



Opinion Dynamic Modeling of Fake News Perception

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Abstract. Fake news diffusion represents one of the most pressing issues of our online society. In recent years, fake news has been analyzed from several points of view, primarily to improve our ability to separate them from the legit ones as well as identify their sources. Among such vast literature, a rarely discussed theme is likely to play uttermost importance in our understanding of such a controversial phenomenon: the analysis of fake news' perception. In this work, we approach such a problem by proposing a family of opinion dynamic models tailored to study how specific social interaction patterns concur to the acceptance, or refusal, of fake news by a population of interacting individuals. To discuss the peculiarities of the proposed models, we tested them on several synthetic network topologies, thus underlying when/how they affect the stable states reached by the performed simulations.

Keywords: Fake news · Opinion dynamics · Polarization

1 Introduction

Nowadays, one of the most pressing and challenging issues in our continuously growing and hyperconnected (online) world is identifying fake/bogus news to reduce their effect on society. Like all controversial pieces of information, fake news usually polarizes the public debate - both online and offline - with the side effect of radicalizing population opinions, thus reducing the chances of reaching a synthesis of opposing views. Moreover, such phenomena are usually amplified due to the existence of stubborn agents, individuals that foster - either for personal gain, lack of knowledge, or excessive ego - their point of view disregarding the existence of sound opposing arguments or, even, debunking evidence. So far, the leading efforts to study such a complex scenario was devoted to: (i) identifying fake news, (ii) debunk them, (iii) identifying the sources of fake news, and (iv) studying how they spread. Indeed, all such tasks are carriers of challenges as well as opportunities: each costly, step ahead increasing out knowledge on this complex phenomenon, a knowledge that can be applied to reduce its effect on the public debate. Among such tasks, the analysis of how fake news diffuse is

probably the most difficult to address. Even by restricting the analysis on the online world, tracing the path of a content shared by users of online platforms is not always feasible (at least extensively): it becomes even impossible when we consider that such content can diffuse across multiple services, of which we usually have only a partial view. However, we can argue that - in the fake news scenario - it is important how a given controversial content spreads (e.g., how different individuals get in touch with it) and how the population reached by such content perceives it. Dangerous fake news cannot only reach a broad audience, but it is also capable of convincing it of its trustworthiness. The latter component goes beyond the mere spreading process that allows it to become viral: it strictly relates to individuals' perception, opinions that are formed not only to the news content itself but also through the social context of its users.

In this work, moving from such observation, we propose a family of opinion dynamics models to understand the role of specific social factors on the acceptance/rejection of fake news. Assuming a population composed of agents aware of a given piece of information - each starting with its attitude toward it - we study how different social interaction patterns lead to the consensus or polarization of opinions. In particular, we model and discuss the effect that stubborn agents, different levels of trusts among individuals, open-mindedness and, attraction/repulsion phenomena have on the population dynamics of fake news perception.

The paper is organized as follows. In Sect. 2, the literature relevant to our work is discussed. Subsequently, in Sect. 3, we describe the opinion dynamics models we designed to describe and study the evolution of Fake news perception. In Sect. 4, we provide an analysis of the proposed models on synthetic networks having heterogeneous characteristics. Finally, Sect. 5 concludes the paper by summarizing our results and underlying future research directions.

2 Related Works

We present the literature review by dividing this Section into two subparagraphs: first, we try to characterize fake news, and we illustrate the main areas of research for these. Then, we introduce opinion dynamics, and we describe the most popular methods.

Fake News Characterization. Before examining the central studies in the literature on the topic of fake news, it is appropriate to define the term itself. There is no universal definition of fake news, but there are several explanations and taxonomies in the literature. We define “fake news” to be news articles that are intentionally and verifiably false and could mislead readers, as reported in [1]. Indeed, identifying the components that characterize fake news is an open and challenging issue [2]. Moreover, several approaches have been designed to address the problem of unreliable content online: most of them propose methods for detecting bogus contents or their creators. Focusing on the target of the analysis involving fake news, we can distinguish different areas of research:

creator analysis (e.g., bots detection [3]), content analysis (e.g., fake news identification [4]), social context analysis (e.g., the impact of the fake news and their diffusion on society [5]).

Opinion Dynamics. Recently, opinion formation processes have been attracting the curiosity of interdisciplinary experts. We hold opinions about virtually everything surrounding us, opinions influenced by several factors, e.g., the individual predisposition, the possessed information, the interaction with other subjects. In [6], opinion dynamics is defined as the process that “*attempts to describe how individuals exchange opinions, persuade each other, make decisions, and implement actions, employing diverse tools furnished by statistical physics, e.g., probability and graph theory*”. Opinion dynamics models are often devised to understand how certain assumptions on human behaviors can explain alternative scenarios, namely consensus, polarization or fragmentation. The consensus is reached when the dynamic stable state describes the population agreement toward a single and homogeneous opinion cluster; polarization describes a simultaneous presence of more than one, well defined, separated opinion clusters of suitable sizes; finally, fragmentation corresponds to a disordered state with an even higher set of small opinions’ clusters.

Agent-based modeling is often used to understand how these situations are achieved. In these models, each agent has a variable corresponding to his opinion. According to the way opinion variables are defined, models can be classified in discrete or continuous models. Among the classic models, we can distinguish: the Voter model [7], the Majority rule model [8], and the Sznajd model [9], which are discrete models that describe scenarios in which individuals have to choose between two options on a given topic (for example, yes/no, true/false, iPhone/Samsung). For the continuous models, on the other hand, the most prominent ones are the Hegselmann-Krause (HK) model [10] and Deffuant-Weisbuch model [11] that describe the contexts in which an opinion can be expressed as a real value - within a given range - that can vary smoothly from one extreme to the other, such as the political orientation of an individual.

3 Fake News: Opinion Dynamic Modeling

To model opinion dynamics of fake news perception, we assume a scenario in which a set of agents shares their position w.r.t a given piece of news (that we assume to be bogus) posted on a social platform. Agents are allowed to interact only with the contents posted by their friends, updating their point of view to account for their distance in opinions. Thus, our effort is not in estimating how the fake news spread but, conversely, in understanding how agents perceive them as a function of the social environment that surrounds them.

Due to the peculiar nature of the phenomena we are analyzing - e.g., how fake news is perceived by individuals and how such perception fosters their spreading - we opted for a continuous modeling framework, extending the well-known Hegselmann-Krause model.

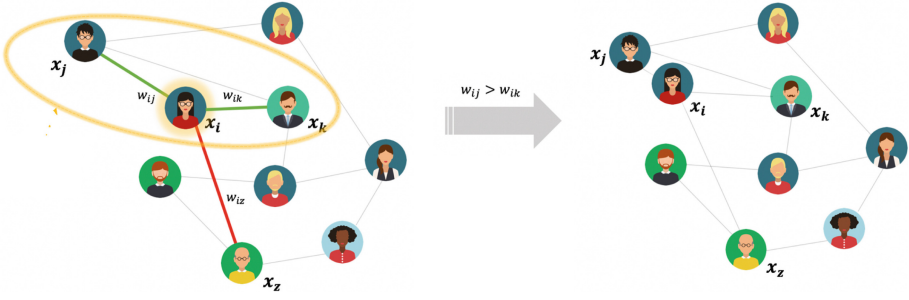


Fig. 1. *Weight example.* Opinion x_i is influenced by the opinions of agents with the opinion more similar to its opinion; e.g., the agents in the yellow elliptical. At the end of the interaction, x_i approaches the opinions of the agents with heavier weights (as visually shown x_i change of position).

Definition 1 (Hegselmann-Krause (HK)). *The HK model considers N agents - each one having an internal status representing its opinion in the continuous range $[-1, 1]$ - that interact during discrete time events, $T = \{0, 1, 2, \dots\}$. Agents can only interact if their opinions differ up to a user-specified threshold ϵ , namely their confidence level. During each interaction event $t \in T$ a random agent i is selected and the set $\Gamma_\epsilon(i)$ of its neighbors j whose opinions differ at most $d_{i,j} = |x_i(t) - x_j(t)| \leq \epsilon$ is computed. Leveraging $\Gamma_\epsilon(i)$, when selected, agent i changes its opinion following the update rule:*

$$x_i(t + 1) = \frac{\sum_{j \in \Gamma_\epsilon(i)} a_{i,j} x_j(t)}{\sum_{j \in \Gamma_\epsilon(i)} a_{i,j}} \tag{1}$$

where $a_{i,j}$ is 1 if i, j are connected by an edge, 0 otherwise. As an outcome, i 's opinion at time $t + 1$ becomes the average of its ϵ -neighbors' opinions.

The HK model converges in polynomial time, and its behavior is strictly related to the expressed confidence level: the higher the ϵ value, the higher the number of opinions clusters when model stability is reached.

Given its definition, the HK model does not consider the strength of the ties of the agents. In a fake news scenario, we can suppose that when an agent i reads a post on his Facebook wall concerning a news A the reliability attributed from i to the content of the post is closely linked to the user that shared it - as exemplified in Fig. 1. To adapt the HK model to include such specific information, we extend it to leverage weighted, pair-wise, interactions.

Definition 2 (Weighted-HK (WHK)). *Conversely from the HK model, during each iteration WHK consider a random pair-wise interaction involving agents at distance ϵ . Moreover, to account for the heterogeneity of interaction frequency*

among agent pairs, WHK leverages edge weights, thus capturing the effect of different social bonds' strength/trust as it happens in reality. To such extent, each edge $(i, j) \in E$, carries a value $w_{i,j} \in [0, 1]$. The update rule then becomes:

$$x_i(t + 1) = \begin{cases} x_i(t) + \frac{x_i(t)+x_j(t)w_{i,j}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0 \\ x_i(t) + \frac{x_i(t)+x_j(t)w_{i,j}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0 \end{cases} \quad (2)$$

The idea behind the WHK formulation is that the opinion of agent i at time $t + 1$, will be given by the combined effect of his previous belief and the average opinion weighed by its, selected, ϵ -neighbor, where $w_{i,j}$ accounts for i 's perceived influence/trust of j .

Moreover, we can further extend the WHK model to account for more complex interaction patterns, namely attractive-repulsive effects.

Definition 3 (Attraction WHK - (AWHK)). By "attraction", we identify those pair-wise interactions between agents that agree on a given topic. At the end of the interaction, agent i begins to doubt his position and to share some thoughts of j . For this reason his opinion will tend to approach that of his interlocutor, so $d_{ij}(t) > d_{i,j}(t + 1)$.

After selecting the pair of agents i and j , the model has the following update rule:

$$x_i(t + 1) = \begin{cases} x_i(t) - \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) \geq 0, x_i(t) > x_j(t) \\ x_i(t) + \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) \geq 0, x_i(t) < x_j(t) \\ x_i(t) + \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) < 0, x_i(t) > x_j(t) \\ x_i(t) - \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) < 0, x_i(t) < x_j(t) \\ x_i(t) - \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) < 0, sum_{op} > 0 \\ x_i(t) + \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) < 0, sum_{op} < 0 \\ x_i(t) + \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) \geq 0, sum_{op} > 0 \\ x_i(t) - \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) \geq 0, sum_{op} < 0 \end{cases} \quad (3)$$

where $sum_{op} = x_i(t) + x_j(t)w_{i,j}$.

The used criterion is always the same: the new opinion of i is the result of the combined effect of his initial opinion and that of the neighbor j , but each case applies a different formula depending on whether the opinions of i and j show discordant or not, so we can guarantee that the difference between the respective opinions is reduced after the communication.

However, when observing real phenomena, we are used to identifying more complex interactions where individuals influence each other despite their initial opinions, getting closer to the like-minded individuals and moving apart from ones having opposite views.

Definition 4 (Repulsive WHK - (RWHK)). This circumstance is called a "repulsion": two agents' opinions will tend to move them apart. Consider the

situation where agent i communicates with j with an opposite belief. At the end of the interaction, i will continue to be more convinced of his thoughts and his new opinion will be further away from that of j . So, when the communication between the two agents ends, the opinion of i will move away from that of j by following:

$$x_i(t+1) = \begin{cases} x_i(t) + \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) \geq 0, x_i(t) > x_j(t) \\ x_i(t) - \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) \geq 0, x_i(t) < x_j(t) \\ x_i(t) - \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) < 0, x_i(t) > x_j(t) \\ x_i(t) + \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) < 0, x_i(t) < x_j(t) \\ x_i(t) + \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) < 0, sum_{op} > 0 \\ x_i(t) - \frac{sum_{op}}{2}(1 - x_i(t)) & \text{if } x_i(t) \geq 0, x_j(t) < 0, sum_{op} < 0 \\ x_i(t) - \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) \geq 0, sum_{op} > 0 \\ x_i(t) + \frac{sum_{op}}{2}(1 + x_i(t)) & \text{if } x_i(t) < 0, x_j(t) \geq 0, sum_{op} < 0 \end{cases} \quad (4)$$

with $sum_{op} = x_i(t) + x_j(t)w_{i,j}$.

Once again, we proceed for cases, each of which defines a particular situation given by the sign of agents' opinions. The updated opinion of i will ensure that $d_{i,j}(t) < d_{i,j}(t + 1)$.

Indeed, AWHK and RWHK can be combined to obtain a comprehensive model that accounts for both behaviors.

Definition 5 (Attraction-Repulsion WHK - (ARWHK)). *To model the attraction and repulsion of opinions, during each iteration an agent i is randomly selected along with one of its neighbors, j - not taking into account the ϵ threshold. Once identified the pair-wise interaction, the absolute value of the difference between the opinions of i and j is computed. If such a value is lower than ϵ AHK is applied to compute $x_i(t + 1)$, otherwise RHK. If the difference between $x_i(t)$ and $x_j(t)$ exceeds ϵ then the repulsive interaction occurs and the update rule 4 is applied.*

The ARWHK model allows us to describe several complex scenarios and, among them, the changes of mind that individuals experience when confronted with a piece of news, either fake or not, shared by a trusted/trusted peer.

However, such a model still does not consider the existence of *stubborn* individuals - e.g., agents having fixed opinions that, despite communicating with neighboring ones, are not subject to external influence acting to influence their peers. Stubborn agents are representative of different types of individuals and are used to model those who spread misinformation.

This type of agent can correspond to prominent individuals in society, such as media, companies, or politicians. [12] and [13] are among the first studies in which the presence of this type of agent has been introduced. In the former, the system behavior is studied on homogeneous graphs for mean-field approximation; in the latter, there is an analysis based on the average of random networks and the mean-field approximation.

To integrate this idea into the model presented above, we add a binary flag to each agent to denote it as “stubborn” or not. The update rule changes are then straightforward: if the randomly selected agent is a stubborn one, he will not update his opinion and, therefore, $x_i(t) = x_i(t+1)$; otherwise, the previously discussed update strategy is applied.

4 Experimental Analysis

This Section describes the performed experimental analysis, focusing on its main components: the selected network datasets, the designed experimental protocol, and the obtained results. To foster experiments reproducibility, the introduced models have been integrated within the NDlib¹ python library [14].

Datasets. We simulate the AWHK and ARWHK models on three scenarios: (i) mean-field (e.g., complete graph), (ii) random network, and (iii) scale-free network. In all scenarios, since we are not interested in studying the proposed models’ scalability, we set the number of nodes to 100. Moreover, due to lack of space, we show the results obtained only for the networks generated with the following parameter setup: (i) Random network (Erdős and Rényi) [15], $p = 0.4$; (ii) Scale-free network (Barabasi-Albert) [16], $m = 3$.

To simulate a more realistic mesoscale network topology (e.g., presence of communities), we also tested the model against a network generated through the LFR benchmark [17]. The LFR graph is composed by 300 nodes, assigned to 4 non overlapping communities. The parameters used for its construction have been set as follows: (i) power law exponent for the degree distribution, $\gamma = 3$; (ii) power law exponent for the community size distribution, $\beta = 1.5$; (iii) fraction of intra-community edges incident to each node, $\mu = 0.1$; (iv) average degree of nodes, $\langle k \rangle = 7$; (v) minimum community size $min_s = 40$.

Analytical Protocol. The proposed model is analyzed while varying the bounded confidence, ϵ , and the percentage of stubborn agents in the network. The simulation results are then discussed through opinion evolution plots representing the evolution through each agent’s opinion.

Results. We report the results obtained by AWHK and ARWHK on the previously described synthetic scenarios and, after that, we discuss the impact of community structure on them. Edge weights, representing trust values among agent pairs, are drawn from a normal distribution.

Attraction & Stubbornness. Figure 2 shows the results obtained by AWHK on the scale-free network for different values of ϵ while maintaining constant the percentage of stubborn agents (90% of the individuals assume and maintain a positive opinion). Different colors represent the agent’s initial opinion (positive,

¹ NDlib: Network Diffusion library. <https://ndlib.readthedocs.io/>.

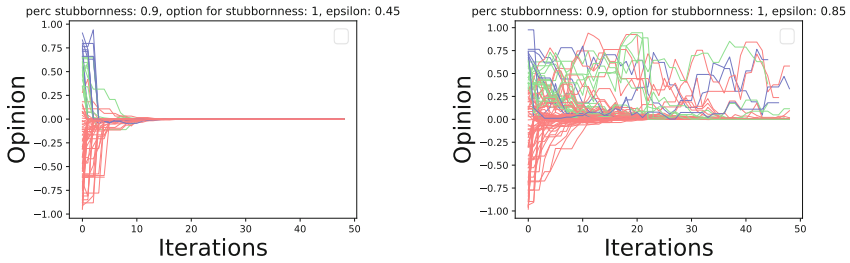


Fig. 2. Effect of the stubborn agents varying ϵ on scale-free network in the AWHK model. Stubborn population opinion evolution lines are omitted.

negative, or neutral). We can observe that in the selected scenarios, the increase of the bounded confidence interval results in a more chaotic regime, characterized by a subset of agents whose opinions heavily fluctuates toward the critical mass introduced by the stubborn agents. The presence of stubborn agents affects opinions' evolution since they act as pivots for those open to change their minds. We executed the same simulation varying the percentage of stubborns and the set of initial stubborns' opinion. As expected, we observed a similar result when stubborns are tied to negative opinions and even a more chaotic regime when such class of agents equally distributes over the opinion spectrum (we do not report the figures for limited space). So stubborns act as persuaders, bringing the opinion of the population closer to theirs. The higher their number, the more evident appears their action on the remaining population. As previously stated, Fig. 2 reports the results observed in a scale-free scenario: however, our experimental investigation underlines that the observed trends can also be identified in random and mean-field scenarios (with a significant reduction of the chaotic regime due to the more regular topological structure).

Attraction/Repulsion & Stubbornness. Figure 3 shows the simulation results obtained while introducing repulsion between the interacting subjects - as defined in the ARWHK model - while maintain constant the percentage of stubborn agents (30% of the individuals assume and maintain a negative opinion). In these settings, the overall observed while running the simulation on the scale-free network is different from what happens in the random one. In the former (highlighted in the first row of Fig. 3), we observe a fragmentation in three clusters of opinions, with the central group (the one generated by the attractive interactions), which tends to disappear by increasing the confidence parameter. In latter, when the ϵ value increases, the opinion groups tend to converge into a single one, obtaining a situation very similar to consensus. We can thus observe how the more complex scenario described by ARWHK results in more erratic behaviors. An extensive analysis of simulation results underlines that ϵ acts as a razor that implicitly separates the probability of observing either attractive or repulsive pair-wise interactions: low ϵ values will favor the application of RWHK - thus leading to a more fragmented equilibrium - while higher ones will results in

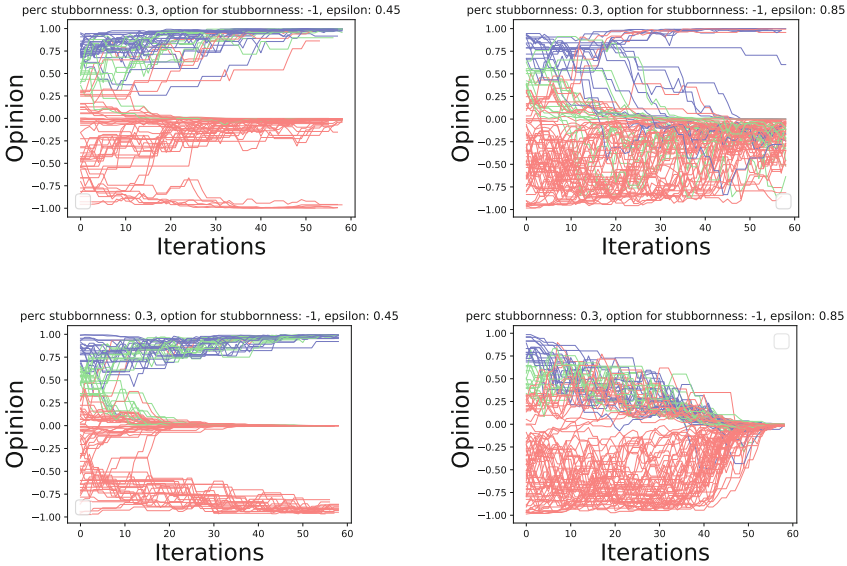


Fig. 3. Effect of the stubborn agents varying *epsilon* on scale-free (first row) and random network (second row) in the ARWHK model. Stubborn population opinion evolution lines are omitted.

a more likely application of AWHK - thus leading to consensus. However, disregarding the network topology simulating the social tissue, ARWHK convergence will require a higher number of iterations than the previously analyzed models. Moreover, even when accounting for repulsive behaviors, stubborn agents play an important role in the opinion dynamics. Our experiments suggest that their presence (i) foster the repulsive behavior for lower values of ϵ (thus increasing opinion fragmentation) and, (ii) slow-down the convergence process to a neutral opinion for higher values of such parameter.

Community Structure. To better underline node clusters' effect to the unfolding of the opinion dynamic process, we report network visualization instead of the previously adopted opinion dynamic plots. In such visualizations, nodes with positive opinions are shown in red. In contrast, the ones with negative opinions in blue: the darker the shade of colors, the more extreme opinion². In this scenario, we study the opinion spreading process while varying the number of stubborn agents and the distribution of initial opinions in the network communities. As a general remark, we observed that the stubborn agents' effect plays a relevant role only in the presence of high bounded confidence values and only when they reach high critical mass. Such a behaviour can be explained by the modular

² All images are taken from animations that reproduce the unfolding of the simulated dynamic processes. Animations, as well as the python code to generate them, are available at <https://bit.ly/3jzp1Qs>.

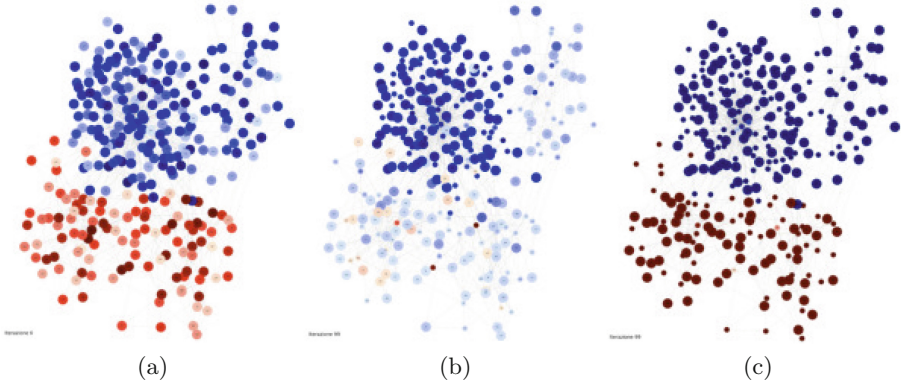


Fig. 4. Network visualizations. (a) Nodes initial conditions - three communities, two prevalently negative (blue node), two positive (red nodes); (b) AWHK final equilibrium; (c) ARWHK final equilibrium.

structure of the analyzed network that acts as boundaries for cross-cluster diffusion. The network topologies considered in this analysis are exemplified in the toy example of Fig. 4, that we will use to summarize the observed outcomes of our analysis. Such a particular case study describes a setup in which network nodes are clustered in four loosely interconnected blocks - two composed by agents sharing opinions in the negative spectrum, the others characterized by an opposite reality. In Fig. 4(a), we report the initial condition shared by two simulations (one based on AWHK, the other on ARWHK) that will be further discussed. Both simulations assume the same value for $\epsilon = 0.85$ and a fixed set of stubborn agents (e.g., the 6 less community embedded nodes - namely, the ones with the higher ratio among their intra-community degree and their total degree) - which are prevalently allocated to the bigger negative (blue) community. While performing a simulation that involves attraction, using AWHK, we can observe how the resulting final equilibrium (Fig. 4(b)) converges toward a common spectrum. In particular, in this example, we can observe how stubborn agents can make their opinion prevail, even crossing community boundaries. Indeed, such a scenario can be explained in terms of the prevalence of negative stubborn agents and the relative size of the negative communities (covering almost 3/5 of the graph). Conversely, when applying the ARWHK model, we get a completely different result, as can be observed in Fig. 4(c). Two strongly polarized communities characterize the final equilibrium. In this scenario, stubbornness has a two-fold role: (i) they increase the polarization of their community by radicalizing agents' opinions and, (ii) as a consequence, make rare the eventuality of cross-community ties connecting moderate agents, thus ideologically breaking apart the population. While varying the models' parameters, our experimental analysis confirms the results obtained on the scale-free and random graphs: well-defined mesoscale clusters prevalently slow-down convergence in case of a population-wide agreement while accelerating the process of fragmentation.

5 Conclusion

In this paper, we modeled the response of individuals to fake news as an opinion dynamic process. Modeling some of the different patterns that regulate the exchange of opinions regarding a piece of given news - namely, trust, attraction/repulsion and existence of stubborn agents - we were able to drive a few interesting observations on this complex, often not properly considered, context. Our simulations underlined that: (i) differences in the topological interaction layer reflect on the time to convergence of the proposed models; (ii) the presence of stubborn agents significantly affects the final system equilibrium, especially when high confidence bounds regulates pair-wise interactions; (iii) attraction mechanisms foster convergence toward a common opinion while repulsion ones facilitate polarization.

As future work, we plan to extend the experimental analysis to real data to understand the extent to which the proposed models can replicate observed ground truths. Moreover, we plan to investigate the effect of higher-order interactions on opinion dynamics, thus measuring the effect that peer-pressure has on the evolution of individuals' perceptions.

Acknowledgment. This work is supported by the scheme 'INFRAIA-01-2018-2019: Research and Innovation action', Grant Agreement n. 871042 'SoBigData++: European Integrated Infrastructure for Social Mining and Big Data Analytics'.

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