Basic Radiation Physics as Relevant to Nuclear Medicine

1

Gauri Shankar Pant and Anil Kumar Pandey

Contents

G. S. Pant (✉)
Retried Professor, Department of Nuclear Medicine,
All India Institute of Medical Sciences (AIIMS),
New Delhi, India

A. K. Pandey
Department of Nuclear Medicine, All India Institute
of Medical Sciences (AIIMS), New Delhi, India

The peaceful applications of atomic radiations have been well established, in many areas particularly in the field of medicine for diagnosis and treatment. The basic understanding of atomic radiations and their characteristics is therefore important for those working in nuclear medicine and other allied disciplines. A brief description on basic physics as relevant to nuclear medicine is given here.

1.1 Atom

It is well known that matter is comprised of atoms, and atom is the smallest unit of a chemical element. An atom rarely exists alone and often combines with other atoms to form the molecule, the component of a chemical compound.

1.2 Modern Atomic Theory

1.2.1 Wave–Particle Duality

According to classical physics, the particles cannot be wave and vice versa. However, Einstein, while explaining the photoelectric effect, postulated that electromagnetic radiation has wave–particle nature. He used the term photon for the packet of electromagnetic radiation and proposed a simple formula to relate the energy of the photon E to its frequency ν or the wavelength.

$$E = h\nu = h\frac{c}{\lambda} \qquad (1.1)$$

In this equation, h is the Planck's constant $(6.634 \times 10^{-34}\,\text{J s})$ and c is the velocity of light in vacuum.

De Broglie generalized the idea and postulated that all sub-atomic particles have wave–particle nature. In some phenomena, the particle behaves as a particle, and in others, it behaves as a wave but in none as both (wave–particle duality). He suggested the following equation to relate the momentum of the particle P to its wavelengths λ:

$$\lambda = \frac{h}{p} \qquad (1.2)$$

Only when the particles are with extremely small mass (subatomic particles), the associated wave is appreciable. Electron microscope is an instrument that proves the accuracy of the wave–particle duality. In macroscopic scale, De Broglie theory is not applicable. Erwin Schrodinger derived an equation in 1925, which describes not only the subatomic, atomic and molecular system but also the macroscopic systems. This discovery was a significant landmark in quantum mechanic for which he was awarded Nobel Prize in Physics in 1933. The detailed discussion on Schrodinger equation is beyond the scope of this chapter.

1.2.2 Electron Configuration

Electrons around a nucleus can be described with wave functions. Wave functions determine the location, energy and momentum of the particle. The square of a wave function gives probability distribution of the particle. At a given time, electrons can be anywhere around the nucleus, but different locations have different probabilities. The space around the nucleus in which the probability is highest is called an orbital. In quantum mechanics, orbital is a mathematical concept that suggests the average location of an electron around the nucleus. If the energy of the electron changes, this average location also changes. For the single electron of hydrogen atom, an infinite number of wave functions and therefore infinite number of orbitals exist.

Orbital can completely be described using the corresponding wave function, but the process is tedious and difficult. In simple terms, an orbital can be described by four quantum numbers.

- The principal quantum number n characterizes the energy and shell size in an atom. It is an integer and can have value from 1 to ∞, but practically n is always less than 8. Maximum number of electrons in the orbital n, is given as $2n^2$. The shells of electrons are labelled alphabetically as $K(n = 1)$, $L(n = 2)$, $M(n = 3)$, etc. based on the principal quantum number.
- The orbital quantum number l relates to the angular momentum of the electron; l can take integer values from 0 to $n − 1$. In a stable atom, its value does not go beyond 3. Orbital quantum number characterizes the configuration of the electron orbital. In the hydrogen atom, the value of l does not appreciably affect the total energy, but in atoms with more than one electron, the energy depends on both n and l. The sub-shells or orbitals of electrons are labelled as $s(l = 0)$, $p(l = 1)$, $d(l = 2)$ and $f(l = 3)$.
- The azimuthal or magnetic quantum number m_l relates to the direction of the electron's

angular momentum and takes on integer values from $-l$ to $+l$.

- The spin quantum number m_s relates to electron angular momentum and can have only two values $-½$ or $+½$.

In 1925 Wolfgang Pauli added a complementary rule for the arrangement of electrons around the nucleus. The postulation is now called **Pauli's exclusion principle** and states that no two electrons can have all quantum numbers same or exist in identical quantum states.

The filling of electrons in orbitals obeys the so-called Aufbau principle. The Aufbau principle assumes that electrons are added to an atom, one at a time, starting with the lowest energy orbital, until all of the electrons have been placed in an appropriate orbital. The sequence of energy states and electron filling in orbitals of a multi-electron atom can be represented as:

$$1s-2s-2p-3s-3p-4s-3d-4p-5s$$
$$-4d-5p-6s-4f-5d-6p-7s-5f-6d-7p$$

1.3 Electron Binding Energies

The bound electrons need some external energy to make them free from the nucleus. It can be assumed that electrons around a nucleus have negative potential energy. The absolute value of the potential energy is called binding energy and is the minimum energy that is required to make an electron free of the atom.

1.3.1 Atomic Emissions

For stability, electrons are required to be in the minimum possible energy level or in the innermost orbitals. However, there is no restriction for an electron to transfer into outer orbitals if it gains sufficient energy. If an electron absorbs external energy that is more than or equal to the binding energy of its orbital, the electron is freed from the atom. A pair of ion, the electron and the atom with positive charge, is created. This process is termed as *ionization*. If the external energy

is more than the binding energy of the electron, the excess energy is divided between the two in such a way that conservation of momentum is preserved. The energy level of a free electron is not necessarily discrete.

If an electron absorbs energy and is either elevated to outer orbits or becomes free to move out of the atom, the original orbital does not remain vacant. Soon the vacancy will be filled by electron from outer layers. This is a random process, and occupier may be any electron from outer orbital. However, closer electron has more chance to occupy the vacancy. In each individual filling-up process, a quantum of energy equal to the difference between the binding energies $E_2 - E_1$ of the two involved orbitals is released, usually in the form of single photon. The frequency ν and wavelength λ of the emitted photon (radiation) are as follows:

$$E_2 - E_1 = \Delta E = h\nu = h\frac{c}{\lambda} \qquad (1.3)$$

When an atom has excess energy, it is in an unstable or *exited state*. The excess energy is released usually in the form of electromagnetic radiation until the atom is again in its natural *stable state*. The frequency spectrum of the radiation emitted from an exited atom can be used as the fingerprint of atom. Such radiation is called characteristic radiation.

1.3.2 Nucleus

In simple terms, the nucleus is made up of two fundamental particles, protons and neutrons. These particles together are called nucleons. The protons are positively charged, and neutrons are electrically neutral. Since electrons are light particles (with mass about 1/1840 times the mass of either neutron or protons), the entire mass of the atom is concentrated in the nucleus.

1.4 Nuclear Structure

There are several notations to summarize nuclear composition of an atom. The most commonly used is ${}^{A}_{Z}X_{N}$, where X represents the chemical

Table 1.1 Mass and charge of proton, neutron and electron

Particle	Symbol	Charge[a]	Mass[b]	Mass (kg)	Energy (MeV)	Relative mass
Proton	P	+1	1.007276	1.6726×10^{-27}	938.272	1836
Neutron	N	0	1.008665	1.6749×10^{-27}	939.573	1839
Electron	e^-	−1	0.000548	9.1093×10^{-31}	0.511	1

[a]Unit charge = 1.6×10^{-11} coulombs
[b]Mass expressed in Universal mass unit (mass of 1/12 of ^{12}C atom)
(Data from "Particles and nuclei 1999)

symbol of the element. Chemical symbol and atomic number carry the same information, and neutron number is the difference between A and Z. Hence, for the sake of simplicity, the notation is briefed to $^A X$ that is quite comprehensible and popular now. For example, in ^{137}Cs, where 137 is the mass number ($A + Z$), and the symbol Cs represents the 55th element in the periodic table. The neutron number can easily be calculated ($A - Z = 82$). Table 1.1 shows the mass, charge and energy of proton, neutron and electron.

1.5   Nuclear Forces

Protons in a nucleus are fairly at close distance ($\approx 10^{-15}$ m). This closeness results in an enormously strong repulsive force between them. They still remain within the nucleus due to the existence of a very strong (short range) nuclear force between nucleons that dominate the repulsive force and make the atom stable. The force is effective in a very short range, and neutrons must have an essential role in creating such force. Without neutrons, protons cannot stay in close distances.

In 1935, Yukawa proposed that the short-range *strong force* came about from the exchange of particles that he called *mesons*. The strong nuclear force is one of the four fundamental forces in nature that is created between nucleons by the exchange of mesons. This exchange can be compared to constantly hitting a tennis ball back and forth between two people. As long as this meson exchange is happening, the strong force holds the nucleons together. Neutrons also participate in the meson exchange and are even a bigger source of strong force. Neutrons have no charge so they approach other nuclei without adding extra repulsive force, and meanwhile, they increase the average distance between protons and help to reduce the repulsion between them within a nucleus.

1.5.1   Nuclear Binding Energy and Mass Defect

Nuclear strong force is the resultant of the phenomenon known as *mass defect*. Direct measurements show that the mass of a nucleus is always less than the sum of the individual masses of the constituent protons and neutrons. Using the Einstein relationship, the deficient mass Δm is exactly equal to the energy required to separate the nucleons or *binding energy* E_b of the nucleus.

$E_b = \Delta m c^2$, c is the speed of the light in vacuum.

The **nuclear mass defect (Δm)** is the nuclear binding energy holding the nucleons together. The **average binding energy** per nucleon is a measure of nuclear stability. The higher the average binding energy, the more stable is the nucleus.

1.5.2   Radioactivity

Radioactivity is the spontaneous emission of particles and/or photons as a result of nuclear instability. The stability of a nucleus depends upon the arrangement of its nucleons particularly the ratio of the number of neutrons to the number of protons. Presence of adequate number of neutrons in relation to protons is essential for stability. Amongst the many possible combinations of protons and neutrons, only around 260 are stable, and
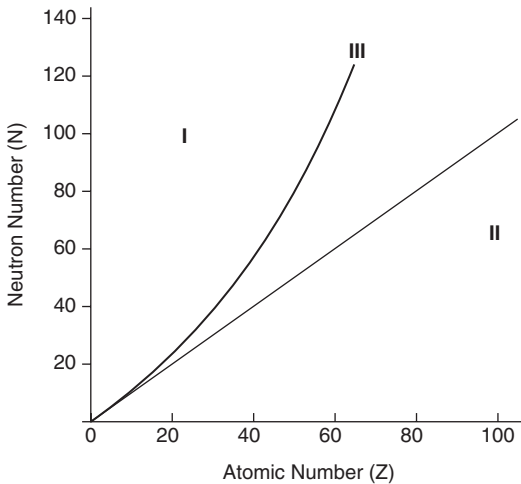
Fig. 1.1 The line of stability and different regions around it. (Reproduced from [2])

the rest all are unstable. It seems that there are favoured neutron-to-proton ratios amongst the stable nuclides. Figure 1.1 shows the function of the number of neutron (N) against the number of proton (Z) for all available nuclides. The stable nuclides gather around an imaginary line, called the line of stability. For light elements ($A < 50$), this line corresponds to $N = Z$, but with increasing atomic number, neutron-to-proton ratio increases up to 1.5 ($N = 1.5Z$). The line of stability ends at $A = 209$ (Bi), and all nuclides above that and those that are not close to this line are unstable. Nuclides that lie on the left of the line of stability (Area I) have excess of neutrons, those lying on the right of the line (Area II) are neutron deficient, and those above the line (Area III) are too heavy (excess of both neutrons and protons) to be stable.

An unstable nucleus sooner or later (nanoseconds to years) changes to a more stable proton–neutron combination by emitting particle(s) such as alpha, beta and gamma. As mentioned earlier, the phenomenon of spontaneous emission of such particles from the nucleus is called radioactivity, and nuclides involved in this phenomenon are called radionuclides. The change from the unstable nuclide (parent) to more stable nuclide (daughter) is called radioactive decay or disintegration. During disintegration, in addition to the emission of nuclear particle(s) energy is released. The process is spontaneous and remains unaf-

fected by external factors. It is also not possible to predict as to which radioactive atom will disintegrate first.

1.6 Modes of Decay

The radionuclide, which decays to another nuclide in order to attain stability, is called the 'parent' nuclide, and the nuclide so formed is called 'daughter'. The unstable nuclide may undergo transformation by any one of the following modes:

1.6.1 Nuclides Having an Excess Number of Neutrons

1.6.1.1 Beta Emission

Nuclides having excess number of neutrons attempt to acquire a stable form by the emission of beta particle (also called negatron). In this process, a neutron coverts to a proton. Along with an electron (negatron or beta minus), an antineutrino is also emitted. The nuclear equation may be given as follows:

$$n \rightarrow p + \beta^- + \bar{v} + \text{energy}$$
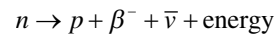
Here n, p, β^- and $\bar{v}$ represent the neutron, the proton, the negatron (beta minus) and the antineutrino, respectively. The proton stays in the nucleus, but the beta minus and the antineutrino are ejected out carrying the released energy as their kinetic energy. In this mode of decay, atomic number of the daughter nuclide is one more than that of the parent, but the mass number remains the same. The daughter may or may not be stable. The mass of the neutron is very slightly more than the masses of the proton, the beta and the antineutrino combined (the daughter is lighter than her parent). This difference in mass is converted into energy and randomly shared between beta particle and antineutrino. Hence, the beta particle may have energy between zero to a certain maximum level. The antineutrino has no mass and charge.

Radionuclides in which the daughter attains a ground or stable state by just emitting beta parti-

cles only are called pure beta emitters such as ^{3}H, ^{14}C, ^{32}P and ^{35}S. Those that cannot attain a stable state after beta emission and that still remain in one of the excited states of the daughter emit gamma photon(s) either in a single transition or through cascades to attain the ground state. In the latter situations, we get more than one gamma photon per beta emission. For example, ^{131}I, ^{132}Xe and ^{60}Co all of which emit beta particle followed by number of gamma emissions.

1.6.1.2 Nuclides Which Lack in Neutrons

In such cases, there are two alternatives for the nucleus to come to a stable state:

(a) **Positron emission and subsequent emission of annihilation photons**

In this mode of decay, a proton transforms to a neutron with the emission of a positron and a neutrino.

$$p \rightarrow n + \beta^+ + v + \text{energy}$$
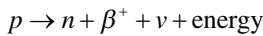
The neutron stays in the nucleus, but a positron and a neutrino are ejected out carrying the emitted energy as their kinetic energy. The positron is the antiparticle of the electron. They are very much alike but with opposite charge. In this mode of decay, the atomic number of the daughter is one less than that of the parent, but the mass number remains the same. The daughter may or may not be stable. The mass of the proton is slightly less than the combined mass of the neutron, the positron and the neutrino. Energy for the creation of this mass is supplied by the whole nucleus; hence, the difference between masses of parent and daughter elements must be sufficient to produce this energy ($E > 1.022$ MeV). Therefore, for positron emission to take place, the energy of parent nuclide must exceed that of the daughter by at least 1.022 MeV. The excess energy, if any, is randomly shared by positron and neutrino. Energy spectrum of the positron is like that of the beta minus (from zero up to a certain maximum). Some of the positron emitting radionuclides are ^{11}C, ^{13}N, ^{15}O and ^{18}F.

Just a few nanoseconds after its production, when its kinetic energy comes to zero, the positron combines with an electron at rest. Their masses are converted into energy in the form of two equal energy photons (0.511 MeV each) which leave the site of their creation in exactly opposite directions to conserve the momentum. This phenomenon is called annihilation reaction, and the photons so created are called annihilation photons.

(b) **Electron captures**

For a nucleus with excess protons has an alternative way to get a stable configuration by capturing one of its own orbital electrons (usually k electron). The electron combines with the proton producing a neutron and a neutrino in the process.

$$p + e \rightarrow n + v$$
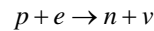
The electron capture is an important phenomenon because an electron from outer orbit fills the vacancy of the attracted electron. Photons (characteristic radiation) are emitted in the process, which may have a considerable amount of energy. These photons may knock out orbital electrons from outer orbits before leaving the atom creating the so-called Auger electrons. Auger electrons may be useful for therapeutic application (targeted therapy) due to their short range in the medium.

Electron capture is more likely to occur in heavy elements (electrons more close to the nucleus), whereas positron emission is more likely in lighter elements provided the energy of the parent exceeds that of the daughter by at least 1.022 MeV. Radionuclides such as ^{67}Ga, ^{111}In, ^{123}I and ^{125}I decay partially or fully by electron capture.

1.6.1.3 Nuclides Having an Excess of Protons and Neutrons

When the nuclides are too heavy (many nucleons) (region III in Fig. 1.1) and unstable, there are two ways for them to become stable, either by alpha emission or by fission process.

(a) **Alpha decay**

There are some nuclides, which have excess protons and neutrons and try to get rid of them by emitting an alpha particle (two neutrons and two protons). The atomic number of the daughter in such decay is reduced by 2, and the mass number is reduced by 4. The alpha particle emission may be sufficient to bring the daughter to the stable state or may follow with gamma emission. Naturally occurring radionuclides such as ^{238}U, ^{232}Th and ^{226}Ra not only decay by alpha emission but also form a chain of decay process as their daughter products are also radioactive.

(b) **Fission**

It is the spontaneous fragmentation of very heavy nuclei into two lighter nuclei usually with the emission of two to three neutrons. A large amount of energy (hundreds of MeVs) is also released in this process. Fissile nuclides themselves have no clinical application, but some of their fragments are found quite useful because of being carrier free. Thus, specific activity of fission-produced radionuclides is always higher than those produced by nuclear reactor.

1.7 Gamma Radiation and Internal Conversion

In some decay processes, the daughter nuclei are not in their ground state. This happens when all the energy associated with decay process is not carried away by the emitted particles. Such nuclei can be either in an excited state or in a metastable (isomeric) state. In both the situations, the excess energy is often released in the form of one or more gamma photons. Average life time of excited states is very short, and energy is released within a fraction of a nanosecond, but the average life time of metastable states is very much longer, and emission may vary from few milliseconds to few days or even more. During this period, the isomeric state behaves as an independent isotope that becomes a pure gamma emitter. Some of the metastable states have great clinical application in nuclear medicine. The decay scheme of ^{99}Mo–^{99m}Tc is an example of isomeric transition and shown in Fig. 1.2.

There are situations when the excited nuclei instead of emitting a gamma photon utilise the energy in knocking out one of the orbital electrons from their own atom. This process is called

Fig. 1.2 Decay scheme of ^{99}Mo. (Reproduced from [2])
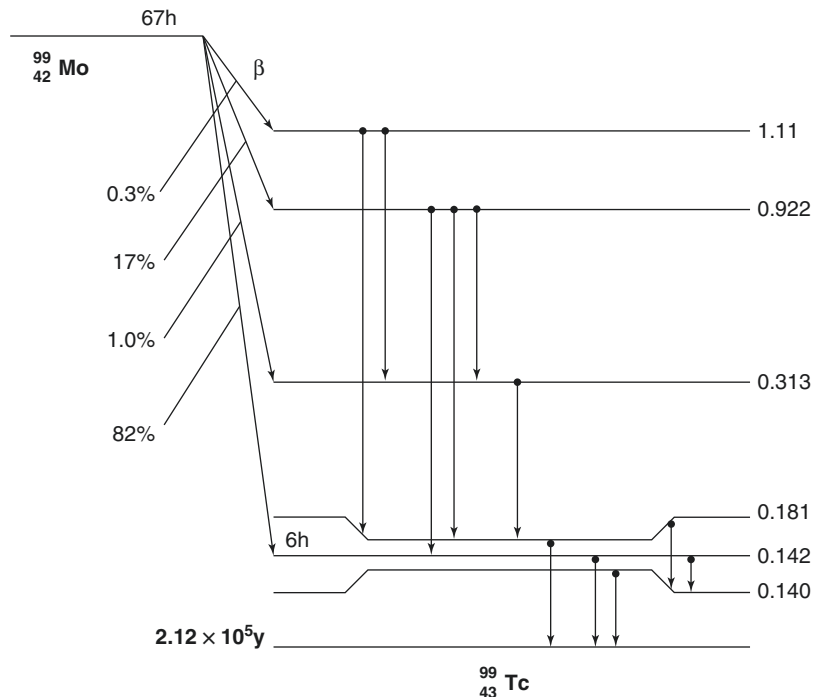
internal conversion, and the emitted electron is called a conversion electron. The probability of *K* conversion electron is more than *L* or *M* conversion electrons, and the phenomenon is more common in heavy atoms. The internal conversion is followed by emission of characteristic X-rays and Auger electrons as the outer shell electrons move in to fill the inner shell vacancies. It should be noted here that the X-ray emission is an atomic phenomenon even though there is no difference between an X-ray and a gamma photon of equal energy.

1.8   Laws of Radioactive Decay

There is no way to predict the time of disintegration of an atom nor should one be really interested about the fate of an individual atom. The number of atoms disintegrating within a given time interval can however be estimated which is of our interest. The radioactive decay has been found to be a spontaneous and random process independent of any environmental factor. In other words, nothing can influence the process of radioactive disintegration. Any random process just can be described in terms of probabilities and average constants.

In a sample containing a large number of identical radioactive atoms, during a short period of time (∂t) the number of decayed atoms ($-\partial N$) is proportional to the total number of atoms (N) present at that time. Mathematically, it can be expressed as:

$$-\partial N \propto N \partial t$$
$$-\partial N = \lambda N \partial t \qquad (1.4)$$
$$\text{or} \quad \frac{\partial N}{N} = -\lambda \partial t$$

In this equation, the constant λ (known as decay constant) has a characteristic value for each radionuclide. Decay constant is the fraction of atoms undergoing decay per unit time in a very large number of atoms. Its unit is inverse of time.

In simple terms, the decay constant can be defined as the probability of disintegration of a nucleus per unit time. Thus $\lambda = 0.01$ per second means that the probability of disintegration of each atom is 1% per second. It is important to note that this probability does not change with time.

The exact number of parent radionuclides in a sample at any time can be calculated by integrating Eq. (1.4), which takes the following form:

$$N = N_o * \exp(-\lambda t) \qquad (1.5)$$

where N_o is the initial number of atoms in the sample, and N is the number present at time t.

The term $\frac{\partial N}{\partial t}$ shows the number of disintegration per unit time and is known as activity. The SI unit of activity is Bq (one decay per second). The conventional unit of activity is Curie (Ci) which is equal to 3.7×10^{10} disintegrations per second (dps).

1.8.1   Half-Life

The time after which 50% of the atoms in a sample undergo disintegration is called the half-life. The half-life and decay constant are related to each other by the following equation:

$$T_{1/2} = \frac{0.693}{\lambda} \quad \text{or} \quad \lambda = \frac{0.693}{T_{1/2}} \qquad (1.6)$$

1.8.2   Average Life

The actual lifetimes of individual atoms in a sample are quite different. Some have very short and some have very long lifetime. The average lifetime, characteristics of the atoms, is related to the half-life by:

$$T_{av} = 1.44 * T_{1/2} \qquad (1.7)$$

The average life is a useful parameter for calculating the cumulated activity in source organ in internal dosimetry in nuclear medicine.

1.8.3   Radioactive Equilibrium

In many cases, the daughter element is also radioactive and immediately starts disintegrating after its formation. Although the daughter obeys the general rule of radioactive decay, its activity does not follow the exponential law of decay of its own while existing with the parent. The reason is that the daughter is produced (mono-exponentially) by disintegration of its parent, and at the same time, it disintegrates (mono-exponentially) as a radioactive element. So the activity of such elements changes bi-exponentially, first the activity increases, reaches a maximum value and then starts decreasing. The rate at which the activity changes in such a mixture of radionuclides depends on the decay constant of both the parent and the daughter.

If we start with a pure sample of a parent with a half-life of T_1 and decay constant λ_1 and contains $(N_1)_0$ atoms initially, the decay of this parent can be expressed by:

$$N_1 = \left(N_1\right)_0 e^{-\lambda_1 t} \qquad (1.8)$$

The rate of decay of the parent is the rate of formation of the daughter. Let the daughter decays at the rate $\lambda_2 N_2$, where λ_2 is the decay constant of the daughter and N_2 is the number of atoms of the daughter present. The net rate of formation of the daughter can be given by:

$$\frac{\partial N_2}{\partial t} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (1.9)$$

The solution of this equation in terms of activity can be given as:

$$A_2 = A_1 \frac{T_1}{T_1 - T_2}\left(1 - e^{-0.693\left(\frac{T_1 - T_2}{T_1 T_2}\right)t}\right) \qquad (1.10)$$

where A_1 and A_2 are the activity of the parent and the daughter, respectively, T_1 and T_2 are their physical half-lives and t is the elapsed time. This equation is for a simple parent–daughter mixture. In general, three different situations arise from Eq. (1.10).

(a) **Secular equilibrium**

When the half-life of the parent (T_1) is too long in comparison to that of the daughter (T_2), Eq. (1.10) may be expressed as:

$$A_2 = A_1\left(1 - e^{\frac{-0.693t}{T_2}}\right) \qquad (1.11)$$

After one half-life of the daughter $(t = T_2; A_2)$ will become nearly $A_1/2$; after two half-lives, the daughter may grow up to 3/4 of the parent, and after four half-lives (of the daughter), it rises to about 94% of the parent activity. Thus, activity of the daughter gradually increases, and after few half-lives, the activity of the parent and the daughter become almost equal (Fig. 1.3). The parent and the daughter are said to be in secular equilibrium.

(b) **Transient equilibrium**

Half-life of the parent is few times (>10 times) longer than that of the daughter but not as much long as in secular equilibrium. In this case, the activity of daughter increases, eventually exceeds the activity of parent, reaches a maximum and then decays with the half-life of the parent, but the activity of the
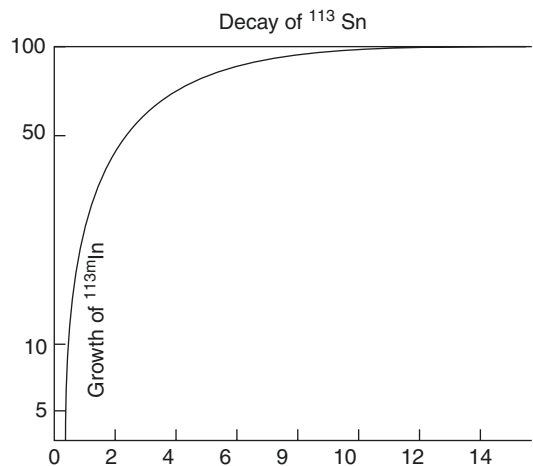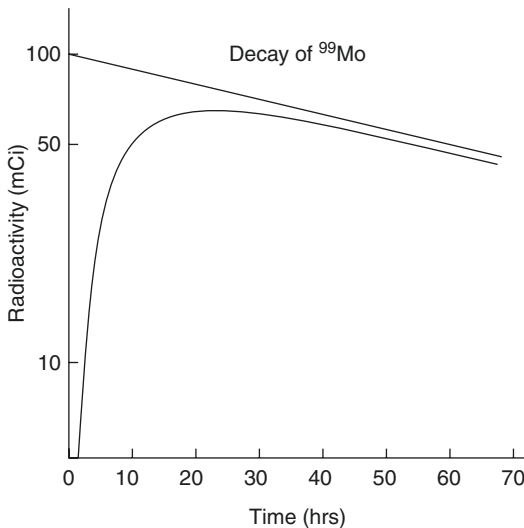

Fig. 1.3 Secular equilibrium

Fig. 1.4 Transient equilibrium. In case of 100% abundance of the daughter product and 100% extraction efficiency in elution, the growth curve of ^{99m}Tc will first cross over the decay curve of ^{99}Mo slightly then will decay in parallel to the parent. However, ^{99m}Tc has an abundance of 89% and extraction during elution is also less than 100%; therefore, the actual curve of transient equilibrium will look as given above

daughter slightly exceeds the parent activity after its maximum growth but practically the activity of daughter remains less than the parent because the extraction efficiency is normally less than 1 or 100%, as can be seen in Fig. 1.4a, b. Equation (1.10) in such a case can be written as:

$$A_2 = A_1 \frac{T_1}{T_1 - T_2} \quad \text{for} \quad t \gg T_2 \quad (1.12)$$

The growth of the daughter for multiples of $T_2(T_2, 2T_2, 3T_2, 4T_2,$ etc.) will be nearly 50%, 75%, 87.5% and 94%, respectively, of the activity of the parent. It is therefore advisable to elute the activity from technetium generator after every 24 h (Mo-99 with 67 h half-life and Tc-99m with 6 h half-life).

1.8.4 No Equilibrium

When the half-life of the daughter is longer than the half-life of the parent, there would be no equilibrium between them.

1.9 Interaction of Radiation with Matter

Ionizing radiations transfer their energy in full or part to the medium through which they pass by way of interactions. The significant types of interactions are excitation and ionization of atoms or molecules of the matter by both the charged particles and electromagnetic radiation (X- or gamma rays).

1.9.1 Interaction of Charged Particles with Matter

The charge particles lose some of their energy by way of the following interactions:

(a) Ejection of electrons from the target atoms (ionization)
(b) Excitation of electrons from lower to higher energy state
(c) Molecular vibrations along the path (elastic collision) and conversion of energy into heat
(d) Emission of electromagnetic radiation

For the charged particles in the energy range of 10 keV to 10 MeV, ionization predominates over excitation. The probability of absorption of charged particles is so high that even a small thickness of the material can stop them completely.

The nature of interaction of all charged particles in the energy range mentioned above is similar to each other. Light particles such as electrons deflect at larger angles than heavier particles, and there is a wide variation in their tortuous path. The path of a heavier particle is more or less a straight line. When electrons are deflected at large angles, they transfer more energy to the target atom and eject electrons from it. These electrons while passing through the medium produce secondary electrons along their track (delta rays). The charged particles undergo a large number of interactions before they come to rest. In each interaction, they lose a small amount of energy. These losses are called collision losses.

Energetic electrons can approach the nucleus where they get decelerated and produce brems-

strahlung radiation (X-rays). The chance of such an interaction increases with increase in electron energy and atomic number of the target material. Loss of electron energy by this mode is termed as radiative loss. The energy lost per unit path length along the track is known as the linear energy transfer (LET) and is generally expressed in keV/μm.

1.9.2 Range of a Charged Particle

After travelling through a distance in the medium, the charged particle loses all its kinetic energy and comes to rest. The average distance travelled in a given direction by a charged particle is known as its 'range' in that medium and is influenced by the following factors:

(a) Energy: Higher the energy of the particle more is the range
(b) Mass: Higher the mass of the charged particle smaller is the range
(c) Charge: The range is inversely proportional to square of the charge
(d) Density of the medium: Higher the density of the medium shorter is the range of charged particle

1.9.3 Interaction of Electromagnetic Radiation with Matter

When a beam of X- or gamma rays passes through an absorbing medium, some of the photons are completely absorbed, some are scattered, and the rest pass through the medium almost unchanged in energy and direction. The transferred energy results in excitation and ionization of atoms or molecules of the medium and also in the production of heat. The attenuation of the beam through a given medium may be summarized as follows:

– Greater the thickness of the absorbing material more is the attenuation.
– Greater the atomic number of the material more is the attenuation.

– Higher the photon energy lesser is the attenuation produced by a given thickness of material.

1.9.4 Linear Attenuation Coefficient

The linear attenuation coefficient (μ) is defined as the fractional reduction in the beam per unit thickness as determined by a thin layer of the material.

$$\mu = \frac{\text{Fractional reduction in a thin layer}}{\text{Thickness of the layers}(\text{cm})}$$

The unit of μ is cm^{-1}.

1.9.4.1 Exponential Attenuation

Exponential law can explain the attenuation of radiation beam intensity. The mathematical derivation is given below.

Let N_o be the initial number of photons in the beam, and N is the number recorded by the detector placed behind the absorber (Fig. 1.5).

The number δN, which gets attenuated, will be proportional to the thickness (δx) of the absorber and to the number of photons N present in the beam.

Mathematically:

$$\begin{aligned} \delta N &\propto N \cdot \delta x \\ \text{or } \delta N &= -\mu \cdot N \cdot \delta x \end{aligned} \tag{1.13}$$

where μ is a constant called linear attenuation coefficient for the radiation. The negative sign indicates that as δx increases, the number of photons in the beam decreases. Equation 1.13 can be rearranged as:

$$\mu = -\frac{\delta N}{N \cdot \delta x} \tag{1.14}$$

The formal definition of attenuation coefficient is derived from Eq. 1.14, integration of which gives the following relationship:

$$N = N_o \cdot e^{-\mu x} \tag{1.15}$$

Fig. 1.5 Attenuation of a radiation beam by an absorber. The transmitted beam is measured by the detector '*P*'. (Reproduced from [3])
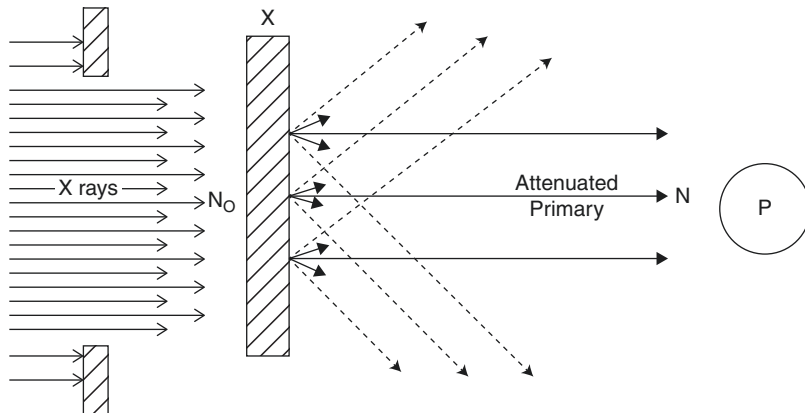

Table 1.2 Attenuation coefficients (cm^{-1}), mass attenuation coefficients (cm^2/g) and HVL (cm) in lead and concrete for 140 and 511 keV photons

Radiation type	HVL in cm lead (Pb 11.35 g/cm^3)	HVL in cm concrete (2.35 g/cm^3)	Attenuation coefficient (μ cm^{-1}) in lead (11.35 g/cm^3)	Attenuation coefficient (μ cm^{-1} in concrete)	Mass attenuation coefficient (cm^2/g) in lead	Mass attenuation coefficient (cm^2/g) in concrete
Tc-99m 140 keV	0.027	1.99	25.67	0.348	2.26	0.148
PET annihilation photons 511 keV	0.4	3.4	1.7325	0.206	0.1526	0.0877

Equation 1.15 can also be expressed in terms of beam intensity as:

$$I = I_o \cdot e^{-\mu x} \qquad (1.16)$$

I and I_o are the intensities of the beam as recorded by the detector with and without absorbing material, respectively. The attenuation coefficient may vary for a given material due to non-uniformity in its thickness. This is particularly so if the absorbing material is malleable. It is therefore better to express the **mass absorption coefficient**, which is independent of the thickness of the absorbing material. Mass absorption coefficient is obtained by dividing the linear attenuation coefficient with the density of the material. The unit of mass attenuation coefficient is cm^2/gm. Table 1.2 gives the values of attenuation coefficient, mass attenuation coefficient and HVL in lead and concrete for 140 keV photons and 511 keV photons. The electronic and atomic attenuation coefficients are also defined accordingly. The electronic attenuation coefficient is the

fractional reduction in X- or gamma ray intensity produced by a layer of thickness of 1 electron per cm^2, whereas the atomic attenuation coefficient is the fractional reduction by a layer of thickness of 1 atom per cm^2. Thus, the atomic attenuation coefficient will be *Z* times the electronic one.

1.9.5 Half-Value Layer (HVL)

From Eq. 1.16, it can be seen that for a certain thickness ($d_{1/2}$) of the absorbing material the intensity becomes half of its original value, i.e., $I = I_o/2$. Substituting these values, Eq. 1.4 can be rearranged as:

$$d_{1/2}\left(\mathrm{HVL}\right) = \frac{0.693}{\mu} \qquad (1.17)$$

Half-value layer (thickness) can be defined as the thickness of an absorbing material, which reduces the beam intensity to half of its

original value. Depending upon the energy of radiation, various materials are used for the measurement of HVL such as aluminium, copper, lead, brick and concrete. Similarly tenth-value layer (TVL) can also be defined that reduces the intensity of the beam to 1/10th of its initial value.

1.9.6 Mechanism of Attenuation

There are many types of interactions between a photon and the matter, but only few are important for us as described below.

1.9.6.1 Photon Scattering
The scattering may or may not result in transfer of energy during interaction of photon with atom of the medium.

1.9.6.2 Elastic Scattering
In elastic scattering or unmodified scattering, the photons are scattered in different directions without any loss of energy. The process, thus, attenuates the beam without absorption. In this process, the photon interacts with a tightly bound electron in an atom. The electron later releases the photon in any direction without absorbing energy from it. The contribution of this mode of interaction is relatively insignificant in medical applications of radiation. However, it has tremendous application in X-ray crystallography.

1.9.6.3 Inelastic (Compton) Scattering
Compton elucidated the mechanism of this type of scattering. In this process, photon interacts with loosely bound (free) electrons. Part of the energy of the photon is used in ejecting out the electron and rest is scattered in different direction (Fig. 1.6).

In a so-called head on collision, the photon turns back along its original track (scattered through 180°), and energy is transferred to the recoil electron. The change in wavelength $\delta\lambda$ of the photon is given by:

$$\delta\lambda = 0.024\left(1-\cos\varphi\right)A^0 \qquad (1.18)$$
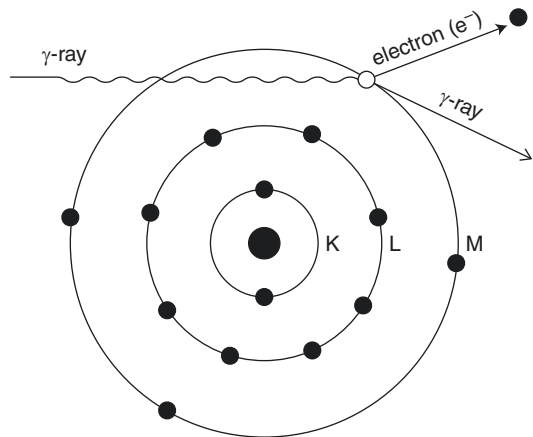
Fig. 1.6 Process of compton scattering. The incoming photon ejects the electron from outer orbit and gets scattered with reduced energy in a different direction. (Reproduced from [3])

where φ is the angle of scattering of gamma photon, and A^0 is angstrom unit for wave length. The energy of scattered photon is expressed as:

$$E_1 = E_0\Big/\left[1+\left(E_0/m_e c^2\right)\left\{1-\cos\varphi\right\}\right] \qquad (1.19)$$

where E_0 is the energy of incident photon and E_1 is that of the scattered photon, m_e is the mass of the electron, and c is the velocity of light in vacuum/space. Scattered photons at all angles therefore need appropriate shielding all around in X-ray rooms. The Compton scattering involves interaction between photons and electrons. The probability therefore depends upon the number of electrons present (electron density). With the exception of hydrogen, all elements contain nearly same number of electrons per gram (practically the same electron density). The Compton scattering thus depends upon the electron density and is the choice of interaction required in conventional radiation therapy, where the delivered dose is aimed to be homogeneous in spite of the tissue inhomogeneity within the body. The electron densities of some of the important elements are listed below:

Elements	Electron density
O_2	3.01×10^{23}
Ca	3.00×10^{23}
Pb	2.38×10^{23}
H_2	5.997×10^{23}

The total probability (σ) for Compton process is given by:

$$\sigma = \sigma_s + \sigma_a$$

where σ_s and σ_a are the probabilities for scattering and absorption, respectively.

1.9.7 Summary of Compton Scattering

- Involves interaction between a photon and a free electron.
- Is independent of Z (depends on electron density).
- Probability of occurrence decreases with increase in radiation energy.
- More the incident photon energy, more is the energy transferred as kinetic energy to the electron.
- Most common type of interaction in soft tissue in the energy ranges from 100 keV to 10 MeV.

1.9.8 Photoelectric Effect (PEE)

In this process, the photon disappears when it interacts with the bound electron. The photon energy has to be higher than the binding energy of the electron for this type of interaction to take place.

$$hv = \text{BE} + \text{Kinetic energy}$$

where hv is the energy of the photon, and BE is the binding energy of the electron in the shell (Fig. 1.7). If the photon energy is slightly higher than the binding energy (BE), then the chance of PEE is more. For example, a photon of energy 100 keV has high probability of undergoing PEE when it interacts with Pb atom for which the K shell binding energy is 88 keV. Rest of the (100–88) 12 keV energy will be carried away by the
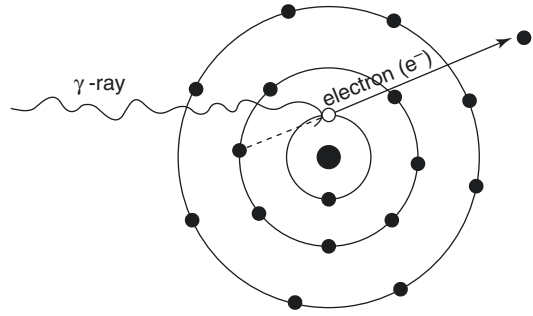
Fig. 1.7 Process of photoelectric absorption. The incoming photon disappears (absorbed) and orbital electron is knocked out. Electron from the outer shell falls (dotted line) into the inner shell to fill up the vacancy. (Reproduced from [3])

ejected electron as its kinetic energy. The ejection of electron creates a hole in the inner shell, which is filled by an electron from any one of the outer shells. Since the electron in outer shells possess higher energy than those in the inner shells, the difference in their energy is released as X-ray photon. Such photons are characteristic to the atom from which they are emitted. The K, L, M, etc. shells of a given atom have fixed energy so is the difference in their energies is also fixed. The radiation emitted are therefore termed as characteristic X-rays.

Three types of possibilities exist during photoelectric effect:

(i) **Radiative transitions**

As has been explained above, during the electron transition from the outer orbit to the inner one, a photon is emitted with energy equal to the difference of the binding energies of the orbits involved. The vacancy moves to a higher shell and consequently a characteristic photon of lower energy follows. The probability of emission of a photon is expressed as the fluorescent yield.

$$\text{Fluorescent yield} = \frac{\text{Number of X} - \text{ray photons emitted}}{\text{Number of orbital vacancies created}}$$

Mostly it is the *K* shell that is responsible for the fluorescent yield:

$$K \text{ shell fluorescent yield}(\omega_k) = \frac{\text{Number of } K \text{ X} - \text{ray photons emitted}}{\text{Number of } K \text{ shell vacancies}}$$

The yield increases with increase in the atomic number.

(ii) **Auger electrons**

The characteristic X-ray photon, instead of being emitted out, has a probability to eject another orbital electron from the atom. These electrons are called Auger electrons (Fig. 1.8). The energy of Auger electron is equal to the difference of the characteristic X-ray photon energy and the binding energy of the shell involved in the process. The Auger yield is expressed as the ratio of electrons emitted due to vacancies in subshell i and the total number of atoms with a vacancy in subshell i.

1.9.9 Coster–Kronig Electrons

The process is exactly like Auger transition except that the electron filling the vacancy comes from the subshell of the same principal shell in which the vacancy lies. The kinetic energy of the emitted electrons can be calculated exactly as for Auger electrons. The energy of Coster–Kronig electrons is so small that they are quickly absorbed in the medium.

1.9.10 Summary of the Photo Electric Effect

- The probability is very high when the photon has just enough energy to eject out electron from the shell.
- The process involves bound electrons.
- Effect is proportional to cube of the atomic number (Z^3) and is inversely proportional to the cube of the photon energy ($1/E^3$).
- Even though the PEE decreases with energy but suddenly increases near the *K* shell binding energy of the absorber. It is because of knocking out of *K* shell electron and emission of characteristic X-rays. The *K* edge depends upon the type of the absorber. In case of lead, the *K* edge is seen at 88 keV energy.

1.9.11 Pair Production

When a photon with energy in excess of 1.022 MeV passes close to the nucleus of an atom, it may disappear and in its place two anti-particles (negatron and positron) may be produced as shown in Fig. 1.9. In this process, energy is converted into mass in accordance with
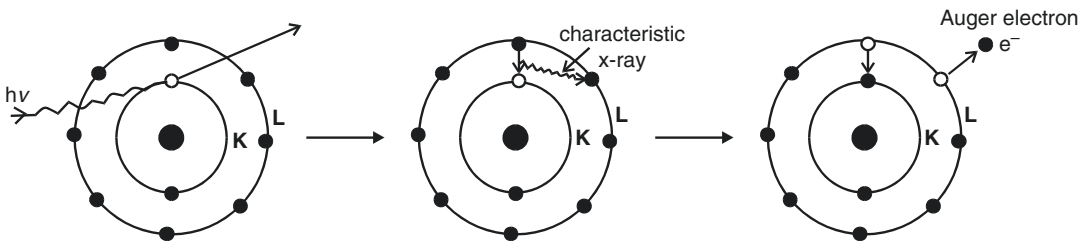

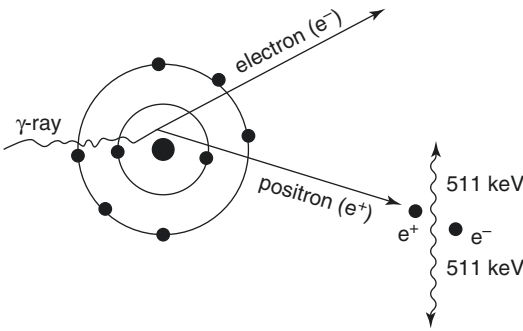**Fig. 1.8** Mechanism of Auger electron emission. (Reproduced from [3])

Fig. 1.9 Schematic representation of pair production. (Reproduced from [3])

Einstein mass energy equation ($E = mc^2$). After traversing some distance through the medium, the positron loses its energy and combines with an electron at rest and annihilates. During this process, both the antiparticles disappear (annihilate), and two photons of 0.511 MeV are emitted in opposite direction to obey the law of conservation of momentum.

1.9.12 Summary of Pair Production

- This involves interaction between photon and nucleus.
- Threshold energy for this process is 1.022 MeV, and the interaction is proportional to the energy in excess of 1.022 MeV.
- The probability of this type of interaction is proportional to the atomic number (Z) and density of the material.
- Two annihilated photons each of 0.511 MeV are produced per interaction and emitted from the site of interaction in the opposite directions.

1.9.13 Photo Nuclear Reaction

When photon energy is too high, either a neutron or proton may be knocked out (more likely the neutron) from the nucleus. For majority of atoms, the threshold energy for this effect is around 10 MeV, and the probability increases with increasing energy till a maximum is reached above which the probability falls rapidly.

1.10 Statistics in Radiation Measurements (Counting Statistics)

The purpose of counting statistics in nuclear medicine is to check the variability in the acquired data and to test the significance of observed variations. Let us first mention the types of error in the acquired data in nuclear medicine. There are three types of errors that are normally encountered:

1. Random error
2. Systematic error
3. Outliers or blunders

The random error is due to the random nature of physical processes involved in acquiring the data in nuclear medicine, such as radioactive disintegration, interaction of radiation with the detector material, etc. The nuclear medicine data usually suffers from random variation, resulting in imprecise results. Two measurements are from the same radiation source and, for exactly the same time period, are not identical. The data may however be accurate but not precise. The random nature of nuclear disintegration can be expressed in terms of Poisson statistics.

In systematic error, the result of several measurements may be same but not accurate. This error is mostly related to the instruments and may primarily be due to improper calibration of instruments or manufacturing defect.

The third type of error, not very common, is outlier or blunder, which results in grossly inaccurate results. Examples of blunders in radiation counting measurements result from:

- Incorrect energy window setting
- Counting heavily contaminated samples
- Using contaminated detectors
- High activities leading to excessive dead time effects
- Selecting wrong patient orientation during imaging

Blunders or outliers can be easily detected by experienced staff.

The measurement may be precise but inaccurate, or vice versa. Results of a radiation counting measurements may be imprecise (because of the random nature of emission) but still their (average) value may be accurate. Randomness is always present in counting measurements and during interactions of radiation with the scintillation detector. It is therefore necessary to know the statistical methods to analyse the data if so required. The statistical parameters and their application have been concisely described in the following text.

1.10.1 Measures of Central Tendency

It is convenient to summarize the results of a set of measurement with a single value in the form of **mean** or average value.

1.10.1.1 Mean

The mean of individual data points is simple to calculate. However, in grouped data, the central value of a group represents the class and the sum of their product with the respective frequency divided by the sum of frequencies results in the mean of the data.

Mathematically,

$$\text{Mean} = \sum X_i / N$$

where X_i is the ith data, and N is the total number of data points.

Simple mean cannot describe the data; therefore, two more terms, mode and median, are also defined.

1.10.1.2 Mode

Mode represents the most frequent value in the data distribution.

1.10.1.3 Median

Median divides the data into two equal parts. Each part can further be divided into two more parts. Thus, there are three data points which divide the distribution into four parts. The first data point is called first quartile or 25th(P25) percentile, the second is median or 50th percentile,

and the third is 3rd quartile or 75th(P75) percentile. Twenty five percent values lie below P25 (first quartile).

1.10.1.4 Skewness

When distribution curve is exactly symmetrical, then mean, mode and median are equal. The same is not true, when there is asymmetry in the distribution curve (skewness) as shown in Fig. 1.10. For positive skewness in the distribution curve, the mode is greater than median, and the median is greater than mean. In case of negative skewness, the mean is greater than median, and the median is greater than mode. Skewness is defined in terms of its first and second coefficients.

- The first coefficient of skewness = (mean − mode)/standard deviation
- The second coefficient = 3(mean − median)/ standard deviation

1.10.1.5 Standard Deviation

One of the important statistical parameters which describes the variability in a set of data is the standard deviation. The standard deviation for a series of measurements determines the precision or reproducibility of the measured data. When data follows Poisson probability distribution, such as nuclear counting data, the standard deviation equals the square root of the mean. In some situations, where a single measurement is acquired as in digital imaging, the total pixel counts in the image is taken as mean for all practical purposes. This helps in estimating the noise in the image. For sample data, the standard deviation can be given by following equation:

$$\text{s.d.} = \sqrt{\left(\sum \left(x_i - \bar{x} \right)^2 / \left(N - 1 \right) \right)} \qquad (1.20)$$

where x_i is the ith data, $\bar{x}$ is the mean, and N is the total number of data points in the distribution.

Example The observed counts against each serial number and intermediate steps for the calculation of mean, standard deviation and coefficient of variation in a tabulated form is given below.
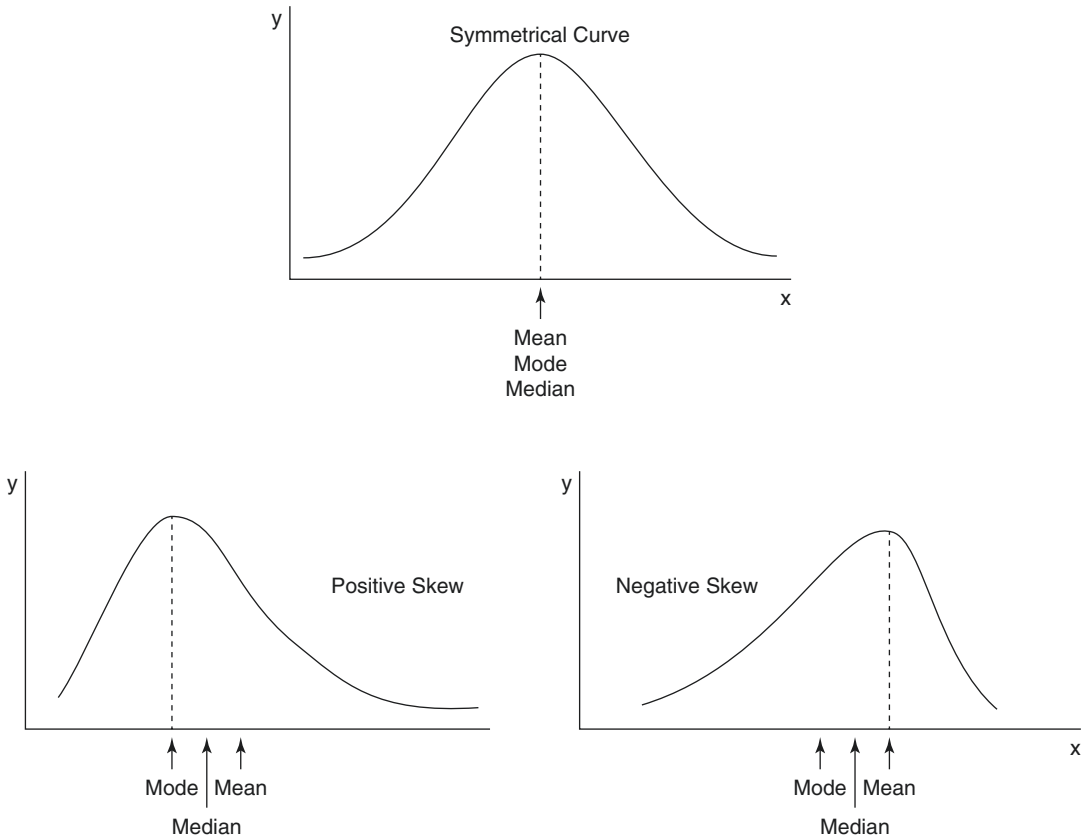
Fig. 1.10 Symmetrical distribution (top), asymmetrical distribution with positive skewness (bottom left) and asymmetric skewness (bottom right)

S.No.	Counts observed (x_i)	Deviation $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
1.	1000	−50	2500
2.	1100	50	2500
3.	970	−80	6400
4.	980	−70	4900
5.	1150	100	10,000
6.	1060	10	100
7.	950	−100	10,000
8.	1200	150	22,500
9.	1000	−50	2500
10.	1090	40	1600
11.	Sum = 10,500		63,000

$$\text{Mean}\left(\bar{x}\right) = \sum x_i / n = 10,500 / 10 = 1050$$

$$\text{Standard deviation}\left(\text{s.d.}\right) = \sqrt{\left(\sum \left(x_i - \bar{x}\right)^2 / \left(n-1\right)\right)}$$
$$= \sqrt{\left(63,000 / 9\right)} = 83.6$$

Variance = $(\text{s. d.})^2 = 6989$
Coefficient of variation (%) = (s.d./mean) × 100 = (83.6/1050) × 100% = 7.96%

1.10.2 Standard Error

The spread of observations in a given measurement is estimated by the standard deviation. If such measurements are repeated several times, a mean of all means can be estimated. The spread of means in such situations is measured in terms of standard error. Distribution of these means is usually normal.

$$\text{Standard error} = \text{standard deviation} / \sqrt{n}$$

where n is the number of individual measurements.

1.10.3 Confidence Interval

The confidence interval is the estimate of a parameter ±Margin of Error. The margin of error of a confidence level is determined by a couple of

factors. We can see this by examining the formula for margin of error. A margin of error is of the form:

$$\text{Margin of Error} = (\text{Statistic for Confidence Level}) * (\text{Standard Error})$$

The statistics for the confidence level depends upon what probability distribution is being used and what level of confidence has been chosen. For example, if C is our confidence level and we are working with a normal distribution, then C is the area under the curve between $-z^*$ and z^*. This number z^* is the number in the margin of error formula. The other term necessary in the margin of error is the standard error. In Fig. 1.11, a density plot of normal distribution data is given, the two vertical lines marking the 1.96 standard error above and below the mean. This is the z-score for two-tailed significance level of 0.05. In a normal distribution, there are 2.5% probability above and 2.5% probability below. It turns out that 94.4% of the confidence intervals captures the population mean. This is what confidence interval really means: *if we repeat the sampling procedures infinitely many times, about 95% of the confidence intervals will contain the population mean.* In other words, approximately 5% of the confidence intervals fail to capture the population mean. In normal distribution, the area under 1 (one) standard deviation on either side of the mean is about 68.3% of the total area as shown in Fig. 1.11c. Similarly, the lines drawn at 2σ and 3σ on either side will cover 95% and 99.7%, respectively, of the total area under the normal distribution curve. Other important characteristics of normal distribution curve are illustrated graphically in Fig. 1.11.

1.10.4 Normal Distribution

The normal distribution is also widely used in counting statistics. It should be emphasized that the binomial, Poisson and normal distribution become nearly identical for processes with a small probability for individual success and with a large enough number of trials.

In normal distribution, the area under 1 (one) standard deviation on either side of the mean is about 68.3% of the total area as shown in Fig. 1.11c. Similarly, the lines drawn at 2σ and 3σ on either side will cover 95% and 99.7%, respectively, of the total area under the normal distribution curve. Other important characteristics of normal distribution curve are illustrated graphically in Fig. 1.11.

The graphical representation shows the correct picture of the data distribution. It is not necessary that the two sets of data with the same mean and standard deviation will have the same graphical display. There are several methods of graphical representation of the data such as bar diagram, histogram and pie charts. Statistical parameters with the same mean and standard deviations can have different distribution (Fig. 1.12).

Generally the statistical properties of the data can be described by the frequency distribution, mean and variance (square of standard deviation). However, it is to understand the probability distribution function (pdf) to which the data belongs.

1.10.5 Probability Distribution Functions (pdf)

It describes the probability of each outcome from a measurement. There are three pdfs that are relevant to nuclear medicine:

1. Binomial
2. Poisson
3. Gaussian or normal

1.10.5.1 Binomial Distribution

In binomial distribution, a trial is an event that may have more than one outcome, and a measurement consists of counting the number of successes from a specified number of trials. Details of binomial distribution function are not described here as the following two probability distributions are more important in nuclear medicine.

THE NORMAL DISTRIBUTION CURVE
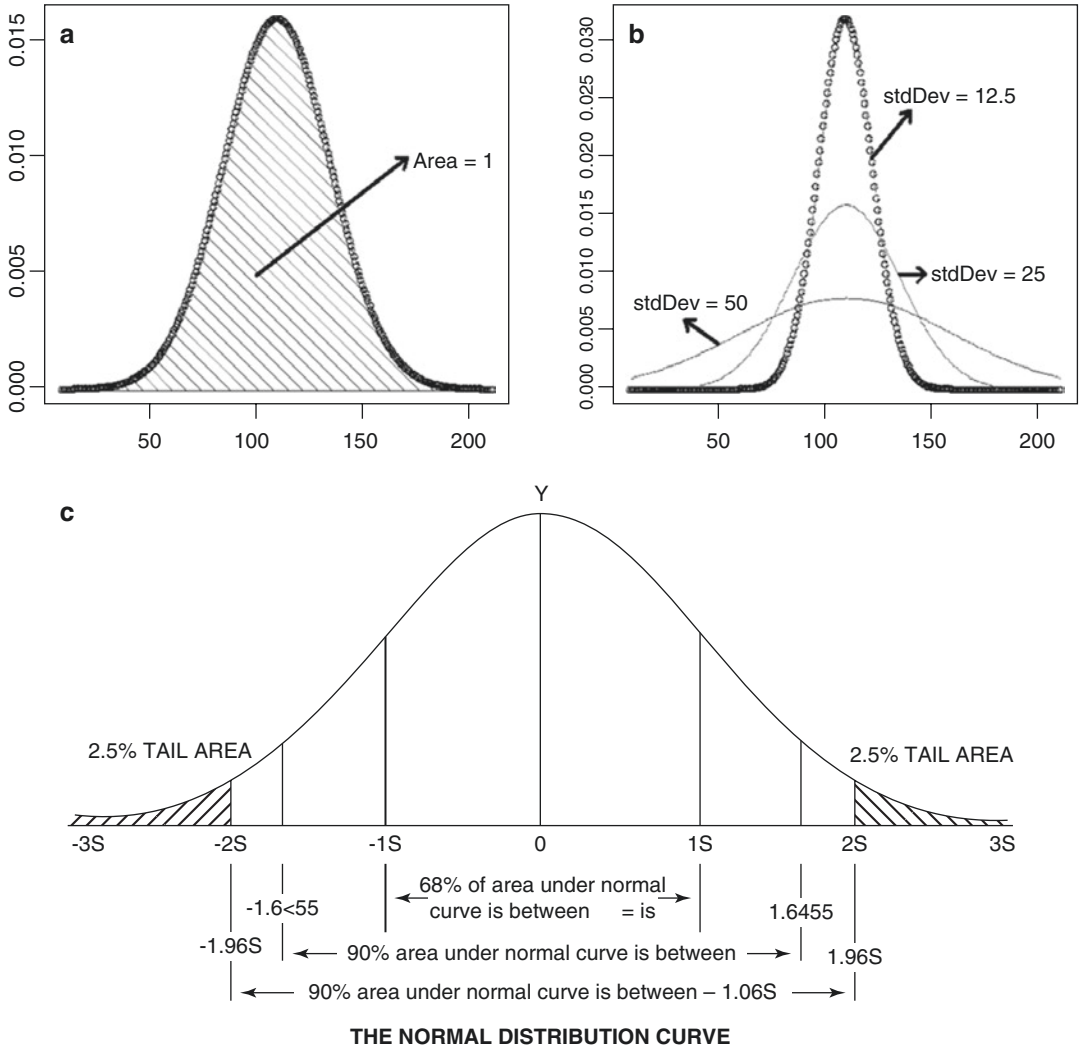
Fig. 1.11 Important characteristics of normal probability distribution. (**a**) Area under normal distribution is unity. (**b**) Standard deviation affects the shape of the normal dis-tribution. (**c**) The percentage of areas included under the various standard deviation

1.10.5.2 Poisson Distribution

The Poisson distribution is normally applied in the following conditions when:

1. The events are discrete or discontinuous.
2. The events are independent, i.e., the first event has no influence on the second event.
3. In a given interval, any number of events can occur.
4. The probability of a single occurrence of the event is very rare, but in a given interval of time, it is proportional to the length of the time interval.
5. In an infinitesimally small interval of time, the probability of an event to occur more than once is negligible.

The Poisson probability density function $p(x)$ at each value of x is given by

$$p(x,\mu) = \frac{\mu^x}{x!} e^{-\mu} \qquad (1.21)$$

Fig. 1.12 The chi-square, Poisson, uniform and normal distribution of simulated data having mean = 11 and standard deviation = 7

where μ is the mean, and $p(x)$ is the probability of observing count x when the mean count is μ. In Poisson distribution, the mean is equal to the variance.
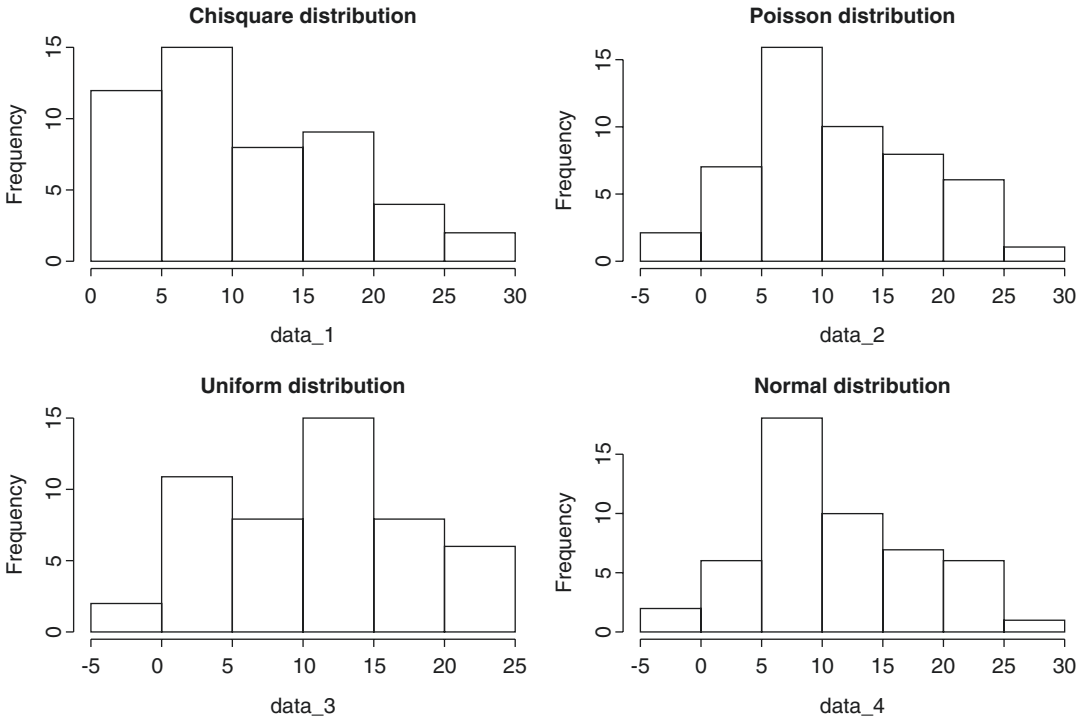
Example What is the probability of getting a count 15 in a measurement when the true or average count is 10.

Using Eq. 1.21

$$P(15,10) = \left(e^{-10}\ 10^{15}\right)/15! = \left(4.54 \cdot 10^{-5}\right)$$
$$\times \left(9.999 \cdot 10^{14}\right)/\left(1.3 \cdot 10^{12}\right)$$
$$= 3.49 \cdot 10^{-2} = 0.0349$$

The probability is 0.0349 or 3.49% that a count 15 will be obtained when the average count is 10.

In Poisson distribution, the variance is equal to the mean. Variance is a number that is equal to square of standard deviation.

$$\text{Varience}\left(\sigma^2\right) = \text{mean}\left(\mu\right)$$

Thus, as outlined above, the standard deviation $(\sigma) = \sqrt{\mu}$.

In counting measurements, the measured counts in a given time is always assumed as mean. This is very important and useful statements in nuclear medicine as the image data are acquired in one go, and the acquired counts are taken as a mean or variance. If 10,000 counts are acquired in a given length of time, the standard deviation will be ±100. The observer will have a confidence level of 68% that the data will lie between 9900 and 10,100 with an accuracy of ±1%. It is important to acquire more counts to reduce error in any counting measurement.

1.11 Propagation of Error in Counting Measurements

Any parameter that is estimated from counting measurement is associated with some degree of uncertainty. If the final result involves arithmetic operation of such parameters, as is usually

the case with nuclear medicine data, then the error gets propagated. The error introduced depends on the type of arithmetic operation performed and on the value of uncertainty in the original data.

For sum and difference of two terms N_1 and N_2, the error will be propagated as:

$$\left(N_1 \pm N_2\right) = \left(N_1 \pm N_2\right) \pm \surd\left(N_1 + N_2\right) \quad (1.22)$$

The error in case of multiplication and division is propagated as:

$$N_1 \times N_2 \text{ or } N_1 / N_2 = \left(N_1 \times N_2\right) \text{ or}$$
$$N_1 / N_2 \pm \sqrt{(1/N_1 + 1/N_2)} \quad (1.23)$$

Example for Addition or Subtraction of the Counts A radioactive sample is measured in a counter. The sample holder gives 10,000 and 100 counts in 10 s with and without sample. Find true counts in the sample.

- Background counts = 100
- Measured counts in the sample = 10,000
- True sample counts = $(10,000 - 100) \pm \surd(10,000 + 100) = 9900 \pm 100.5$
- Percentage uncertainty = (100.5/9900) × 100 = 1.01%

To have better results in terms of accuracy and precision, it is always necessary to have measured counts many fold more than the background counts.

Example for Product or Quotient The integrated counts between two cursors in the rising portion of a renogram at 1 and 3 min for right and left kidney were found to be 3000 and 2800 counts, respectively. Calculate the relative function of the kidneys.

$$\text{Relative function of right kidney} = \text{Integrated counts for right kidney}(R) / \text{Iintegrated counts for}$$
$$\text{right and left kidney for the same time interval}(R+L)$$
$$= R/\left(R+L\right) \pm \text{error} = 3000/5800 \pm \surd\left(1/3000 + 1/5800\right)$$
$$= 3000/5800 \pm 0.024 = 0.517 \pm 0.024 = \left(51.7 \pm 2.4\right)\%$$

- Percentage right kidney function = 51.7 ± 2.4
- Similarly, percentage left kidney function = 48.3 ± 2.4%

While using count rate instead of counts, one can use the same formula (Eqs. 1.22 and 1.23), but the uncertainty will have to be estimated for the count rate 'R' instead of counts 'N'.

$$\text{Count rate}\left(C\right) = \frac{\text{Counts}\left(N\right)}{\text{Time interval}\left(t\right)}$$
$$\text{Uncertainty in count rate} = \frac{\text{Uncertainty in counts}\left(N\right)}{t}$$
$$\text{Uncertainty in count rate} = \frac{\sqrt{N}}{t} \quad (1.24)$$
$$\text{Uncertainty in count rate} = \frac{\sqrt{N}}{\left(\sqrt{t}\right)\left(\sqrt{t}\right)} = \frac{1}{\sqrt{t}} \times \sqrt{\frac{N}{t}} = \frac{1}{\sqrt{t}} \times \sqrt{C} = \sqrt{\frac{C}{t}}$$
$$\text{Uncertainty in count rate} = \sqrt{\frac{C}{t}}$$

Example In a measurement with 1500 counts in 10 s. Calculate the uncertainty in count rate?

Given $N = 1500$ counts and $t = 10\,\text{s}$

$$\text{Count rate} = \frac{1500\,\text{counts}}{10\text{s}} = 150\,\text{counts per second}$$

$$\text{Uncertainty in count rate} = \sqrt{\frac{150}{10}} = \sqrt{15} = 3.87$$

Thus, the count rate $= 150 \pm 3.87$ counts/s.

1.12 Optimum Time of Measurement for Maximum Precision

For a given time of measurement $(t_g + t_b)$, the maximum precision can be obtained with the following condition:

$$t_g / t_b = \sqrt{\left(C_g / C_b\right)} \qquad (1.25)$$

Example In an experiment, the total time available for measurement is 5 min. Gross counts from the sample and background for 10 s are 5000 and 250, respectively. Calculate the optimum time for the sample and background for maximum precision.

Solution Gross sample count rate $= 5000/10 = 500$

Background count rate $= 250/10 = 25$

$$t_g / t_b = \sqrt{\left(C_g / C_b\right)} \quad \left(\text{see Eq.1.25}\right)$$
$$= \sqrt{\left(500 / 25\right)}$$
$$= 4.47$$
$$t_g = 4.47 t_b$$

Given, $t_g + t_b = 5$ min

$$4.47 t_b + t_b = 5\text{min}$$
$$5.47 t_b = 5\text{min}$$
$$t_b = 0.91\text{min}$$

Therefore $t_g = 4.47 \times 0.91 = 4.09$ min

Example The total time allotted for measuring a sample and background is 5 min. Gross counts from the sample and background for 10 s were 50,000 and 30,000 respectively. Calculate the optimum time for the sample and background for maximum precision?

Solution Gross count rate $= 50,000/10 = 5000$

Background count rate $= 30,000/10 = 3000$

$$t_g / t_b = \sqrt{\left(C_g / C_b\right)}$$
$$= \sqrt{\left(5000 / 3000\right)}$$
$$= 1.29$$
$$t_g = 1.29 t_b$$

Given, $t_g + t_b = 5$

$$1.29 t_b + t_b = 5$$
$$2.29 t_b = 5$$
$$t_b = 2.18\text{min}$$

Therefore, $t_g = 2.82$ min

1.12.1 The Chi Square Test

The reliability of counting instruments in nuclear medicine can be evaluated by chi square (χ^2) statistics. For this test, a minimum of 20 measurements are recommended. The mean of the measurement is calculated and chi square is then estimated from the mean as follows:

$$\text{Mean}\left(\bar{x}\right) = \sum x_i / n$$
$$\chi^2 = \sum \left(x_i - \bar{x}\right)^2 / \bar{x}$$

From χ^2 table, p value is obtained for a given set of measurements. A p value of 0.02–0.98 is treated as acceptable. In fact p value of 0.5 would be ideal to indicate that observed chi square value is in the middle of the range expected for Poisson distribution. A low p value shows a small probability that a Poisson distribution would give the chi square value as large as actually observed and indicates the presence of additional sources of

random error. With a *p* value just smaller than acceptable, the experiment suggests repeat measurement, as the results are suspicious in such cases. Similarly a high *p* value between 0.98 and 0.99 is also considered to be suspicious, and the experiment needs to be repeated. For $p > 0.99$, the random variations are much smaller than expected for a Poisson distribution.

Example A radioactive sample was placed in a well counter and pre-set time for counting the sample was kept as 10 s. Without disturbing the geometrical condition, 20 measurements were recorded as given in Table 1.3:

Based on the chi square test, comment on the precision of the well counter performance.

Mean = 3239.75 and chi square value = 28,531.75/3239.75 = 8.806.

The degrees of freedom are equal to $n - 1$ where *n* equals the number of measurements made. Thus, the degree of freedom in this case is 19. The largest critical value for df = 19 is 36.1909. The obtained chi square value lies between the 7.63 (*p*-value = 0.99) and 9.59 (*p*-value = 0.97) as shown in Table 1.4. As mentioned earlier, the

value between 0.02 and 0.98 is considered acceptable. Thus, the precision of the well counter performance is within acceptable limit.

Table 1.3 Counts observed in 20 measurements of a radioactive sample

S.No.	Total counts (10 s)
1.	3242
2.	3201
3.	3283
4.	3198
5.	3286
6.	3291
7.	3249
8.	3189
9.	3295
10.	3160
11.	3324
12.	3169
13.	3333
14.	3170
15.	3134
16.	3172
17.	3312
18.	3345
19.	3290
20.	3152

Table 1.4 Chi square table; this table has been created with *R* statistical software

df	*p*-values							
	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01
1	0.0002	0.001	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349
2	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2103
3	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449
4	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767
5	0.5543	0.8312	1.1455	1.6103	9.2364	11.0705	12.8325	15.0863
6	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119
7	1.239	1.6899	2.1673	2.8331	12.017	14.0671	16.0128	16.4753
8	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902
9	2.0879	2.7004	3.3251	4.1682	14.6837	16.919	19.0228	21.666
10	2.5582	3.247	3.9403	4.8652	15.9872	18.307	20.4832	23.2093
11	3.0535	3.8157	4.5748	5.5778	17.275	19.6751	21.92	24.725
12	3.5706	4.4038	5.226	6.3038	18.5493	21.0261	23.3367	26.217
13	4.1069	5.0088	5.8919	7.0415	19.8119	22.362	24.7356	27.6882
14	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412
15	5.2293	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5779
16	5.8122	6.9077	7.9616	9.3122	23.5418	26.2962	28.8454	31.9999
17	6.4078	7.5642	8.6718	10.0852	24.769	27.5871	30.191	33.4087
18	7.0149	8.2307	9.3905	10.8649	25.9894	28.8693	31.5264	34.8053

Table 1.4   (continued)

df	p-values							
	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01
19	7.6327	8.9065	10.117	11.6509	27.2036	30.1435	32.8523	36.1909
20	8.2604	9.5908	10.8508	12.4426	28.412	31.4104	34.1696	37.5662
21	8.8972	10.2829	11.5913	13.2396	29.6151	32.6706	35.4789	38.9322
22	9.5425	10.9823	12.338	14.0415	30.8133	33.9244	36.7807	40.2894
23	10.1957	11.6886	13.0905	14.848	32.0069	35.1725	38.0756	41.6384
24	10.8564	12.4012	13.8484	15.6587	33.1962	36.415	39.3641	42.9798
25	11.524	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141
26	12.1981	13.8439	15.3792	17.2919	35.5632	38.8851	41.9232	45.6417
27	12.8785	14.5734	16.1514	18.1139	36.7412	40.1133	43.1945	46.9629
28	13.5647	15.3079	16.9279	16.9392	37.9159	41.3371	44.4608	48.2782
29	14.2565	16.0471	17.7084	19.7677	39.0875	42.557	45.7223	49.5879
30	14.9535	16.7908	18.4927	20.5992	40.256	43.773	46.9792	50.8922
31	15.6555	17.5387	19.2806	21.4336	41.4217	44.9853	48.2319	52.1914
32	16.3622	18.2908	20.0719	22.2706	42.5847	46.1943	49.4804	53.4858
33	17.0735	19.0467	20.8665	23.1102	43.7452	47.3999	50.7251	54.7755
34	17.7891	19.8063	21.6643	23.9523	44.9032	48.6024	51.966	56.0609
35	18.5089	20.5694	22.465	24.7967	46.0588	49.8018	53.2033	57.3421
36	19.2327	21.3359	23.2686	25.6433	47.2122	50.9985	54.4373	58.6192
37	19.9602	22.1056	24.0749	26.4921	48.3634	52.1923	55.668	59.8925
38	20.6914	22.8785	24.8839	27.343	49.5126	53.3835	56.8955	61.1621
39	21.4262	23.6543	25.6954	26.1958	50.6598	54.5722	58.1201	62.4281
40	22.1643	24.433	26.5093	29.0505	51.8051	55.7585	59.3417	63.6907

Suggested Reading

1. Pant GS, Rajabi H. Basic atomic and nuclear physics. In: Basic physics and radiation safety in nuclear medicine. 2nd ed. Mumbai: Himalaya Publishing House; 2018.
2. Pant GS, Pandey AK. Radioactivity. In: Basic physics and radiation safety in nuclear medicine. 2nd ed. Mumbai: Himalaya Publishing House; 2018.
3. Pant GS. Basic interaction of radiation with matter. In: Basic physics and radiation safety in nuclear medicine. 2nd ed. Mumbai: Himalaya Publishing House; 2018.
4. Cherry SR, Sorenson JA, Phelps ME. Physics in nuclear medicine. 3rd ed. Philadelphia: Saunders; 2003.
5. Chandra R. Introductory physics of nuclear medicine. Philadelphia: Lea & Febiger, Publisher; 1992.
6. Meredith WJ, Massey JB. Fundamental physics of radiology. Bristol: John Wright & Sons Limited; 1974.
7. Johns HE, Cunningham JR. The physics of radiology. Springfield: Charles C Thomas Publisher; 1969.
8. Henkin R, editor. Nuclear medicine. St Louis: Mosby; 1996.
9. Brown PH. Mathematics and statistics. In: Bernier DR, Christian PE, Langan GK, editors. Nuclear medicine: technology and techniques. 4th ed. St. Louis: Mosby; 1997. p. 1–35.
10. Johnson TE, Birky BK. Health physics and radiological health. 4th ed. Philadelphia: Wolters Kluwer/Lippincott Williams and Wilkins; 2012.
11. Environmental health and safety. Radiological safety guidance. University of Michigan; 2018.