Evaluation of Packet Transmission Delay Variation in the G/G/1 System

Igor Kartashevskiy, Marina Buranova(✉), and Dinara Ergasheva

Povolzhskiy State University of Telecommunications and Informatics, 23 L. Tolstoy, 443010 Samara, Russia
mburanova@yandex.ru

Abstract. Analyzing the performance parameters of IP-networks when processing multimedia streams is a very important task. There are many approaches to evaluating the quality of service parameters in the G/G/1 system.

Changing the packet delay in the network is a very significant parameter that determines the quality of traffic processing. It is particularly important for multimedia streams. The delay variation is generally defined as a packet jitter.

However, the analysis of the delay variation is often based on assumptions that do not allow the parameters to be determined with the required accuracy. This paper presents a new approach to defining packet delay variation in the G/G/1 system as delay variation. The presented approach is based on approximation of arbitrary distributions by hyperexponential distributions, i.e. modeling the G/G/1 system by the $H_2/H_2/1$ system. The EM algorithm is used to estimate the parameters of hyperexponential distributions. The paper presents the results of simulation. The packet delay variation was evaluated when processing traffic registered on a real network, CBR traffic, traffic with Pareto distribution of time intervals between packets and packet lengths, and traffic with exponential distribution of time intervals between incoming packets. Due to the fact that CBR traffic has explicit correlated properties, it can be noted that the presence of correlation inherent in CBR traffic leads to a decrease in delay variation.

Keywords: Delay variation · EM-algorithm · Hyperexponential distribution

1 Introduction

When designing and organizing infocommunication networks, it is necessary to take into account the heterogeneous nature of modern traffic. Algorithms for processing such traffic should account for the high requirements for various parameters when determining the required level of quality of processing streams of different types (data, voice, multimedia streams, etc.). The main parameters considered when determining the quality of service (QoS), are delay, delay variation (jitter variation) and loss probability. Research on this topic focuses on the problem of delay retention at the required level [1–3].

It should be noted that certain types of traffic, for example, multimedia streams, are highly critical not only to delay transmission, but also to the change in packet delay during

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transmission. In [4–8], the problem of estimating change in packet delay is raised. They show that such an assessment involves a number of difficulties. Traditionally, the main mechanism to determine these parameters was the principal tool of queuing theory, which with high accuracy allows determining the parameters of network functioning only when processing simple flows. Systems processing such flows are described by the M/M/1 model [9]. At the same time, a feature of modern processed flows is the presence of self-similarity properties characterized by heavy tail distributions for random time intervals between packets and packet durations (model G/G/1). The effect of self-similarity is largely determined by the nature of user behavior, the organization of requests and the peculiarity of the TCP protocol. Statistical models based on heavy tail distributions such as Pareto and Weibull show more accurate estimates for the characteristics describing the rate of arrival of packets and their duration.

In this case, one should take into account the fact that the particular type of distribution underlying the mathematical model significantly depends on the specific traffic implementation and requires careful analysis. Another problem associated with the use of distributions with heavy tails is the complexity of their analysis and use. When using this type of distribution, it is required to obtain Laplace transformations of these distributions, at the same time, certain problems are caused by the lack of a convenient expression for the Laplace transforms of the Pareto and Weibull distributions.

Jitter estimation is associated with certain difficulties due to the lack of accurate estimation techniques, including the lack of adequate analytical models for jitter estimation in non-Poisson flows processing systems. Previously [4, 10], solved the problem of evaluating jitter in G/G/1 systems and ensuring packet jitter at a given level. Some assumptions make it possible to determine jitter with sufficient accuracy in systems such as M/M/1, G/M/1, but in the system where random time intervals between packets and packet lengths are described by arbitrary distributions, the jitter definition is associated with great computational difficulties. It was shown in [2, 3] that a sufficiently accurate approximation of the G/G/1 system allows one to use the approximation by the $H_l/H_k/1$ system. In this case, the problem is reduced to determining the parameters of hyperexponential distributions. To develop this topic, the paper proposes to use the $H_2/H_2/1$ approximation to model G/G/1 systems.

The $H_2/H_2/1$ model can be used in various approaches to describing systems. For example, [11] provides a technique for analyzing network performance when processing self-similar traffic using a hyperexponential distribution, where the first component of the distribution shows an exponential component and the rest describe the behavior of the heavy tail. But this approach is also labor-intensive enough to estimate network parameters and requires analysis of distributions with heavy tails. It is more convenient to take approaches using approximations by the sum of two exponentials [2, 3].

In this paper, we use an approximation of arbitrary distributions of G by hyperexponential distributions. The EM-algorithm is used to determine the parameters of hyperexponential distributions. This algorithm is a fairly convenient tool for implementing an iterative search procedure using numerical methods of extremum of the objective function in various optimization problems.

There are many works devoted to the description of the EM-algorithm procedures and possible ways of its application. The EM-algorithm is very effective for finding approximations of the observed realizations of both one-dimensional and multidimensional distributions. The EM-algorithm solves the problem of statistical estimation of mixture parameters. For example, [5–7] defines the procedure for implementing the EM-algorithm in the framework of cluster analysis, particularly in relation to problems of mixture separation.

The practice of using the EM-algorithm is usually associated with the separation of a mixture of normal distributions [17]. While in queuing theory, all distributions describing traffic behavior in a modern IP network refer to random variables that take non-negative values (for example, an exponential distribution is most often used). Therefore, the development of an EM algorithm for separating a mixture of exponential distributions is relevant when using the approximation of the G/G/1 system by the $H_2/H_2/1$ system.

2 Analysis of Delay Variation

The arbitrary probability density used in the G/G/1 system is denoted by $f(x)$. Then, the approximation $f(x)$ obtained using a mixture of exponential distributions will take the form:

$$f(x) = H_N(x) = \sum_{i=1}^{N} p_i h_i(x). \tag{1}$$

where $p_i \geq 0$ is probability of the i-th component of the mixture, $h_i(x)$,

$$\sum_{i=1}^{N} p_i = 1.$$

For the $H_2/H_2/1$ model, expression (1) is obtained in the form:

$$f(x) = p\alpha_1 e^{-\alpha_1 x} + (1-p)\alpha_2 e^{-\alpha_2 x}. \tag{2}$$

The change in packet transmission delay can be defined as a random variable defined as [12].

$$J_{i+1} = |T_{i+1} - T_i|,$$

where T_i is the delay time of the i-th packet in the network node, which is determined as $T_i = W_i + Q_i$. Here W_i is the waiting time of the i-th packet in the queue, Q_i is its service time, V_{i+1} is the time interval between the arrival of the (i+1)-th and i-th packets.

The general methodology for solving the problem of jitter determining according to this approach is shown in [4–8].

In this paper, the delay variation can be determined according to [13] as a variation of the packet delay. For variation of packet delay write

$$\sigma(X) = \sqrt{D(X)},$$

where $D(X)$ is dispersion of the delay of packets.

If we assume that random variables T_i, Q_i and V_i are independent of each other and independent in the structure of each sequence of a random quantity, the index i of the corresponding probability densities can be discarded and the notation: $f_T(x)$ is probability density of random variable T, $f_V(y)$ is probability density of random variable V and $f_Q(z)$ is probability density of random variable Q.

Given the independence of the considered time intervals, the G/G/1 system can be designated as GI/GI/1.

We use hyperexponential distributions to approximate the densities under consideration. For the probability density of time intervals between packets is $f_V(y)$:

$$f_V(\tau) = p_1\gamma_1 e^{-\gamma_1\tau} + p_2\gamma_2 e^{-\gamma_2\tau}, \tag{3}$$

for the service time is $f_Q(z)$, that is determined by the parameter μ:

$$f_Q(\tau) = q_1\mu_1 e^{-\mu_1\tau} + q_2\mu_2 e^{-\mu_2\tau}, \tag{4}$$

for transit time is $f_T(x)$, that is defined by δ:

$$f_W(\tau) = \Delta_1\delta_1 e^{-\delta_1\tau} + \Delta_2\delta_2 e^{-\delta_2\tau}. \tag{5}$$

Given that the packet delay is determined by the random value of T, the dispersion of the delay can be determined according to the expression:

$$D(X) = \int_0^\infty [x - M(X)]^2 f_T(x)dx = \int_0^\infty x^2 f_T(x)dx - \left(\int_0^\infty x f_T(x)dx\right)^2, \tag{6}$$

Since for the delay time T of the packet in the system is $T = W + Q$, the probability density $f_T(y)$ is determined by the convolution of the distributions of random variables W and Q (taking into account their independence):

$$f_T(y) = \int_0^\infty f_W(u) f_Q(y-u)du \tag{7}$$

Given (4) and (5), we obtain:

$$f_T(x) = D\mu_1 e^{-\mu_1 x} + C\mu_2 e^{-\mu_2 x} \tag{8}$$

where

$$C = (1-q)\frac{g\delta_1}{\delta_1 - \mu_1} + (1-\Delta)\frac{\delta_2}{\delta_2 - \mu_2},$$

$$D = (1-\Delta)\frac{q\delta_2}{\delta_2 - \mu_2} + \Delta q\frac{\delta_1}{\delta_1 - \mu_1}.$$

As a result, for the packet delay variation taking into account (6) and (8), we can obtain:

$$\sigma = \sqrt{\frac{2D - D^2}{\mu_1^2} + \frac{2C - C^2}{\mu_2^2} - \frac{2DC}{\mu_1\mu_2}}. \tag{9}$$

Thus, the solution to the problem of estimating changes in packet delay is reduced to determining the distribution parameters (3), (4) and (5) ($q, p, \Delta, \mu_1, \mu_2, \gamma_1, \gamma_1, \delta_1, \delta_2$).

There are various approaches to determining the parameters of exponential distributions, for example, the method of determining parameters by two points (average, dispersion) of the initial distribution for independent random variables [8, 14]. Using this approach, it is possible to obtain analytical expressions of the initial moments of hyperexponential distributions up to the second order. The method is based on the use of the Laplace transform property.

If the researcher has a traffic implementation obtained in the experiment, then to determine the parameters of hyperexponential distributions, one can use an approach based on the use of the EM-algorithm (expectation-maximization) [14–19]. This method has proven itself and successfully provides reliable estimates of maximum likelihood for many applications, including estimating the density of a mixture.

The algorithm consists of two steps: E-step (expectation) and M-step (maximization). The initial data is the observed sequence $x_1, x_2, \ldots, x_N$ with a one-dimensional probability density $f(x, \theta_1, \ldots \theta_m)$ having m parameters. In this case, the implementation of the EM-algorithm will be associated with parameter estimation $\theta_1, \ldots \theta_m$.

If each element of the sample $x_1, x_2, \ldots, x_N$ can belong to the distribution of a mixture of K random variables with probability densities:

$$f^1\left(x, \theta_1^1, \ldots \theta_m^1\right), \ldots f^j\left(x, \theta_1^j, \ldots \theta_m^j\right), \ldots f^k\left(x, \theta_1^k, \ldots \theta_m^k\right),$$

the process will be associated with the assessment of the main distribution parameters of each of the indicated probability densities ($\theta_1^1, \ldots \theta_m^1, \theta_1^j, \ldots \theta_m^j, \theta_1^k, \ldots \theta_m^k$), as well as the relative shares of observations of each random variable - ($\pi^1, \ldots \pi^k$).

If the sequence in question $x_1, x_2, \ldots, x_N$ is a realization of a random variable with a probability density of distribution $f(x, \theta_1, \ldots \theta_m)$ having m parameters, then the likelihood function of the sample will take the form:

$$\mathcal{L}(\theta_1, \ldots \theta_m) = \prod_{i=1}^{N} f(x, \theta_1, \ldots \theta_m).$$

The likelihood function $\mathcal{L}$ represents the total density of individual observations for any given set of distribution parameters. The maximum likelihood score is the value of the distribution parameters that maximize $\mathcal{L}$:

$$\left(\hat{\theta}_1, \ldots \hat{\theta}_m\right) = \arg max(\mathcal{L}).$$

For Gaussian distributions, this approach gives quick and good results. Using in this case a mixture of exponential distributions ($f^j\left(x_i, \theta_1^j, \ldots \theta_m^j\right)$) complicates the problem and does not allow to use this approach in the form presented [15].

Obviously, if we cannot determine exactly which sample the observations belong to, we cannot determine which random variable generates each observation. This means that we do not know the relative distribution of observations belonging to each variable.

Therefore, the likelihood function of the sample will be:

$$\mathcal{L}\Big(\pi_1, \ldots \pi_k, \theta_1^1, \ldots \theta_m^1, \ldots \theta_1^k, \ldots \theta_m^k\Big) = \prod_{i=1}^{N} \sum_{j=1}^{k} \pi_j f^j\Big(x_i, \theta_1^j, \ldots \theta_m^j\Big),$$

taking into account that $\sum_{j=1}^{K} \pi_j = 1$.

In this case, the estimate of the maximum likelihood of the distribution parameters of the mixture will be:

$$\Big(\hat{\pi}_1, \ldots \hat{\pi}_k, \hat{\theta}_1^1, \ldots \hat{\theta}_m^1, \ldots \hat{\theta}_1^j, \ldots \hat{\theta}_m^j, \ldots \hat{\theta}_1^k, \ldots \hat{\theta}_m^k\Big) =$$

$$= \arg max\Big\{\mathcal{L}\Big(\pi_1, \ldots \pi_k, \theta_1^1, \ldots \theta_m^1, \ldots \theta_1^k, \ldots \theta_m^k\Big)\Big\}$$

Taking into account the introduced notation, the likelihood functions will take the form:

$$g(x, p, \lambda_1, \lambda_2) = p\lambda_1 e^{-\lambda_1 x} + (1-p)\lambda_2 e^{-\lambda_2 x} = p_1\lambda_1 e^{-\lambda_1 x} + p_2\lambda_2 e^{-\lambda_2 x}$$

In this case, the distribution parameters:$\pi_j = \Big(p_1^j, p_2^j\Big)$, $\theta^j = \Big(\lambda_1^j, \lambda_2^j\Big)$.

At each step of the algorithm (ν), the mixture component will be used $f_j^{(\nu)}(x_i) = \lambda_j e^{-\lambda_j x_i}$.

Then the density of the mixture is $g^{(\nu)}(x_i) = \sum_{j=1}^{k} p_j \lambda_j e^{-\lambda_j x_i}$

Accordingly, for a two-component mixture

$$p_1 = p,\ p_2 = 1 - p$$

M-step of the algorithm—the values of the distribution parameters at the current step are specified

$$p^{(\nu+1)} = \frac{f_j^{(\nu)}(x_i) p_j^{(\nu)}}{\sum_{i=1}^{N} g^{(\nu)}(x_i)} / N \tag{10}$$

$$\lambda_j^{(\nu+1)} = \frac{\sum_{i+1}^{N} \frac{f_j^{(\nu)}(x_i) x_i}{g^{(\nu)}(x_i)}}{\sum_{i+1}^{N} \frac{f_j^{(\nu)}(x_i)}{g^{(\nu)}(x_i)}} \tag{11}$$

Schematically, the principle of the EM-algorithm is presented in Fig. 1.

It is advisable to take stabilization of the values of the estimated parameters as the calculation stop criterion. Using this approach to the implementation of the EM-algorithm, the parameters of the components of the hyperexponential distributions (3) and (4) can be obtained. By defining the parameters μ_1 and μ_2 and taking into account the contribution of each of them to the average value μ, it is possible to determine δ_1

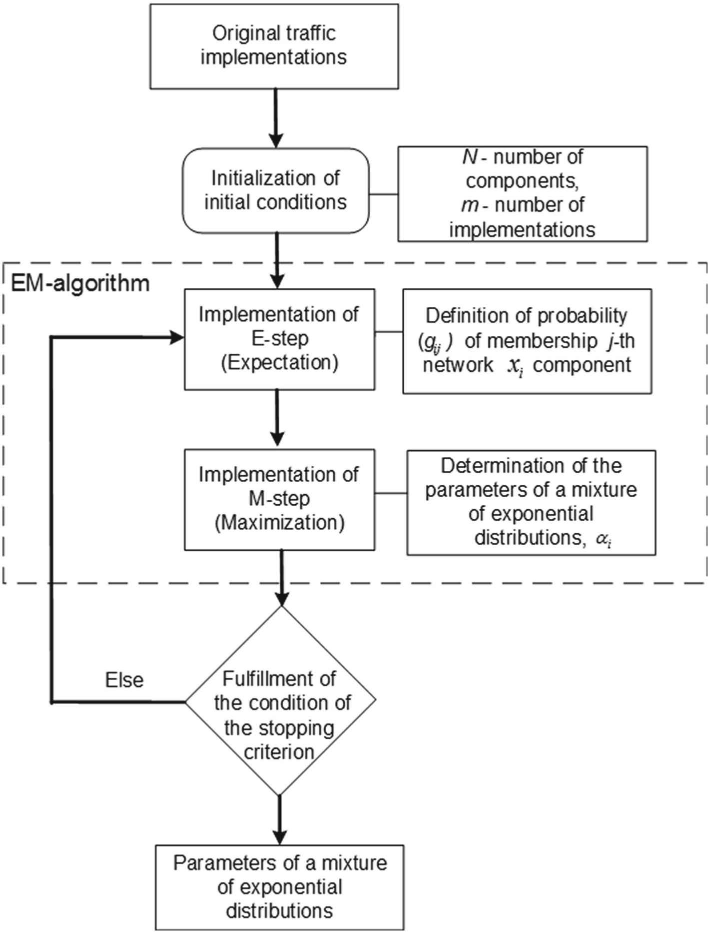

Fig. 1. EM algorithm for separating a mixture of exponential distributions

and δ_2 with shares corresponding to the contribution μ_1 and μ_2 to (4). At the same time, the need to determine the parameter value δ is obvious.

As shown in [9], the value can be determined from the equation:

$$\delta = \mu(1 - \xi), \tag{12}$$

where ξ is root of the equation $\xi = \Lambda_Q(\mu - \mu\xi)$, Λ_Q is Laplace transform of density $f_Q(\cdot)$, μ is average packet processing rate in the G/G/1 system. Given the above and taking into account (4), we obtain

$$\xi = \int_0^\infty e^{-s\tau} f_Q(\tau) d\tau$$

$$\begin{aligned} \xi = \int_0^\infty e^{-(\mu - \mu\xi)\tau} \left(p\mu_1 e^{-\mu_1\tau} + (1-p)\mu_2 e^{-\mu_2\tau}\right) d\tau \\ = p\mu_1 \frac{1}{s-\mu_1} + (1-p)\mu_2 \frac{1}{s-\mu_2}. \end{aligned} \tag{13}$$

Therefore, ξ can be defined as the root of Eq. (13).

To determine the parameter μ, consider that this is the inverse of the average packet processing time in the system—$\bar{\tau} = \frac{1}{\bar{\mu}}$. The average packet processing time is determined according to the expression

$$\bar{\tau} = \int_0^\infty \tau f_Q(\tau) d\tau$$

Given (4) we get

$$\bar{\tau} = \int_0^\infty \left(q\tau\mu_1 e^{-\mu_1\tau} + (1-q)\tau\mu_2 e^{-\mu_2\tau}\right) d\tau = \frac{q}{\mu_1} + \frac{1-q}{\mu_2}$$

As a result, we have

$$\bar{\tau} = \frac{q\mu_2 + (1-q)\mu_1}{\mu_1\mu_2}$$

$$\bar{\mu} = \frac{\mu_1\mu_2}{q\mu_2 + (1-q)\mu_1}$$

$$\delta = \frac{\mu_1\mu_2}{q\mu_2 + (1-q)\mu_1}(1-\xi)$$

3 Analysis of Statistical Characteristics of Multimedia Traffic

For analysis, we used multimedia traffic, the statistical characteristics of which are given in [7]. Given the limitations on the independence of random variables within the sequence, it is obvious that verification of compliance with this condition is necessary.

Analysis of the distribution of random time intervals between packets showed that the Weibull distribution with parameters $\alpha = 0{,}32$, $\beta = 167$ is most accurate. For packet lengths the result was obtained in the form of Pareto distribution with the parameters: $\alpha = 0{,}3$, $\beta = 60$ (Fig. 2).

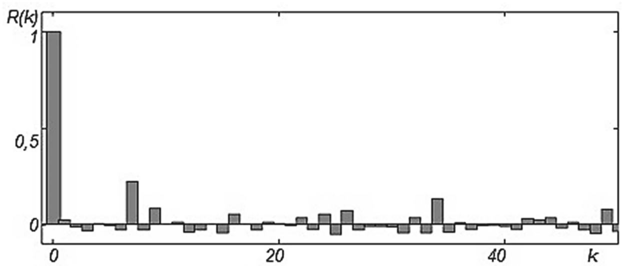

Fig. 2. Graph of correlation coefficients $R(k)$ of packet lengths

For the studied samples of the considered multimedia stream, the following dependence of the correlation coefficients was determined

The similar result was obtained for the time intervals between packets. An analysis of the dependency graphs of the correlation coefficients $R(k)$ for the considered samples shows that there are practically no correlations. This allows you to make an assumption on the independence of these random variables when analyzing the functioning parameters.

4 Results of Analytical and Simulation Modeling

4.1 Results of Analytical Modeling

When using the EM algorithm to analyze the parameters of hyperexponential distributions (3) and (4), according to the logic described above in Sect. 2, the following notation should be introduced:

- for (3), in the probability density of time intervals between packets, component weights—$\left(p_1^j, p_2^j\right) = \left(P_1^j, P_2^j\right)$, distribution parameters—$\left(\lambda_1^j, \lambda_2^j\right) = \left(\gamma_1^j, \gamma_2^j\right)$;
- for (4), in the probability density of packet processing durations, component weights $\left(p_1^j, p_2^j\right) = \left(q_1^j, q_2^j\right)$, distribution parameters—$\left(\lambda_1^j, \lambda_2^j\right) = \left(\mu_1^j, \mu_2^j\right)$.

To initialize the operation of the EM-algorithm, it is necessary to establish the initial parameters of the component weights $\left(P_1^0, P_2^0\right)$, $\left(q_1^0, q_2^0\right)$ and the parameters of the components of the mixture γ_1^0, μ^0. To establish these values, you can use standard methods [15–19], according to which it is assumed at the initial stage that in the case of a two-component mixture, the weight of each component $P_j^0 = 1/2$ and $q_j^0 = 1/2$. The average values of the sample are taken as the component parameters. To evaluate the distribution parameters (3) and (4), it should be taken into account that the distribution density of the sequences of time intervals between packets is characterized by the parameter $\bar{\gamma} = \frac{1}{\bar{t}}$, where $\bar{t}$ is the average value of the time intervals between packets, and the distribution density of the transmission duration of the packet is characterized by the parameter $\bar{\mu} = \frac{1}{\bar{\tau}}$, where $\bar{\tau}$ is the packet processing duration.

Taking into account the characteristics of the traffic investigated in Sect. 3, the following parameters can be used to initialize the algorithm:

$P_j^0 = 1/2$—for both components, similarly $q_j^0 = 1/2$;
$\gamma^0 = 0{,}000539$, s^{-1}—for time intervals between packets;
$\mu^0 = 0{,}2166$, s^{-1}—for packet transmission durations.

The parameters of hyperexponential distributions by the EM-algorithm were obtained according to expressions (9), (10) and (11). The results are presented in the Table 1.

Using the δ parameter values to calculate δ_1 and δ_2, we have $\delta = 0{,}002\ \text{ms}^{-1}$.

The obtained values of the parameters for $f_V(y)$, show that the hyperexponential distribution degenerates into exponential with the parameter: $\gamma = 558$.

Based on the obtained parameter values and formula (9), the delay variation is determined—$\sigma = 0{,}005$ ms.

Table 1. Parameters of the hyperexponential distributions

Probability density of random variable	Parameters component of the mixture, ms^{-1}	Weight component of the mixture
$f_V(\tau)$	$\gamma_1 = 558{,}09$, $\gamma_2 = 558{,}06$	$P_1 = 0{,}96157$, $P_2 = 0{,}0{,}3843$
$f_Q(\tau)$	$\mu_1 = 1229{,}06$, $\mu_2 = 101{,}7$	$q_1 = 0{,}9989$, $q_2 = 0{,}0011$
$f_W(\tau)$	$\delta_1 = 0{,}0199$, $\delta_2 = 2{,}2 \times 10^{-6}$	$\Delta_1 = 0{,}9989$, $\Delta_2 = 0{,}9989$

It seems interesting to analyze the effect of network load depending on the load factor $\rho = \frac{\gamma}{\mu}$, the result of which is shown in Fig. 3. The load change in the model was realized by varying the time intervals between packets (which corresponds to the parameter γ, the parameter δ is recalculated accordingly) with a constant packet length (which corresponds to the parameter μ), while taking into account that the initial traffic implementation was obtained when the channel load was 0,4 (Fig. 4).

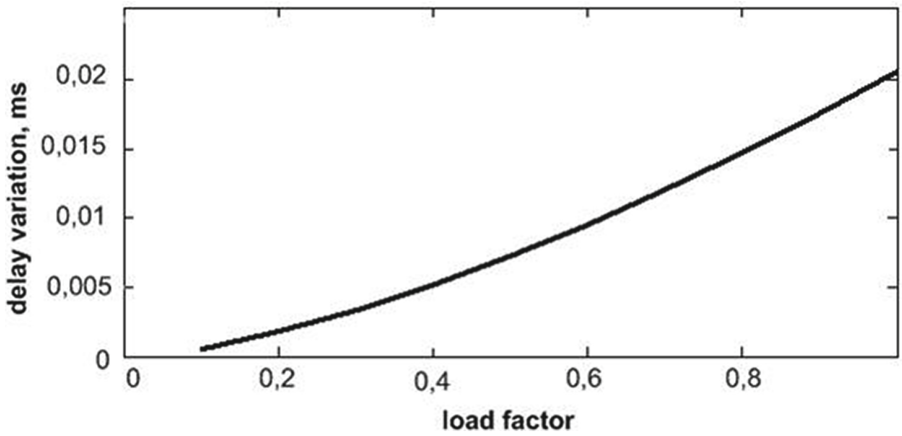

Fig. 3. The dependence of delay variation on the network load in analytical modeling

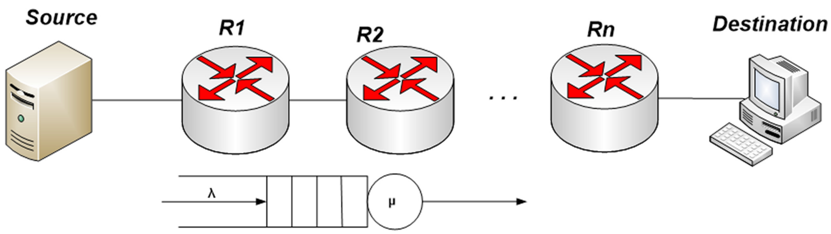

Fig. 4. Scheme of modeling

4.2 Results of Simulation Modeling

When choosing a modeling environment, there is a problem with the possible processing of different types of traffic; it is desirable to start streams that are registered on the real network. The ns2 software environment meets these requirements.

In the simulation environment, the following streams were processed:

- traffic that are registered on the real network, for which the results of analytical modeling were obtained;
- CBR-traffic;
- exponential stream;
- traffic with Pareto distribution of time intervals between packets and packet lengths.

The channel load in the model is set to 0.4.

For processing in a simulation environment, the flows were selected that are most characteristic for infocommunication networks [1, 7–9]. It is known that the exponential traffic processing system corresponds to the M/M/1 model. This stream is characterized by the absence of correlation within the sequence of the stream implementation.

On the contrary, CBR stream is characterized by strong correlations.

Based on the analysis of simulation results, the dependences of the delay variation on the network load were obtained (Fig. 5).

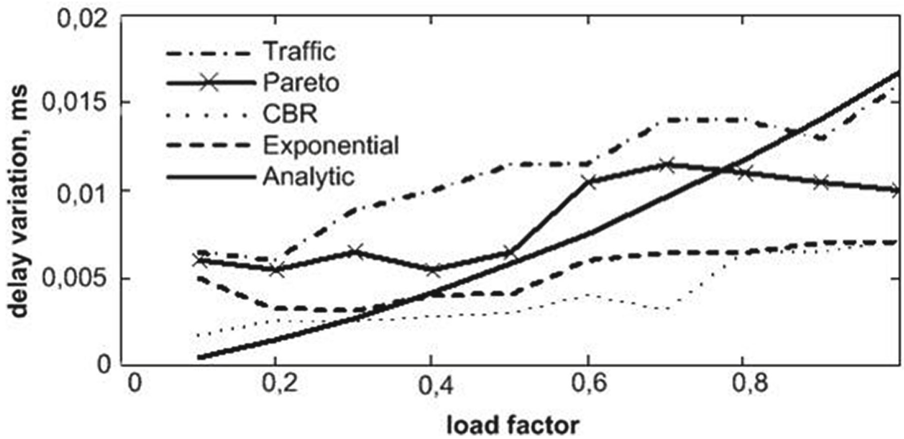

Fig. 5. The dependence of delay variation on the network load during simulation

From the results, it can be seen that for real traffic the delay variation is larger than for CBR and exponential flows. CBR traffic has delay variation values less than exponential. As you know, exponential flows are characterized by the absence of correlation. Considering the presence of correlations within the sequences of CBR traffic, as well as the absence of correlation for multimedia traffic established in the analysis, it can be stated that with an increase in the degree of correlation, the delay variation decreases.

5 Conclusion

1. Analytical evaluation of delay variation in the G/G/1 system simulated as $H_2/H_2/1$ is obtained. The obtained approach allows estimating the variation in packet delay during transmission, regardless of what actual distribution describes the waiting time of the packet in the queue.
2. The approach to determining the parameters of the hyperexponential distribution based on the EM-algorithm is proposed. Estimates are obtained of the variation in packet delay during network loading $\rho = 0{,}4$ as a result of analytical modeling—$\sigma = 0{,}005$ ms and simulation—$\sigma = 0{,}007$ ms. According to the results, the dependences of the delay variation on the network load were obtained, which showed that with an increase in the load, the delay variation increases.
3. Based on the analysis of the simulation results, it was found that with an increase in the degree of correlation, the variation of the delay decreases.

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