaneous equations could be solved, and how the area of various geometric shapes such as rectangles, trapezia, and circles could be computed. Chinese mathematicians were familiar with the formula to calculate the volume of a sphere. The best approximation that the Chinese had of π was 3.14159, and this was obtained by approximations from inscribing regular polygons with 3×2^n sides in a circle.

The Chinese made contributions to number theory including the summation of arithmetic series and solving simultaneous congruences. The Chinese remainder theorem deals with finding the solutions to a set of simultaneous congruences in modular arithmetic. Chinese astronomers made accurate observations, which were used to produce a new calendar in the sixth century. This was known as the Taming Calendar and it was based on a cycle of 391 years.

Indian mathematicians have made important contributions such as the development of the decimal notation for numbers that is now used throughout the world. This was developed in India sometime between 400 BC and 400 AD. Indian mathematicians also invented zero and negative numbers, and also did early work on the trigonometric functions of sine and cosine. The knowledge of the decimal numerals reached Europe through Arabic mathematicians, and the resulting system is known as the *Hindu-Arabic numeral system*.

The Shulba Sutras is a Hindu text that documents Indian mathematics, and it dates from about 400 BC. It includes a summary of early Hindu trigonometry and their rules. The Indians were familiar with the statement and proof of Pythagoras's theorem, rational numbers, quadratic equations, as well as the calculation of the square root of 2 to five decimal places.

2.8 Review Questions

1. Discuss the strengths and weaknesses of the various number systems.
2. Describe ciphers used during the Roman civilization and write a program to implement one of these.
3. Discuss the nature of an algorithm and its importance in computing.
4. Discuss the working of an abacus and its application to calculation.
5. What are the differences between syllogistic logic and propositional and predicate logic??

2.9 Summary

The last decades of the twentieth century have witnessed a proliferation of high-tech computers, mobile phones, and information technology. Software is now pervasive and technology has become an integral part of the western world, with the pace of change and innovation quite extraordinary. It has led to increases in industrial productivity and potentially allows humans the freedom to engage in more creative and rewarding tasks.

This chapter considered the contributions of early civilizations to computing, including contributions of the Babylonians, the Egyptians, the Greeks, and the Romans and Islamic scholars.

The Babylonian civilization flourished from about 2000 BC, and they produced clay cuneiform tablets containing mathematical texts. These included tables for multiplication, division, squares, and square roots, as well as the calculation of area and the solution of linear and quadratic equations.

The Egyptian civilization developed along the Nile from about 4000 BC, and they used mathematics for practical problem solving. The Greeks made major contributions to western civilization, with Euclid developing a systematic treatment of geometry. Aristotle's syllogistic logic remained in use until the development of propositional and predicate logic in the late nineteenth century.

The Islamic contribution helped to preserve western knowledge during the dark ages in Europe. Islamic scholars in Baghdad, Cairo, and Cordoba translated Greek texts into Arabic. They also added to existing knowledge in mathematics, science, astronomy, and medicine.