A Simulation Approach to Reliability Assessment of a Redundant System with Arbitrary Input Distributions

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Abstract. With the rapid development and spread of computer networks and information technologies, researchers are faced with new complex challenges of both applied and theoretical nature in investigating the reliability and availability of communication networks and data transmission systems. In the current paper, we perform the system-level reliability analysis for a redundant system with arbitrary distributions of uptime and repair time of its elements using a simulation approach. Also, we obtained the values of the relative recovery speed at which the desired level of reliability is achieved, presented dependency plots of the probability of system uptime and plots of the uniform difference of the obtained simulation results against the relative speed of recovery; also plots of the empirical distribution function $F^*(x)$ and reliability function $R^*(x)$ relative to the reliability assessment. Software implementation of simulation algorithms was carried out on the basis of the R language.

Keywords: Simulation · Stochastic modeling · Reliability of redundant systems · Redundant communications · Relative repair rate · Probability of the failure-free operation · Sensitivity analysis

1 Introduction

With the rapid development of computer networks and information technologies, researchers are faced with new complex applied and theoretical problems on studying the reliability and availability of networks and data transmission systems [1].

Currently, simulation is effectively utilized in modeling of info-communication network systems, validating mathematical methods, testing information technologies, elaborating new computational models for analysis of functioning of computer networks, modeling teletraffic, etc. Previously in [2], it was shown

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that explicit analytical expressions for the stationary distribution of the system under consideration cannot always be obtained. The simulation model developed in this work allowed to investigate the reliability of the system, defined as the stationary probability of its failure-free operation, as well as to assess the reliability measures of the system; also numerical research and graphical analysis have shown that this dependence becomes vanishingly small under a “fast” recovery, that is, with the growth of the relative repair rate ρ.

Recently, the functioning of various aspects of modern society has become critically dependent on communication networks [3,4]. With the migration of critical communications tools, it has become vital to ensure the reliability and accessibility of data networks and systems.

A number of previous studies [5–9] have focused on analyzing the reliability of various complex telecommunications systems. In particular, a study was conducted on the reliability of cold-standby data transmission systems. Paper [10] focused on reliability analysis of a combined power plant running on a gas turbine engine. In a series of works by Enrico Zio et al. [11–13] the Monte Carlo simulation method was applied to reliability assessment and risk analysis of multi-state physics systems. The aim of [14] was to develop a model for studying system reliability and analyzing the sensitivity of system availability. In stochastic systems stability often means insensitivity or low sensitivity of their output characteristics to the shapes of some input distributions. The proof of the insensitivity can significantly simplify the model of the system under study by using more convenient distributions (from the exponential family). This urges the importance of the sensitivity analysis. Actually, the term “sensitivity analysis” can be understood differently in civil engineering than in basic sciences [15]. In operations research, sensitivity analysis is developed as a method of critical assessment of decisional variables, and is capable to identify those sensitive variables that influence the final desired result [16]. Another suitable complement to probabilistic reliability analysis is structural sensitivity analysis [17,18].

In [19], a simulation method was considered to simulate the reliability of a task by a complex system by modeling a task cyclogram, modeling a run-time profile and a method of dynamic reliability modeling. In [20], modeling and estimation methods were presented that allow temperature optimization of the reliability of a multiprocessor system on a chip for specific applications.

The current paper summarizes the results of previous studies of the authors in the case of cold standby of the system $\langle GI_N/GI/1\rangle$ with an arbitrary distribution function (DF) of uptime and an arbitrary DF of repair time of its elements. The aim of the work is to conduct simulation to find the value of the coefficient ρ (relative repair rate), at which a given level of reliability is achieved and to graph the dependence of the probability of failure-free operation of the system on the relative repair rate. The results of calculating the reliability estimate for different input distributions are presented.

2 Problem Statement and Model Description

As a simulation model of a redundant data transmission system consisting of N different types of data transmission channels, we consider a repairable multiple cold standby system $\langle GI_N/GI/1\rangle$ with one repair device, with an arbitrary distribution function (DF) of uptime and an arbitrary DF of repair time of its elements.

In this paper, we consider the dependence of the probability of failure-free operation of the system $\langle GI_N/GI/1\rangle$ on the relative repair rate. The task is to develop a simulation model for calculating the steady-state probabilities of the system, to find the stationary probability of failure-free operation of the system for some special cases of distributions and assess the reliability of the system, for $N = 3$.

2.1 Simulation Model for Calculating Steady-State Probabilities of $\langle GI_N/GI/1\rangle$ System

Let's define the following states of the simulated system:

- State 0: One (main) element works, $N - 1$ are in a cold standby;
- State 1: One element failed and is being repaired, one – works, $N - 2$ are in a cold standby;
- State 2: Two elements have failed, one is being repaired, the other is waiting for its turn for repair, one – works, $N - 3$ are in a cold standby;
- State N: All the items have failed, one is being repaired, the rest are waiting their turn for repair.

To describe the reliability modeling algorithm for the $\langle GI_N/GI/1\rangle$ system we introduce the following variables:

- double t - simulation clock; changes in case of failure or repair of the system's elements;
- int i, j - system state variables; when an event occurs, the transition from i to j takes place;
- double $t_{nextfail}$ – service variable, which stores the time until the next element failure;
- double $t_{nextrepair}$ – service variable, which stores the time until the next repair of the failed element;
- int k - counter of iterations of the main loop.

For clarity, the simulation model is presented graphically in Fig. 1 in the form of a flowchart. The criterion for stopping the main cycle of the simulation model is to achieve the maximum model execution time T.

For a better understanding and reproducibility of the simulation model, in addition to the flowchart, the algorithm of the discrete-event process of simulation modeling is also provided in the form of pseudo-code with comments (Algorithm 1).

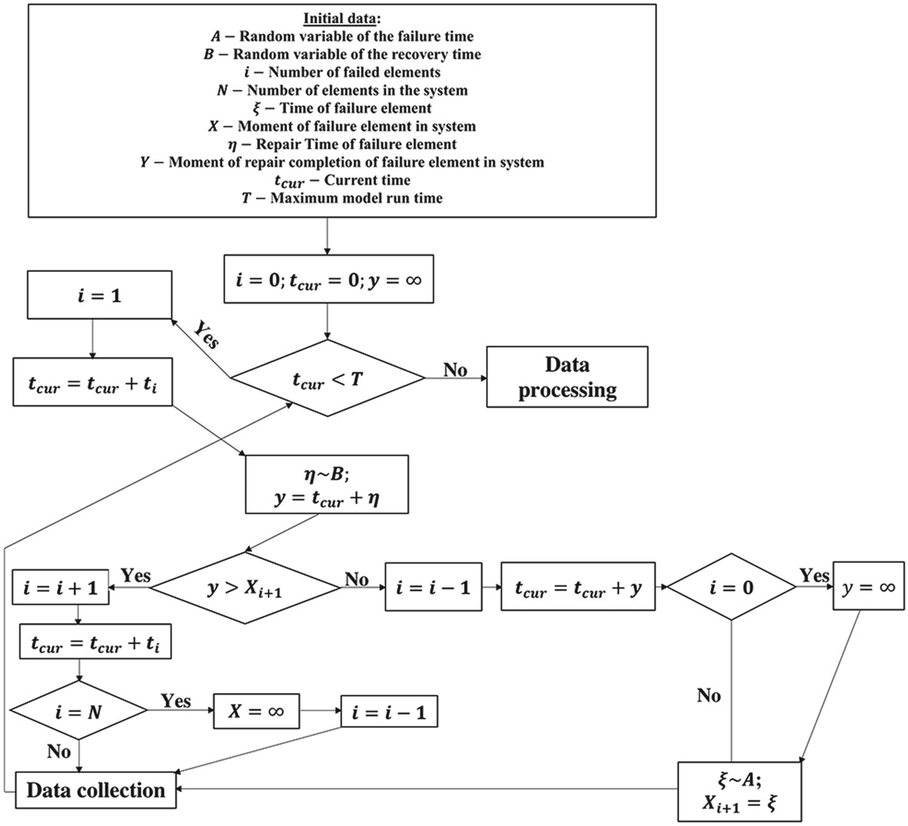

Fig. 1. Flowchart of the simulation model for estimating stationary probabilities.

Algorithm 1. Pseudocode of the simulation process of the system $\langle GI_N/GI/1\rangle$.

Input: a1, b1, N, T, NG, GI.

a1 - Average time between element failures,
b1 - Average repair time,
N - Number of elements in the system,
T - Maximum model run time,
NG - Number of Trajectory (Path) Graphs,
"GI" - Arbitrary Distribution function.

Output: steady-state probabilities $P_0, P_1, P_2, \ldots, P_N$.

Begin

array $r[\,] := [0, 0, 0]$; // multi-dimensional array containing results, k-step of the main cycle (loop)

double $t := 0.0$; // time clock initialization

int $i := 0$; $j := 0$; // system state variables double $t_{nextfail} := 0.0$; // variable in which time until the next element failure

double $t_{nextrepair} := 0.0$; // variable in which time is stored until the next repair is completed

int $k := 1$; // count of iterations of the main loop

$s := rf\_GI(\lambda_i)$; // generation of an arbitrary random variable $s-$ time to the first event (failure)

$ss := rf\_GI(\delta(x))$; // generation of an arbitrary random variable $ss-$ time of repair of the failed element)

$t_{nextfail} := t + s$;

$t_{nextrepair} := t + ss$;

while $t < \infty$ **do**

  if $i = 0$ **then**

    $t_{nextrepair} := \infty$; $j := j + 1$; $t := t_{nextfail}$;

  else

    for(v in $1 : (N - 1)$)

      if $i = v$ **then**

        $S_1 := rf\_GI(\lambda_i)$;$S_2 := rf\_GI(''\delta(x)'')$;

        $t_{nextfail} := t + S_1$; $t_{nextrepair} := t + S_2$;

        if $t_{nextfail} < t_{nextrepair}$ **then**

          $j := j + 1$; $t := t_{nextfail}$;

        else

          $j := j - 1$; $t := t_{nextrepair}$;

        end

      else

        $i = N$; $t_{nextfail} := \infty$; $j := j - 1$; $t := t_{nextrepair}$;

      end

      if $t > T$**then**

        $t = T$

      end

    $r[,,k] := [t, i, j]$; $i := j$; $k := k + 1$;

  end do

Calculate the duration of stay in each state $i, i = 0, 1, 2, \ldots N$. The formula for calculating stationary probabilities is:

$$\widehat{P}_i = \frac{1}{NG} \sum_{j=1}^{NG} (\text{duration of stay in state } i/T)_j$$

end

Table 1 shows the values of the coefficient $\rho = \frac{a_1}{b_1}$ — the relative repair rate (i.e. the ratio of the average uptime of the main element to the average repair time of the failed element), at which the specified level of stationary reliability $1 - \pi_3 = 0.9; 0.99; 0.999$. To analyze and compare the results, the following distributions were chosen: Exponential (M), Weibull-Gnedenko (WB), Lognormal (LN).

We consider particular cases of the model at $\rho = 25$; $N = 3$; $NG = 100$; $T = 1000$; where $b_1 = 1$; T_1 - system uptime; T_2 - repair time of a failed element.

Table 1. Values of the relative repair rate, at which a given level of the system's stationary reliability is achieved.

T_1	T_2								
	$M(1/b_1)$			$WB(W)$			$LN(sig)$		
	0.9	0.99	0.999	0.9	0.99	0.999	0.9	0.99	0.999
$M(1/a_1)$	1.6	4.2	9.1	1.5	4.7	11.3	1.6	4.4	9.7
$WB(W)$	3.2	12.2	25	2.9	11.6	25	3.3	11.9	25
$LN(sig)$	1.6	3.9	7.6	1.6	4.5	9.7	1.6	4	7.2

A sufficiently high level of system reliability is achieved with a relatively small excess of the average values of the uptime by the repair time, except when the uptime of the system elements is distributed according to the Weibull-Gnedenko distribution.

Figure 2 presents graphs of the probability of system uptime; and Fig. 3 shows the uniform difference of the results of the simulation model between the exponential and the non-exponential cases.

The obtained results demonstrate a high asymptotic insensitivity of the stationary reliability of the system. It can be seen that the differences between the curves during "fast" recovery become vanishingly small for all the considered distributions of the repair time of the system elements. For example, already starting from the value $\rho = 10$, all the curves are almost indistinguishable.

Graphical results from Fig. 3 show that the uniform difference between the models $\langle M_3/M/1\rangle$ and $\langle LN_3/M/1\rangle$; $\langle M_3/M/1\rangle$ and $\langle LN_3/LN/1\rangle$ tends to zero with a small increase in ρ.

2.2 Simulation Model for Assessment of the $\langle GI_N/GI/1\rangle$ System Reliability

In this case, the system stops functioning after all N elements have failed, and the maximum model run time T is equal to ∞. For clarity, the simulation model is presented graphically in Fig. 4.

For a better understanding and reproducibility of the simulation model, in addition to the flowchart, the algorithm of the discrete-event process of simulation modeling is also provided in the form of pseudo-code with comments (Algorithm 1).

Algorithm 2. Pseudocode of the simulation process of the system $\langle GI_N/GI/1\rangle$.

Input: a1, b1, N, NG, GI.

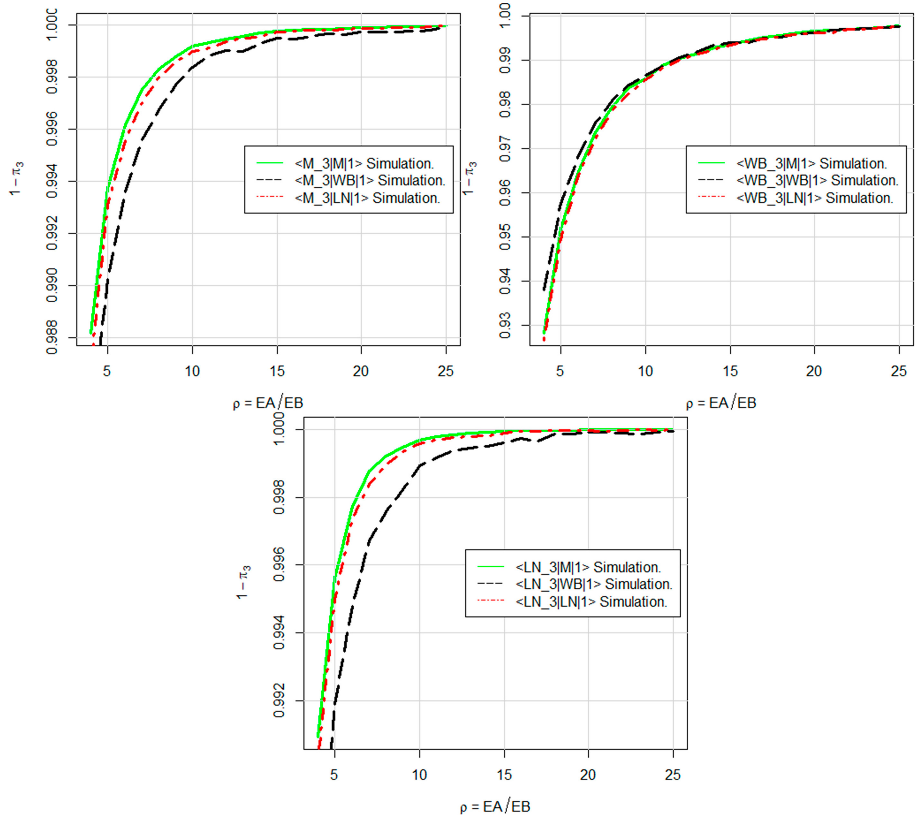

Fig. 2. Graphs of the probability of system uptime versus relative recovery rate for the systems $\langle M_3/GI/1\rangle$, $\langle WB_3/GI/1\rangle$ and $\langle LN_3/GI/1\rangle$.

Output: Reliability assessment $\widehat{ET}$.

Begin

array $r[\,] := [0, 0, 0]$; // multi-dimensional array containing results, k-step of the main cycle (loop)

double $t := 0.0$; // time clock initialization

int $i := 0$; $j := 0$; // system state variables

double $t_{nextfail} := 0.0$; // variable in which time until the next element failure

double $t_{nextrepair} := 0.0$; // variable in which time is stored until the next repair is completed

int $k := 1$; // count of iterations of the main loop

$s := rf\_GI(\lambda_i)$; // generation of an arbitrary random variable $s-$ time to the first event (failure)

$ss := rf\_GI(\delta(x))$; // generation of an arbitrary random variable $ss-$ time of repair of the failed element)

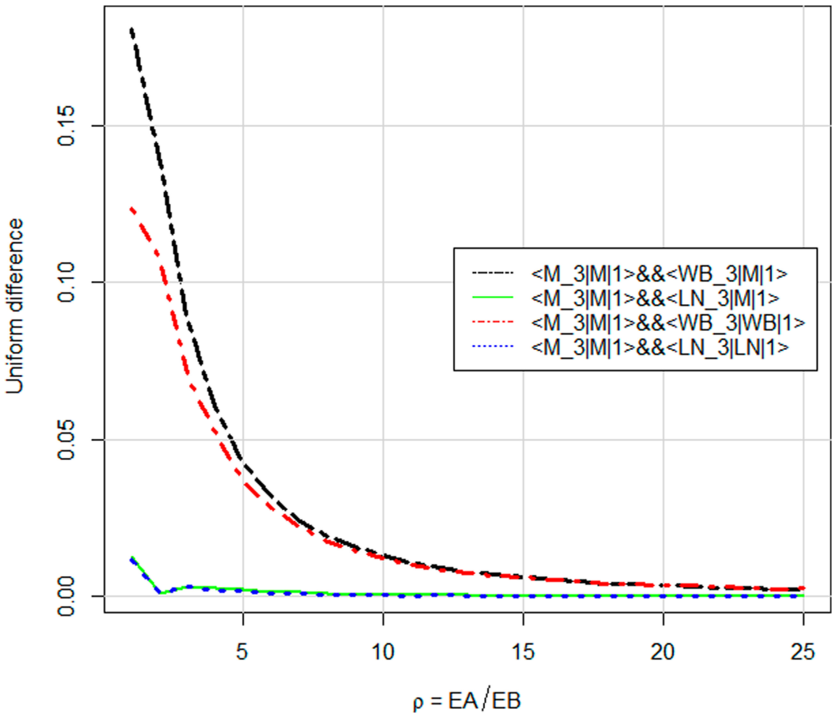

Fig. 3. Graphs of the uniform difference in the results of the simulation model as a function of ρ.

$t_{nextfail} := t + s$;
$t_{nextrepair} := t + ss$;
while $t < \infty$ **do**
  if $i = 0$ **then**
    $t_{nextrepair} := \infty$; $j := j + 1$; $t := t_{nextfail}$;
  else
    for$(v$ in $1 : (N - 1))$
      if $i = v$ **then**
        $S_1 := rf\_GI(\lambda_i)$;$S_2 := rf\_GI(''\delta(x)'')$;
        $t_{nextfail} := t + S_1$; $t_{nextrepair} := t + S_2$;
        if $t_{nextfail} < t_{nextrepair}$ **then**
          $j := j + 1$; $t := t_{nextfail}$;
        else
          $j := j - 1$; $t := t_{nextrepair}$;
        end
      else
        $i = N$; **then break** ;
      end
      end

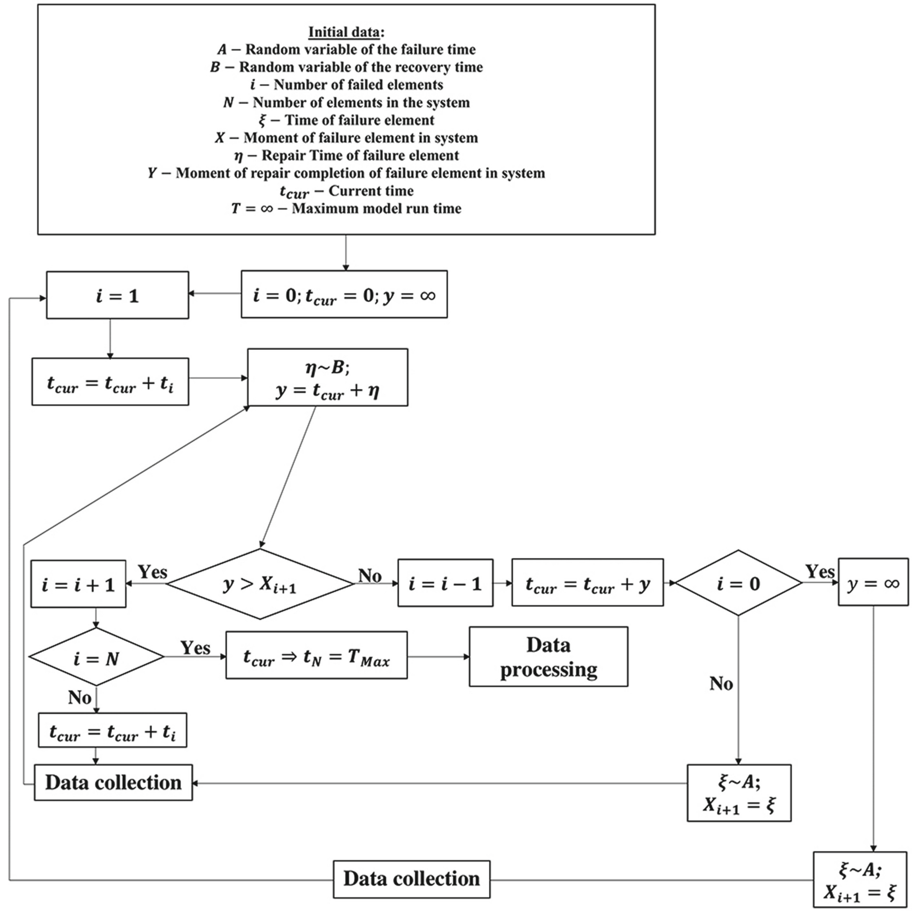

Fig. 4. Flowchart of the simulation model for evaluating system reliability.

$r[,,k] := [t, i, j];\ i := j;\ k := k + 1;$

end do

Calculate the duration of stay in state N. The formula for calculating the reliability measure is:

$$\widehat{ET} = \frac{1}{NG} \sum_{i=1}^{NG} (\text{duration of stay in state } N)_i$$

end

Table 2 shows the values of the reliability estimates of the system (estimates of the mean time to failure of the system) with the time spent on modeling. The same distributions were chosen: Exponential, Weibull-Gnedenko, Lognormal.

We consider particular cases of the model at $\rho = 25$; $N = 3$; $NG = 10000$; where $b_1 = 1$; T_1 - system uptime; T_2 - repair time of a failed element.

Table 2. Values of the estimates of the mean time to failure of the $\langle GI_3/GI/1\rangle$ system.

T_1	T_2		
	$M(1/b_1)$	$WB(W)$	$LN(sig)$
$M(1/a_1)$	16530.34	19566.77	25.18033
$WB(W)$	28.57675	927.8087	564.099
$LN(sig)$	249458.5	71212.42	190780.8

As it can be seen from Table 2, the most reliable model is a model with a lognormal distribution of uptime and an exponential distribution of the repair time of a failed element.

Figure 5 presents graphs of the empirical distribution function $F^*(t)$ and the empirical reliability function $R^*(t)$.

The results also show the high asymptotic insensitivity of the empirical distribution function and the corresponding empirical reliability function of the system to the shapes of the uptime and repair time distributions of the system's elements.

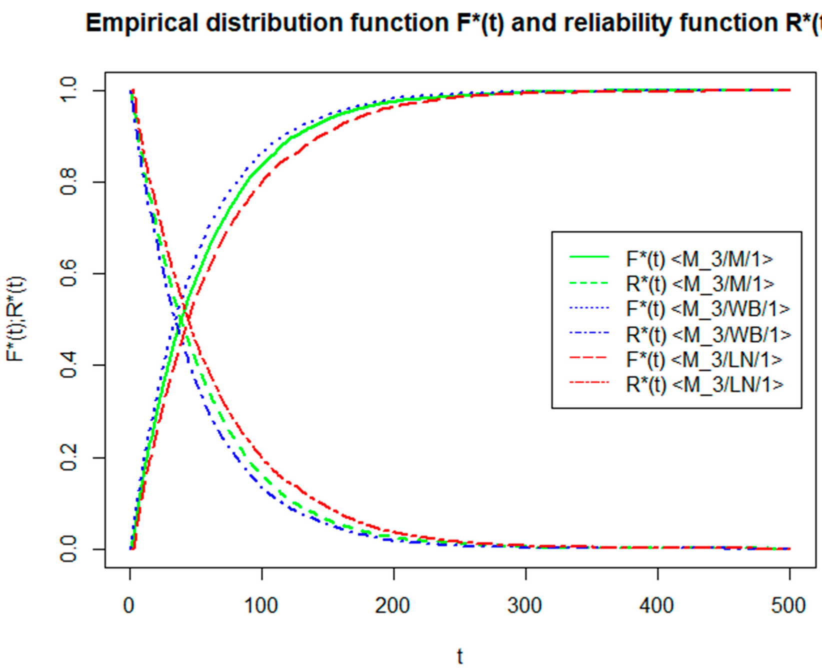

Fig. 5. Graphs of the empirical distribution function $F^*(t)$ and the empirical reliability function $R^*(t)$

3 Conclusion

In practice, redundancy is a common approach to enhance the reliability of communication systems, which may be designed to avoid communication failure by including redundant components that are active upon the failure of a primary component. To address these practical issues, in the current work we considered a repairable multiple cold standby system $\langle GI_N/GI/1\rangle$ with one repair device, with an arbitrary distribution function of uptime and an arbitrary distribution function of repair time of its elements. This paper is a continuation of the previous studies in this area that were focused on analytical models. For the considered system we applied the discrete-event simulation approach to perform the assessment of the system-level reliability and obtained the values of the relative repair rate at which the given level of the system's stationary reliability is achieved. Graphic and numerical results show a high asymptotic insensitivity of the stationary system reliability to the input distributions. The differences between the curves under "fast" recovery become vanishingly small for all the studied special cases of distributions. It was shown that the most reliable case is the model with a lognormal distribution of uptime and an exponential distribution of the repair time of a failed element. The graphic results also show a high asymptotic insensitivity of the empirical distribution function and the empirical reliability function of the system.

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References

1. Houankpo, H.G.K., Kozyrev, D.V., Nibasumba, E., Mouale, M.N.B.: Mathematical model for reliability analysis of a heterogeneous redundant data transmission system. In: 12th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), Brno, Czech Republic, vol. 2020, pp. 189–194 (2020). https://doi.org/10.1109/ICUMT51630.2020.9222431
2. Houankpo, H.G.K., Kozyrev, D.V.: Sensitivity analysis of steady state reliability characteristics of a repairable cold standby data transmission system to the shapes of lifetime and repair time distributions of its elements. In: Information and Telecommunication Technologies and Mathematical Modeling of High-Tech Systems, Moscow, Russia, pp. 107–113 (2017)
3. Ahmed, W., Hasan, O., Pervez, U., Zadir, J.: Reliability modeling and analysis of communication networks. J. Netw. Comput. Appl. **78**, 191–215 (2017)
4. Ometov, A., Kozyrev, D.V., Rykov, V.V., Andreev, S., Gaidamaka, Y.V., Koucheryavy, Y.: Reliability-centric analysis of offloaded computation in cooperative wearable applications. In: Wireless Communications and Mobile Computing, vol. 2017, p. 15 (2017). Article ID 9625687. https://doi.org/10.1155/2017/9625687

5. Rykov, V., Kozyrev, D., Zaripova, E.: Modeling and simulation of reliability function of a homogeneous hot double redundant repairable system. In: Proceedings of the 31st European Conference on Modelling and Simulation, ECMS2017, pp. 701–705 (2017). https://doi.org/10.7148/2017-0701
6. Efrosinin, D., Rykov, V.: Sensitivity analysis of reliability characteristics to the shape of the life and repair time distributions. In: Dudin, A., Nazarov, A., Yakupov, R., Gortsev, A. (eds.) ITMM 2014. CCIS, vol. 487, pp. 101–112. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-13671-4_13
7. Efrosinin, D., Rykov, V.V., Vishnevskiy, V.: Sensitivity of reliability models to the shape of life and repair time distributions. In: 9th International Conference on Availability, Reliability and Security (ARES 2014), pp. 430–437. IEEE (2014). Published in CD: 978-I-4799-4223-7/14. https://doi.org/10.1109/ARES.40
8. Rykov, V.V., Kozyrev, D.V.: Analysis of renewable reliability systems by markovization method. In: Rykov, V.V., Singpurwalla, N.D., Zubkov, A.M. (eds.) ACMPT 2017. LNCS, vol. 10684, pp. 210–220. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-71504-9_19
9. Rykov, V., Kozyrev, D.: On sensitivity of steady-state probabilities of a cold redundant system to the shapes of life and repair time distributions of its elements. In: Pilz, J., Rasch, D., Melas, V.B., Moder, K. (eds.) IWS 2015. SPMS, vol. 231, pp. 391–402. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-76035-3_28
10. Lisnianski, A., Laredo, D., Haim, H.B.: Multi-state markov model for reliability analysis of a combined cycle gas turbine power plant, Published. In: Second International Symposium on Stochastic Models in Reliability Engineering. Life Science and Operations Management (SMRLO) (2016). https://doi.org/10.1109/SMRLO.2016.31
11. Wang, W., Di Maio, F., Zio, E.: Three-loop Monte Carlo simulation approach to multi-state physics modeling for system reliability assessment. Reliab. Eng. Syst. Saf. **167**, 276–289 (2017)
12. Li, X.-Y., Huang, H.-Z., Li, Y.-F., Zio, E.: Reliability assessment of multi-state phased mission system with non-repairable multi-state components. Appl. Math. Model. **61**, 181–199 (2018)
13. Lin, Y.-H., Li, Y.-F., Zio, E.: A comparison between Monte Carlo simulation and finite-volume scheme for reliability assessment of multi-state physics systems. Reliab. Eng. Syst. Saf. **174**, 1–11 (2018)
14. Tourgoutian, B., Yanushkevich, A., Marshall, R.: Reliability and availability model of offshore and onshore VSC-HVDC transmission systems. In: 11th IET International Conference on AC and DC Power Transmission, 13 July 2015. https://doi.org/10.1049/cp.2015.0101
15. Kala, Z.: Sensitivity analysis in probabilistic structural design: a comparison of selected techniques. Sustainability **12**(11), 19 (2020). https://doi.org/10.3390/su12114788
16. Kala, Z.: Quantile-oriented global sensitivity analysis of design resistance. J. Civil Eng. Manage. **25**(4), 297–305 (2019). https://doi.org/10.3846/jcem.2019.9627. ISSN 1392–3730. E-ISSN 1822–3605
17. Kala, Z.: Estimating probability of fatigue failure of steel structures. Acta et Commentationes Universitatis Tartuensis de Mathematica **23**(2), 245–254 (2019). https://doi.org/10.12697/ACUTM.2019.23.21. ISSN 1406–2283. E-ISSN 2228–4699
18. Kala, Z.: Global sensitivity analysis of reliability of structural bridge system. Eng. Struct. **194**, 36–45 (2019). https://doi.org/10.1016/j.engstruct.2019.05.045. ISSN 1644–9665

19. Cao, J., Wang, Z., Shen, Y.: Research on modeling method of complex system mission reliability simulation. In: 2012 International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering (2012). https://doi.org/10.1109/IC'R2MSE.2012.6246242
20. Gu, J., Zhu, C., Shang, L., Dick, R.: Application-specific multiprocessor system-on-chip reliability optimization. IEEE Trans. Very Large Scale Integr. (VLSI) Syst. **16**(5) (2008). https://doi.org/10.1109/TVLSI.2008.917574