



# Effect of Very-Large-Scale Motions on One- and Two-Point Statistics in Turbulent Pipe Flow Investigated by Direct Numerical Simulations

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**Abstract.** Very-large-scale motions appear in the bulk region of turbulent pipe flow. They become increasingly energetic with the Reynolds number and interact with the near-wall turbulence. These structures appear either in the shape of positive (high-speed) or negative (low-speed) streamwise velocity fluctuation. The impact of the sign of the structures on the pipe flow turbulence is analysed in this study by means of conditionally averaged one- and two-point statistics, using data from direct numerical simulations of turbulent pipe flow in a flow domain of length  $L = 42R$  and friction Reynolds numbers of  $180 \leq Re_\tau \leq 1500$ . Conditionally averaged two-point velocity correlations reveal that low-speed motions are longer and more energetic than their high-speed counterparts. The latter are predominately responsible for the Reynolds number dependency of turbulence statistics in the vicinity of the wall, which is in good agreement with observations of the so-called amplitude modulation in wall-bounded turbulence.

**Keywords:** Turbulent pipe flow · DNS · VLSM

## 1 Introduction

Very long coherent regions of energetic streamwise velocity fluctuations, also referred to as very-large-scale motions (VLSM), play an important role in high Reynolds number wall-bounded turbulence, since they carry a substantial fraction of turbulent energy [2, 6]. Although the maximum energy content of VLSM is located in the outer flow region, they penetrate deep into the buffer layer. The interaction of the large- and very-large-scale outer flow motions with the near-wall coherent structures and their impact on high-order turbulence statistics has recently been discussed by Bauer et al. [1]. Due to this interaction, logarithmic Reynolds number dependencies of different statistical moments of the

velocity distribution were found and reported. In the current study, the different contributions of high- and low-speed VLSM to one- and two-point statistics are investigated in more detail. Therefore, conditionally averaged statistics are computed from the DNS data used in Bauer et al. [1] and subsequently analyzed.

## 2 Numerical Methodology

The incompressible Navier-Stokes equations in their dimensionless form

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \frac{1}{Re_\tau} \nabla^2 \mathbf{u}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

are integrated in time using a leapfrog-Euler scheme, after being discretised by means of a fourth-order finite volume method. The governing equations as well as all quantities presented below are normalised in viscous units, i.e. the friction Reynolds number  $Re_\tau = u_\tau R / \nu$ , based on friction velocity, pipe radius and kinematic viscosity, and the viscous length  $\delta_\nu = \nu / u_\tau$ . The flow geometry is a smooth annular pipe with length  $L$  and radius  $R$ . Table 1 lists the different considered cases for which DNS were performed in Bauer et al. [1].

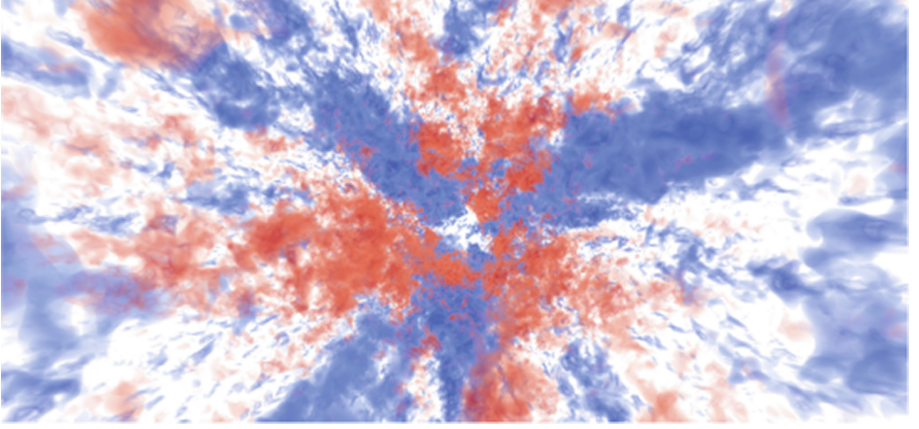
**Table 1.** Turbulent pipe flow simulation cases.  $N_z$ ,  $N_\varphi$  and  $N_r$  are the number of grid points with respect to the axial, azimuthal and radial direction, respectively.  $\Delta z^+$ , streamwise grid spacing;  $R^+ \Delta \varphi$ , azimuthal grid spacing at the wall;  $\Delta r_{min}^+$  and  $\Delta r_{max}^+$ , minimal and maximal radial grid spacing, respectively. All grid spacings normalised by wall units.

Case	$Re_\tau$	$L/R$	$N_z$	$N_\varphi$	$N_r$	$\Delta z^+$	$R^+ \Delta \varphi$	$\Delta r_{min}^+$	$\Delta r_{max}^+$
<i>P180</i>	180	42	1536	256	84	4.9	4.4	0.31	4.4
<i>P360</i>	360	42	3072	512	128	4.9	4.4	0.39	4.4
<i>P720</i>	720	42	4608	1024	222	6.6	4.4	0.49	6.6
<i>P1500</i>	1500	42	8192	2048	408	7.7	4.6	0.49	7.8

Statistical quantities are computed as follows

$$\langle u \rangle(r) = \frac{1}{L} \frac{1}{2\pi r} \frac{1}{\Delta t} \int_{t=t_0}^{t_0+\Delta t} \int_{z=0}^L \int_{\varphi=0}^{2\pi} u(z, \varphi, r, t) r d\varphi dz dt, \quad (3)$$

where angle brackets indicate averaging in both homogeneous directions and time with  $\Delta t$  being the averaging interval in time.



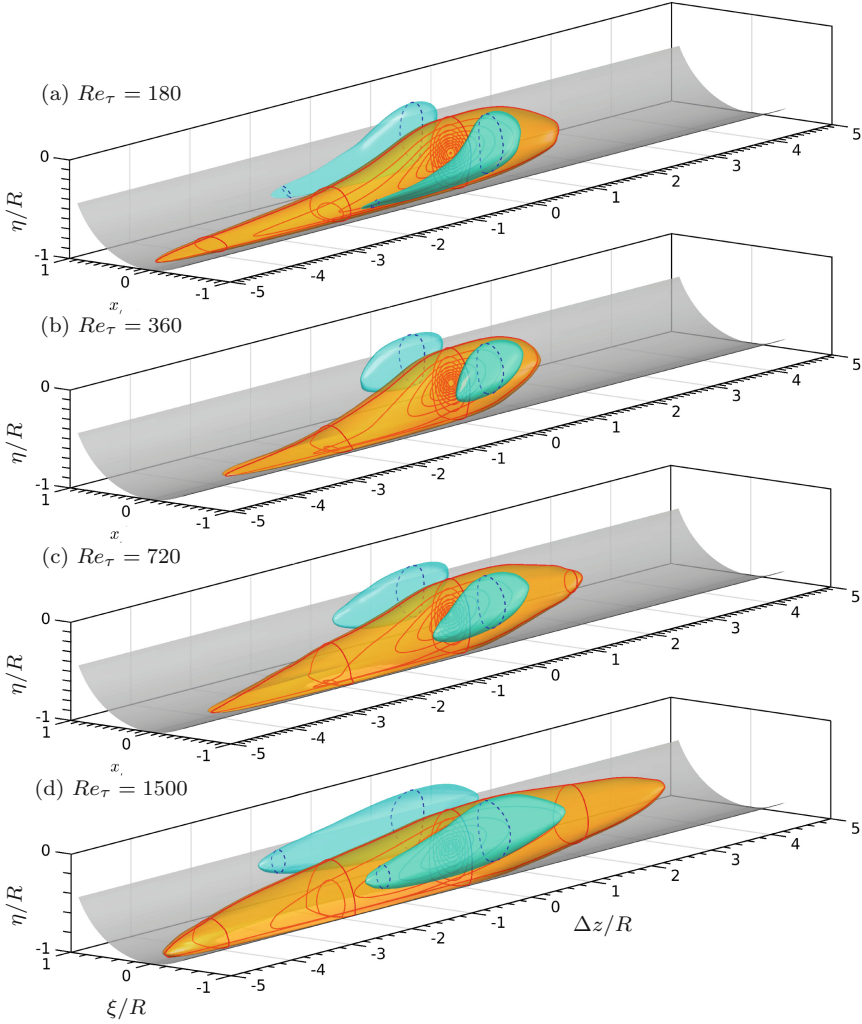
**Fig. 1.** Iso-volumes of the streamwise velocity fluctuation  $u_z'^+$  inside the pipe. Red (blue) structures exhibit values ranging from  $+(-)3$  to  $+(-)5$ .

### 3 Results

Instantaneously, VLSM can be visualised by means of iso-volumes of the streamwise velocity fluctuation  $u_z'$  shown in Fig. 1. The statistical counterpart of these turbulent coherent structures are iso-contours of the streamwise two-point velocity correlation

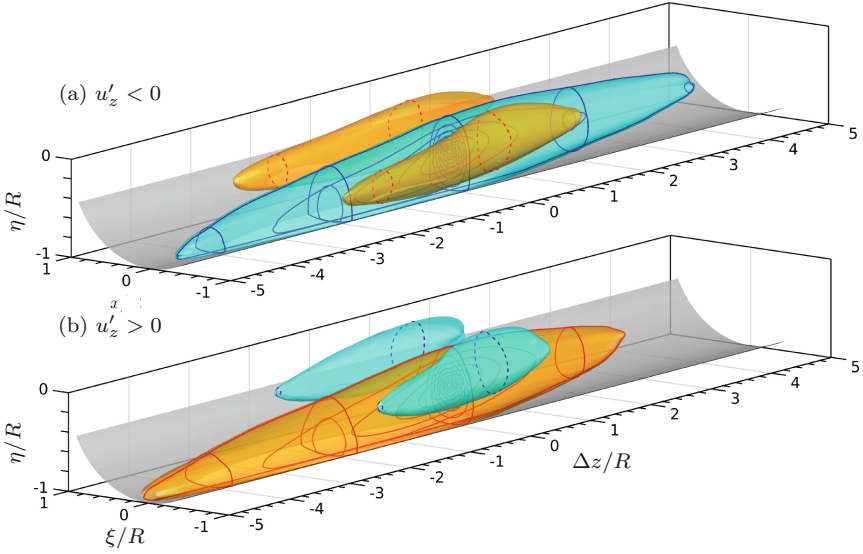
$$R_{zz}(\Delta z, \Delta \varphi, r_0 + \Delta r) = \frac{\langle u_z'(0, 0, r_0) u_z'(\Delta z, \Delta \varphi, r_0 + \Delta r) \rangle}{\langle u_z'(0, 0, r_0) u_z'(0, 0, r_0) \rangle}, \quad (4)$$

where  $\Delta z$ ,  $\Delta \varphi$  and  $\Delta r$  are the axial, azimuthal and radial separation lengths, respectively, and  $r_0$  is the reference point. Iso-contours of the streamwise three-dimensional two-point velocity correlation with  $R_{zz} = +(-)0.1$  are depicted in Fig. 2 for the different considered Reynolds numbers. The reference point of the velocity correlations is located at  $r_0 = 0.6R$ . For the lowest Reynolds number,  $Re_\tau = 180$  (Fig. 2 (a)), the footprint of near-wall velocity streaks—located at a wall distance of  $y^+ \approx 15$ —is visible in the form of near-wall tails of the iso-contours. With increasing Reynolds number these structures become less pronounced, since they scale in viscous units. VLSM, on the contrary, appear at Reynolds numbers around  $Re_\tau \approx 720$  and become increasingly energetic and clearly visible through the iso-contours shown in Fig. 2 (d). VLSM appear as either high- or low-speed motions [3]. The latter are of larger streamwise extent than the former, as the iso-contours of the conditionally averaged streamwise velocity correlations depicted in Fig. 3 show. The logarithmic Reynolds number dependency of statistical moments of the velocity distribution up to the fourth is related to the interaction of the large- and very-large outer scale motions with the near-wall turbulence [1]. In order to determine the effect of high- and low-speed motions separately, the variance and skewness of the streamwise velocity distribution are computed conditionally for regions of positive and negative

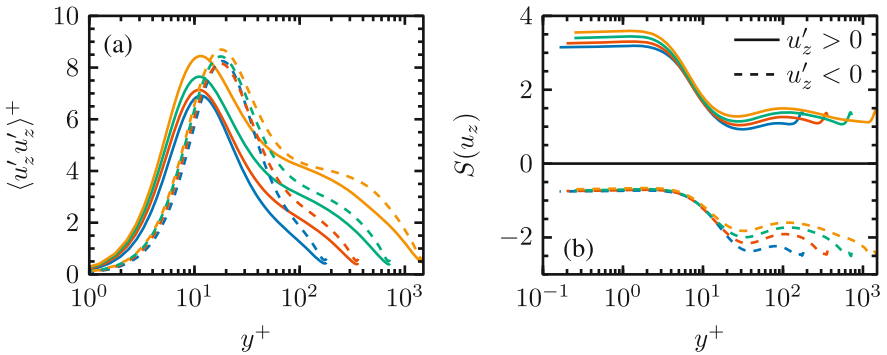


**Fig. 2.** Iso-surfaces and contours of the three-dimensional two-point velocity correlation of the streamwise velocity component  $R_{zz}(\Delta z, \varphi, r_0 + \Delta r)$  as a function of the axial, azimuthal and radial separation lengths. Reference point of the correlation at  $r_0 = 0.6R$ . Cartesian cross-sectional coordinates  $\xi = (r_0 + \Delta r)\sin(\Delta\varphi)$ ,  $\eta = (r_0 + \Delta r)\cos(\Delta\varphi)$ . (a)  $Re_\tau = 180$ , (b)  $Re_\tau = 360$ , (c)  $Re_\tau = 720$ , (d)  $Re_\tau = 1500$ . The orange (cyan) iso-surfaces exhibit values of  $+(-)0.1$ . Iso-contours values range from  $+(-)0.1$  to  $+(-)1$ , increment of  $0.1$ .

velocity fluctuations. The results are shown in Fig. 4 for both variance (a) and skewness (b) for regions of positive (solid lines) and negative (dashed lines) streamwise velocity fluctuations. For both quantities the profiles for high-speed



**Fig. 3.** Iso-surfaces and contours of the conditionally averaged three-dimensional two-point velocity correlation of the streamwise velocity component  $R_{zz}(\Delta z, \varphi, r_0 + \Delta r)$  depicted as in Fig. 2. (a) Condition  $u'_z < 0$ , cyan (orange) iso-surfaces exhibit values of  $+(-)0.1$ . (b) Condition  $u'_z > 0$ , orange (cyan) iso-surfaces exhibit values of  $+(-)0.1$ .  $Re_\tau = 1500$ .



**Fig. 4.** Variance  $\langle u'_z u'_z \rangle^+$  (a) and Skewness  $S(u_z)$  (b) of the streamwise velocity distribution. Conditionally averaged for regions of positive (solid lines) and negative (dashed lines) streamwise velocity fluctuations. —,  $Re_\tau = 180$ ; —,  $Re_\tau = 360$ ; —,  $Re_\tau = 720$ ; —,  $Re_\tau = 1500$ .

regions reflect a significantly stronger dependency on the Reynolds number in the vicinity of the wall as the ones for low-speed regions. This means that the modulation of the small scales in the vicinity of the wall is primarily caused by

high-speed outer flow motions, which is in good agreement with observations of the amplitude modulation in turbulent boundary layer flows [4, 5].

## 4 Conclusion

Observations from DNS data of turbulent pipe flow show that high- and low-speed VLSM appearing in the outer layer of the flow, interact differently with the near-wall turbulence. High-speed motions modulate the small-scale motions in the vicinity of the wall considerably more than their low-speed counterparts, which can be seen in the analysis of conditionally averaged turbulence statistics. This is consistent with the general observations of sweeps and ejections in wall-bounded turbulence. High-speed fluid is predominately moving towards the wall (sweep), whereas low-speed fluid is ejected away from the wall. Since the small scales near the wall are modulated by the large scales in the outer flow, it is plausible that this modulation is related to high-speed motions and, thus, to the sweeps.

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