Modeling of Vibration Separation of Bulk Materials Based on the Theory of Random Processes

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Abstract The separation of granular materials into specific fractions by size on sieve classifiers is a large and complex system both in terms of the separation process and hardware design. The separation process depends on many design and operating parameters, the shape and size of the sieve cells, the number of sieves, as well as the fractional composition, shape and particle size of the material to be separated, i.e. is a cyber-physical system (CPS). The key to CPS is the mathematical model of the separation process, which is used in the control system. A mathematical model of the process of separation of granular materials on sieve classifiers based on the theory of random processes is developed. As a random process, the linear particle density of the considered fractions on the sieve surface is considered and a system of stochastic differential equations is constructed for its determination. The obtained solutions allow us to determine the recovery coefficient and evaluate the separation efficiency. Based on the constructed mathematical model, the design and operational parameters of the classifier were optimized. The performance criteria and separation efficiency are considered as optimization criteria. All this allows us to control the process of separation of granular materials by determining the optimal values of the operating parameters of the classifier depending on the fractional composition, shape and size of the particles of the original material to be separated and its other characteristics.

Keywords Bulk material · Separation · Stochastic modeling · Optimization

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A. G. Kravets et al. (eds.), *Cyber-Physical Systems: Modelling and Intelligent Control*, Studies in Systems, Decision and Control 338,
https://doi.org/10.1007/978-3-030-66077-2_6

1 Introduction

The separation of granular materials into specific fractions by size, shape, density and other characteristics is a common technological process. Such processes can be used as an independent operation to isolate the target product of a given fractional composition, as well as an auxiliary operation to remove impurities before grinding the material. To separate bulk materials by size, vibration devices are often used, in particular, multilevel sieve classifiers [1, 2]. The processes of separation of bulk materials using similar equipment depend on a large number of design and operating parameters, the shape and size of the sieve cells, the number of sieves, as well as the fractional composition, shape and particle size of the material to be separated, i.e. It is a large and complex system both in terms of the process of separation and hardware design. Thus, the process of separating granular materials can be considered as a cyber-physical system (CPS). The key to CPS is the mathematical model of the separation process, which is used to determine the optimal design parameters of the equipment used and the operational parameters of its operation to control the process itself [3–8].

Mathematical models of the processes that occur during the separation of granular materials are the basis for the optimal design and technological calculation of the sieve classifier. In the article [1], particle motion in an oscillating medium was studied, various models of vibrational motion were considered, and dependences were obtained for the average velocity and segregation velocity. In the articles [2, 3, 5], the process of isolating target products on sieve classifiers was studied using the theory of random processes. In particular, in the article [3], the process of separating granular media on sieve classifiers was considered as a diffusion process and a change in the concentration of the number of passage particles along the thickness of a layer of granular material depending on time towards a vibrating surface was studied from the standpoint of Markov processes and described by the Kolmogorov-Fokker-Plank (K-F-P) equation.

In the article [5], the theory of Poisson processes is used to describe the separation process on screens. In many works, for example [4–7], the kinetics of the separation process on sieve classifiers has been considered. For example, in the article [6] the stochastic process of motion of small particles in a large medium in the direction of a sieve is considered on the basis of the theory of Markov chains. The work [7] is devoted to the study of segregated flows during the organization of various processes for processing granular materials. These works propose the principle of organizing technological processes with controlled segregated flows, the formation of which is accompanied by most of the processes of processing dispersed materials associated with the mutual movement of particles. In the article [8], the process of separating granular medium by density on a sieve device was studied. This device consists of a pair of mesh screens with 1 mm square apertures mounted above the surface of a vibrated fluidized bed. The upper mesh contains a large central hole referred to as a Sink-Hole. This device allows you to effectively separate bulk material with particle sizes in the range of 2.8–8.0 mm. The Monte Carlo simulation screening probability as described in the works [9, 10].

The probabilities of sifting particles into sieve cells were studied in the articles [2, 4, 5]. The probability of passage of particles through the openings of the sieves is determined depending on the geometric dimensions and shape of the particles and the openings of the mesh sieves, as well as the speed of vibrational motion. The movement of granular materials on a vibrating surface depending on the hydrodynamic properties of the material to be separated, particle size distribution, the shape of individual particles, the presence of specific properties, etc. can be simulated both in the approximation of a single material point and on the basis of methods of mechanics of heterogeneous media [11–16]. For example, the article [15] describes the main characteristics of the state of bulk material and an overview of the physical phenomena observed by vibration, which contributes to a better understanding of the behavior of bulk material to improve separation efficiency.

Modeling of the separation of granular materials on sieves using Markov processes was carried out in the article [17]. A system of stochastic differential equations is constructed with respect to the density distribution of the number of particles on the sieve surface. This allows you to find all the interesting characteristics of the process. However, this approach makes it possible to describe the processes of separation of granular materials into only a small number of fractions with a relatively small number of tiers of the classifier. This is due to difficulties in solving a system of equations (K-F-P) of large dimension with respect to the distribution density of a random process.

The optimization of processes associated with the separation of granular materials was considered in the works [18, 19]. As optimization criteria, equipment performance, separation efficiency, and other economic indicators can be considered.

Thus, various approaches can be used to simulate the process of separating granular materials into specific fractions by size, but taking into account the random nature of the process as a whole, the stochastic approach is most preferable [2–11, 20–23].

Despite the obvious achievements in the quantitative description of the processes of separation of granular materials, which was facilitated by the development of mathematical modeling and the widespread use of computer technology based on the theory of stochastic processes and methods of mechanics of heterogeneous media, a description of these processes taking into account discreteness, stochasticity and their optimal design are not complete. It should also be noted that there are few publications related to the optimization of separation processes, especially in a multicriteria setting. Therefore, mathematical modeling of the processes of separation of granular materials according to various criteria, taking into account stochasticity, the development of calculation methods and their optimal design are an urgent task for chemical technology and related industries.

The aim of this work is the mathematical modeling of the vibrational separation of bulk materials by size on multilevel sieve classifiers to control this process, considering it as a large and complex system.

2 Mathematical Modeling

The process of vibrational separation of bulk materials into specific fractions by size on a multi-tiered sieve classifier is considered. The working body of the multi-tiered classifier is several oscillating screening surfaces, which can be made in the form of a sieve or a bolter. Moreover, they are located one above the other, forming tiers. The initial granular material, which is characterized by a size distribution function, is fed to the beginning of the upper tier and is divided into a passage and a descent part in the process of vibrational movement. At the same time, the first largest fraction is removed from the first tier, and the product for separation at the next i-th tier is granular material sifted from the upper $i-1$-th tier.

A description of the process of isolating target products from bulk material on a sieve classifier will be carried out taking into account the stochastic nature of such processes. As a random process, we consider the value of $N_i = N_i(x, t; d_j)$, which determines the linear density of particles with dimensions d_j at a distance x from the beginning of the i-th sieve at time t. Then the system of kinetic equations describing the process of thin-layer separation of bulk materials on a sieve classifier can be represented in the form [2]:

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = \frac{\partial N_i}{\partial t} + V_{i\,av}\frac{\partial N_i}{\partial x} = \alpha_{i-1}N_{i-1} - \alpha_i N_i + \beta_i \eta_i(t), \quad \alpha_0 \equiv 0, \ i = \overline{1, n} \quad (1)$$

where α_i are the kinetic equation coefficients, n is the number of classifier screens, $\eta_i(t)$—are time-delta-correlated random functions (white noise) with known numerical characteristics $M[\eta_i] =< \eta_i(t) >= 0$ and $K[\eta_i] =< \eta_i(t)\eta_i(t+\tau) >= \Delta_i\delta(\tau)/2$, β_i is the intensity, $\Delta_i/2$ is the spectral density of white noise. A feature of Eq. (1), which allows us to call them stochastic, is the presence of an effect in the form of white noise.

The number of particles of the selected fraction on the surface of the i-th sieve at any time $t > t_i = L_i/V_{i\,av}$ is determined by the expression: $\overline{N}_i(t) = \int_0^{L_i} N_i(x, t; d_j)dx$, where L_i is the length of the sieve, $V_{i\,av}$ is the average speed of vibrational motion. The deviation of the number of $\overline{N}_i(t)$ particles from the average value at any time is related to the probability of sieving particles into sieve cells. The probability of sifting into a cell depends on the size and shape of the particles, the particle size distribution of the material to be separated, constraint conditions and other factors, as well as on the relative speed of the vibrational movement of the material. Therefore, the number of $\overline{N}_i(t)$ particles is considered as a random process. Approximation of the random process N_i by white noise is possible, because the correlation time of the random process is much shorter than the average residence time of the selected particles on the surface of the sieve. Taking into account the properties of white noise, process N_i is a Markov process; therefore, to study it, one can use the mathematical apparatus of the theory of Markov processes. Then the distribution density of the random process $W_i(N_1, \ldots, N_i, x, t)$ is determined from the solution of the system of equations (K-F-P):

$$\frac{\partial W_i}{\partial t} = -\frac{\partial}{\partial x}(V_{i\,av} W_i) - \sum_{k=1}^{i} \frac{\partial}{\partial N_k}(F_i W_i) + \frac{1}{2}\sum_{k=1}^{i+1}\sum_{j=1}^{i+1} \frac{\partial^2}{\partial x_k \partial x_j}(B_{kj} W_i),\ i = \overline{1, n} \tag{2}$$

where $x_k \equiv N_k,\ k = \overline{1, i};\ x_{i+1} \equiv x,\ B_{kj}$—is the diffusion coefficient. Solving equations (K-F-P) for large values of n is a difficult task.

The coefficients of kinetic equations in a first approximation are calculated by the dependence:

$$\alpha_i = V_{i\,av} p_i / 2a_i,$$

where p_i is the probability of sifting into the cell, $2a_i$ is the step of the i-th sieve. The coefficients α_i determine the number of particles passing through the cell in one second, thereby characterizing the rate of change of the random process.

The probability of sifting into the cell is considered as a complex event [2]: $p = p_g p_v$, where p_g is the probability, which depends on the size and shape of the sieve cell and particles of the material to be separated, and p_v is the probability, which depends on the relative speed of the particle on the vibrating surface. The calculation of the probability of sifting into a cell was considered in the article [2]. The probability of speed is determined by the formulas:

$$p_v = 2 - (\Phi(z) + \Phi(z_0)),\ z = (V_a - V_k)/\sigma,\ z_0 = V_k/\sigma,\ z = (V_a - V_k)/\sigma$$

where V_a is the particle velocity amplitude relative to the sieve, $\Phi(x)$ is the standard normal distribution function, V_k, σ are the parameters of the normal law, which are determined in the process of identifying the constructed models. To do this, the calculated values of the extraction coefficient are compared with the experimental values obtained at some well-defined high-speed modes of operation of the apparatus.

The vibrational motion of granular media was studied in sufficient detail in the article [1], the calculation of the average speed, the relative velocity amplitudes for some modes of vibrational motion are given in the work [2].

Consider the solution of differential Eq. (1) under the following initial and boundary conditions:

$$N_i(0, x) = 0 \text{ for } i = \overline{1, m} \text{ and } N_1(t, 0) = \overline{N}_{10}(t),\ N_i(t, 0) = 0 \text{ for } i = \overline{2, m}. \tag{3}$$

Conditions (3) determine the supply of the material to be separated only at the beginning of the upper tier of the multilevel classifier, $\overline{N}_{10}$ is the number of selected particles (1/m) that arrive at the beginning of the first sieve. Using the replacement $\tau_i = t - x/V_i,\ z = x$, Eq. (1), taking into account conditions (3), can be solved by reduction to ordinary differential equations. Then the average value of the random process N_1^j for the first sieve, taking into account conditions (3), has the form: $\overline{N}_1^j(\tau_1, z) = \overline{N}_{10}^j(\tau_1)\exp(-\alpha_1^j z/V_1)$, and the general solution of the differential equation with respect to the average value of the random process for the *i*-th sieve can be written in the form:

$$\overline{N}_i^j(\tau_i, z) = \overline{N}_{10}^j(\tau_i)\frac{\alpha_1^j \ldots \alpha_{i-1}^j}{V_2 \ldots V_i}[\Psi_{1,i}\exp(-\alpha_1^j z/V_1) + \ldots + \Psi_{i,i}\exp(-\alpha_i^j z/V_i)] \tag{4}$$

where

$$\Psi_{k,j} = 1\Big/\Big[(\alpha_1^j/V_1 - \alpha_k^j/V_k) \times \ldots \times (\alpha_{k-1}^j/V_{k-1} - \alpha_k^j/V_k) \times (\alpha_{k+1}^j/V_1 - \alpha_k^j/V_k) \times \ldots \times (\alpha_i^j/V_i - \alpha_k^j/V_k)\Big], \quad i \geq 2$$

These results can also be obtained on the basis of the theory of Poisson processes.

In the general case, the solution of the system of equations (K-F-P) regarding the distribution density of a random process is carried out by numerical methods. When approximated by white noise, a random N_i process is normal. Therefore, knowing the numerical characteristics of the random process, we can write an approximate solution for the distribution density and transition probabilities, which are fundamental solutions to the Cauchy problem under delta-shaped initial conditions:

$$W_1^j\left(N_1^j, \tau_1\right) = \frac{1}{\sqrt{2\pi\,\sigma^2_{N_1^j}(\tau_1, z)}}\exp\left(-\frac{\left(N_1^j(\tau_1, z) - \overline{N}_1^j(\tau_1, z)\right)^2}{2\sigma^2_{N_1^j}(\tau_1, z)}\right),$$

$$W_{i/i-1}^j\left(N_i^j, \tau_i \middle| N_{i-1}^j, \tau_{i-1}\right) = \frac{1}{\sqrt{2\pi\,\sigma^2_{N_i^j}(\tau_i, z)}}\exp\left(-\frac{\left(N_i^j(\tau_i, z) - \overline{N}_i^j(\tau_i, z)\right)^2}{2\sigma^2_{N_i^j}(\tau_i, z)}\right).$$

For the first sieve, an analytical solution to Eq. (2) can be obtained. By replacing $\varphi = N_1\exp(-\alpha_1 x/V_1)$, $\xi = \frac{\beta_1^2\Delta_1}{2\alpha_1}(1 - \exp(-2\alpha_1 x/V_1))$, the equation (K.-F.-P.) for the first sieve is reduced to the simplest diffusion equation: $\frac{\partial \tilde{W}_1}{\partial \xi} = \frac{1}{2}\frac{\partial^2 \tilde{W}_1}{\partial \phi^2}$ and the solution of the resulting equation is written as an integral convolution:

$$\tilde{W}_1(\varphi, \xi, \tau_1) = \frac{1}{\sqrt{2\pi\xi}}\int_0^\infty \overline{W}_{10}(\theta, \tau_1)\left(\exp\left(-\frac{(\varphi-\theta)^2}{2\xi}\right) - \exp\left(-\frac{(\varphi+\theta)^2}{2\xi}\right)\right)d\theta,$$

where $\overline{W}_{10}(N_1, \tau_1)$ is the distribution density of N_1 in the initial section.

Then the general solution of the equation (K-F-P) for the first sieve has the form: $W_1(N_1, x, \tau_1) = \tilde{W}_1(\varphi, \xi, \tau_1)\exp(-\alpha_1 x/V_1)$.

For the remaining screens, starting from the second, obtaining an analytical solution to the equations of system (2) is a difficult task.

The coefficients of the kinetic equations of system (1) α_i^j determine the probability of passage of particles of the *j*-th fraction through the holes of the *i*-th sieve per unit time, thereby characterizing the rate of the process. Then the average value of the number of passes of the *j*-th fraction from the *i*-th sieve, taking into account solution

(4) for the time Δt_i, is determined by the formula:

$$\int_0^{\Delta t_i}\int_0^{L_i} \alpha_i^j \overline{N}_{i0}^j \exp\left(-\alpha_i^j z/V_i\right) dz\,dt = V_i\,\Delta t_i\,\overline{N}_{i0}^j\left(1-\exp(-\alpha_i^j L_i/V_i\right), \quad (5)$$

where $\overline{N}_{i0}^j$ is the number of particles of the *j*-th fraction per unit length of the *i*-th sieve in its initial section. The expression $V_i\,\Delta t_i\,\overline{N}_{i0}^j$ in (5) determines the total number of particles of the *j*-th fraction entering the *i*-th sieve during the time Δt_i. Then, the extraction coefficient of the *j*-th fraction from the *i*-th sieve, taking into account (5), is determined by the formula:

$$\eta_i^j = \exp(-\alpha_i^j L_i/V_i),\ i = \overline{1, n}\,,\ j = i,\ i+1,$$

where $j = i$ is the coarse (passing) fraction, $j = i + 1$ is the next largest (passing) fraction. The classification process will be the more effective the more you can get the extraction and less "pollute" the products. The separation efficiency on the *i*-th sieve is calculated by the dependence:

$$E_i = \eta_i^i(1-\eta_i^{i+1}) \times 100\%,\ i = \overline{1, n}\,,$$

where η_i^{i+1} is the fractional fraction of the coarse fraction in the coarse fraction, which for the *i*-th fraction is considered as the fraction of impurities in the target product.

Figure 1 shows the results of calculations of the distribution of the linear particle density of the target products along the first and second sieves of the multilevel classifier. Based on the obtained solutions, the extraction coefficients of the target products are calculated. Also, the quality of the sieved residues from the sieves is evaluated. For this, the proportion of non-target products (small) that are in the target product is determined. Based on the constructed mathematical models to determine the optimal values of the structural and operational parameters of the classifier, it is possible to formulate and solve the optimization problem in a multi-criteria setting [19].

3 Separation Process Optimization

The efficiency of the separation process can be evaluated by various criteria [18]. Separation results can be characterized by indicators such as, for example, recovery, pollution, concentration, etc. This creates some uncertainty when evaluating the operation of the equipment used and the quality of separation. In practice, the quality of separation is most fully evaluated by two indicators—the degree of extraction of the desired fraction from the starting material and its contamination.

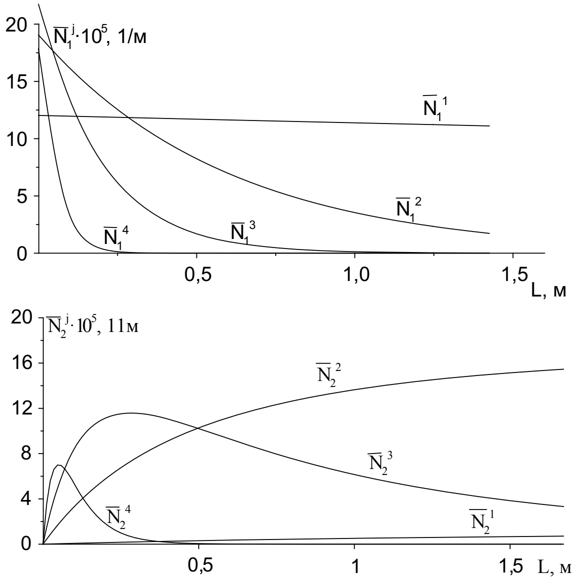

Fig. 1 Change in the linear particle density of the target products along the first and second sieves of the multi-tiered classifier: $\alpha_1^1 = 5.78\text{E}-3$; $\alpha_1^2 = 8.42\text{E}-2$; $\alpha_1^3 = 2.57\text{E}-1$; $\alpha_1^4 = 9.08\text{E}-1$; $\alpha_2^1 = 3.66\text{E}-4$; $\alpha_2^2 = 2.77\text{E}-3$; $\alpha_2^3 = 7.18\text{E}-1$; $\alpha_2^4 = 6.93\text{E}-1$ (sek^{-1}); $V_{cp} = 0.05$ m/sek; sizes of fractions: 1–$(0.8 \div 0.9) \cdot 10^{-3}$; 2–$(0.7 \div 0.8) \cdot 10^{-3}$; 3–$(0.6 \div 0.7) \cdot 10^{-3}$; 4–$(0.5 \div 0.6) \cdot 10^{-3}$ (m)

The processes of separation of granular materials on sieve classifiers depend on a large number of design and operating parameters, characteristics of the shared material, i.e. is a large and complex system. To control this process, on the basis of the constructed mathematical model, the optimization problem is constructed in a multi-criteria setting [19]. The optimization results will allow you to control the separation process by adjusting the operating parameters of the process depending on the fractional composition, shape and particle size of the shared material. The performance of the apparatus and the coefficients of separation efficiency on screens are considered as criteria:

$$\max Q(A, \omega, \alpha, \beta, h, B) = \rho_c h B V_{av} ,$$
$$\max Ef_i(A, \omega, \alpha, \beta, D_i, L_i, r_i) = \eta_i (1 - R_i) \times 100\% , \quad i = \overline{1, m} ,$$

$$\text{under conditions:} x_j^{\min} \le x_j \le x_j^{\max}, \ \varphi_k^{\min} \le \varphi_k(A, \omega, \alpha, \beta) \le \varphi_k^{\max}, \qquad (6)$$

where $x_j^{\min}$, $x_j^{\max}$ is the smallest and greatest value of the component of the vector $\overline{x} = (A, \omega, \alpha, \beta, D, L, h, r)$, ϕ_k are the functional limitations associated with the selected speed regime, A is the amplitude of the oscillations of the sieves, ω is the frequency of the oscillations of the sieves, α, β are the angles of inclination and vibration of the sieves, ρ_c is the bulk density, L_i is the length of the i-th sieve, B is the width of the sieve, D_i is the diameter of the cells of the i-th sieve, V_{av} is the average velocity of the granular material on the first sieve, h is the thickness of the layer of granular material at the beginning of the first sieve, R_i is the proportion of non-target products in the target hopper, r_i—requirements for separation products, for example, $R_i < r_i$.

Optimization on a multi-criteria basis makes it possible to select design and operational parameters of equipment, taking into account several criteria necessary for the production. For example, along with efficiency, it is necessary to take into account the productivity indicator, as well as other economic indicators. But it is important to understand that for the classification process, these indicators from a certain moment, as they individually approach the extreme values, come into conflict. This is the essence of multi-criteria tasks.

The multicriteria problem (6) is solved by known methods [2, 19, 24]. For the practical organization of the classification process, depending on the type (shape) of particles, fractional composition, the type and size of the cell hole are selected, and the length of the sieves, their angle of inclination, and the amplitude of oscillations as a result of the corresponding calculations given in the article [2]. The optimal values of the angle of vibration and vibration frequency are determined from the solution of the optimization problem. The device should be able to be controlled without significant changes in design parameters by adjusting the operating parameters depending on changes in the characteristics of the material being shared.

4 Results and Discussion

A computational experiment was carried out using a complex of programs [25]. A polymer-based granular material with linear dimensions in the range of (0.65–2.65) $\times$ 10^{-3} m and the same diameters of 1.5 $\times$ 10^{-3} m was considered. The bulk density of the material was $\rho_c = 1160$ kg/m^3, porosity – 23%. The dimensions of the first fraction are (2.15–2.65) $\times$ 10^{-3} m, the second (1.65–2.15) $\times$ 10^{-3} m. To select the first fraction, a lattice of length L = 1.5 m and width B = 1.0 m with mesh sizes of (3.2 $\times$ 3.2) $\times$ 10^{-3} m with a circular hole of diameter D = 2 $\times$ 10^{-3} was selected. The angle of inclination of α sieves was varied within 00–50, the amplitude of the oscillations A is within (3–5) $\times$ 10^{-3} m, the height of the exit slit of the loading hopper h = 4 $\times$ 10^{-3} m, the requirement for the purity of separation $r_1 \leq 8\%$. As a result of solving the multicriteria problem, a compromise solution was obtained: $Ef_1 = 87.4\%$, $Q = 1320$ kg/h. Optimum parameter values: $\omega = 48.54$ s^{-1} and $\beta = 10.06^0$. The average velocity was $V_{av} = 7.94 \times 10^{-2}$ m/s, the amplitude of the relative velocity was $V_a = 0.285$ m/s, the extraction coefficient of the first fraction was $\eta_1 = 0.95$. Thus, the constructed mathematical models make it possible to determine all the interesting characteristics of the separation process and, based on the solution of the multicriteria problem, to establish the optimal values of the design and operating parameters of the multi-tiered classifier.

5 Conclusion

The theory of random processes using the experimental results to determine the model parameters allows you to build mathematical models of the vibrational separation of bulk materials into specific fractions by size on multi-level sieve classifiers, to evaluate the separation efficiency and determine the optimal values of the design and operating parameters of the classifier. The mathematical model and the experimental results make it possible to control the separation process by adjusting its operating parameters depending on changes in the characteristics of the shared material, considering the separation process on sieves as a large and complex system.

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